INFN
Istituto Nazionale di Fisica Nucleare

Exploring fundamental interactions by analog models

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QFC
Pisa 27 Oct 2022

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I have always been more interested in experiment, than in accomplishment.

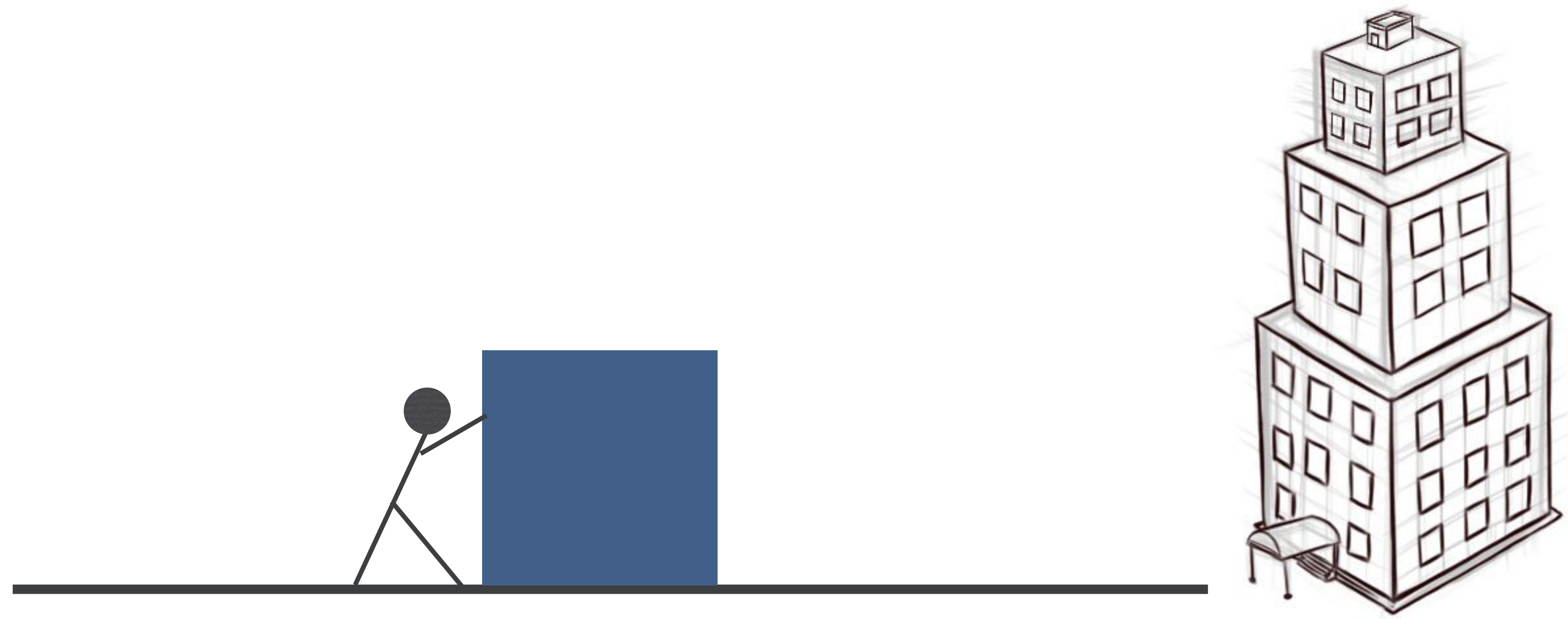
Orson Welles

Outline

- **Introduction to analogs**
- **Analogies for quark matter**
- **Color superconductors**
- **Viscosity**
- **Viscosity in a gravity analog model**
- **Conclusions**

Analogies at work for solving problems…

Standard model



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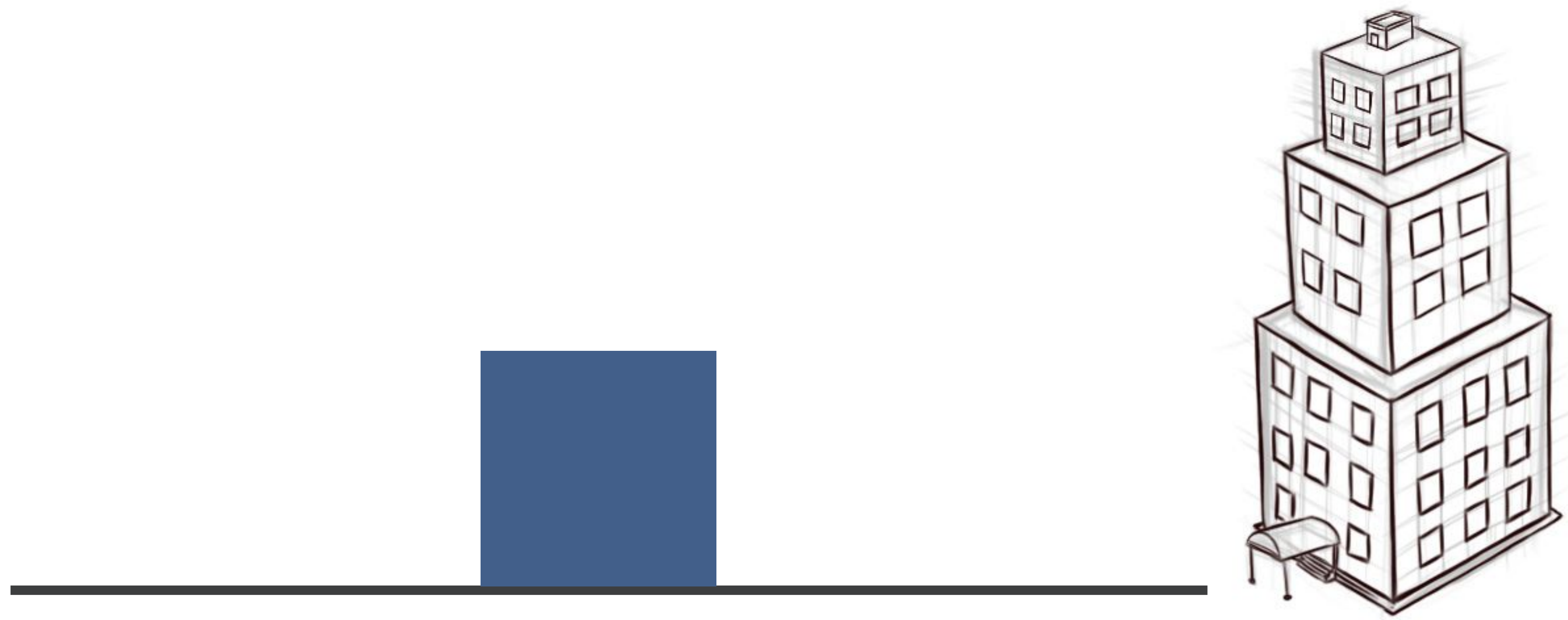

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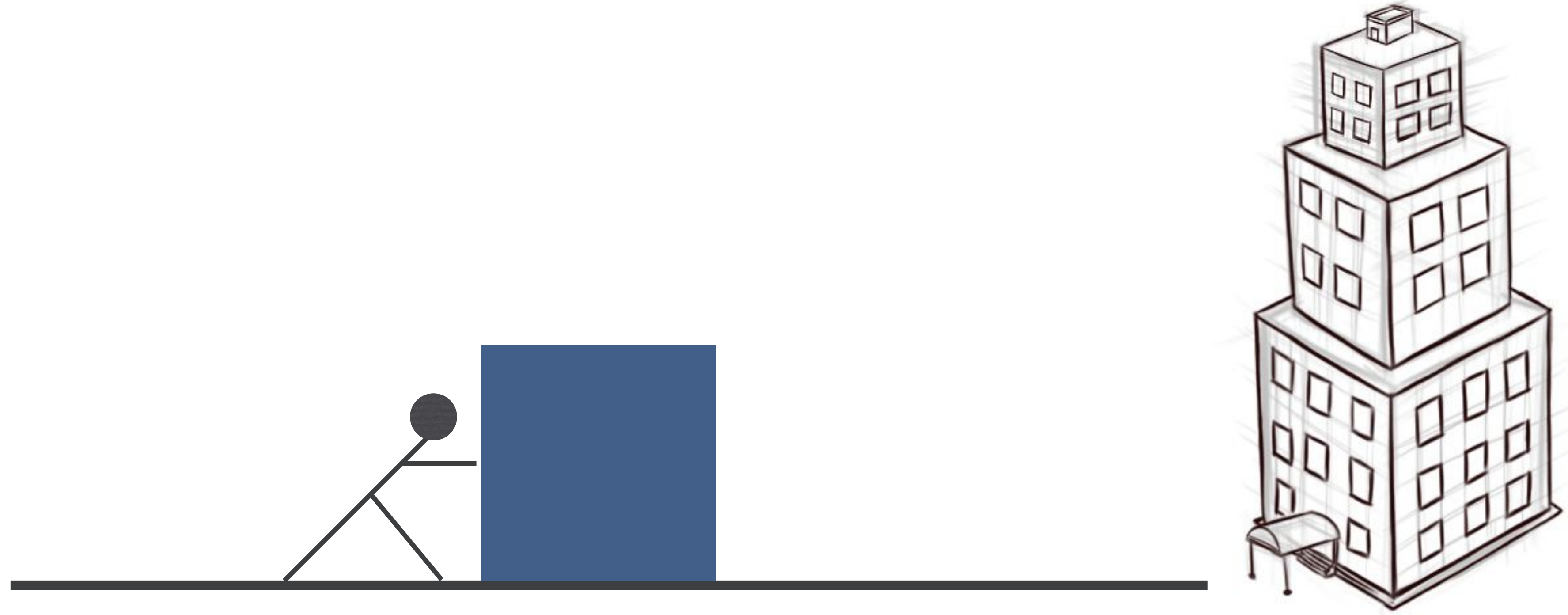
Standard model

Try harder!



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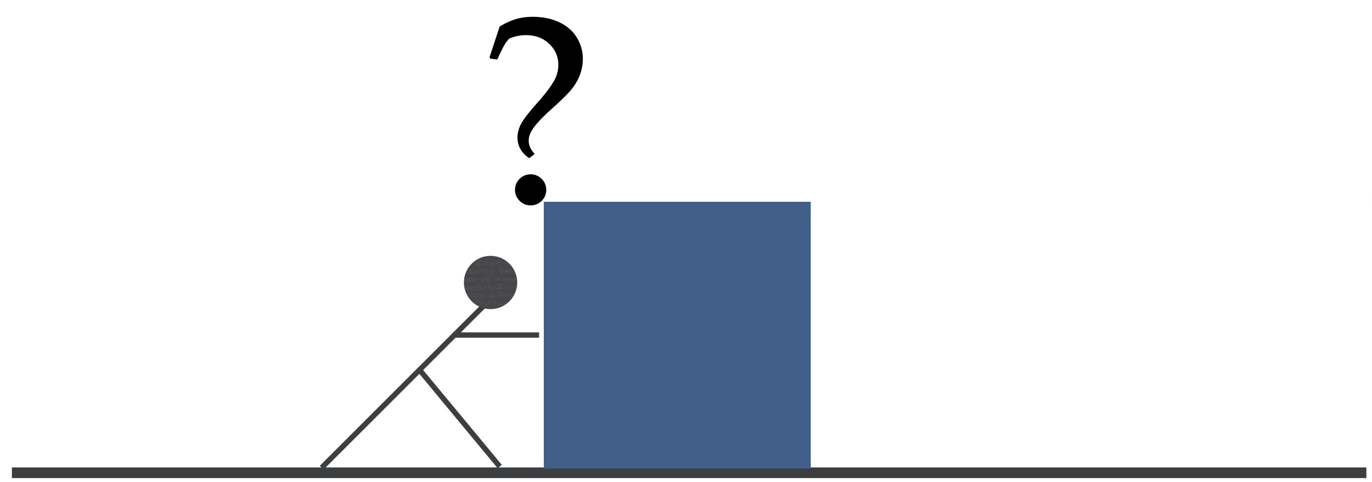

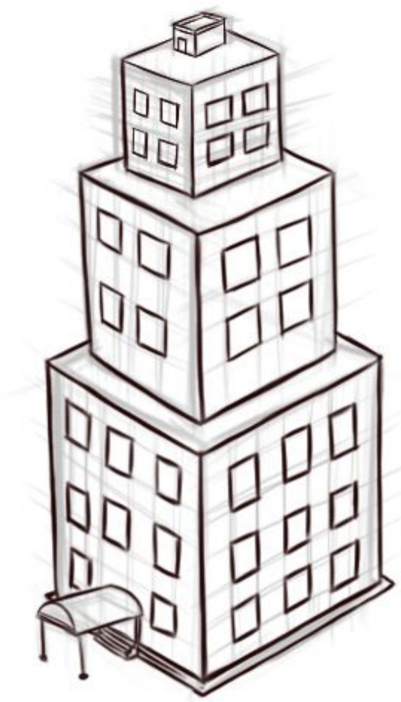

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Use approximations!

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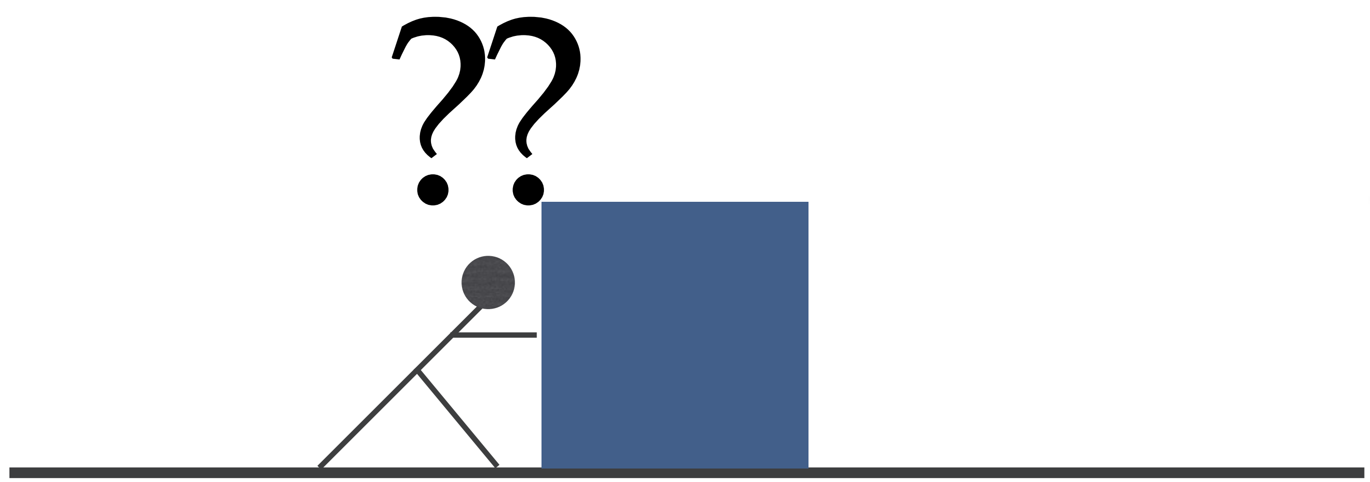

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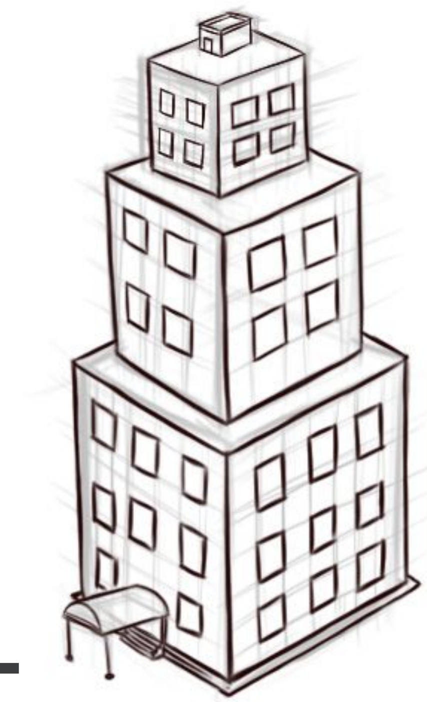

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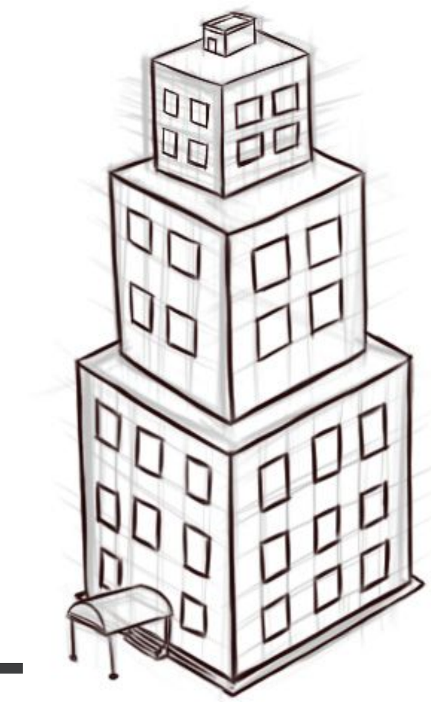

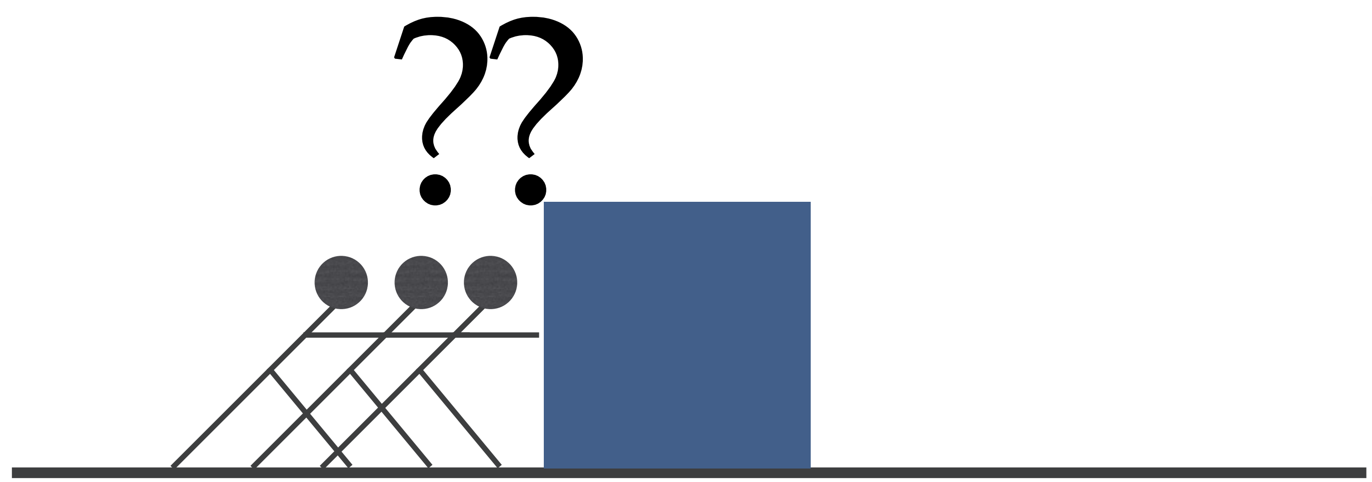

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Ask others to join!

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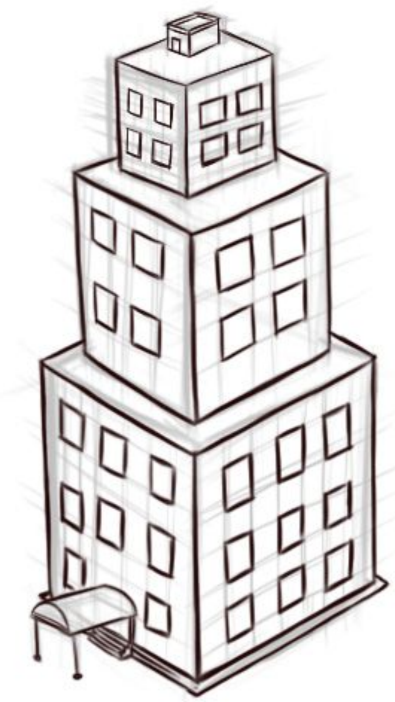
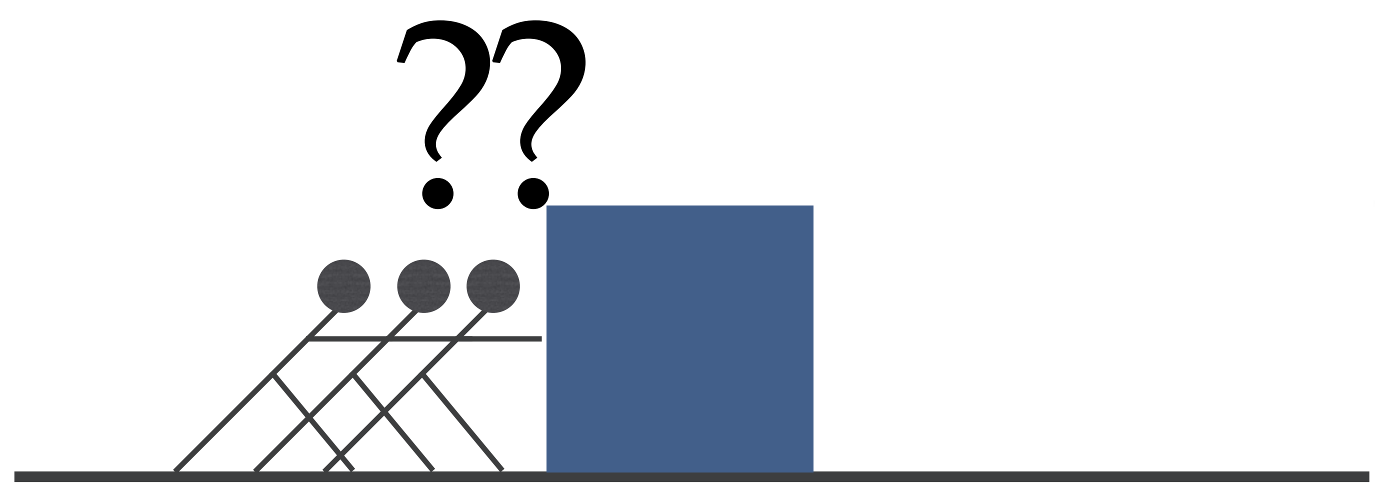




Try harder!

Use approximations!

Use numerical methods!

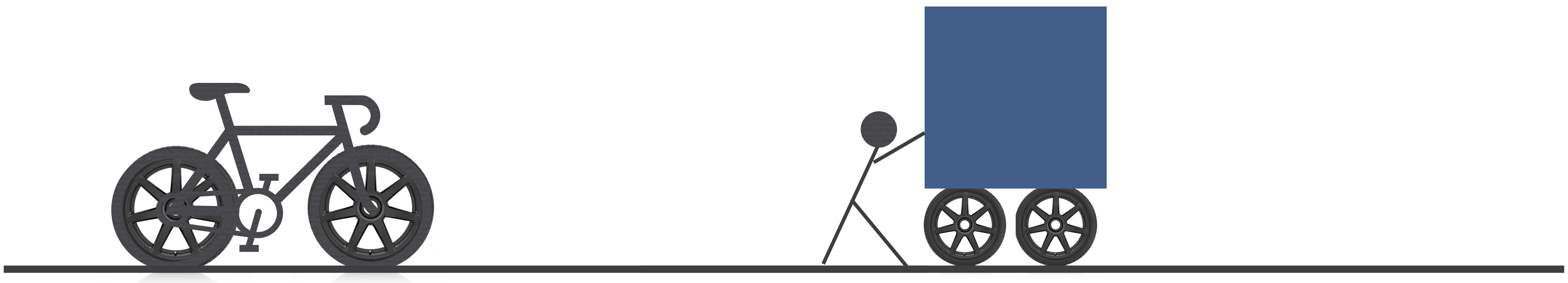
Ask others to join!

Suppose you succeed, did you really understand the problem?

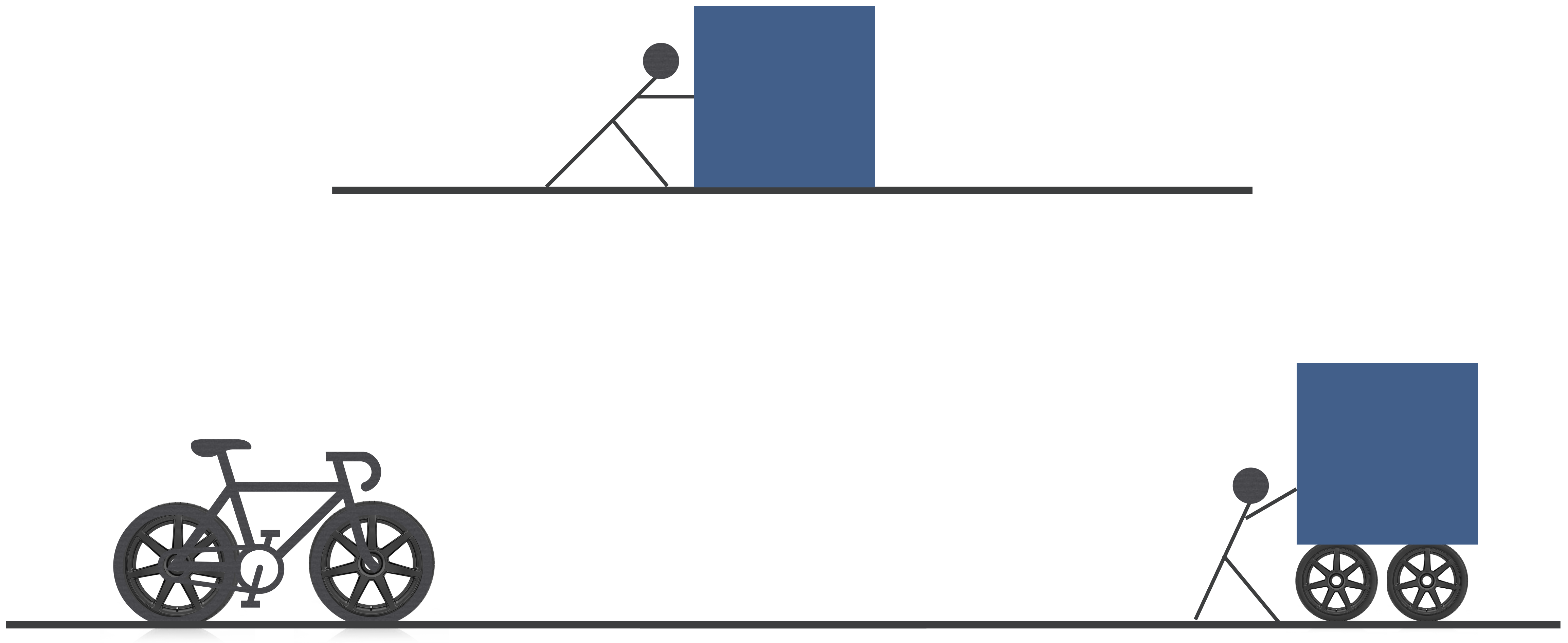
Take a different perspective: use analogies

Take a different perspective: use analogies

Take a different perspective: use analogies



Take a different perspective: use analogies



Take a different perspective: use analogies

We do not have to reinvent the wheel…

Using the wheel in different ways we better understand how it works

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson[3] has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

The simplest theory which exhibits this behavior is a gauge-invariant version of a model used by Goldstone[2] himself: Two real[4] scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density

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where

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$$\partial_\nu F^{\mu\nu} = e\varphi_0\{\partial^\mu(\Delta\varphi_1) - e\varphi_0 A^\mu\}. \qquad (2c)$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^2)\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_\mu = A_\mu - (e\varphi_0)^{-1}\partial_\mu(\Delta\varphi_1),$$

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Equation (4) describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e = 0$) the situation is quite different: Equations (2a) and (2c) describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of (2c) is just the linear approximation to the conserved current: It is linear in the vector potential, gauge invariance being maintained by the presence of the gradient term.[5]

When one considers theoretical models in which spontaneous breakdown of symmetry under a semisimple group occurs, one encounters a variety of possible situations corresponding to the various distinct irreducible representations to which the scalar fields may belong; the gauge field always belongs to the adjoint representation.[6] The model of the most immediate interest is that in which the scalar fields form an octet under SU(3): Here one finds the possibility of two nonvanishing vacuum expectation values, which may be chosen to be the two $Y = 0$, $I_3 = 0$ members of the octet.[7] There are two massive scalar bosons with just these quantum numbers; the remaining six components of the scalar octet combine with the corresponding components of the gauge-field octet to describe

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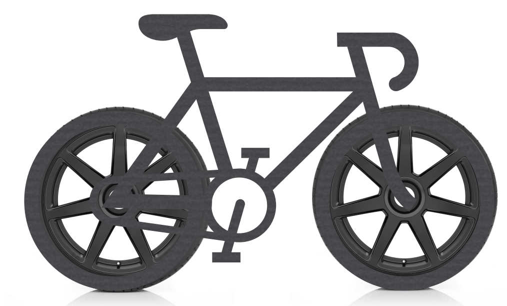
Analog of what was proposed in superconductors



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In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson[3] has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

The simplest theory which exhibits this behavior is a gauge-invariant version of a model used by Goldstone[2] himself: Two real[4] scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density

$$L=-\tfrac{1}{2}(\nabla\varphi_1)^2-\tfrac{1}{2}(\nabla\varphi_2)^2-V(\varphi_1^{\,2}+\varphi_2^{\,2})-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad (1)$$

where

$$\nabla_\mu\varphi_1=\partial_\mu\varphi_1-eA_\mu\varphi_2,$$

$$\nabla_\mu\varphi_2=\partial_\mu\varphi_2+eA_\mu\varphi_1,$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,$$

e is a dimensionless coupling constant, and the metric is taken as $-+++$. L is invariant under simultaneous gauge transformations of the first kind on $\varphi_1 \pm i\varphi_2$ and of the second kind on A_μ. Let us suppose that $V'(\varphi_0^{\,2})=0$, $V''(\varphi_0^{\,2})>0$; then spontaneous breakdown of U(1) symmetry occurs. Consider the equations [derived from (1) by treating $\Delta\varphi_1$, $\Delta\varphi_2$, and A_μ as small quantities] governing the propagation of small oscillations about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1)-e\varphi_0A_\mu\}=0, \qquad (2a)$$

$$\{\partial^2-4\varphi_0^{\,2}V''(\varphi_0^{\,2})\}(\Delta\varphi_2)=0, \qquad (2b)$$

$$\partial_\nu F^{\mu\nu}=e\varphi_0\{\partial^\mu(\Delta\varphi_1)-e\varphi_0A^\mu\}. \qquad (2c)$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^{\,2})\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_\mu=A_\mu-(e\varphi_0)^{-1}\partial_\mu(\Delta\varphi_1),$$

$$G_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu=F_{\mu\nu}, \qquad (3)$$

into the form

$$\partial_\mu B^\mu=0,\quad \partial_\nu G^{\mu\nu}+e^2\varphi_0^{\,2}B^\mu=0. \qquad (4)$$

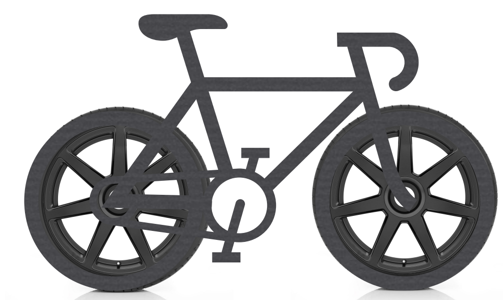
Equation (4) describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e=0$) the situation is quite different: Equations (2a) and (2c) describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of (2c) is just the linear approximation to the conserved current: It is linear in the vector potential, gauge invariance being maintained by the presence of the gradient term.[5]

When one considers theoretical models in which spontaneous breakdown of symmetry under a semisimple group occurs, one encounters a variety of possible situations corresponding to the various distinct irreducible representations to which the scalar fields may belong; the gauge field always belongs to the adjoint representation.[6] The model of the most immediate interest is that in which the scalar fields form an octet under SU(3): Here one finds the possibility of two nonvanishing vacuum expectation values, which may be chosen to be the two $Y=0$, $I_3=0$ members of the octet.[7] There are two massive scalar bosons with just these quantum numbers; the remaining six components of the scalar octet combine with the corresponding components of the gauge-field octet to describe

The hard problem: failing of the Goldstone theorem for gauge symmetries

Massive gauge bosons appear

Analog of what was proposed in superconductors



Plasmons, Gauge Invariance, and Mass

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 8 November 1962)

Schwinger has pointed out that the Yang-Mills vector boson implied by associating a generalized gauge transformation with a conservation law (of baryonic charge, for instance) does not necessarily have zero mass, if a certain criterion on the vacuum fluctuations of the generalized current is satisfied. We show that the theory of plasma oscillations is a simple nonrelativistic example exhibiting all of the features of Schwinger's idea. It is also shown that Schwinger's criterion that the vector field $m\neq 0$ implies that the matter spectrum before including the Yang-Mills interaction contains $m=0$, but that the example of superconductivity illustrates that the physical spectrum need not. Some comments on the relationship between these ideas and the zero-mass difficulty in theories with broken symmetries are given.

RECENTLY, Schwinger[1] has given an argument strongly suggesting that associating a gauge transformation with a local conservation law does not necessarily require the existence of a zero-mass vector boson. For instance, it had previously seemed impossible to describe the conservation of baryons in such a manner because of the absence of a zero-mass boson and of the accompanying long-range forces.[2] The problem of the mass of the bosons represents the major stumbling block in Sakurai's attempt to treat the dynamics of strongly interacting particles in terms of the Yang-Mills gauge fields which seem to be required to accompany the known conserved currents of baryon number and hypercharge.[3] (We use the term "Yang-Mills" in Sakurai's sense, to denote any generalized gauge field accompanying a local conservation law.)

The purpose of this article is to point out that the familiar plasmon theory of the free-electron gas exemplifies Schwinger's theory in a very straightforward manner. In the plasma, transverse electromagnetic waves do not propagate below the "plasma frequency," which is usually thought of as the frequency of long-wavelength longitudinal oscillation of the electron gas. At and above this frequency, three modes exist, in close analogy (except for problems of Galilean invariance implied by the inequivalent dispersion of longitudinal and transverse modes) with the massive vector boson mentioned by Schwinger. The plasma frequency is equivalent to the mass, while the finite density of electrons leading to divergent "vacuum" current fluctuations resembles the strong renormalized coupling of Schwinger's theory. In spite of the absence of low-frequency photons, gauge invariance and particle conservation are clearly satisfied in the plasma.

In fact, one can draw a direct parallel between the dielectric constant treatment of plasmon theory[4] and Schwinger's argument. Schwinger comments that the commutation relations for the gauge field A give us one sum rule for the vacuum fluctuations of A, while those for the matter field give a completely independent value for the fluctuations of matter current j. Since j is the source for A and the two are connected by field equations, the two sum rules are normally incompatible unless there is a contribution to the A rule from a free, homogeneous, weakly interacting, massless solution of the field equations. If, however, the source term is large enough, there can be no such contribution and the massless solutions cannot exist.

The usual theory of the plasmon does not treat the electromagnetic field quantum-mechanically or discuss vacuum fluctuations; yet there is a close relationship between the two arguments, and we, therefore, show that the quantum nature of the gauge field is irrelevant. Our argument is as follows:

The equation for the electromagnetic field is

$$p^2A_\mu=(k^2-\omega^2)A_\mu(\mathbf{k},\omega)=4\pi j_\mu(\mathbf{k},\omega).$$

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The Fourier transform of the corresponding retarded Green's function is our response function:

$$K'(p) = \int \frac{dm^2\, m^2 B_1(m^2)}{p^2-m^2}[p_\mu p_\nu - g_{\mu\nu}p^2],$$

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Thus, (aside from a factor 4π which Schwinger has not used in his field equation) his criterion is also that the polarizability α', here expressed in terms of a dispersion integral, have its maximum possible value, 1.

The polarizability of the vacuum is not generally considered to be observable[6] except in its p dependence (terms of order p^4 or higher in K). In fact, we can remove (11) entirely by the conventional renormalization of the field and charge

$$A_r = AZ^{-1/2}, \quad e_r = eZ^{1/2}, \quad j_r = jZ^{1/2}.$$

Z, here, can be shown to be precisely

$$Z = 1 - 4\pi\alpha' = 1 - \int_0^\infty dm^2\, B_1(m^2).$$

Thus, the renormalization procedure is possible for any merely polarizable "vacuum," but not for the special case of the conducting "plasma" type of vacuum. In this case, no net true charge remains localized in the region of the dressed particle; all of the charge is carried "at infinity" corresponding to the fact, well known in the theory of metals, that all the charge carried by a quasi-particle in a plasma is actually on the surface. Nonetheless, conservation of particles, if not of bare charge, is strictly maintained. Note that the situation does not resemble the case of "infinite" charge renormalization because the infinity in the vacuum polarizability need only occur at $p^2=0$.

Either in the case of the polarizable vacuum or of the "conducting" one, no low-energy experiment, and even possibly no high-energy one, seems capable of directly testing the value of the vacuum polarizability prior to renormalization. Thus, we conclude that the plasmon is a physical example demonstrating Schwinger's contention that under some circumstances the Yang-Mills type of vector boson need not have zero mass. In addition, aside from the short range of forces and the finite mass, which we might interpret without resorting to Yang-Mills, it is not obvious how to characterize such a case mathematically in terms of observable, renormalized quantities.

We can, on the other hand, try to turn the problem around and see what other conclusions we can draw about possible Yang-Mills models of strong interactions from the solid-state analogs. What properties of the vacuum are needed for it to have the analog of a conducting response to the Yang-Mills field?

Back to Yang-Mills model

Certainly the fact that the polarizability of the "matter" system, without taking into account the interaction with the gauge field, is infinite need not bother us, since that is unobservable. In physical conductors we can see it, but only because we can get outside them and apply to them true electromagnetic fields, not only internal test charges.

More serious is the implication—obviously physically from the fact that α has a pole at $p^2=0$—that the "matter" spectrum, at least for the "undressed" matter system, must extend all the way to $m^2=0$. In the normal plasma even the final spectrum extends to zero frequency, the coupling rather than the spectrum being affected by the screening. Is this necessarily always the case? The answer is no, obviously, since the superconducting electron gas has no zero-mass excitations whatever. In that case, the fermion mass is finite because of the energy gap, while the boson which appears as a result of the theorem of Goldstone[7,8] and has zero unrenormalized mass is converted into a finite-mass plasmon by interaction with the appropriate gauge field, which is the electromagnetic field. The same is true of the charged Bose gas.

Superconductors work as analogs

It is likely, then, considering the superconducting analog, that the way is now open for a degenerate-vacuum theory of the Nambu type[9] without any difficulties involving either zero-mass Yang-Mills gauge bosons or zero-mass Goldstone bosons. These two types of bosons seem capable of "canceling each other out" and leaving finite mass bosons only. It is not at all clear that the way for a Sakurai[3] theory is equally uncluttered. The only mechanism suggested by the present work (of course, we have not discussed non-Abelian gauge groups) for giving the gauge field mass is the degenerate vacuum type of theory, in which the original symmetry is not manifest in the observable domain. Therefore, it needs to be demonstrated that the necessary conservation laws can be maintained.

I should like to close with one final remark on the Goldstone theorem. This theorem was initially conjectured, one presumes, because of the solid-state analogs, via the work of Nambu[10] and of Anderson.[11] The theorem states, essentially, that if the Lagrangian

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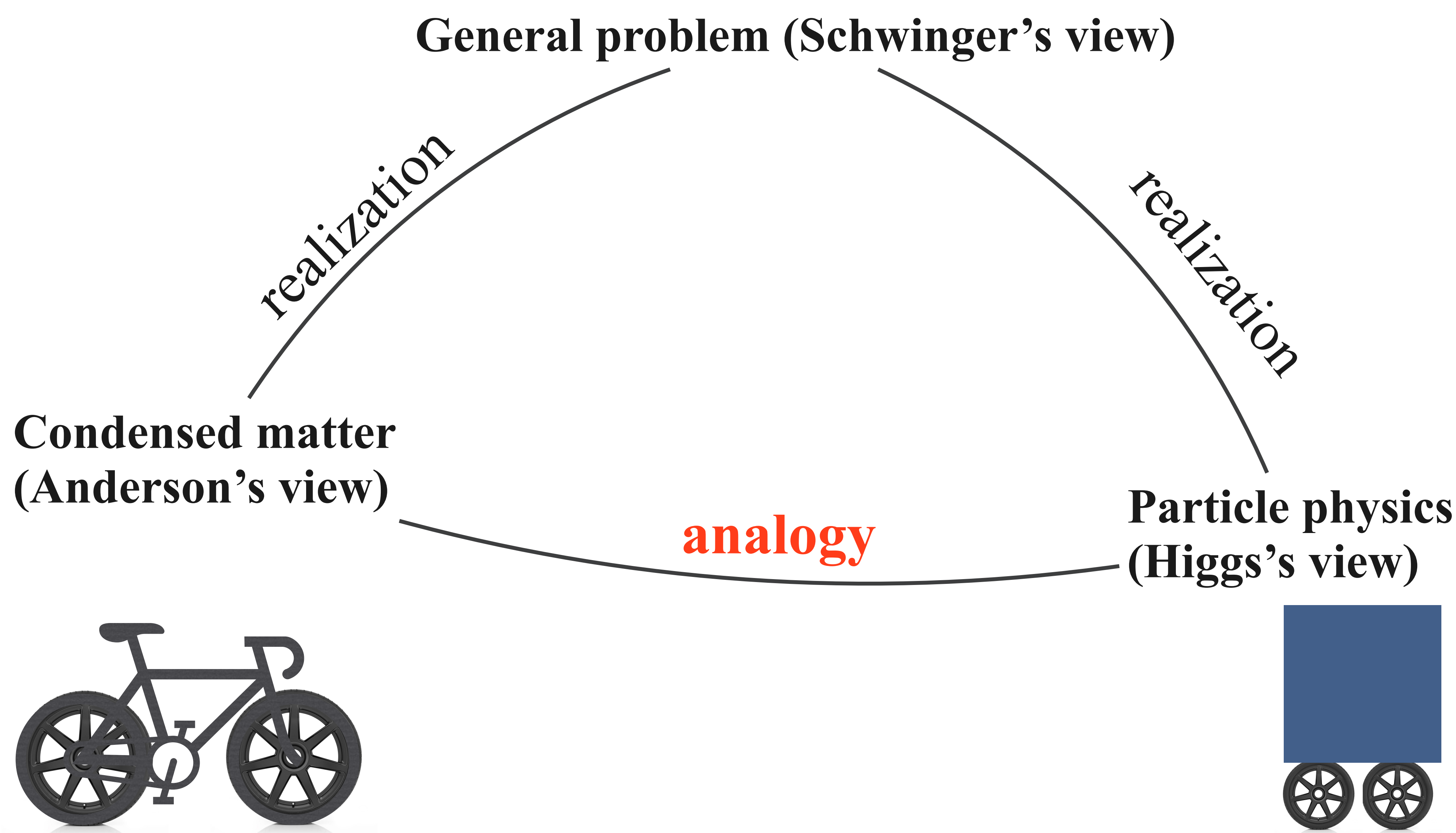

Using analogies different approaches are **intertwined**

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An **analogy** is a correspondence between **phenomena,** realized in two or more different physical systems, such that their descriptions, at least within *some energy range*, relies on the **same physical process**.

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- The analogy can be experimentally exploited realizing systems that have common features. This is particularly useful when one of the two systems is unreachable.

