The Nuclear EoS: from experiments to astrophysical observations

Isaac Vidaña, INFN Catania

QFC2022 – Quantum gases, Fundamental Interactions & Cosmology
Pisa (Italy), October 26th-28th 2022

In this talk …

I will review different experimental and astrophysical observational (NSs) constraints of the nuclear EoS (*i.e.,* thermodynamical relation between pressure & energy density P=P(ε)) as well as some of the ab-initio theoretical many-body approaches & phenomenological models commonly used in its description

Three recent reviews on the topic are

- M. Oertel, M. Hempel,T. Klahn & S. Typel, Rev. Mod. Phys. 89, 015007 (2017)
- F. Burgio & A. Fantina, in “The Physics & Astrophysics of Neutron Stars”, L. Rezzolla, P. Pizzochero, I. Jones, N. Rea & I.V. Eds, Springer-Verlag 2018
- F. Burgio, H.-J. Schulze, I.V. & J. B. Wei, Prog. Part. Nucl. Phys. 120, 103879 (2021)

The Nuclear EoS

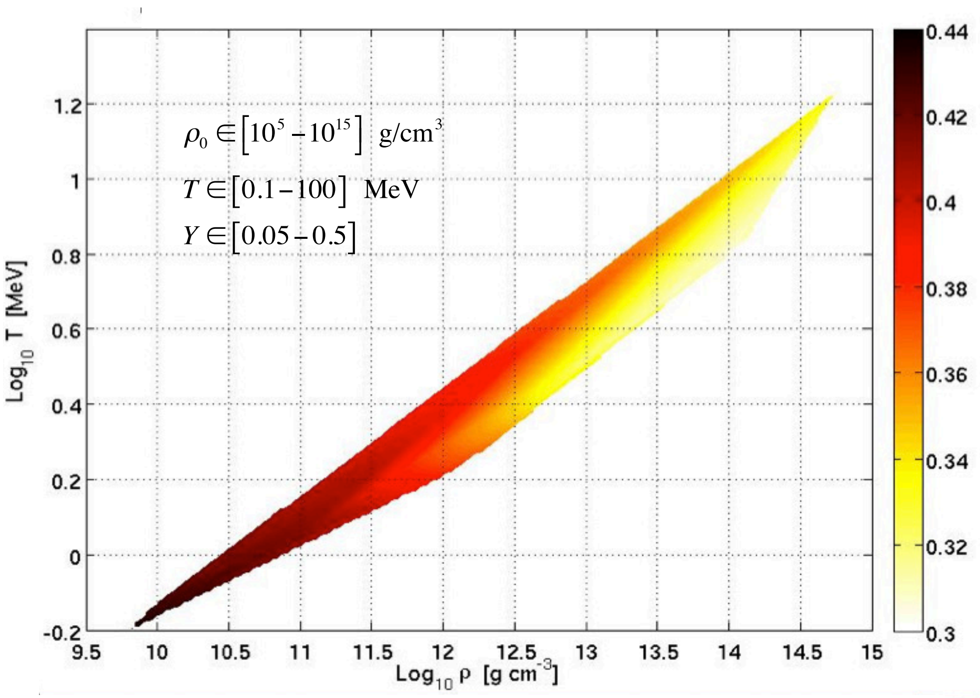
The Nuclear EoS is a fundamental ingredient for the understanding of the static & dynamical properties of NS, core-collapse SN & compact star mergers

However, its determination is very challenging due to the wide range of densities, temperatures & isospin asymmetries found in these astrophysical scenarios.

Main difficulties associated to:

- ✓ Complexity of the bare baryon-baryon interaction
- ✓ Very complicated resolution of the so-called nuclear many-body problem

Conditions in the center of the star from the onset of the collapse up to 25 ms after bounce (15 M_{sun} progenitor)

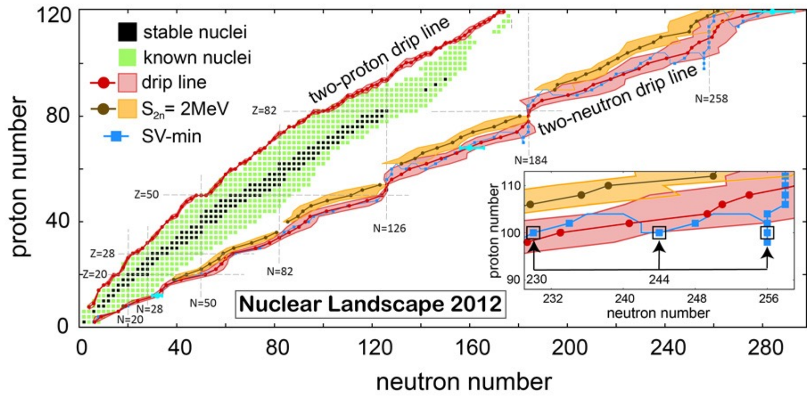


What do we know to build the nuclear EoS ?

J. Erler et al., Nature 486, 509 (2012)

✧ Masses, radii & other properties of more than 3000 isotopes

✧ Scattering (cf. > 4000 NN data for E_{lab} < 350 MeV)

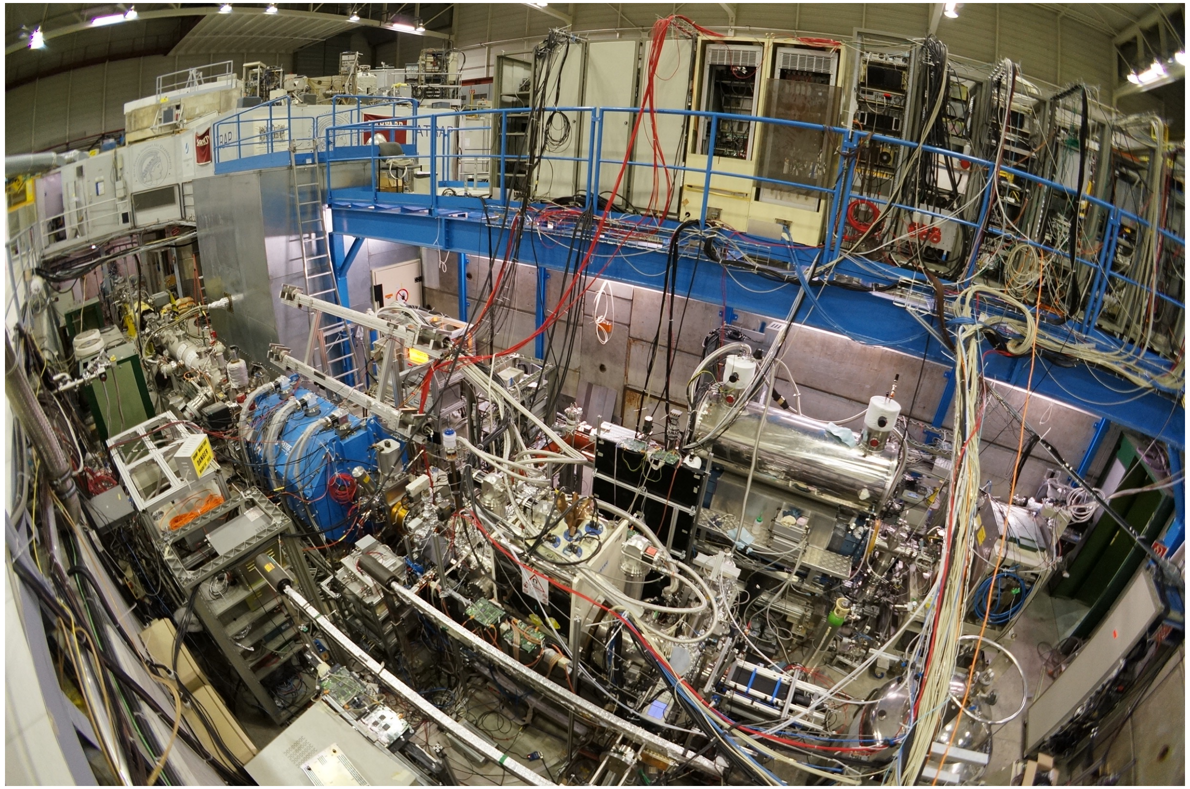


➢ Around ρ_0 & β=0 the nuclear EoS can be characterized by a few isoscalar (E_0 ,K_0, Q_0) & isovector (E_{sym}, K_{sym}, Q_{sym}) parameters which can be constrained by nuclear experiments & astrophysical observables

$$\frac{E}{A}(\rho,\beta) = E_0 + \frac{1}{2}K_0 x^2 + \frac{1}{6}Q_0 x^3 + \left(E_{sym} + Lx + \frac{1}{2}K_{sym} x^2 + \frac{1}{6}Q_{sym} x^3\right)\beta^2 + \cdots \quad , \quad x = \frac{\rho - \rho_0}{3\rho_0}$$

➢ Extrapolation to high densities should rely on theoretical models to be tested with astrophysical observations

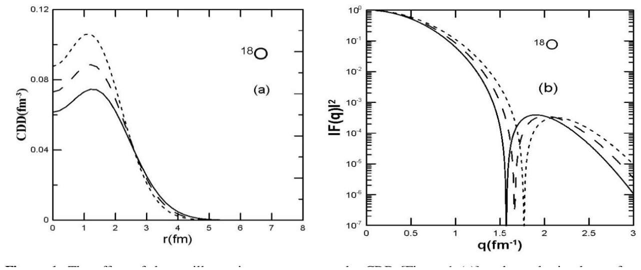
Constraints from Nuclear Physics Experiments



Density Distributions & Nuclear Binding Energies

✧ **Density distributions:**

(e,e') elastic scattering, hadron proves $\xrightarrow{A=N+Z\to\infty}$ $\rho_0 \sim 0.16 \ \ \mathrm{fm}^{-3}$



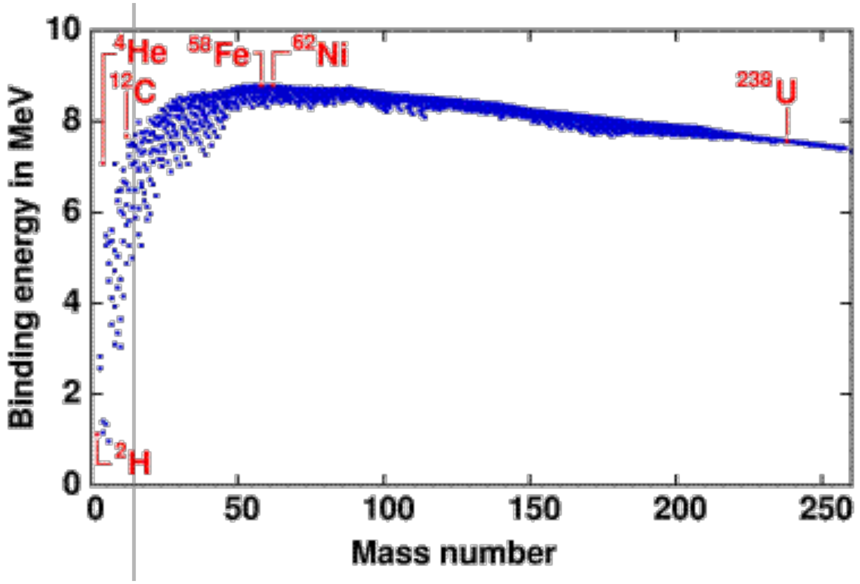
✧ **Nuclear binding energies:**

$$B(N,Z) = a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + \left(a_{Av} A + a_{As} A^{2/3}\right)\frac{(N-Z)^2}{A^2} + \delta a_p A^{-1/2}$$

Measurements of nuclear binding energies allow the identification

$$a_V \Leftrightarrow B_{sat} = -E_0$$
$$a_{Av} \Leftrightarrow E_{sym}$$

(in the limit $A = N + Z \to \infty$)



Recent fits of binding energies with non-relativistic & relativistic EDF give

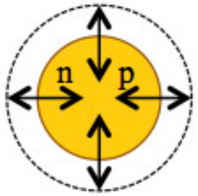
SHF models: $B_{sat} = (15.96 \pm 0.31)$ MeV, $E_{sym} = (31.2 \pm 6.7)$ MeV

RMF models: $B_{sat} = (16.13 \pm 0.51)$ MeV, $E_{sym} = (33.4 \pm 4.7)$ MeV

Dutra et al., PRC 85, 035201 (2012); PRC 90, 055203 (2014)

Nuclear Resonances

✧ ISGMR



$\Delta L=0$
$\Delta S=0,\ \Delta T=0$

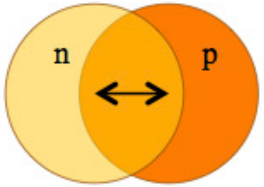
Collective monopole mode oscillation of all neutrons & protons in a nucleus vibrating in phase

K_0 from the measurement of excitation energy E_{ISGMR}

Typical values in the range ~ 210 – 270 MeV

Phys. Rep. 64, 171 (1980); PRC 90, 055203 (2014)

