Quantum gases with long-range magnetic interactions

FRANCESCA

FERLAINO

IQOQI-Institute for Quantum Optics and Quantum Informations

UIBK-University of Innsbruck (Austria)

www.erbium.at

universität innsbruck

Why Many-Body Quantum systems are keeping us so busy and fascinated?

$$\zeta = \frac{\text{Range of Interaction}}{\text{Mean interp. Distance}} = \frac{a}{d}$$

QUANTUM FLUID

STRONG

^{4}He Droplets

$\zeta \sim 1$

Quantum Gases

$\zeta \sim 0.01$

WEAK

Tunable contact interaction

$$U_c = \frac{4\pi\hbar^2 a}{m}\delta(r)$$

Only effective at short distance

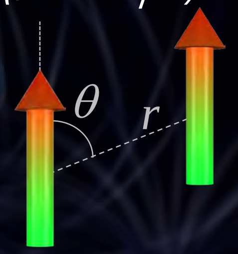
Directionally independent (isotropic)

Why Many-Body Quantum systems are keeping us so busy and fascinated?

Dipolar interaction

$$V_{\mathrm{dd}} = \frac{\mu_0 \mu^2}{4\pi} \frac{1 - 3\cos^2\theta}{r^3}$$

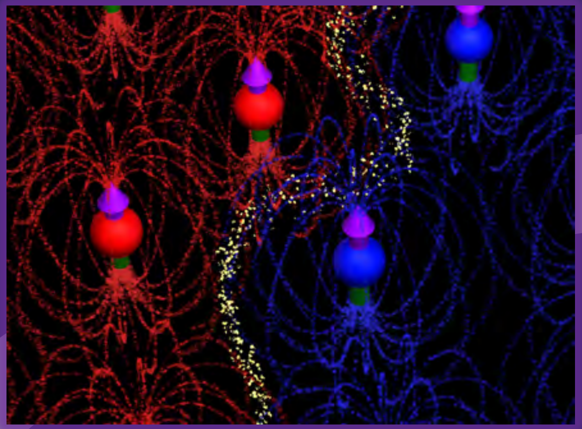
Directional (anisotropic)



Effective at long distance

Long-range interacting systems

Lanthanides

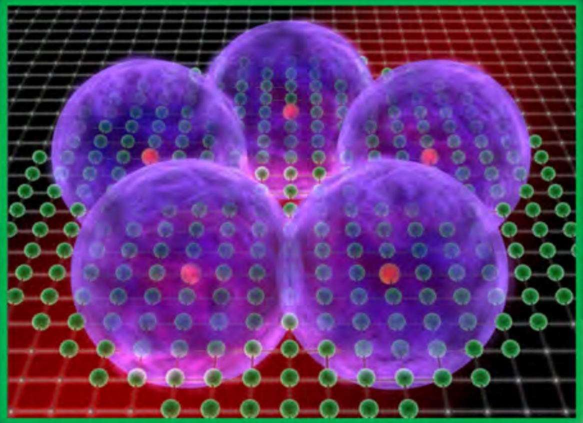


© M. J. Mark / uibk

Hetero-nuclear molecules

© Jose-Luis Olivares/MIT

Rydberg atoms



© MPI of Quantum Optics

Alkalis: 10^{-2}

Chromium: 10^{-1}

Lanthanides: 1

Hetero-nuclear molecules: 10^{2}

Rydberg atoms: 10^{6}

Light-induced interactions

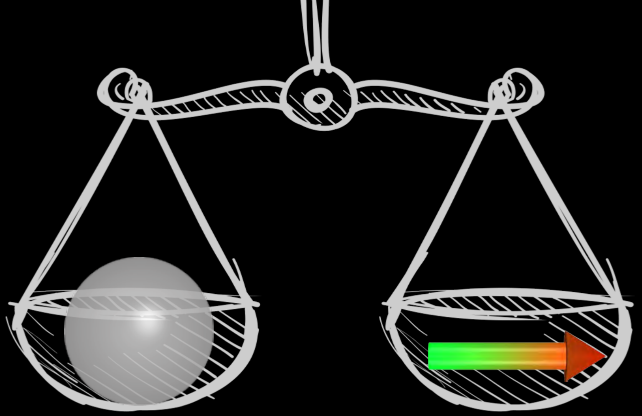
© Haslinger group/ tuwien

Ions

© Harald Ritsch/ uibk

$$\varepsilon_{dd} = \frac{a_{dd}}{a_s}$$

$(a_s = 100a_0)$

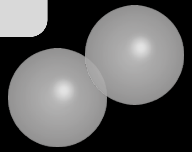


Tunable contact interaction

$$U_c = \frac{4\pi\hbar^2 a}{m}\delta(r)$$

Only effective at short distance

Directionally independent (isotropic)

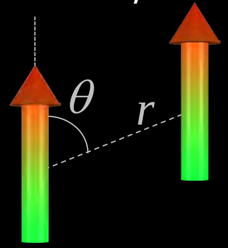


Dipolar interaction

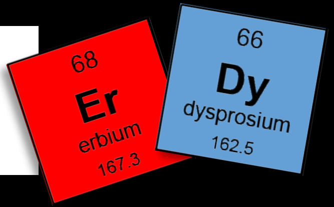
$$V_{\rm dd} = \frac{\mu_0\mu^2}{4\pi}\frac{1-3\cos^2\theta}{r^3}$$

Directional (anisotropic)

Effective at long distance



Ultracold



BECs of highly magnetic atoms now available!

- Dysprosium (Dy), Lev's Group (2011)
- Erbium (Er), Our Group (2012)
- Dysprosium-Erbium Q-mixture, Our Group (2018)

Outline

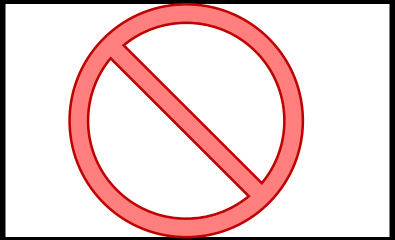
I. Dipolar fermions

II. Dipolar bosons

I. Identical fermions

Interaction and collisions

Scattering in odd partial waves
(*e.g.* l=1, p-wave)



"short-range" fermions
(e.g. alkali as K, Li)

Wigner threshold law for $1/r^n$ potentials

$$\sigma_l \propto \frac{\delta_l^2}{k^2} \qquad V_{\mathrm{vdW}} \propto 1/r^6 \qquad \delta_l = k^{2l+1} \qquad \sigma_{l=1} \propto k^4 \to 0$$

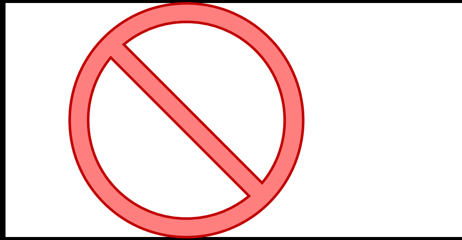
At low T, identical fermions stop colliding

I. Identical fermions

D. Jin, Enrico Fermi Summer School (2006)

Interaction and collisions

Scattering in odd partial waves
(*e.g.* l=1, p-wave)



"short-range" fermions
(e.g. alkali as K, Li)

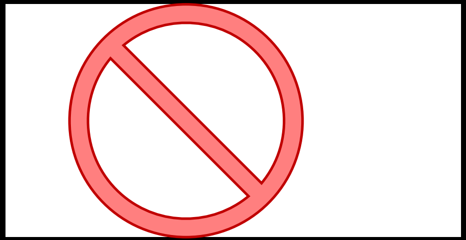
"distinguishable" fermions
(e.g. spin, isotope or heteronuclear mixtures)

I. Identical fermions

D. Jin, Enrico Fermi Summer School (2006)

Interaction and collisions

Scattering in odd partial waves
(*e.g.* l=1, p-wave)



"short-range" fermions
(e.g. alkali as K, Li)

"distinguishable" fermions
(e.g. spin, isotope or heteronuclear mixtures)

B. DeMarco and D. S. Jin, Science (1999)
G. Truscott et al. , Science (2001)
F. Schreck et al., PRL (2001)
G. Roati et al., PRL (2002)
J. M. McNamara et al., PRL (2006)
T. Fukuhara et al., PRL (2007)
B. J. DeSalvo et al., PRL (2010)
M. Lu et al., PRL (2012)
…

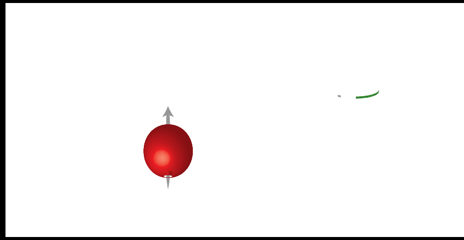
Degenerate Fermi gas of 7 different species

K • Li • ^{3}He • Yb • Sr • Dy • Cr

I. Identical fermions

Interaction and collisions

Scattering in odd partial waves
(*e.g.* l=1, p-wave)



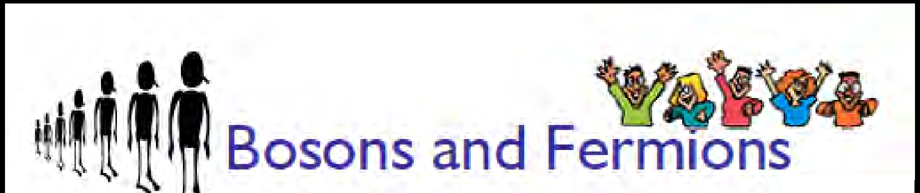
"short-range" fermions
(e.g. alkali as K, Li)

"distinguishable" fermions
(e.g. spin, isotope or heteronuclear mixtures)

"long-range" fermions
(e.g. dipolar fermions)

A third way to Fermi degeneracy

I. Identical fermions

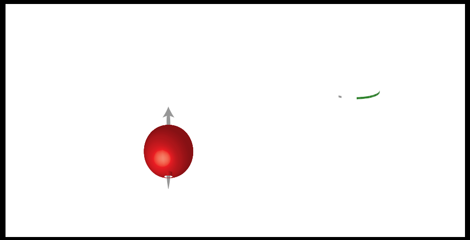


D. Jin, Enrico Fermi Summer School (2006)

Interaction and collisions

Scattering in odd partial waves
(*e.g.* l=1, p-wave)

$$\sigma_l \propto \frac{\delta_l^2}{k^2} \qquad V_{\rm DDI} \propto 1/r^3 \qquad \delta_l = k^{n-1} \qquad \sigma_{l=1} = Cost.$$



$$\sigma_{\rm el} = \frac{16\pi}{30} \times a_{dd}^2 \quad \propto m^2 \mu^4$$

"long-range" fermions
(e.g. dipolar fermions)

Intensive theoretical work: L.D. Landau, E.M. Lifshitz, Quantum Mechanics (1999); B. Deb and L. You, PRA (2001);
C. Ticknor, PRL 100, 133202 (2008) , J. L. Bohn, M. Cavagnero, and C. Ticknor, New J. Phys. 11, 055039 (2009), Paul S. Julienne et al., Phys. Chem. Chem. Phys. 13, 19114 (2011), and many more

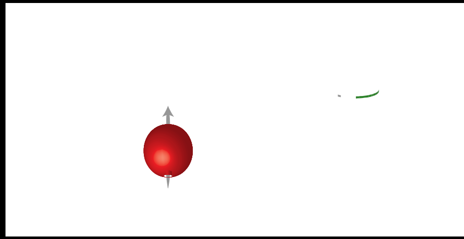
I. Identical fermions

- **Dipolar cooling** via evaporation of identical fermions

$$\sigma_l \propto \frac{\delta_l^2}{k^2} \qquad V_{\mathrm{DDI}} \propto 1/r^3 \qquad \delta_l = k^{n-1} \qquad \sigma_{l=1} = Cost.$$

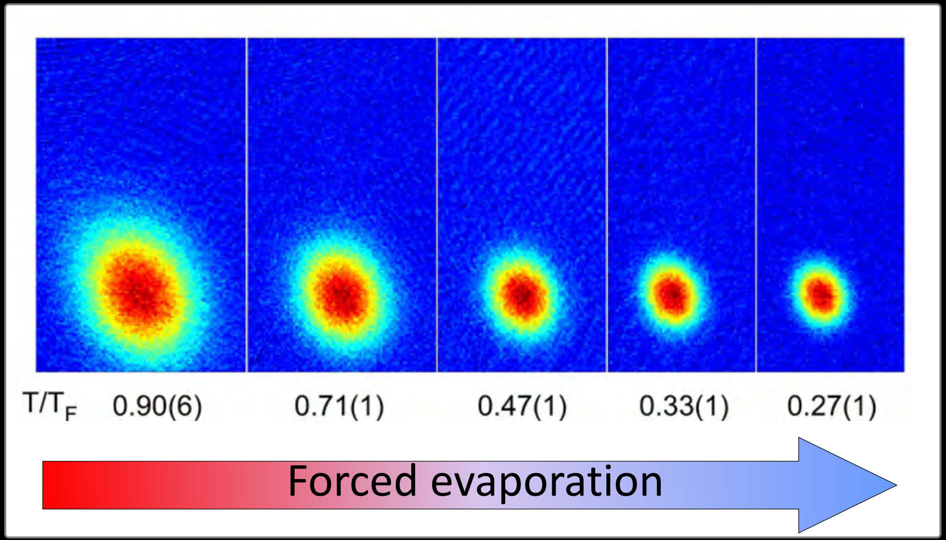
$$\sigma_{\mathrm{el}} = \frac{16\pi}{30} \times a_{dd}^2 \quad \propto m^2 \mu^4$$

- **Many-body dipolar interaction** in spin polarized Fermi gas



"long-range" fermions
(e.g. dipolar fermions)

Dipolar cooling of identical fermions



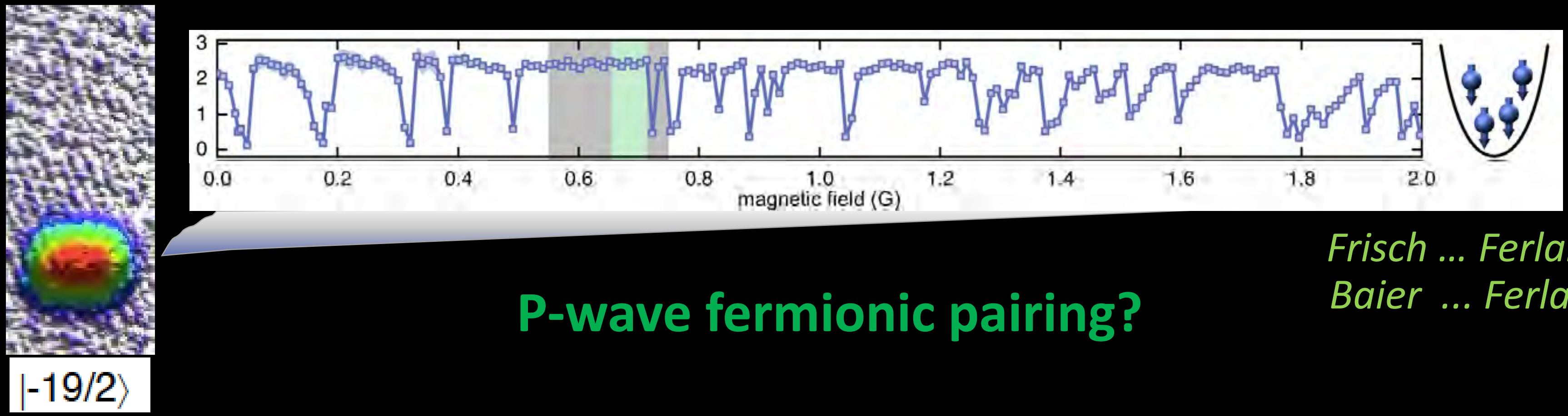
The peak density is a factor of two higher that the one in our BEC!!

$n = 4 \times 10^{14}\ cm^{-3}$

$N = 3 \times 10^{4}$

$\mathbf{T/T_F \approx 0.11}$

Er dFg: *K. Aikawa, et al. PRL **112** (2014)*

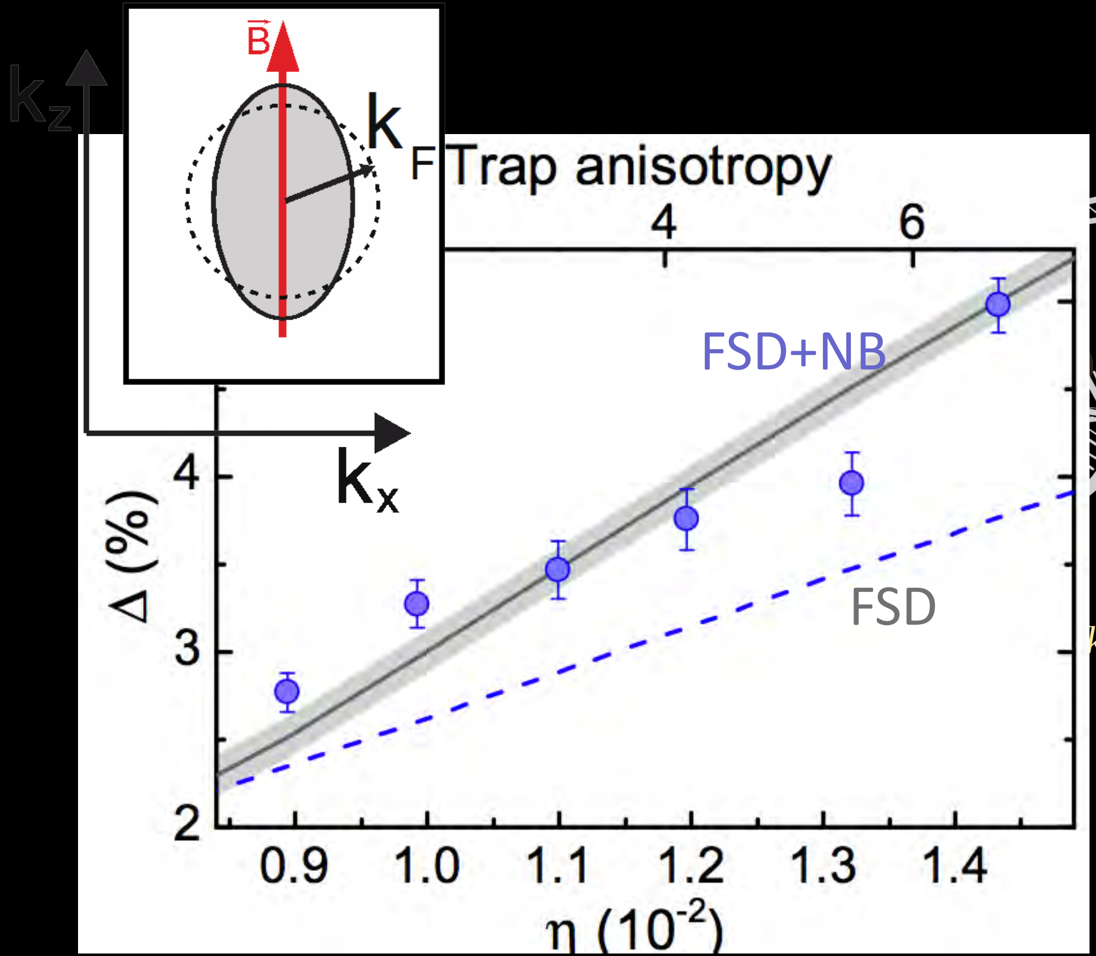


Frisch … Ferlaino, Nature, 507, 475-479, 2014
Baier … Ferlaino PRL (2018)

P-wave fermionic pairing?

*Many-body interaction between **identical dipolar** fermions*

Fermi surface



kin and nd^2

K. Aikawa, Science 345 (2014)
Veljic et al., New. J. Phys. (2018)

Very intense theoretical work

T. Miyakawa et al., PRA (2008)◆J.-N. Zhang and S. Yi, PRA (2009) ◆T. Sogo et al. New J. Phys. (2009)◆S. Ronen and J. Bohn, PRA PRA (2010)◆C.-K. Chan, C. Wu, W.-C. Lee, and S. Das Sarma, PRA (2010)◆U. Baillie and P. Blakie, PRA (2010)◆ F. Wächtler, et al., arXiv:1311.5100 (2013)◆… *and many more*

Outline

I. Dipolar fermions

II. Dipolar bosons

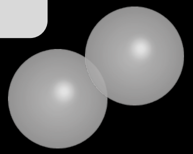
II. Dipolar bosons

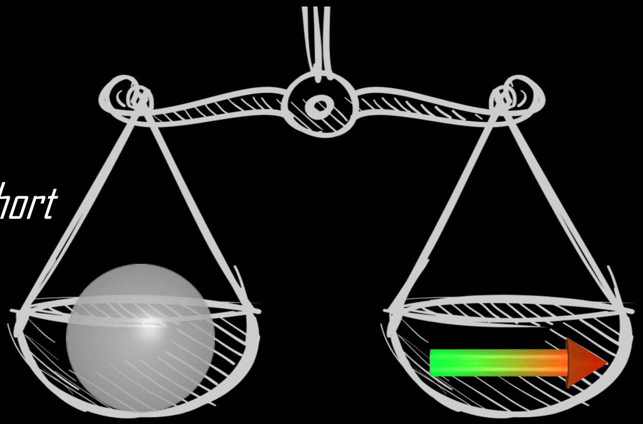
Tunable contact interaction

$$U_c = \frac{4\pi\hbar^2 a}{m}\delta(r)$$

Only effective at short distance

Directionally independent (isotropic)



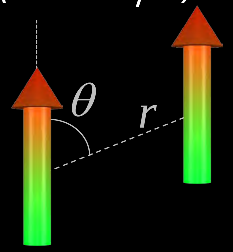


Dipolar interaction

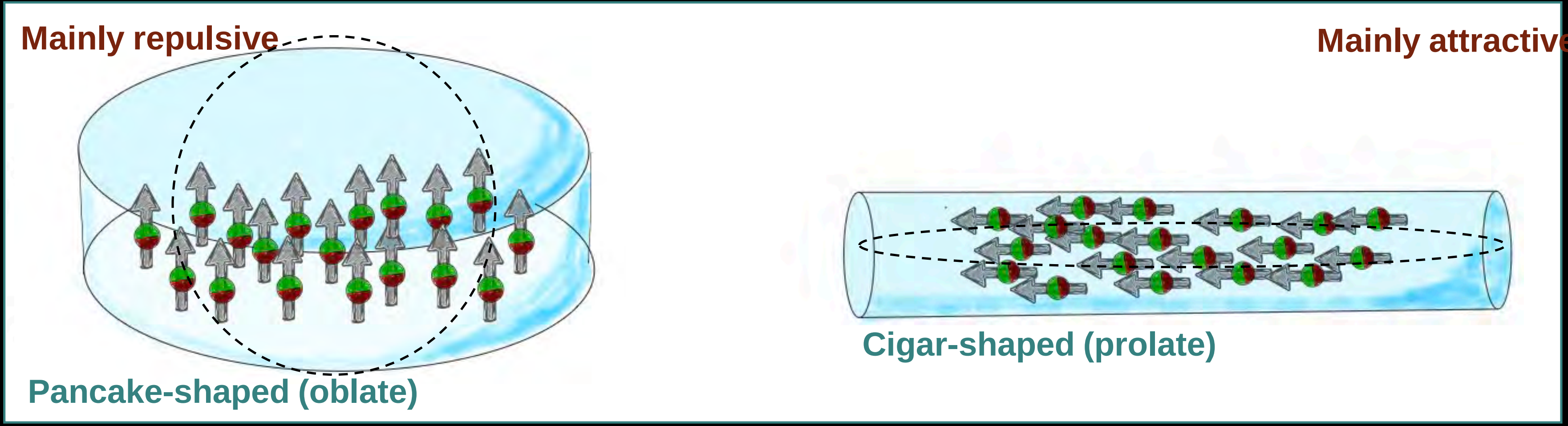
$$V_{\mathrm{dd}} = \frac{\mu_0 \mu^2}{4\pi}\frac{1 - 3\cos^2\theta}{r^3}$$