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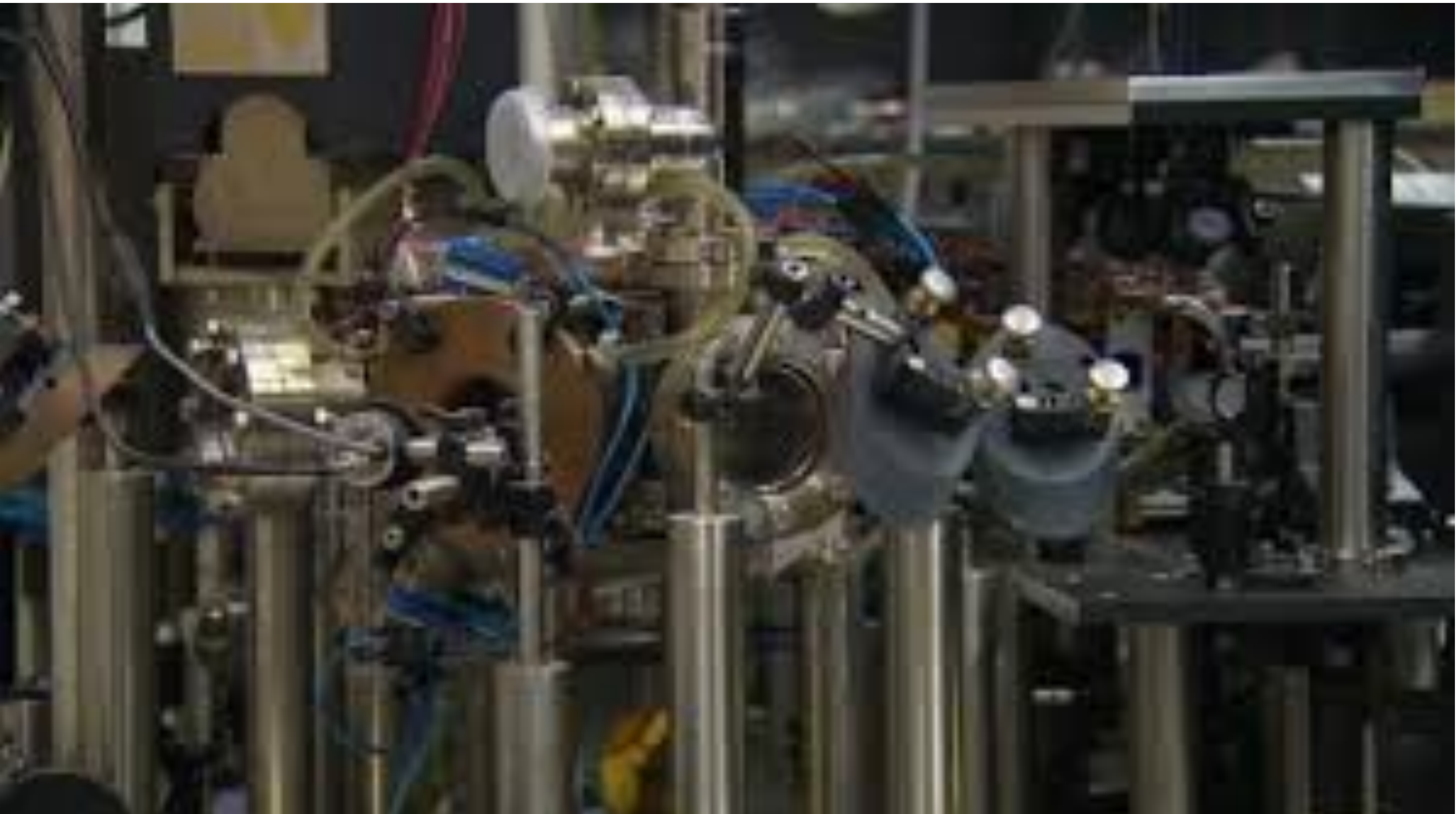
Unreachable system

BH analog in a lab



Some analogies

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Particle propagation ⟷ ***Wave propagation***

Some analogies

Particle propagation ⟷ *Wave propagation*

Higgs mechanism ⟷ *Meissner effect*

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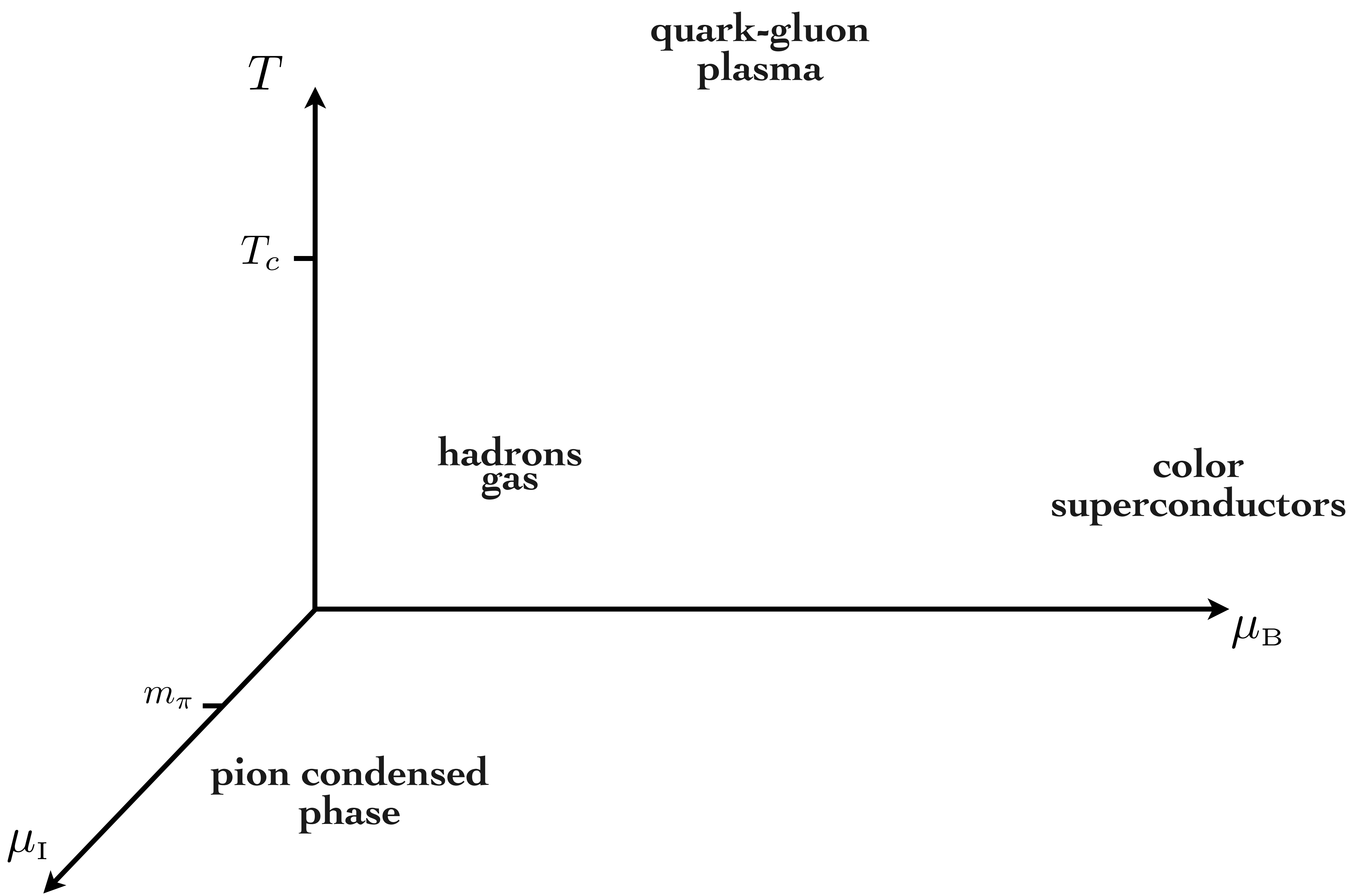
Phonon in a fluid ⟷ *Scalar fields in GR*

Analogies for quark matter

Mark G. Alford et al. “Color superconductivity in dense quark matter”, Rev.Mod.Phys. 80 (2008) 1455-1515

MM, “Meson condensation”, MDPI-Particles 2 (2019) no.3, 411

Phases of hadronic matter



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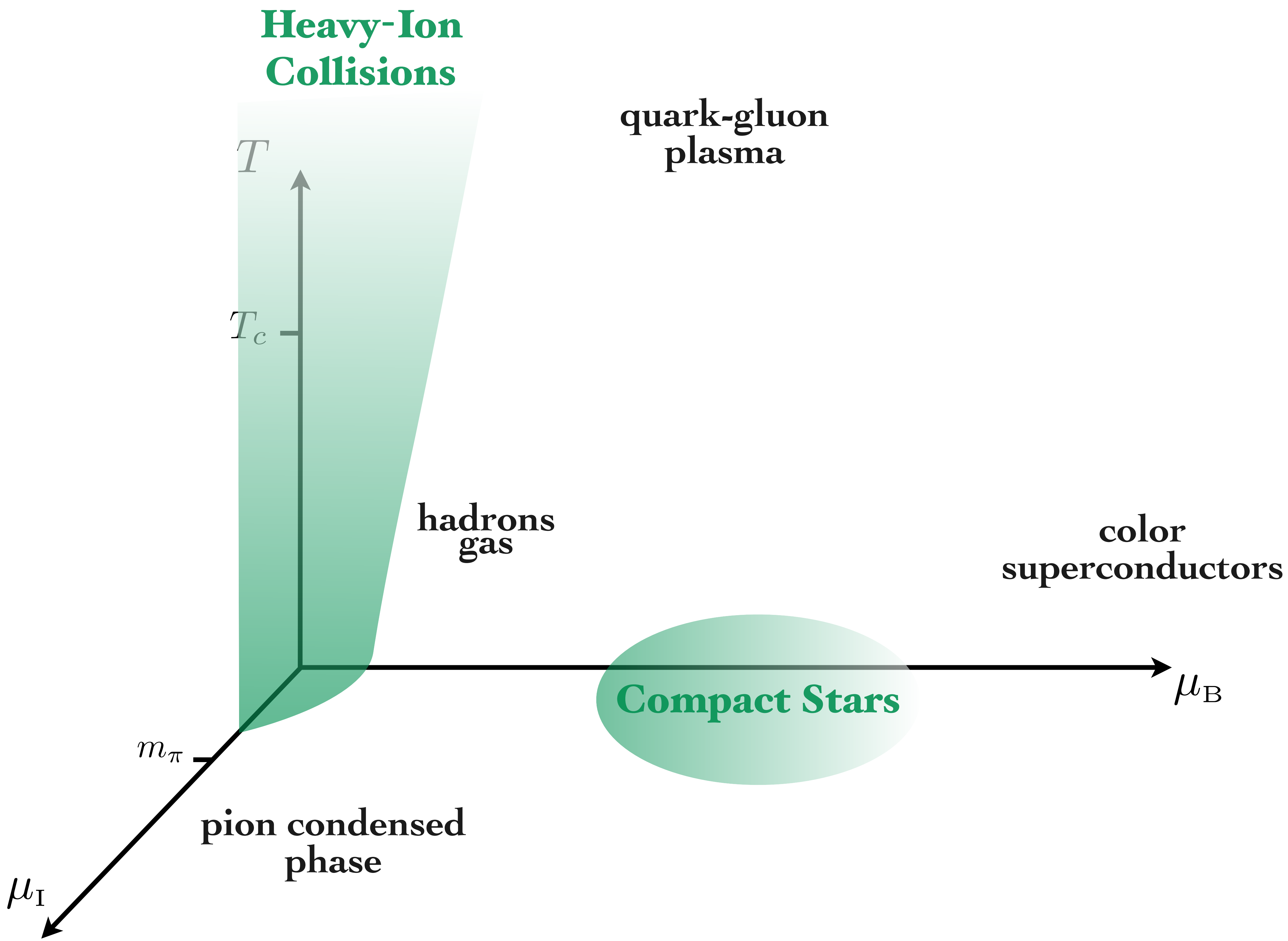

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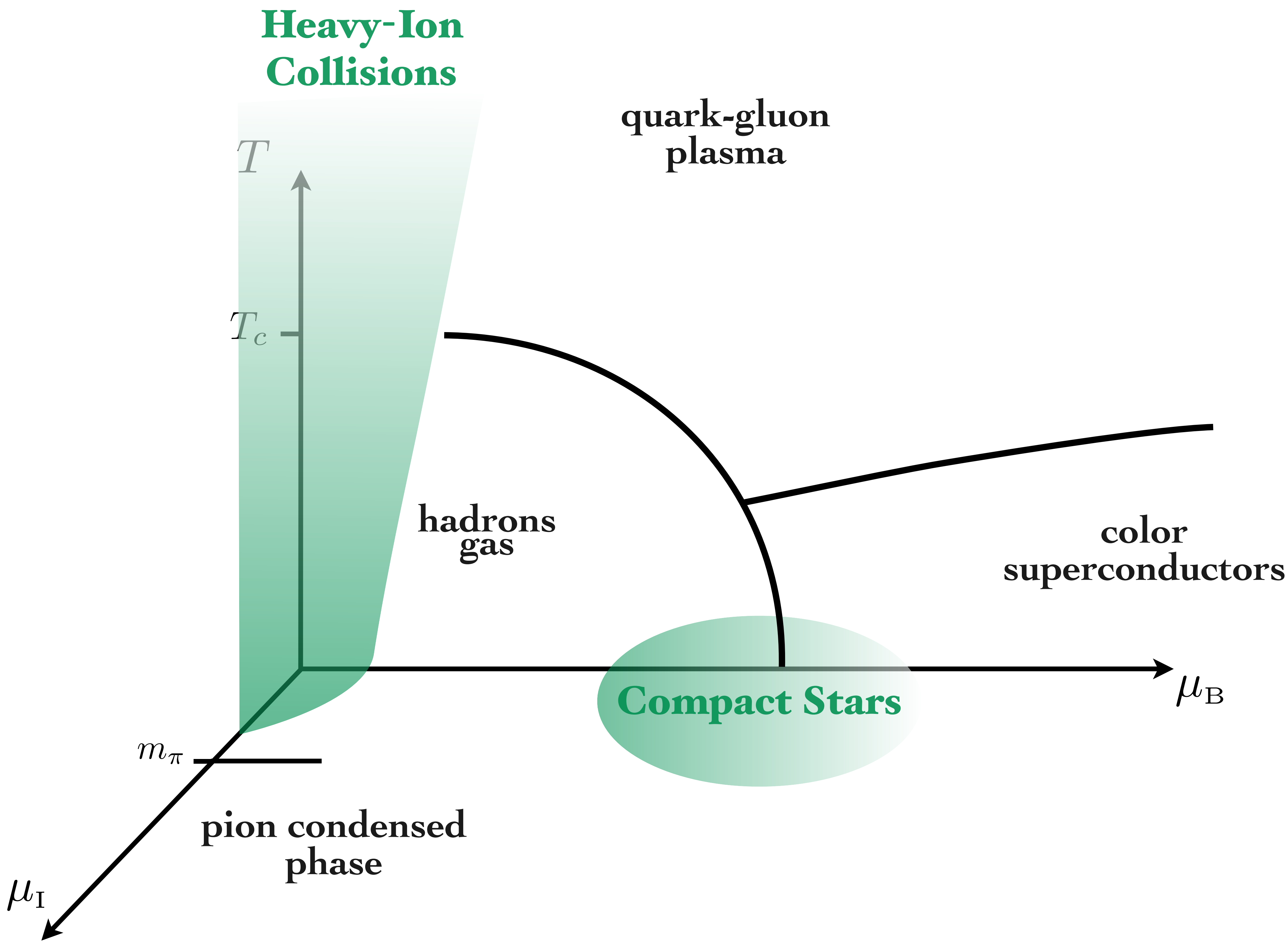

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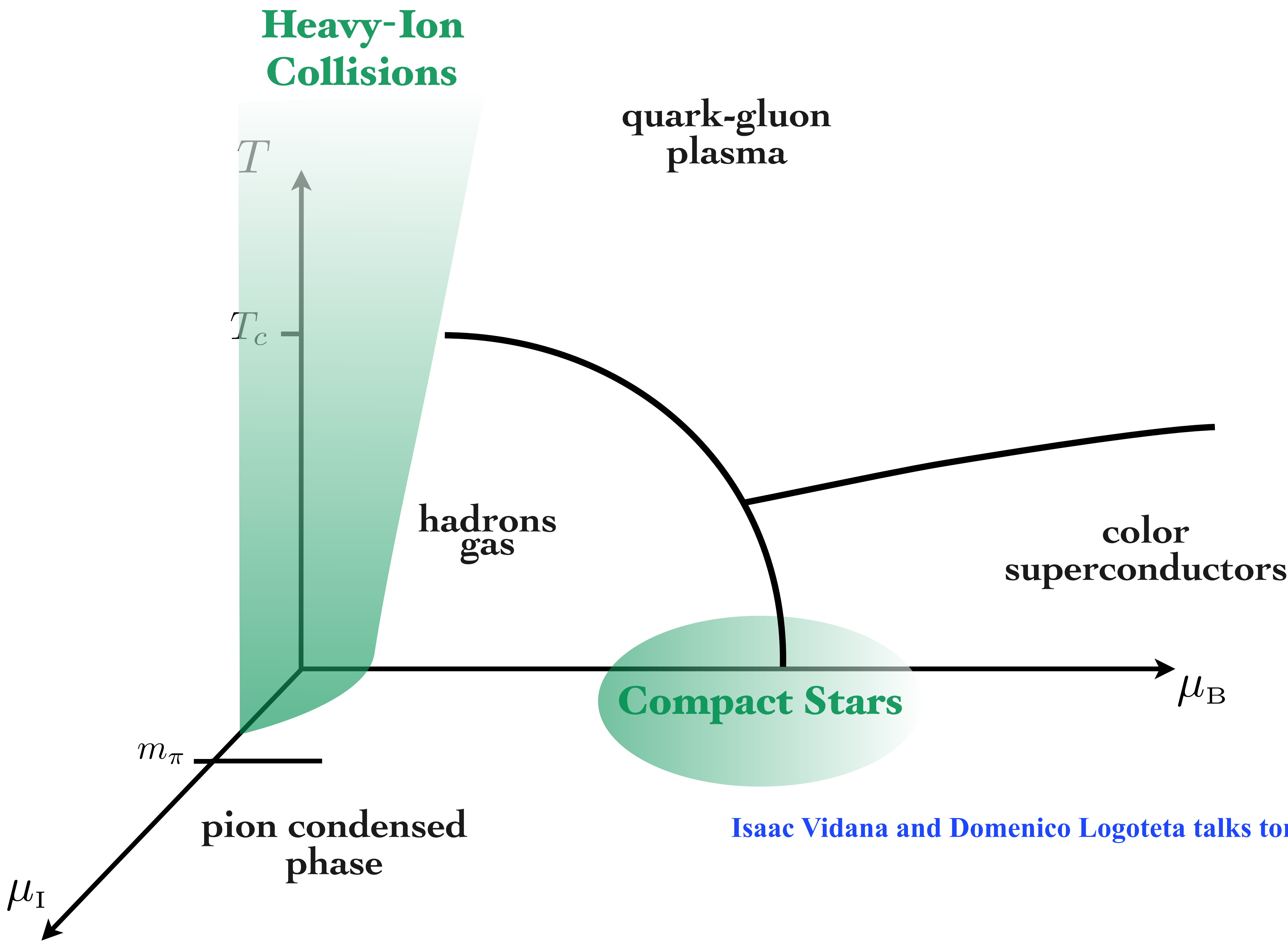
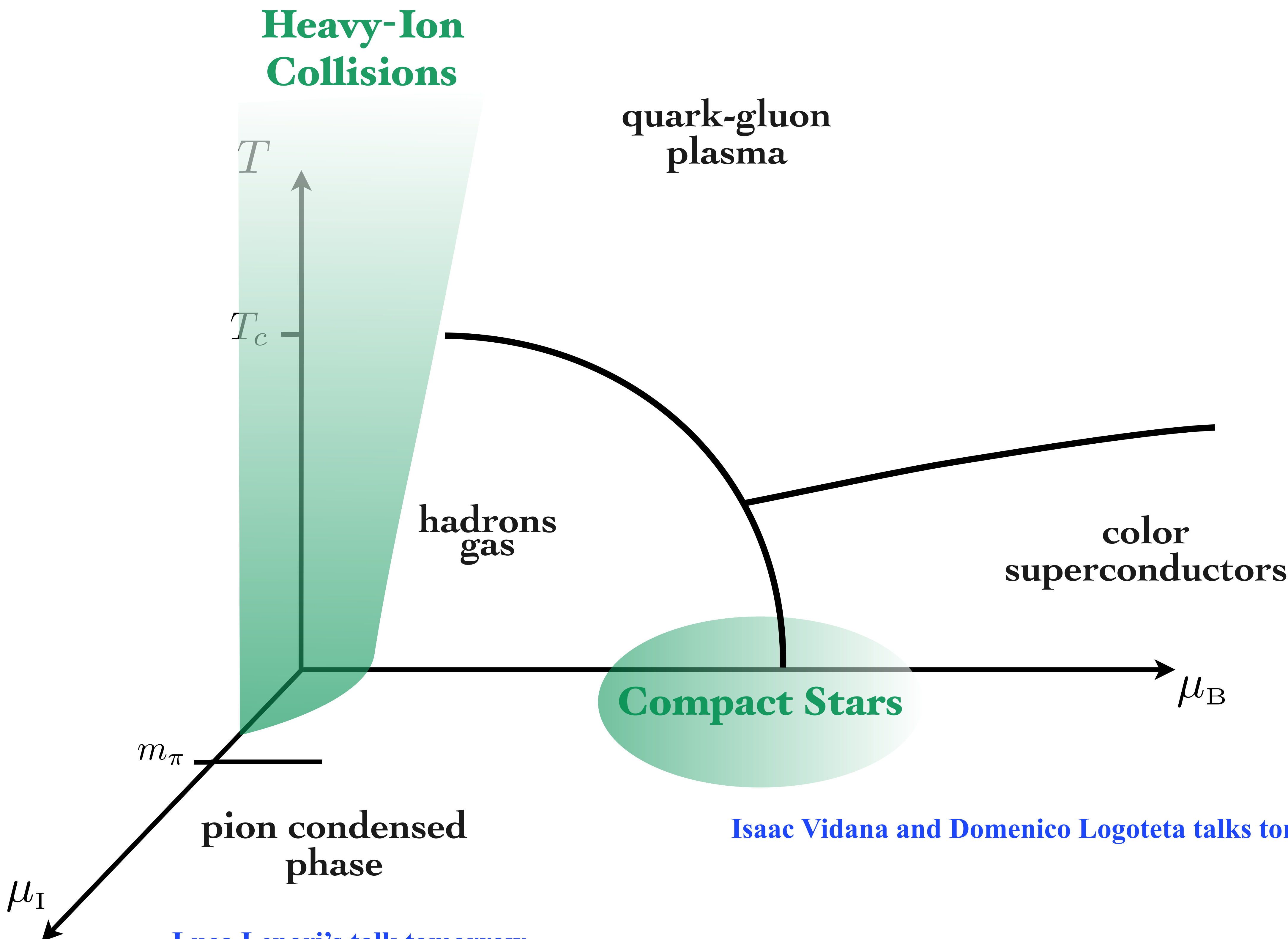


Isaac Vidana and Domenico Logoteta talks tomorrow

Phases of hadronic matter



Isaac Vidana and Domenico Logoteta talks tomorrow

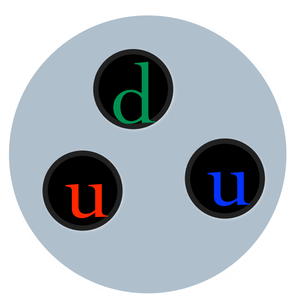
Luca Lepori's talk tomorrow

Quark matter

Building blocks of hadrons are **quarks and gluons**

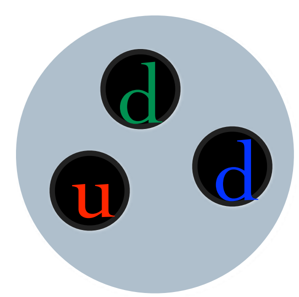
Q	Quarks (mass in MeV)		
$+2/3$	u (3)	c (1300)	t (170000)
$-1/3$	d (5)	s (130)	b (4000)

proton



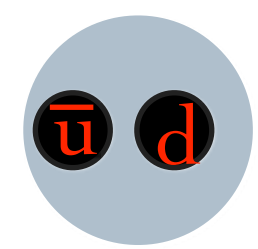
$p =$ "uud"

neutron



$n =$ "udd"

π^-



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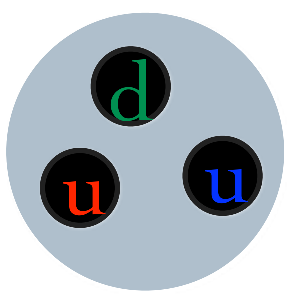

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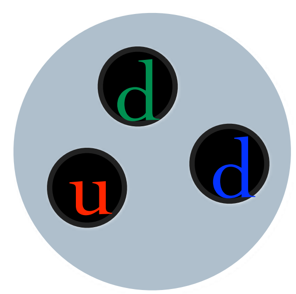

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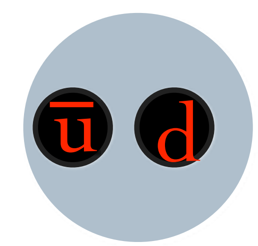

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...

Open problems: Where does the proton mass come from?
Does an analog of confinement exist?

Symmetries of QCD

Neglecting u, d and s quark masses

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B$$

gauge color

global chiral symmetry

global baryonic number

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A large symmetry group can be broken in a zoo of possible phases

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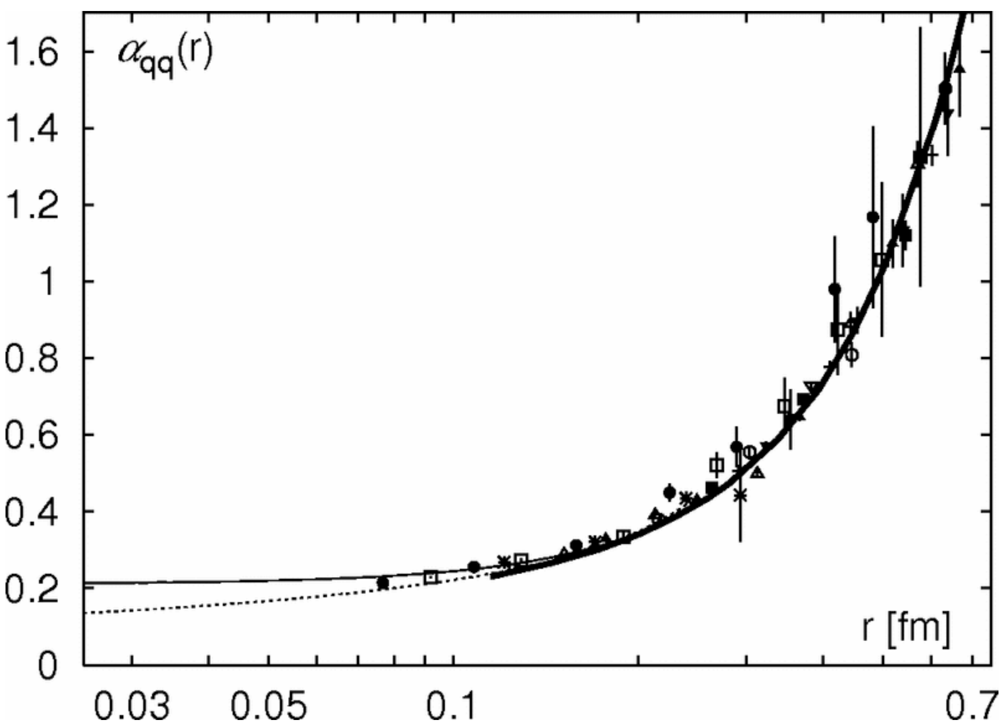
gauge color **global chiral symmetry** **global baryonic number**

A large symmetry group can be broken in a zoo of possible phases

The **analogy** with "standard" fermionic systems may serve as **guidance**

The dawn of color superconductors

QCD is an asymptotic free theory: in the UV interactions are perturbative



"Running" of the QCD interaction strength

Kaczmarek and Zantow
Physical Review D 71(11):114510 (2005)

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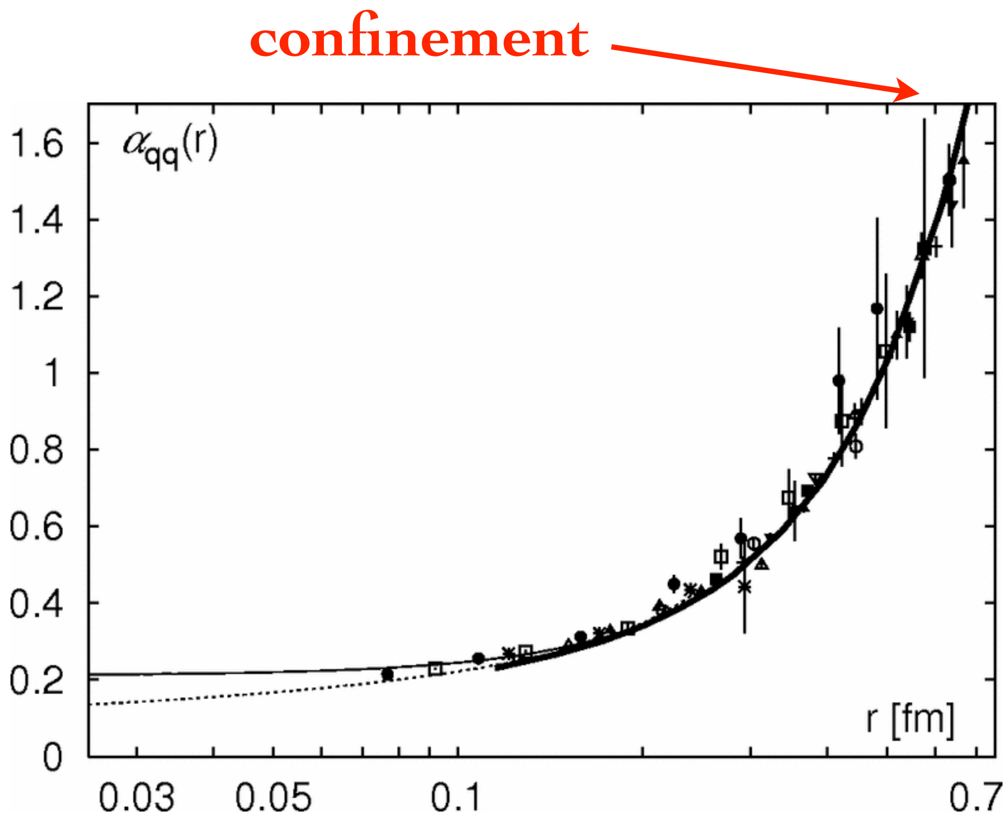

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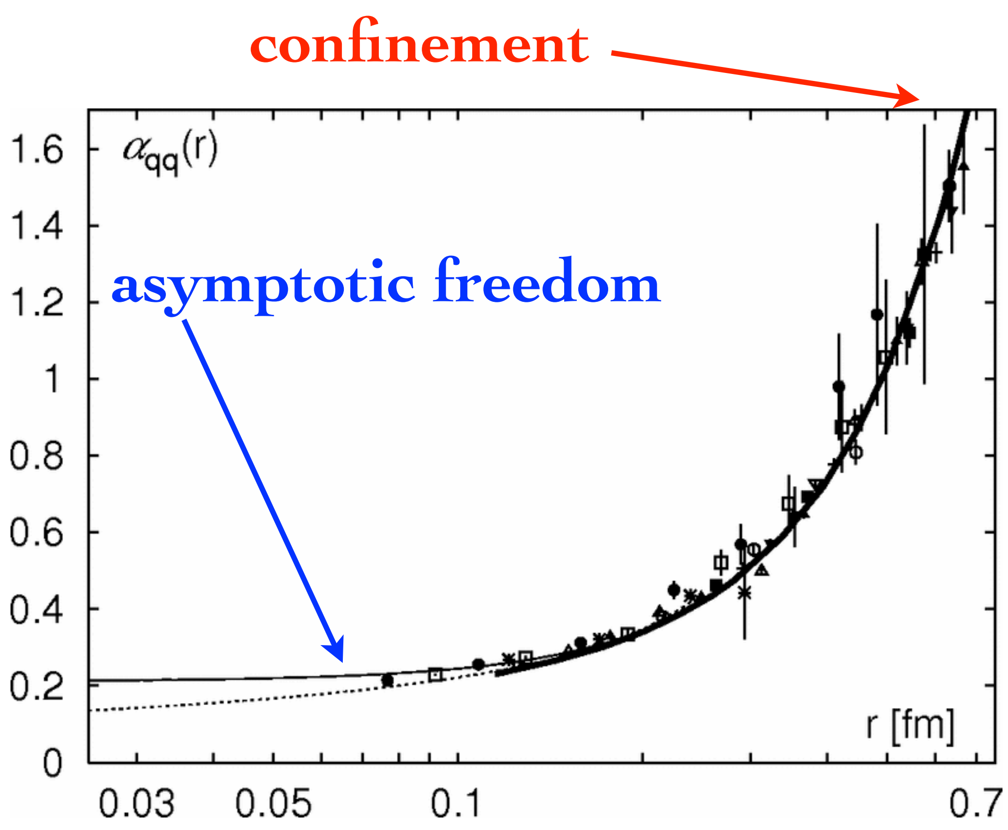

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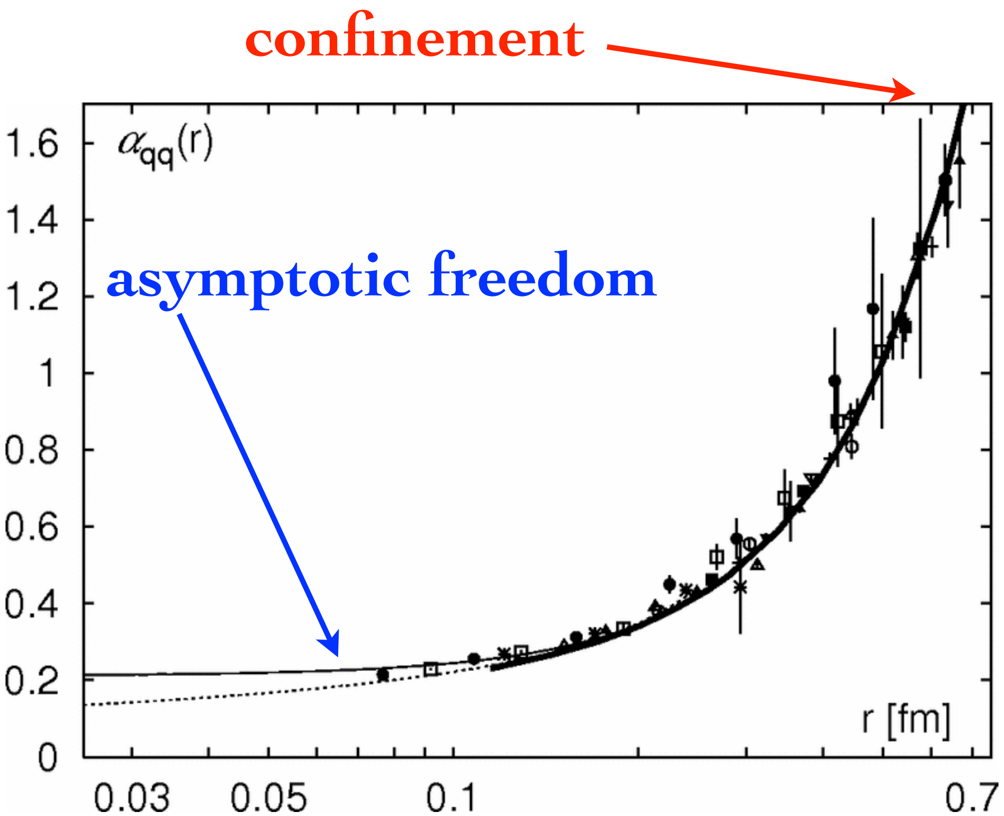

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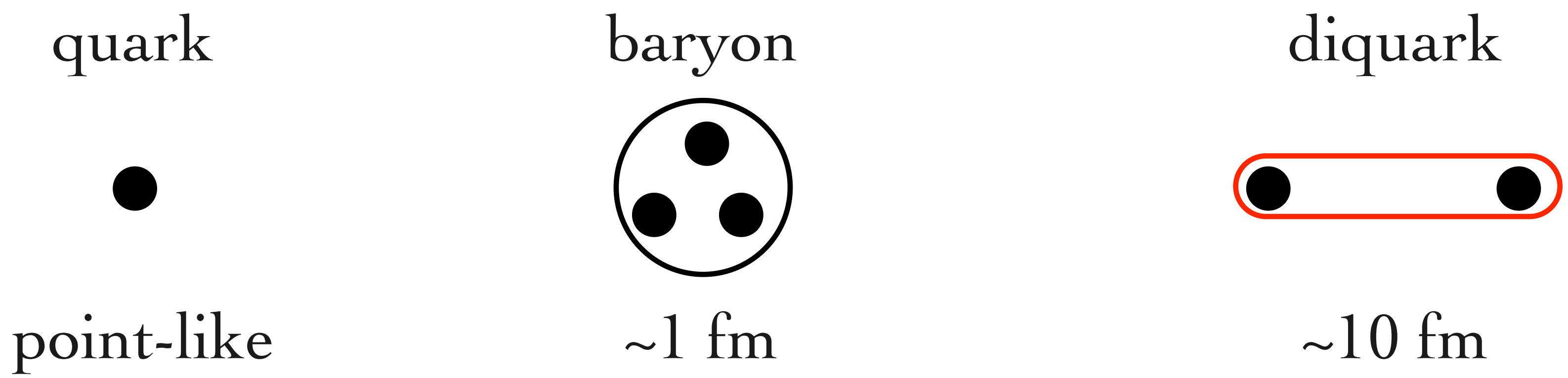
Also we might except [sic]
superfluidity and superconductivity, since the interquark forces are attractive in at least some channels.