✧ IVGDR



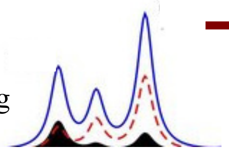
$\Delta L=1$
$\Delta S=0,\ \Delta T=1$

Collective dipole model oscillation of all neutrons & protons in a nucleus vibrating in opposite phase

Symmetry energy influences the excitation energies of IVGDR. Their analysis allows to determine E_{sym}

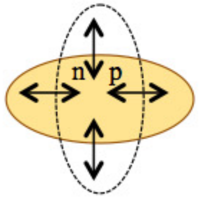
$$23.3 < E_{sym}\left(\rho = 0.1\text{fm}^{-3}\right) = 24.9 \quad \text{MeV}$$

Trippa et al., PRC 77, 061304 (R) (2008)

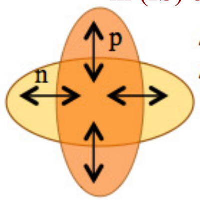


✧ ISGQR & IVGQR

Collective quadrupole mode oscillation of all neutrons & protons in a nucleus vibrating in (IS) & out (IV) of phase



$\Delta L=2$
$\Delta S=0,\ \Delta T=0$



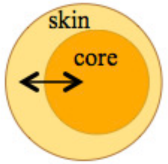
$\Delta L=2$
$\Delta S=0,\ \Delta T=1$

Correlation of Δr_{np} with ISGQR & IVGQR excitation energies from which

$$\Delta\left({}^{208}\text{Pb}\right) = 0.14 \pm 0.03 \quad \text{fm}, \qquad L = 37 \pm 18 \quad \text{MeV}$$

Roca-Maza et al., PRC 87, 037301 (2013)

✧ PDR



Collective oscillation of neutron skin against the core

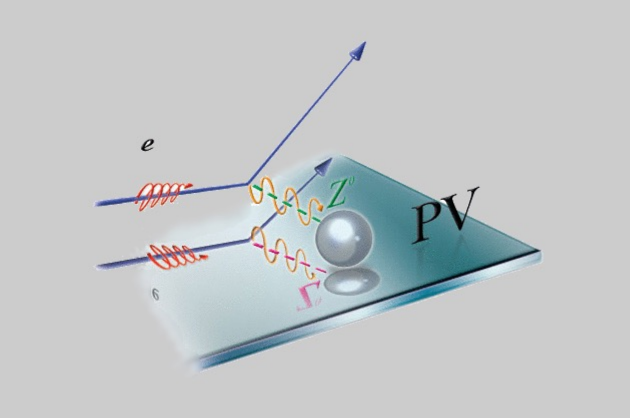
Sensitive to the symmetry energy. A recent analysis of PDR in ^{68}Ni & ^{132}Sn using RPA models for the dipole response based in Skyrme & RMF give

$$E_{sym} = 32.3 \pm 1.3 \quad \text{MeV}, \qquad L = \quad 64.8 \pm 15.7 \quad \text{MeV}$$

Carbone et al., PRC 81, 041301 (R) (2010)

Neutron Skin Thickness & Symmetry Energy

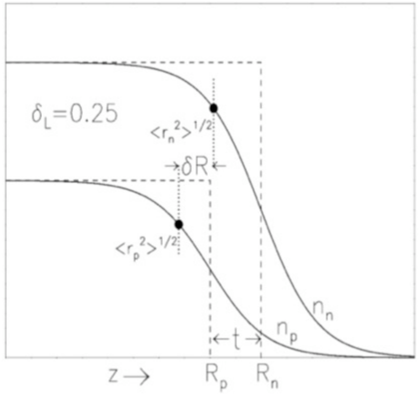
Accurate measurements of Δr_{np} via parity-violating electron scattering at JLAB can constrain $E_{sym}(\rho)$, particulary L via its strong correlation with Δr_{np}



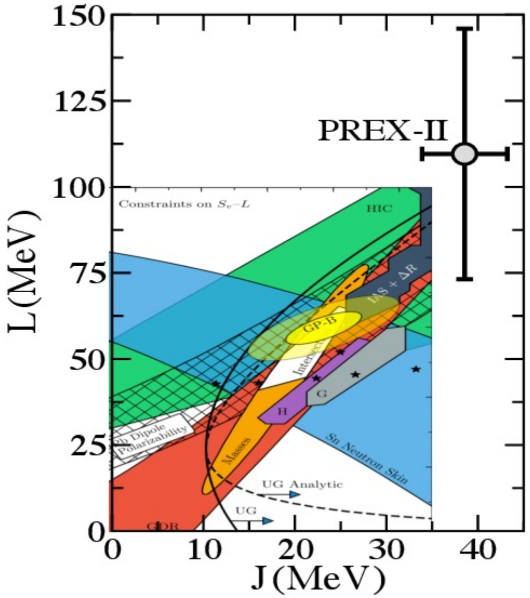
neutron form factor

$$A_{pv} = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}} \approx \frac{G_F q^2}{4\pi\alpha\sqrt{2}} \frac{F_n(q)}{F_p(q)}$$

proton form factor



$$\Delta r_{np} = \sqrt{\langle r_n \rangle^2} - \sqrt{\langle r_p \rangle^2}$$



<u>CREX & PREX-II experiments</u>

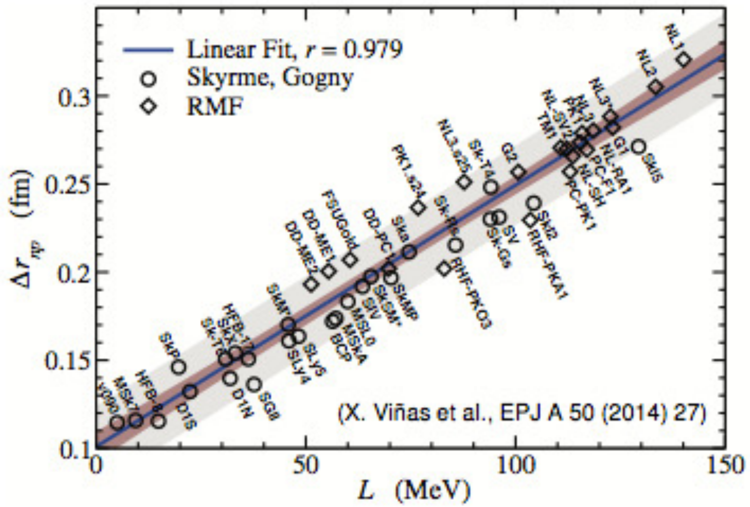
Δr_{np}(48Ca) = 0.121 ± 0.026 fm

Δr_{np}(^{208}Pb) = 0.283 ± 0.071 fm

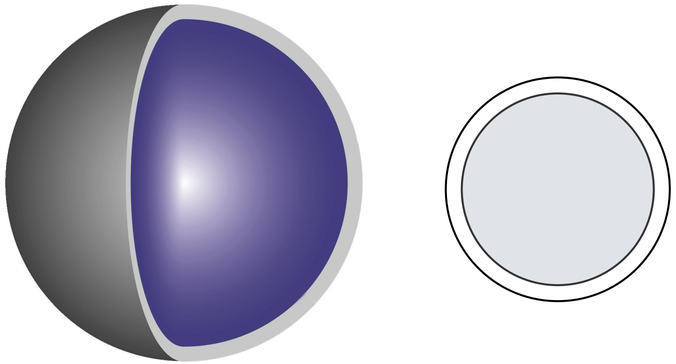
E_{sym}=34-42 MeV

L=74-149 MeV

PRL 126, 172502 (2021



Neutron Skin Thickness & Crust-Core Transition Density in Neutron Stars



Neutron Star   Heavy nucleus

Neutron Star Crust & Neutron Skin are made out of neutron rich matter at similar densities

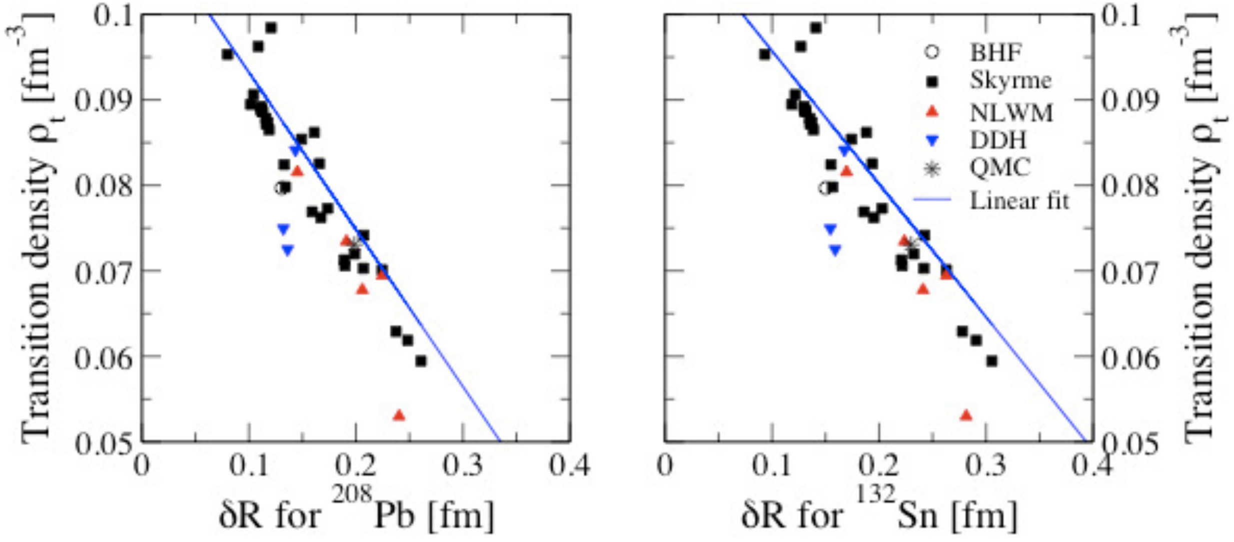
↓

Both are governed by EoS at subnuclear densities in particular by $E_{sym}(\rho)$ & its derivatives

Inverse correlation between δR and ρ_t (Horowiz & Piekarewicz)

↓

Accurate measurements of neutron skin in neutron rich nuclei such as the ones performed at JLAB can provide considerable & valuable information on the crust-core transition density

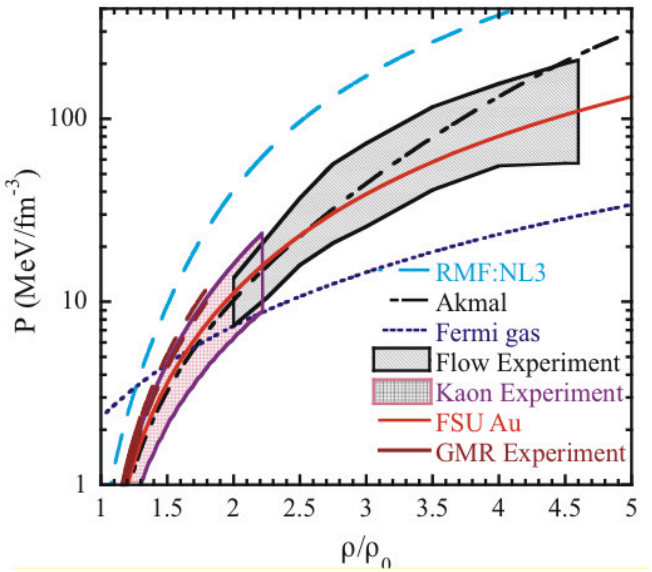


EoS from Heavy Ion Collisions

The analysis of data from HIC requires the use of transport models which do not depend directly on the EoS but rather on the mean field of the participant particles & the in-medium cross sections of the relevant reactions

However, there are several transport codes in the market. A natural question arises: How much the results depend on the transport codes ?