Directional (anisotropic)

Effective at long distance



Trap as the integration volume



Magnetostriction

Cr Exp. Bosons: Stuhler... Pfau, J. of Magn. Magn. Mater. **316**, 429, (2007)

Er Exp. Fermion: Aikawa…Ferlaino, Science (2014)

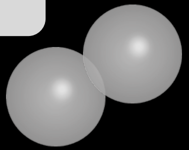
II. Dipolar bosons

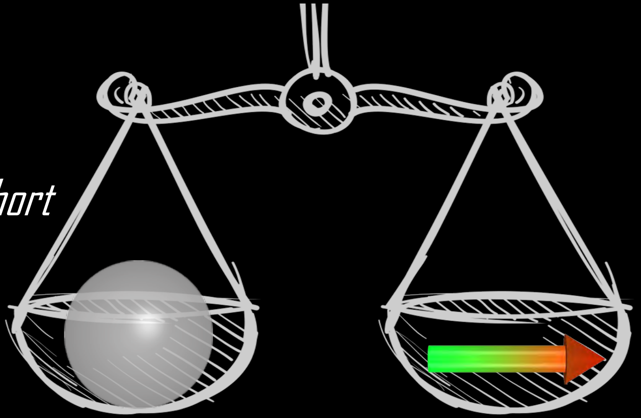
Tunable contact interaction

$$U_c = \frac{4\pi\hbar^2 a}{m}\delta(r)$$

Only effective at short distance

Directionally independent (isotropic)



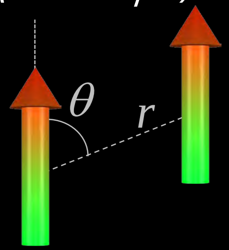


Dipolar interaction

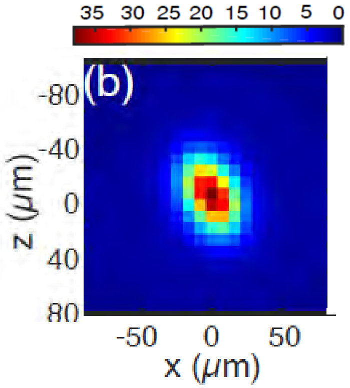
$$V_{dd} = \frac{\mu_0\mu^2}{4\pi}\frac{1-3\cos^2\theta}{r^3}$$

Directional (anisotropic)

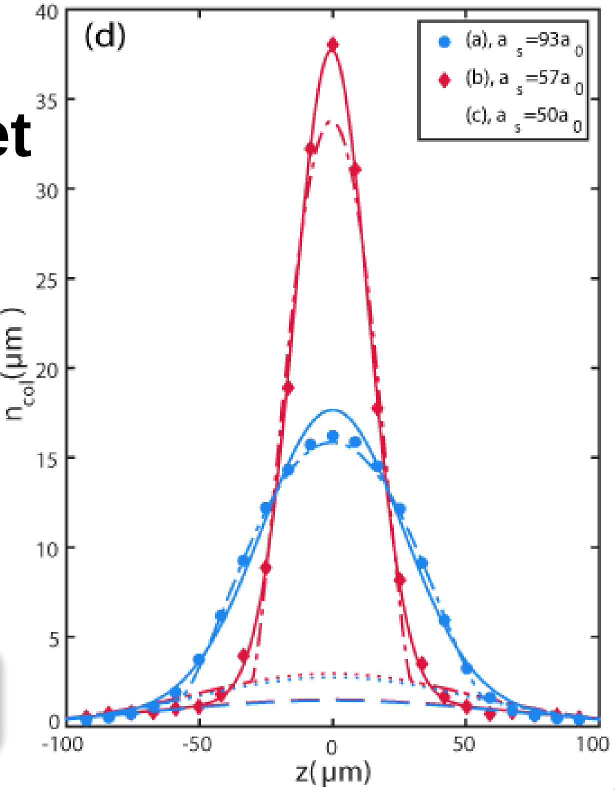
Effective at long distance

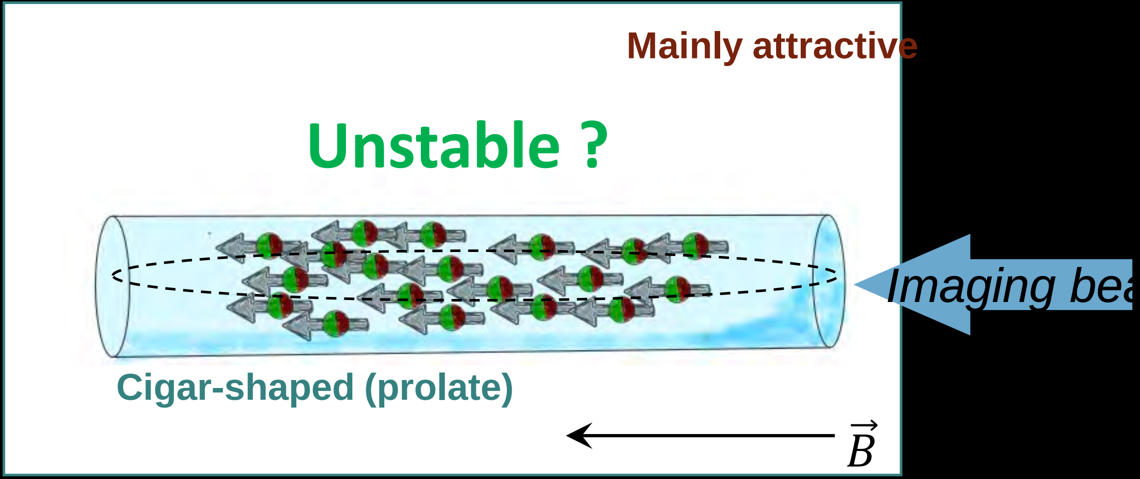


New Phase: Macrodroplets



ε_{dd} =1.15





Chomaz, ... Ferlaino PRX 6 (4), 041039 (2016)

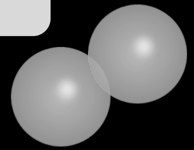
II. Dipolar bosons

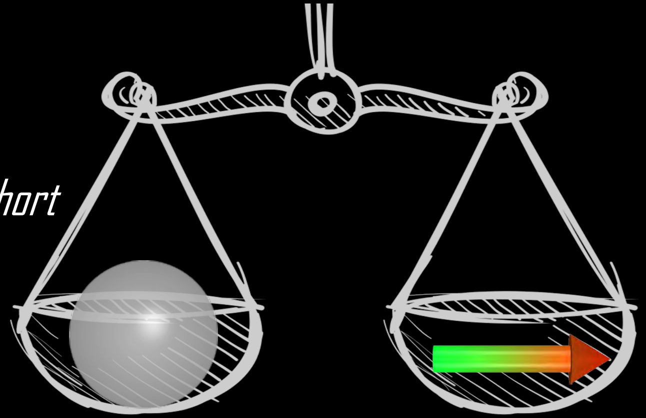
Tunable contact interaction

$$U_c = \frac{4\pi\hbar^2 a}{m}\delta(r)$$

Only effective at short distance

Directionally independent (isotropic)

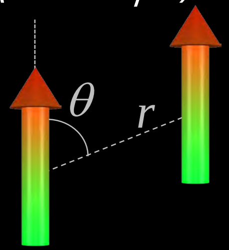




Dipolar interaction

$$V_{dd} = \frac{\mu_0 \mu^2}{4\pi}\frac{1 - 3\cos^2\theta}{r^3}$$

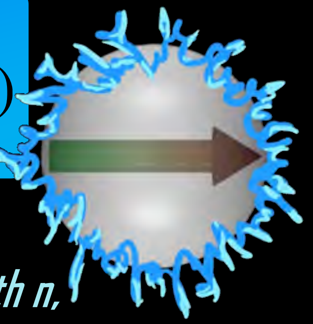
Directional (anisotropic)



Effective at long distance

+

$$\frac{128\sqrt{\pi}\hbar^2 a_s}{3m} n(\mathbf{r})\sqrt{n(\mathbf{r})a_s^3}F(\epsilon_{dd})$$



Quantum fluctuations

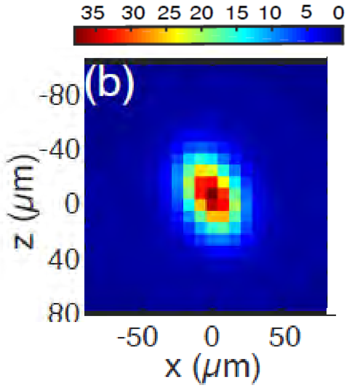
Only repulsive, rapid growth with n, effective at short distance, isotropic

A. Lima and A. Pelster, PRA 84, 041604(R) (2011)
A. Lima and A. Pelster, PRA 86, 063609 (2012)

New Phase: Macrodroplets

Experimental works:

Dipolar Gases - Stuttgart (© Pfau) & Innsbruck (© Ferlaino)
Non-dipolar mixture: ICFO (© Tarruell) & LENS (© Fattori) & Palaiseau (© Bourdel)



D. S. Petrov, PRL 115, 155302 (2015)
Kandau et al., Nature (2016)
L. Chomaz, & al. PRX 6 (4), 041039 (2016)
I. Ferrier-Barbut, & al. PRL 116, 215301 (2016)
F. Wächtler & L. Santos, PRA 93, 061603 (2016)
R. N. Bisset, & al. PRA 94, 033619 (2016)
F. Wächtler & L. Santos, PRA 94, 043618 (2016)
D. Baillie, & al. PRA 94, 021602(R) (2016)....

L. Chomaz ... Ferlaino PRX 6 (2016)

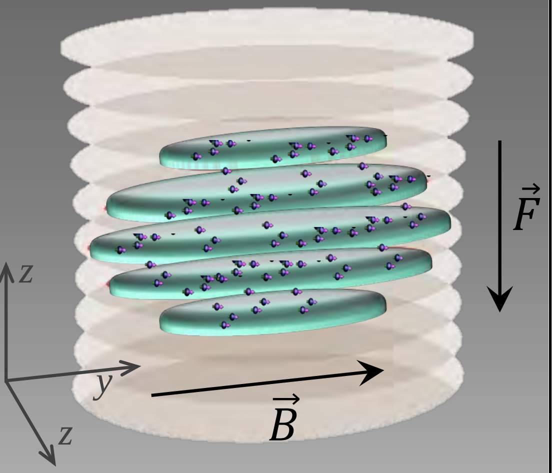
It‘s a density regulator!

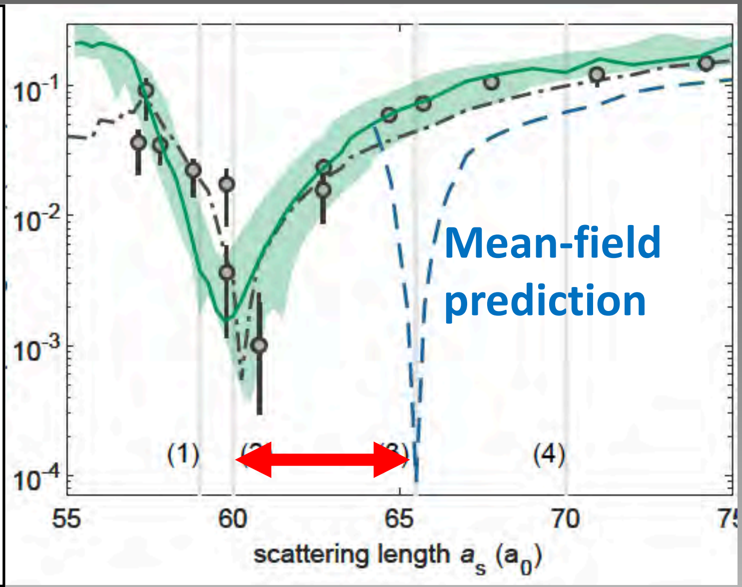
D. S. Petrov, PRL 115, 155302 (2015)

Interferometry of Quantum Fluctuations

Bloch oscillations (non dipolar: Salomon (1996), Arimondo (2001), Inguscio/Modugno (2004), Tino (2006), Kasevich (2007), Nägerl (2008), ...)

Array of 2D systems





Natale, ..Mark, Ferlaino arXiv:2205.03280 (2022)
Comm. Phys 5, 227 (2022)

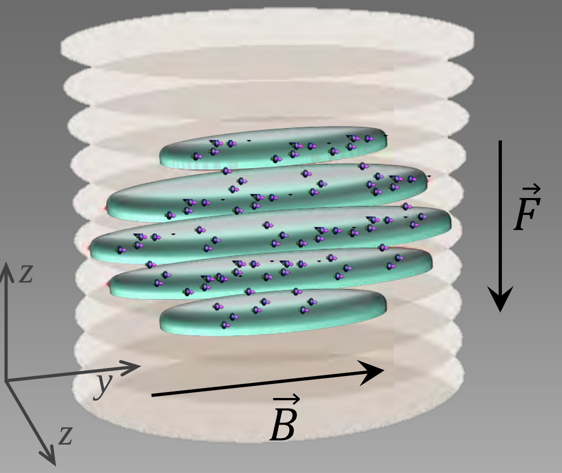
Direct measurement of Quantum Fluctuation

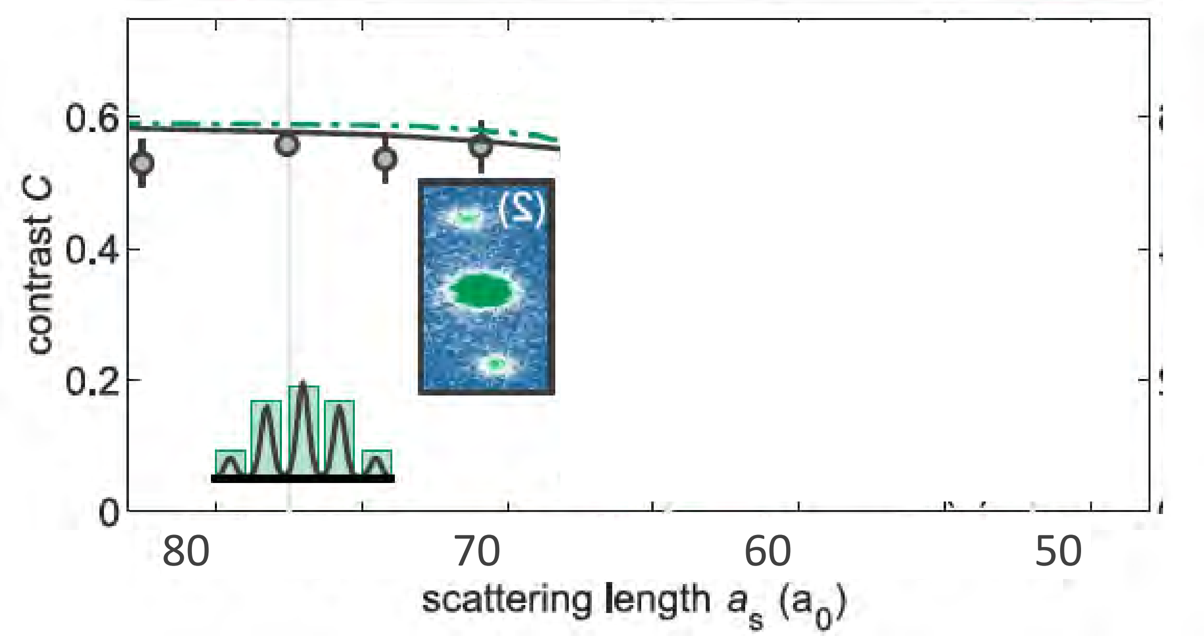
Roati .. Inguscio PRL **92** (2004)
Ferrari ...Tino, PRL **97**, 060402 (2006)
Li .... Kasevich PRL **98** (2007)

M.Gustavsson ... Naegerl, PRL **100**, 080404 (2008)
Fattori ... Modugno, PRL **101**, 190405 (2008)
.....

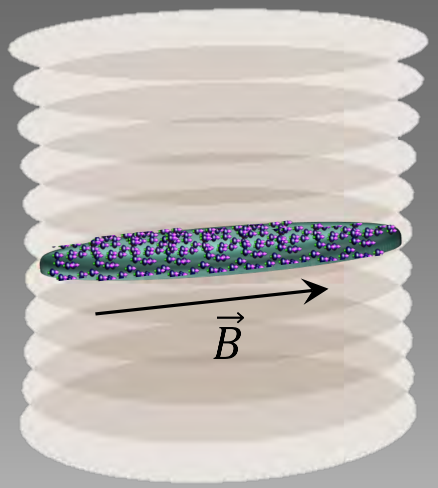
Interferometry of Quantum Fluctuations

Array of 2D systems





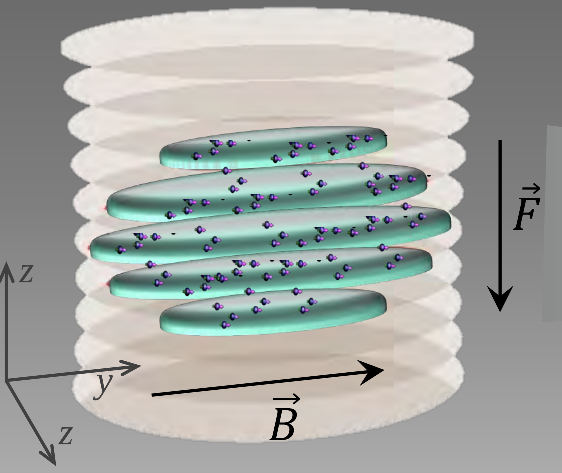
Single 2D plane



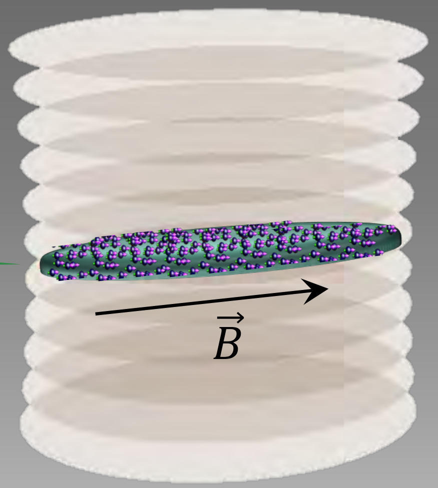
Natale, ..Mark, Ferlaino, Comm. Phys 5, 227 (2022)

Interferometry of Quantum Fluctuations

Array of 2D systems



Single 2D plane



SELF-FOCUSING in a single lattice plane! by entering in the **macrodroplet** regime

New way to create dense 2D systems !!

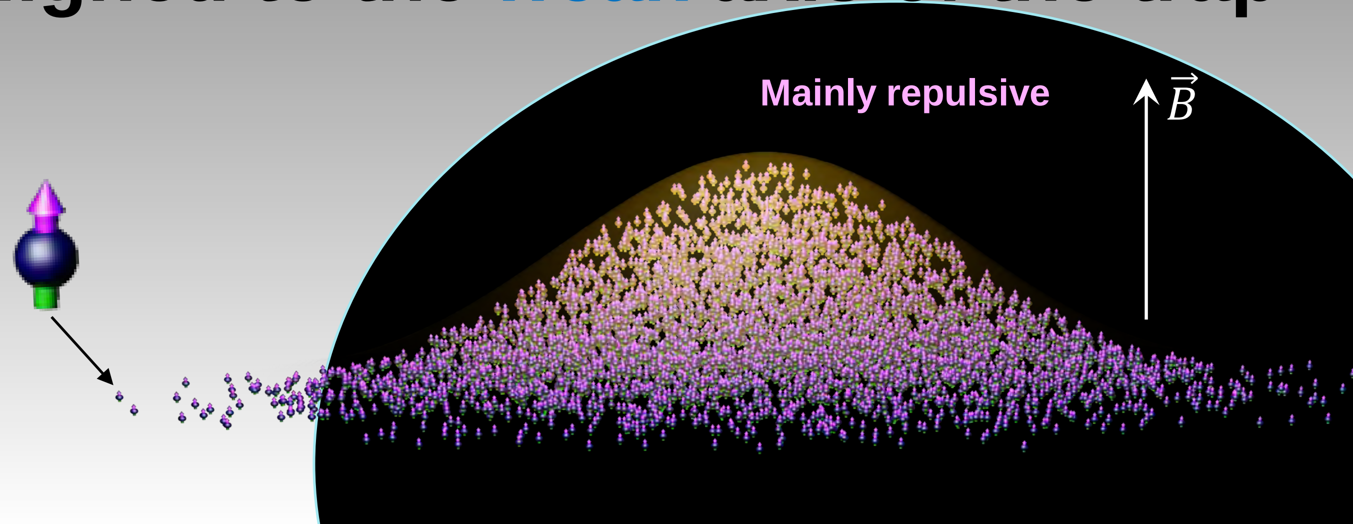
Natale, ..Mark, Ferlaino, Comm. Phys 5, 227 (2022)