J. C. Collins and M.J. Perry Phys.Rev.Lett. 34 (1975) 1353


quark
baryon
diquark
point-like
~1 fm
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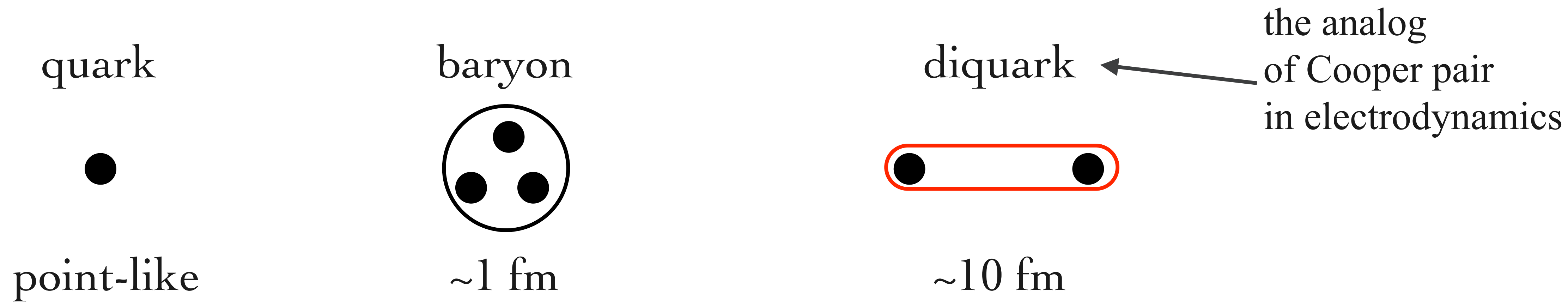
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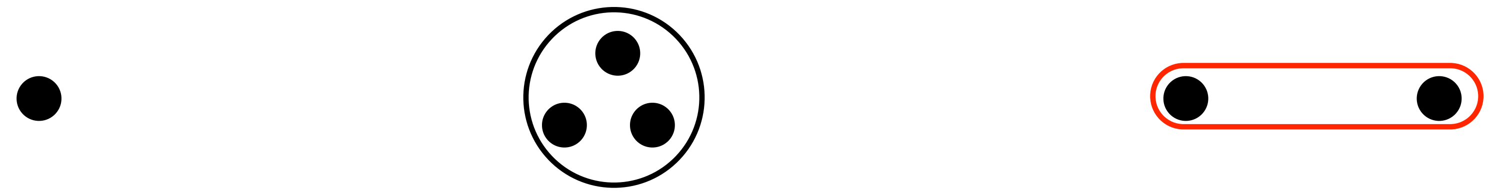
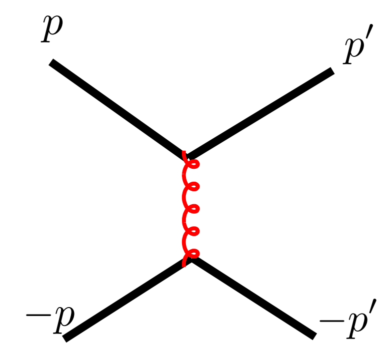


point-like ~1 fm ~10 fm

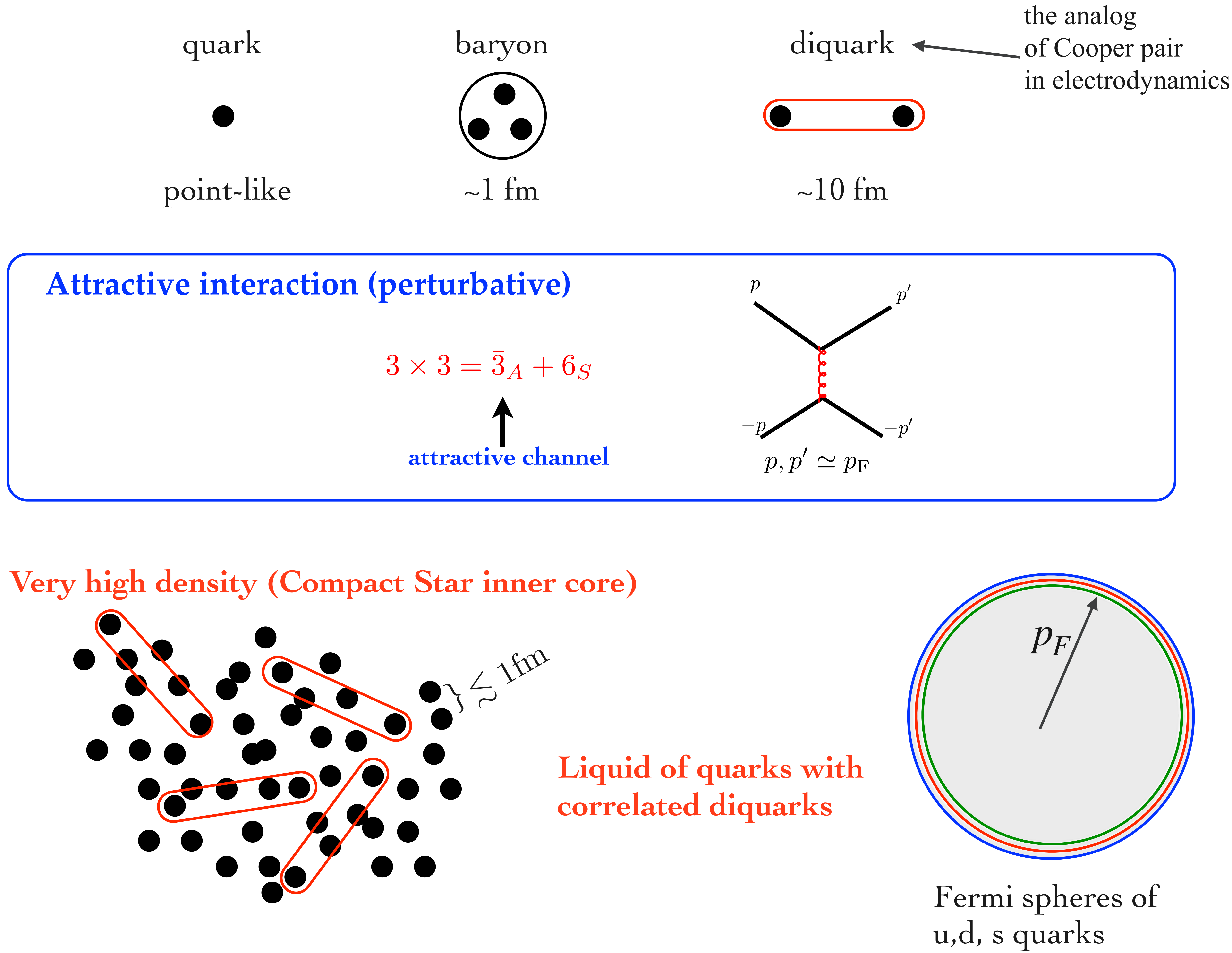
Attractive interaction (perturbative)

$$3 \times 3 = \bar{3}_A + 6_S$$

attractive channel



$p, p' \simeq p_{\mathrm{F}}$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Very high density (Compact Star inner core)
$\} \lesssim 1\mathrm{fm}$
Liquid of quarks with
correlated diquarks
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Fermi spheres of
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Color Flavor Locked phase

Pairing of quarks of all flavors and colors

Alford, Rajagopal, Wilczek Nucl.Phys. B537 (1999) 443

Symmetry breaking

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\mathrm{Q}}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\mathrm{Q}}}} \times Z_2$$

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The system is at the same time a (color) superconductor and a (baryonic) superfluid

Supersolid quark matter

R. Anglani, MM et al. “Crystalline color superconductors”, Review of Modern Physics 86, 509 (2014)

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See Giovanni Modugno’s talk on supersolids tomorrow

Bulk quark matter in compact stars

sizable strange quark mass
+
weak equilibrium
+
electric neutrality

→

mismatch of Fermi momenta

Bulk quark matter in compact stars

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+
electric neutrality

$\longrightarrow$ mismatch of Fermi momenta

No pairing case

weak interactions

$$u \to d + \bar{e} + \nu_e$$
$$u \to s + \bar{e} + \nu_e$$
$$u + d \leftrightarrow u + s$$

$\longrightarrow$

$$\mu_u = \mu_d - \mu_e$$
$$\mu_d = \mu_s$$

electric neutrality $\longrightarrow$ $\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$

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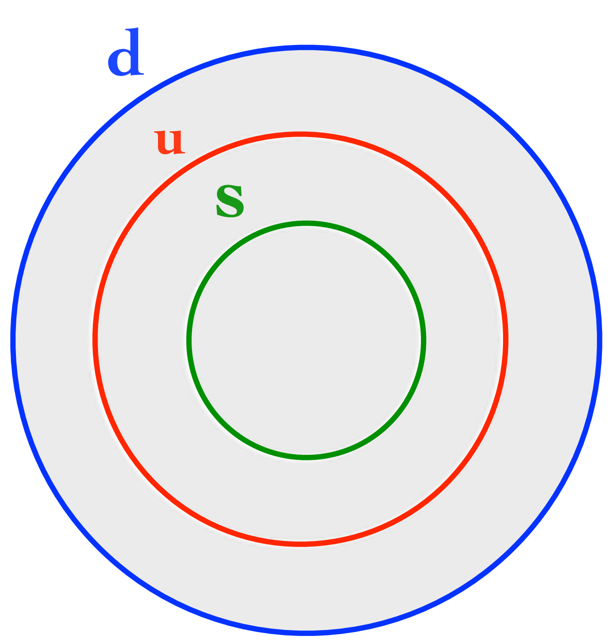

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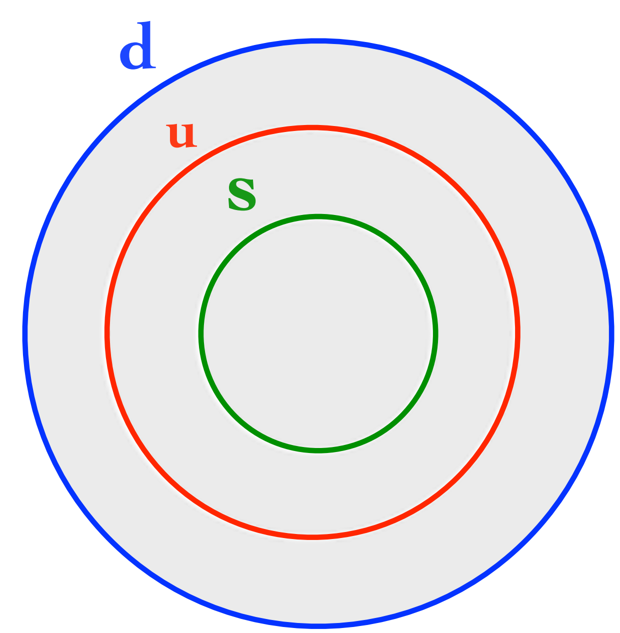

Fermi spheres of u,d, s quarks

Fermi momenta $\quad k_u^F = \mu_u \quad k_d^F = \mu_d \quad k_s^F = \sqrt{\mu_s^2 - m_s^2}$

The FFLO-phase analog

Inhomogeneous superconductor with a spatially modulated condensate

P. Fulde, R.A Ferrell "Superconductivity in a Strong Spin-Exchange Field". Phys. Rev. 135 (3A): A550–A563 (1964).
A.I. Larkin, Yu.N. Ovchinnikov, "Nonuniform state of superconductors" Zh. Eksp. Teor. Fiz. 47: 1136 (1964), Sov.Phys.JETP 20 (1965) 762

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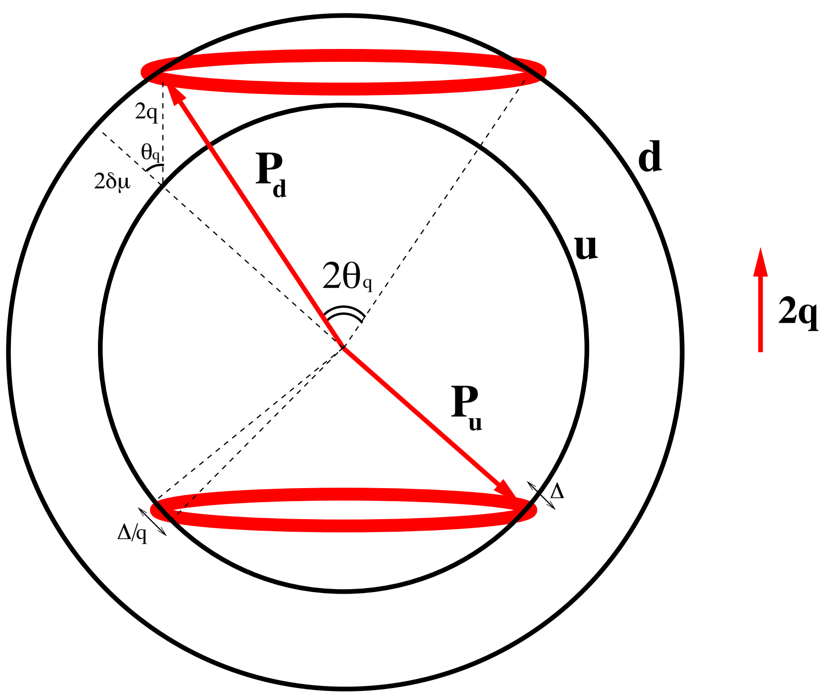

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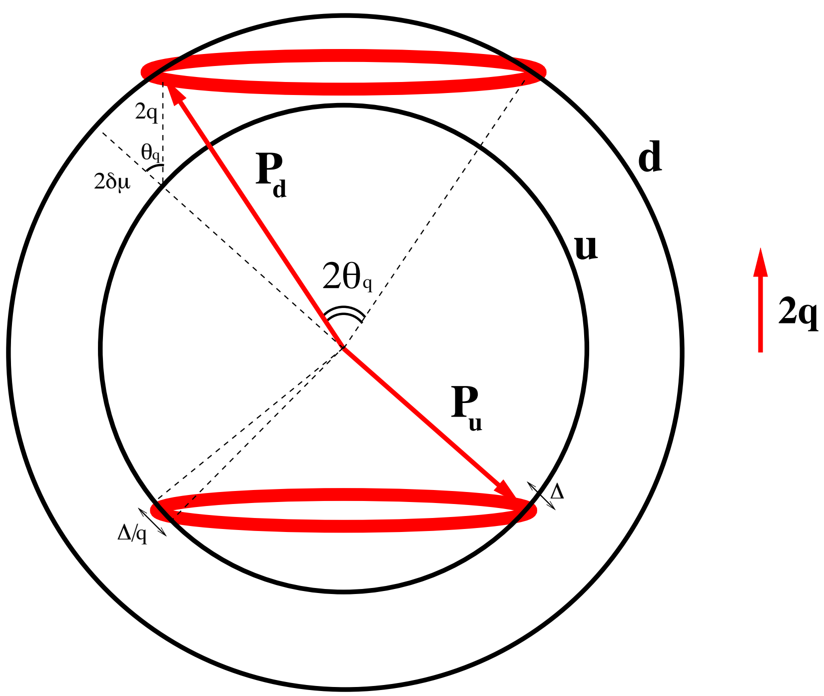

- **In momentum space**

$$< \psi(\boldsymbol{p}_u)\psi(\boldsymbol{p}_d) > \sim \Delta\, \delta(\boldsymbol{p}_u + \boldsymbol{p}_d - 2\boldsymbol{q})$$

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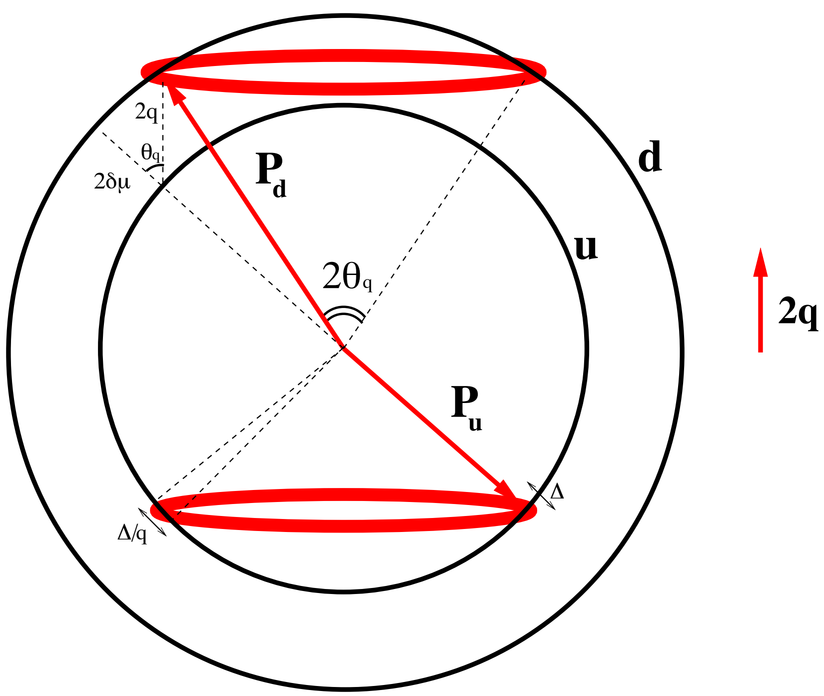

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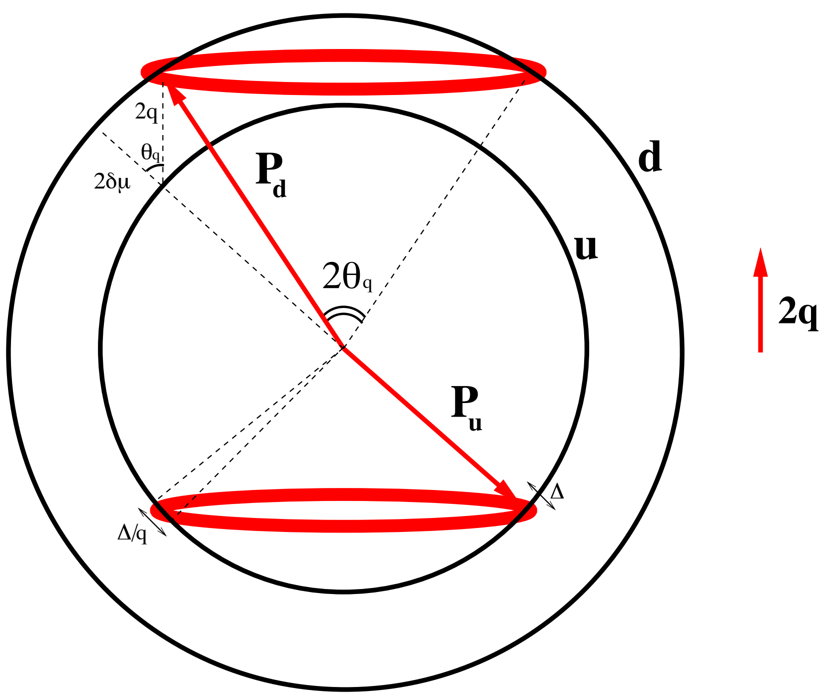

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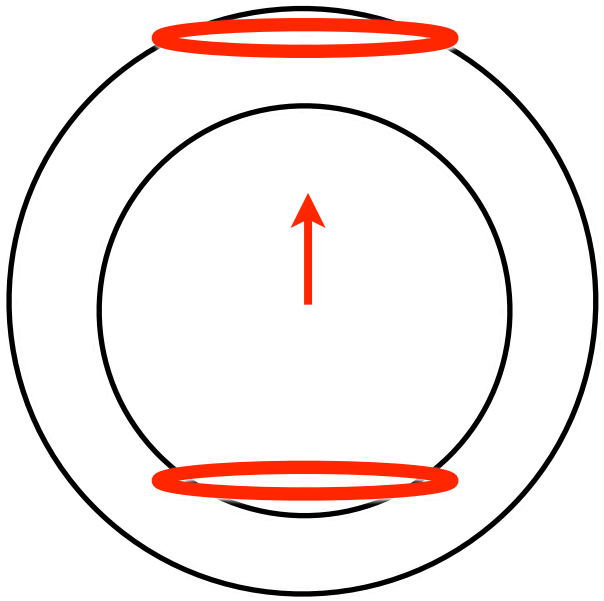
For two flavors in weak coupling

$$\delta\mu_1 \simeq \frac{\Delta_0}{\sqrt{2}} \qquad \delta\mu_2 \simeq 0.75\, \Delta_0$$

Crystalline structures



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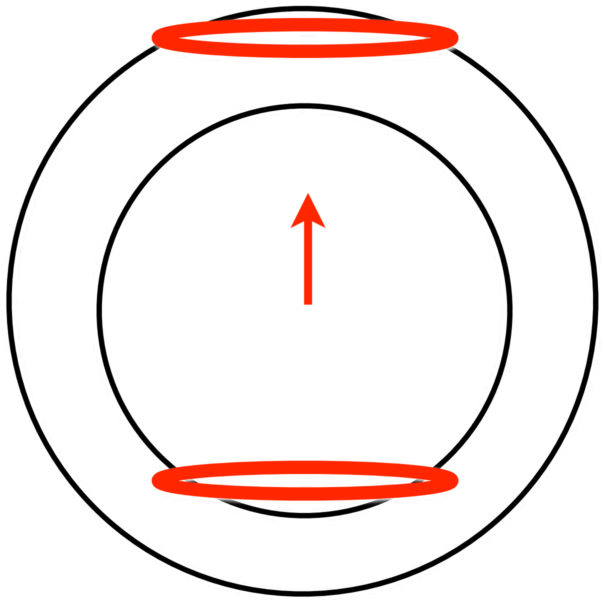

- Complicated structures can be obtained combining more plane waves

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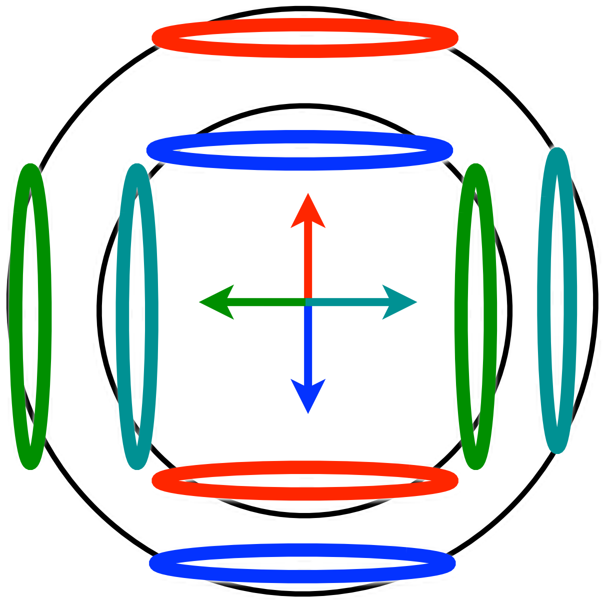

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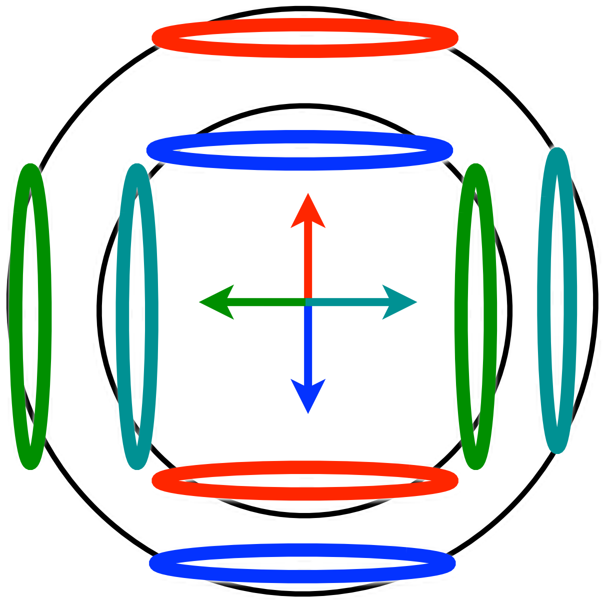

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- "no-overlap" condition between ribbons

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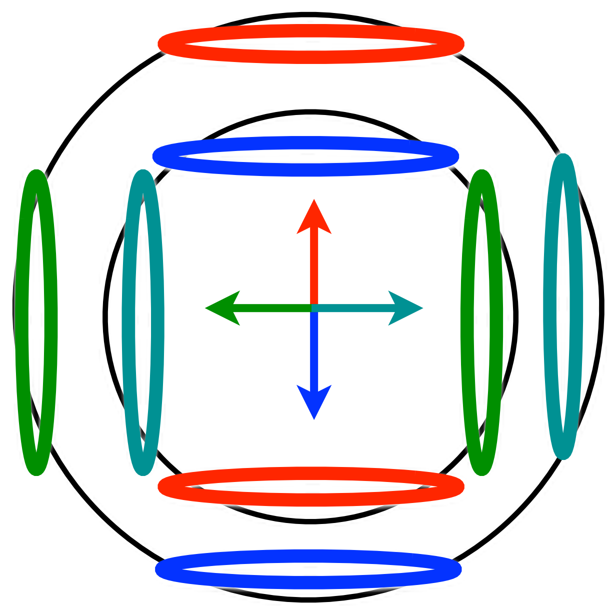

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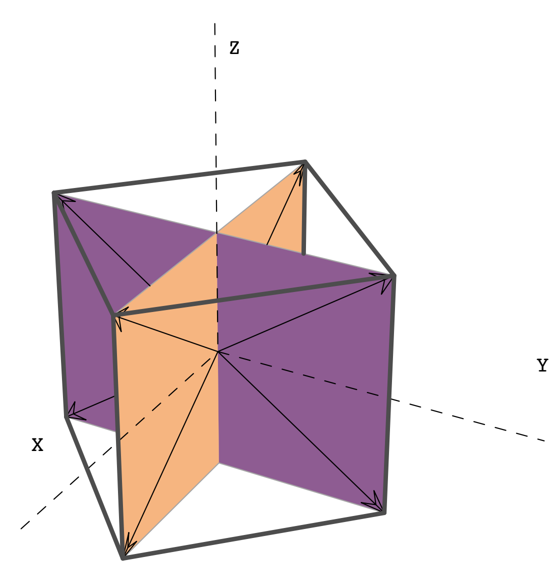
CX



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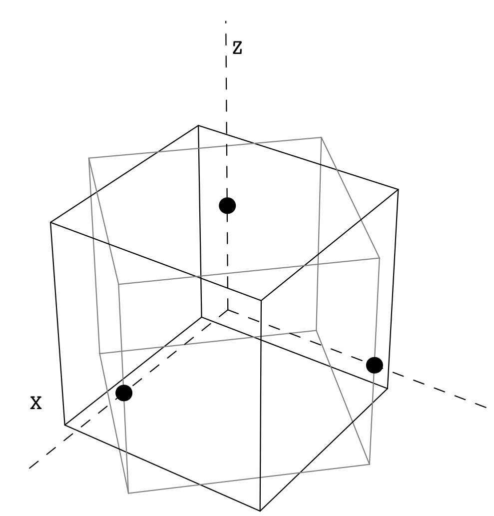


- Three flavors

$$< \psi_{\alpha i} C \gamma_5 \psi_{\beta j} > \sim \sum_{I=2,3} \Delta_I \sum_{\mathbf{q}_I^m \in \{\mathbf{q}_I^m\}} e^{2i\mathbf{q}_I^m \cdot \mathbf{r}} \epsilon_{I\alpha\beta} \epsilon_{Iij}$$

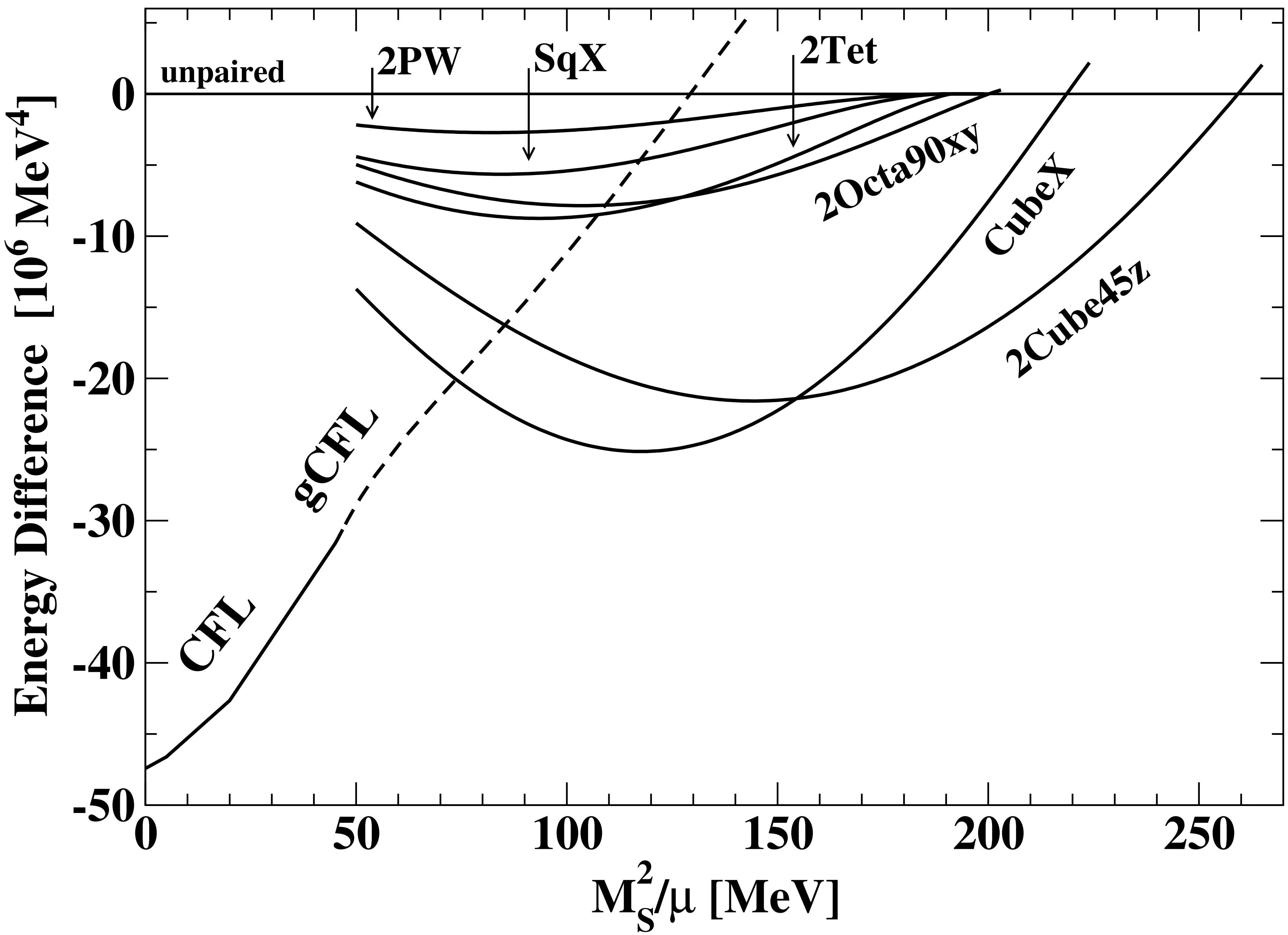
simplifications

$$\mathbf{q}_I^m = q\, \mathbf{n}_I^m$$

Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Free energy estimate

NJL + GL expansion!!



Rajagopal and Sharma Phys.Rev. D74 (2006) 094019
MM, Rajagopal and Sharma Phys.Rev.D 73 (2006) 114012

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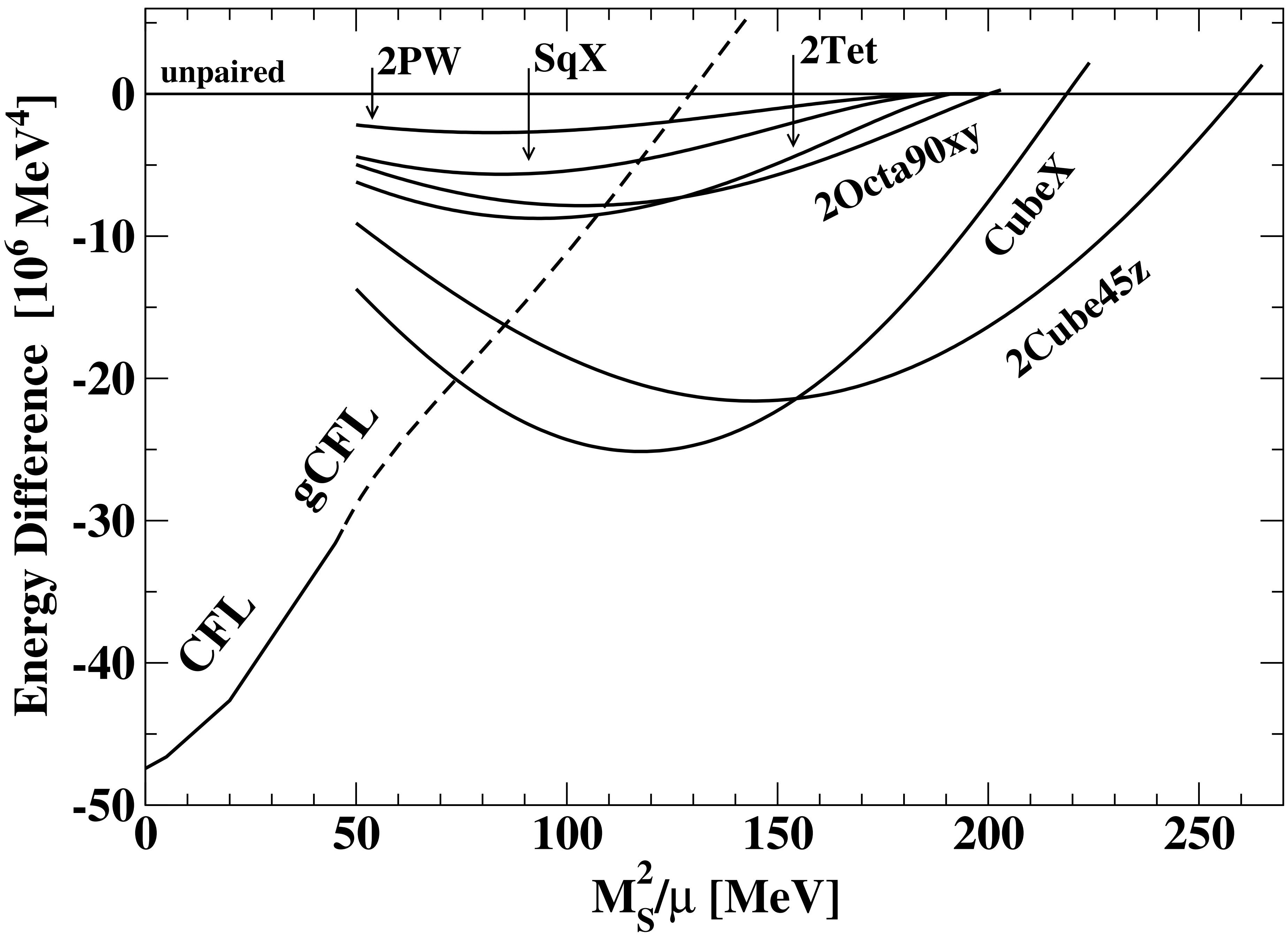
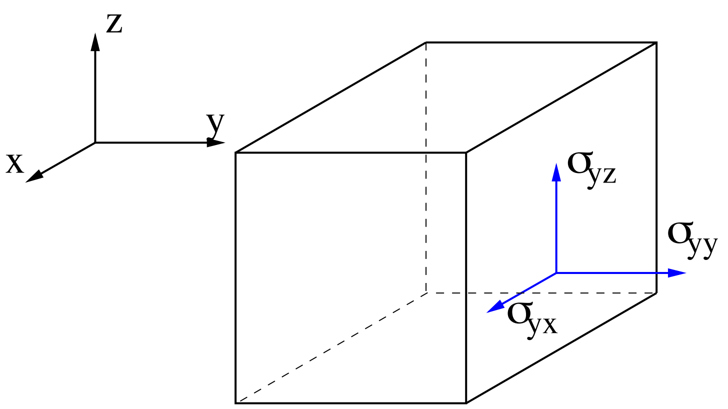


Rajagopal and Sharma Phys.Rev. D74 (2006) 094019
MM, Rajagopal and Sharma Phys.Rev.D 73 (2006) 114012

Improved GL expansion S.Carignano, MM, O.Benhar and F.Anzuini Phys.Rev.D 97 (2018) 3, 036009

Displacement of the crystal

Elastic deformation of a stressed crystal (Landau Lifsits, vol. 7)



displacement vector $u_i = x'_i - x_i$

deformation tensor $u_{ij} = \dfrac{u_{i,j} + u_{j,i}}{2}$

stress tensor $\sigma_{ij} = K u_{kk}\delta_{ij} + 2\nu\left(u_{ij} - \dfrac{1}{3}u_{kk}\delta_{ij}\right)$

compressibility　　shear modulus

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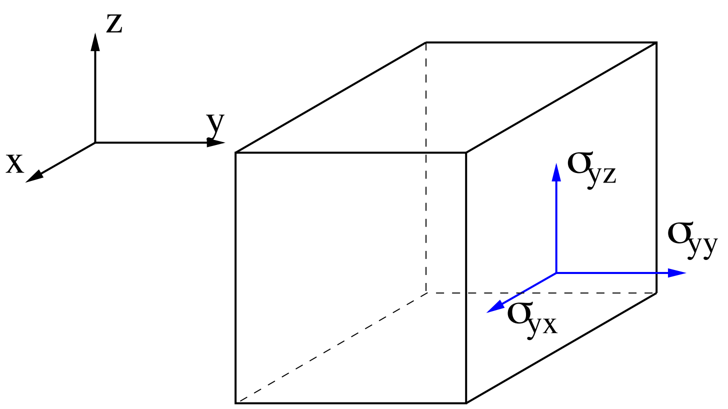

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- **It is this pattern of modulation that is rigid (and can oscillate)**

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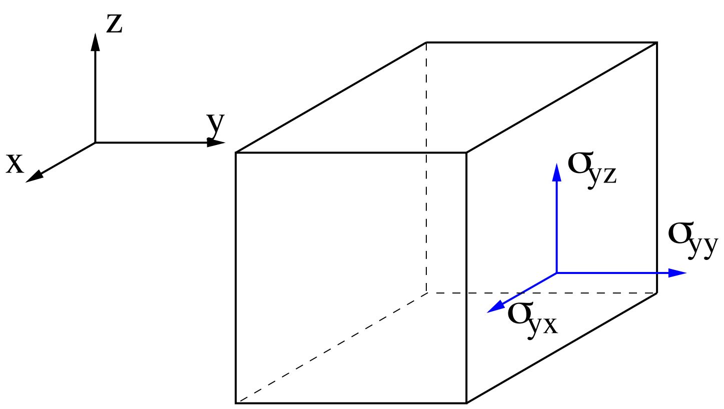

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$\nu_{\rm CCSC} \sim 2.47\,{\rm MeV/fm^3}$

20 to 1000 times more rigid than the crust of neutron stars

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

Shear viscosity

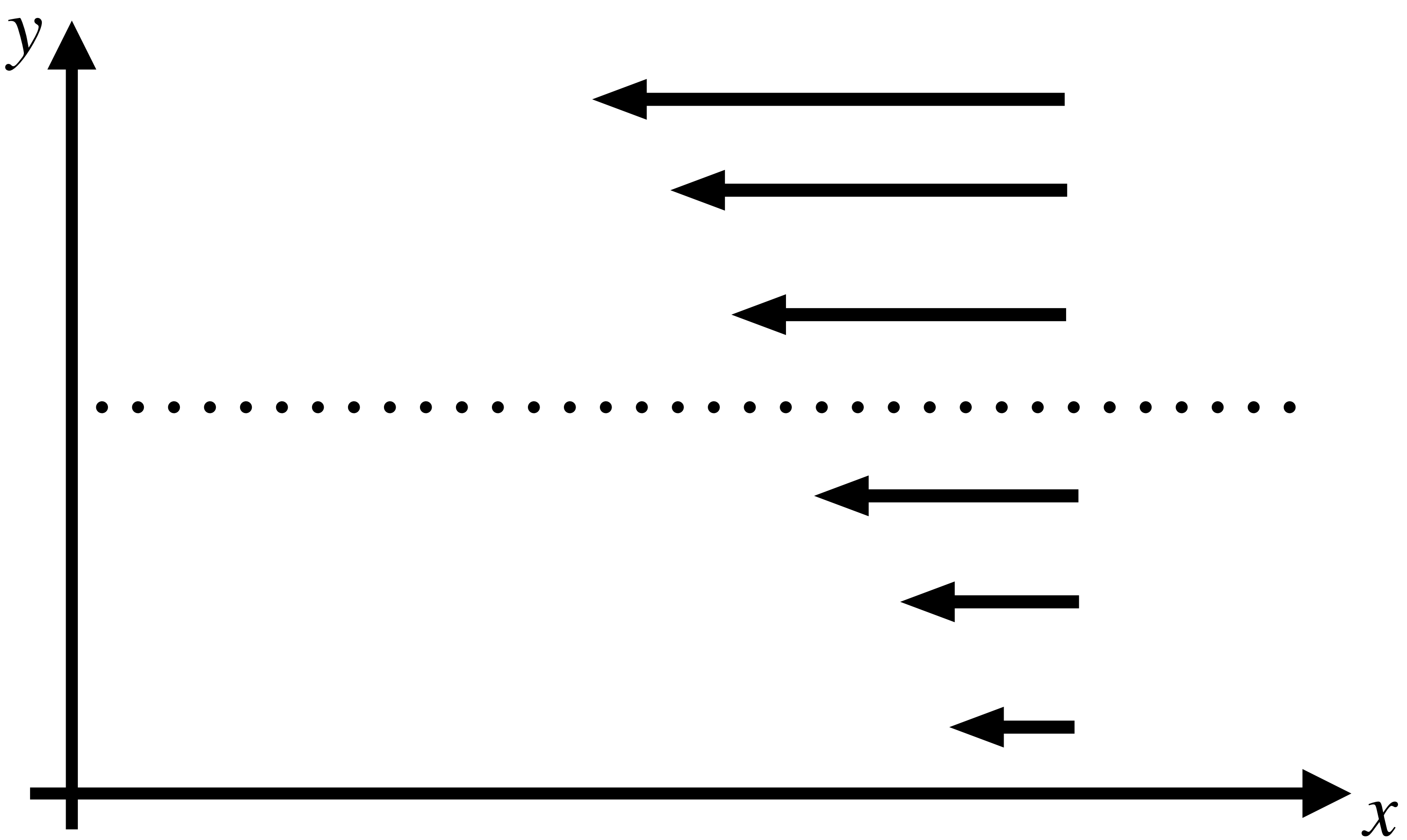
P. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. 94 (2005) 111601

Adams et al. New Journal of Physics 14 (2012) 115009

L. Chiofalo, D. Grasso, MM and S. Trabucco, e-Print: 2202.13790 [gr-qc]

Shear (laminar) flow

$$\mathbf{v} = (v_x, 0, 0) \qquad \frac{\partial v_x}{\partial y} \neq 0$$



In an ideal superfluid the laminar flow persists indefinitely

Shear viscosity η

In an non-ideal fluids the **friction** tends to reduce the laminar flow

$$\eta \sim np\lambda$$

λ is mean free path
p is the average momentum
n is the number density

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In relativisic systems entropy works better. Entropy density $s \propto k_B n$

$$\frac{\eta}{s} \sim p\lambda \geq \frac{\hbar}{k_B}$$

The KSS bound

- **Increasing** the temperature the shear viscosity should increase.
- **Increasing** the interaction strength the shear viscosity should decrease

Does the shear viscosity vanishes in some limit ?

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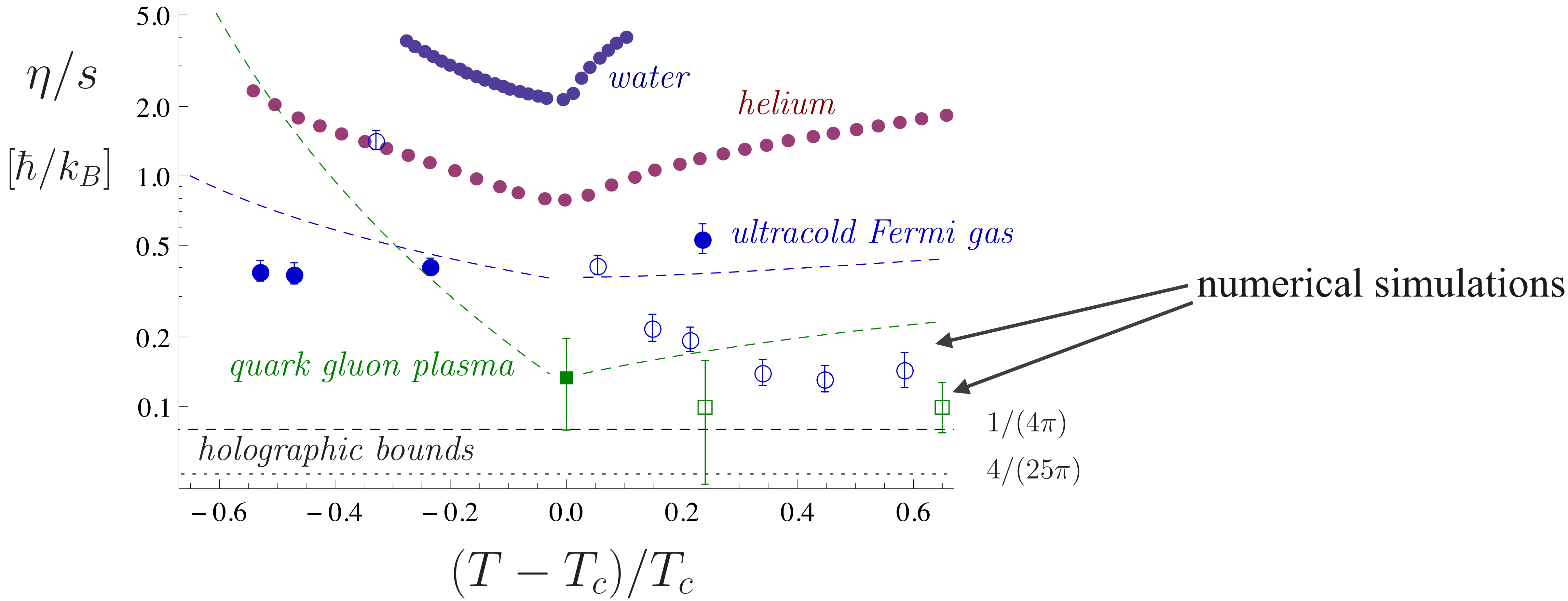
shear viscosity coefficient

entropy density

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

P. Kovtun, D. T. Son, and A. O. Starinets, PRL 94, 111601 (2005)

Shear viscosity to entropy ratio



Adams et al. New Journal of Physics 14 (2012) 115009

It does not exist any real physical system that saturates or violates the KSS bound