P. Danielewicz et al., Science 298, 1592 (2002)



Several observables in HIC are sensitive to the nuclear EoS

sub-saturation densities

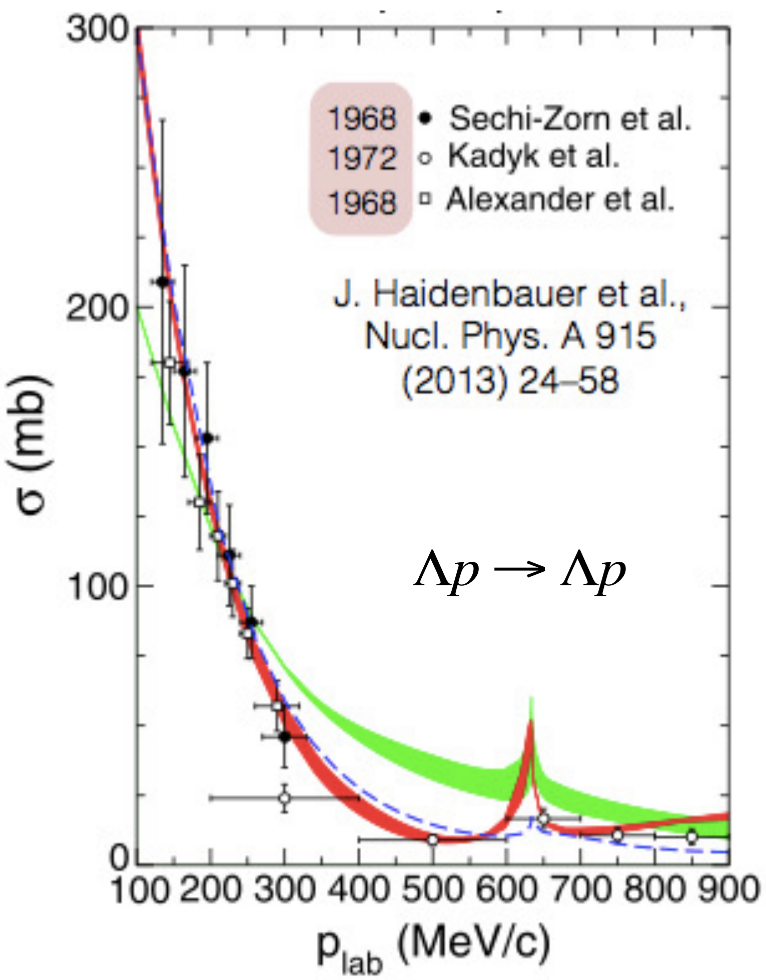
- ✓ n/p & t/^{3}He ratios
- ✓ isospin fragmentation & isospin scaling
- ✓ np correlation functions at low rel. mom.
- ✓ isospin difusion/transport
- ✓ neutron-proton differential flow

supra-saturation densities

- ✓ π^-/π^+ & K^-/K^+ ratios
- ✓ np differential transverse flow
- ✓ nucleon elliptic flow at high trans. mom.
- ✓ n/p ratio of squeezed out nucleons perpendicular to the reaction plane

What do we know to include hyperons in the nuclear EoS ?

Hyperons are expected to appear in the interior of NSs and play an important role on their structure & properties, however, our knowledge of the YN & YY interactions is much more limited than that on the NN one in order to put to put stringent constraints on hypernuclear EoS

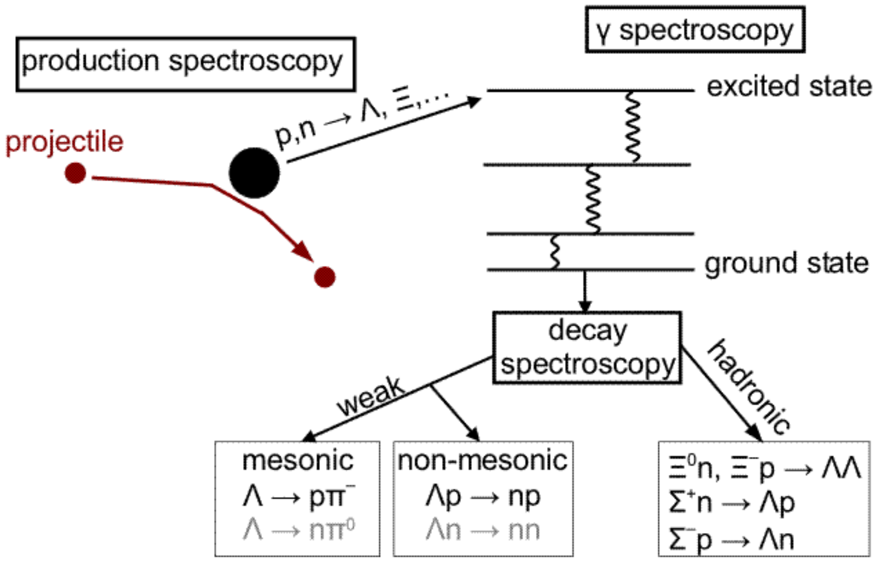


- Very few YN scattering data due to short lifetime of hyperons & low intensity beam fluxes
  - ~ 35 data points, all from the 1960s
  - 10 new data points, from KEK-PS E251 collaboration (2000)
- No YY scattering data exists

(cf. > 4000 NN data for E_{lab} < 350 MeV)

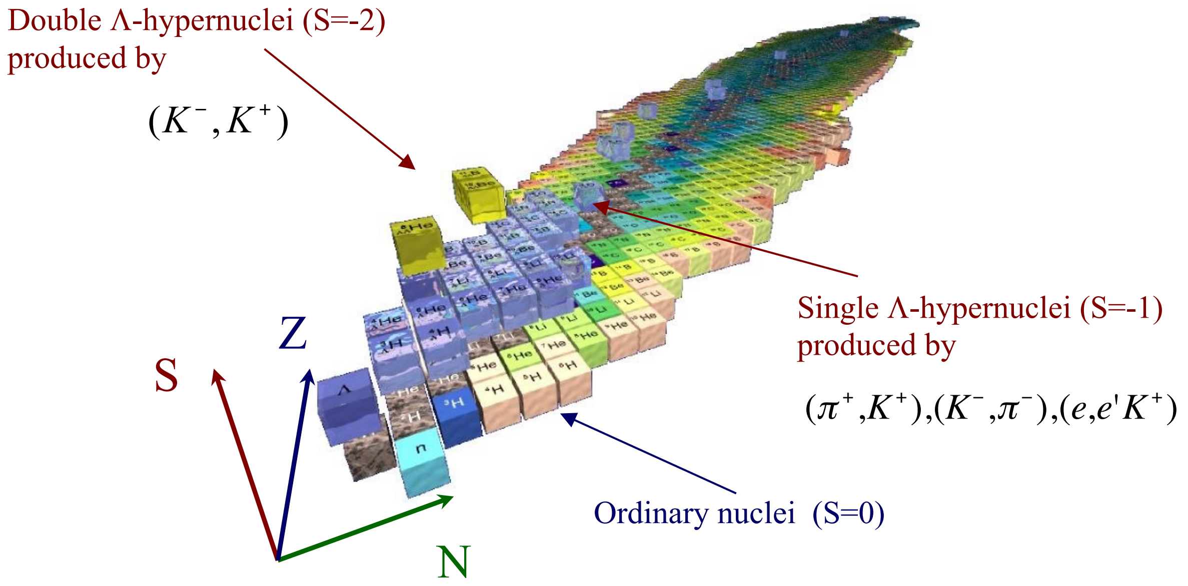
Hypernuclear Physics in a Nutshell

Alternative information can be obtained from the study of hypenuclei (bound nuclear systems of nucleons & hyperons). The goal of hypernucler physics is to relate hypernuclear observables with the underlyning bare YN & YY interactions



- 41 single Λ-hypernuclei ⟶ ΛN attractive ($U_\Lambda(\rho_0) \sim -30$ MeV)
- 3 double-Λ hypernuclei ⟶ weak ΛΛ attraction ($\Delta B_{\Lambda\Lambda} \sim 1$MeV)
- Very few Ξ-hypernuclei ⟶ ΞN attractive ($U_\Xi(\rho_0) \sim -14$ MeV)
- Ambiguous evidence of Σ-hypernuclei ⟶ ΣN repulsive ($U_\Sigma(\rho_0) > +15$ MeV) ?

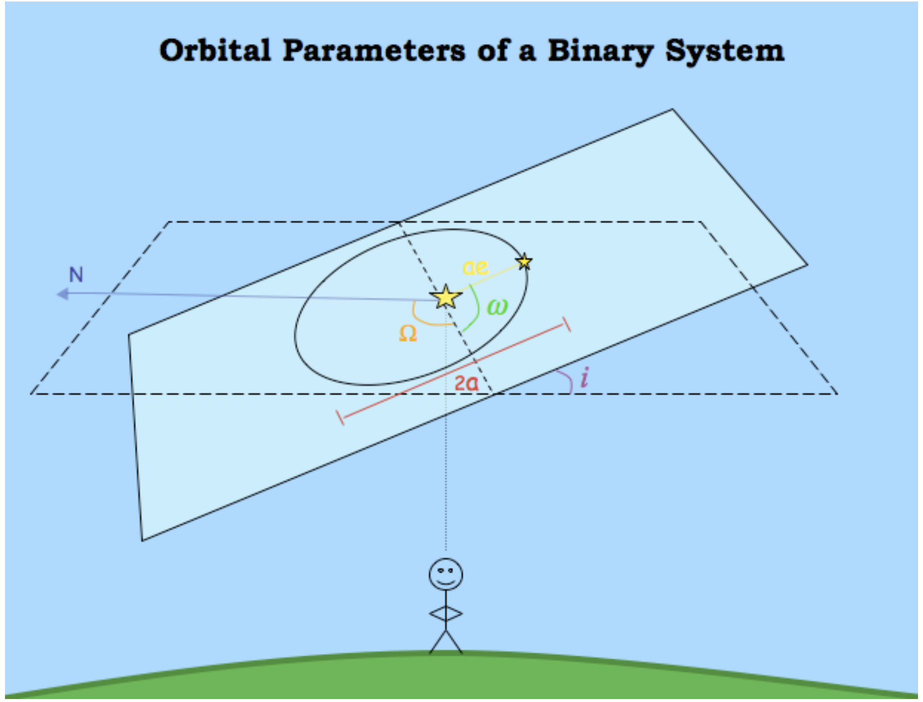
- Strangeness exchange production: $^{A}Z(K^-,\pi^-)^{A}_{\Lambda}Z$
- Associate strangeness production: $^{A}Z(\pi^+,K^-)^{A}_{\Lambda}Z$
- Electroproduction: $^{A}Z(e'K^+)^{A}_{\Lambda}(Z-1)$
- Production in HIC



Astrophysical (Neutron Stars) Constraints

Neutron Star Masses

NS masses can be inferred directly from observations of binary systems



- 5 orbital (Keplerian) parameters can be precisely measured:
  - ✓ Orbital period (P)
  - ✓ Projection of semimajor axis on line of sight (a sin i)
  - ✓ Orbit eccentricity (ε)
  - ✓ Time of periastron (T_0)
  - ✓ Longitude of periastron (ω_0)
- 3 unknowns: M_1, M_2, i

Kepler's 3rd law

$$\frac{G(M_1+M_2)}{a^3} = \left(\frac{2\pi}{P}\right)^2 \rightarrow$$

$$f(M_1, M_2, i) \equiv \frac{(M_2 \sin i)^3}{(M_1+M_2)^2} = \frac{Pv^3}{2\pi G}$$

mass function

In few cases small deviations from Keplerian orbit due to GR effects can be detected

Measure of at least 2 post-Keplerian parameters → High precision NS mass determination

$$\dot{\omega} = 3T_{\otimes}^{2/3}\left(\frac{P_b}{2\pi}\right)^{-5/3}\frac{1}{1-\varepsilon}\left(M_p+M_c\right)^{2/3} \longrightarrow \text{Advance of the periastron}$$

$$\gamma = T_{\otimes}^{2/3}\left(\frac{P_b}{2\pi}\right)^{1/3}\varepsilon\frac{M_c\left(M_p+2M_c\right)}{\left(M_p+M_c\right)^{4/3}} \longrightarrow \text{Time dilation \& grav. redshift}$$

$$r = T_{\otimes}M_c \longrightarrow \text{Shapiro delay “range”}$$

$$s = \sin i = T_{\otimes}^{-1/3}\left(\frac{P_b}{2\pi}\right)^{-2/3}x\frac{\left(M_p+M_c\right)^{2/3}}{M_c} \longrightarrow \text{Shapiro delay “shape”}$$

$$\dot{P}_b = -\frac{192\pi}{5}T_{\otimes}^{5/3}\left(\frac{P_b}{2\pi}\right)^{-5/3}f(\varepsilon)\frac{M_pM_c}{\left(M_p+M_c\right)^{1/3}} \longrightarrow \text{Orbit decay due to GW emission}$$

Recent Measurements of High NS Masses

- <u>PSR J164-2230</u> (Demorest et al. 2010)

✓ binary system (P=8.68 d)

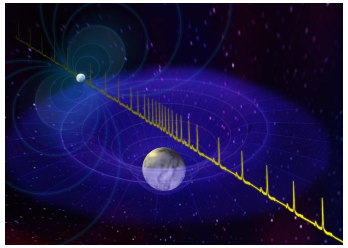
✓ low eccentricity (ε=1.3 x 10^{-6})

✓ companion mass: $\sim 0.5 M_{\odot}$

✓ pulsar mass: $M = 1.928 \pm 0.017 M_{\odot}$

In this decade NS with $2M_{\odot}$ have been observed by measuring Post-Keplerian parameters of their orbits

- Advance of the periastron $\dot{\omega}$
- Shapiro delay (range & shape)
- Orbital decay $\dot{P}_b$
- Grav. redshift & time dilation γ



- <u>PSR J0348+0432</u> (Antoniadis et al. 2013)

✓ binary system (P=2.46 h)

✓ very low eccentricity

✓ companion mass: $0.172 \pm 0.003 M_{\odot}$

✓ pulsar mass: $M = 2.01 \pm 0.04 M_{\odot}$

- <u>MSP J0740+6620</u> (Cromartie et al. 2020)

✓ binary system (P=4.76 d)

✓ low eccentricity (ε=5.10(3) x 10^{-6})

✓ companion mass: $0.258(8) M_{\odot}$

✓ pulsar mass: $M = 2.14^{+0.10}_{-0.0.9} M_{\odot}$ (68.3% c.i.)