$\vec{B}$

$\vec{B}$

aligned to the **strong** axis of the trap

Magnetic field aligned to the weak axis of the trap



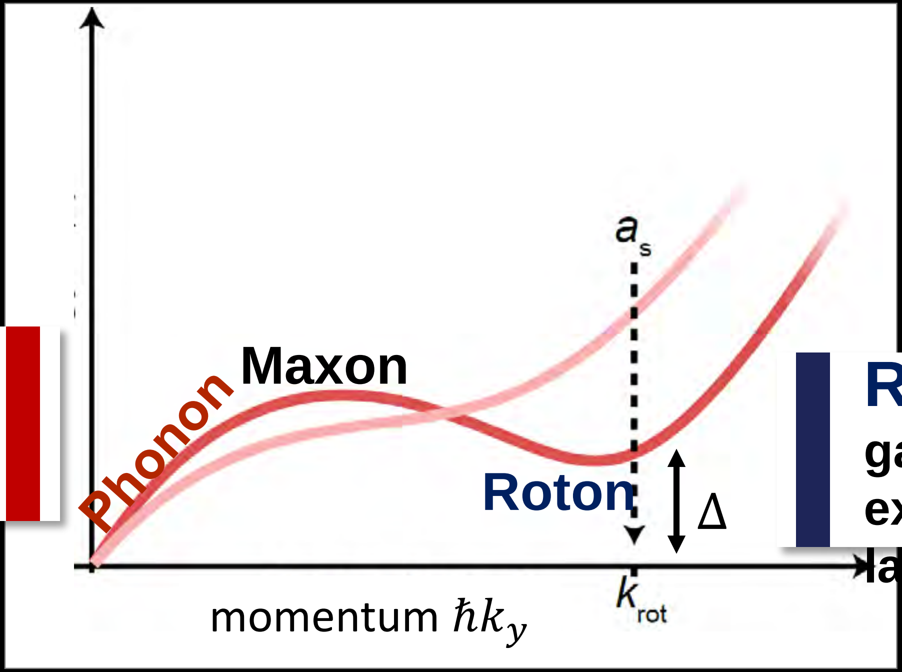
*Dipolar interaction as **k-dependent** interaction*

$$\epsilon(\mathbf{k}) = \sqrt{\frac{\hbar^2\mathbf{k}^2}{2m}\left(\frac{\hbar^2\mathbf{k}^2}{2m} + 2nV_{int}(\mathbf{k})\right)}$$

- **Superfluid phase and excitation become k-dependent (& anisotropic)**

THEORY

Phonon regime (linear): long-wave length excitations, i.e. sound



Santos, Shlyapnikov, Lewenstein PRL 90 (2003)

Roton regime (quadratic; gapped): Short wavelength excitations at the reciprocal

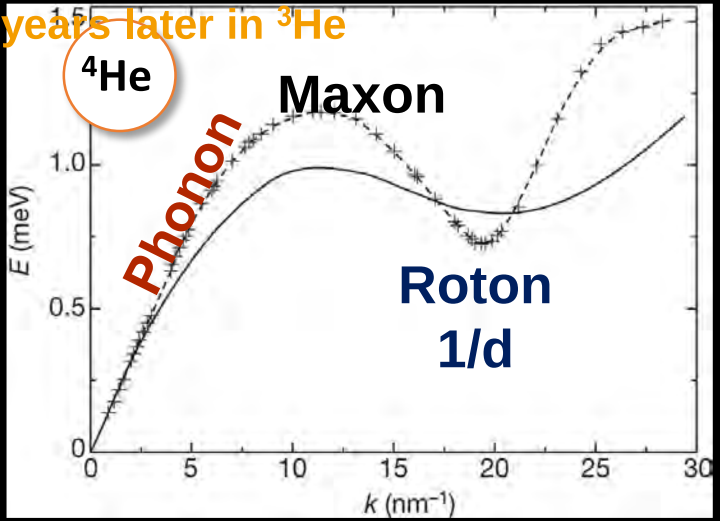
Dipolar interaction as **k-dependent** interaction

$$\epsilon(\mathbf{k}) = \sqrt{\frac{\hbar^2\mathbf{k}^2}{2m}\left(\frac{\hbar^2\mathbf{k}^2}{2m} + 2nV_{int}(\mathbf{k})\right)}$$

- **Superfluid phase and excitation become k-dependent (& anisotropic)**

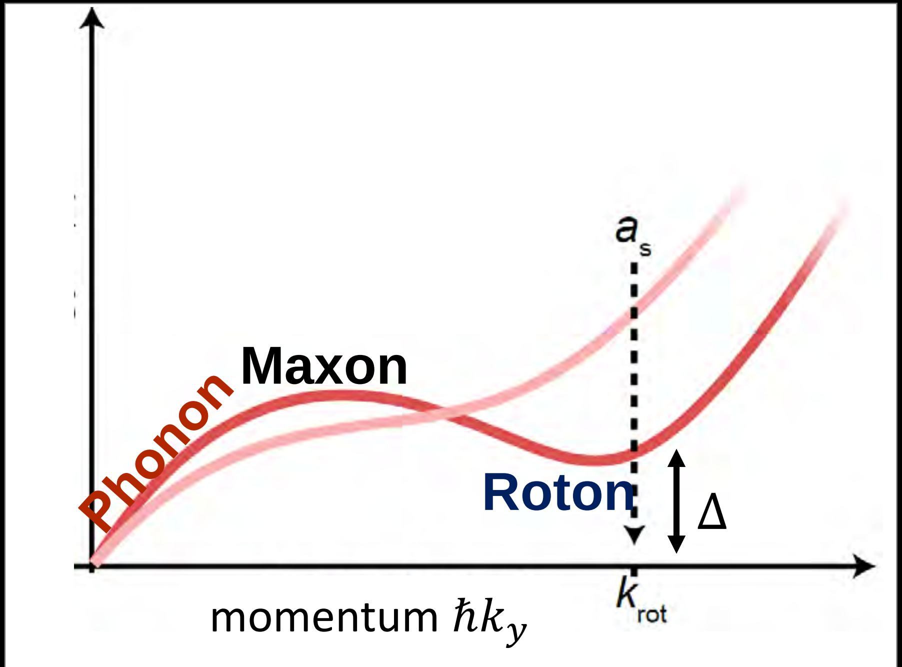
Predicted in 1947 by Landau in He superfluid

Observed 50 years later in ^{4}He and 65 years later in ^{3}He



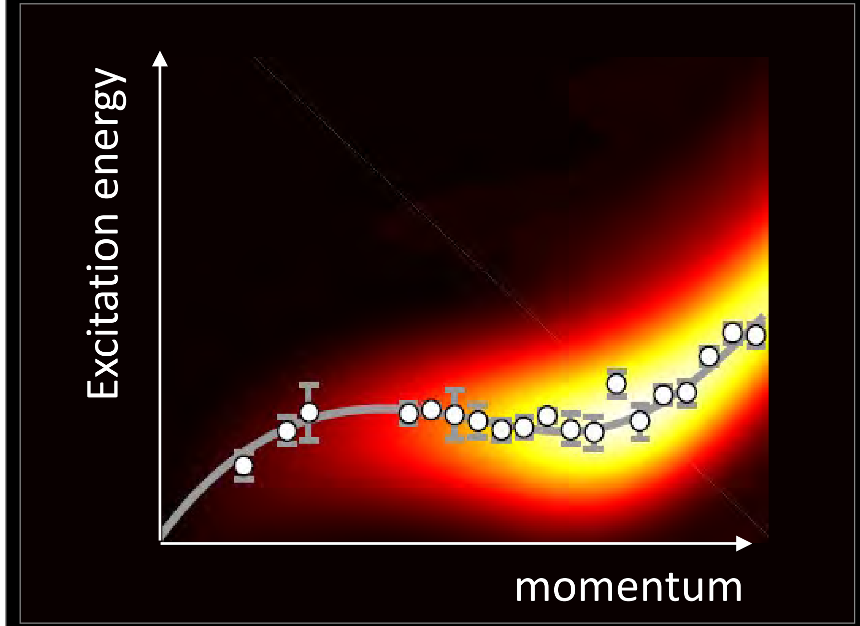
Feynman, R. P. in Progress in Low Temperature Physics Vol. 1 (1955)
Glyde, H. R. Excitations in Liquid and Solid Helium (Clarendon, 1994)

THEORY



Santos, Shlyapnikov, Lewenstein PRL 90 (2003)

EXPERIMENT



Petter ... Ferlaino. PRL 122 (2019)

We actually observed one year before (2018)..

Sposta in...

Lev Pitaevskiy

From Lev Pitaevskii

A: Francesca.Ferlaino@uibk.ac.at

Entrata - FF@UIBK 22 marzo 2018 14:02

LP

Dear Francesca,

I read with great pleasure your beautiful paper about Roton in "Nature". In this relation I would like to attract your attention to two our old papers. Maybe you can observe something in future experiments.

Best regards. Lev.

S. V. Iordanskii and L. P. Pitaevskii, Sov. Phys. Usp. **23**, 317 (1980).

L. P. Pitaevskii, JETP Lett. 39, 511 (1984).

Layered structure of superfluid ^{4}He with supercritical motion

L. P. Pitaevskiĭ

Institute of Physical Problems, Academy of Sciences of the USSR

(Submitted 19 March 1984)

Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 9, pp. 423–425 (10 May 1984)

It is shown that when superfluid ^{4}He flows along a capillary with a velocity exceeding Landau's critical roton velocity, a one-dimensional periodic structure, which is at rest relative to the walls, appears in the helium and the spectrum of excitations is deformed so that the criterion of superfluidity is not violated.

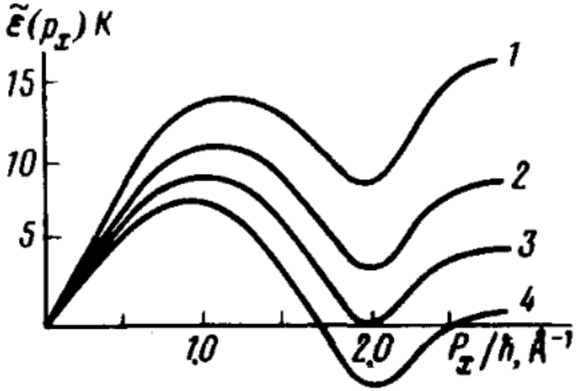


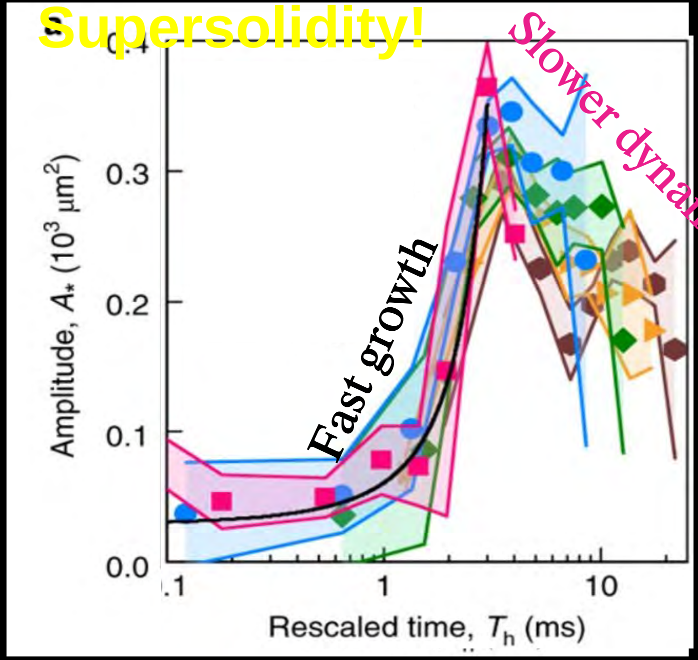
FIG. 1.

… one year before …Tendency towards **crystallization**

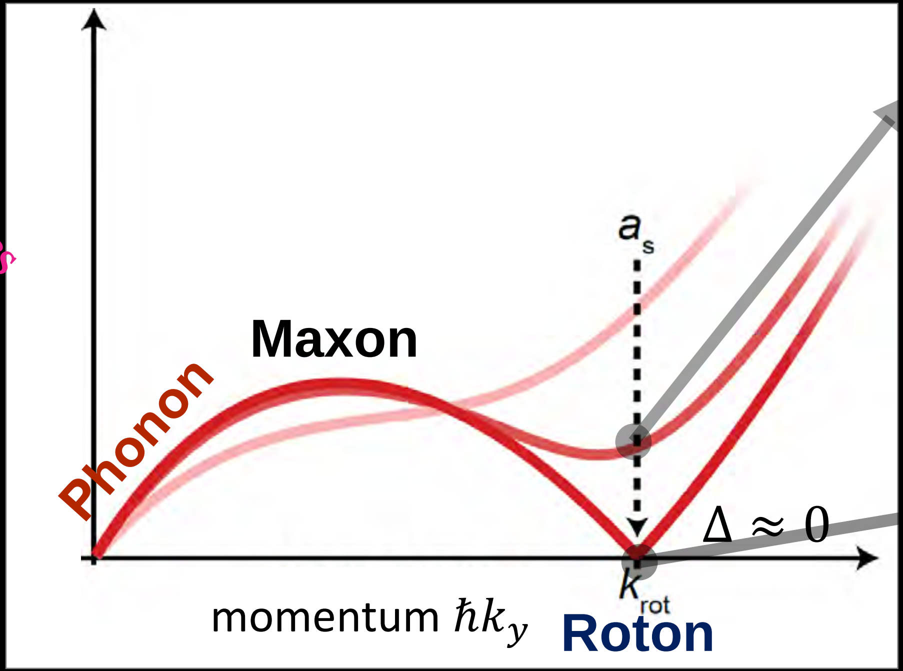
$$\epsilon(\mathbf{k}) = \sqrt{\frac{\hbar^2\mathbf{k}^2}{2m}\left(\frac{\hbar^2\mathbf{k}^2}{2m} + 2nV_{int}(\mathbf{k})\right)}$$

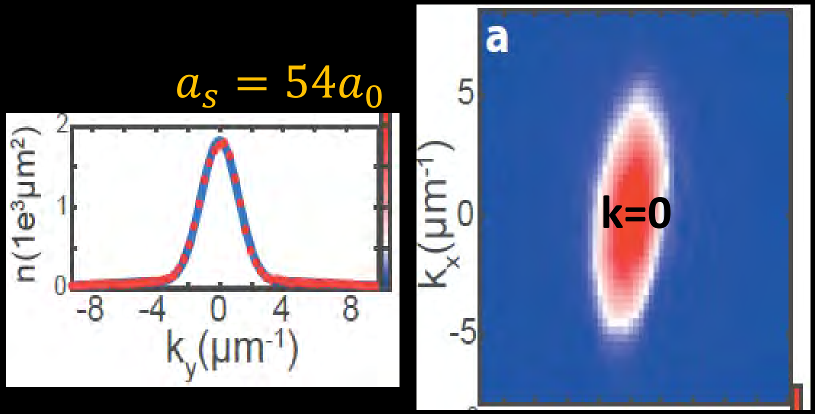
- **Superfluid phase and excitation become k-dependent (& anisotropic)**

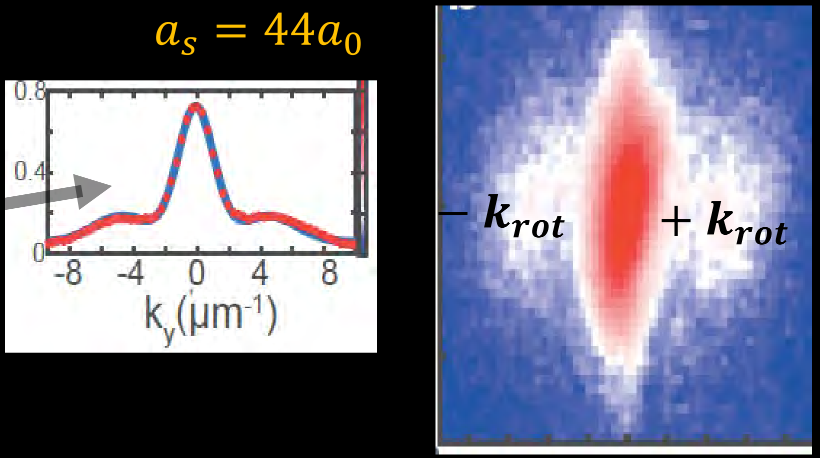
Early Hints of Supersolidity!



THEORY



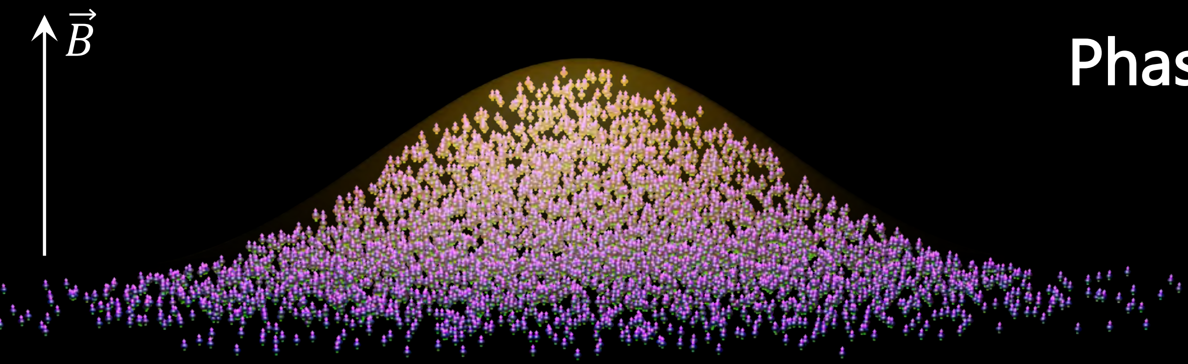




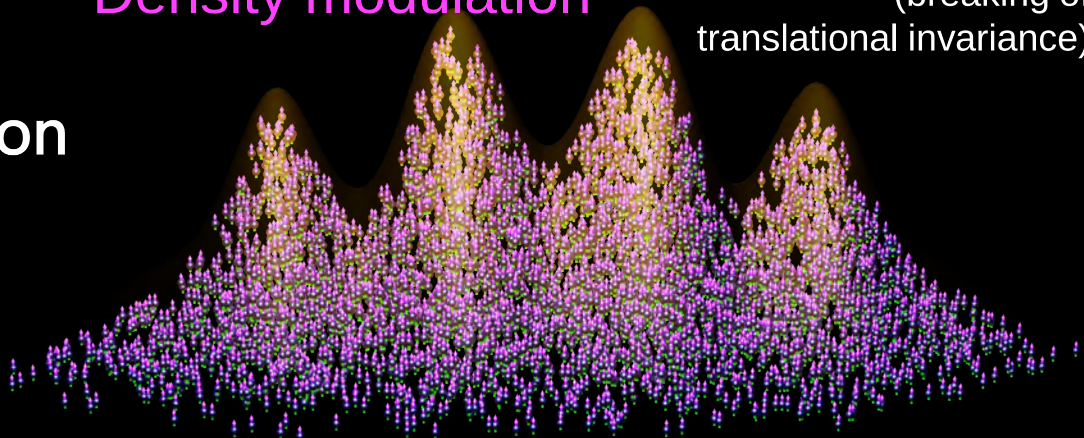
Chomaz … Ferlaino, Nat. Phys. 14 (2018)

Dipolar Supersolid

Dipolar BEC



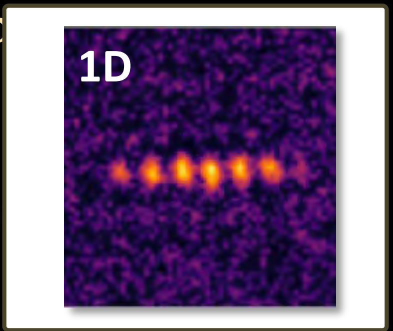
Phase transition →

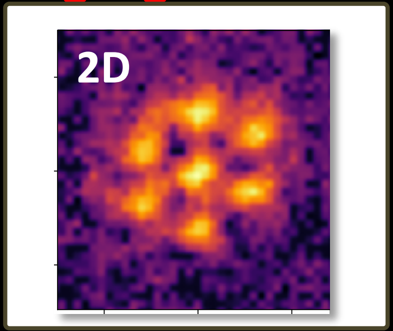


Density modulation (breaking of translational invariance)

Global phase coherence (breaking of gauge invariance)

In-situ imaging





1D: Simultaneous works in dipolar BECs b

Böttcher … Pfau, PRX 9 (2019)

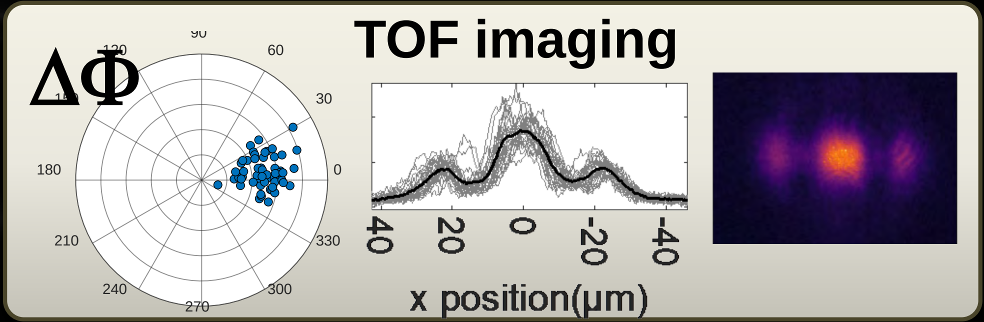
Tanzi … PRL 122 (2019)

Chomaz … Ferlaino PRX 9 (2019)

Now in 2D !