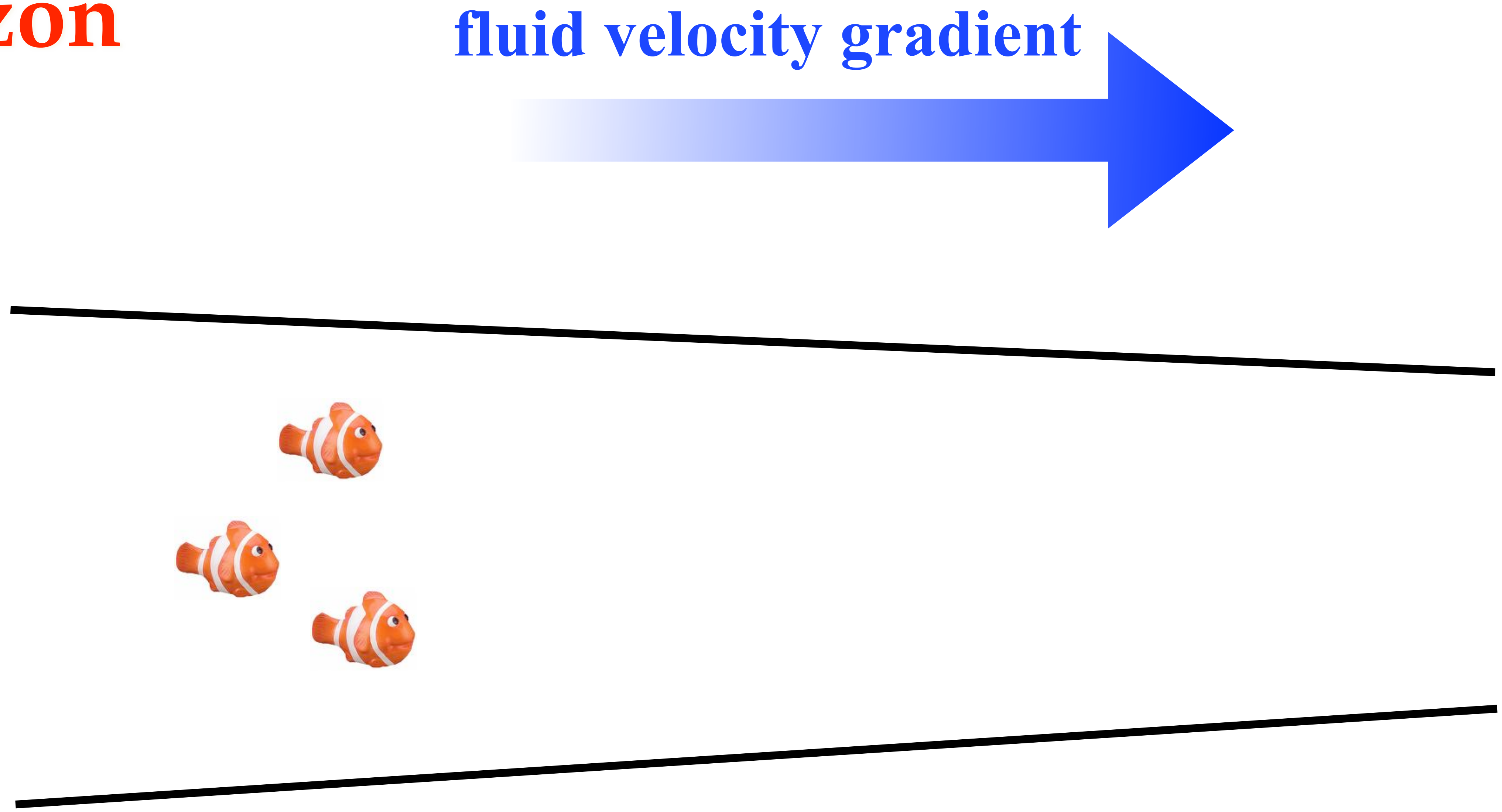
Gravity analogs

W. Unruh, Experimental black hole evaporation, Phys.Rev.Lett. 46 (1981) 1351–1353

M. Visser, Acoustic black holes: Horizons, ergospheres, and Hawking radiation, Class. Quant. Grav. 15 (1998) 1767–1791

C. Barcelo, S. Liberati, and M. Visser, Analogue gravity, Living Rev. Rel. 8 (2005) 12,

Horizon



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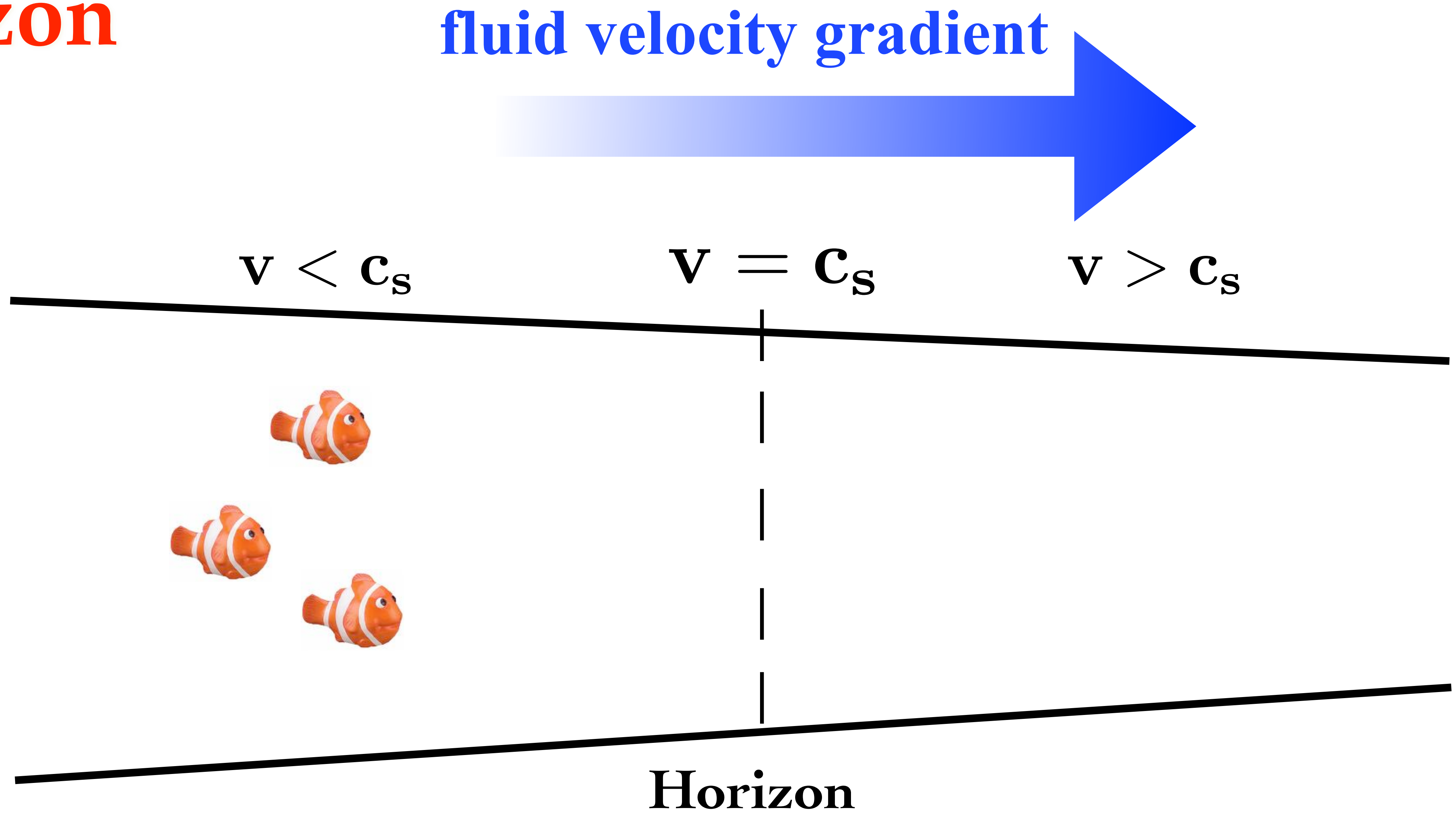
fluid velocity gradient
$\mathbf{v} < \mathbf{c_s}$
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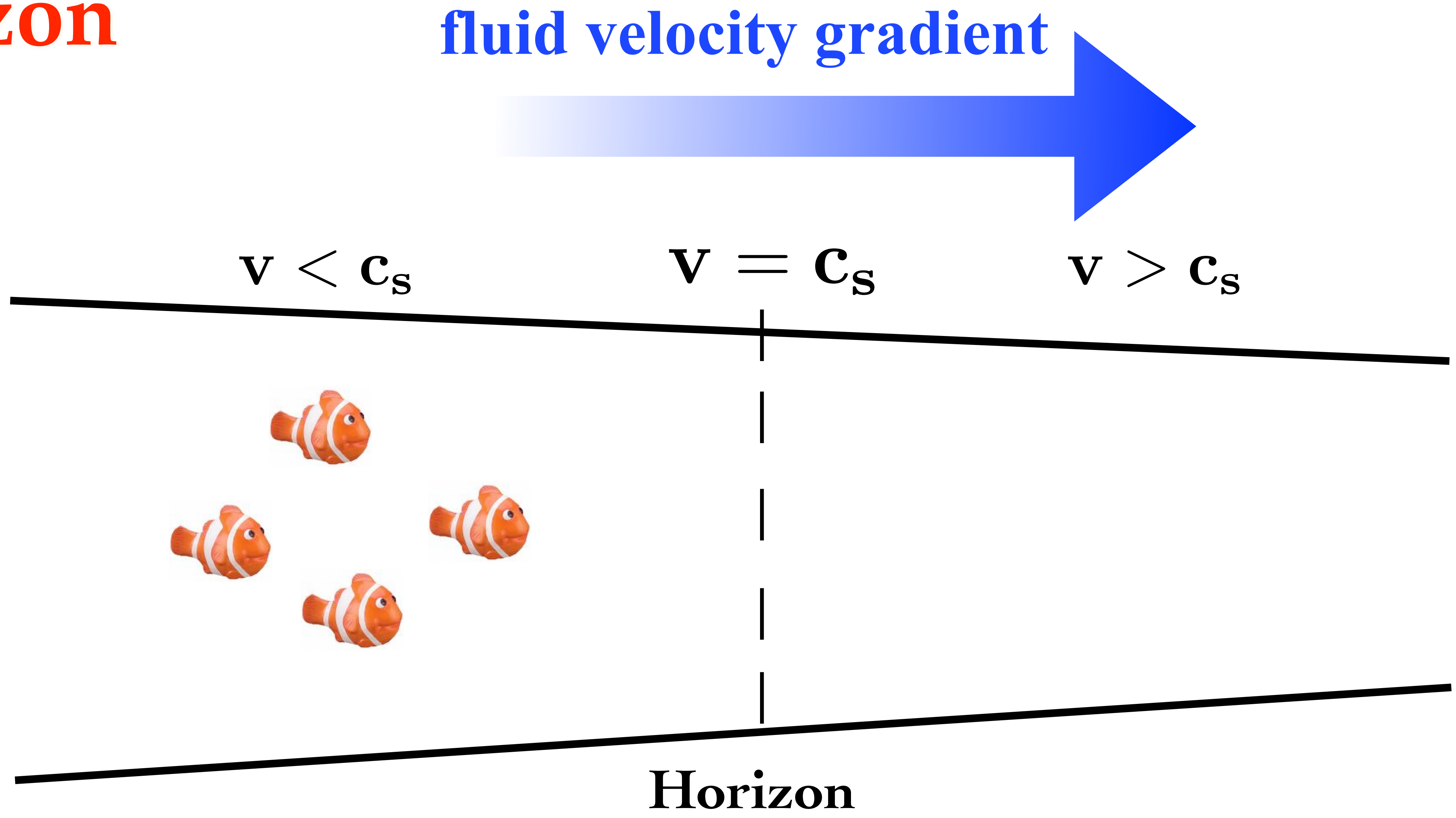
Horizon

Horizon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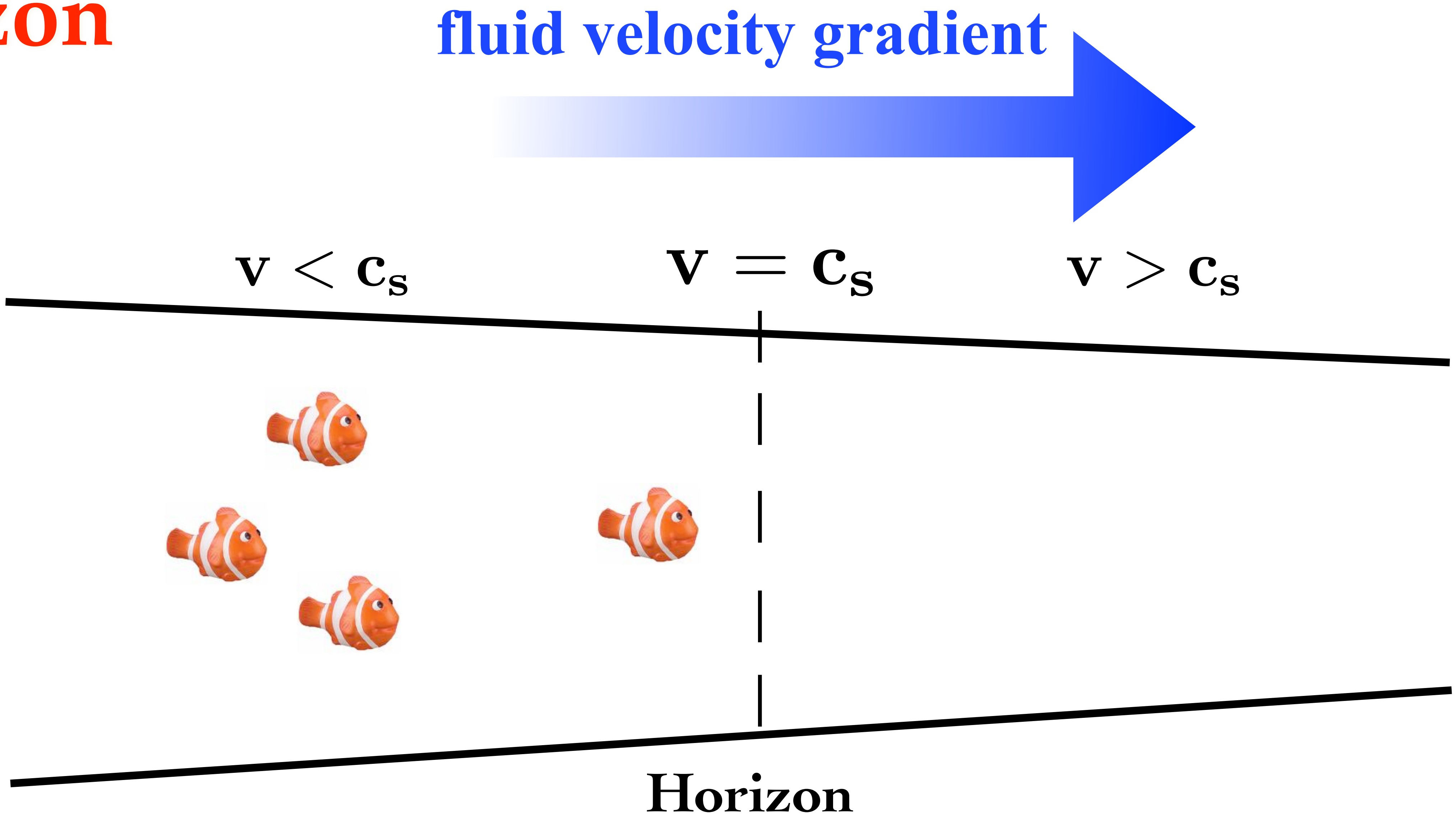

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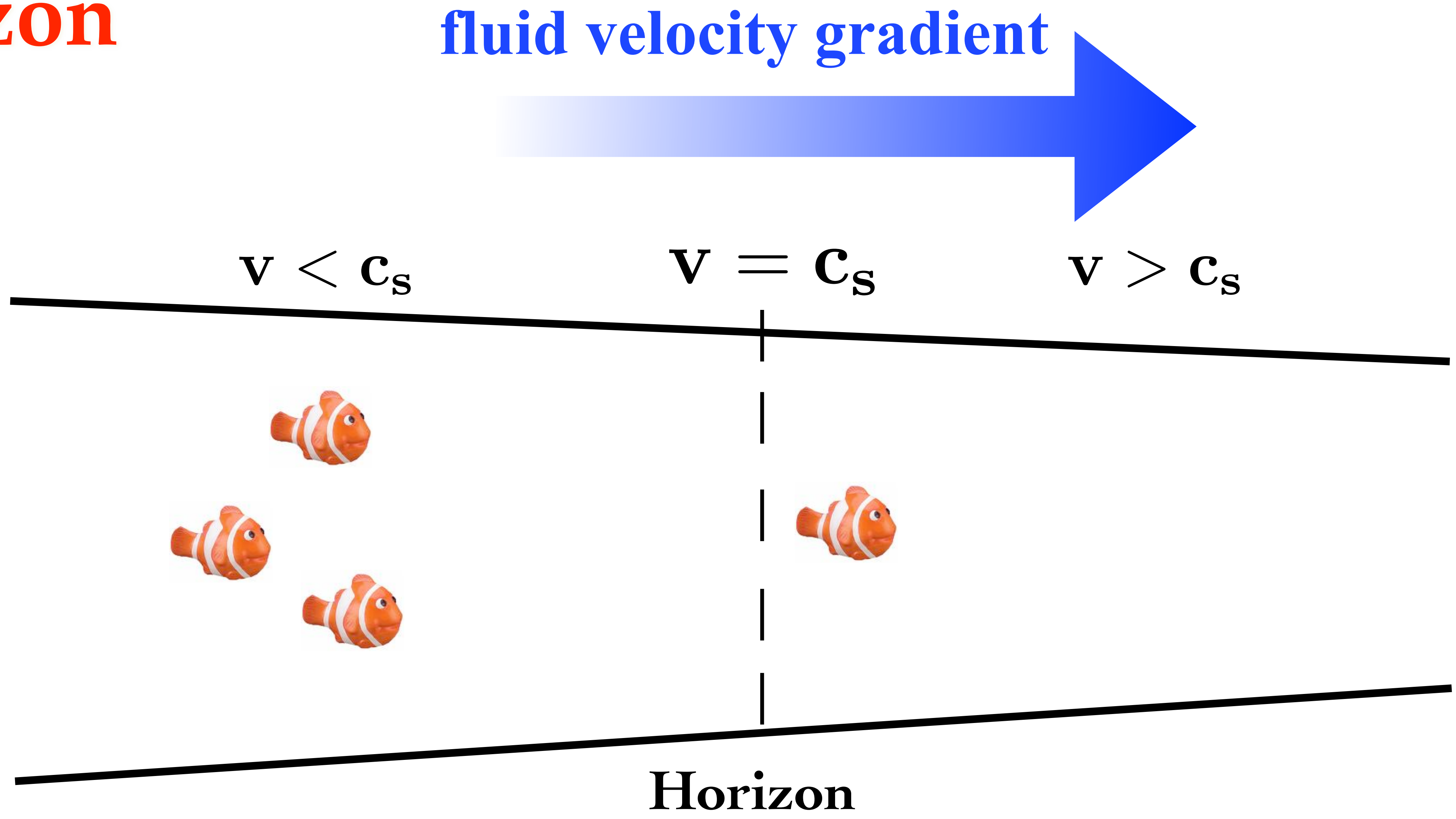
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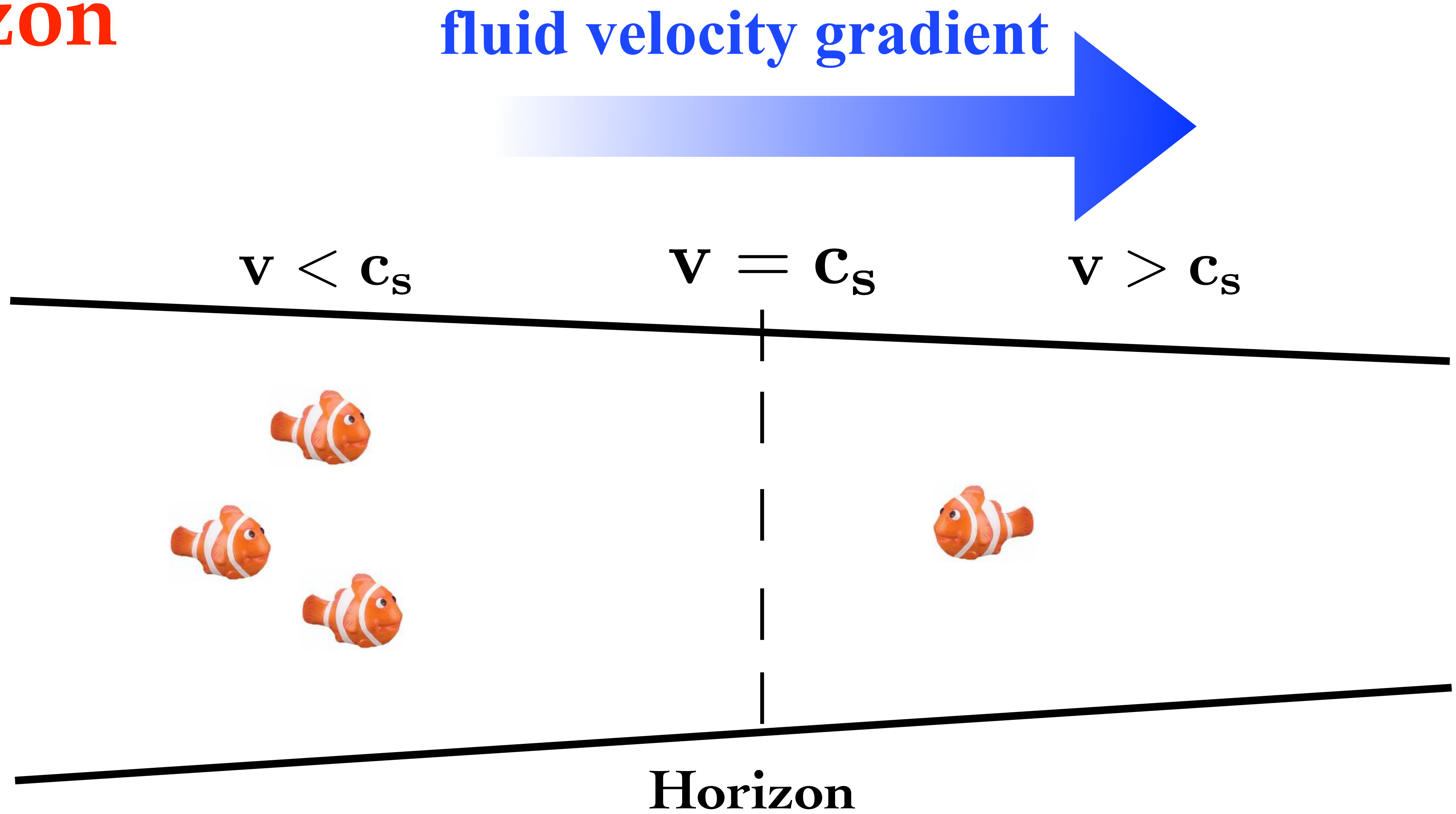

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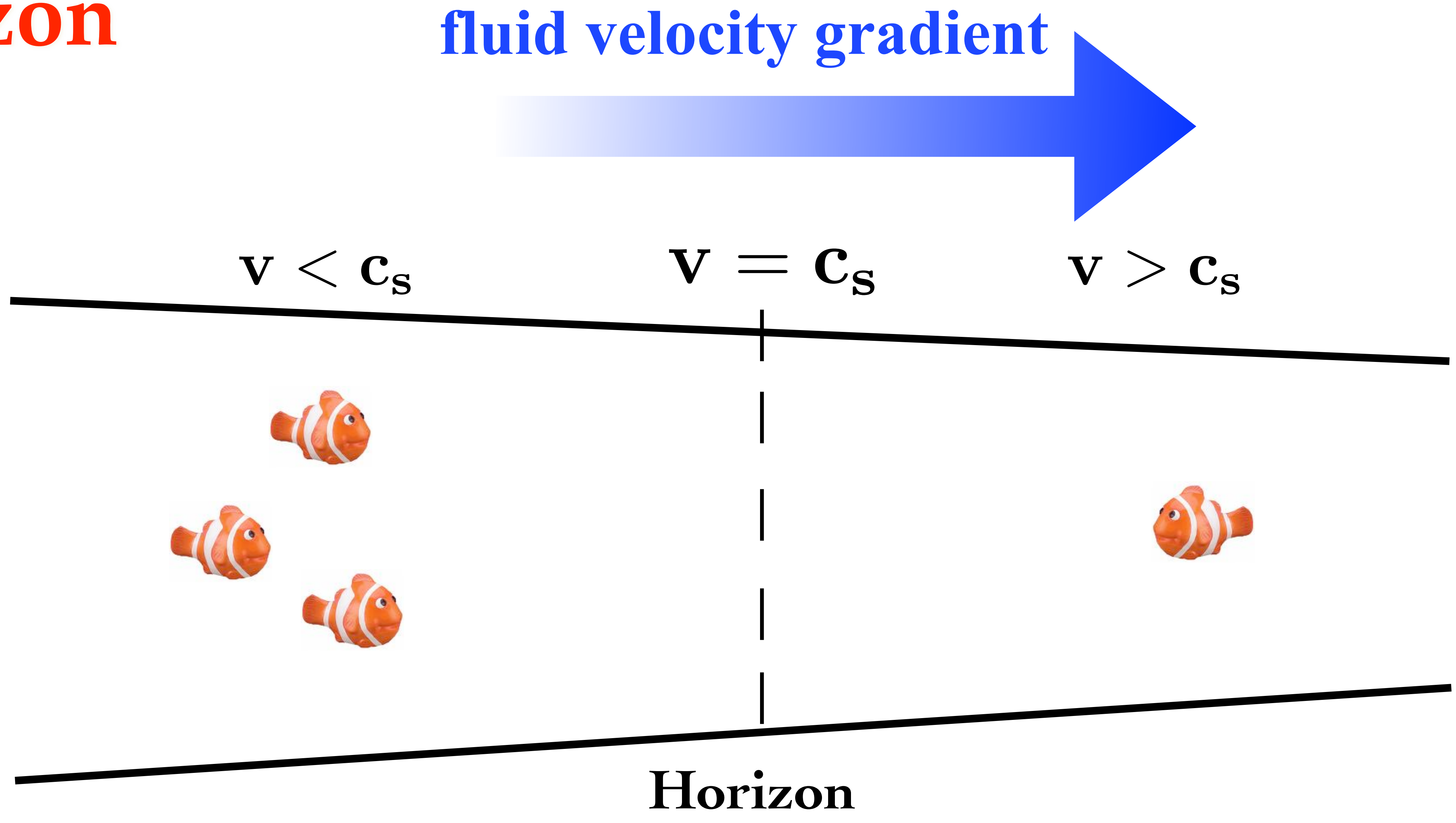

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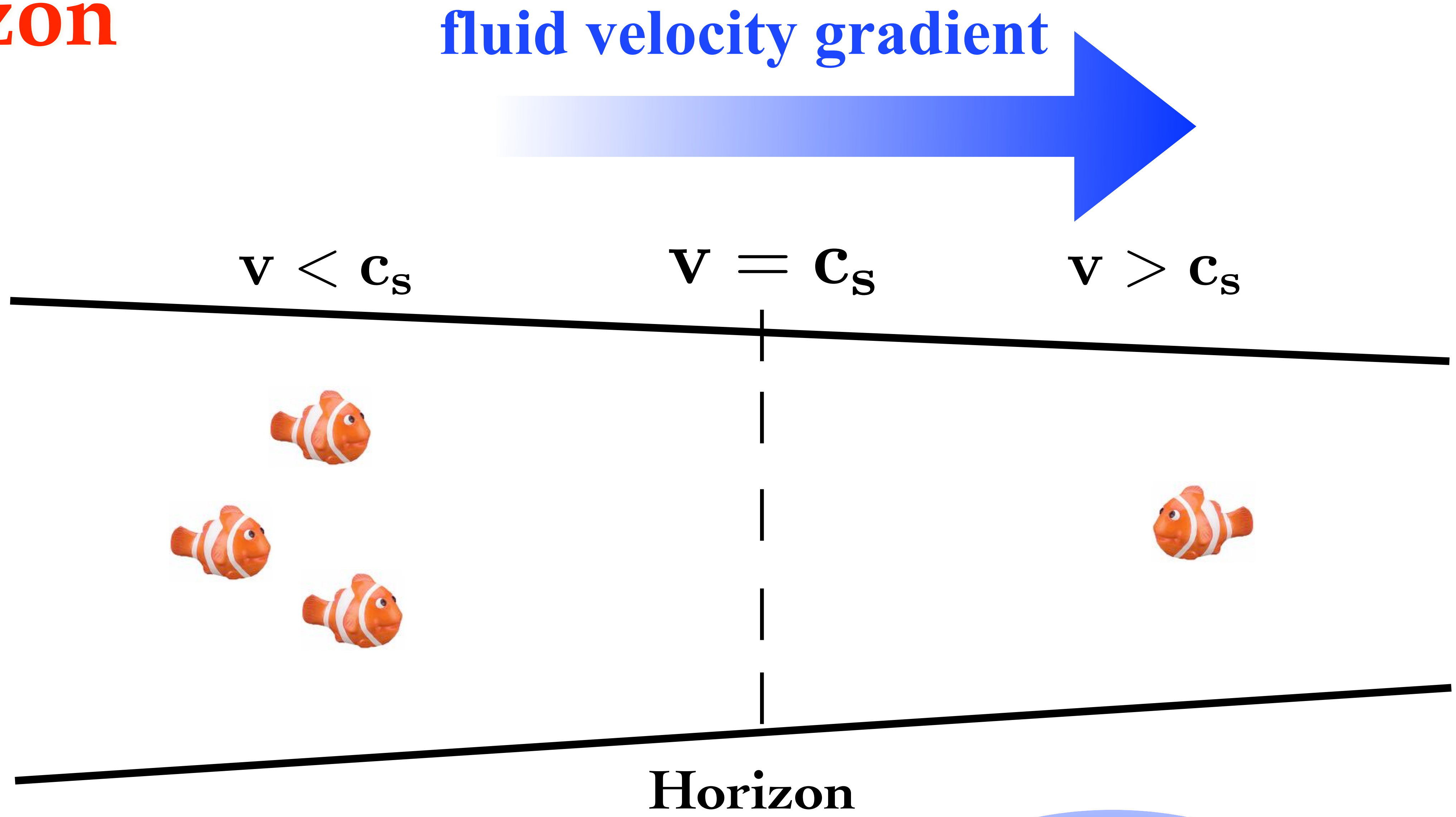
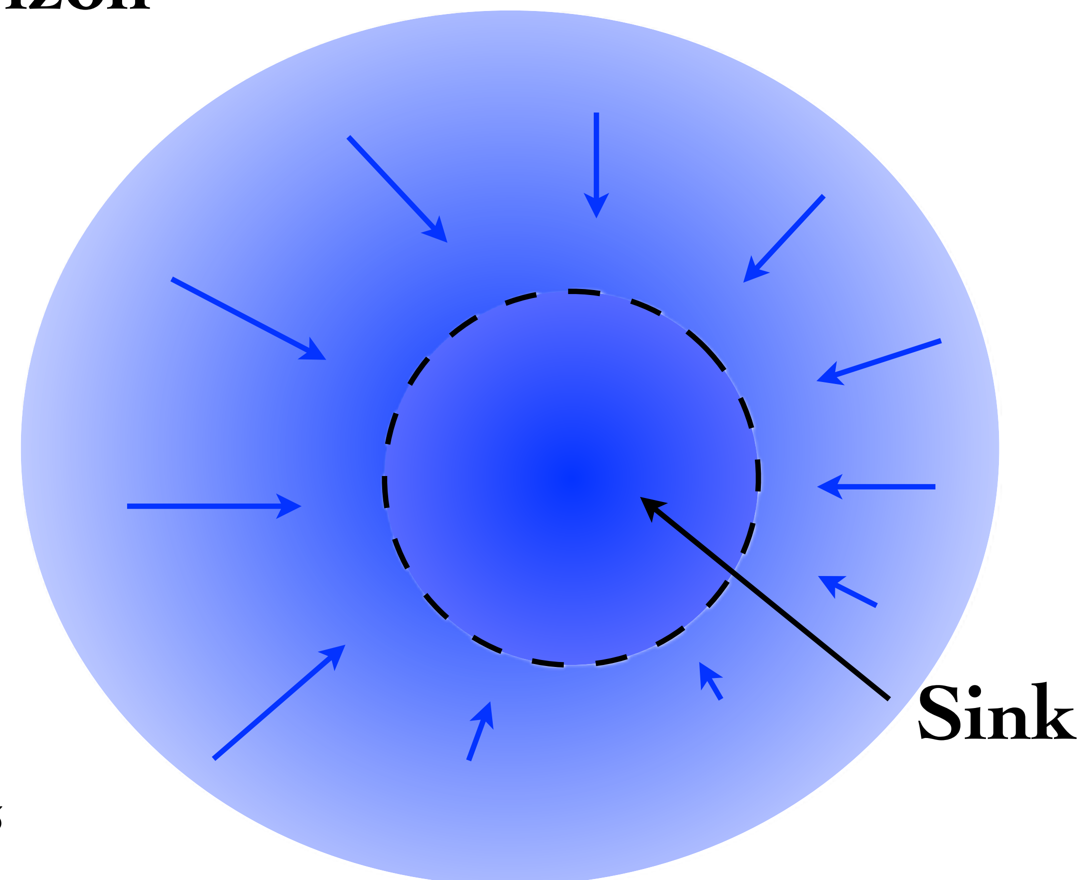
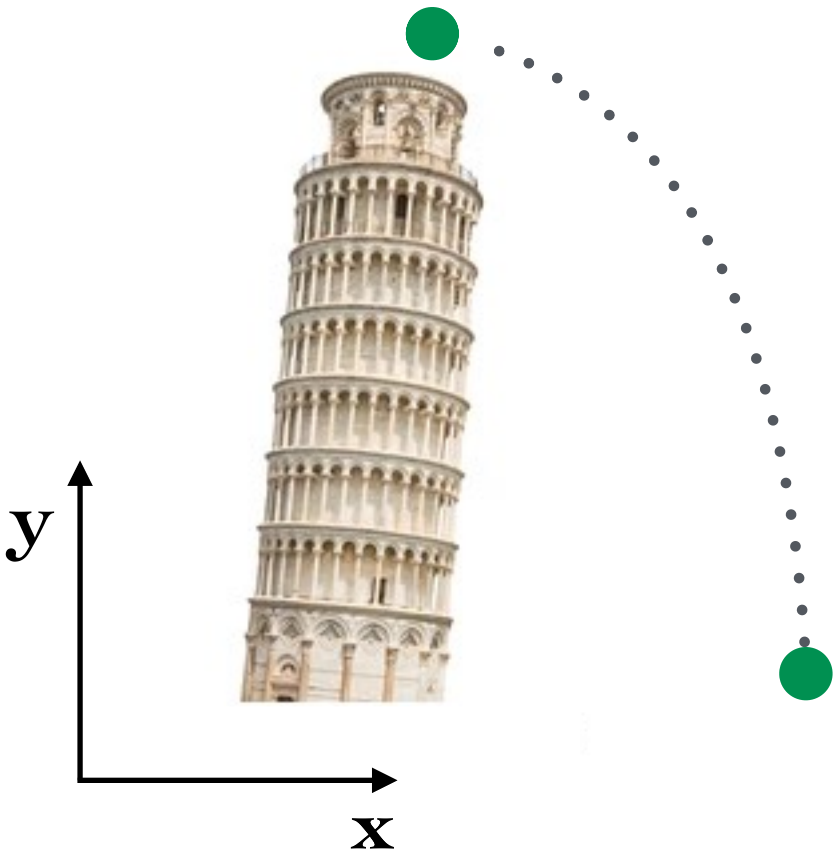


Trapped surface



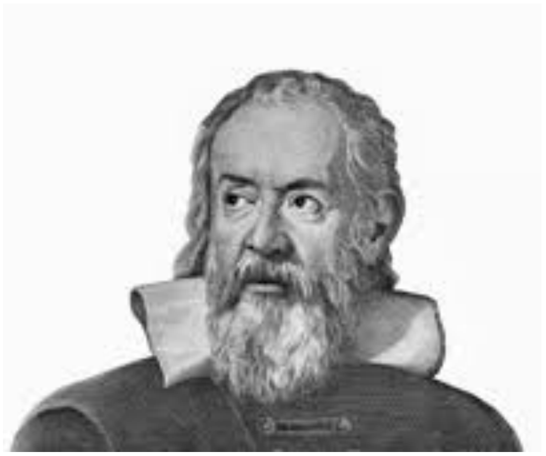
Space gradients to emulate gravity



Gravity

$$v_x = c_s$$

$$v_y = gt$$



Galileo

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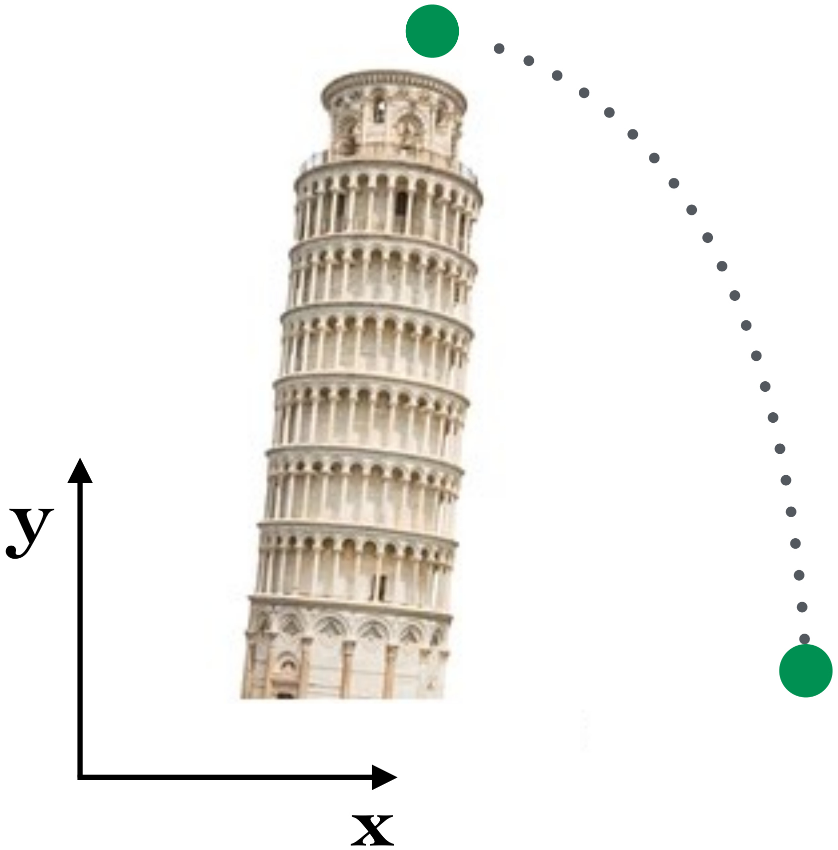

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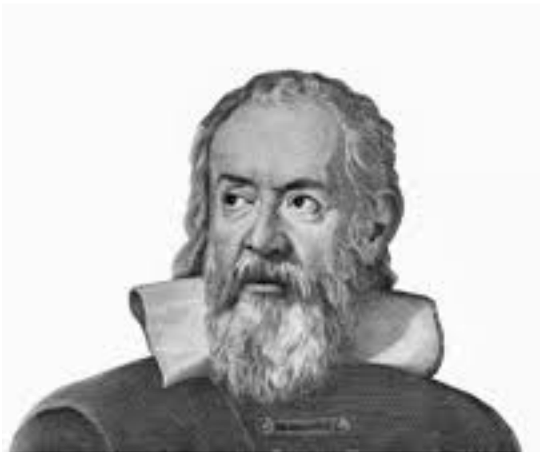

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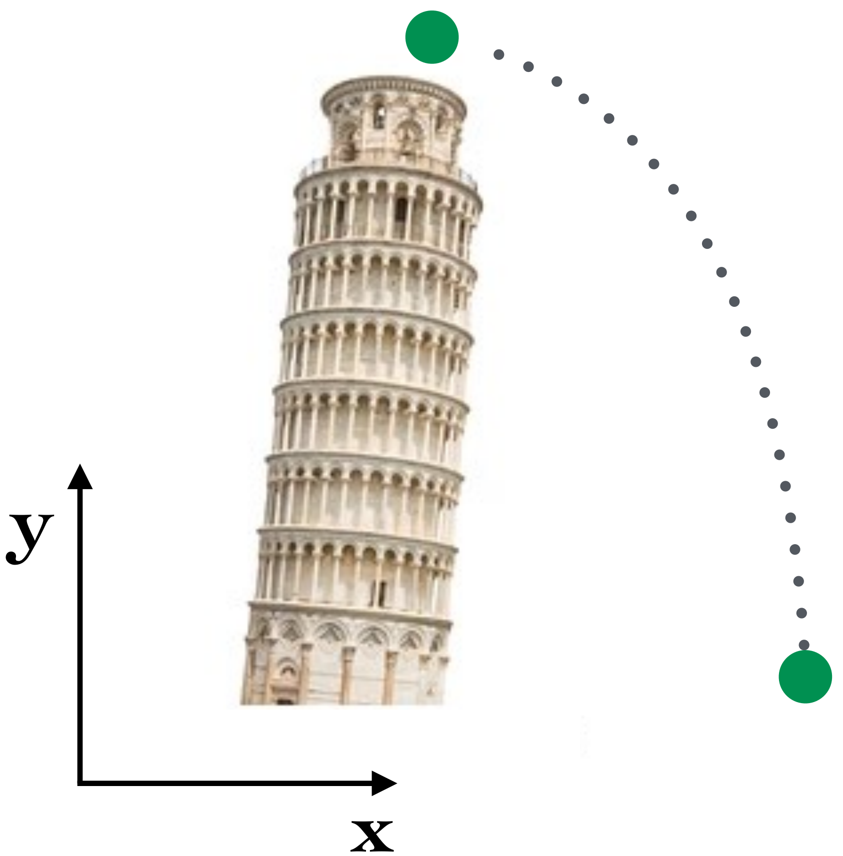

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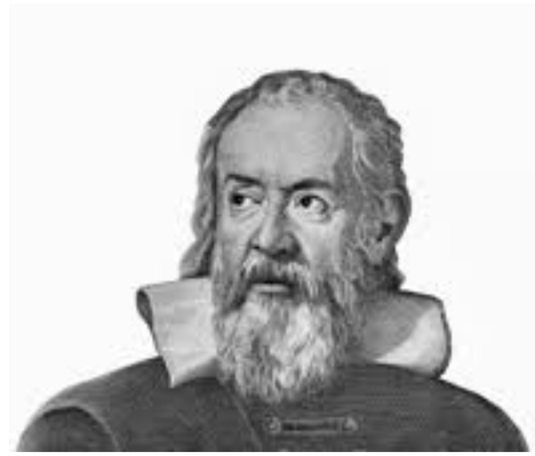

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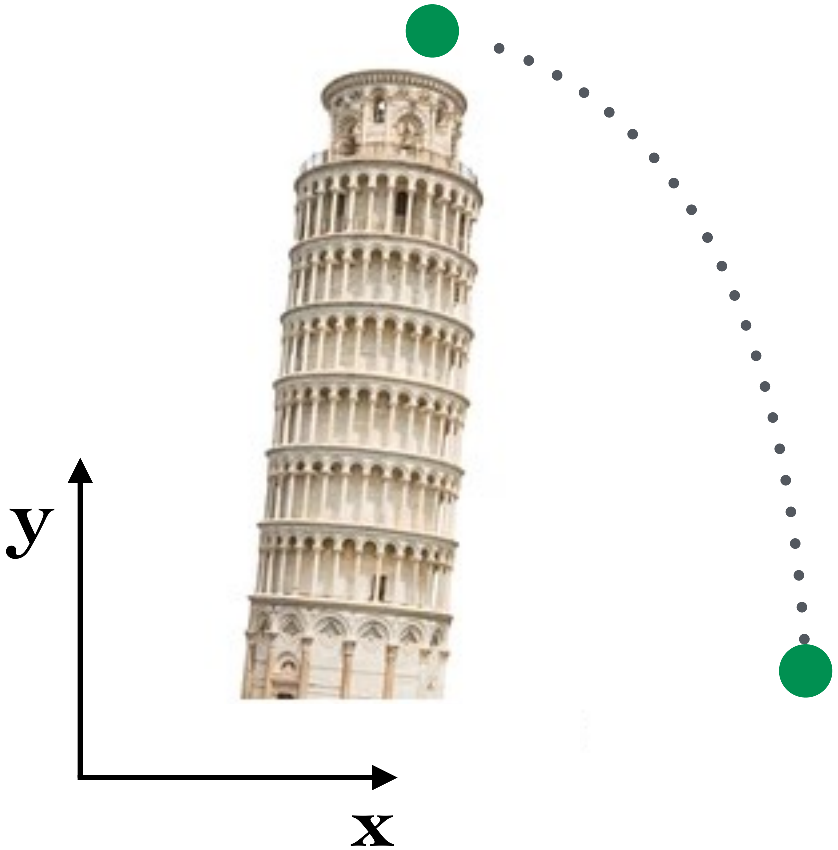

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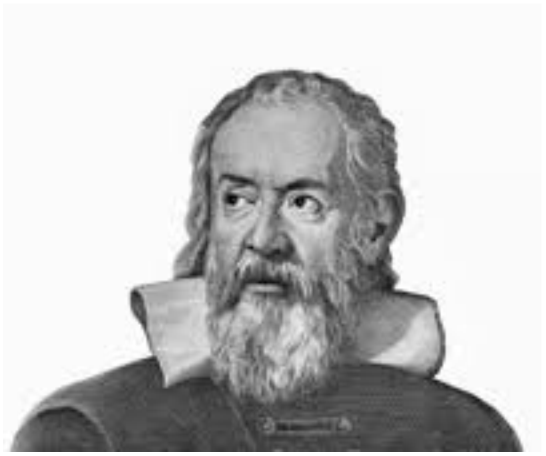