$M = 2.14^{+0.20}_{-0.018} M_{\odot}$ (95.4% c.i.)

Formation of Binary Systems

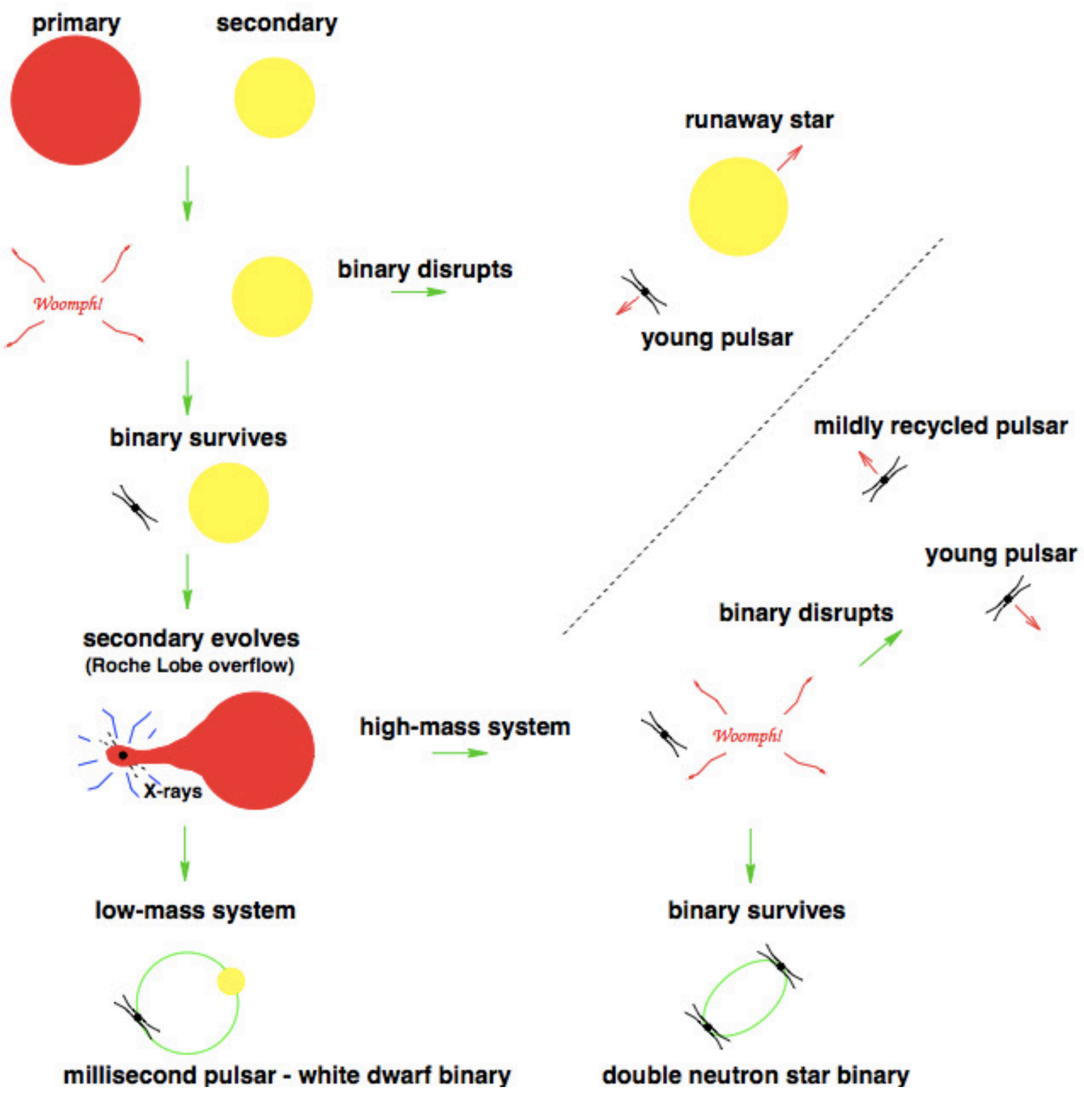
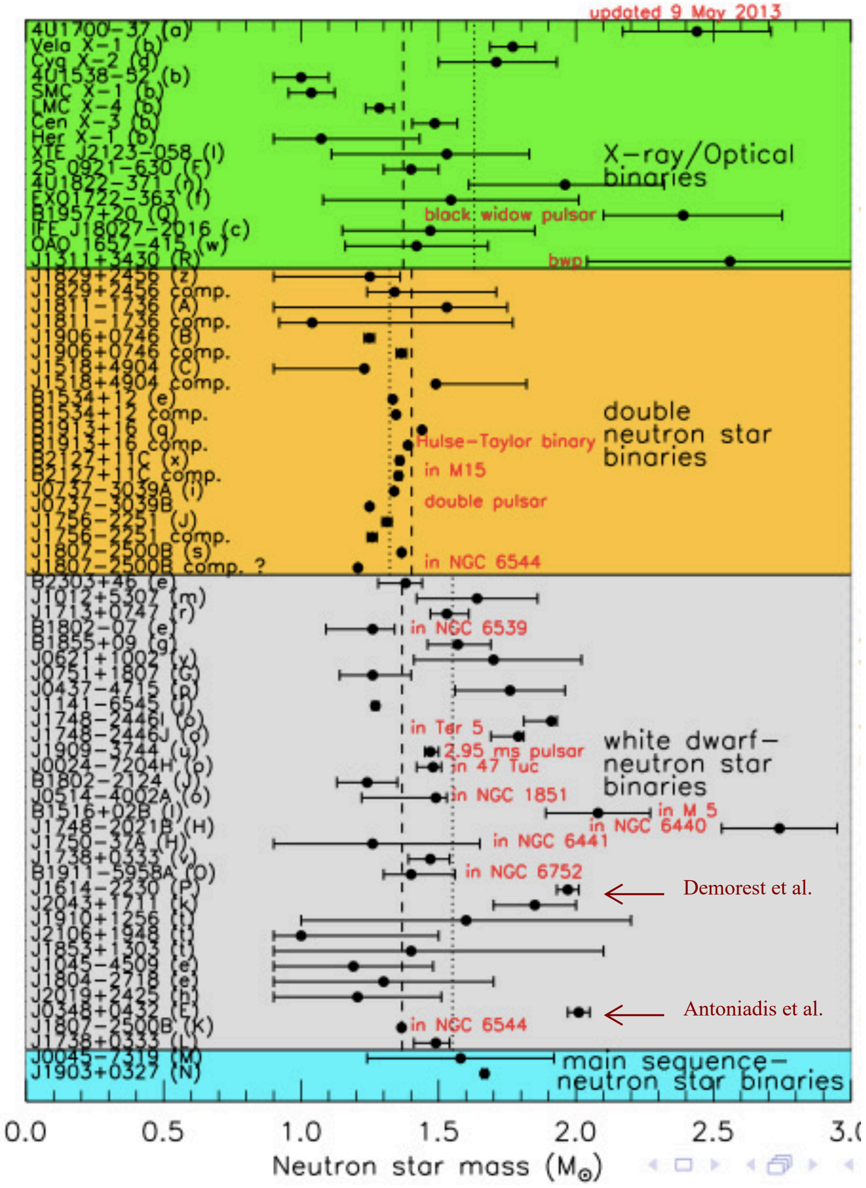


Figure by P.C.C. Freire

Measured Neutron Star Masses (2022)



updated from Lattimer 2013

Observation of ~ 2 $M_\odot$ neutron stars imposes a very stringent constraint

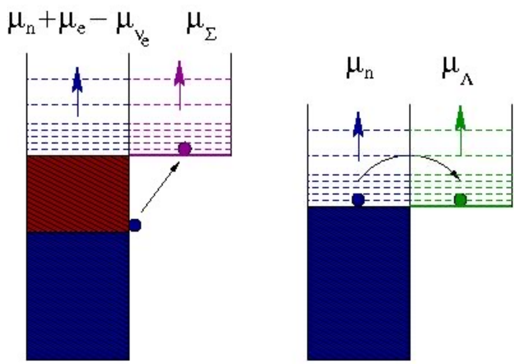
Any reliable nuclear EoS should satisfy

$$M_{max}\left[EoS\right] > 2M_\odot$$

otherwise is rule out

The Hyperon Puzzle

Hyperons are expected to appear in the core of neutron stars at $\rho \sim (2\text{-}3)\rho_0$ when μ_N is large enough to make the conversion of N into Y energetically favorable

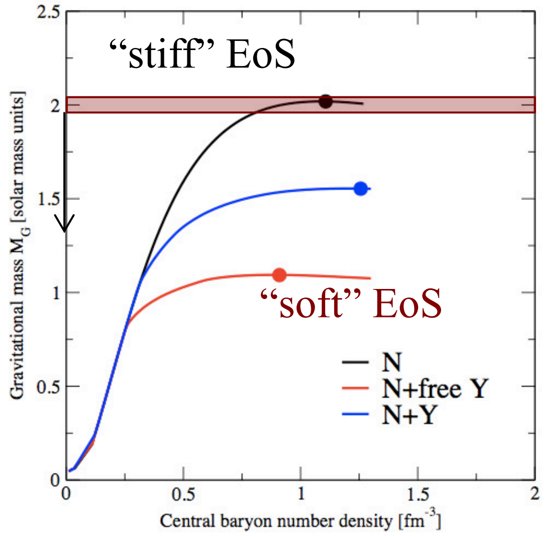


But

The relieve of Fermi pressure due to its appearance leads to a softer EoS and, therefore, to a reduction of the mass to values incompatible with observation

Observation of $\sim 2\,M_\odot$ NS $\longrightarrow$ Any reliable EoS of dense matter should predict $M_{\max}[EoS] > 2M_\odot$

Can hyperons be present in the interior of neutron stars in view of this stringent constraint ?



Limits on the Neutron Star Radius

The radius of a neutron star with mass M cannot be arbitrarily small

General Relativity:
a Neutron Star is not a
Black Hole

$$R > \frac{2GM}{c^2}$$

Finite Pressure:
Neutron Star matter cannot
be arbitrarily compressed

$$R > \frac{9}{4}\frac{GM}{c^2}$$

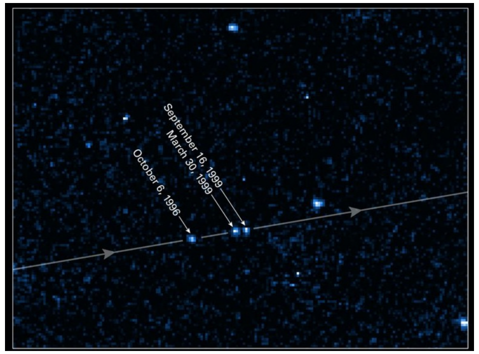
Causality:
speed of sound must
be smaller than c

$$R > 2.9\frac{GM}{c^2}$$

The desired measurement of neutron star radii

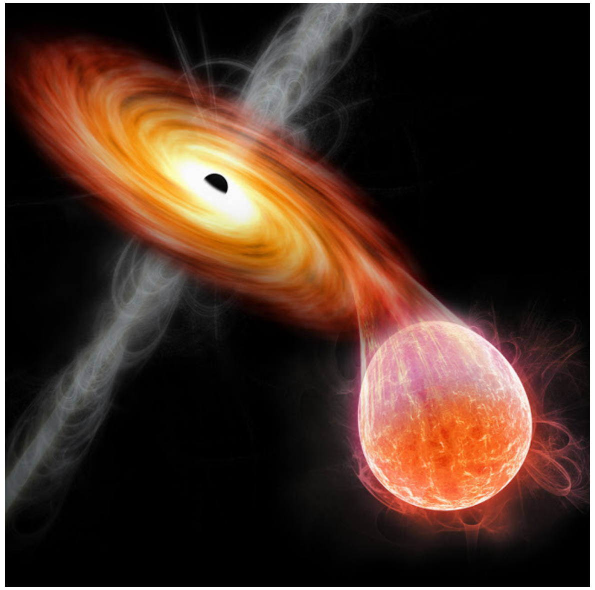
Radii are very difficult to measure because NS:

- ✧ are very small (~ 10 km)
- ✧ are far from us (e.g., the closest NS, RX J185635-3754, is at ~ 200 ly, moving at 100 km/s)



Credit by NASA

A possible way to measure it is to use the thermal emission of low mass X-ray binaries:



NS radius can be obtained from:

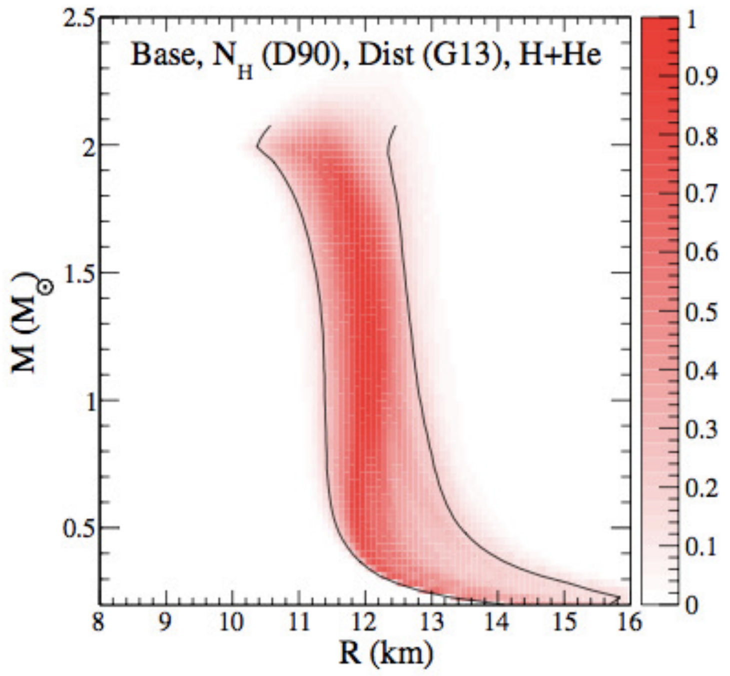
- ✧ Flux measurement +Stefan-Boltzmann's law
- ✧ Temperature (Black body fit+atmosphere model)
- ✧ Distance estimation (difficult)
- ✧ Gravitational redshift z (detection of absorption lines)

$$R_\infty = \sqrt{\frac{FD^2}{\sigma_{SB} T^4}} \rightarrow R_{NS} = \frac{R_\infty}{1+z} = R_\infty \sqrt{1 - \frac{2GM}{R_{NS} c^2}}$$

Estimations of Neutron Star Radii from LMXB

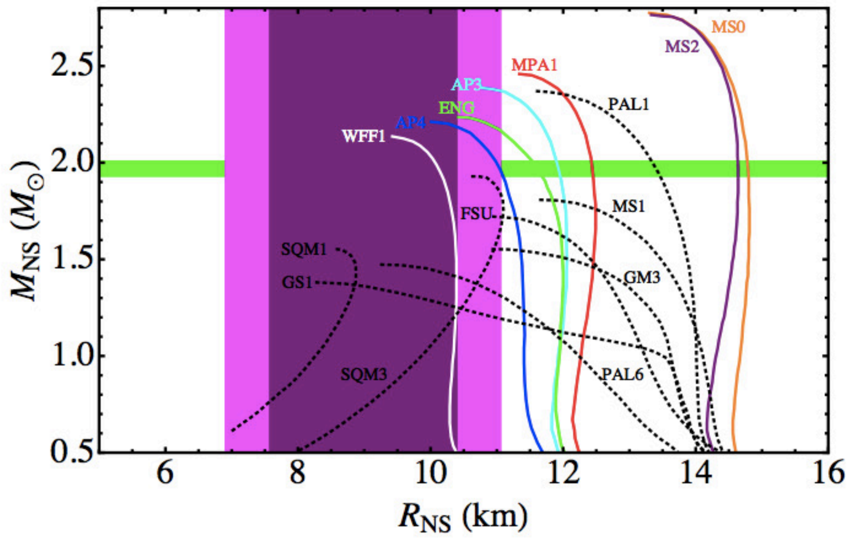
The conclusion from past analysis of the thermal spectrum from 5 quiescent LMXB in globular clusters was controversial

Steiner et al. (2013, 2014)



$$R = 12.0 \pm 1.4 km$$

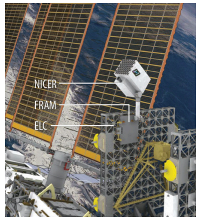
Guillot et al. (2013, 2014)

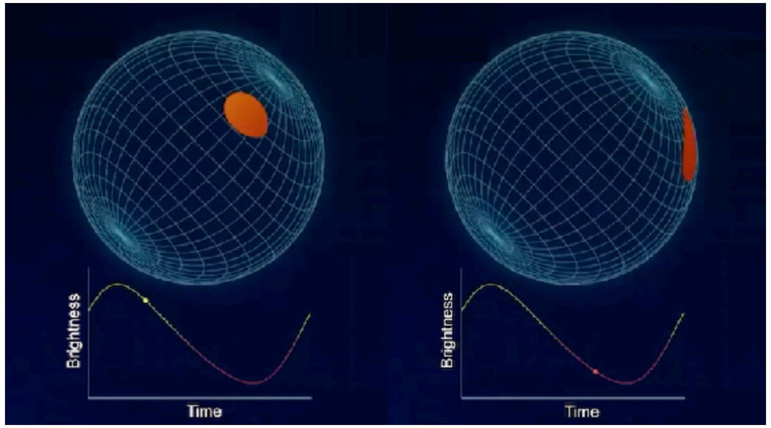


$$R = 9.1^{+1.3}_{-1.5} km$$ 2013 analysis

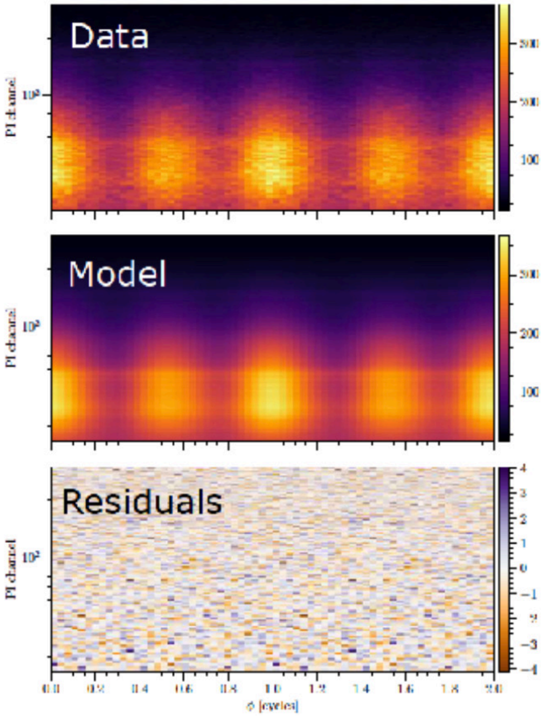
$$R = 9.4 \pm 1.2 km$$ 2014 analysis

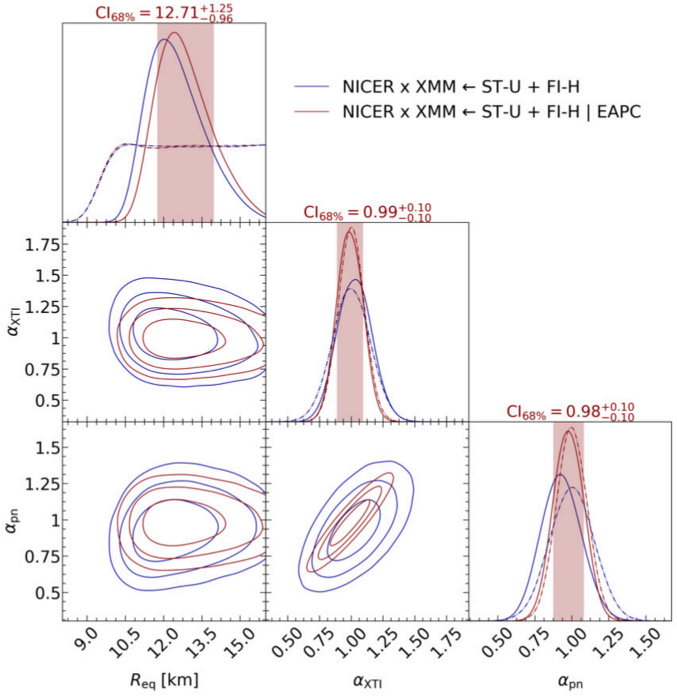
NICER: Neutron Star Interior Composition Explorer





A new way of measuring NS radius by tracking the X-ray emission from “hot spots” on the star’s surface as the star rotates. M/R is extracted by modeling the Pulse Profile of the hot spots





✧ PSR J0740+6620

$$M = 2.072^{+0.067}_{-0.066} M_\odot$$

$$R = 13.7^{+2.6}_{-1.5} \ km$$ Miller et al., arXiv:2105.06979

$$R = 12.39^{+1.30}_{-0.98} \ km$$ Riley et al., arXiv:2105.06980

✧ PSR J0030+0451

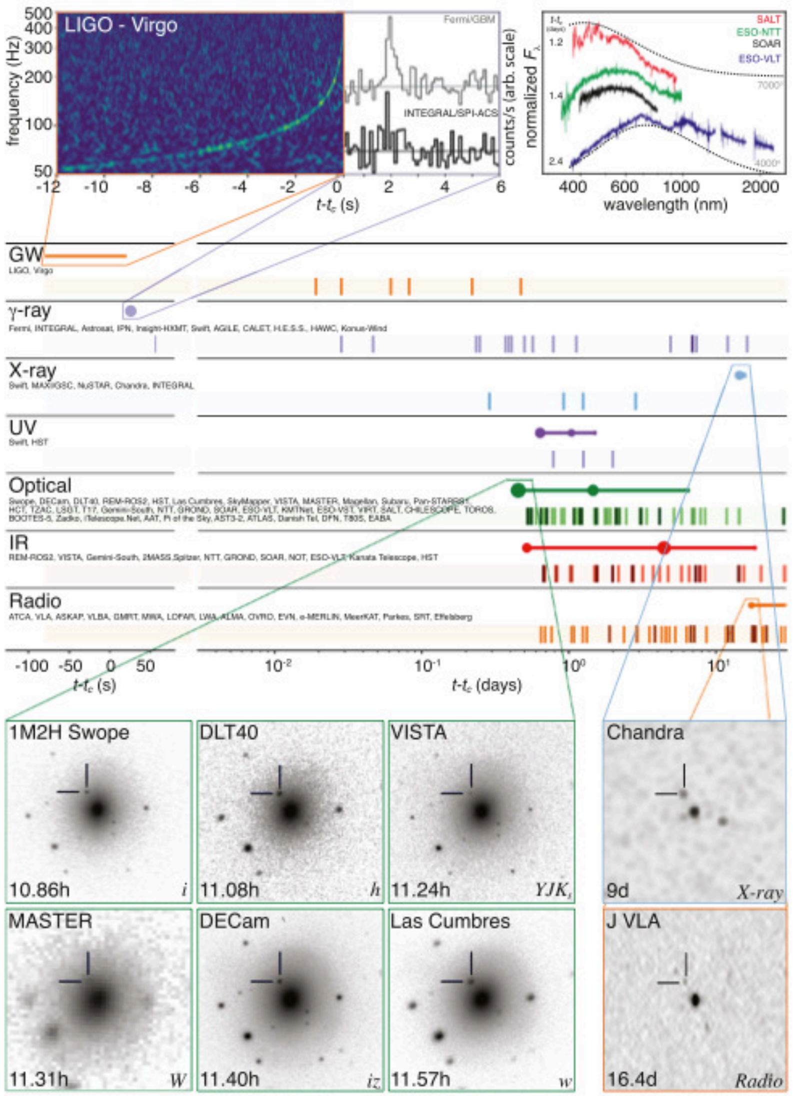
$$M/R = 0.156^{+0.008}_{-0.010}$$

$$R = 13.02^{+1.24}_{-1.06} \ km$$ Miller et al., ApJ 887 L24 (2019)

$$R = 12.71^{+1.14}_{-1.19} \ km$$ Riley et al., APJ 887 L21 (2019)

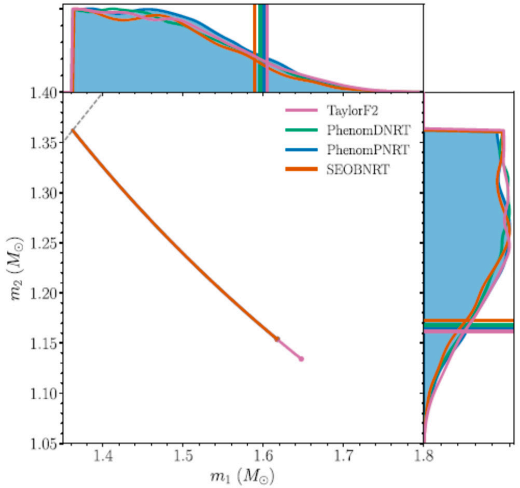
GW170817: the first NS-NS merger

Multi-messenger observations of the event GW170817



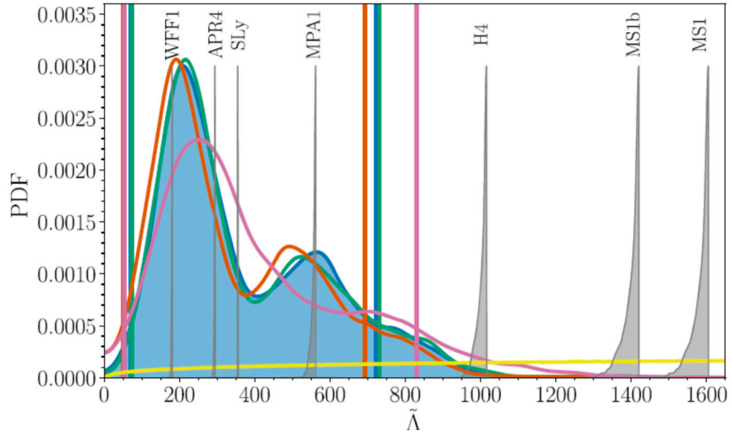
✧ Masses estimated from the chirp mass

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$



✧ Radius from the tidal deformability

$$\tilde{\Lambda} = \frac{16}{13}\frac{(1+12q)\Lambda_1 + (q+12)\Lambda_2}{(1+q)^5}$$