Norcia … Ferlaino, Nature 596 (2021)

TOF imaging



See connected works in Atom-light coupled systems

SOC BECs Ketterle Group: Li et al., Nature (2017)

Atom-cavity experiments : J Léonard et al., Nature (2017)

More from Innsbruck:

Natale et al., PRL 123 (2019)

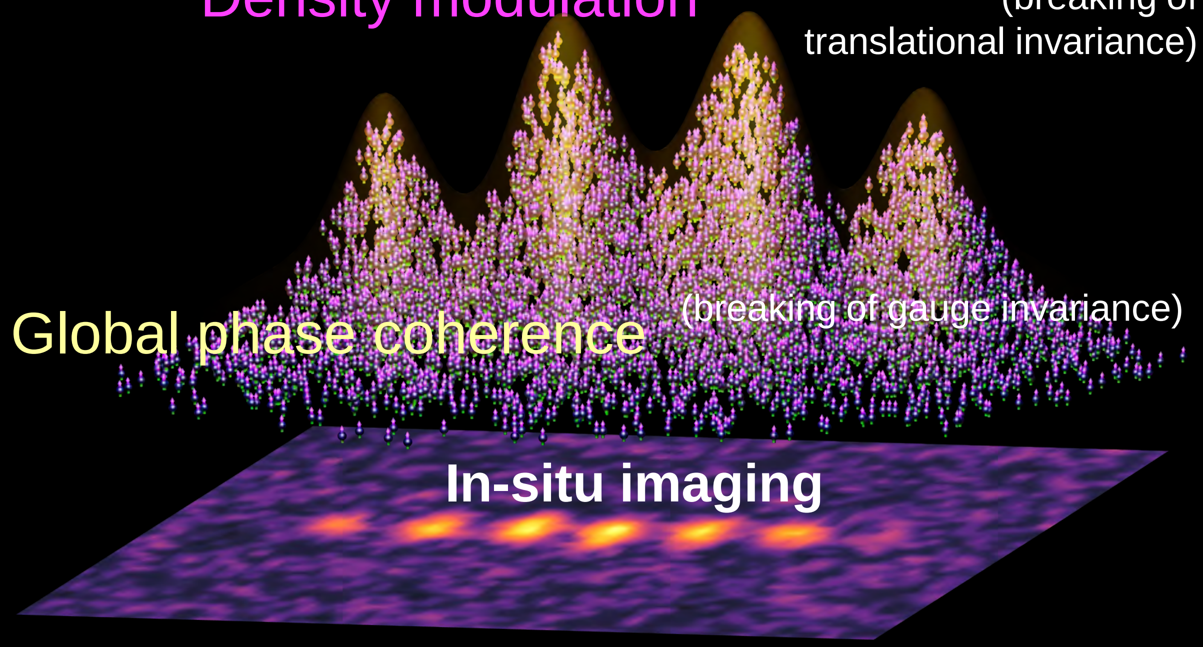
Petter et al., PRA (2021)

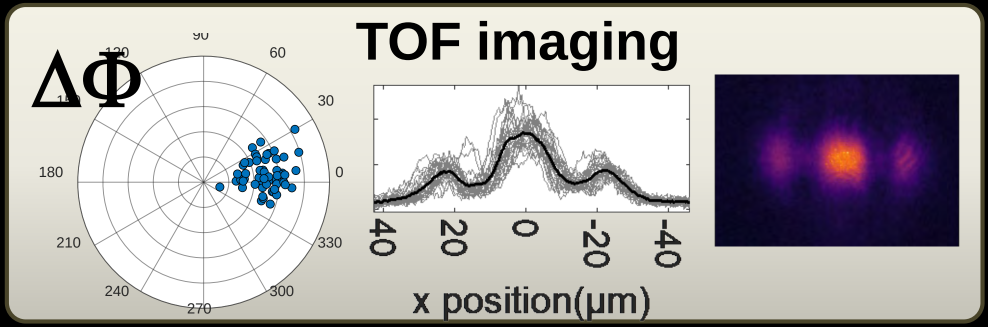
Ilzhoefer et al. Nat. Phys. 17 (2021)

Sohmen et al. Phys. Rev. Lett. 126 (2021)

Norcia et al, PRL (2022)

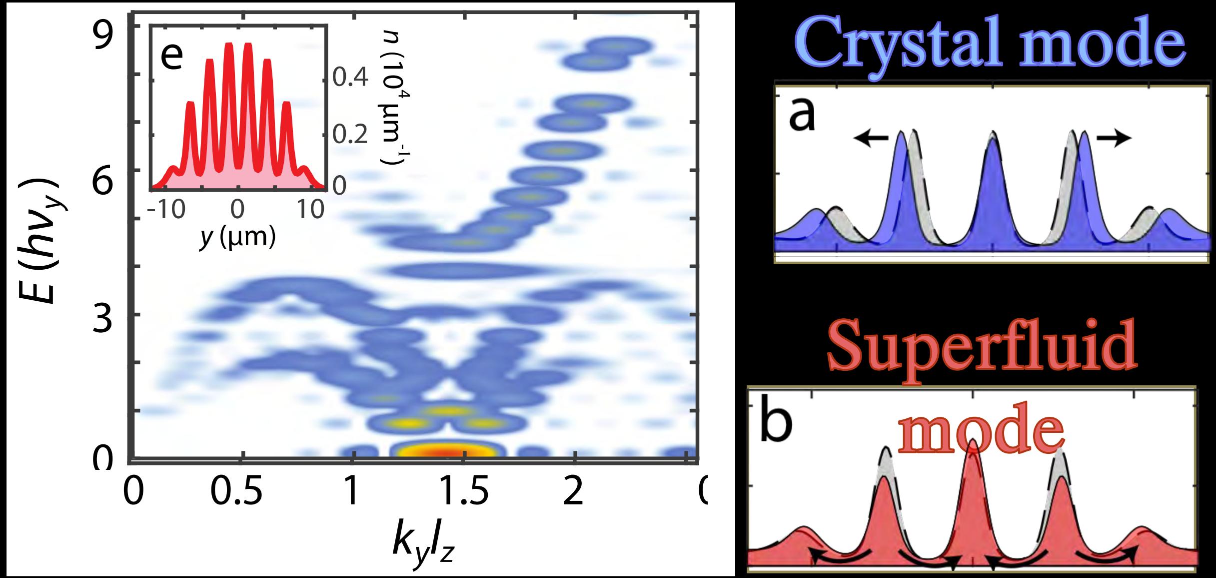
Symmetry breaking





$$P_i = \rho_i \exp\{-i\Phi_i\}$$

Excitation spectrum



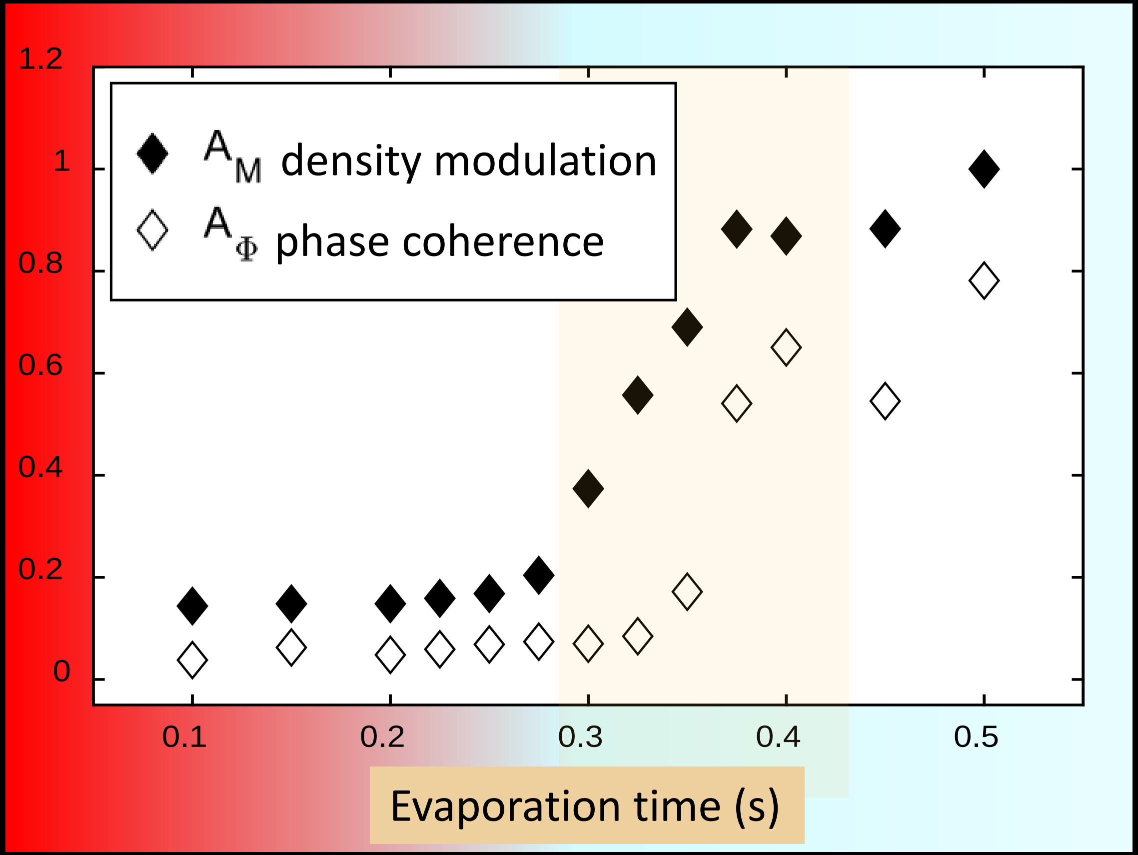
G. Natale …Ferlaino, Phys. Rev. Lett. 123 (2019)

See also:
Tanzi..Modugno, Recati, Stringari, Nature 574 (2019)
Guo ..Pfau Nature 574 (2019)

Symmetry breaking

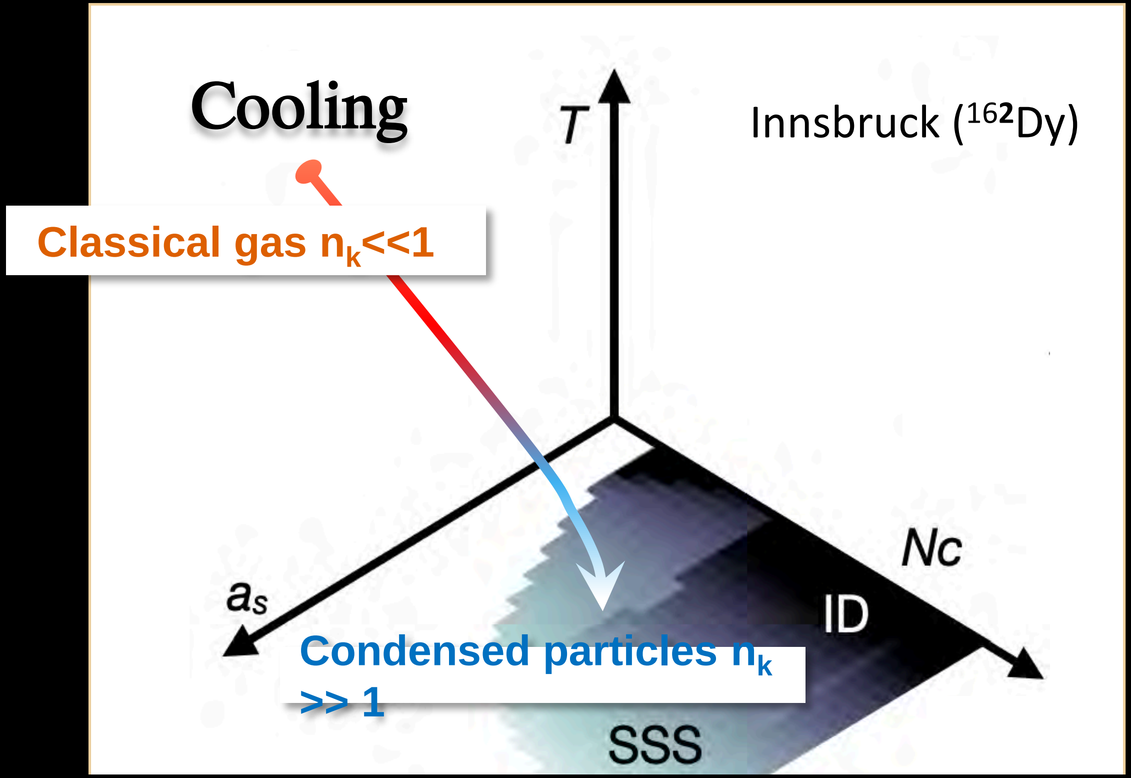
Which symmetry breaks first?

(phase coherence or density modulation ?)



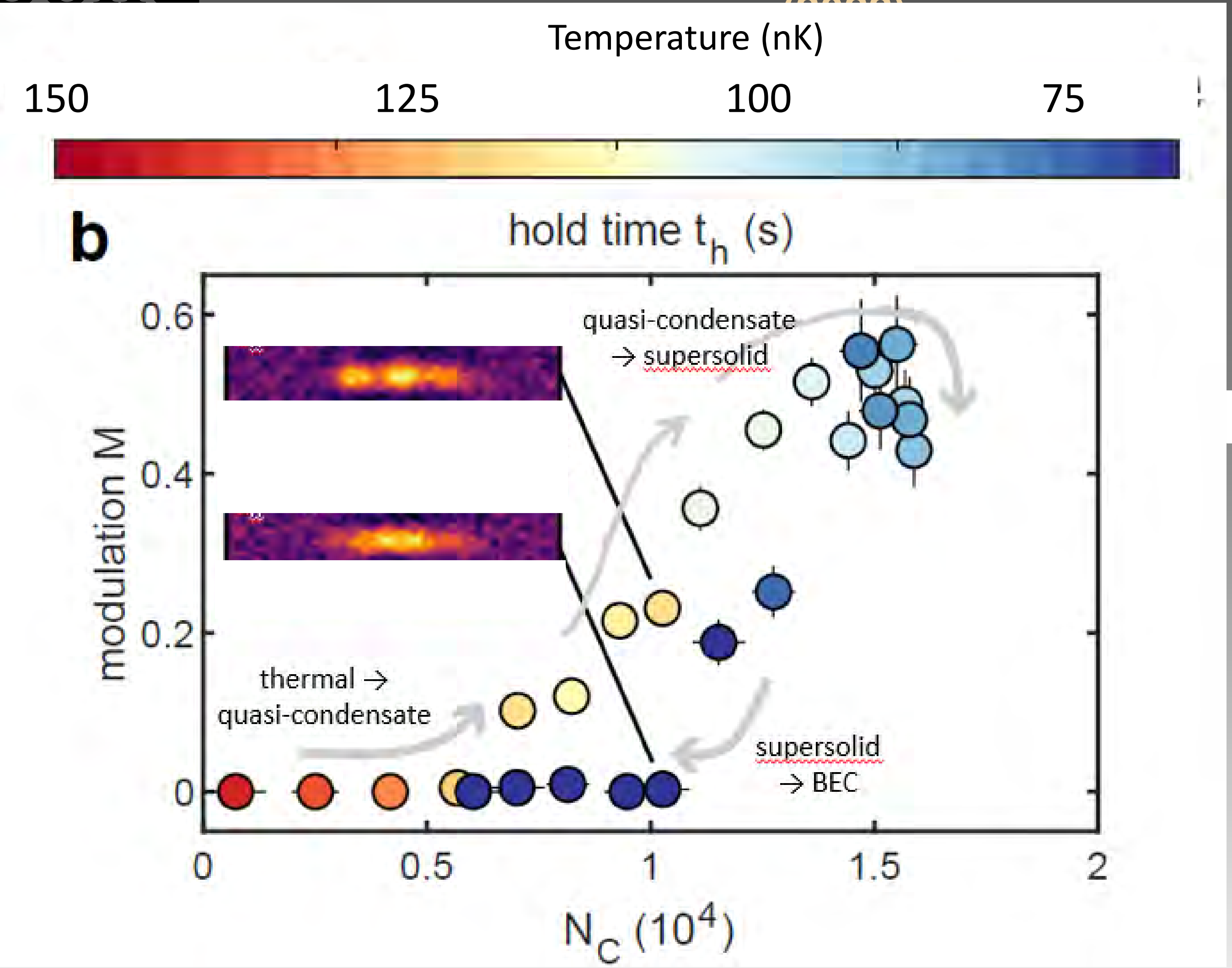
Sohmen … Ferlaino, Phys. Rev. Lett. 126 (2021)

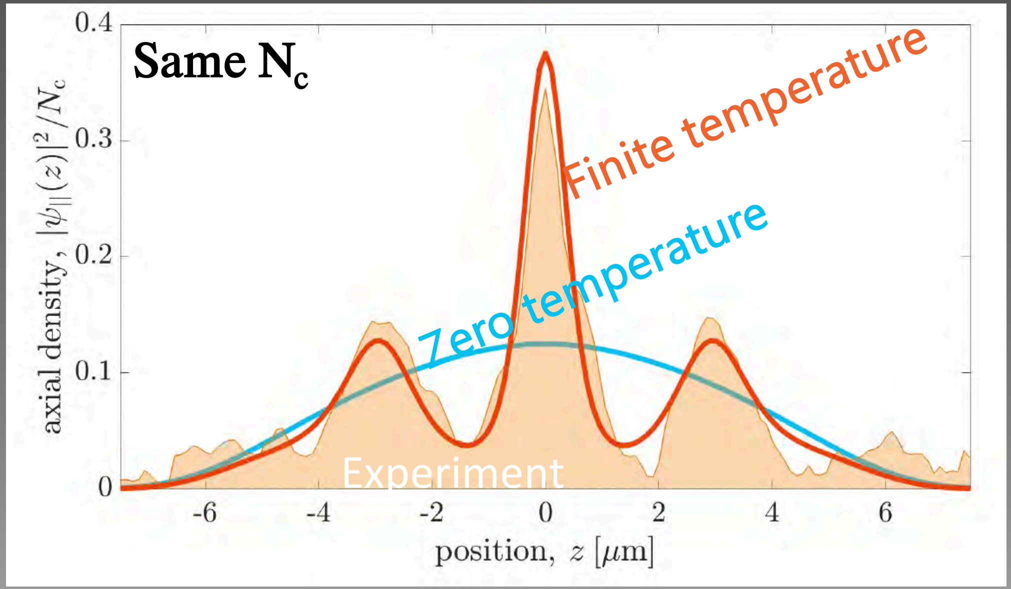
From a thermal cloud to a Supersolid

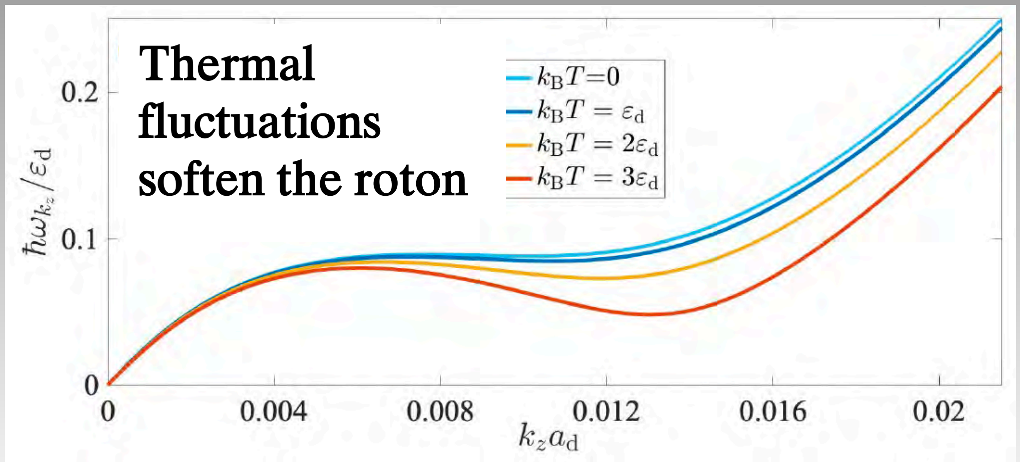


Heating a Quantum Fluid into a Solid

J. Sanchez-Baena, C. Politi, F. Maucher, F. Ferlaino, and T. Pohl, arXiv (2023)



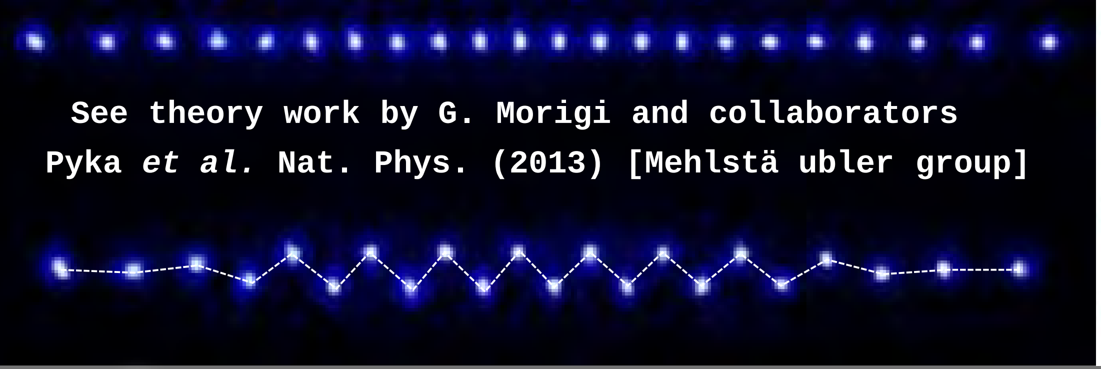




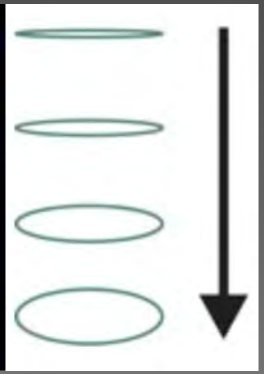
2D Supersolidity

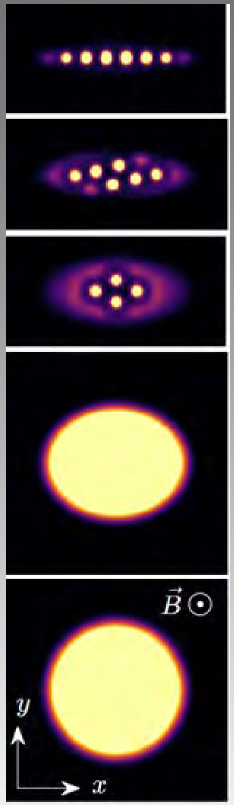
Inspired by..