Galileo

An analog model

Unrhu

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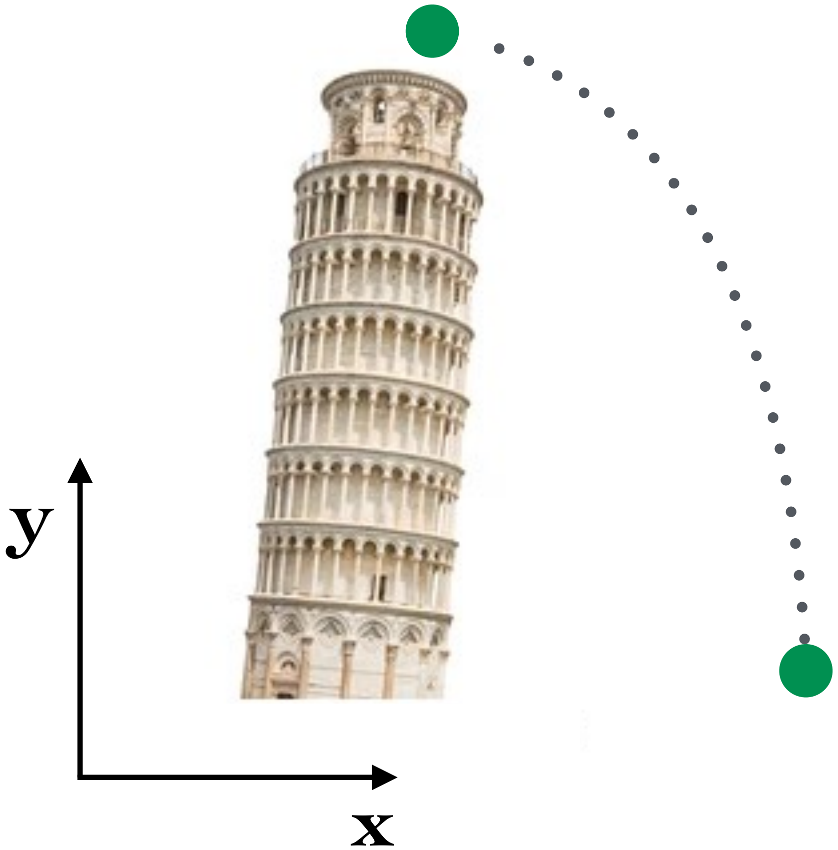

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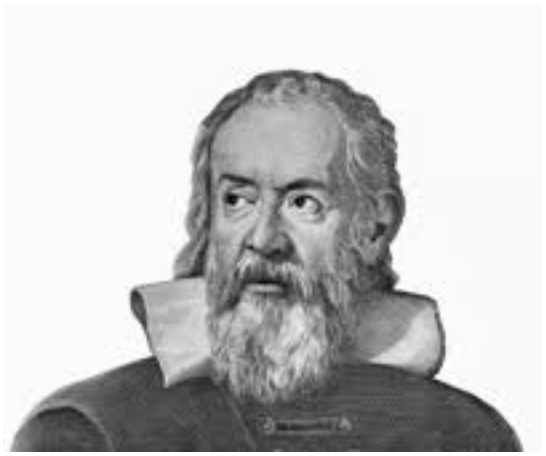

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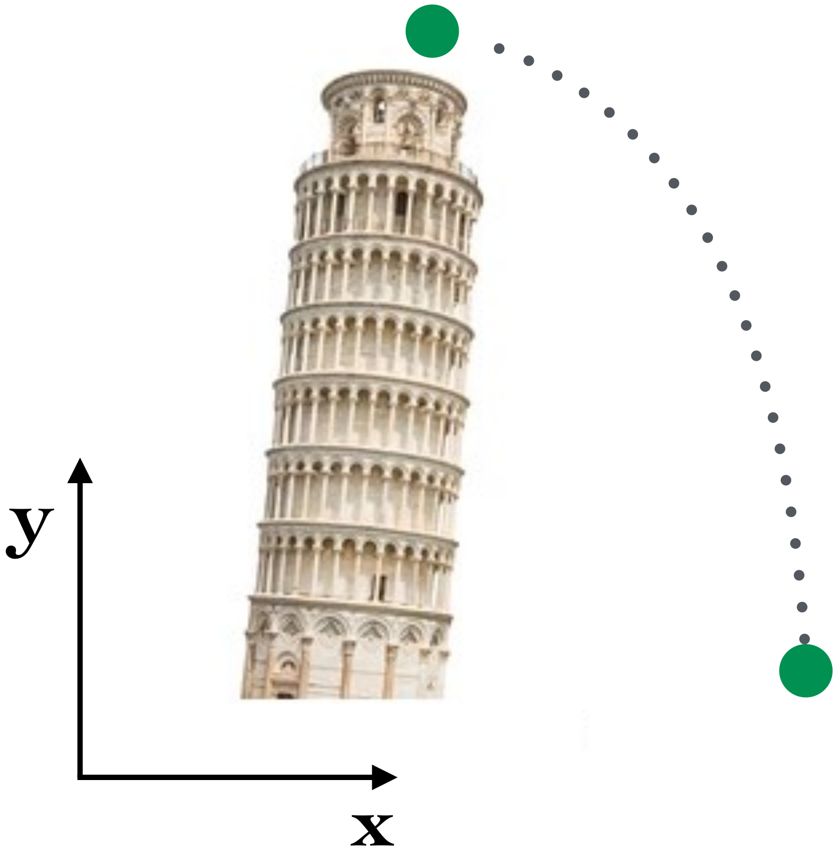

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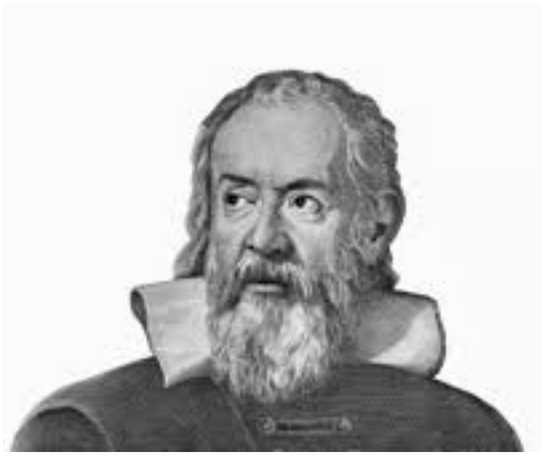

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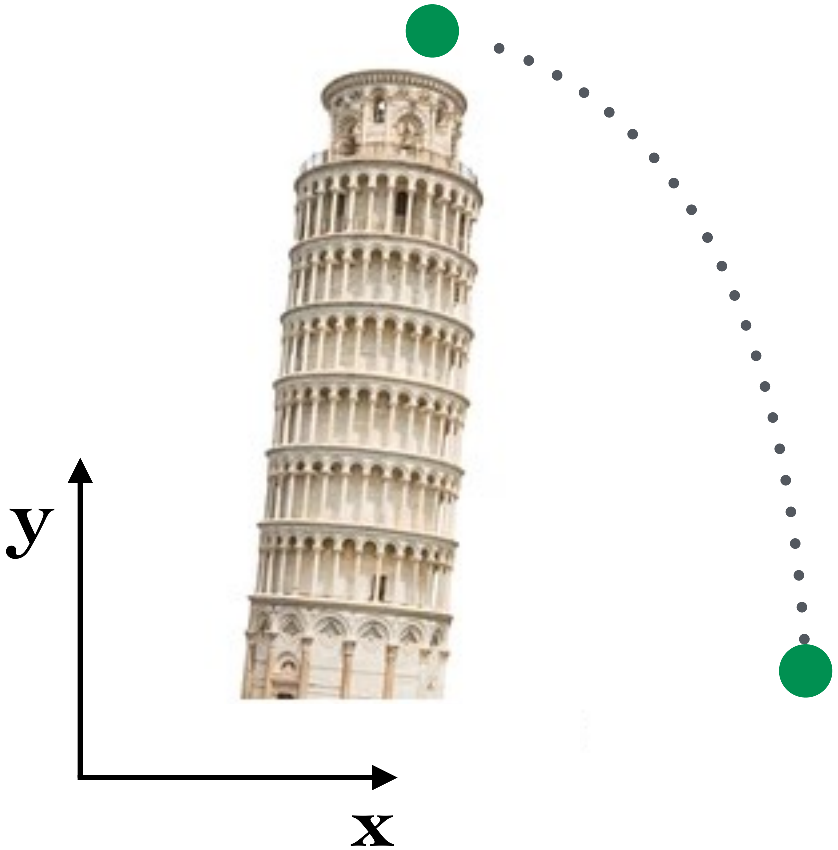

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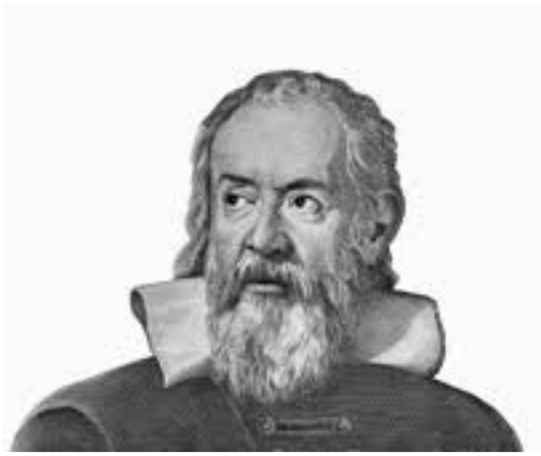

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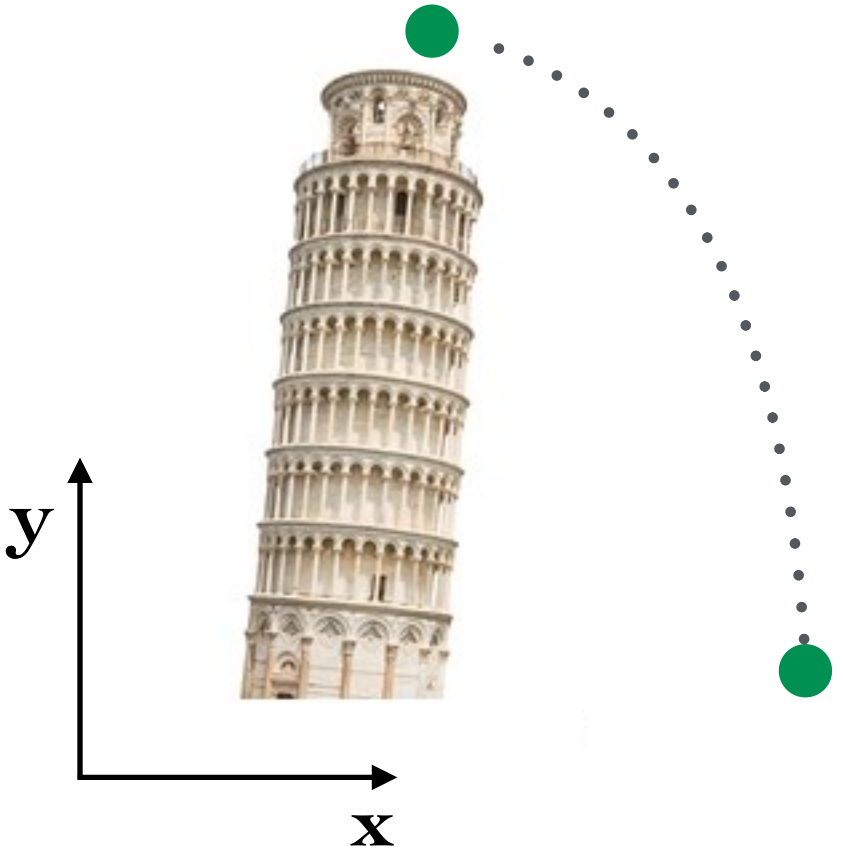

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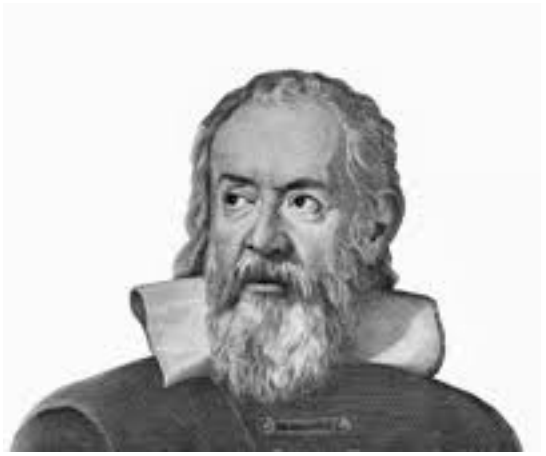

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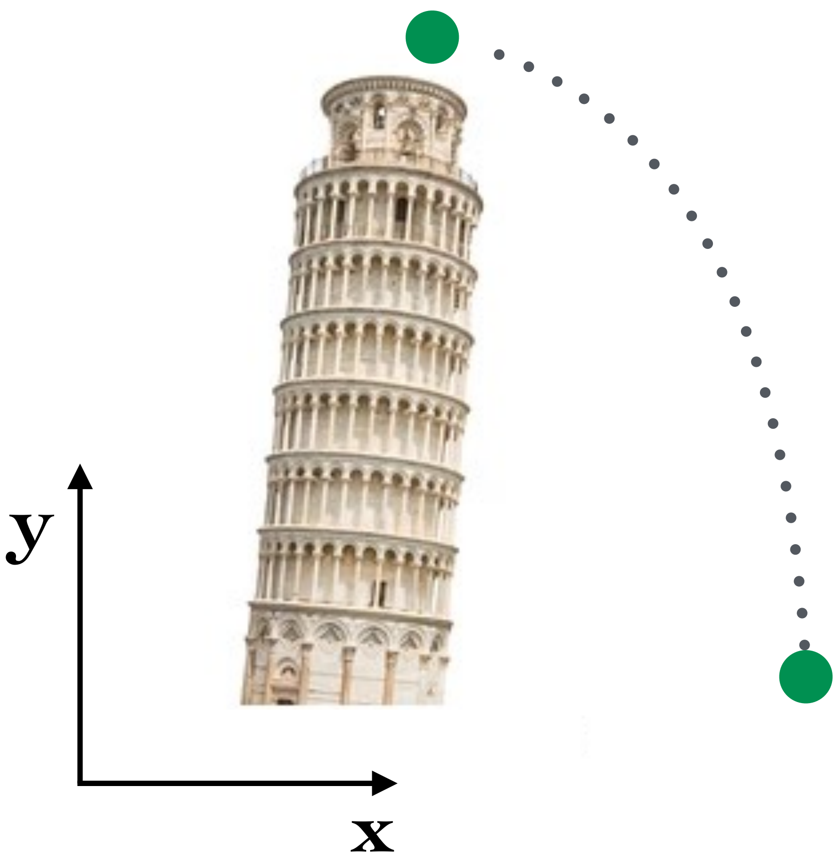

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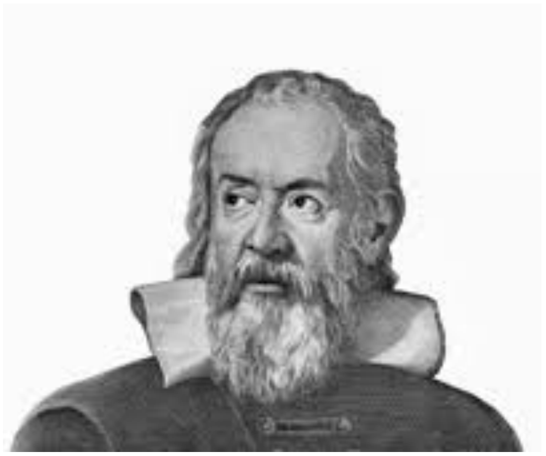

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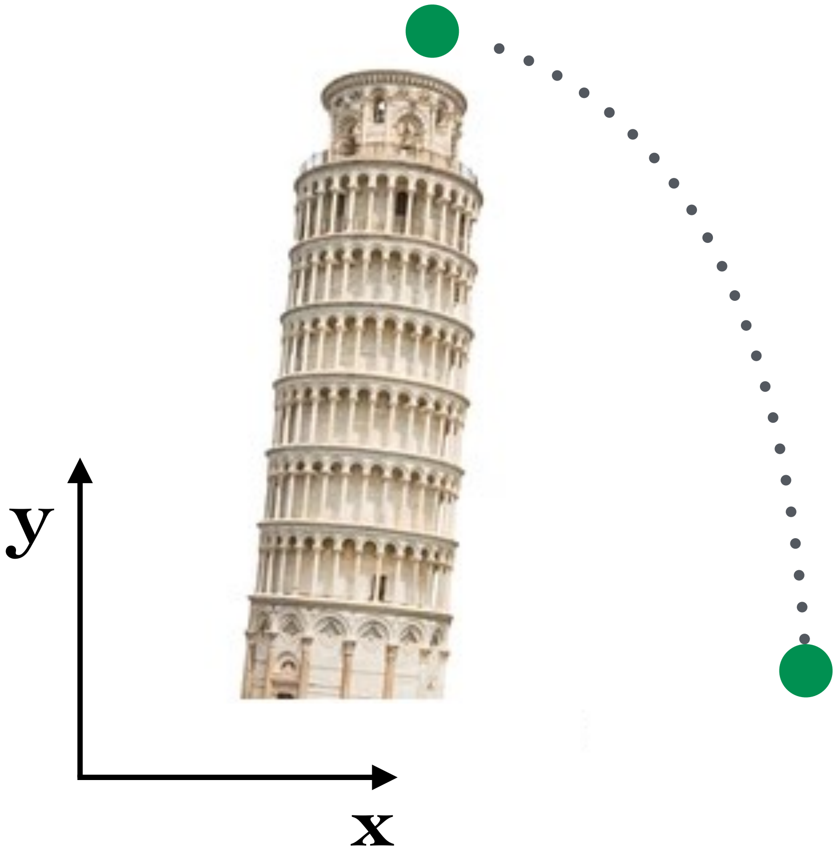

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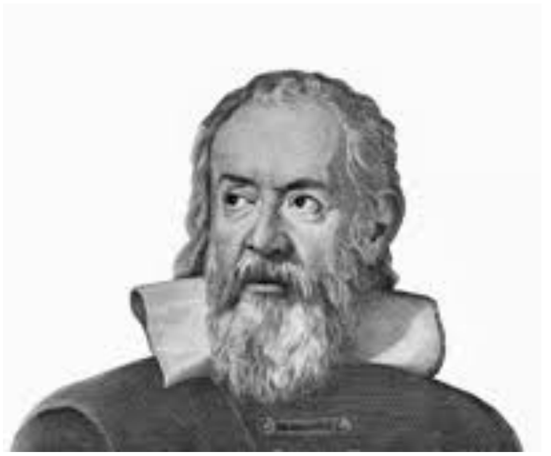

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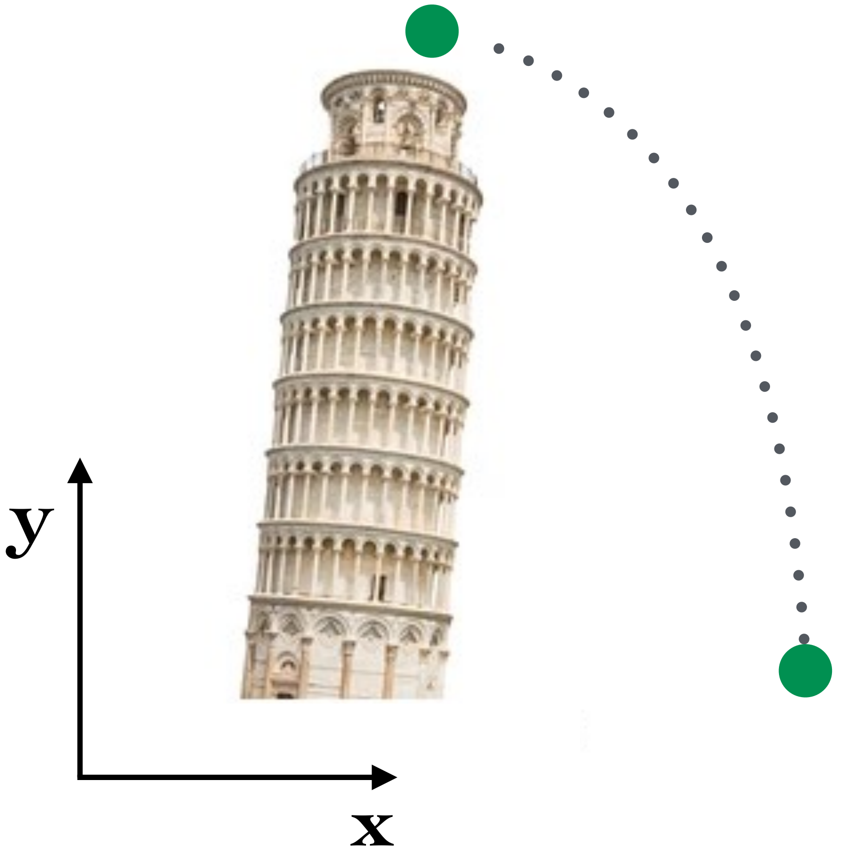

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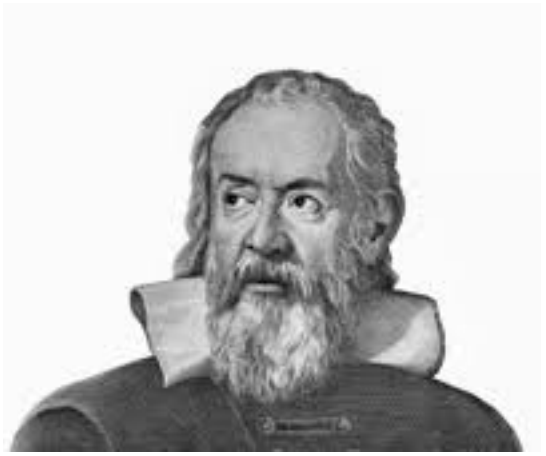

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***k* is related to the "surface" acceleration**

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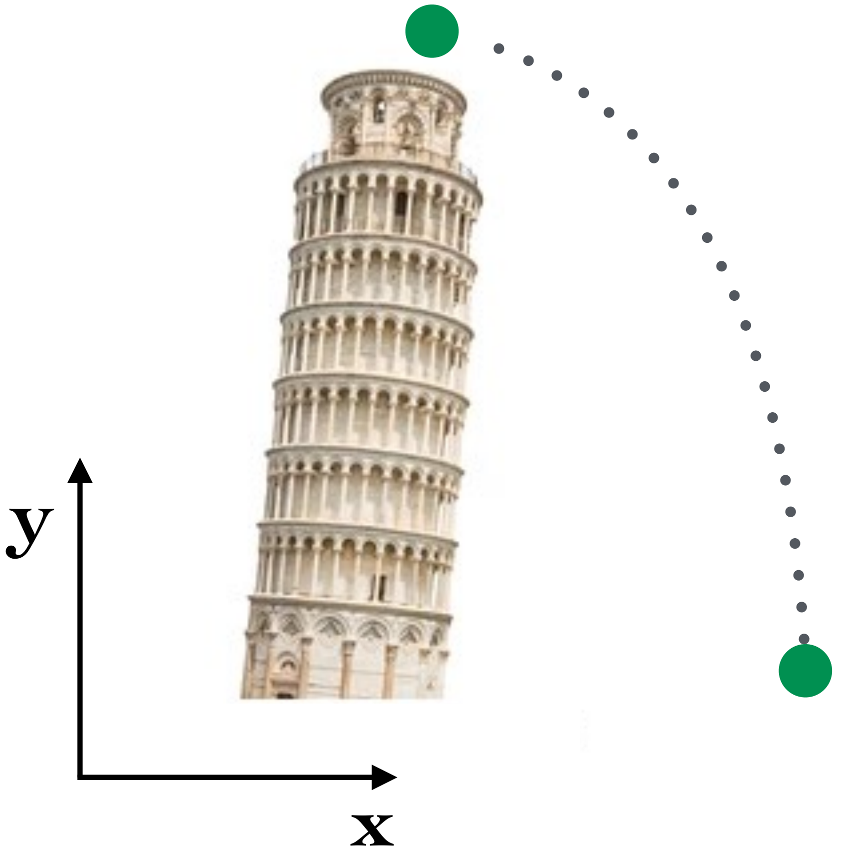

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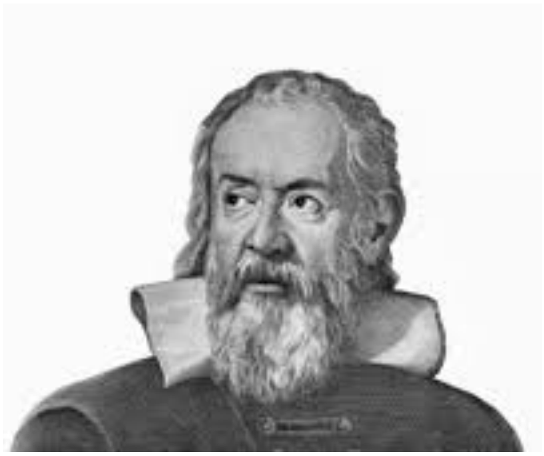

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k **is related to the "surface" acceleration**

Is gravity an emerging phenomenon?

Gravity as emerging theory has been proposed by many, including Sakharov

In Einstein's theory of gravitation one postulates that the action of space-time depends on the curvature (R is the invariant of the Ricci tensor):

$$S(R) = -\frac{1}{16\pi G}\int (\mathrm{d}x)\sqrt{-g}R. \qquad (1)$$

The presence of the action (1) leads to a "metrical elasticity" of space, i.e., to generalized forces which oppose the curving of space.

Here we consider the hypothesis which identifies the action (1) with the change in the action of quantum fluctuations of the vacuum if space is curved. Thus, we consider the metrical elasticity of space as a sort of level displacement effect (cf. also Ref. 1).[1)]

An interesting one page reading

Vacuum quantum fluctuations in curved space and the theory of gravitation

A. D. Sakharov

Dokl. Akad. Nauk SSSR **177**, 70–71 (1967) [Sov. Phys. Dokl. **12**, 1040–1041 (1968). Also S14, pp. 167–169]

Usp. Fiz. Nauk **161**, 64–66 (May 1991)

In Einstein's theory of gravitation one postulates that the action of space-time depends on the curvature (R is the invariant of the Ricci tensor):

$$S(R) = -\frac{1}{16\pi G}\int(\mathrm{d}x)\sqrt{-g}R. \tag{1}$$

The presence of the action (1) leads to a "metrical elasticity" of space, i.e., to generalized forces which oppose the curving of space.

Here we consider the hypothesis which identifies the action (1) with the change in the action of quantum fluctuations of the vacuum if space is curved. Thus, we consider the metrical elasticity of space as a sort of level displacement effect (cf. also Ref. 1).[1)]

In present-day quantum field theory it is assumed that the energy-momentum tensor of the quantum fluctuations of the vacuum $T^i_k(0)$ and the corresponding action $S(0)$, formally proportional to a divergent integral of the fourth power over the momenta of the virtual particles of the form $\int k^3\, dk$, are actually equal to zero.

Recently Ya. B. Zel'dovich[3] suggested that gravitational interactions could lead to a "small" disturbance of this equilibrium and thus to a finite value of Einstein's cosmological constant, in agreement with the recent interpretation of the astrophysical data. Here we are interested in the dependence of the action of the quantum fluctuations on the curvature of space. Expanding the density of the Lagrange function in a series in powers of the curvature, we have (A and $B \sim 1$)

$$(R) = \mathcal{L}(0) + A\int k\mathrm{d}k\cdot R + B\int\frac{\mathrm{d}k}{k}R^2 + \dots \,. \tag{2}$$

The first term corresponds to Einstein's cosmological constant.

The second term, according to our hypothesis, corresponds to the action (1), i.e.,

$$G = -\frac{1}{16\pi A\int k dk}, \quad A \sim 1. \tag{3}$$

The third term in the expansion, written here in a provisional form, leads to corrections, nonlinear in R, to Einstein's equations.[2)]

The divergent integrals over the momenta of the virtual particles in (2) and (3) are constructed from dimensional considerations. Knowing the numerical value of the gravitational constant G, we find that the effective integration limit in (3) is

$$k_0 \sim 10^{28}\ \mathrm{eV} \sim 10^{+33}\,\mathrm{cm}^{-1}.$$

In a gravitational system of units, $G = \hbar = c = 1$. In this case $k_0 \sim 1$. According to the suggestion of M. A. Markov, the quantity k_0 determines the mass of the heaviest particles existing in nature, and which he calls "maximons." It is natural to suppose also that the quantity k_0 determines the limit of applicability of present-day notions of space and causality.

Consideration of the density of the vacuum Lagrange function in a simplified "model" of the theory for noninteracting free fields with particles $M \sim k_0$ shows that for fixed ratios of the masses of real particles and "ghost" particles (i.e., hypothetical particles which give an opposite contribution from that of the real particles to the R-dependent action), a finite change of action arises that is proportional to M^2R and which we identify with R/G. Thus, the magnitude of the gravitational interaction is determined by the masses and equations of motion of free particles, and also, probably, by the "momentum cutoff."

This approach to the theory of gravitation is analogous to the discussion of quantum electrodynamics in Refs. 4 to 6, where the possibility is mentioned of neglecting the Lagrangian of the free electromagnetic field for the calculation of the renormalization of the elementary electric charge. In the paper of L. D. Landau and I. Ya. Pomeranchuk the magnitude of the elementary charge is expressed in terms of the masses of the particles and the momentum cutoff. For a further development of these ideas see Ref. 7, in which the possibility is established of formulating the equations of quantum electrodynamics without the "bare" Lagrangian of the free electromagnetic field.

The author expresses his gratitude to Ya. B. Zel'dovich for the discussion which acted as a spur for the present paper, for acquainting him with Refs. 3 and 7 before their publication, and for helpful advice.

1) Here the molecular attraction of condensed bodies is calculated as the result of changes in the spectrum of electromagnetic fluctuations. As was pointed out by the author, the particular case of the attraction of metallic bodies was studied earlier by Casimir.[2]

2) A more accurate form of this term is $\int(dk/k)(BR^2 + CR^{ik}R_{ik} + DR^{iklm}R_{iklm} + ER^{iklm}R_{iklm})$ where $A, B, C, D, E \sim 1$. According to Refs. 4 to 7, $\int dk/k \sim 137$, so that the third term is important for $R \gtrsim 1/137$ (in gravitational units), i.e., in the neighborhood of the singular point in Friedman's model of the universe.

1. E. M. Lifshits, ZhETF 29:94 (1954); Sov. Phys. JETP 2:73 (1954), trans.
2. H. B. G. Casimir, Proc. Nederl. Akad. Wetensch. 51:793 (1948).
3. Ya. B. Zel'dovich, ZhETF Pis'ma 6:922 (1967); JETP Lett. 6:345 (1967), trans.
4. E. S. Fradkin, Dokl. Akad. Nauk SSSR 98:47 (1954).
5. E. S. Fradkin, Dokl. Akad. Nauk SSSR 100:897 (1955).
6. L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR 102:489 (1955), trans. in Landau's Collected Papers (D. ter Haar, ed.), Pergamon Press, 1965.
7. Ya. B. Zel'dovich, ZhETF Pis'ma 6:1233 (1967).

0038-5670/91/050394-01$01.00

Starting from Euler equations

Description of the fluid

Continuity equation $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Euler equation $$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = \mathbf{f}$$

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Characteristics of the fluid

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Characteristics of the fluid

- barotropic $p \equiv p(\rho)$
- inviscid $\mathbf{f} = -\nabla p$
- irrotational $\mathbf{v} = \nabla \phi$

Small perturbations

Fluctuations around a background configuration

$$
\begin{aligned}
\rho &= \rho_0 + \epsilon\rho_1 + \mathcal{O}(\epsilon^2) \\
p &= p_0 + \epsilon p_1 + \mathcal{O}(\epsilon^2) \\
\phi &= \phi_0 + \epsilon\phi_1 + \mathcal{O}(\epsilon^2)
\end{aligned}
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Bulk

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Bulk **Perturbation**

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Bulk **Perturbation**

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$ general

Small perturbations

Fluctuations around a background configuration

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$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$ general

$$\frac{\partial \rho_0}{\partial t} + \nabla \cdot (\rho_0 \mathbf{v_0}) = 0$$ bulk

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Bulk

Perturbation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$ general

$$\frac{\partial\rho_0}{\partial t} + \nabla\cdot(\rho_0\mathbf{v_0}) = 0$$ bulk

$$\frac{\partial\rho_1}{\partial t} + \nabla\cdot(\rho_0\mathbf{v_1}) + \nabla\cdot(\rho_1\mathbf{v_0}) = 0$$ perturbation

Small perturbations

Combining linearized Euler and continuity equations:

$$\frac{\partial}{\partial t}\left(c_s^{-2}\rho_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)-\nabla\cdot\left(\rho_0\nabla\phi_1-c_s^{-2}\rho_0\mathbf{v}_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)=0$$

where $\quad c_s^2=\dfrac{\partial p}{\partial\rho}$

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check

$\mathbf{v_0} = 0\,,\quad \rho_0 = \text{const},\quad c_s = \text{const}$

$$\frac{\partial^2\phi_1}{\partial t^2} - c_s^2\nabla^2\phi_1 = 0$$

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The non uniform medium changes the propagation

Gravity emerges

$$\frac{\partial}{\partial t}\left(c_s^{-2}\rho_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)-\nabla\cdot\left(\rho_0\nabla\phi_1-c_s^{-2}\rho_0\mathbf{v}_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)=0$$

Solving this equation…

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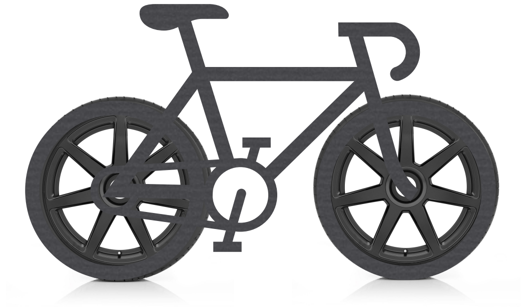
Solving this equation…

GR bike



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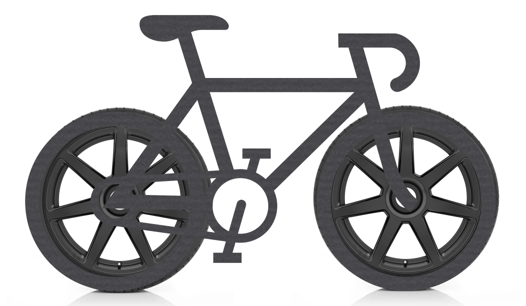

We can rewrite the above equation as

$$\boxed{\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi_1\right)=0}$$

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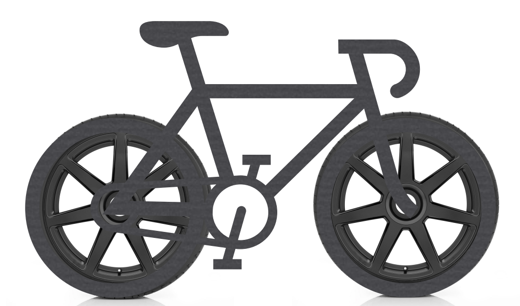

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Schwarzschild acoustic metric?

Acoustic metric

$$ds^2 = \frac{\rho}{c_s}\Big(-(c_s^2 - v^2)dt^2 + 2\mathbf{v}\cdot\mathbf{dx}\,dt + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big)$$

Painleve'–Gullstrand representation of Schwarzschid metric

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 \pm \sqrt{\frac{2GM}{r}}drdt + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

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$v \propto \dfrac{1}{\sqrt{r}}$ divergent flow at the origin

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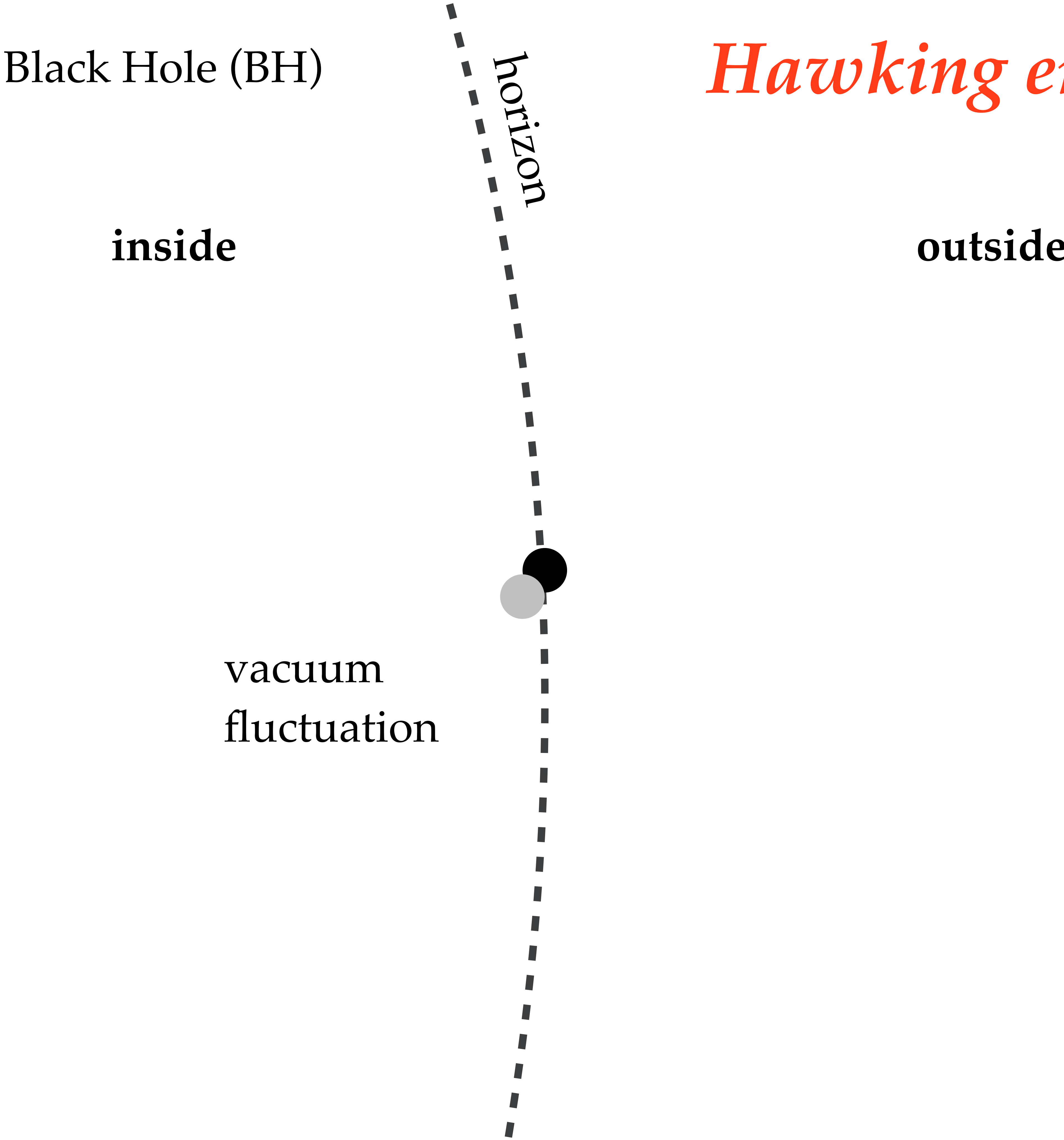
Abandon the 3D spherical geometry

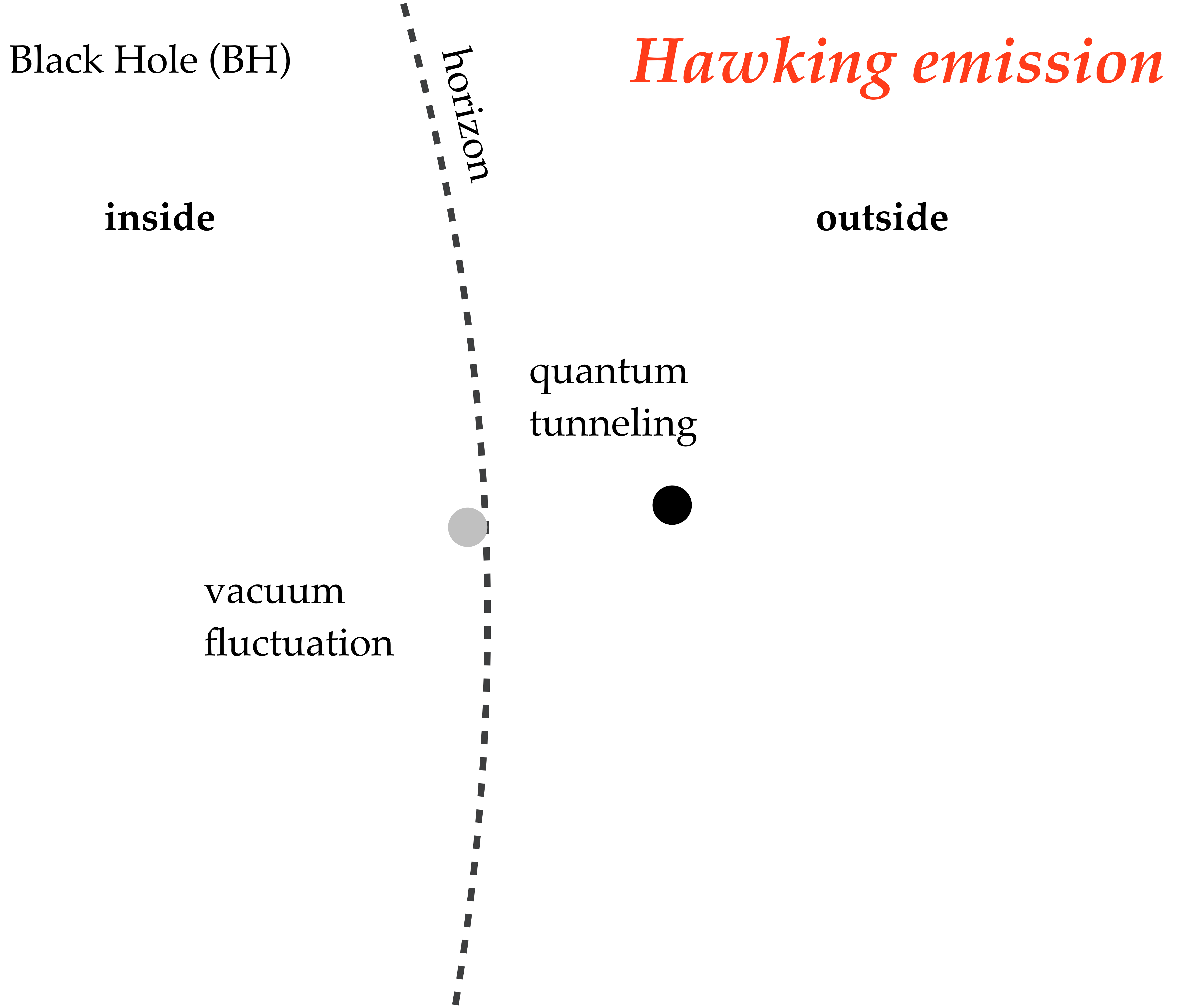
Hawking radiation

S. W. Hawking, Particle creation by black holes, *Commun. Math. Phys. 43, 199 (1975)*

W. Unruh, Experimental black hole evaporation, *Phys.Rev.Lett. 46, 1351 (1981)*.

Hawking emission

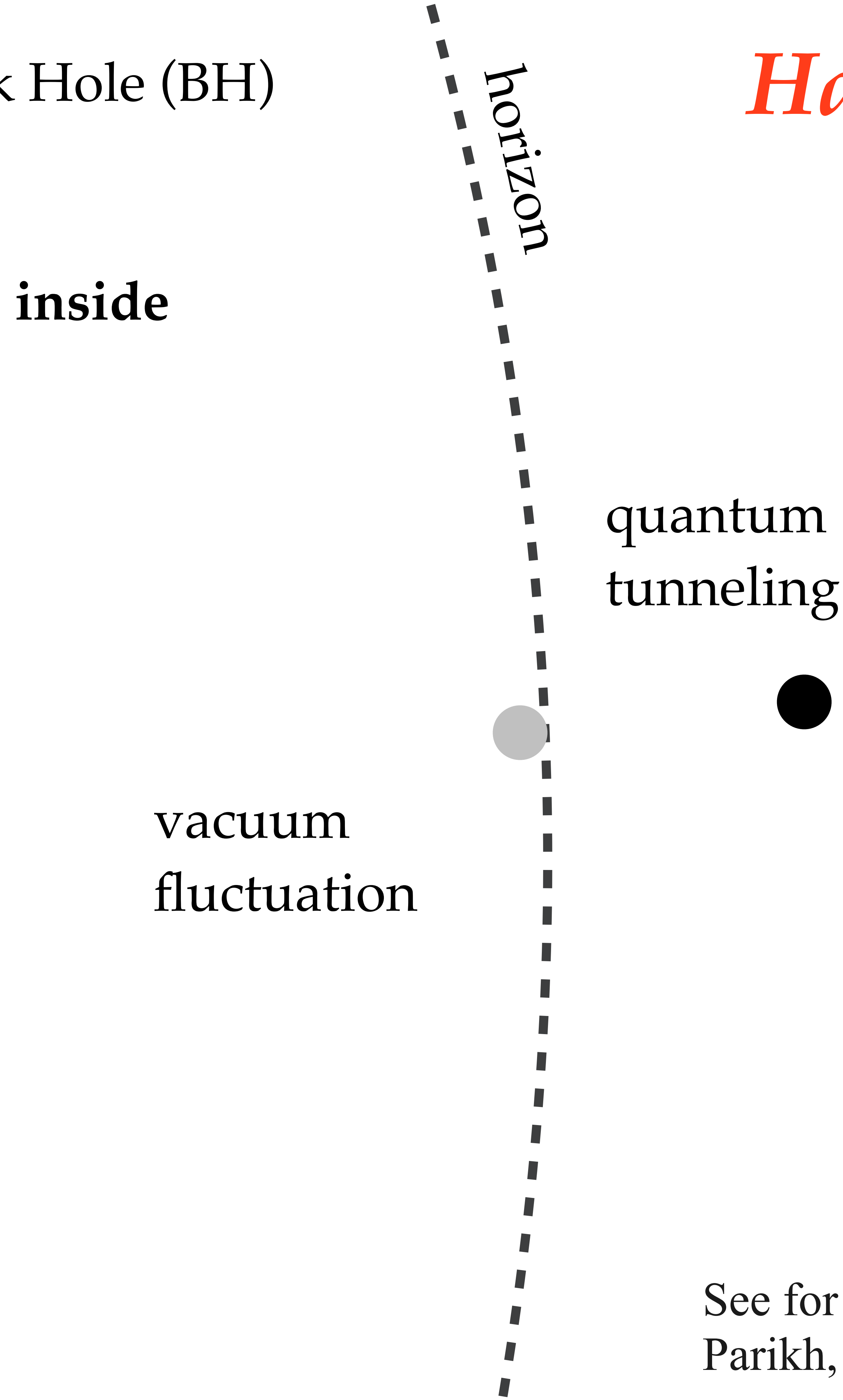




Black Hole (BH)
Hawking emission
horizon
inside
outside
quantum
tunneling
vacuum
fluctuation