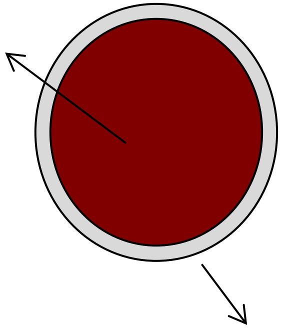
A $1.36 M_\odot$ has a radius of 10.4 km (WFF1), 11.3 km (APR4), 11.7 km (Sly), 12.4 km (MPA1), 14.0 (H4), 14.5 (MS1b) and 14.9 km (MS1)

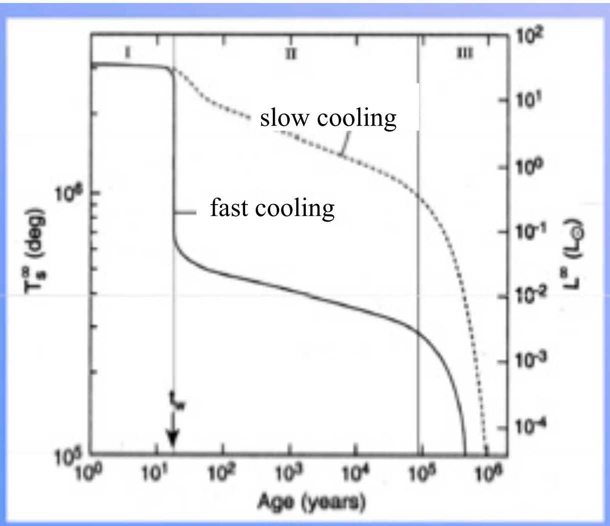
Thermal Evolution of Neutron Stars

Information, complementary to that from mass & radius, can be also obtained from the measurement of the temperature (luminosity) of neutron stars

Two cooling regimes

Slow	Fast
Low NS mass	High NS mass





I. Core relaxation epoch
II. Neutrino cooling epoch
III. Photon cooling epoch

$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu + H$$

✓C_v: specific heat

✓L_γ: photon luminosity

✓L_ν: neutrino luminosity

✓H: “heating”

Strong dependence on the NS composition & EoS

Other neutron star observables

Other NS observables can also help to constraint direct or indirectly the nuclear EoS

Gravitational Redshift:

$$z = \left(1 - \frac{2GM}{c^2 R}\right)^{-1/2} - 1$$

Measurements of z allow to constraint the ratio of M/R

Quasi-periodic Oscillations:

QPO in X-ray binaries measure the difference between the NS rot. freq. & the Keplerian freq. of the innermost stable orbit of matter elements in the accretion disk. Their observation & analysis can put stringent constraints on masses, radii & rotational periods

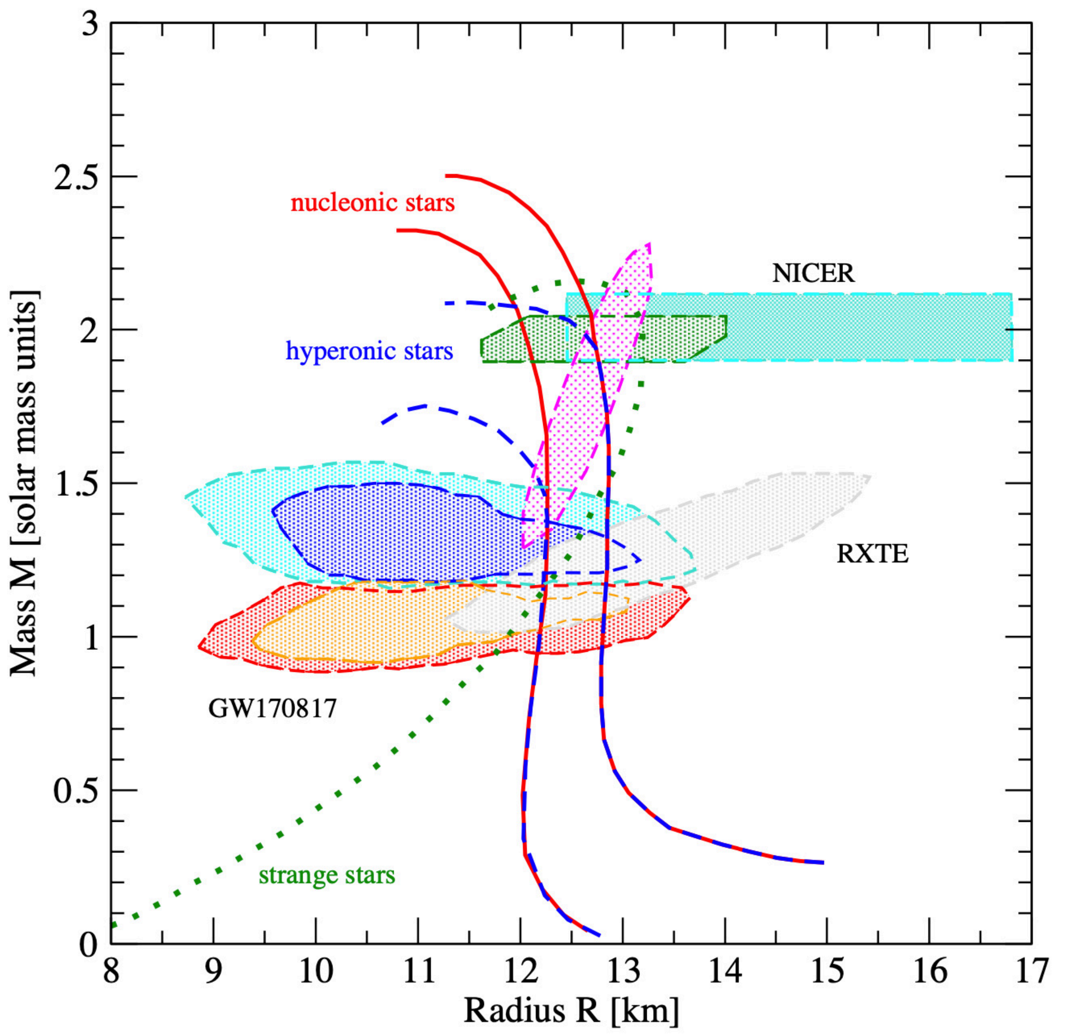
NS moment of inertia:

$$I = \frac{J(\Omega)}{\Omega}; \quad J(\Omega) = \frac{8\pi}{3}\int_0^R dr r^4 \frac{p(r)+\varepsilon(r)}{\sqrt{1-\frac{2M(r)}{r}}}\left(\Omega-\omega(r)\right)e^{-\nu(r)}$$

Measurements of I could also constraint EoS. But not measured yet. Lower bound can be inferred from timing observations of Crab pulsar

Combined analysis of a few astrophysical data

- ✧ NICER PSR J0740+6620 & PSR J0030+0451
- ✧ GW170817
- ✧ Rossi X-ray Timing Explorer (RXTE) results for the cooling tail spectra of 4U1702-429



Building the Nuclear EoS

Approaches to the Nuclear EoS: "Story of Two Philosophies"

Ab-initio Approaches

Based on two- & three-nucleon realistic interactions which reproduce scattering data & the deuteron properties. The EoS is obtained by "solving" the complicated many-body problem

- ✧ Variational approaches: FHNC
- ✧ Diagrammatic: methods: BBG (BHF), SCGF
- ✧ Monte-Carlo techniques: VMC, DMC, GMC, AFDMC
- ✧ RG methods: $V_{\text{low k}}$

Phenomenological Approaches

Based on effective density-dependent interactions with parameters adjusted to reproduce nuclear observables & compact star properties.

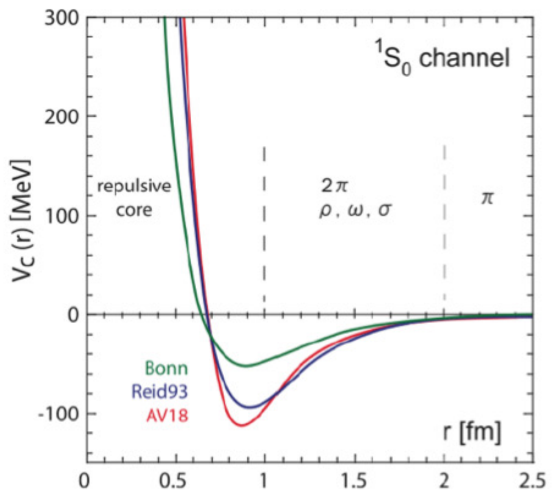
- ✧ Non-relativistic: Skyrme & Gogny
- ✧ Relativistic: RMF

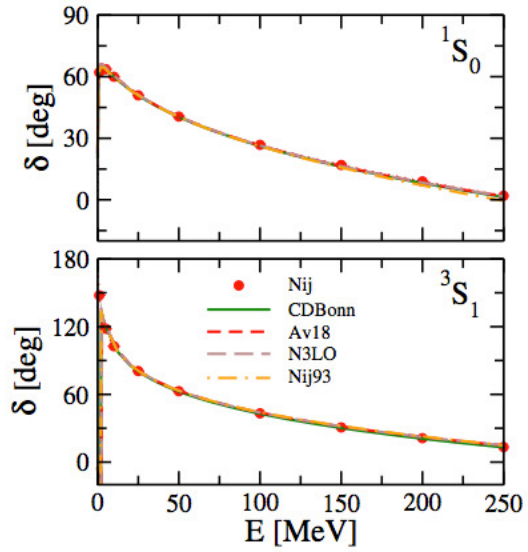
Non-homogeneous matter

- ✧ SN approximation models: Liquid drop models, TF models, Self-consistent models
- ✧ NSE models: NSE, Virial EoS, models with in-medium mass shifts

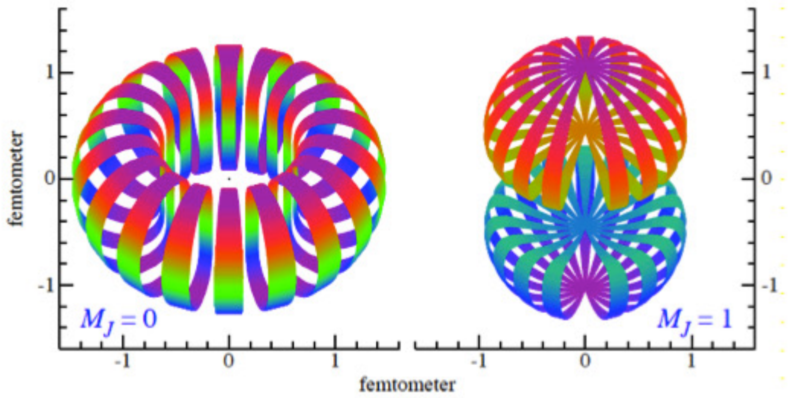
Difficulties of ab-initio approaches

✧ Different NN potentials in the market … but all are phase-shift equivalent





✧ Short range repulsion makes any perturbation expansion in terms of V meaningless. Different ways of treating SRC



✧ Complicated channel & operatorial structure (central, spin-spin, spin-isospin, tensor, spin-orbit, …)

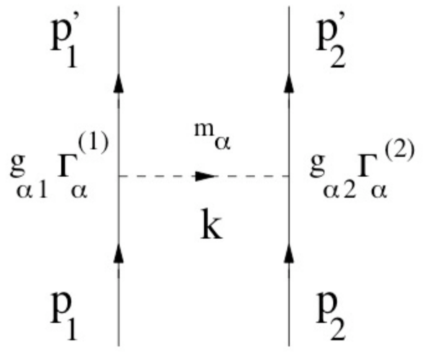
The NN interaction: meson exchange & potential models

✧ Meson Exchange Models:

NN interaction mediated by the exchange of different meson fields (e.g, Bonn, Nijmegen)

- ✧ scalar: σ, δ $\quad \Gamma_s = 1$
- ✧ pseudocalar: π, K, η $\quad \Gamma_{ps} = i\gamma^5$
- ✧ vector: ρ, K, ω, ϕ $\quad \Gamma_v = \gamma^\mu\,, \quad \Gamma_T = \sigma^{\mu\nu}$

$$L = g_M \Gamma_M \left(\bar{\Psi}_B \Psi_B\right) \phi_M$$



$$\langle p_1' p_2' | V_M | p_1 p_2 \rangle = \bar{u}(p_1') g_M^{(1)} \Gamma_M^{(1)} u(p_1) \frac{P_M}{\left(p_1 - p_1'\right)^2 - m_M^2} \bar{u}(p_2') g_M^{(2)} \Gamma_M^{(2)} u(p_2)$$