Structural quantum phase transition in ionic coulomb crystals: linear to zig-zag



See theory work by G. Morigi and collaborators
Pyka *et al.* Nat. Phys. (2013) [Mehlstä ubler group]





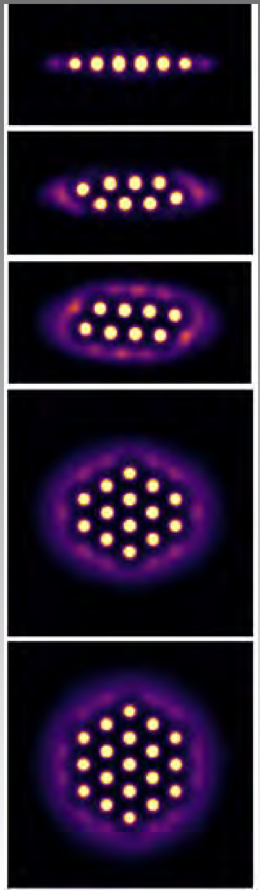
$f_y = 110\text{Hz}$

$f_y = 84.6\text{Hz}$

$f_y = 60\text{Hz}$

$f_y = 40\text{Hz}$

$f_y = f_x = 33\text{Hz}$



$N = 63\text{k}$

$N = 81.9\text{k}$

$N = 115.5\text{k}$

$N = 173.25\text{k}$

$N = 210\text{k}$

^{164}Dy atoms eGPE ground-state solutions

scale up at constant "trap density"

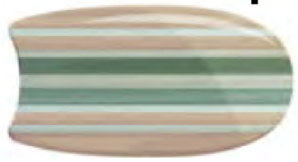
$\varrho = N f_x f_y$

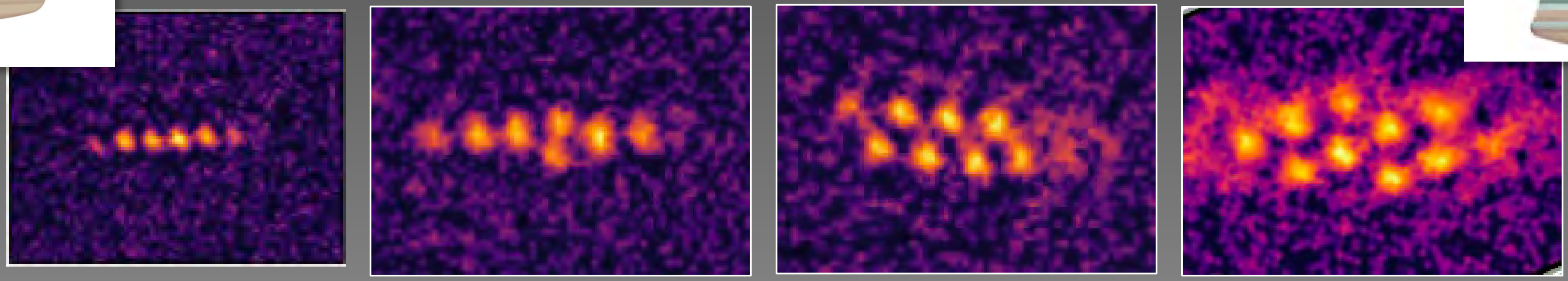
E. Poli,..., F. Ferlaino, PRA 104, 063307 (2021)

2D Supersolidity

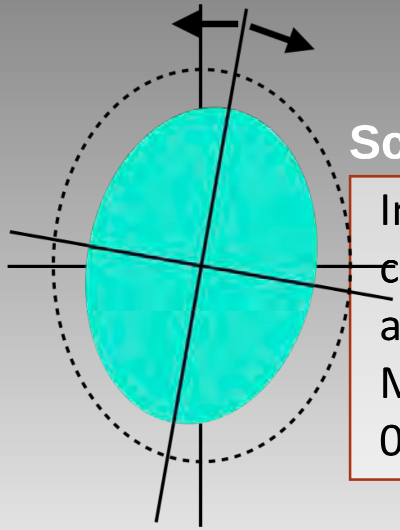
Cigar trap

Surfboard trap





(αH0.29) (αH0.35) (αH0.4) (αH0.43)



Scissors mode

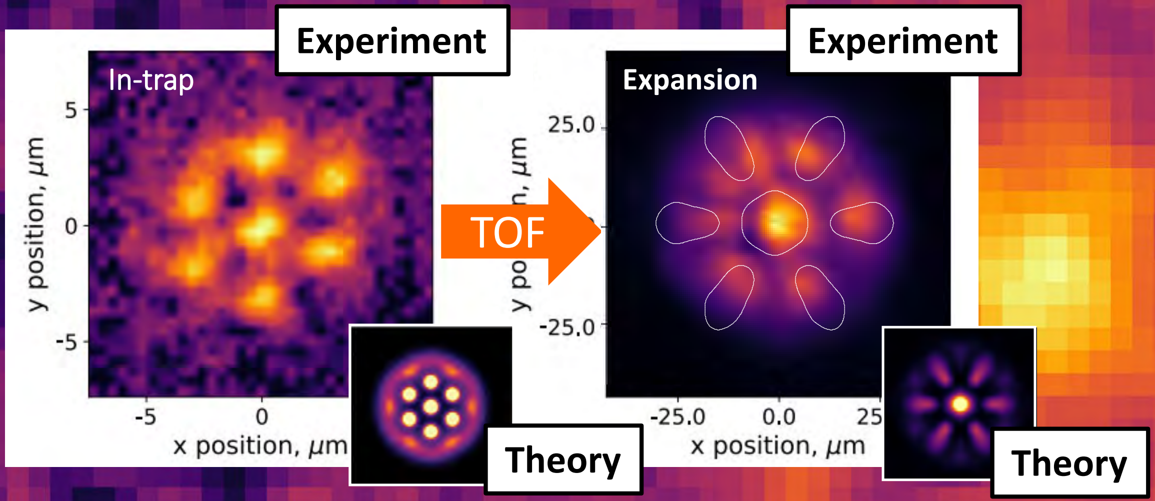
Internal elasticity makes „non-classical moment of inertia“ not to be a robust probe of superfluidity
M. A. Norcia,…, F. Ferlaino PRL. 129, 040403 (2022)

Norcia, Politi, Klaus, Poli, Sohmen, Mark, Bisset, Santos, Ferlaino Nature 596 (2021)

2D Supersolidity in a Circular trap

Norcia...Ferlaino, Nature 596 (2021)

Bland,.. Ferlaino, Santos, Bisset, PRL 128 (2022)



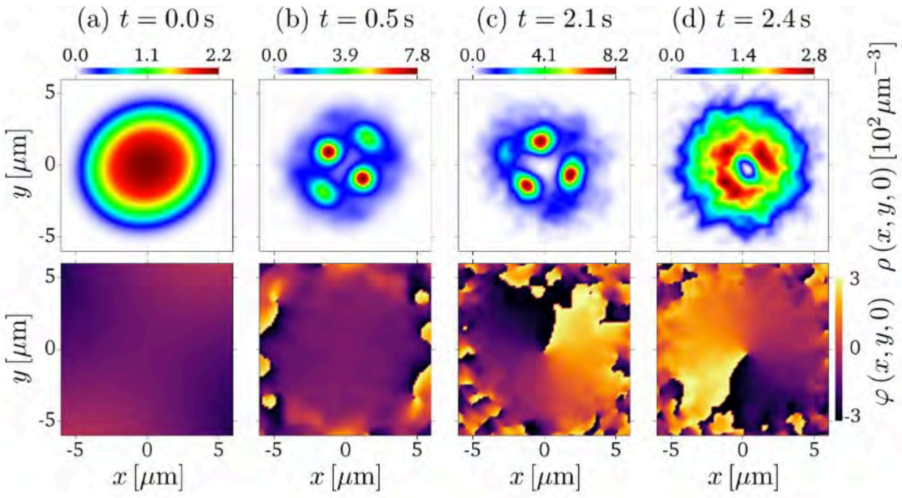
hexagon

2D Supersolidity in a Circular trap ($\alpha \simeq 1$)

Norcia...Ferlaino, Nature 596 (2021)

Bland,.. Ferlaino, Santos, Bisset, PRL 128 (2022)

Vorticity in modulated

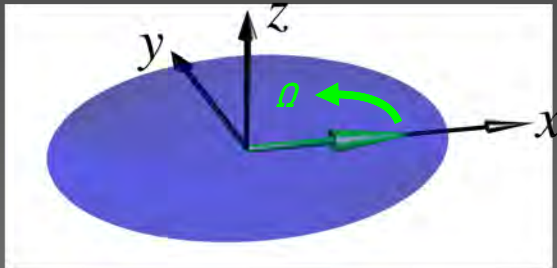


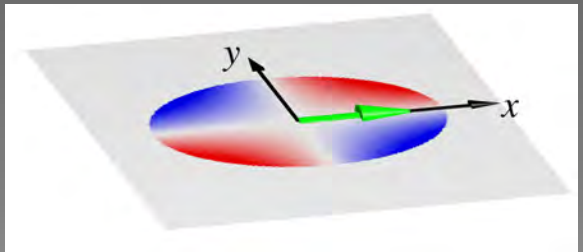
Šindik, … Santos, Stringari, arXiv:2206.14100 (2022)

See also: Ancilotto, Barranco, Pi, Reatto, PRA (2021)

However vortices never observed in dipolar gases even not in NON-MODULATED phase

Vortices in a NON-dipolar gas



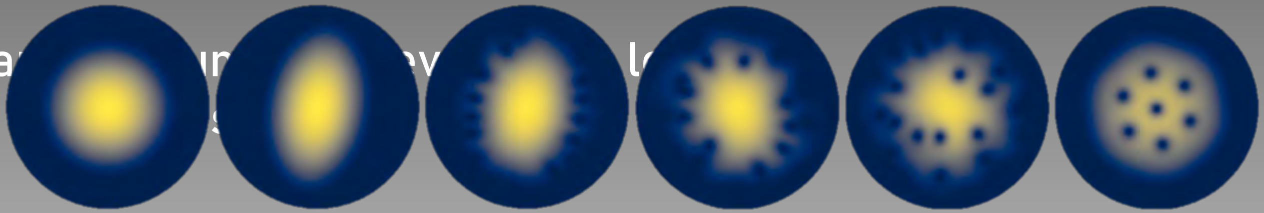


Recipe for vortices:

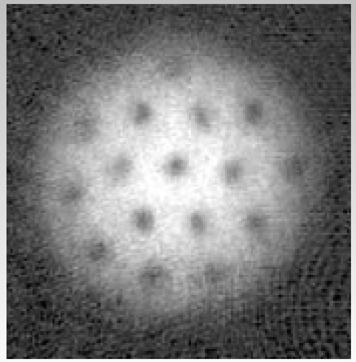
1. Induce small ellipticity in harmonic trap, rapidly rotate trap at Ω.

2. Condensate follows ellipticity and stretches with increasing Ω, with a phase profile of irrotational flow $\alpha\nabla xy$ for amplitude α

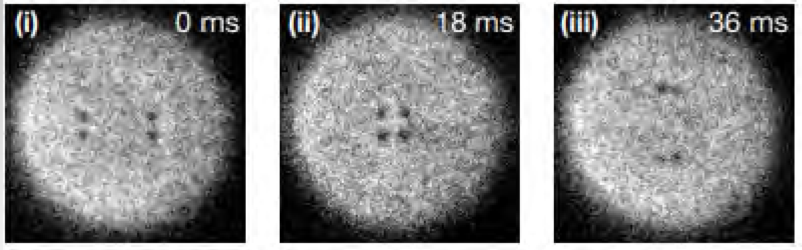
3. This is dyna… generation…



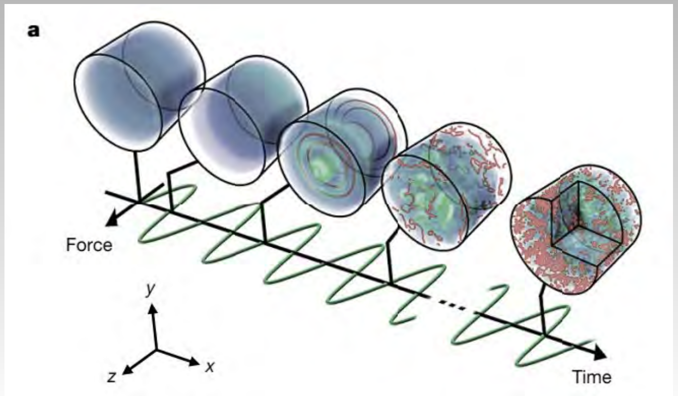
Stirring (Dalibard 2001)



Dragging laser to create dipoles (Roati 2022)

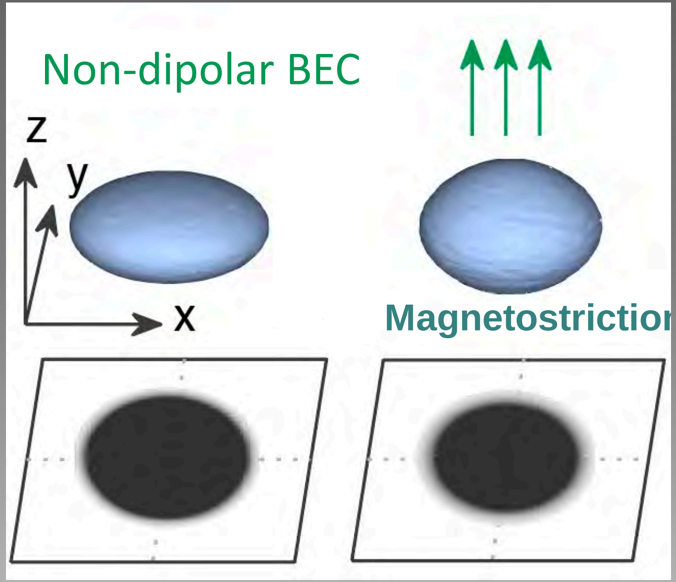


Shaking gas (Hadzibabic 2016



A third way: Magnetostirring a Dipolar BEC

Inspired by Prasad, Bland, Mulkerin, Parker, and Martin PRA 100 (2019).



Top view

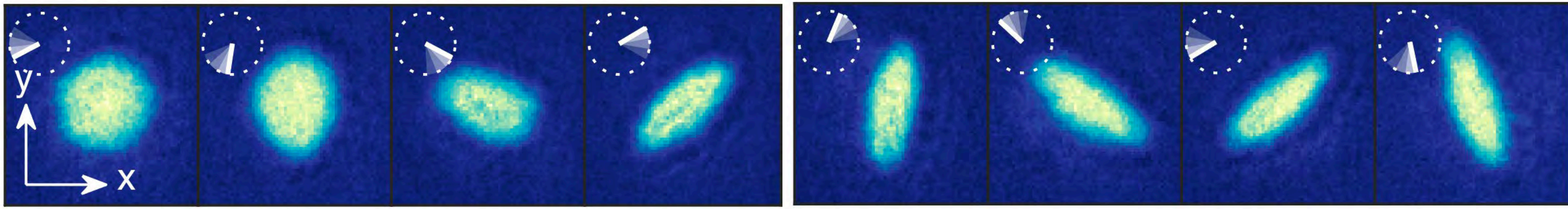
Magnetostirring

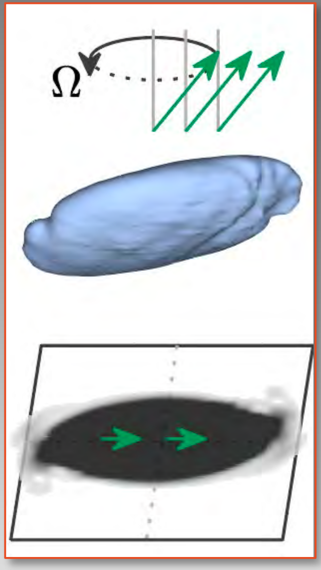
A new «interaction driven» method to transfer angular momentum and nucleate vortices

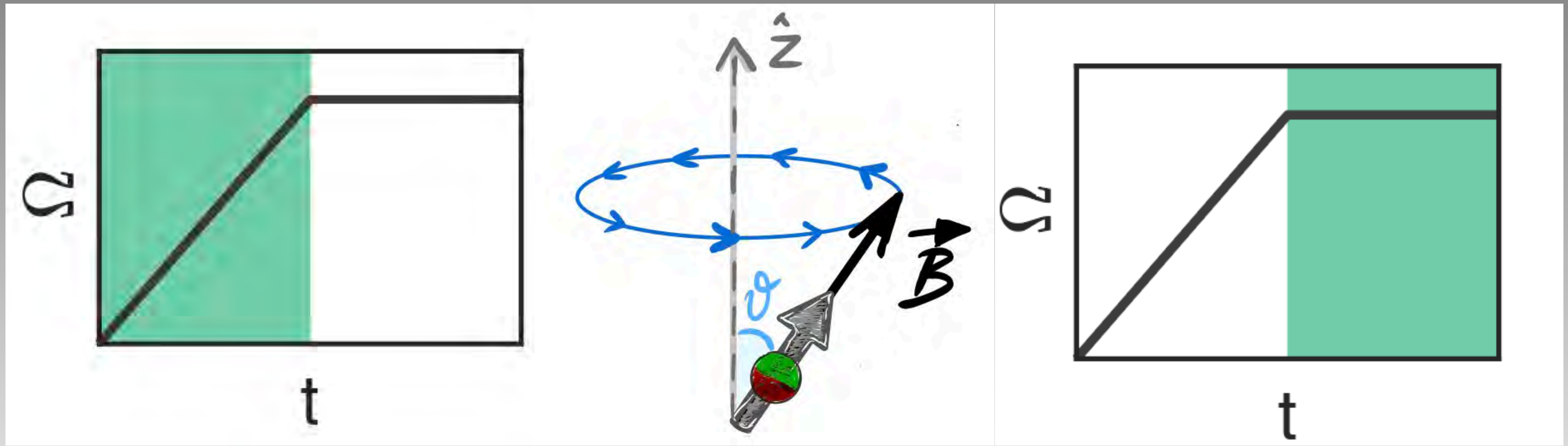
L. Klaus*, T. Bland*, E. Poli, C. Politi, G. Lamporesi, E. Casotti, R. N. Bisset, M. J. Mark, F. Ferlaino, arXiv:2206.12265 (2022), Accepted Nat. Phys. (2022)

Stirring a Dipolar Quantum Gases

Top view





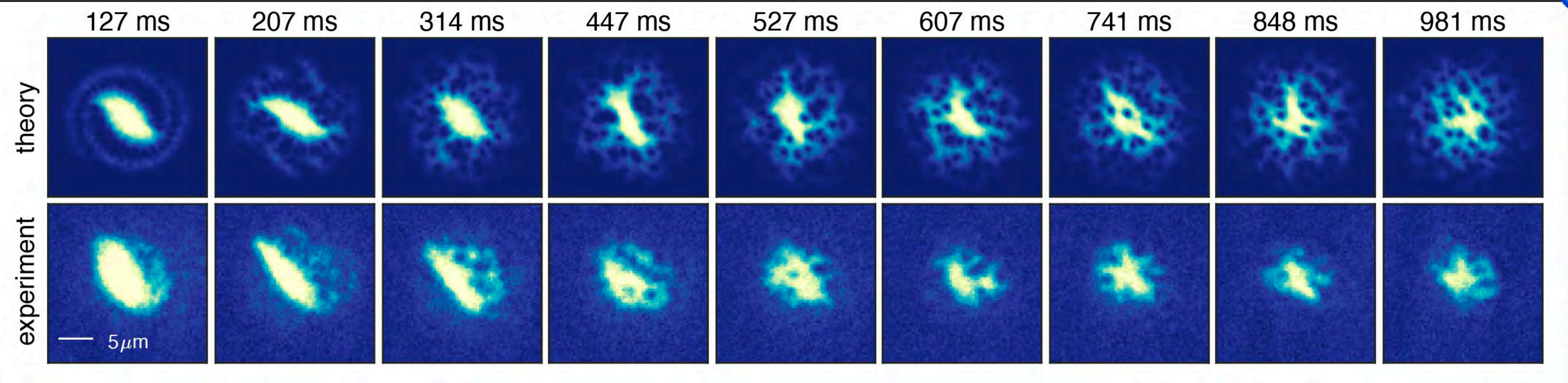


Stirring a Dipolar Quantum Gases

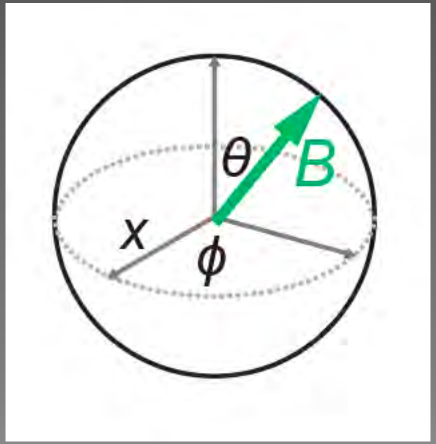
First Observation of Dipolar Vortices

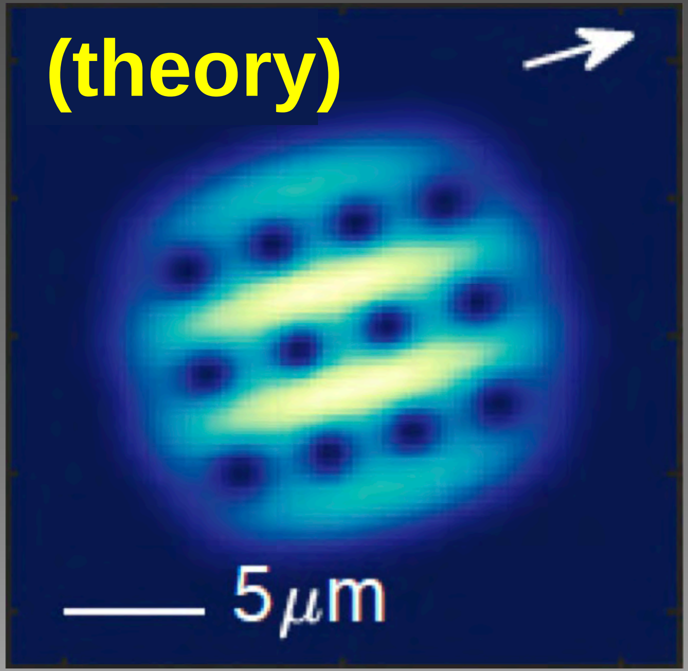
www.erbium.at

L. Klaus*, T. Bland*, E. Poli, C. Politi, G. Lamporesi, E. Casotti, R. N. Bisset, M. J. Mark, F. Ferlaino, arXiv:2206.12265 (2022)
Accepted Nat. Phys.

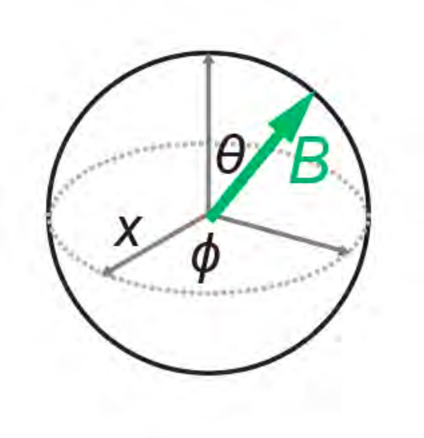


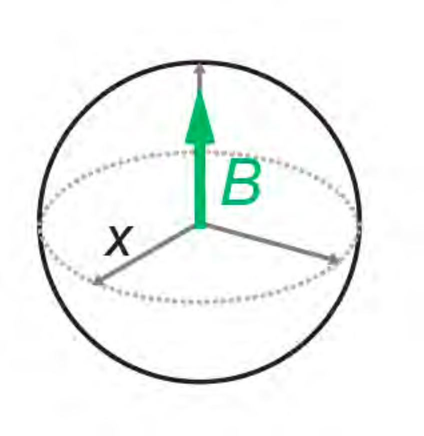
Vortex Stripes

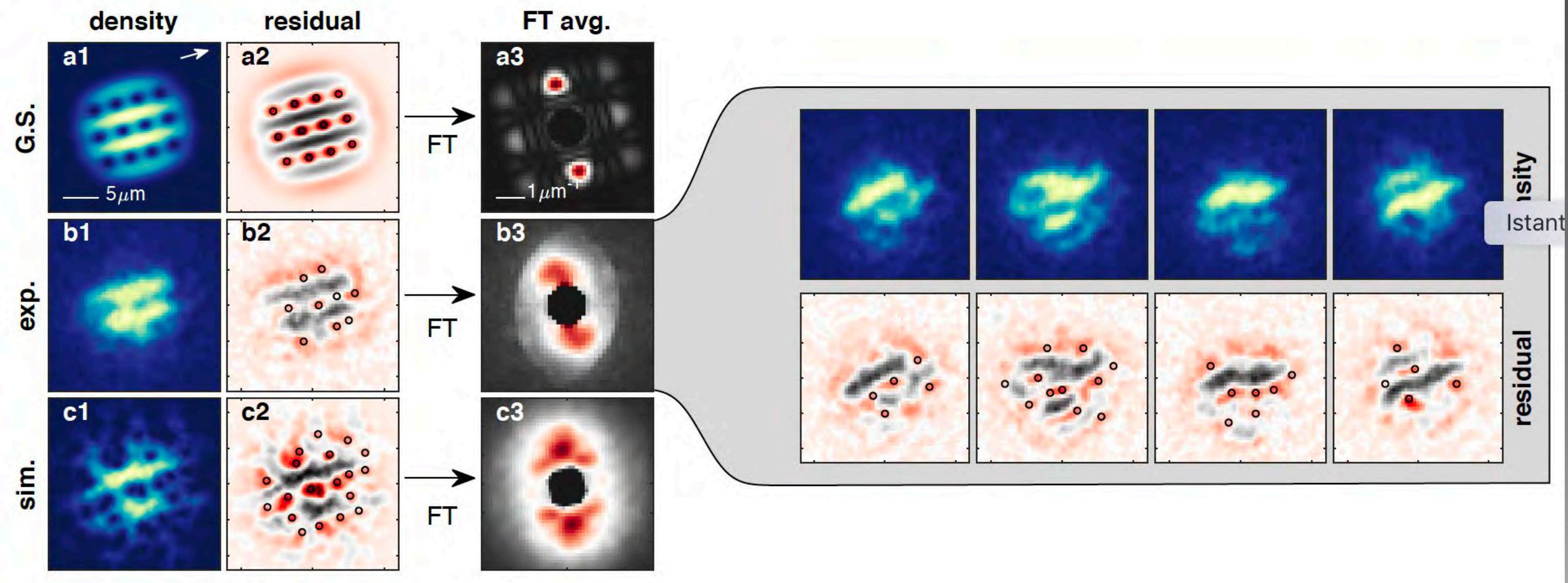




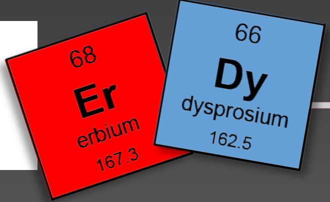
Vortex Stripes







Ultracold Lanthanide

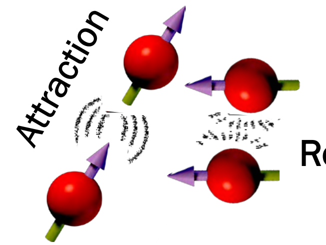


www.erbium.at

Huge permanent magnetic

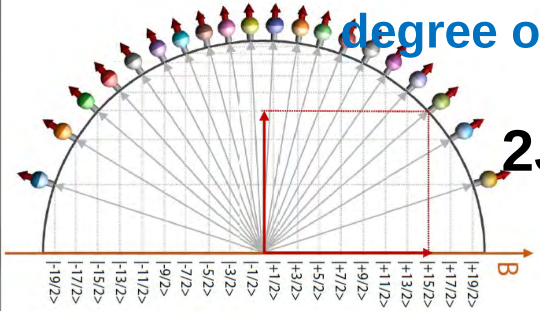
Dipole-dipole interaction

$V_{dd} \propto \mu^2$



Large Spin manifold

A rich reservoir of internal degree of freedom

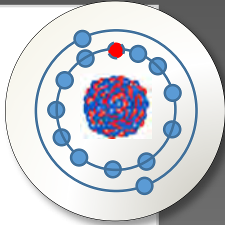


$2J_z+1$

Many valence electrons

Hz-transition at a telecom wavelenght (1299nm)

Optically active Rydberg atoms



INSIGHT | 07 DECEMBER 2021 — nature physics

Ultracold quantum technologies

The impressive achievements made with quantum gases rely on continuous improvements in the underlying methods. This Insight reviews recent technological advances that have deepened and broadened the capabilities of ultracold gas experiments.