Hawking emission

Black Hole (BH)



See for instance
Parikh, Wilczek *Phys.Rev.Lett.* 85 (2000) 5042

BH thermodynamics

A particle/nuclear physics perspective

WKB tunneling amplitude $\Gamma \sim e^{-2\,\mathrm{Im}\,\mathrm{S}}$

BH thermodynamics

A particle/nuclear physics perspective

WKB tunneling amplitude $\Gamma \sim e^{-2\,\mathrm{Im}\,\mathrm{S}}$

using the geodesic equation $\mathrm{Im}\,\mathrm{S} = 4\pi\omega M$

BH thermodynamics

A particle/nuclear physics perspective

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$$\boxed{\Gamma \sim e^{-8\pi M\omega} = e^{-\omega/T} \qquad T = \frac{1}{8\pi M} = \frac{g}{2\pi}}$$

BH thermodynamics

A particle/nuclear physics perspective

WKB tunneling amplitude $\Gamma \sim e^{-2\,\mathrm{Im}\,\mathrm{S}}$

using the geodesic equation $\mathrm{Im}\,\mathrm{S} = 4\pi\omega M$

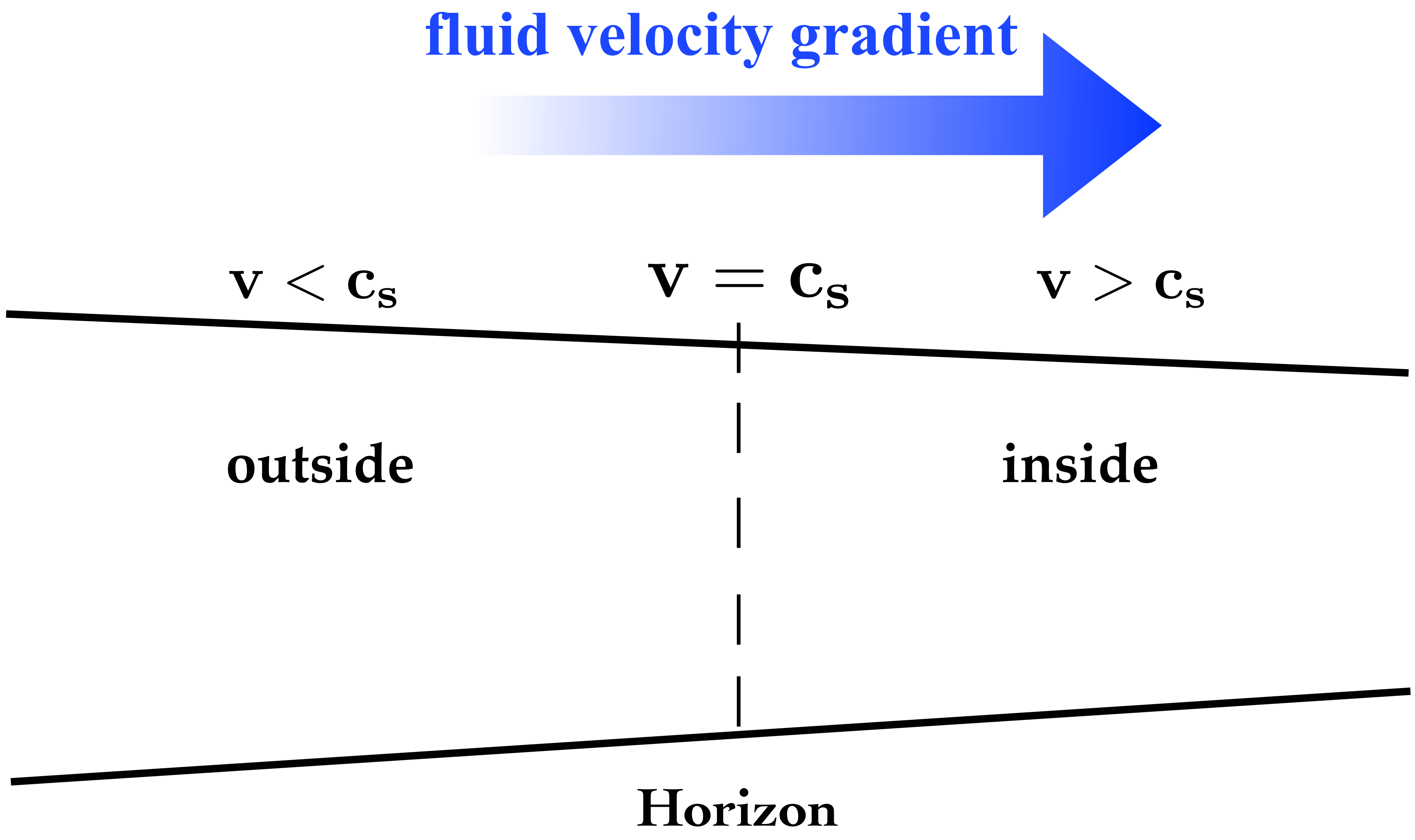
$$\boxed{\Gamma \sim e^{-8\pi M\omega} = e^{-\omega/T} \qquad T = \frac{1}{8\pi M} = \frac{g}{2\pi}}$$

By analogy, the temperature of an acoustic hole $T = \frac{1}{2\pi} \frac{\partial |c_s - v|}{\partial n}\bigg|_H$

$$T \simeq mc_s^2 \simeq 10^{-9} K$$

Quantum effects

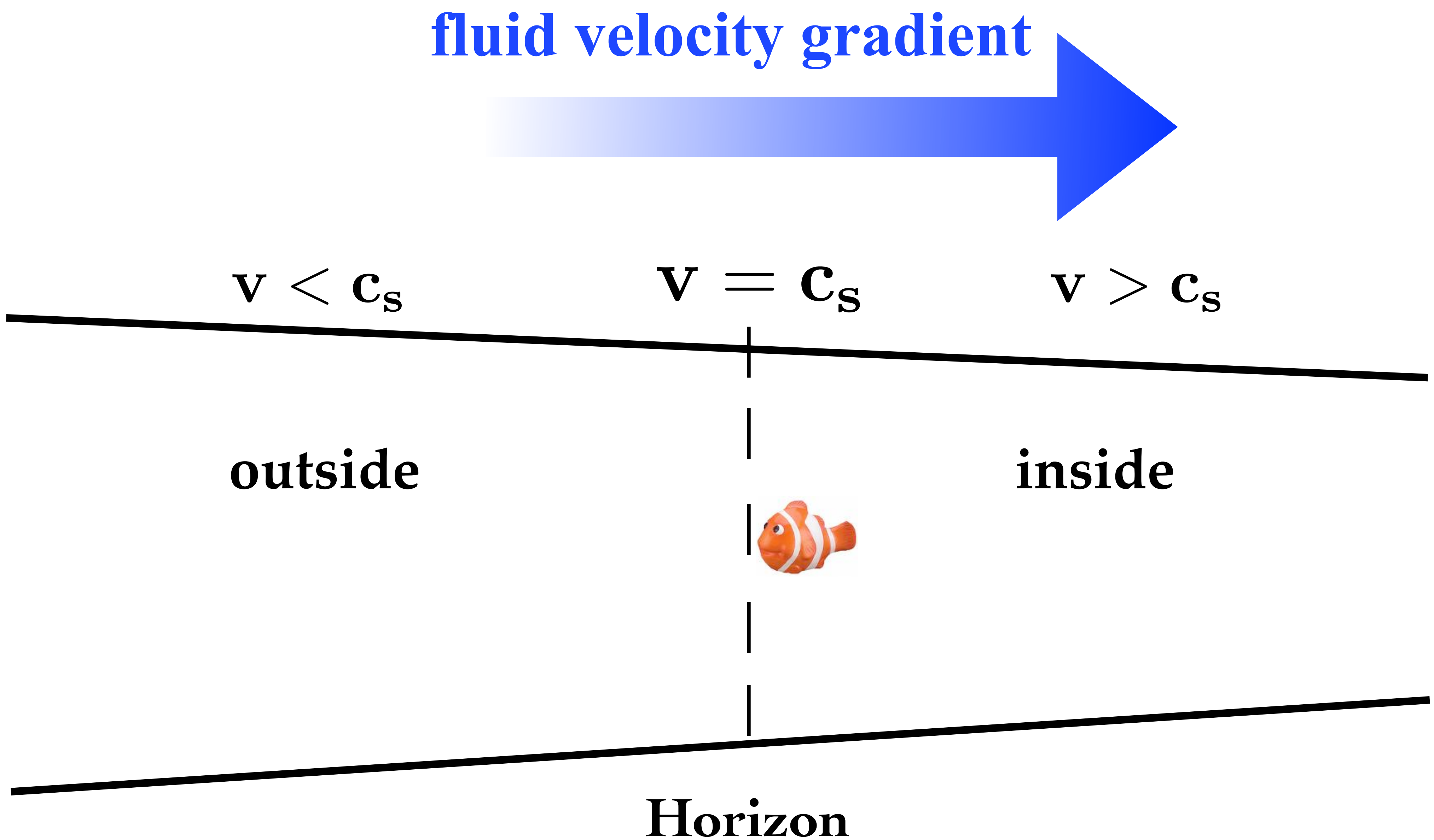
Quantum effects in the analog picture



The phonon escapes by quantum tunneling

Quantum effects

Quantum effects in the analog picture



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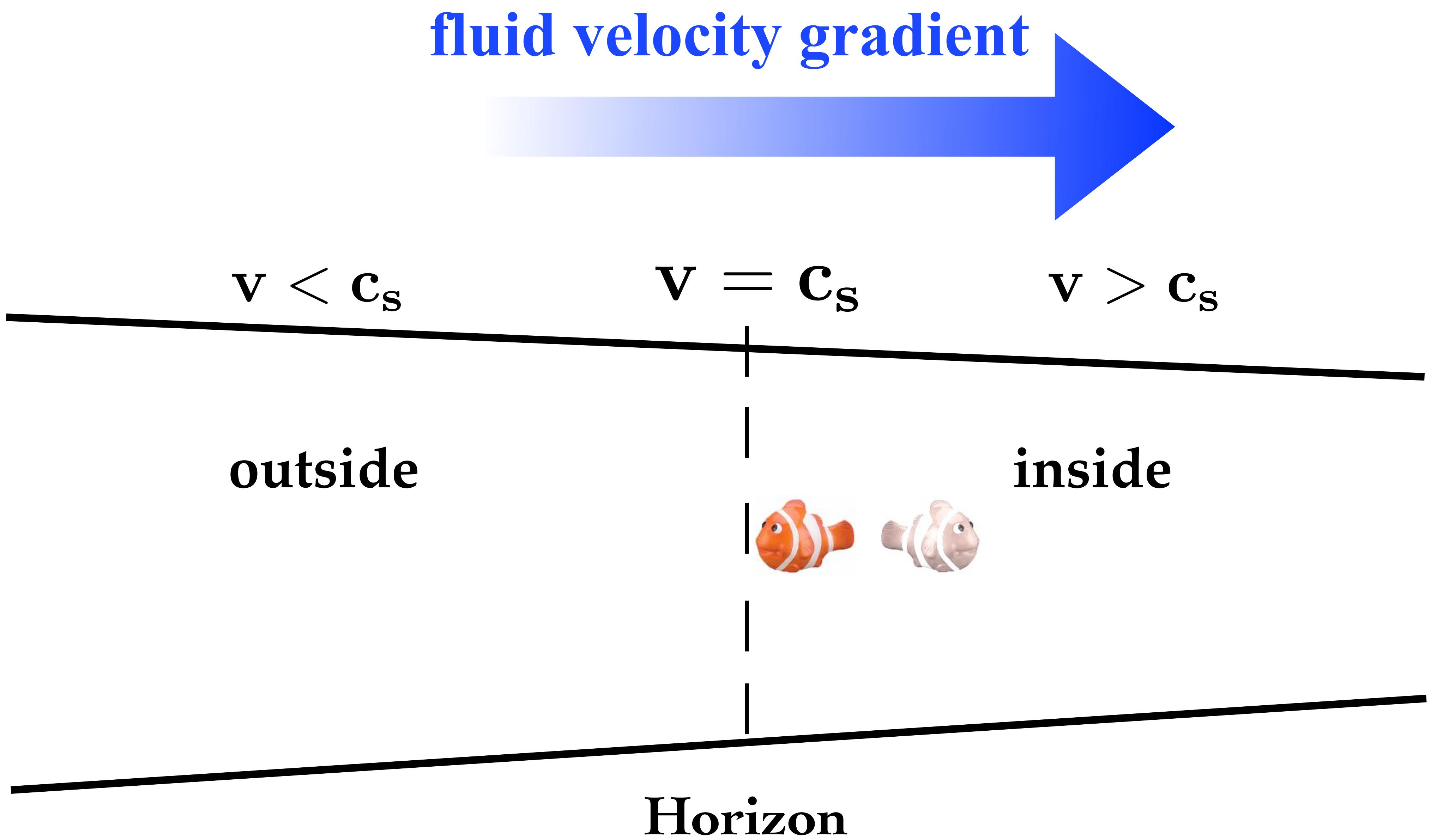

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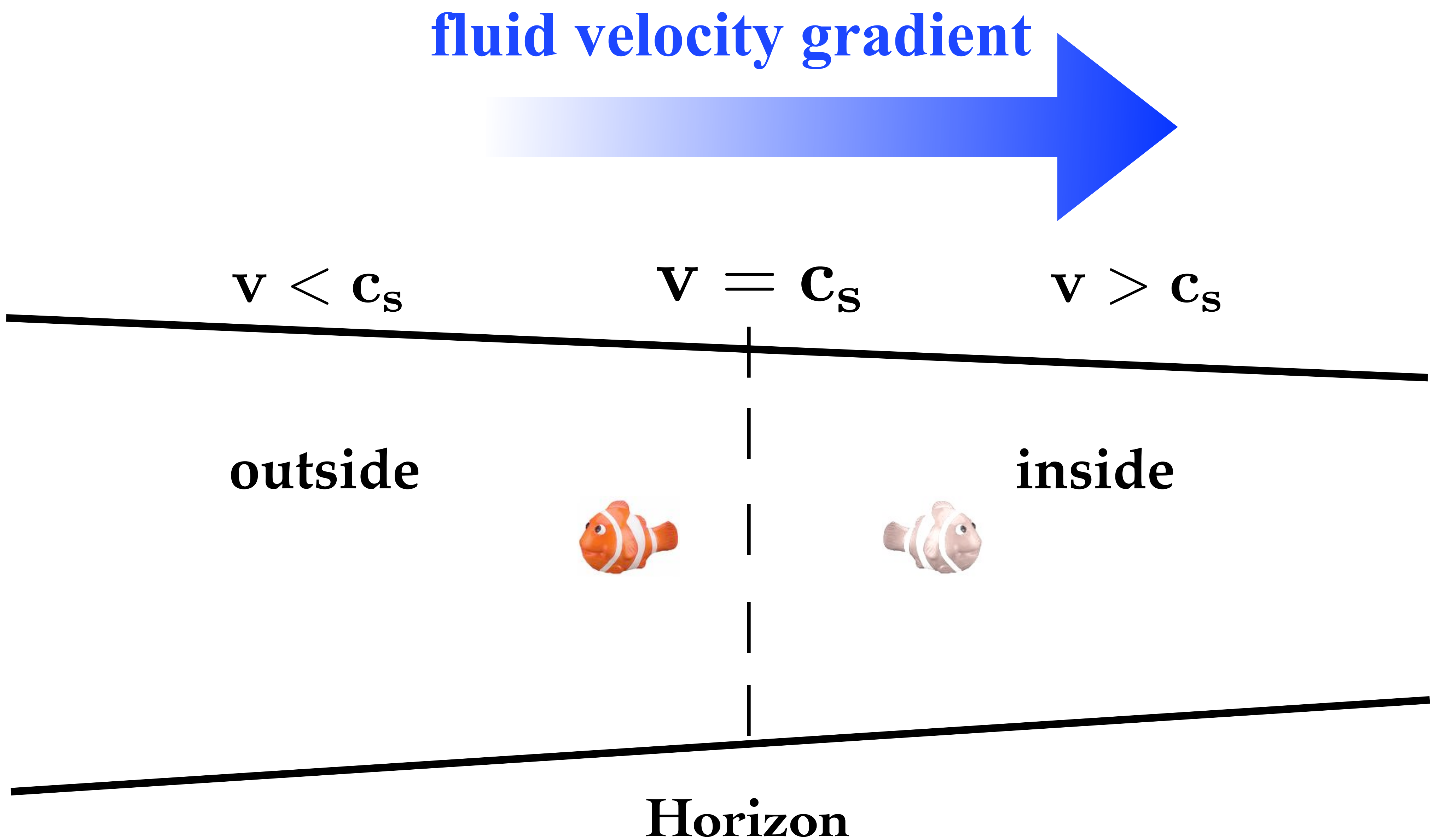

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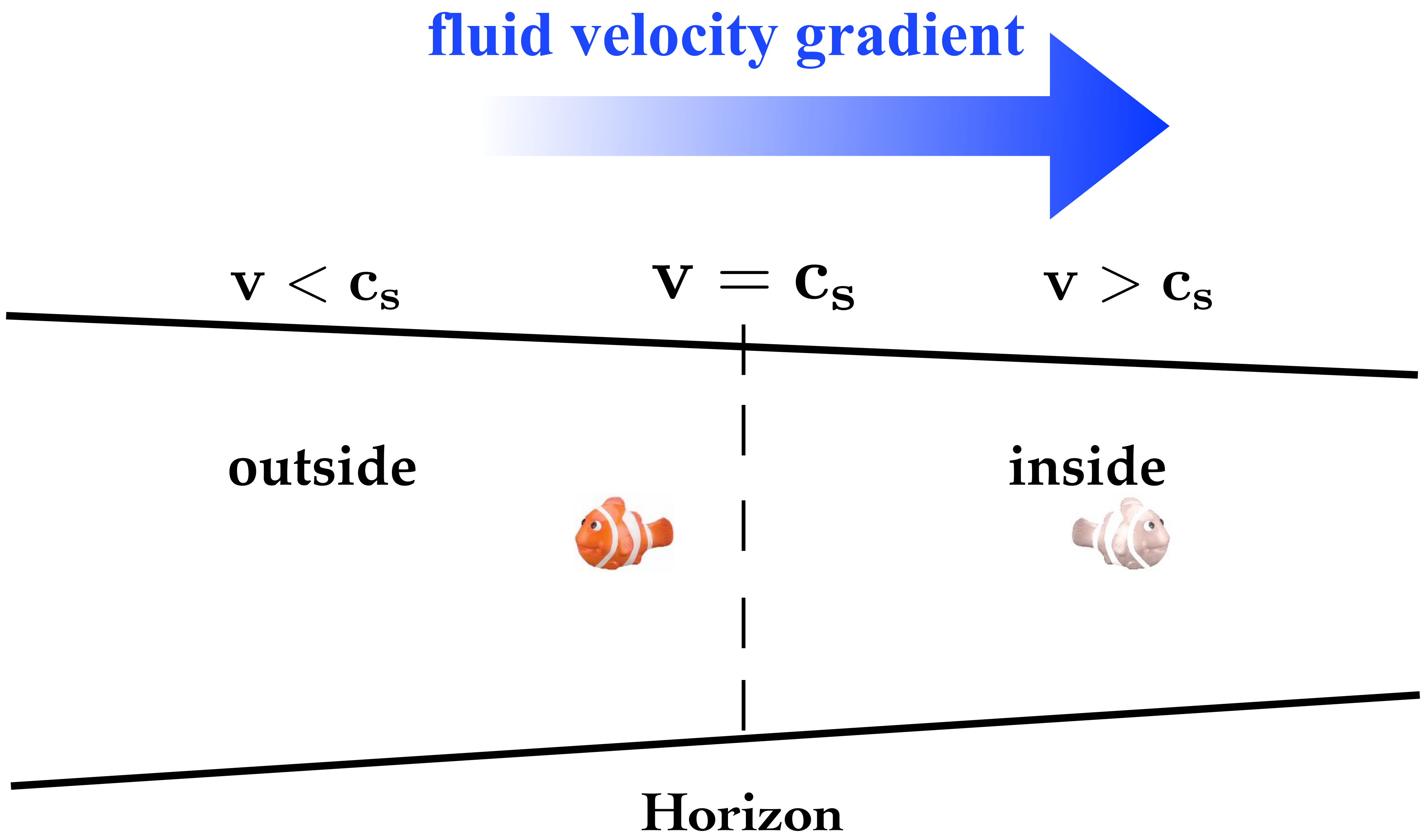
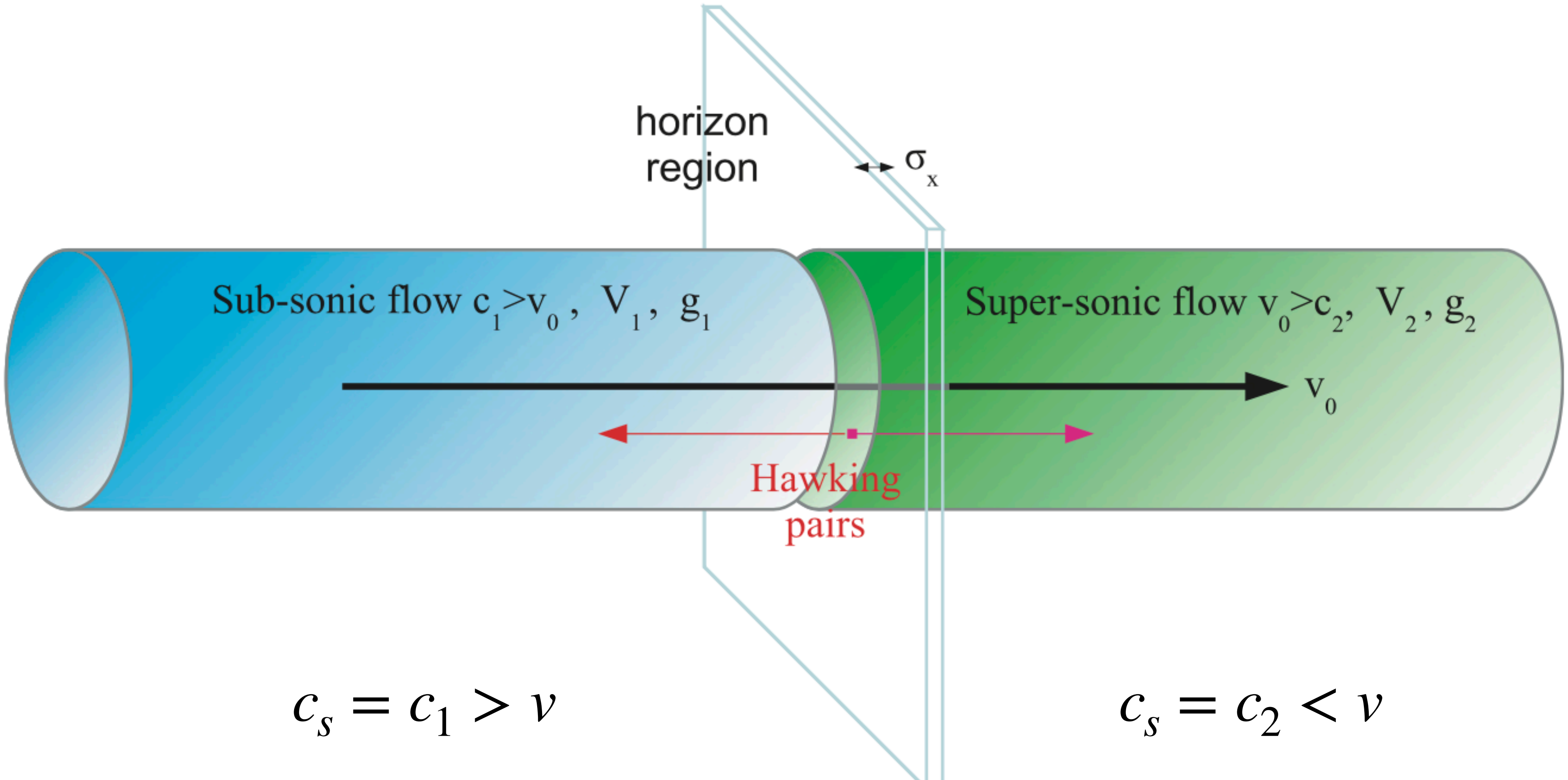


The phonon escapes by quantum tunneling

Setup: trapped BEC condensate



$c_s = c_1 > v$

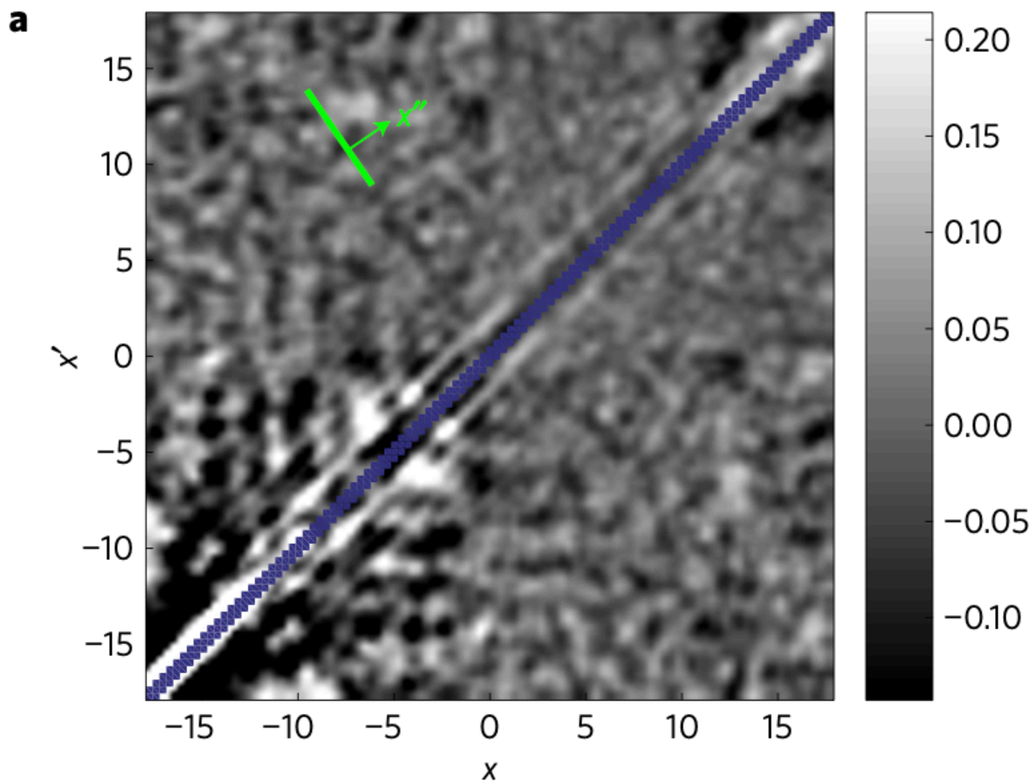
$c_s = c_2 < v$

Carusotto *et al New J. Phys.* 10 103001 (2008)

Instead of changing the velocity, change the speed of sound

Experimental observation

experiment



numeric

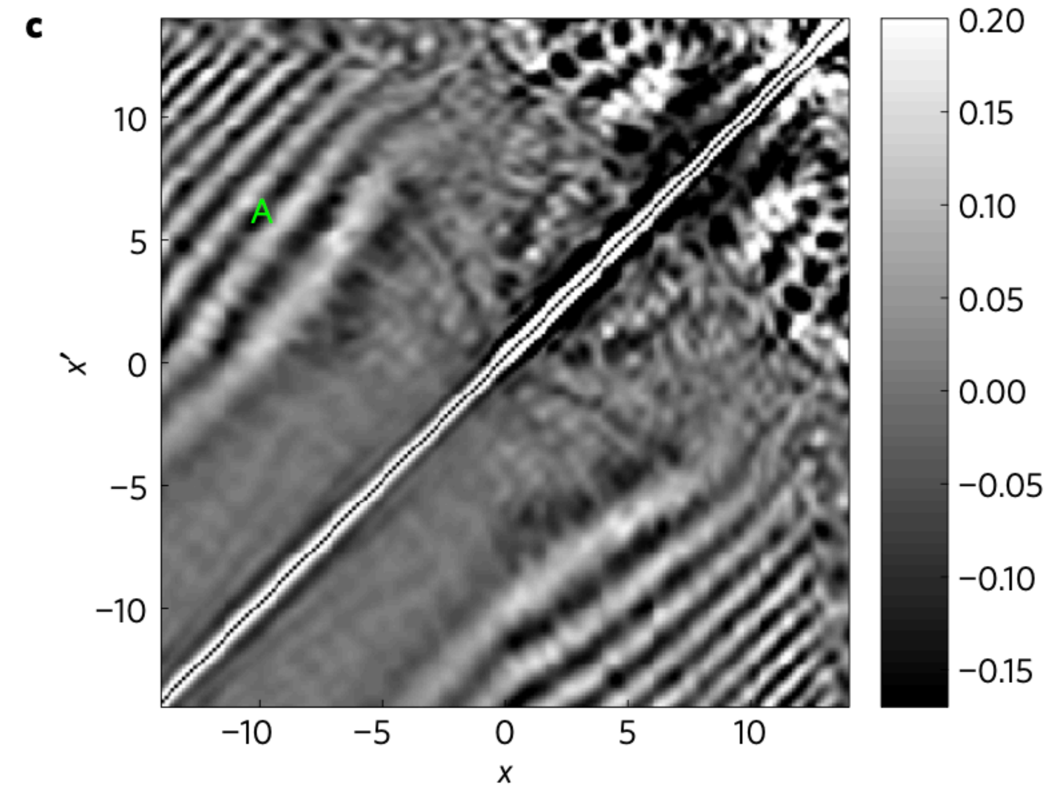
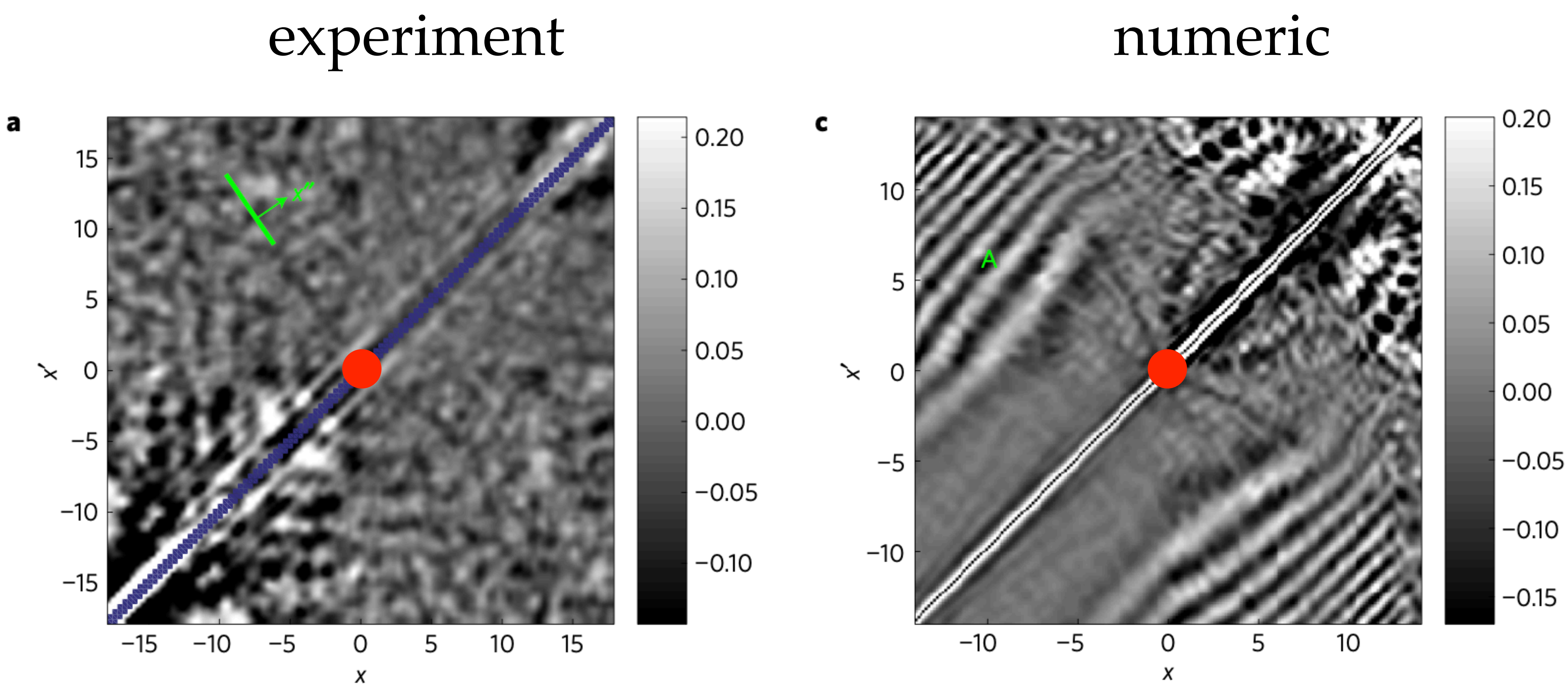


Image obtained by 4600 repetitions of the experiment

Steinhauer, Nature Phys. 12 (2016) 959

Experimental observation



black hole position

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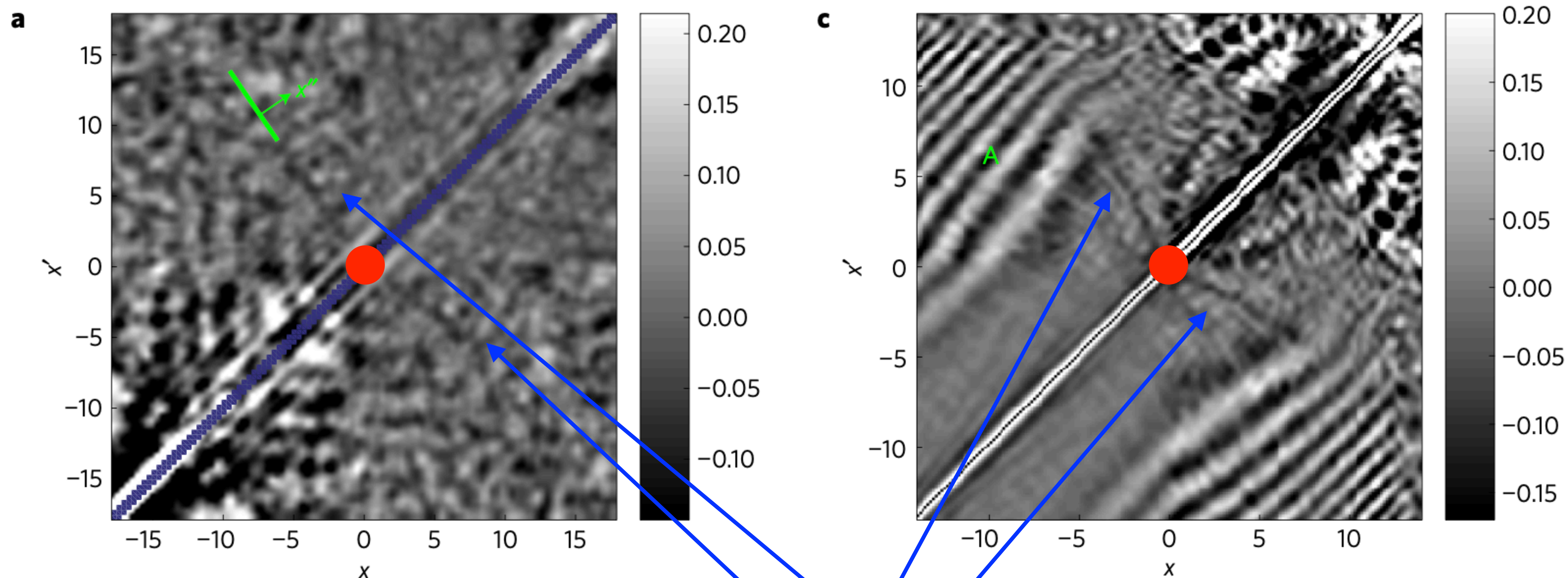
Steinhauer, Nature Phys. 12 (2016) 959

Experimental observation

experiment numeric



black hole position

effect of phonon emission on the density

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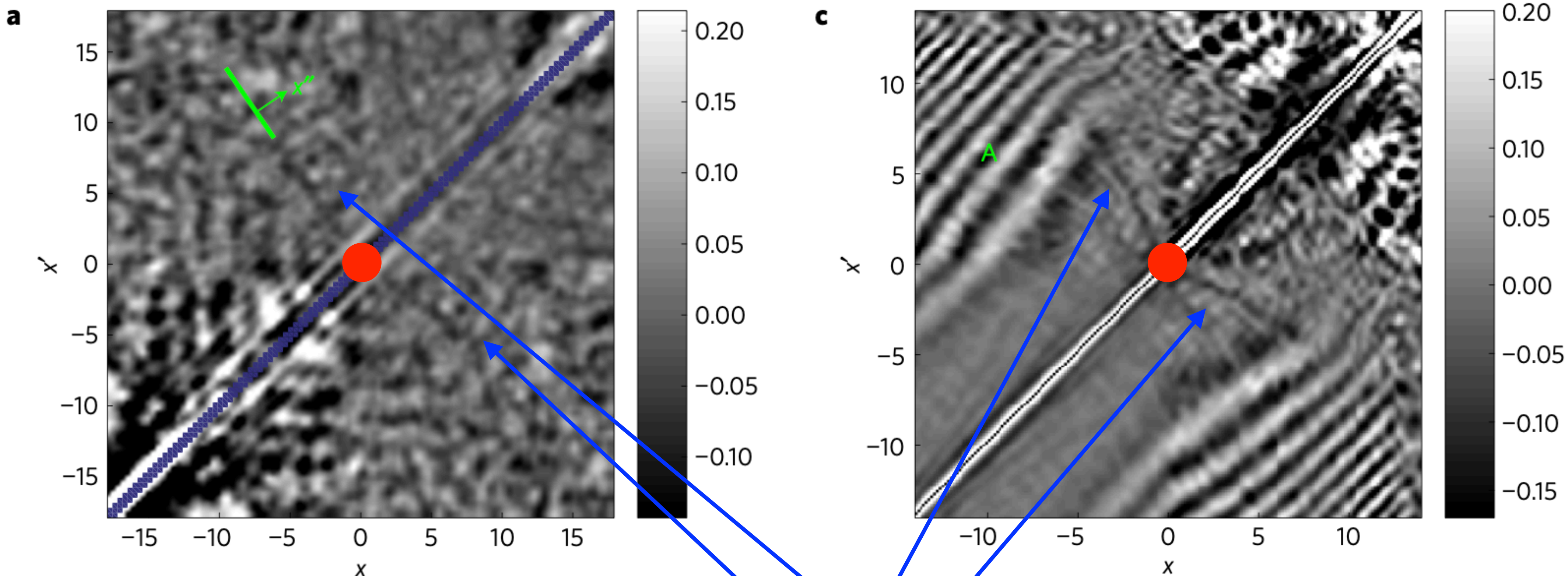

black hole position

effect of phonon emission on the density

Image obtained by 4600 repetitions of the experiment

Fitted Hawking temperature $\sim 10^{-9} K$

Steinhauer, Nature Phys. 12 (2016) 959

Kinetic theory

From GR

R. W. Lindquist, Annals of Physics 37, 487 (1966).
J. Stewart, Lecture Notes in Physics, Lecture Notes in Physics No. v. 10 (Springer-Verlag, 1969).

To the analog model

MM and C. Manuel, Phys.Rev.D 77 (2008) 103014
MM, D. Grasso, S. Trabucco and L. Chiofalo Phys.Rev.D 103 (2021) 7, 076001

Phonon distribution

Phonons emitted at a temperature T have a Bose-Einstein distribution f

Solution of $$L[f] \equiv p^{\alpha}\frac{\partial f}{\partial x^{\alpha}} - \Gamma^{\alpha}_{\beta\gamma}p^{\beta}p^{\gamma}\frac{\partial f}{\partial p^{\alpha}} = C[f]$$

for $C[f] = 0$

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for $C[f] = 0$

Assuming $$f(x,p) = \frac{1}{\exp(p^\mu \beta_\mu) - 1}$$

$$\beta_{\lambda;\rho} + \beta_{\rho;\lambda} = 0$$ solution $$\beta^\mu = (\beta, \mathbf{0})$$

Thermodynamics

Knowing the distribution function we can obtain the thermodynamics

Thermodynamics

Knowing the distribution function we can obtain the thermodynamics

Phonon number $n^{\mu}_{\text{ph}} = \int p^{\mu} f(x, p)\, d\mathscr{P}$

Thermodynamics

Knowing the distribution function we can obtain the thermodynamics

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Transport of "phonon" number

Covariant conservation $$\partial_\nu n^\nu_{\mathrm{ph}} + \Gamma^\mu_{\mu\nu} n^\nu_{\mathrm{ph}} = \int C[f] d\mathcal{P}$$

collision integral

Where $$\Gamma^\mu_{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\nu \sqrt{-g} = \frac{1}{c_s} \frac{\partial c_s}{\partial x^\nu}$$

We keep $C[f] = 0$

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Change in the number of phonons due to the background non uniformity!