Machleidt et al., PR. 149, 1 (1987)
Nagels et al., PRD 17, 768 (1978)

✧ Potential Models:

NN interaction is given by the sum of several local operators (e.g., Urbana, Argonne)

$$V_{ij} = \sum_{p=1,18} V_p(r_{ij}) O_{ij}^p$$

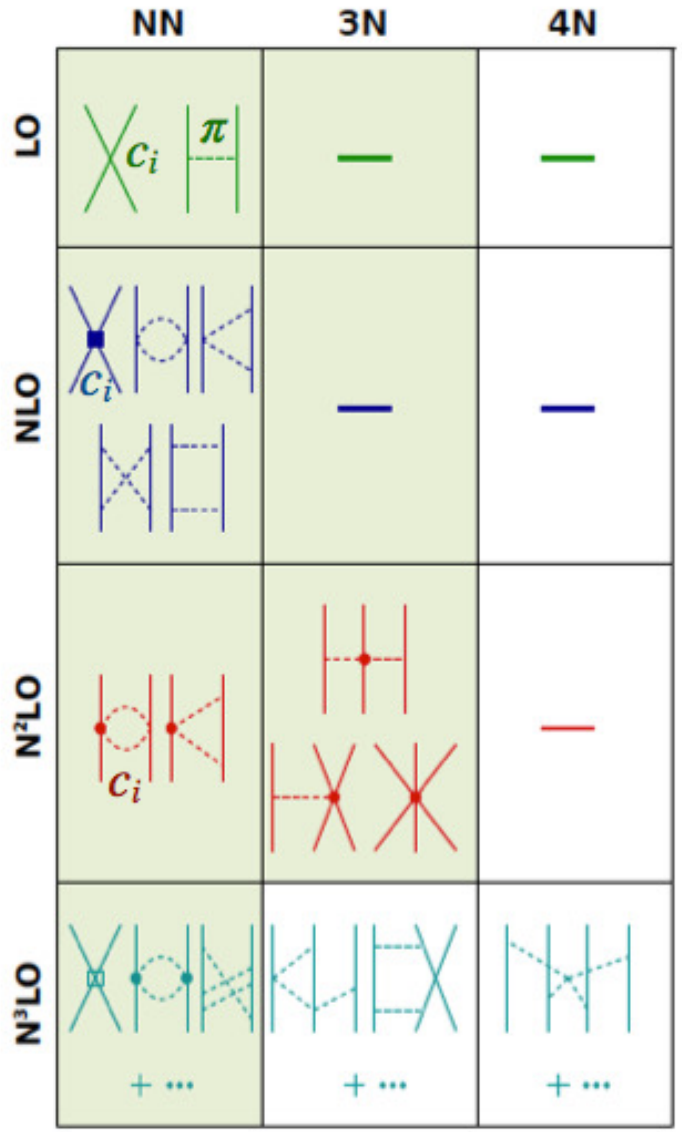
Ex: Local operators of Av18 potential

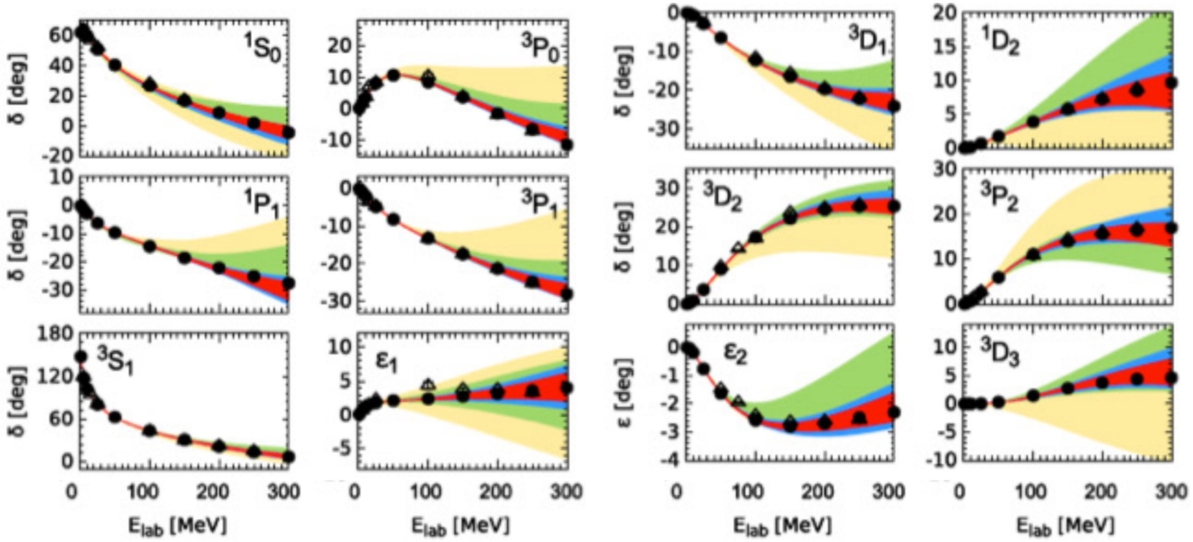
$$O_{ij}^{p=1,14} = \left[1, (\vec{\sigma}_i \cdot \vec{\sigma}_j), S_{ij}, \vec{L}\cdot\vec{S}, L^2, L^2(\vec{\sigma}_i \cdot \vec{\sigma}_j), (\vec{L}\cdot\vec{S})^2\right] \otimes \left[1, (\vec{\tau}_i \cdot \vec{\tau}_j)\right]$$

$$O_{ij}^{p=15,18} = \left[T_{ij}, (\vec{\sigma}_i \cdot \vec{\sigma}_j) T_{ij}, S_{ij} T_{ij}, (\tau_{zi} + \tau_{zj})\right]$$

Wiringa et al., PRC 51, 38 (1995)

The NN interaction: χEFT forces





- Starting point: most general effective chiral Lagrangian that respect required QCD symmetries where π & N (recently also Δ) are the relevant d.o.f. of the theory

- Systematic expansion in powers of Q/Λ_χ [$Q=m_\pi$, k; $\Lambda_\chi \sim 1$ GeV]

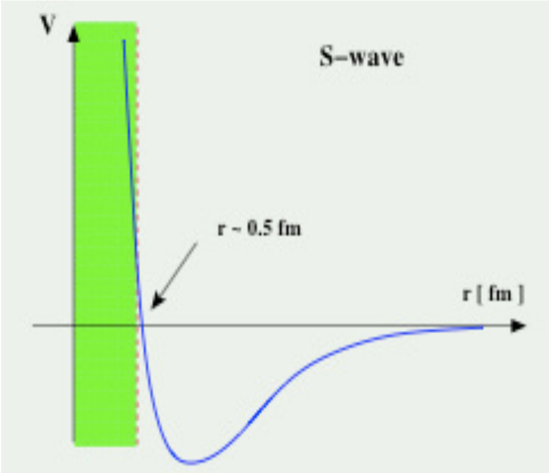
- Consistent derivation of 2N, 3N, 4N, … forces

Weinberg, PLB 251, 288 (1990); NPB 363, 3 (1991)
Entem & Machleidt, PRC 68, 041001(R) (2003)
Epelbaum et al., NPA 747, 363 (2005)

Renormalization Group Method

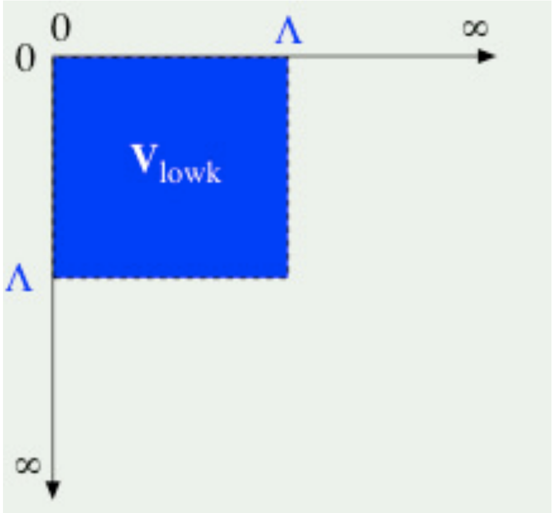
- The presence of a short–range hard core of the nucleon-nucleon interaction V makes any perturbation expansion in terms of V meaningless

- A possible way to soften it consists in integrating out all the momenta q larger than a certain cut-off Λ obtaining in this wat effective interaction $V_{low\ k}$ that is equivalent to the original one for momenta $q < \Lambda$



This results in a modified Lippmann-Schwinger equation with a cut-off dependent effective potential $V_{low\ k}$

$$T\,(k',k\colon E_k) = V_{low\ k}(k',k) + \frac{2}{\pi}\,P\int_0^{\Lambda} dq\,q^2\,\frac{V_{low\ k}(k',q)T(q,k\colon E_k)}{k^2 - q^2 + i\eta}$$



Bogner et al., Phys. Rep. 386, 1 (2003)

Renormalization Group Method

- By demanding $\frac{dT(k',k:E_k)}{d\Lambda} = 0$ one obtains a Renormalization Group equation for $V_{low\ k}$

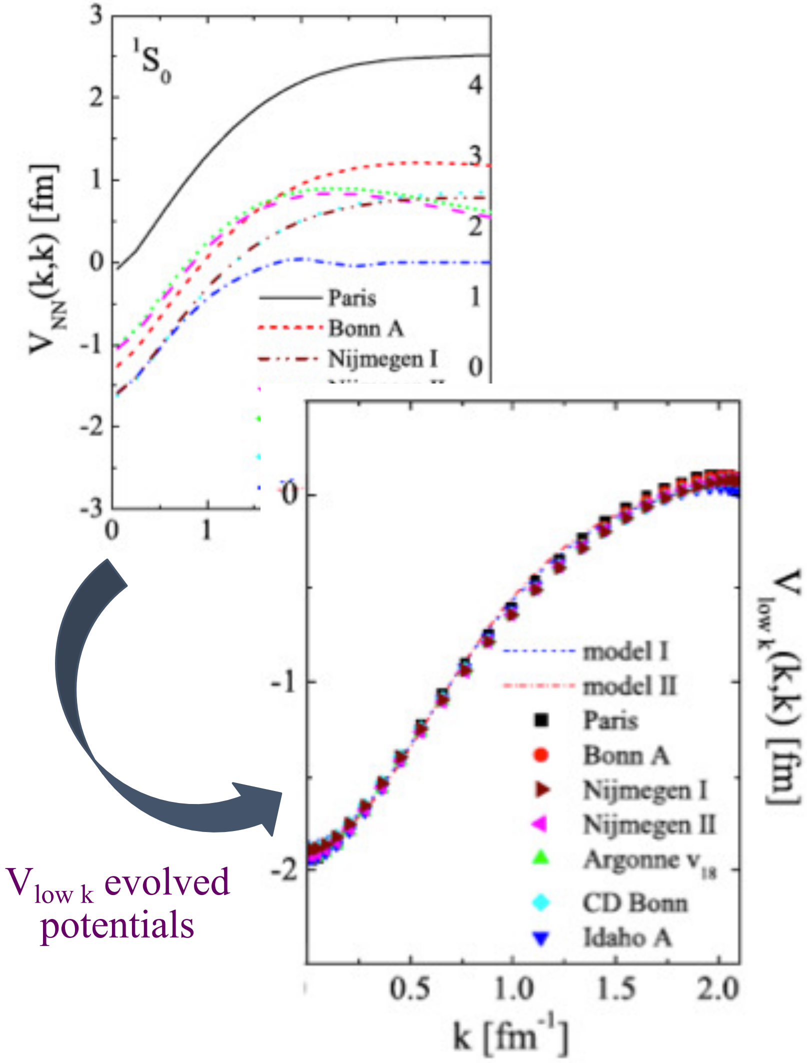
$$\frac{dV_{low\ k}(k',k)}{d\Lambda} = \frac{2}{\pi}\frac{V_{low\ k}(k',k)T(\Lambda,k,\Lambda^2)}{1-k^2/\Lambda^2}$$

- Integrating this flow equation one obtains a "universal" nucleon-nucleon low-momentum potential $V_{low\ k}$ that is:

  - ✓ phase shift equivalent
  - ✓ energy independent
  - ✓ softer (no hard core)
  - ✓ hermitian

- Having a much softer core the $V_{low\ k}$ potential can be used in perturbation expansions and nuclear structure calculations in a more efficient way