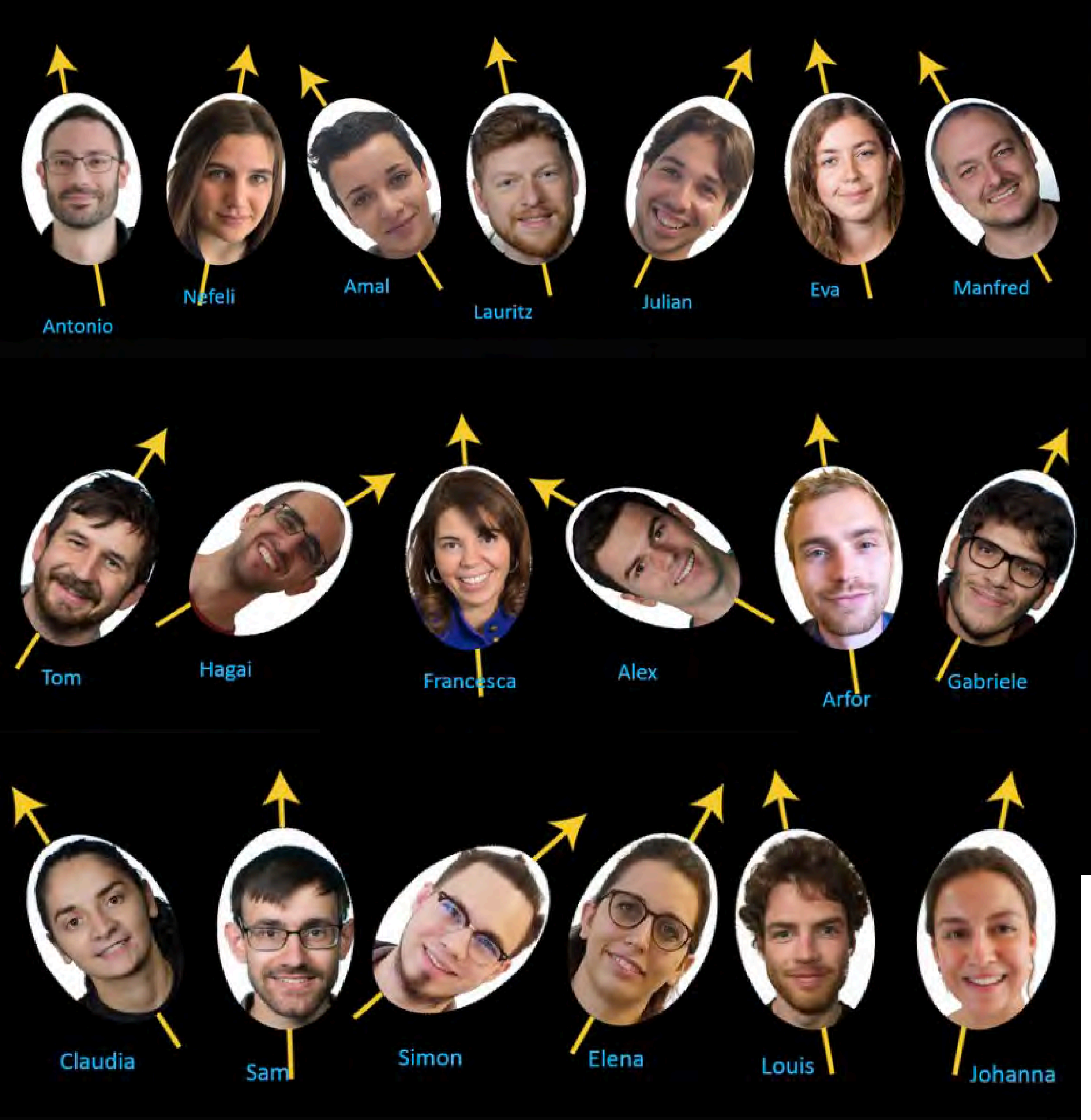
Developments in atomic control using ultracold magnetic lanthanides

The detailed structure of each atomic species determines what physics can be achieved with ultracold gases. This review discusses the exciting applications that follow from lanthanides' complex electronic structure.

Matthew A. Norcia & Francesca Ferlaino — Nature Physics, 17, 1349, 2021

The "*Lanthanide Era*" in the Labs: Stanford (Dy, 2011), Innsbruck (Er, 2012), Stuttgart, Pisa, Bonn, Paris, Harvard, MIT, Mainz, Oxford, Melbourne, …
More: new Europium BEC!!, Thulium, Holmium, .. and more will come

Dipolar physics: A review of experiments with magnetic quantum gases
L. Chomaz, I. Ferrier-Barbut, F. Ferlaino, B. L. Laburthe-Tolra, B. L. Lev, T. Pfau, ArXiv 2201.02672 (2022)



Maybe you?

OPENING 2022 now online!

THEORY&EXPERIMENT

Openings for PhD and Postdoc (Theory/Exp) and Research Associate position with long term perspective (Academy Scientist)

www.erbium.at

@FerlainoGroup

External collaborators

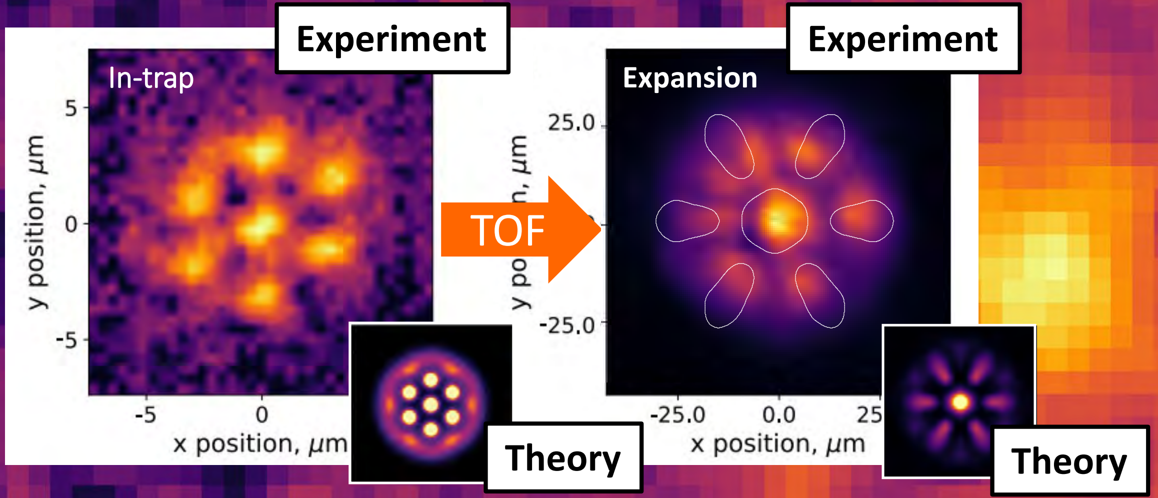
Funding:

IQI, erc, FWF, FOR 2247, Atoms, Light, and Molecules – Innsbruck Physics Research Center, QUANTERA – ERA-NET Cofund in Quantum Technologies

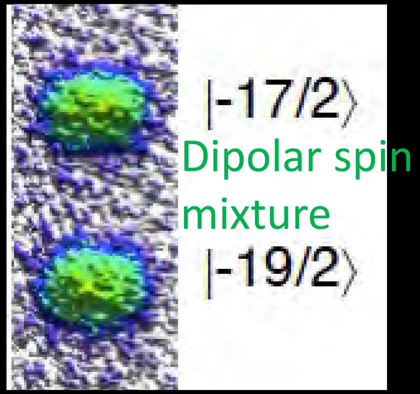
2D Supersolidity in a Circular trap

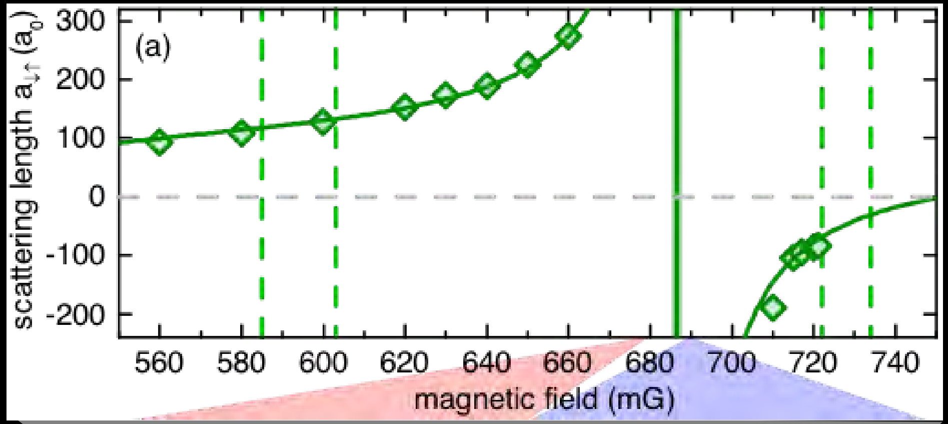
Norcia...Ferlaino, Nature 596 (2021)

Bland,.. Ferlaino, Santos, Bisset, PRL 128 (2022)

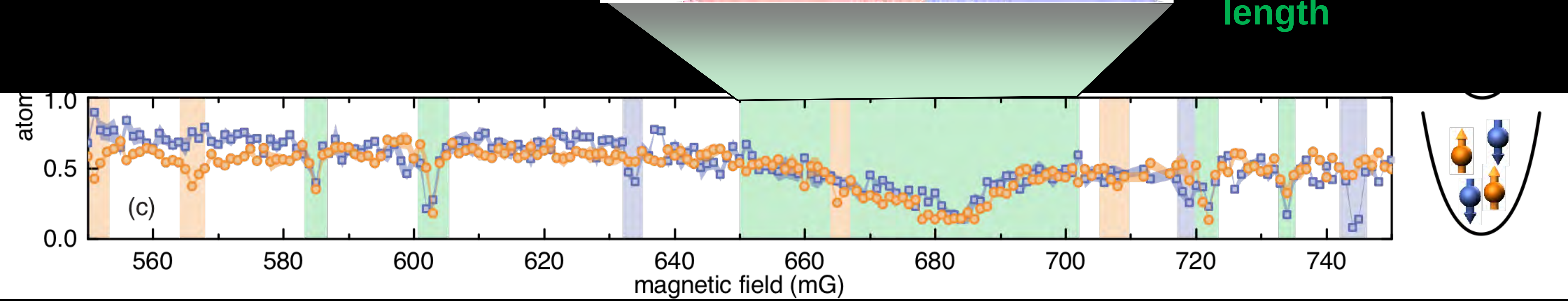


*Strongly interacting **dipolar** Fermi gas*





Precise Measurement of the scattering length

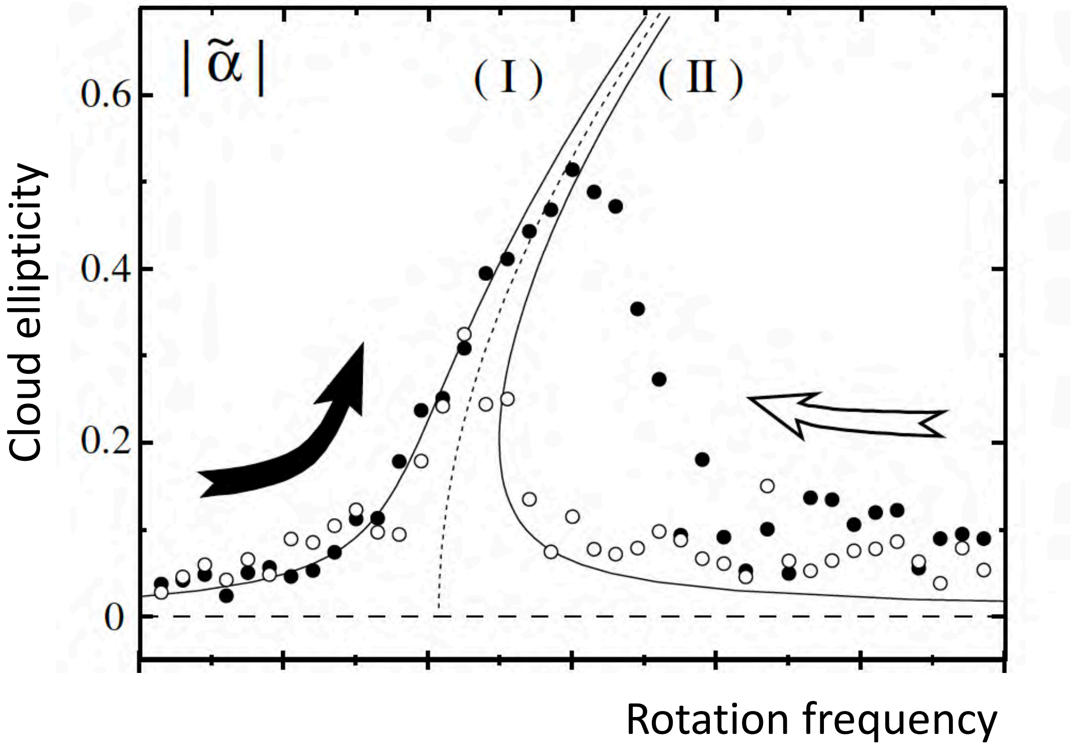


Baier … Ferlaino, Phys. Rev. Lett (2018)

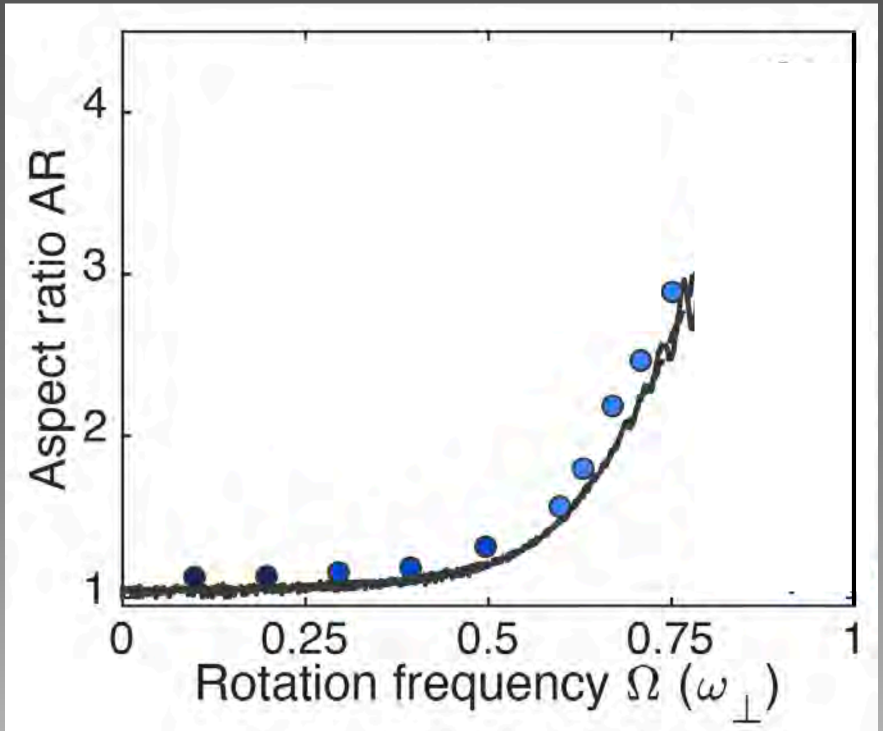
Stirring a Dipolar Quantum

NON-Dipolar BEC

Stationary States of a Rotating Bose-Einstein Condensate: Routes to Vortex Nucleation
K.W. Madison, F. Chevy, V. Bretin, and J. **Dalibard**, PRL (2001)



Dynamically unstable, eventually leads to generation of vortices!

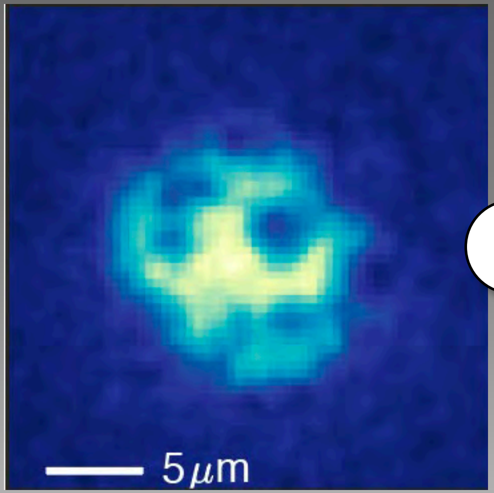


When the field rotates in the (x, y)-plane, ferrofluid magnetization also rotates in this plane with the same speed as the field UNTIL ….

Surface instability and vortex nucleation

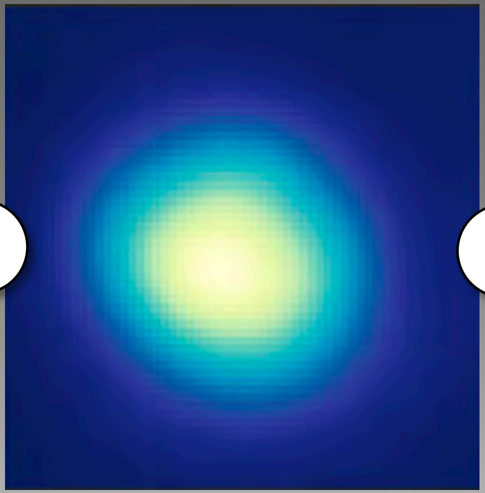
L. Klaus*, T. Bland*, E. Poli, C. Politi, G. Lamporesi, E. Casotti, R. Bisset, M. Mark and F. Ferlaino, in preparation

Single shot



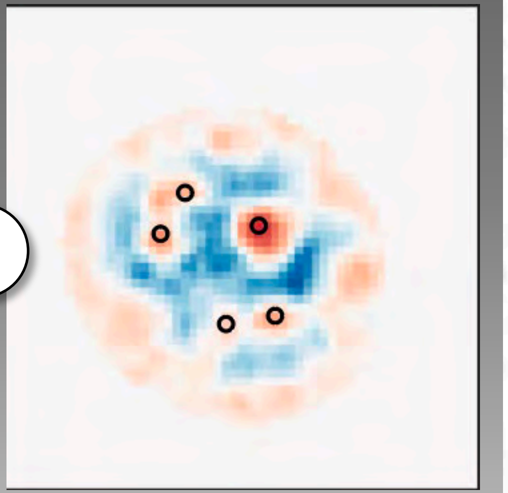
-

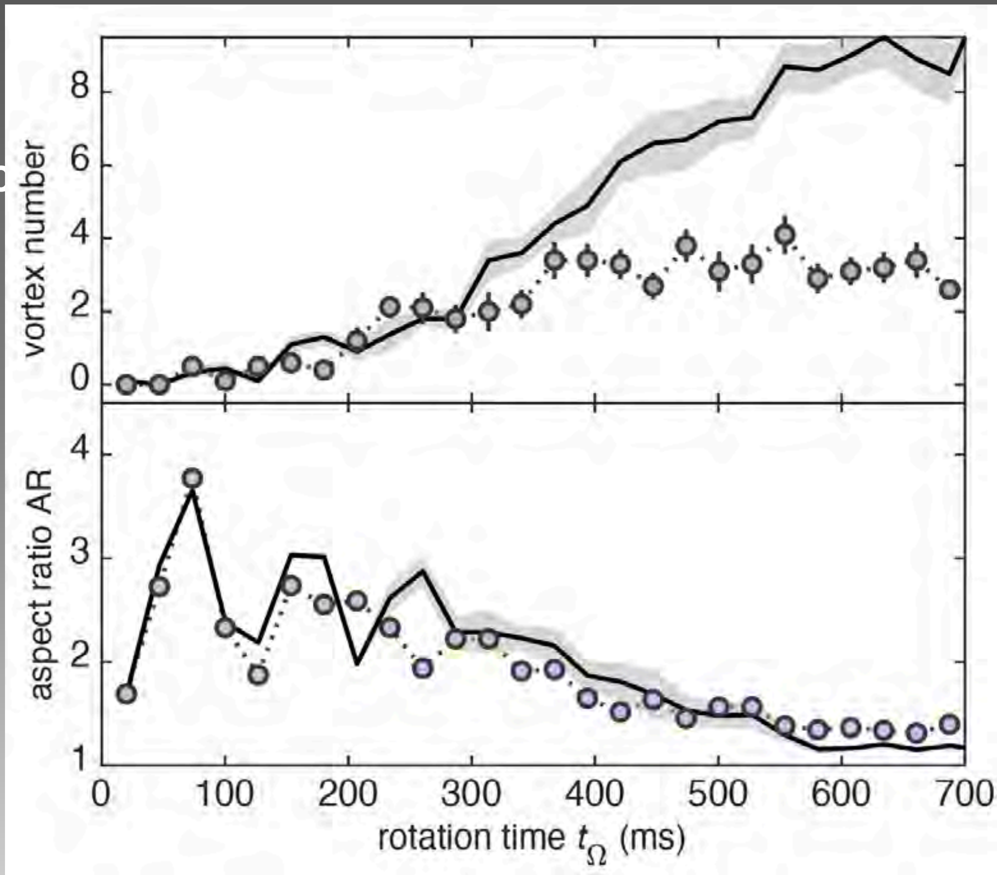
Average over many shots



=

Visualizing the vortic





Solid line: theory, no fitting param

Watching coherence develop

Which comes first, modulation or coherence, or they come together?