The entropy flux

The entropy lost by the horizon is gained by the phonon gas

$$\Delta S_{\mathrm{ph}} = -\,\Delta S_{\mathrm{H}}$$

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From the Fluid $\longrightarrow$ To the Phonon gas

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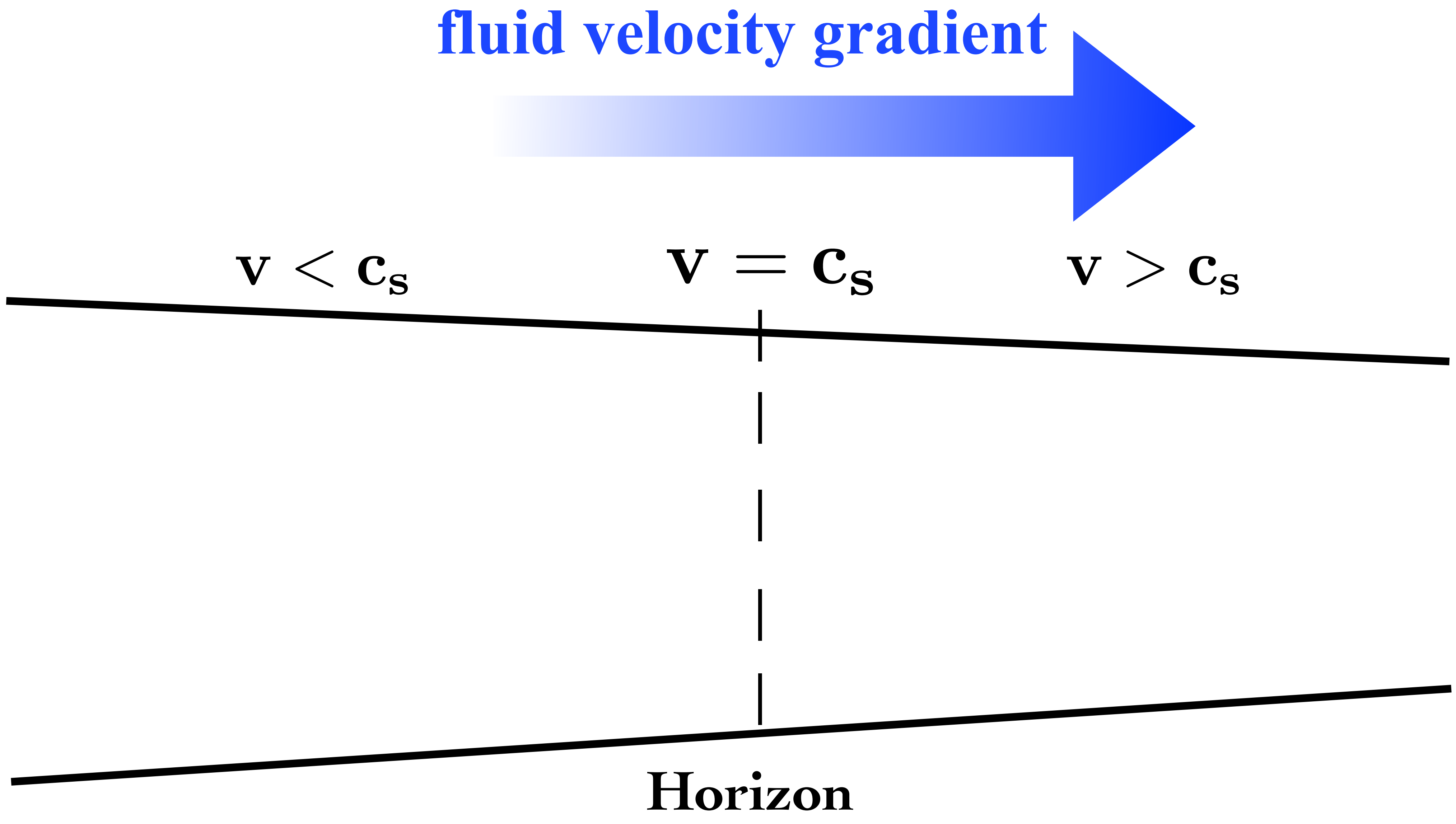
Dissipative processes localized at the horizon

Dissipative processes

M.L. Chiofalo, D. Grasso, MM and S. Trabucco, e-Print: 2202.13790 [gr-qc]

Viscosity of an acoustic hole



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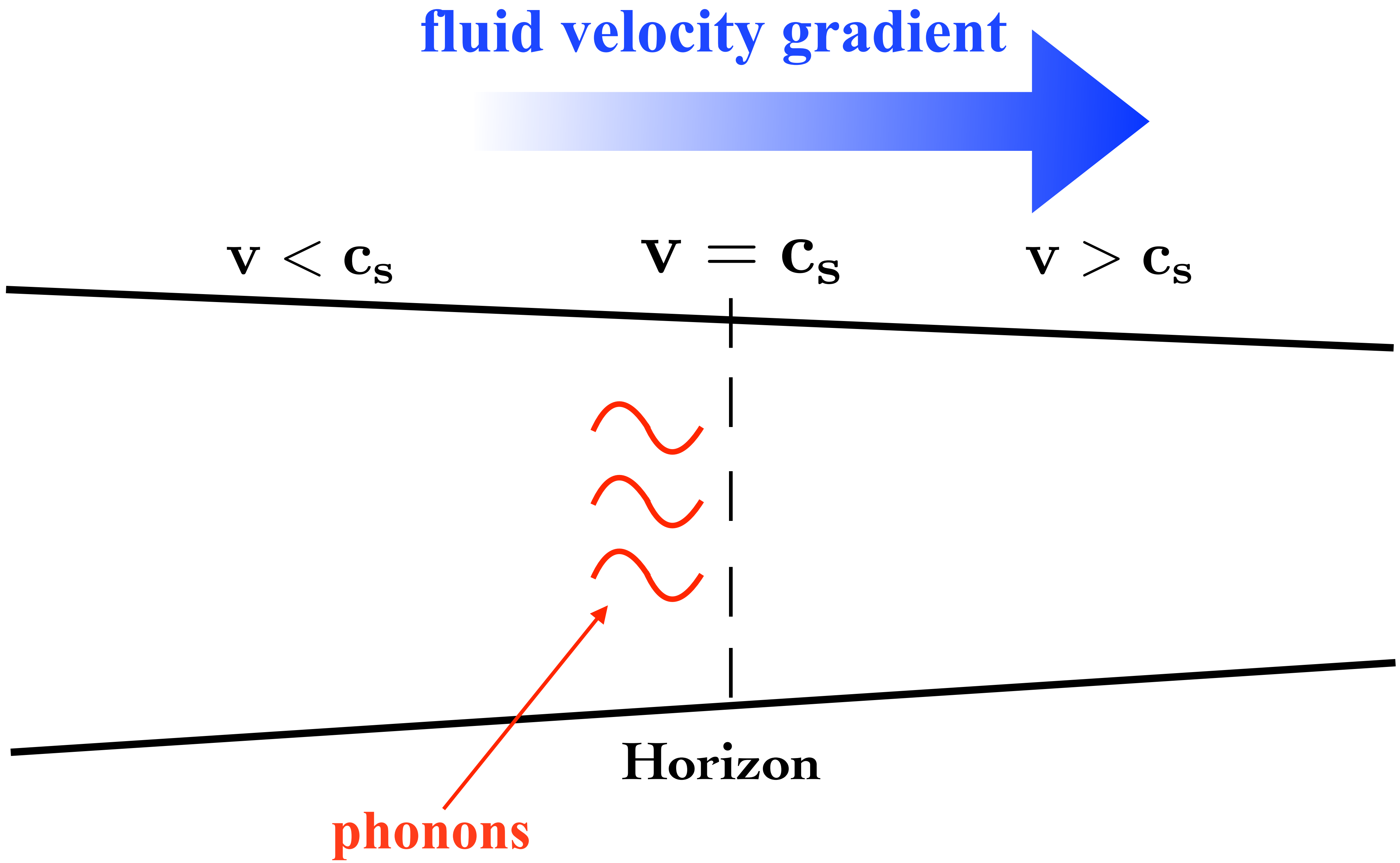

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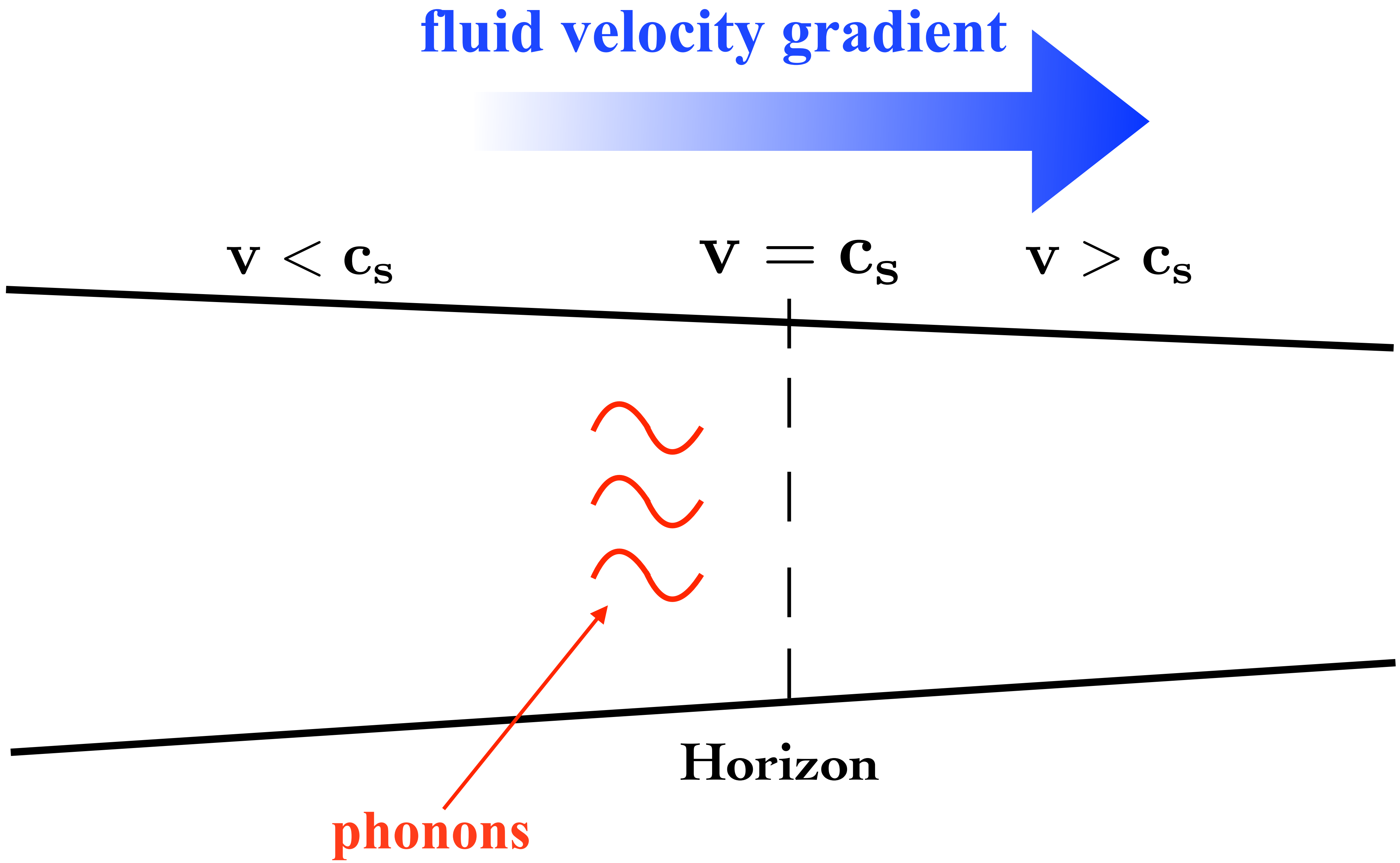

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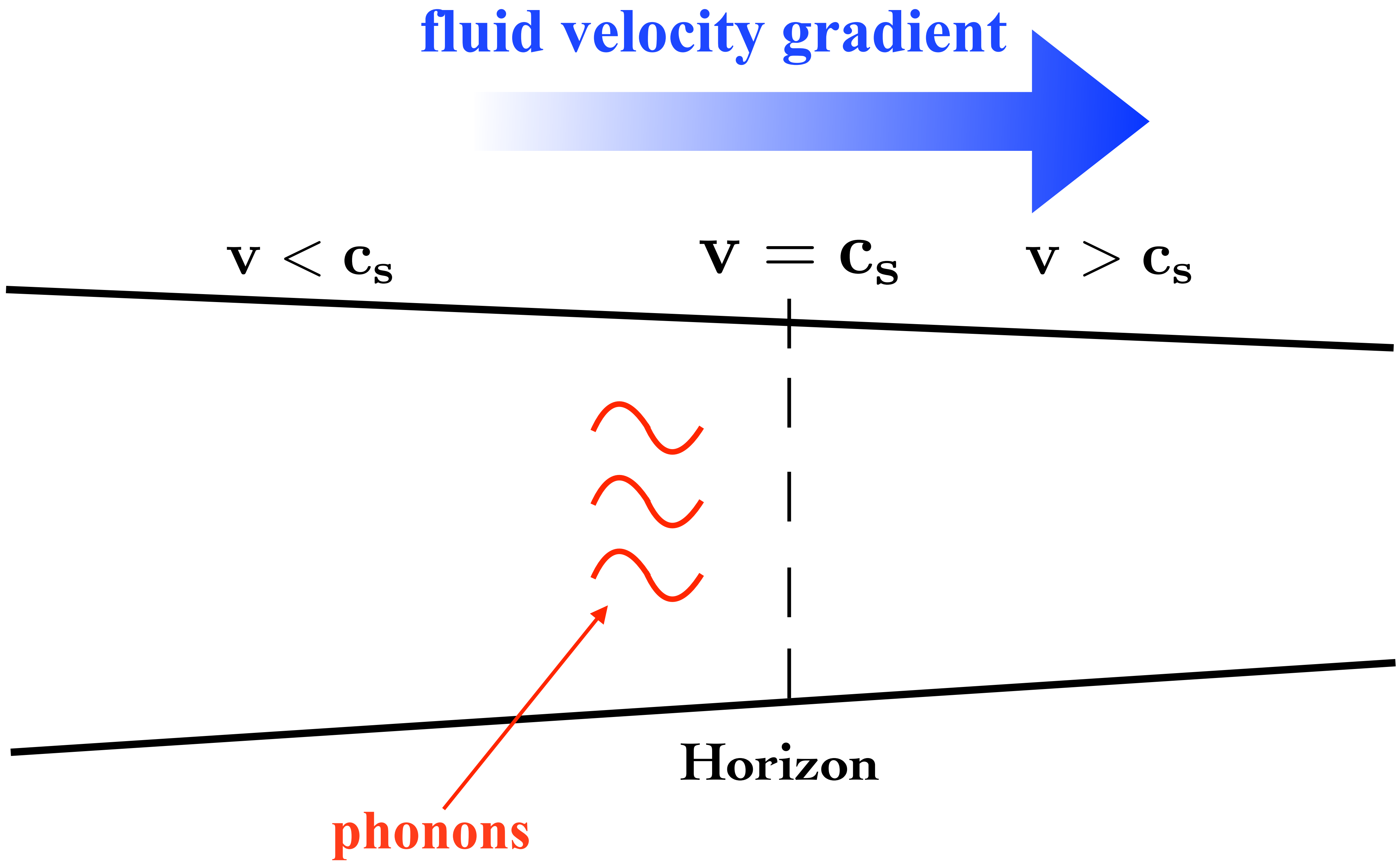
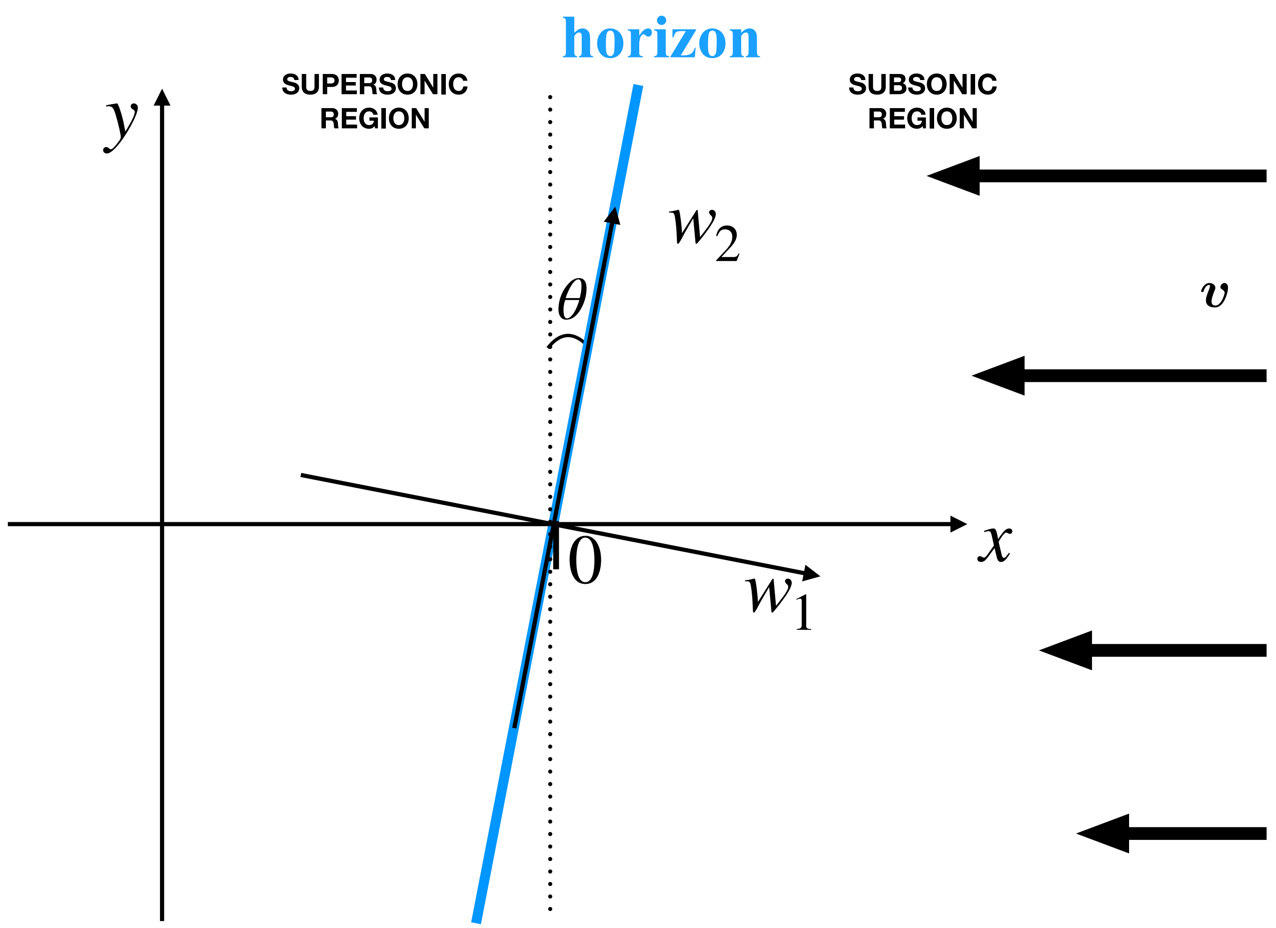


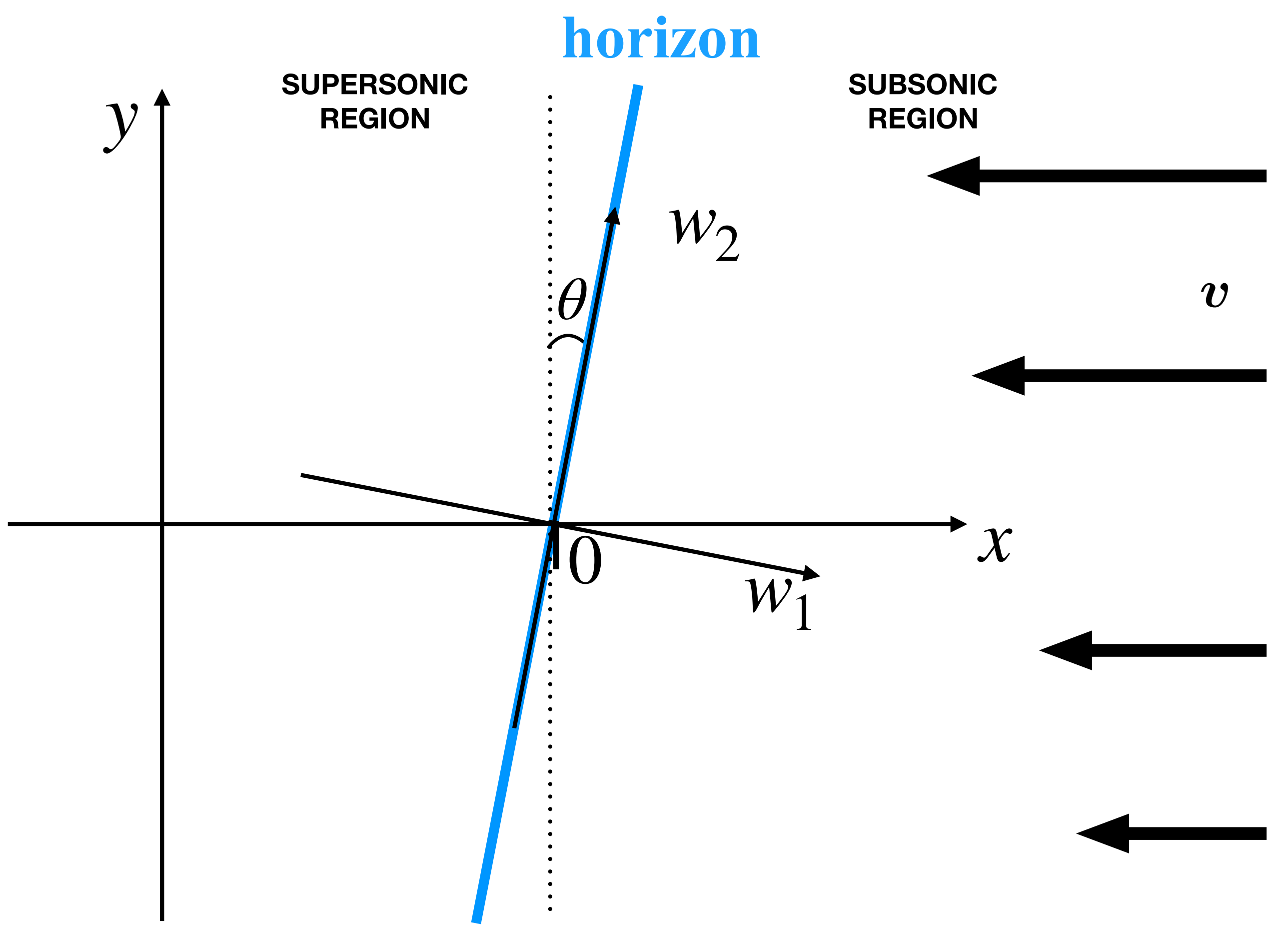
Energy conservation, the phonon emission results in a decrease of the fluid velocity

More formally



$$\mathbf{v} = (v, 0, 0)$$
$$v = c_s - Cx + ky$$

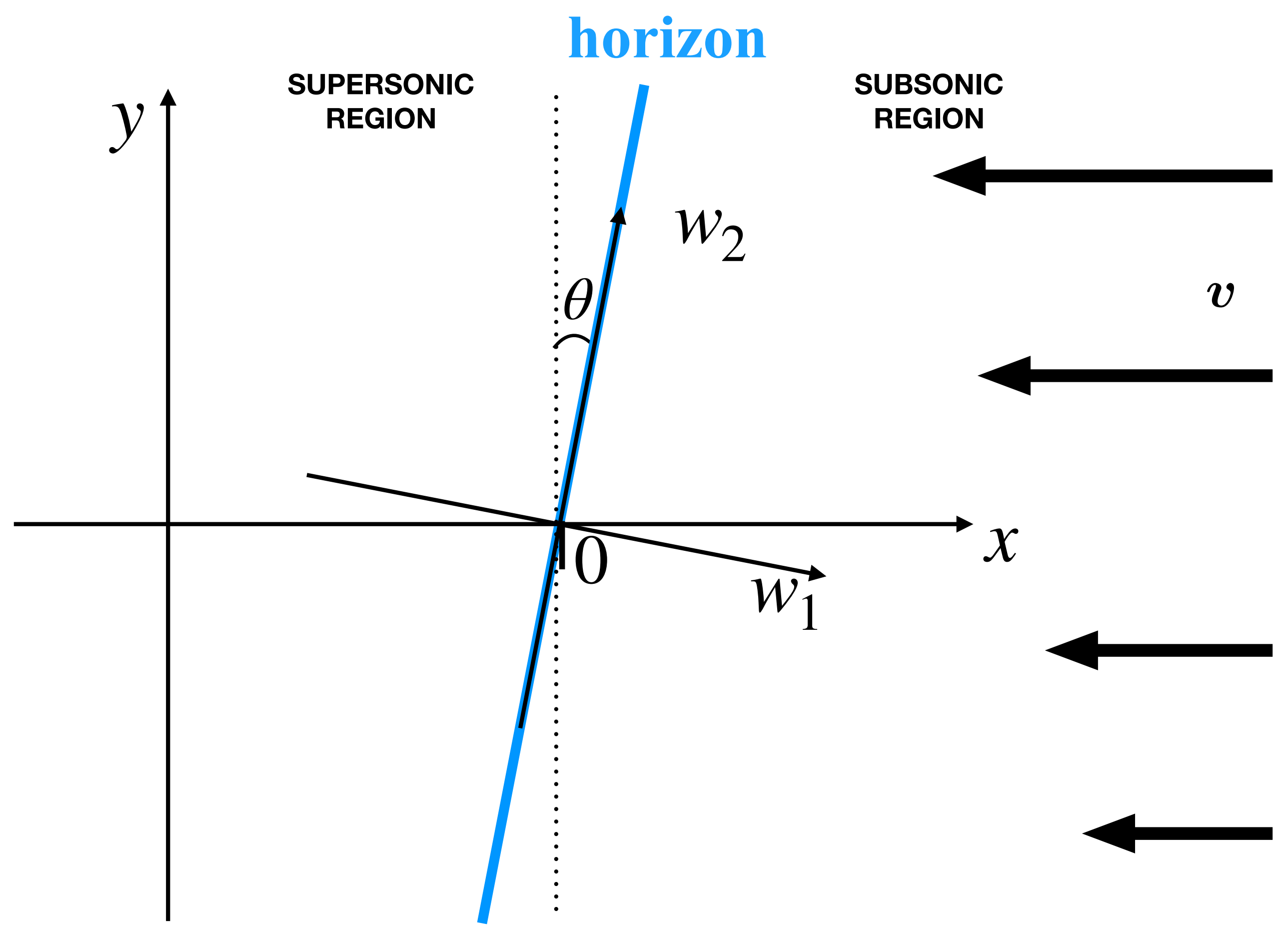
More formally



$$\mathbf{v} = (v, 0, 0)$$
$$v = c_s - Cx + ky$$

Viscous stress-tensor $\qquad \sigma'_{ik} = \eta\left(\partial_i v_k + \partial_k v_i\right) + \zeta \delta_{ix}\delta_{kx}\nabla\cdot\mathbf{v}$

More formally



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Assuming that dissipation is only due to phonon emission

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Saturation of the KSS bounds.

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Yields

$$\frac{\zeta_{\rm eff}}{s_{\rm ph}} = \frac{\eta}{s_{\rm ph}} = \frac{1}{4\pi}$$

Saturation of the KSS bounds.

This is ideal: any phonon scattering would violate the bound.

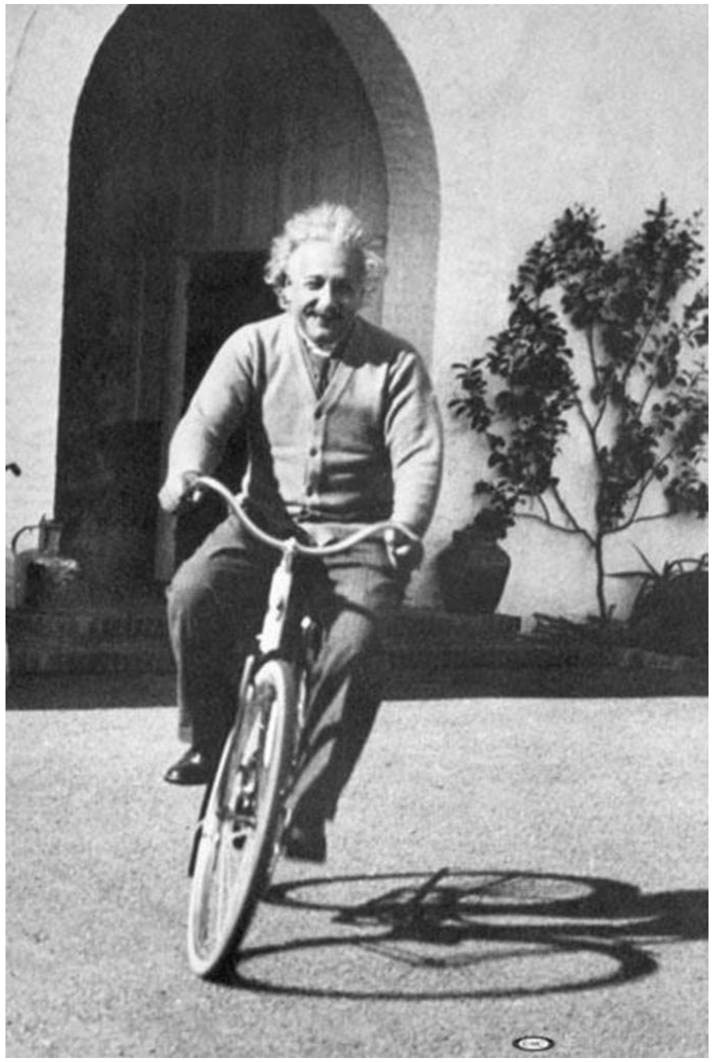
Conclusions

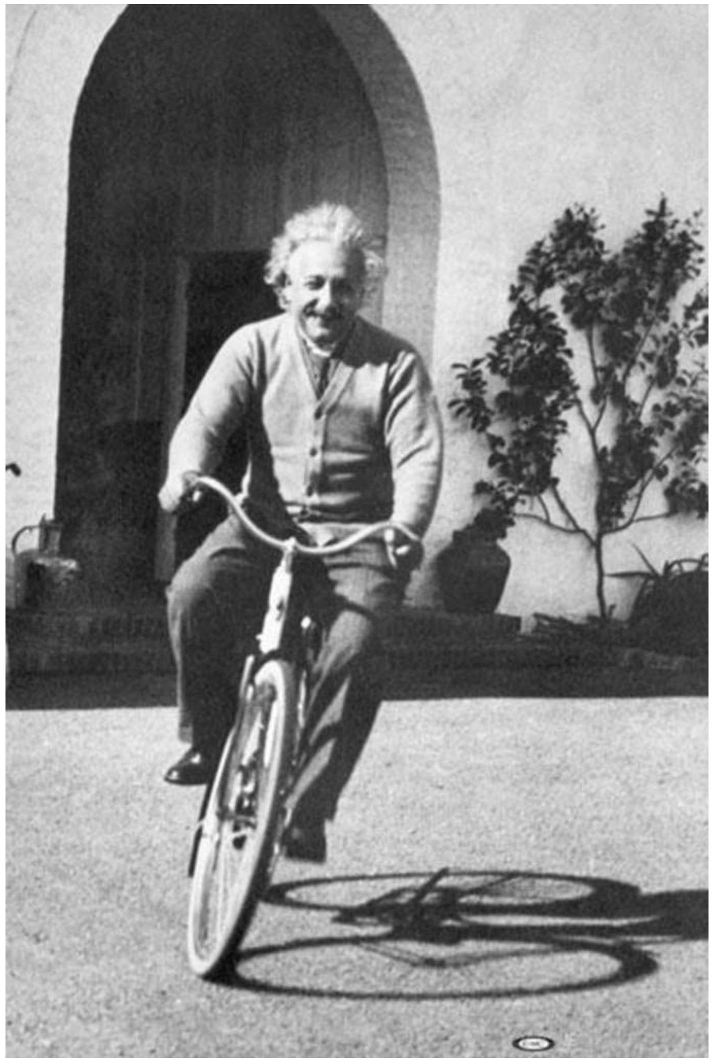
- There is a large number of physical systems linked by analogies

- We can use them to solve/approach hard problems or to reproduce unreachable systems

- Two examples have been discussed:
  1) Color superconductors
  2) Shear viscosity

- The dissipation at the horizon seems to saturate the KSS bound





Thanks for your attention!

massimo@lngs.infn.it

Backup slide

Entropy balance

Entropy loss of the fluid $$\Delta S_H = 2\pi \frac{r_H}{L_c^2} \Delta r_H$$

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number of degrees of freedom

$$s_{\rm ph} = \frac{\pi T}{6 L_c^2 C x}$$

with $C = (v + c_s)'|_H$

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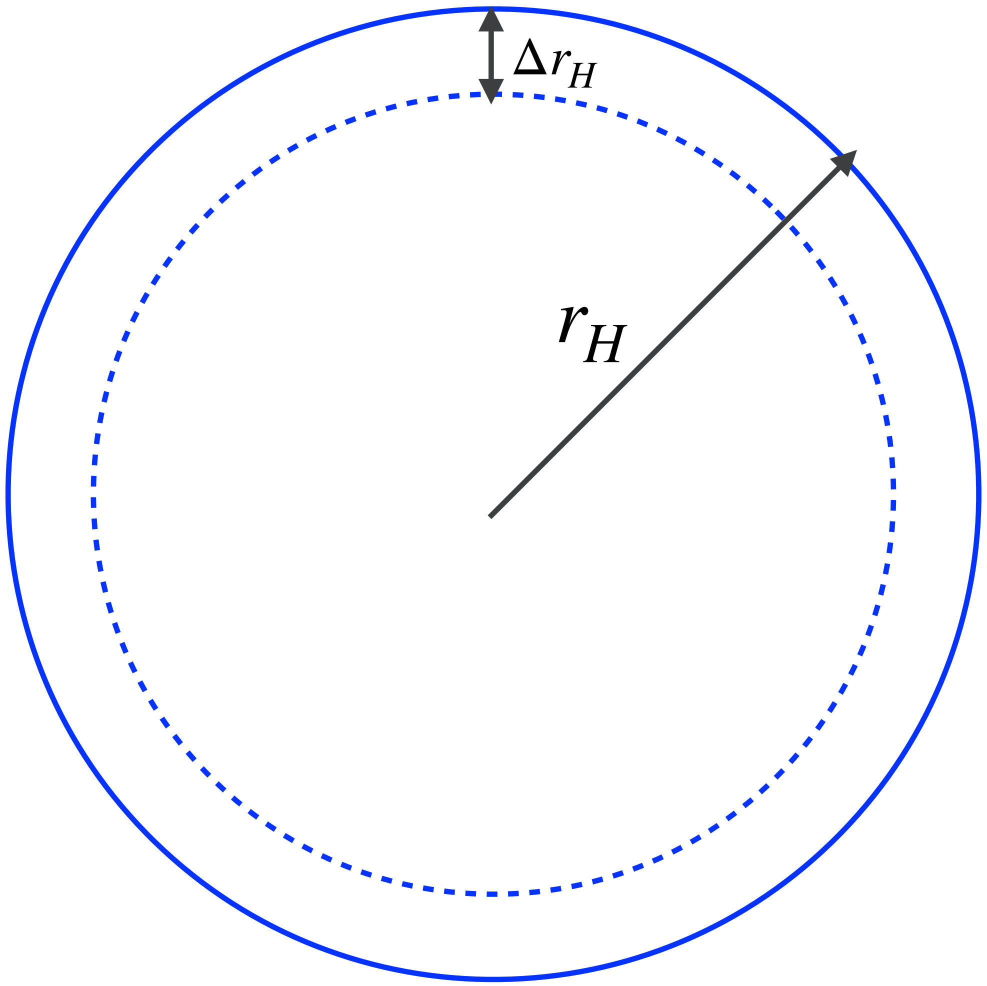
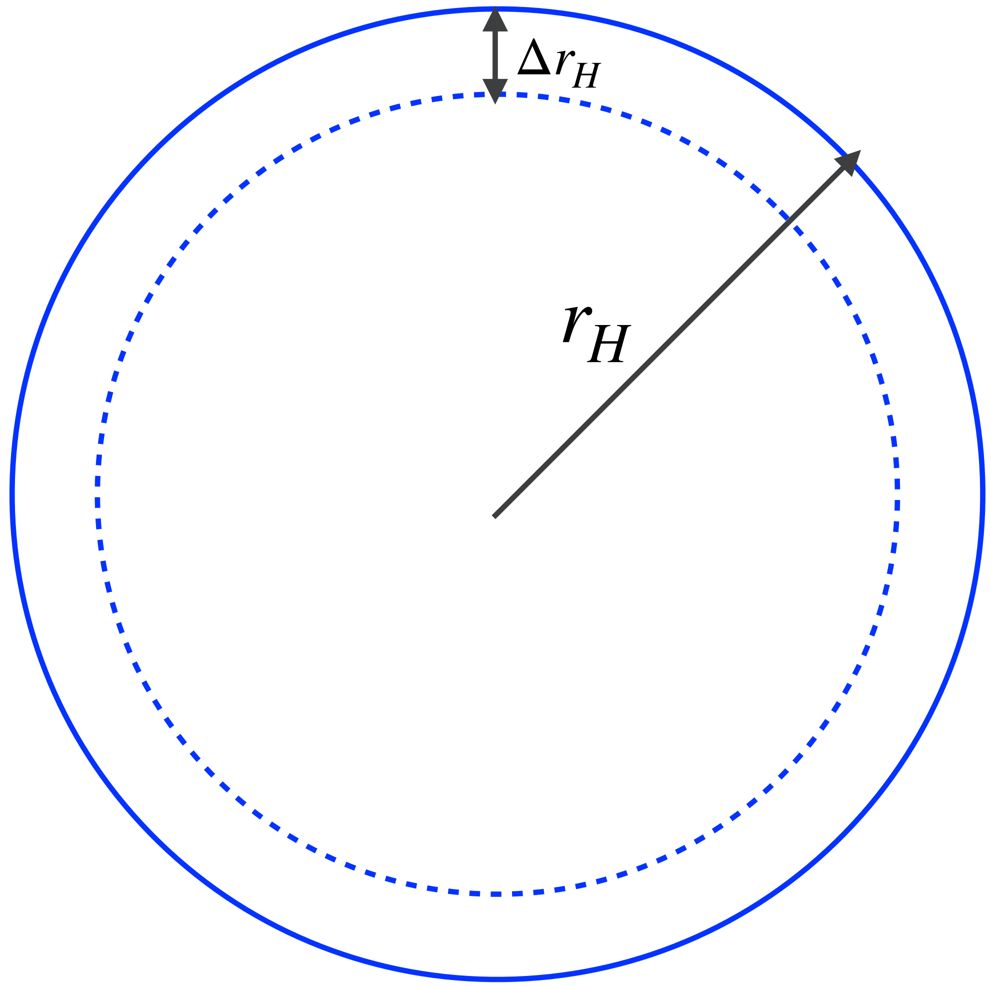
$$\Delta S_{\rm ph} = -\Delta S_{\rm H} \quad \Longrightarrow \quad T = \frac{1}{2\pi} \left(\frac{c_s - |v|}{1 - c_s |v|} \right)' \Bigg|_H$$

Hawking temperature



radius variation due to phonon emission

Hawking temperature



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Associate an entropy to the sonic hole $S_H = \dfrac{A}{4L_c^2}$

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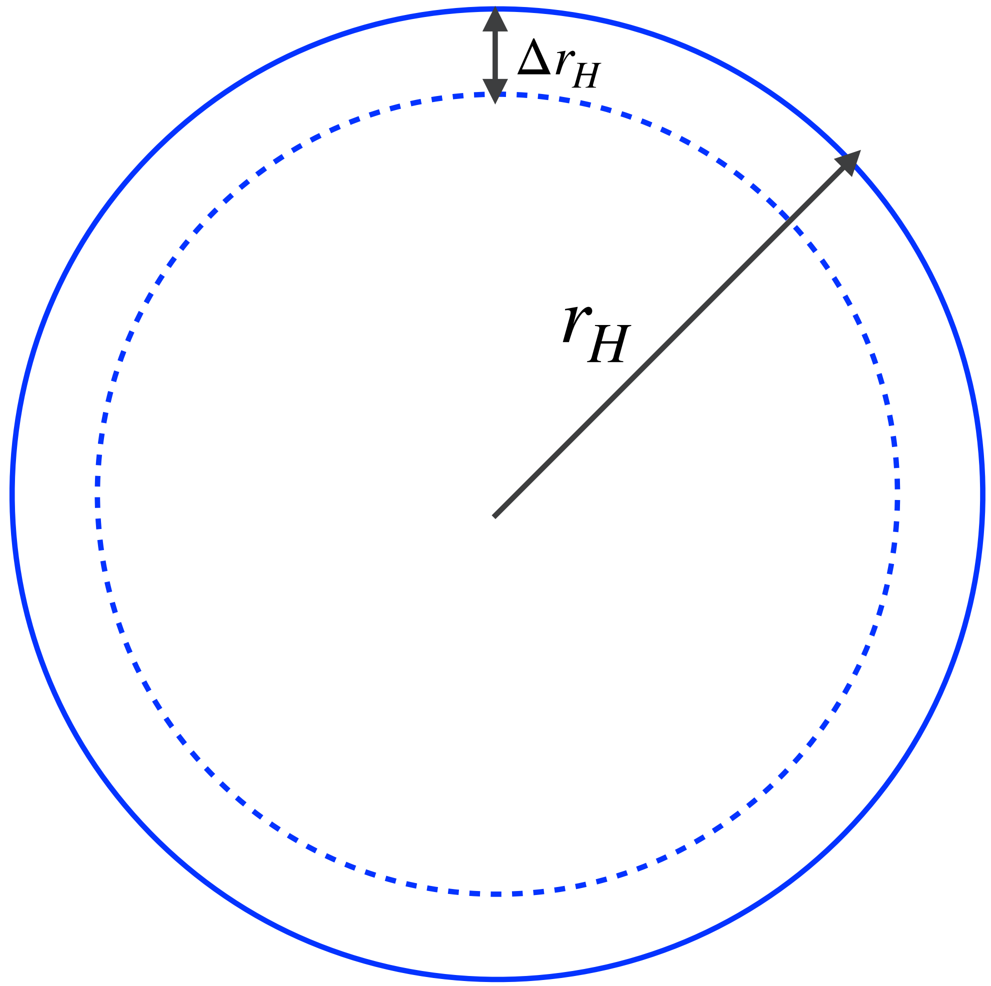

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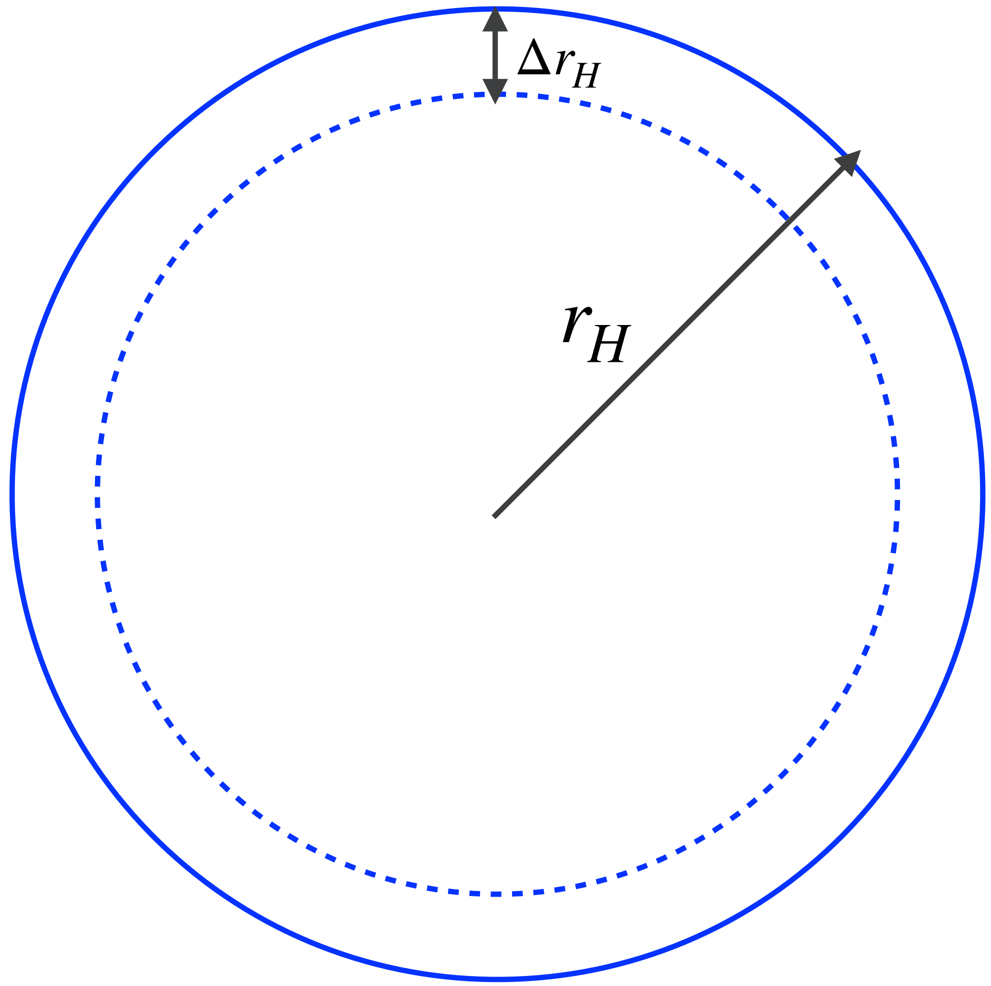

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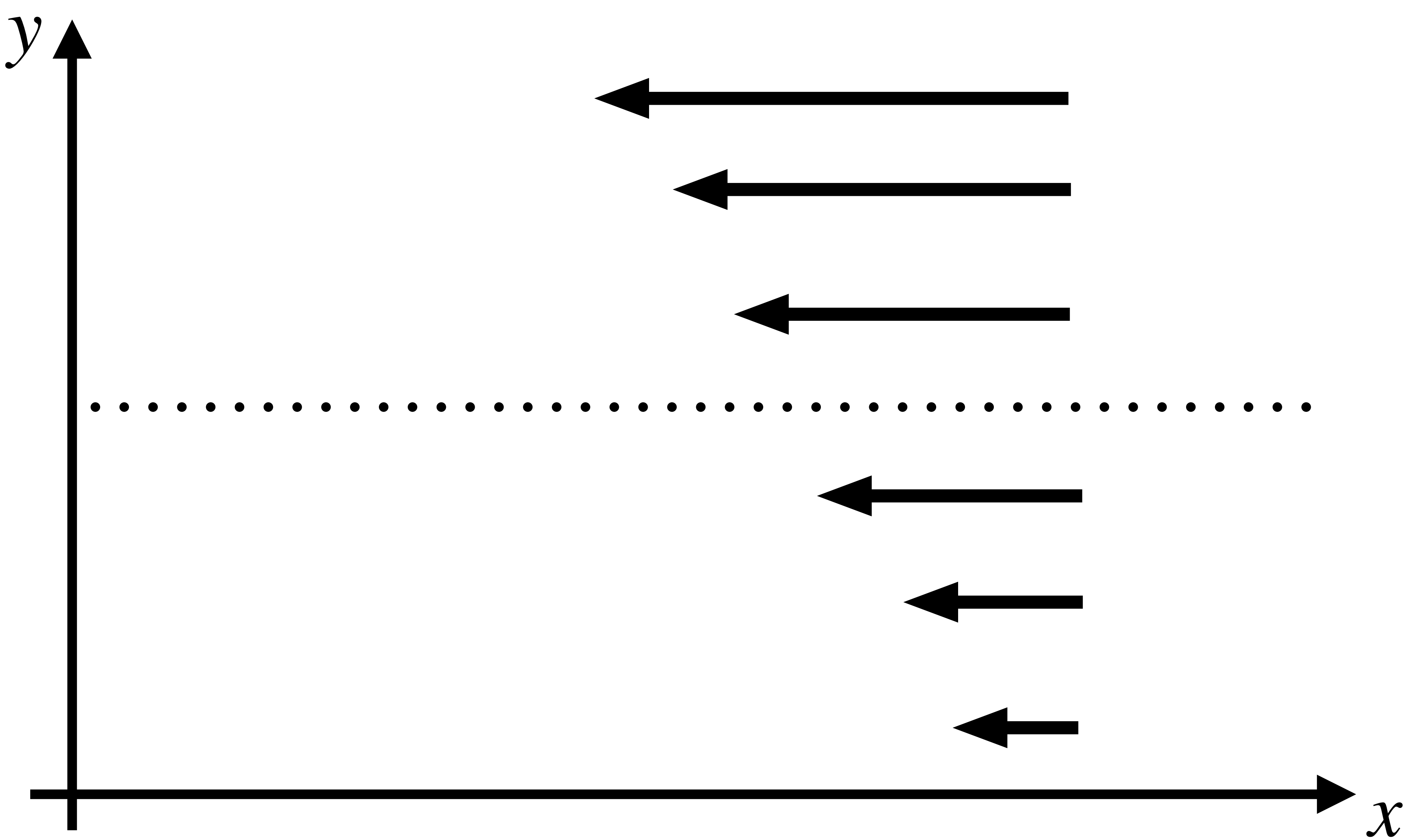
The phonon emission results in an entropy loss of the horizon

The KSS bound

Increasing the temperature the shear viscosity should increase.
Increasing the interaction strength the shear viscosity should decrease



Not only fluids

1. Dielectric media: A refractive index can be reinterpreted as an effective metric, the Gordon metric. (Gordon [2], Skrotskii [3], Balazs [4], Plebanski [5], de Felice [6], and many others.)