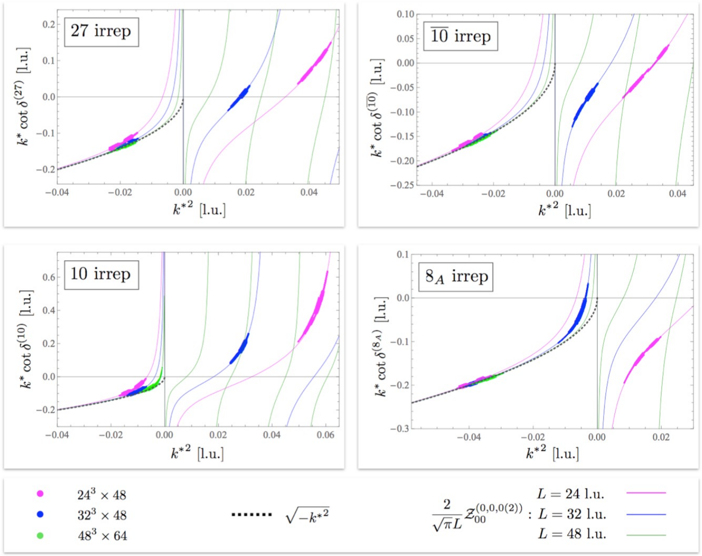
- The method has been applied also to the hyperon-nucleon case. The results seem to indicate a similar convergence to a "universal" softer low-momentum hyperon-nucleon interaction



$V_{low\ k}$ evolved potentials

Baryon-baryon interactions from Lattice QCD

NPLQCD & the HALQCD strategies



- **NPLQCD**

  Combines calculations of correlation functions of two-baryon systems at several light-quark-mass values with low-energy effective field theory to extract scattering phase-shifts

- **HALQCD**

  - Determine the Nambu-Bethe-Salpeter wave function on the lattice

    $$\varphi_{E(r)} = \langle 0|N((x+r,0)N(x,0)|6q,E\rangle \,, N(x) = \varepsilon_{abc}q^a(x)q^b(x)q^c(x)$$

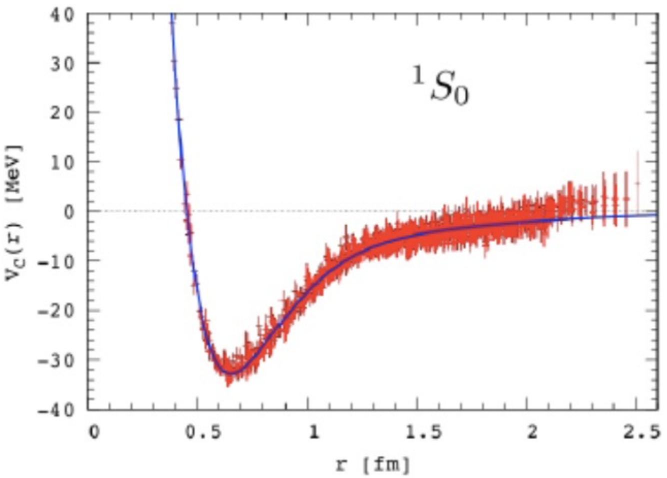
  - Define a local potential $U(x,y)$ from $\varphi_{E(r)}$

    $$\left[E - \frac{\hbar^2\nabla^2}{2\mu_N}\right]\varphi_{E(x)} = \int d^3yU(x,y)\varphi_{E(y)} \,, \qquad U(x,y) = V(x,\nabla)\delta\,(x-y)$$

    $$V(x,\nabla) = V_c(x) + V_T(x)S_{12} + V_{LS}(x)\vec{L}\cdot\vec{S} + \{V_D,\nabla^2\} + \cdots$$

  - Calculate observables (phase shifts, binding energies, …)

a=0.09fm, L=2.9fm $m_\pi \simeq 700$ MeV



Variational & Diagrammatic Approaches

✧ Variational Approach

Based on the variational principle

$$E \le \min\left\{\frac{\langle\Psi_T|\hat{H}|\Psi_T\rangle}{\langle\Psi_T|\Psi_T\rangle}\right\}, \quad |\Psi_T\rangle = \hat{F}|\Phi\rangle, \quad \hat{F} = \prod_{i>j}\sum_p f^{(p)}(r_{ij})\hat{O}_{ij}^{(p)}$$

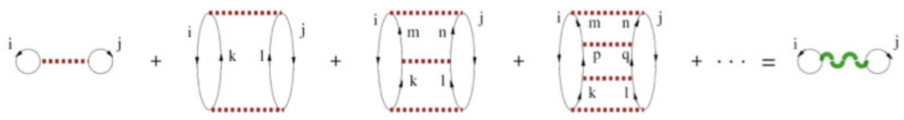
correlation operator   uncorrelated w.f.

$f^{(p)}(r_{ij})$ determined through functional minimization of the energy using techniques like FHNC or VMC

Fantoni & Rosati, Nuov. Cim. 25A, 593 (1975)

✧ BBG theory

Ground state energy of nuclear matter evaluated in terms of the hole-line expansion derived by means of Brueckner reaction matrix



BHF:

$$E_{BHF} = \sum_{i\le A}\langle\alpha_i|K|\alpha_i\rangle + \frac{1}{2}\mathrm{Re}\left[\sum_{i,j\le A}\langle\alpha_i\alpha_j|G(\omega)|\alpha_i\alpha_j\rangle\right]$$

Infinite sumation of two-hole line diagrams

Day, RMF.39, 719 (1967)

✧ SCGF formalism

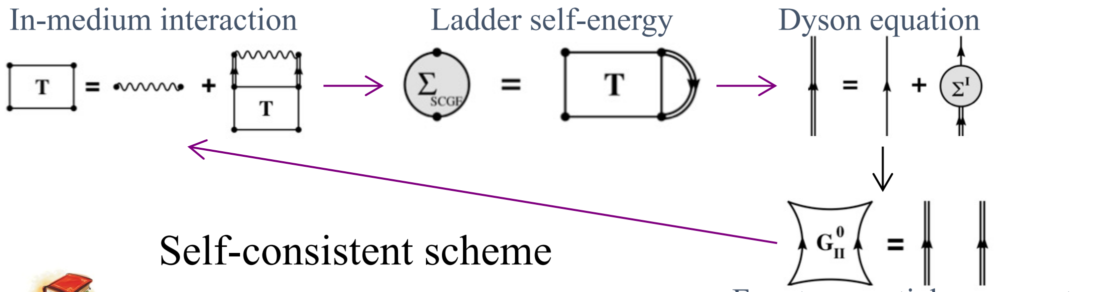
Energy obtained from the Galitskii-Migdal-Koltum (GMK) sum rule

$$E = \frac{\nu}{\rho}\int\frac{d^3k}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{2}\left\{\frac{\hbar^2k^2}{2m}+\omega\right\}A\left(\vec{k},\omega\right)f\left(\omega\right)$$

s. p. spectral function   FD distribution

Spectral function

$$A(\vec{k},\omega) = \frac{-2\,\mathrm{Im}\,\Sigma(\vec{k},\omega)}{\left[\omega-\frac{\hbar^2k^2}{2m}-\mathrm{Re}\,\Sigma(\vec{k},\omega)\right]^2+\left[\mathrm{Im}\,\Sigma(\vec{k},\omega)\right]^2}$$



Carbone et al, PRC 88, 054326 (2013)

Quantum Monte-Carlo Techniques

VMC:

Evaluate energy & other observables using the Metropolis algorithm

$$\langle \hat{O} \rangle = \frac{\sum_i \langle \Psi(\vec{R}_i) | \hat{O} | \Psi(\vec{R}_i) \rangle / W(\vec{R}_i)}{\sum_i \langle \Psi(\vec{R}_i) | \Psi(\vec{R}_i) \rangle / W(\vec{R}_i)}$$

Wiringa et al., PRC 62, 014001 (2000)

DMC:

Model a diffusion process rewriting the Schoedinger equation in imaginary time

$$i\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle \Rightarrow -\frac{\partial}{\partial \tau}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Anderson, J. Chem. Phys. 63, 1499 (19755)

GFMC:

Sample a trial wave function by evaluating path integrals of the form

$$|\Psi(\tau)\rangle = \prod \exp\left[-\left(\hat{H} - E_0\right)\Delta\tau\right]|\Psi_V\rangle$$

$$|\Psi(\tau)\rangle \underset{n\to\infty}{\longrightarrow} |\Psi_0\rangle$$

Carlson et al., PRC 68, 025802 (2003)

AFDMC:

Rewrite Green's function in order to change the quadratic dependence on spin & isospin operators to a linear one by introducing Hubbard-Stratonovich auxiliary fields

Gandolfi et al., PRC 79, 054005 (2009)

Phenomenological Models: Skyrme & Gogny interactions

✧ Skyrme interactions:

Effective zero-range density dependent interaction

$$\hat{V}(\vec{r}_1,\vec{r}_2) = t_0\left(1+x_0\hat{P}_\sigma\right)\delta(\vec{r}_{12}) + \frac{t_1}{2}\left(1+x_1\hat{P}_\sigma\right)\left[\hat{k}^{'}\delta(\vec{r}_{12}) + \delta(\vec{r}_{12})\hat{k}^2\right]$$
$$+t_2\left(1+x_2\hat{P}_\sigma\right)\hat{k}'\delta(\hat{r}_{12})\hat{k} + \frac{t_3}{6}\left(1+x_3\hat{P}_\sigma\right)\rho^\alpha(\vec{R}_{12})\delta(\hat{r}_{12})$$
$$+iW_0\left(\hat{\sigma}_1+\hat{\sigma}_2\right)\left[\hat{k}^{'}\times\delta(\hat{r}_{12})\hat{k}\right]$$

Evaluation of the energy density in the HF approximation yields for nuclear matter a simple EDF in fractional powers of the number densities. Many parametrizations exist

Skyrme, Nucl. Phys. 9, 615 (1959)

✧ Gogny interactions:

Effective finite-range density dependent interaction

$$\hat{V}(\vec{r}_1,\vec{r}_2) = \sum_{j=1,2}\exp\left(-\frac{r_{12}^2}{\mu_j^2}\right)\left(W_j + B_j\hat{P}_\sigma - H_j\hat{P}_\tau - M_j\hat{P}_\sigma\hat{P}_\tau\right)$$
$$+t_0\left(1+x_0\hat{P}_\sigma\right)\rho^\alpha(\vec{R}_{12})\delta(\hat{r}_{12})$$
$$+iW_0\left(\hat{\sigma}_1+\hat{\sigma}_2\right)\left[\hat{k}^{'}\times\delta(\hat{r}_{12})\hat{k}\right]$$

Due to the finite-range terms the evaluation of the energy density is numerically more involved. Less number of parametrizations in the market

Brink & Boeker, NPA 91, 1 (1967)

Phenomenological Models: Relativistic Mean Field Models

Based in effective Lagrangian densities where the interaction is modeled by meson exchanges

$$L = L_{nuc} + L_{mes} + L_{\text{int}} + L_{nl}$$

$$L_{nuc} = \sum_{i=n,p} \bar{\psi}_i \left(\gamma_\mu i \partial^\mu - m_i \right) \psi_i$$

$$L_{mes} = \frac{1}{2}\left(\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \right) + \frac{1}{2}\left(\partial^\mu \vec{\delta} \partial_\mu \vec{\delta} - m_\sigma^2 \right) - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} H_{\mu\nu} H^{\mu\nu} + \frac{1}{2} m_\omega^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu$$

$$L_{\text{int}} = -\sum_{i=n,p} \bar{\psi}_i \left[\gamma_\mu \left(g_\omega \omega^\mu + g_\rho \vec{\tau} \cdot \vec{\rho}^\mu \right) + g_\sigma \sigma + g_\delta \vec{\tau} \cdot \vec{\delta} \right] \psi_i$$

$$L_{nl} = -\frac{A}{3}\sigma^3 - \frac{B}{4}\sigma^4 + \frac{C}{4}\left(\omega_\mu \omega^\mu \right)^2 + D\left(\omega_\mu \omega^\mu \right)\left(\vec{\rho}_\mu \cdot \vec{\rho}_\mu \right)$$

Nucleon & meson equations of motion are derived from the Lagrangian density and usually self-consistently solved in the mean field approximation where mesons are treated as classical fields and negative-energy states of baryons are neglected

Boguta & Bodmer, NPA 292, 413 (1977)

Serot & Walecka, Adv. Nuc. Phys. 16, 1 (1986)

EoS for non-homogeneous nuclear matter

Non-uniform nuclear matter is present in the NS crust and SN cores (low ρ, low T). Till now only two types of phenomenological approaches have been used to describe it:

Single-nucleus approximation models

Composition of matter is assumed to be made of one representative heavy nucleus (the one energetically favored) + light nuclei (α particles) or unbound nucleons

- ✓ (Comprenssible) Liquid-Drop models
- ✓ (Extended) Thomas-Fermi models
- ✓ Self-consistent mean-field models

Nuclear Statistical Equilibrium models

Composition of matter is assumed to be a statistical ensemble of different nuclear species and nucleons in thermodynamical equilibrium

- ✓ (Extended) NSE
- ✓ Virial EoS
- ✓ Models with in-medium mass shifts

The final message of this talk

The Nuclear EoS is a fundamental ingredient for the understanding of the static & dynamical properties of NS, core-collapse SN & compact star mergers

- Major experimental, observational & theoretical advances on understanding the nuclear EoS have been done in the last decades & will be done in the near future

- The isoscalar part of the nuclear EoS is rather well constrained

- Why the isovector part is less well constrained is still an open question whose answer is probably related to our limited knowledge of the nuclear force and, particularly, of its spin & isospin dependence

✧ You for your time & attention

✧ The organizers for their kind invitation & support