Complex phasor

Fourier component at the modulation wavelength in the Fourier transform of the TOF interference profile, normalised by Nc.

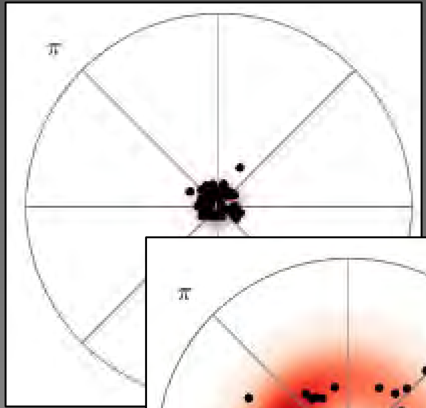
$$P_i = \rho_i \exp\{-i\Phi_i\}$$

Modulation and local degree of coherence within each of the individual droplet

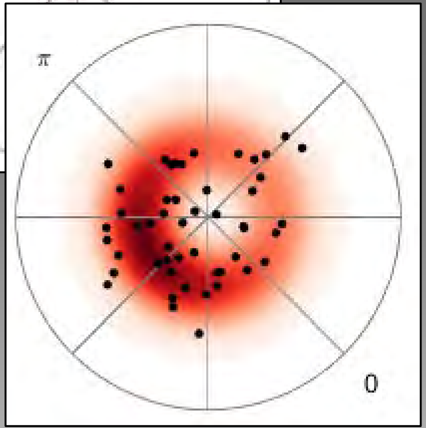
relative phase between the droplets, i.e. **global degree of phase coherence**

$$A_M = \langle |P_{\{i\}}| \rangle \qquad A_\Phi = |\langle P_{\{i\}} \rangle|$$

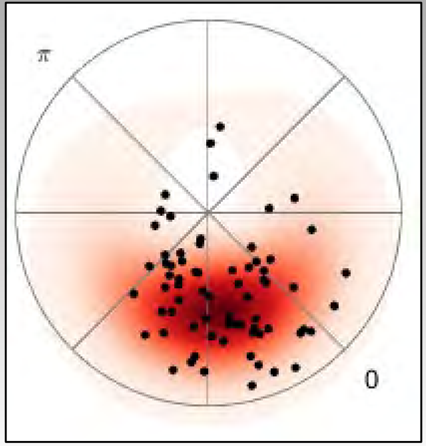
Phase variance $\Delta\Phi \leftrightarrow 1 - \frac{A_\Phi}{A_M}$



No modulation
No global or local phase coherence



Modulation and intra-droplet coherence
No global phase coherence



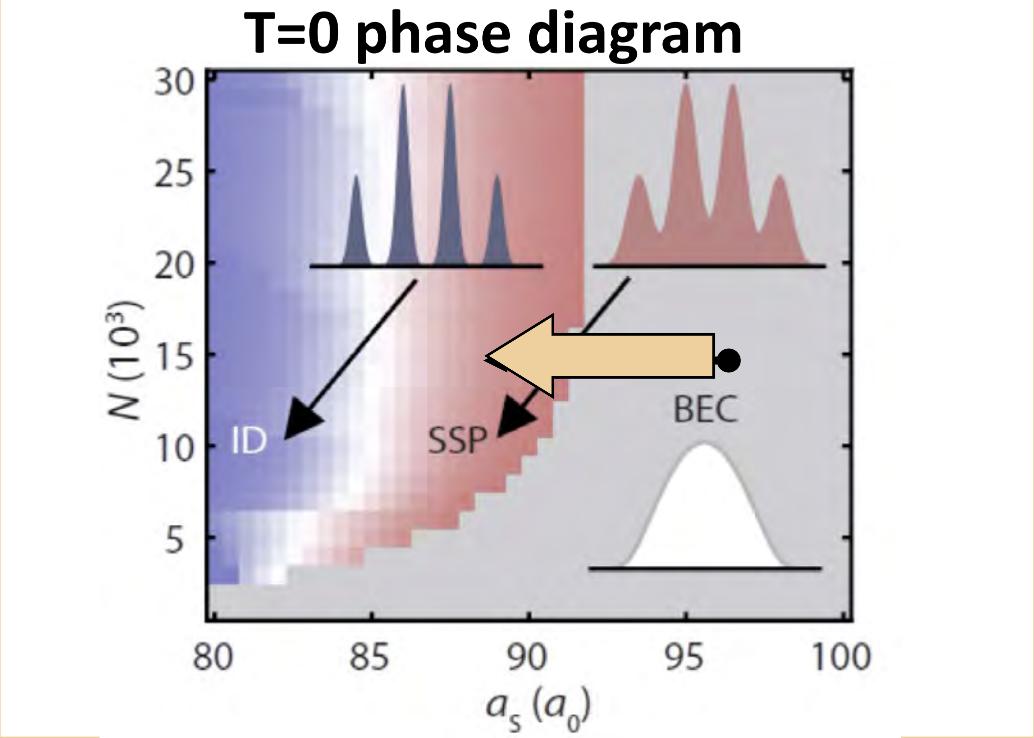
Modulation
Global phase coherence

Sohmen et al. Phys. Rev. Lett. 126, 233401 (2021)

Route to Supersolidity

From Superfluid → to Supersolid

Interaction Quench

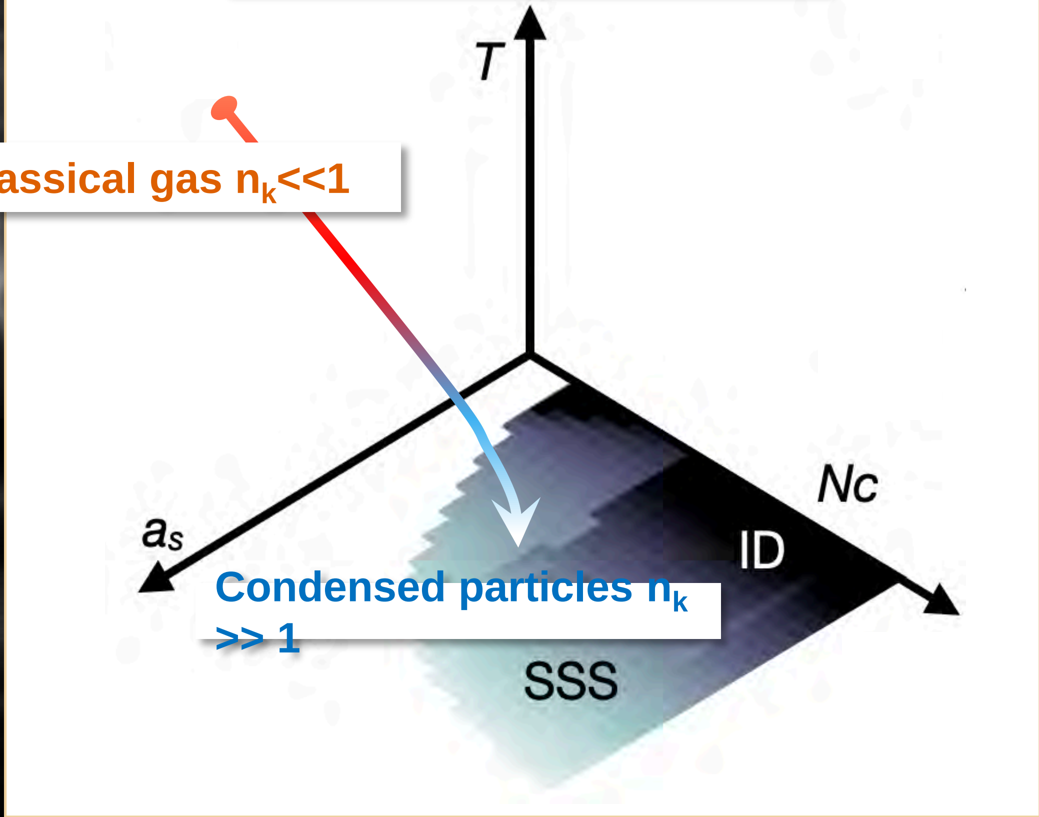


Stuttgart (^{164}Dy) ● Pisa (^{164}Dy) ● Innsbruck (^{162}Dy & ^{166}Er)

Chomaz, Petter et al. PRX 9 (2019)
Ilzhöfer, Sohmen et al. Nat. Phys. (2021)

From thermal gas → to Supersolid

Cooling



Innsbruck (^{162}Dy)

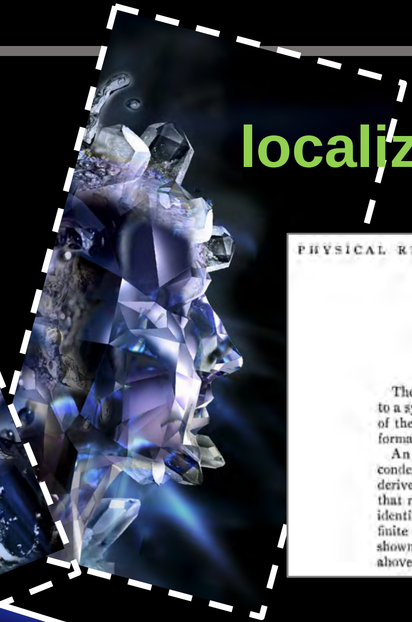
Sohmen et al. PRL 126 (2021)

Supersolid states

Its very existence under theoretical debat

A paradoxal quantum fluid with the crystalline order of a solid and the frictionless flow of a superfluid

superfluidity



localization

Competing orders

Penrose and Onsager 1956; …

PHYSICAL REVIEW VOLUME 104, NUMBER 3 NOVEMBER 1, 1956

Bose-Einstein Condensation and Liquid Helium

OLIVER PENROSE* AND LARS ONSAGER
Sterling Chemistry Laboratory, Yale University, New Haven, Connecticut

(Received July 30, 1956)

The mathematical description of B.E. (Bose-Einstein) condensation is generalized so as to be applicable to a system of interacting particles. B.E. condensation is said to be present whenever the largest eigenvalue of the one-particle reduced density matrix is an extensive rather than an intensive quantity. Some transformations facilitating the practical use of this definition are given.
An argument based on first principles is given, indicating that liquid helium II in equilibrium shows B.E. condensation. For absolute zero, the argument is based on properties of the ground-state wave function derived from the assumption that there is no "long-range configurational order." A crude estimate indicates that roughly 8% of the atoms are "condensed" (note that the fraction of condensed particles need not be identified with ρ_s/ρ). Conversely, it is shown why one would not expect B.E. condensation in a solid. For finite temperatures Feynman's theory of the lambda-transition is applied: Feynman's approximations are shown to imply that our criterion of B.E. condensation is satisfied below the lambda-transition but not above it.

Quantum-mechanically allowed

Gross 1957; Yang 1962; Andreev and Lifshitz 1967; Chester 1970, Leggett 1970; …

REVIEW A VOLUME 2, NUMBER 1 JULY 1970

Speculations on Bose-Einstein Condensation and Quantum Crystals*

G. V. Chester
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850

(Received 13 May 1969)

It is shown, by almost rigorous arguments, that there exist many-body states of a system of interacting bosons which exhibit both crystalline order and Bose-Einstein condensation into the zero-momentum eigenstate of the single-particle density matrix. The implications of this result are discussed in relation to theories of superfluidity and the nature of quantum crystals.

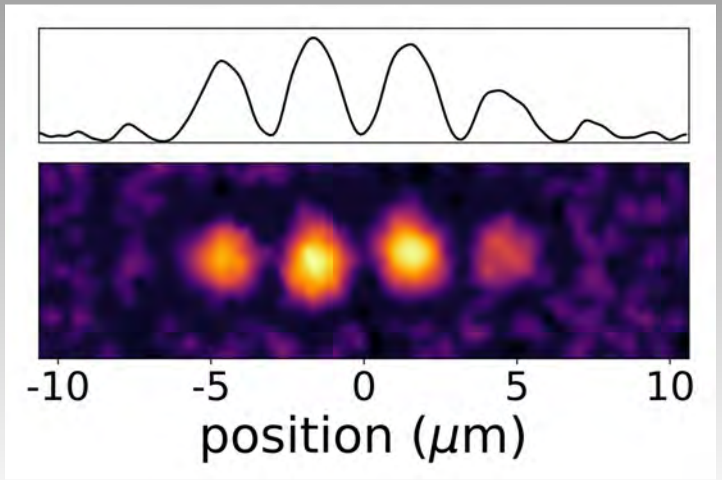
Dipolar Supersolid

$$V_{\rm dd} = \frac{\mu_0 \mu^2}{4\pi} \frac{1 - 3\cos^2\theta}{r^3}$$

- Superfluid state with k-dependent (anisotropic) interaction
- Instability towards crystallization (spontaneous)
- A protection mechanism against collapse

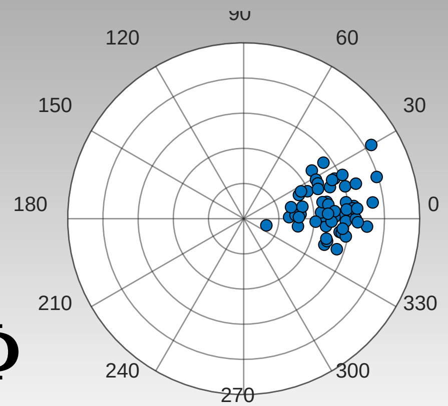
Dipolar Supersolid: The Innsbruck Er and Dy Experiment

In-situ imaging



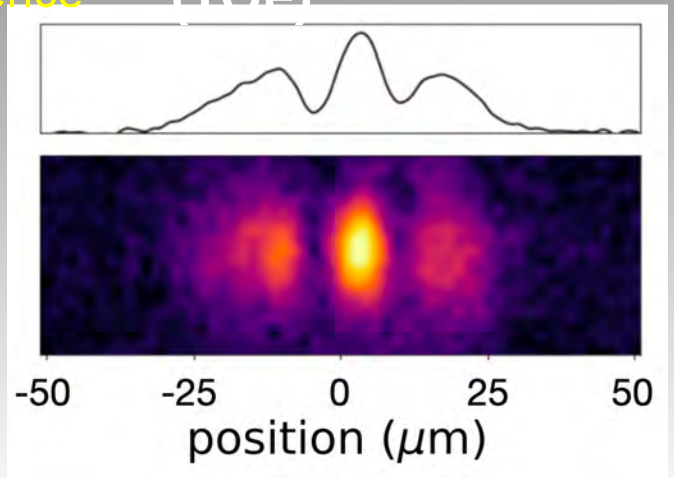
Density modulation

Global phase coherence



$\Delta\Phi$

Interference pattern (TOF)

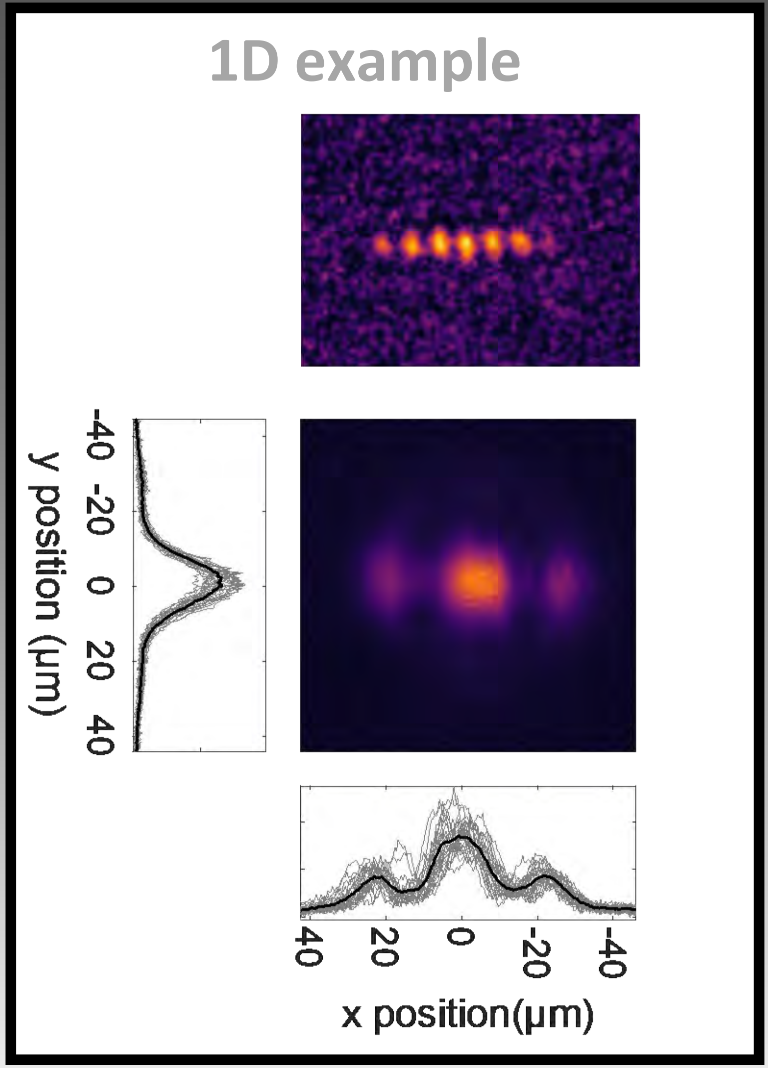


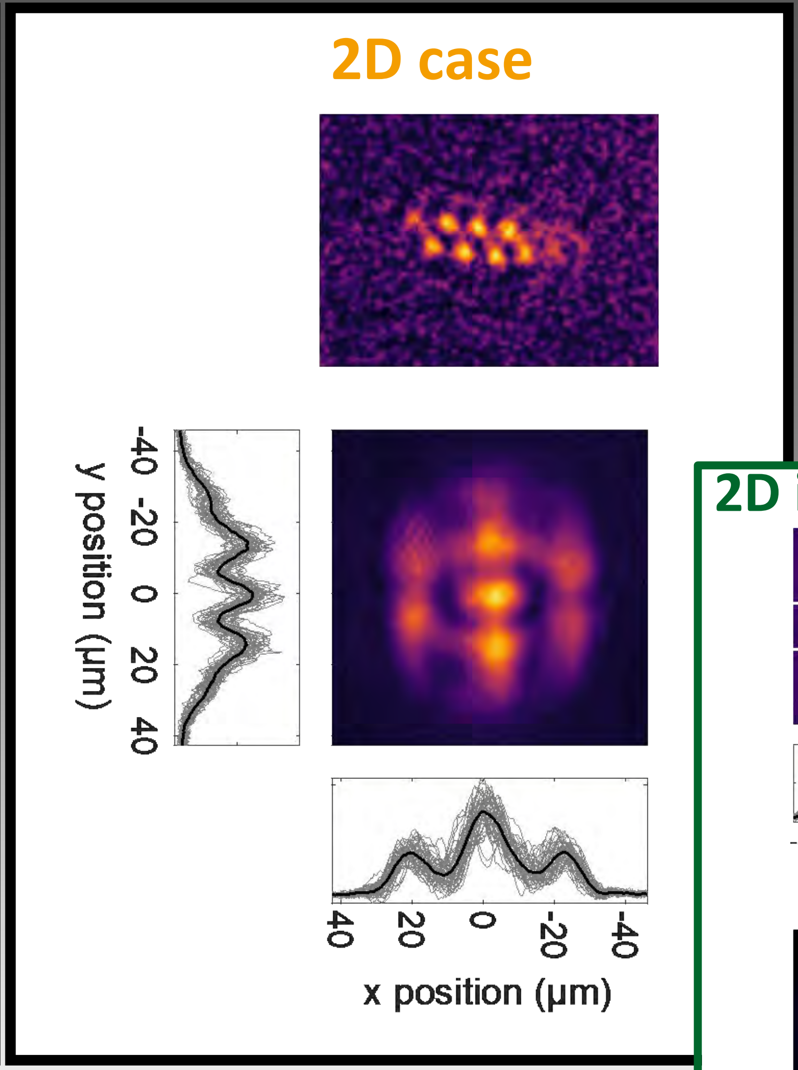
Coherence in 2D state

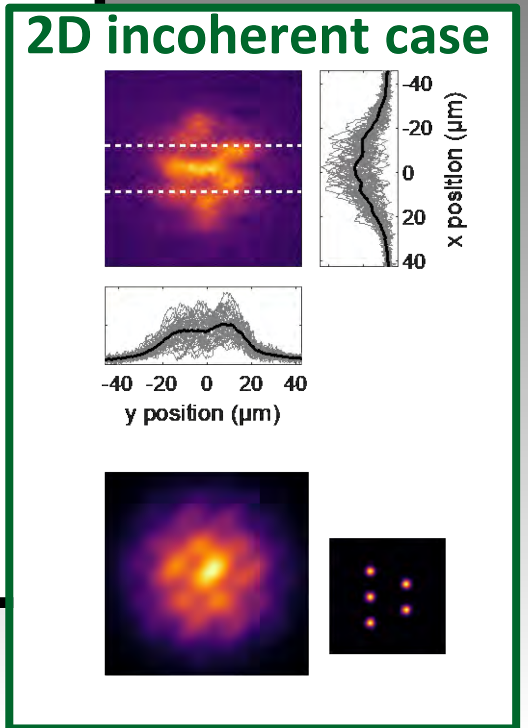
www.erbium.at

- TOF imaging of interference

- recall: stable modulation pattern in TOF indicates coherence

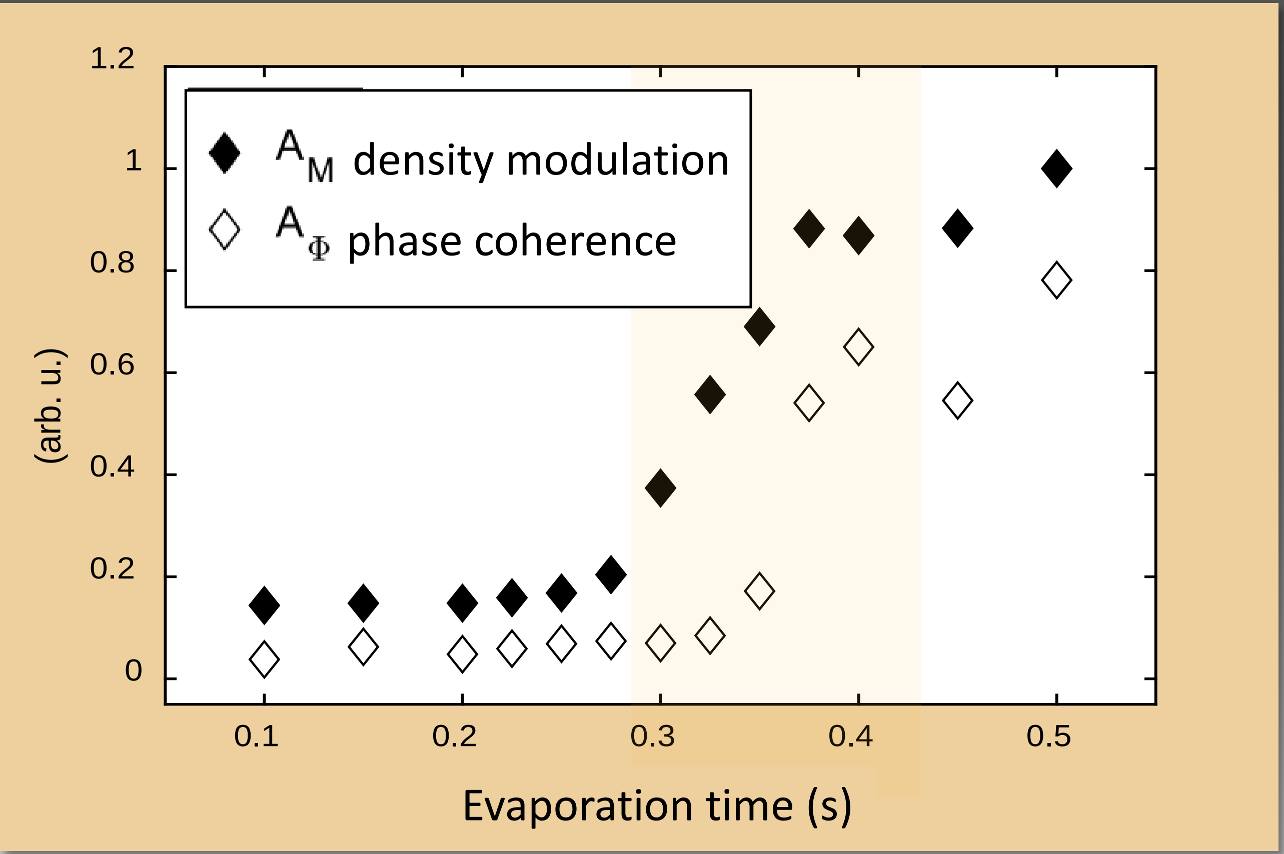






www.erbium.at

Complex phasor



Sohmen et al. Phys. Rev. Lett. 126, 233401 (2021)

Physics Viewpoint : Cooling a Thermal Cloud to a Supersolid

Modulation

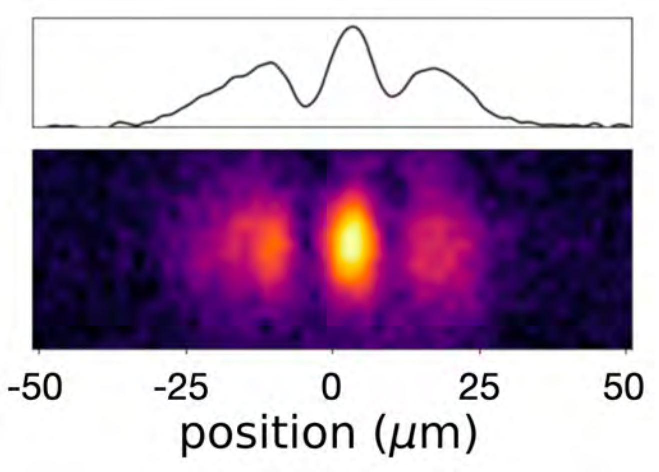
$$A_M = \langle |P_{\{i\}}| \rangle$$

Macroscopic phase coherence

$$A_\Phi = |\langle P_{\{i\}} \rangle|$$

$$P_i = \rho_i \exp\{-i\Phi_i\}$$

Time-of-flight



Interference patter: phases encode relative droplet phase

Describing a dipolar BEC: ground-state

$$\hat{H} = \int dr \hat{\psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2 \nabla^2}{2m} + V_{\mathrm{ext}}(\mathbf{r}) + \int dr' V_{\mathrm{eff}}(\mathbf{r}-\mathbf{r}') \hat{\psi}^\dagger(\mathbf{r}') \hat{\psi}(\mathbf{r}') \right) \hat{\psi}(\mathbf{r})$$

$$V_{\mathrm{eff}}(r) = \frac{4\pi\hbar^2 a_s}{m}\delta(r) + V_{\mathrm{dd}}(r)$$

contact — dipolar

Bogoliubov approach: $\hat{\psi}(\mathbf{r}) = \psi_0(\mathbf{r})\hat{a}_0 + \delta\hat{\psi}_\perp(\mathbf{r}) = \Psi_0(\mathbf{r}) + \delta\hat{\psi}_\perp(\mathbf{r})$

ψ_0 = condensed mode = macroscopically populated with N_0 atoms

$$\Psi_0(r) = \sqrt{N_0}\psi_0(r)$$

$$\hat{a}_0 \approx \hat{a}_0^\dagger \approx \sqrt{\langle \hat{a}_0^\dagger \hat{a}_0 \rangle} \approx \sqrt{N_0}$$

treated as a classical field!

Equation for a dipolar condensate (mean-field): Gross-Pitaevskii Equati

$$\mu\Psi_0(r) = \left[-\frac{\hbar^2}{2m}\Delta + V_{\mathrm{ext}}(r) + \frac{4\pi\hbar^2 a_s}{m}|\Psi_0(r)|^2 + \Phi_{\mathrm{dd}}(r) \right] \Psi_0(r)$$

Describing a dipolar BEC: ground-state

Equation for a dipolar condensate (mean-field): Gross-Pitaevskii Equati

$$\mu\Psi_0(r) = \left[-\frac{\hbar^2}{2m}\Delta + V_{\text{ext}}(r) + \frac{4\pi\hbar^2 a_s}{m}|\Psi_0(r)|^2 + \Phi_{\text{dd}}(r)\right]\Psi_0(r)$$

Usual contact term

Dipolar term: non-local

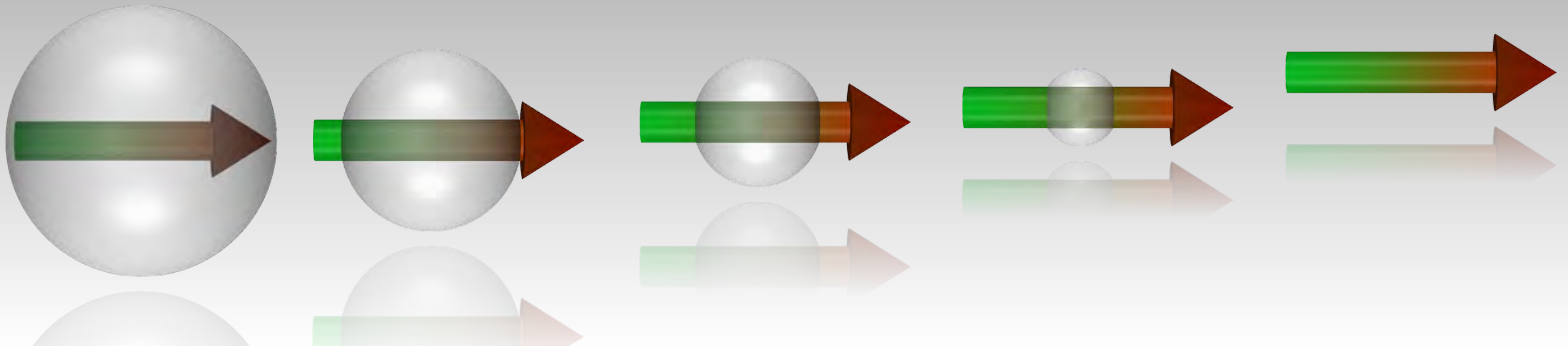
$$\Phi_{\text{dd}}(r) = \int dr' V_{\text{dd}}(r - r')|\Psi_0(r')|^2$$

$$a_d = \frac{\mu_0\mu^2 m}{12\pi\hbar^2} \approx 66a_0$$

Dipolar length (for Er is 66 a_0)

Tuning the relative weight of the two interactions

$$\varepsilon_{\text{dd}} = \frac{a_d}{a_s} \geq 1$$



Origin of the droplets

Energy functional from the Gross-Pitaevskii Equation (mean-field)

$$E[n] = \int dr \left[\frac{\hbar^2}{2m} (\nabla \sqrt{n})^2 + V_{\mathrm{trap}} n + \frac{g}{2} n^2 + \frac{1}{2} n \int dr' V_{\mathrm{dd}} n(r') \right]$$

MF energy

attractive → collapse!

A mystery …

Within the GPE, there is NO WAY to explain the quantum droplets and their stability

Need for an additional mechanism!!

Intense debate…

A realm of experimental and theoretical works followed, which are hard to put chronological

Origin of the droplets

www.erbium.at

GENERALIZED energy functional from the Gross-Pitaevskii Equation (n

$$E[n] = \int dr \left[\frac{\hbar^2}{2m}(\nabla\sqrt{n})^2 + V_{\text{trap}} n + \frac{g}{2} n^2 + \frac{1}{2} n \int dr' V_{\text{dd}} n(r') + \frac{\alpha}{2} n^{2+\gamma} \right]$$

New TERM

Faster in "n" -- Repulsive

Need for an additional mechanism!!

Quantum Fluctuations (QFs)

Lee-Huang-Yang:1st correction to MF

γ=1/2

$$\Delta\mu(n, \epsilon_{dd}) = \frac{32}{3\sqrt{\pi}} g n \sqrt{n a^3} F(\epsilon_{dd}).$$

D. S. Petrov, PRL 115, 155302 (2015)
L. Chomaz, & al. PRX 6 (4), 041039 (2016)
I. Ferrier-Barbut, & al. PRL 116, 215301 (2016)
F. Wächtler & L. Santos, PRA 93, 061603 (2016)
R. N. Bisset, & al. PRA 94, 033619 (2016)
F. Wächtler & L. Santos, PRA 94, 043618 (2016)
D. Baillie, & al. PRA 94, 021602(R) (2016)....

Three-body Forces

γ=1

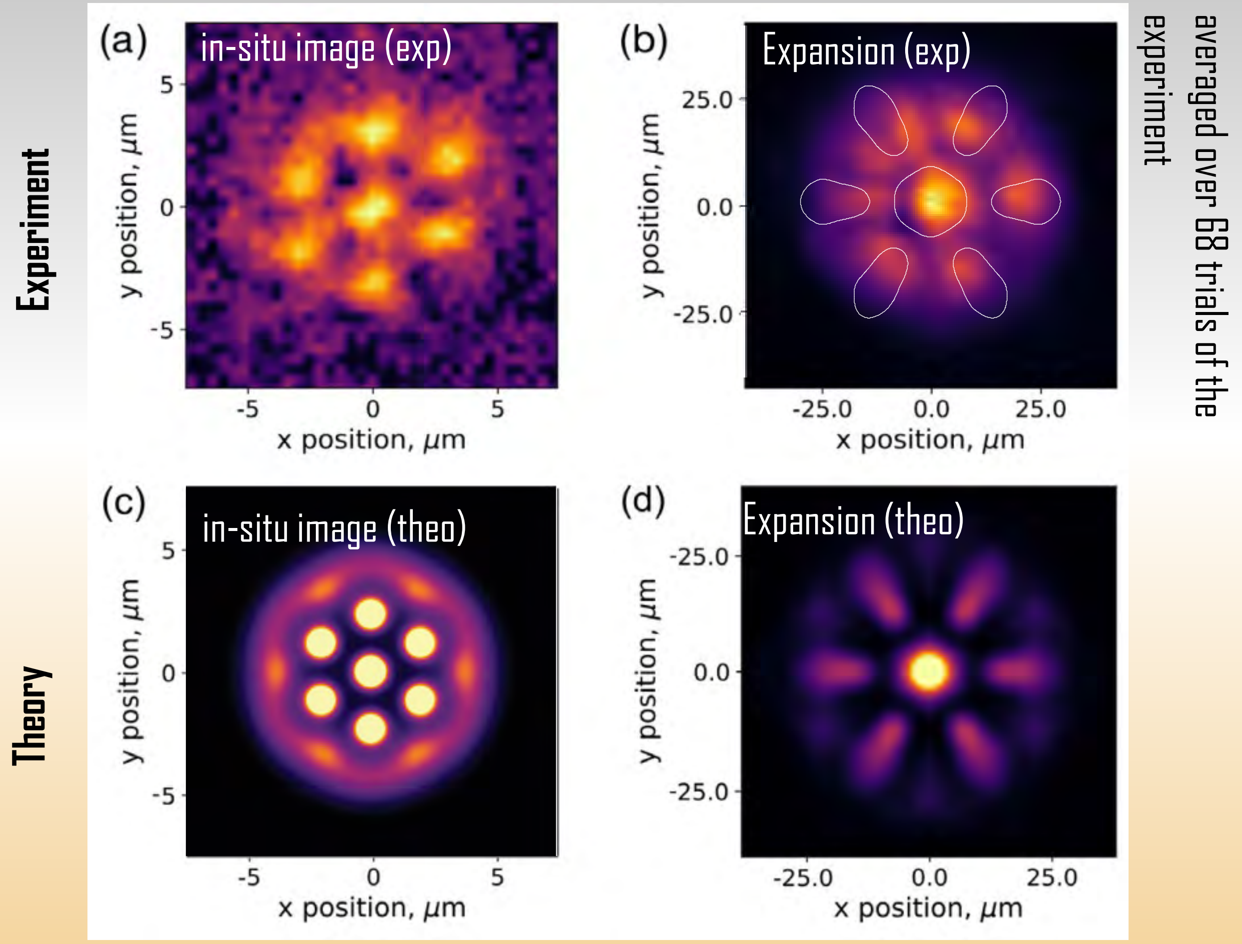
R. N. Bisset & P. B. Blakie, PRA 92, 061603 (2015).
K.-T. Xi & H. Saito, PRA 93, 011604 (2016).
P. B. Blakie, PRA 93, 033644 (2016).

2D Supersolidity in a Circular trap

(a~1)

Bland, Poli, Politi, Klaus, Norcia, Ferlaino, Santos, Bisset, PRL 128 (2022)

Hexagon state with 7 droplets



Dipolar vortices

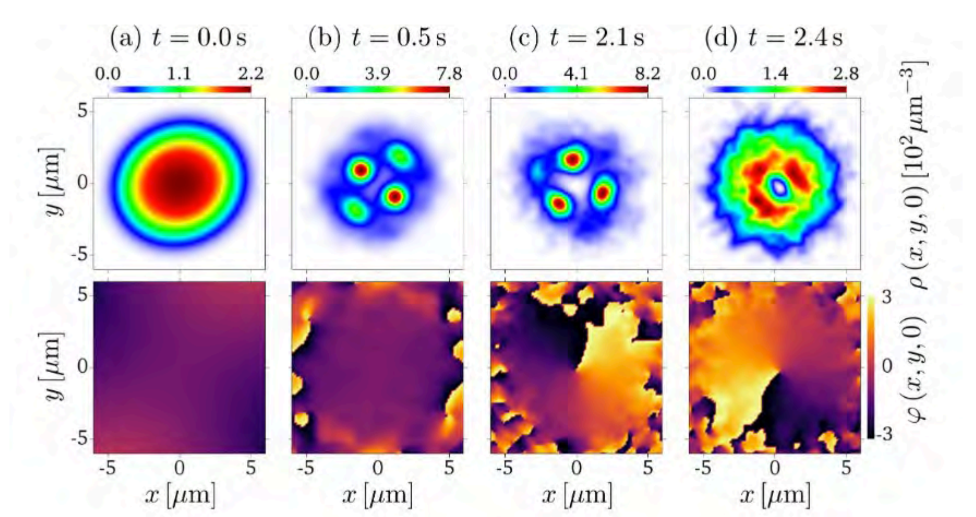
Observations of dipolar vortices in (even non-modulated) dipolar BEC is yet missing!

Many new feautures

Elliptic (giant) core ● Roton modulations around the vortex core ● Stripe vortex lattice

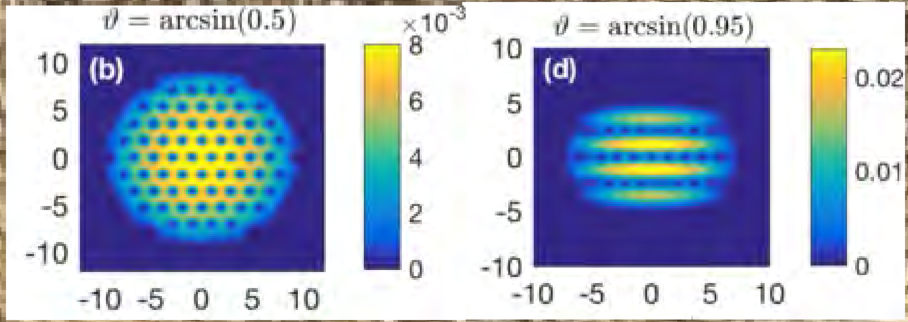
R. Wilson et al., Phys. Rev. A **79**, 013621 (2009)

Vorticity in modulated



Šindik, … Santos, Stringari, arXiv:2206.14100 (2022)

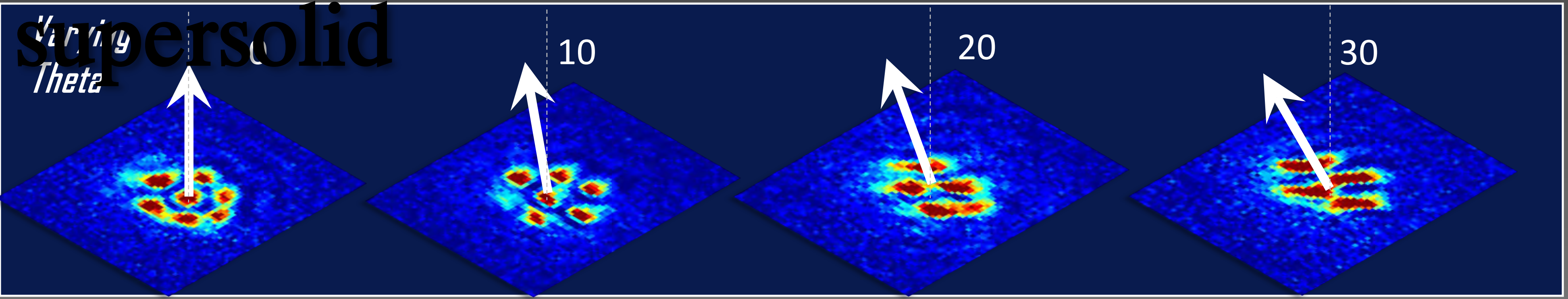
See also: Ancilotto, Barranco, Pi, Reatto, PRA (2021)



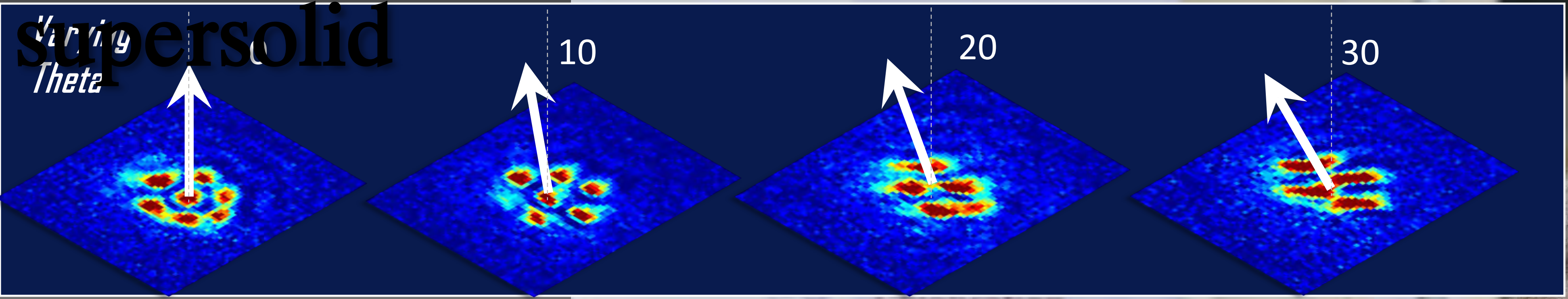
Y. Cai et al., Phys. Rev. A **98**, 023610 (2018)

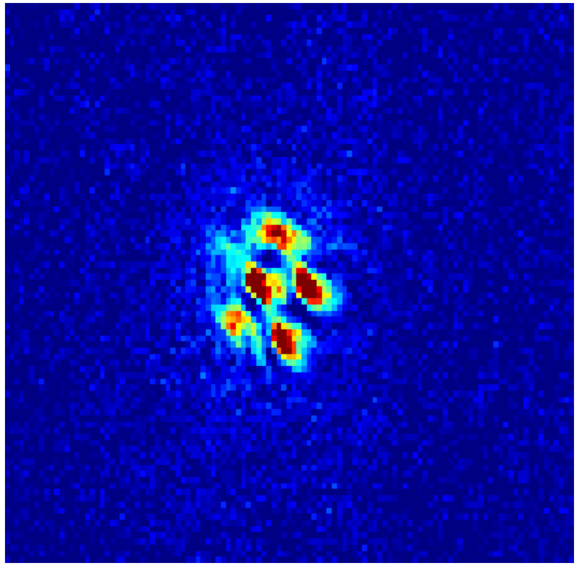
A. M. Martin, N. G. Marchant, D. H. J. O'Dell, and N. G. Parker, Vortices and vortex lattices in quantum ferrofluids, Journal of Physics: Condensed Matter 29, 103004 (2017).

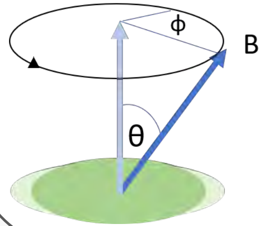
WiP –Magnetostirring of a supersolid



WiP –Magnetostirring of a supersolid







Varying ϕ
(θ = 20)

Challenge Nr.1: How to probe superfluidity and measure its fraction in a supersolid?
And what can we learn from non-dipolar BEC?