2. Acoustics in flowing fluids: Acoustic black holes, *aka* "dumb holes". (Unruh [7], Jacobson [8], Visser [9], Liberati *et al* [10], and many others.)

3. Phase perturbations in Bose–Einstein condensates: Formally similar to acoustic perturbations, and analyzed using the nonlinear Schrodinger equation (Gross–Pitaevskii equation) and Landau–Ginzburg Lagrangian; typical sound speeds are centimetres per second to millimetres per second. (Garay *et al* [11], Barceló [12] *et al.*)

4. High-refractive-index dielectric fluids ("slow light"): In dielectric fluids with an extremely high group refractive index it is experimentally possible to slow lightspeed to centimetres per second or less. (Leonhardt–Piwnicki [13], Hau *et al* [14], Visser [15], and others.)

5. Quasi-particle excitations: Fermionic or bosonic quasi-particles in a heterogeneous superfluid environment. (Volovik [16], Kopnin–Volovik [17], Jacobson–Volovik [18], and Fischer [19].)

6. Nonlinear electrodynamics: If the permittivity and permeability themselves depend on the background electromagnetic field, photon propagation can often be recast in terms of an effective metric. (Plebanski [20], Dittrich–Gies [21], Novello *et al* [22].)

7. Linear electrodynamics: If you do not take the spacetime metric itself as being primitive, but instead view the linear constitutive relationships of electromagnetism as the fundamental objects, one can nevertheless reconstruct the metric from first principles. (Hehl, Obukhov, and Rubilar [23, 24, 25].)

8. Scharnhorst effect: Anomalous photon propagation in the Casimir vacuum can be interpreted in terms of an effective metric. (Scharnhorst [26], Barton [27], Liberati *et al* [28], and many others.)

9. Thermal vacuum: Anomalous photon propagation in QED at nonzero temperature can be interpreted in terms of an effective metric. (Gies [29].)

10. "Solid state" black holes. (Reznik [30], Corley and Jacobson [31], and others.)

11. Astrophysical fluid flows: Bondi–Hoyle accretion and the Parker wind [coronal outflow] both provide physical examples where an effective acoustic metric is useful, and where there is good observational evidence that acoustic horizons form in nature. (Bondi [32], Parker [33], Moncrief [34], Matarrese [35], and many others.)

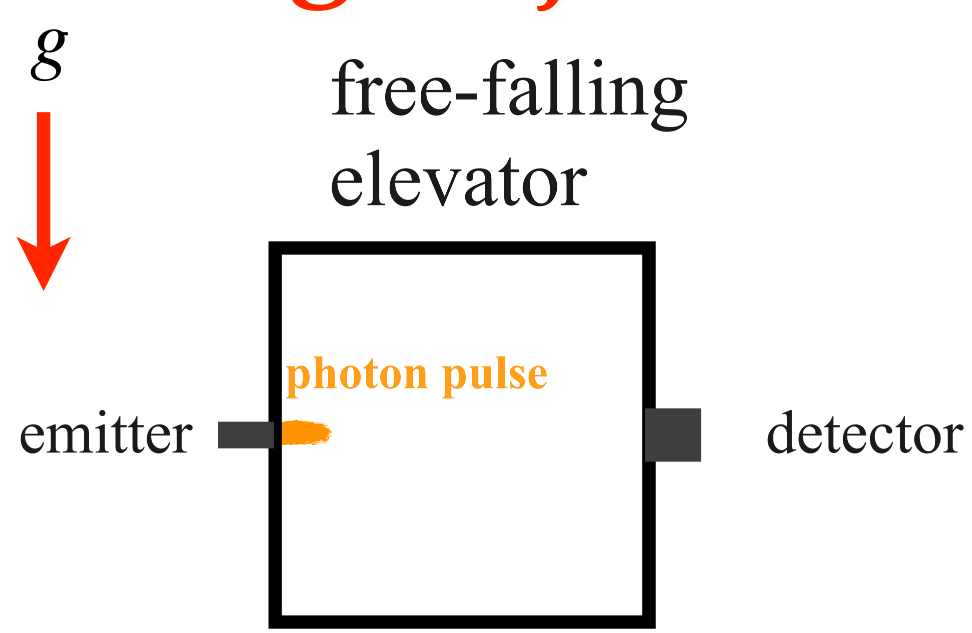
12. Other condensed-matter approaches that don't quite fit into the above classification [36, 37].

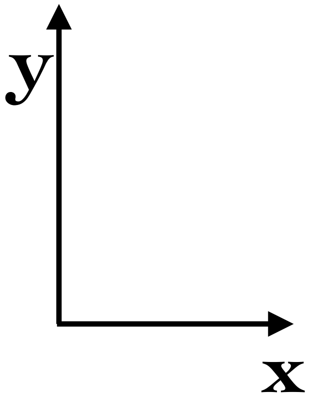
?

Particle in a moving medium $\longleftrightarrow$ Particle in gravity

To which extent does it hold?

Bending trajectories





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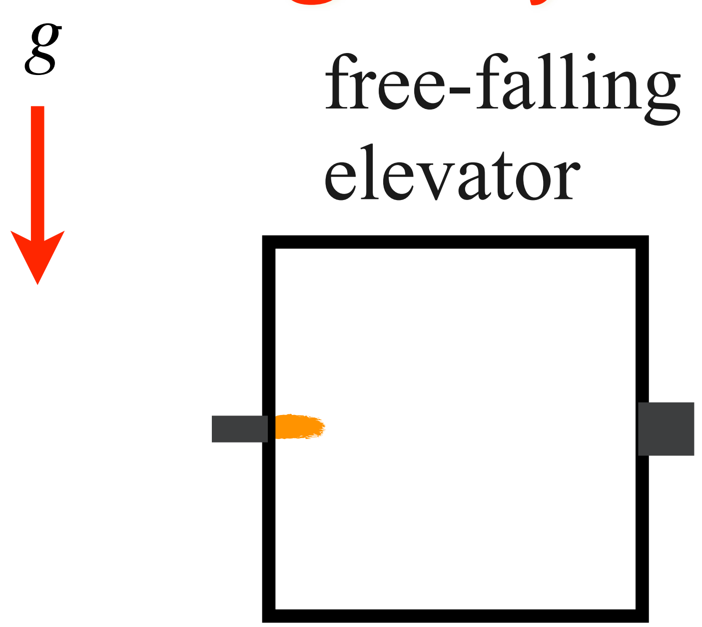
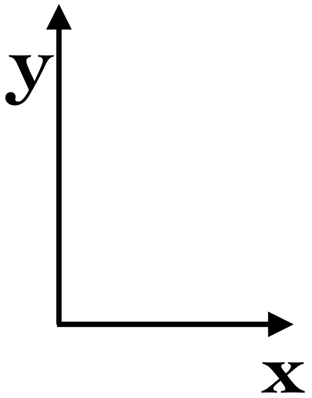
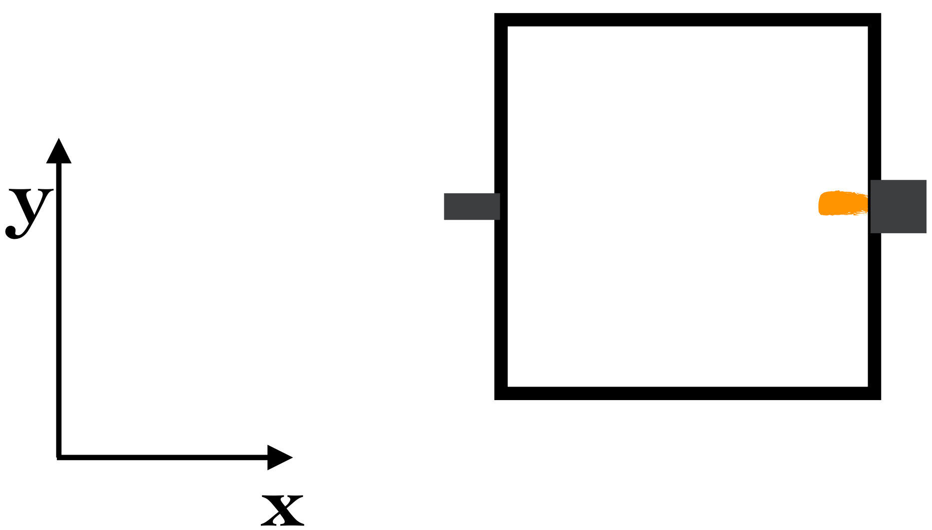




Bending trajectories

free-falling
elevator



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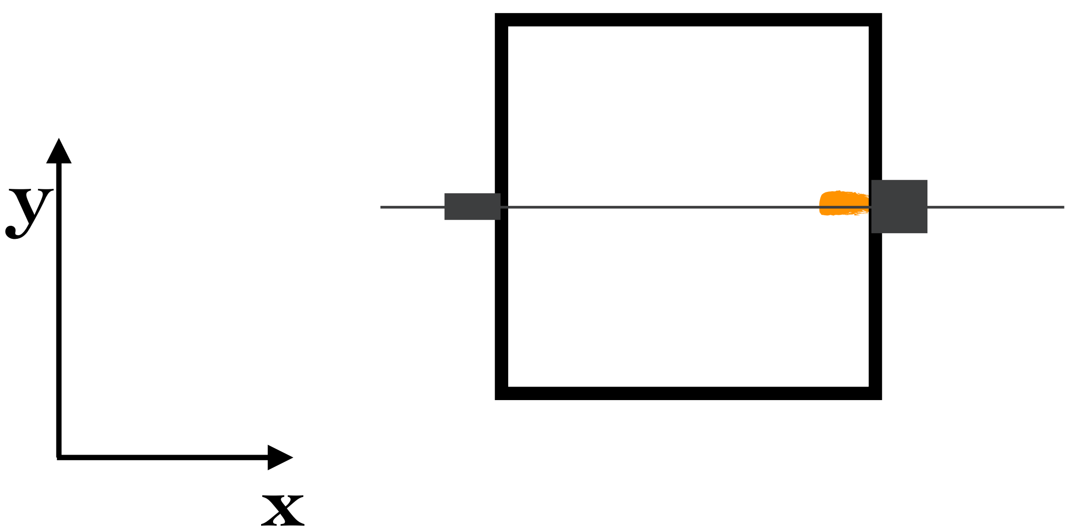

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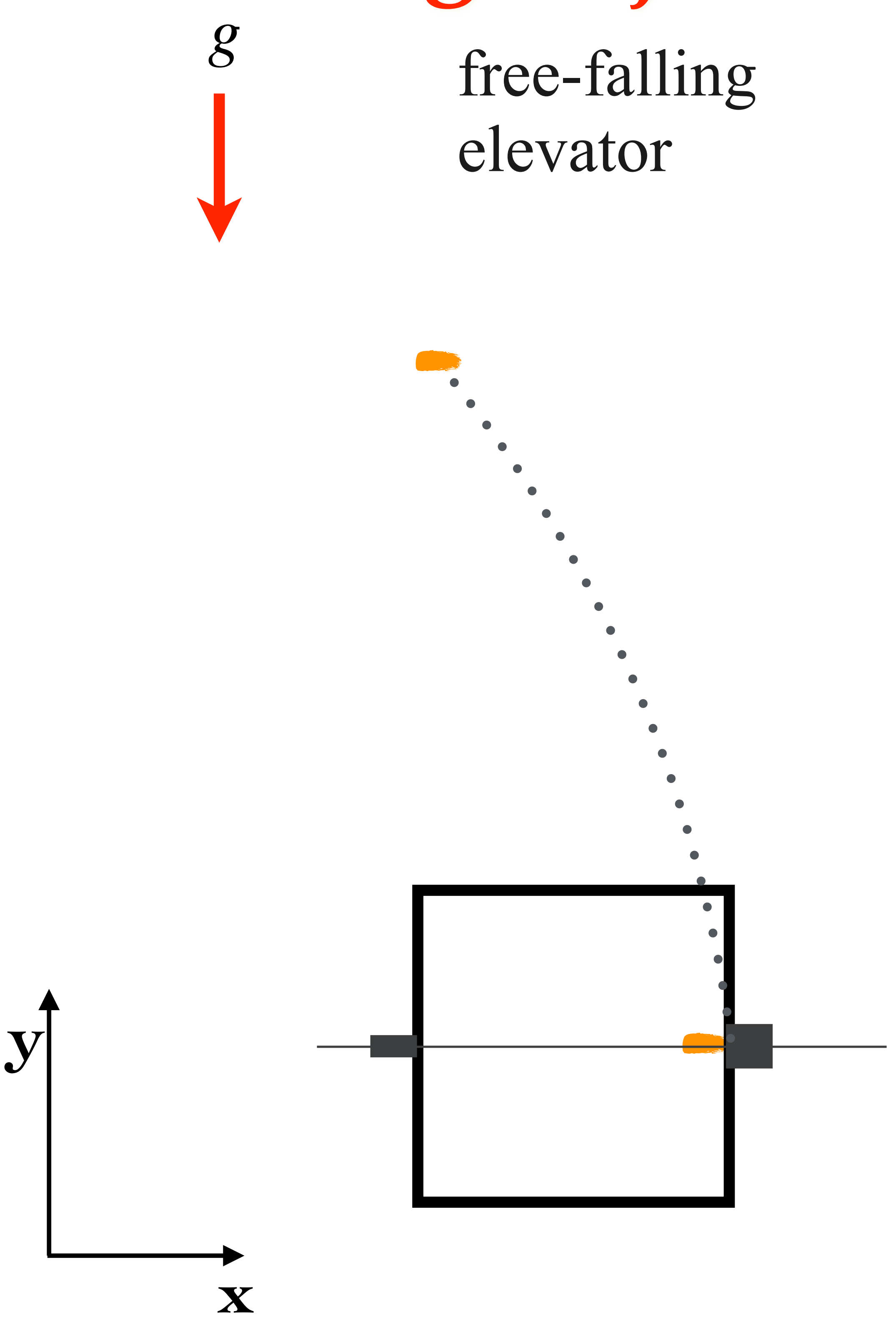

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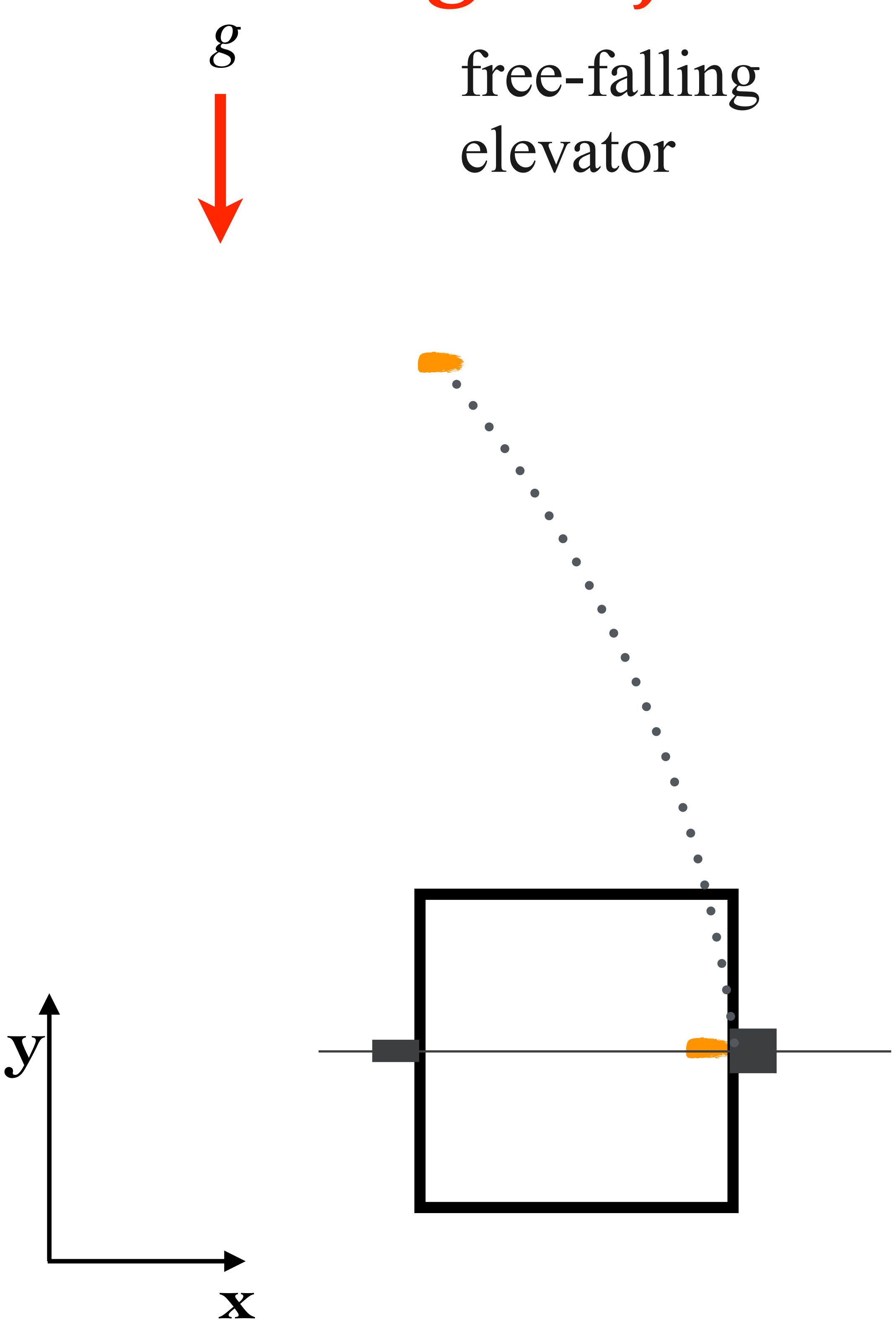

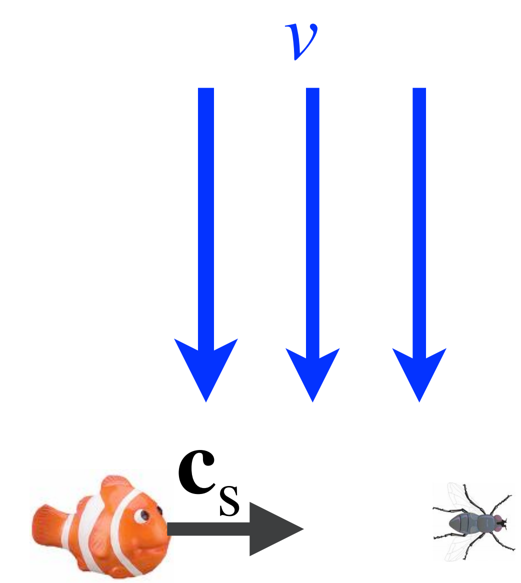

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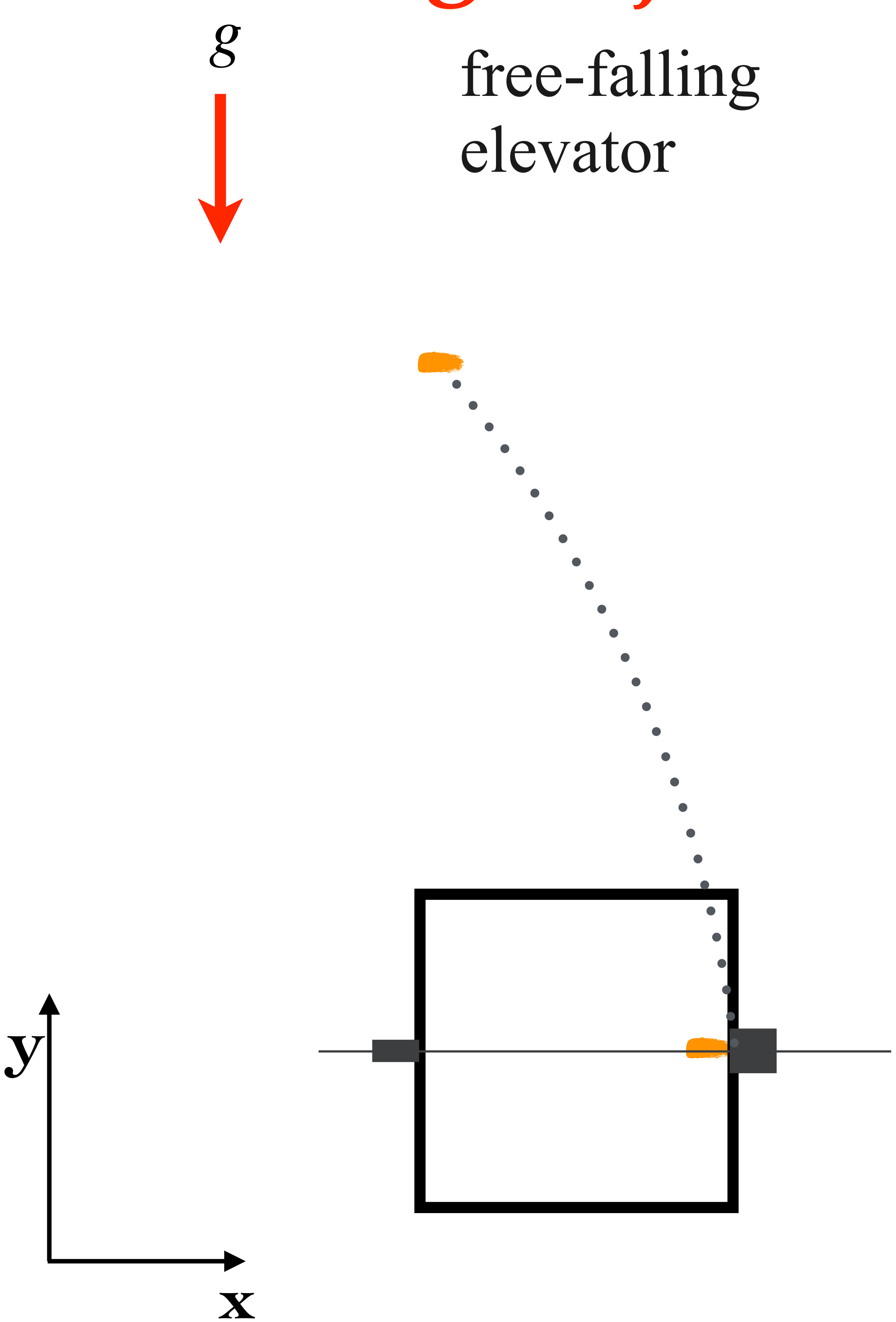

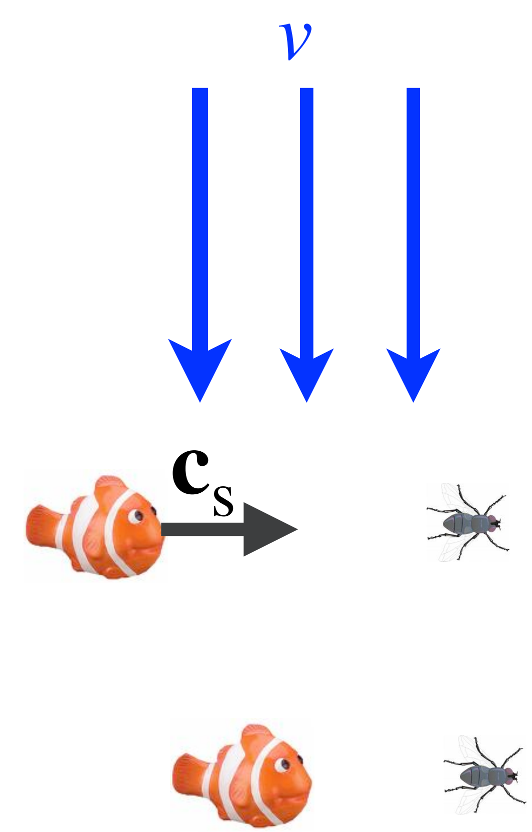

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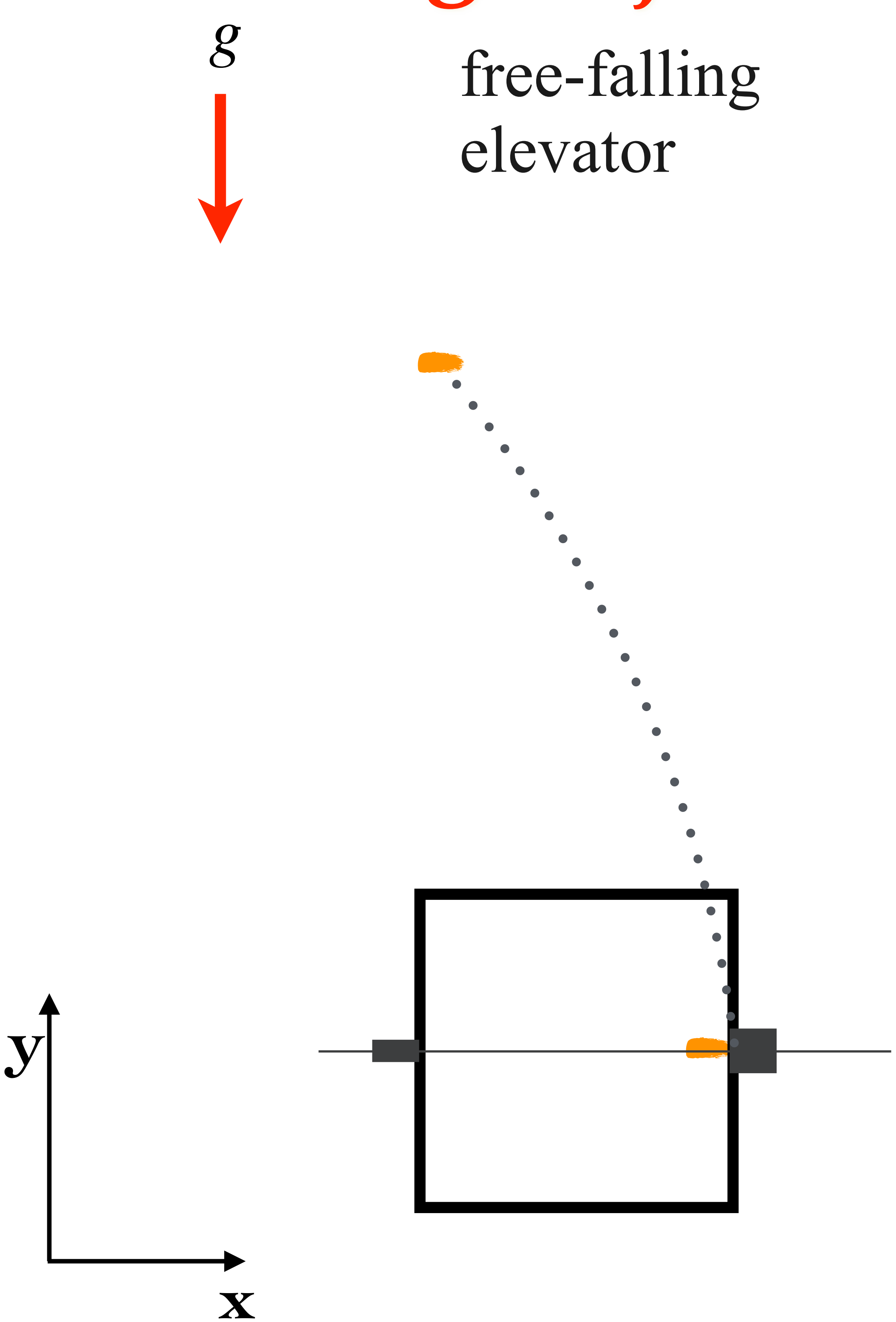

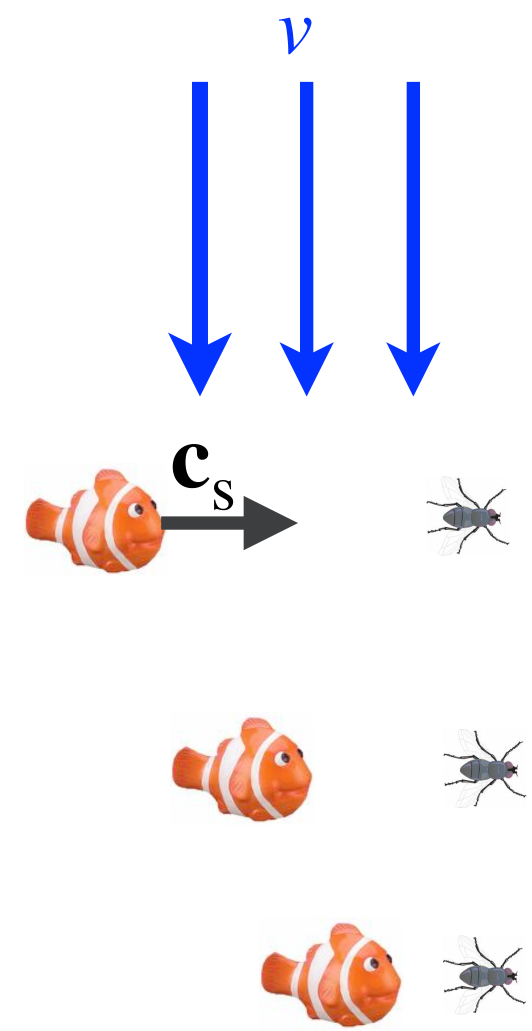

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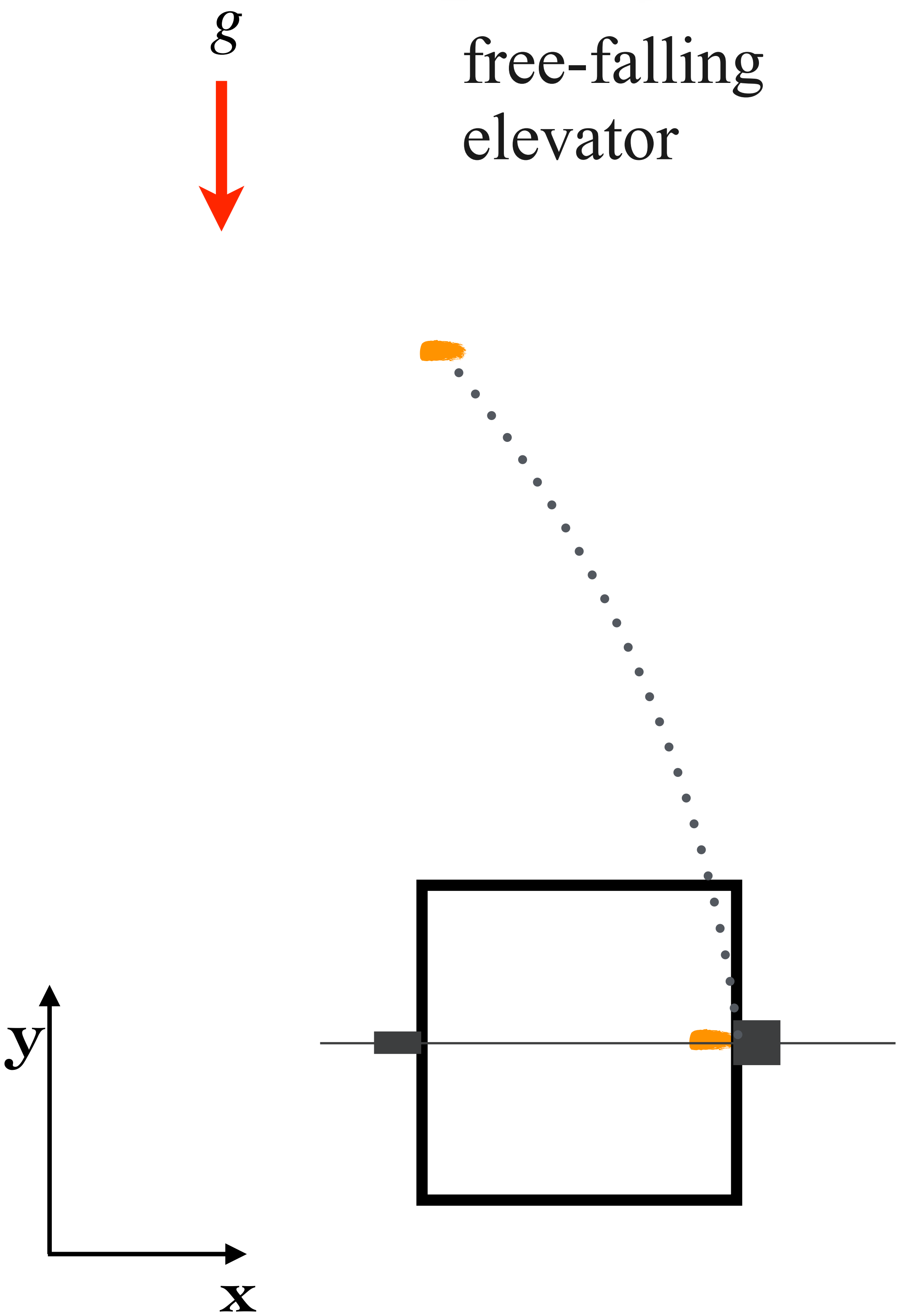

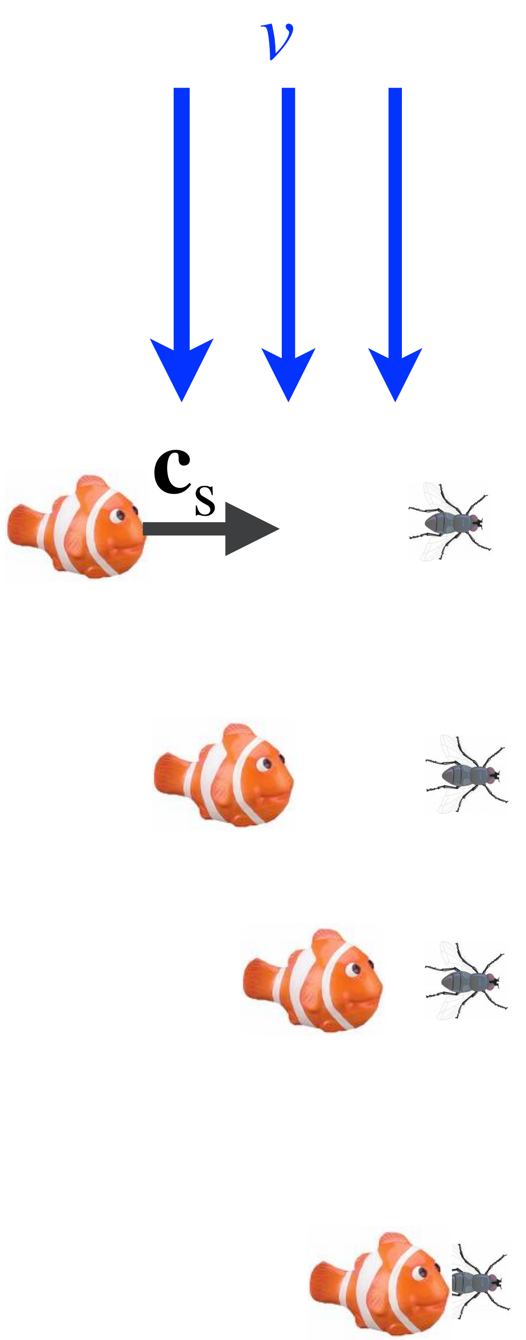

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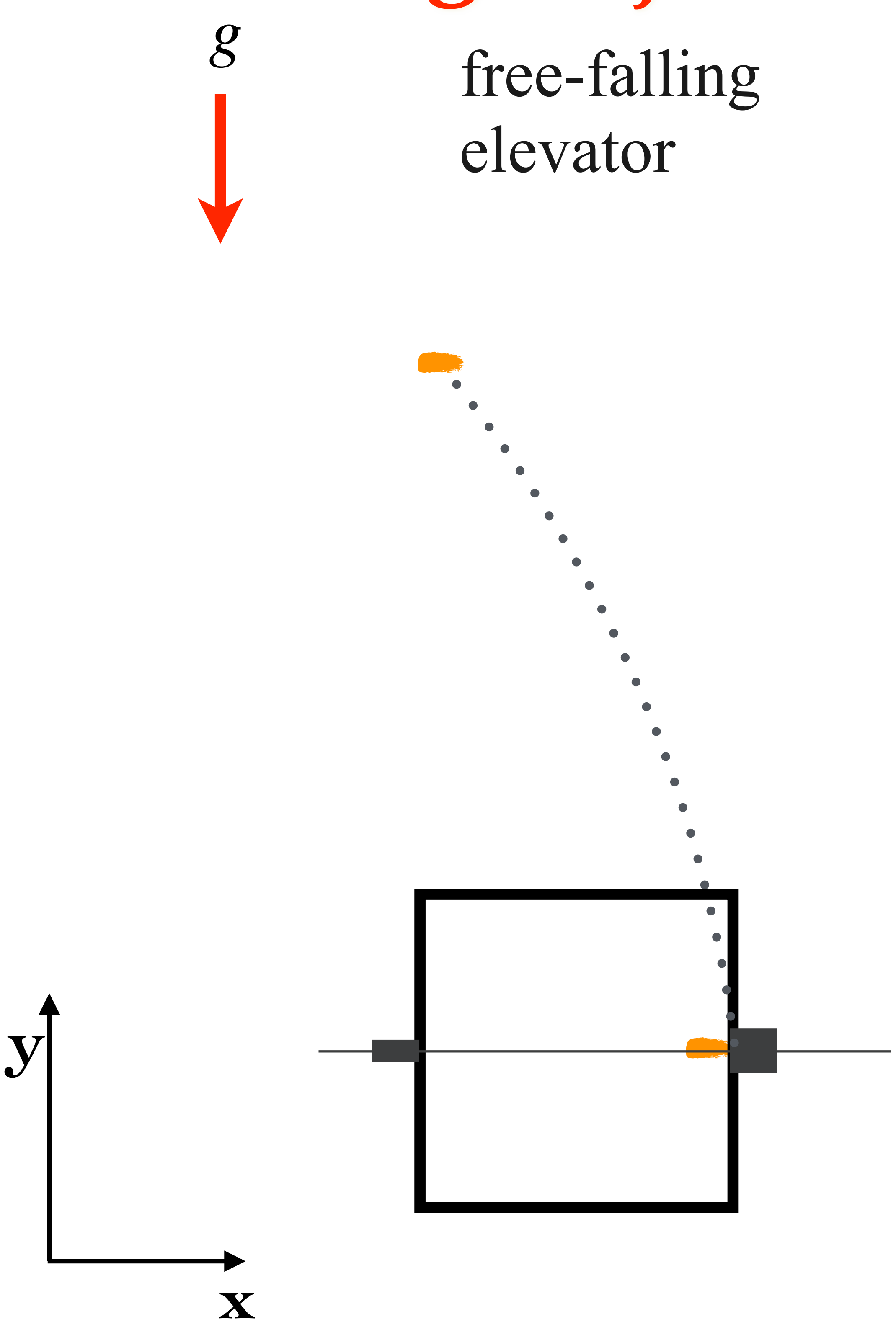
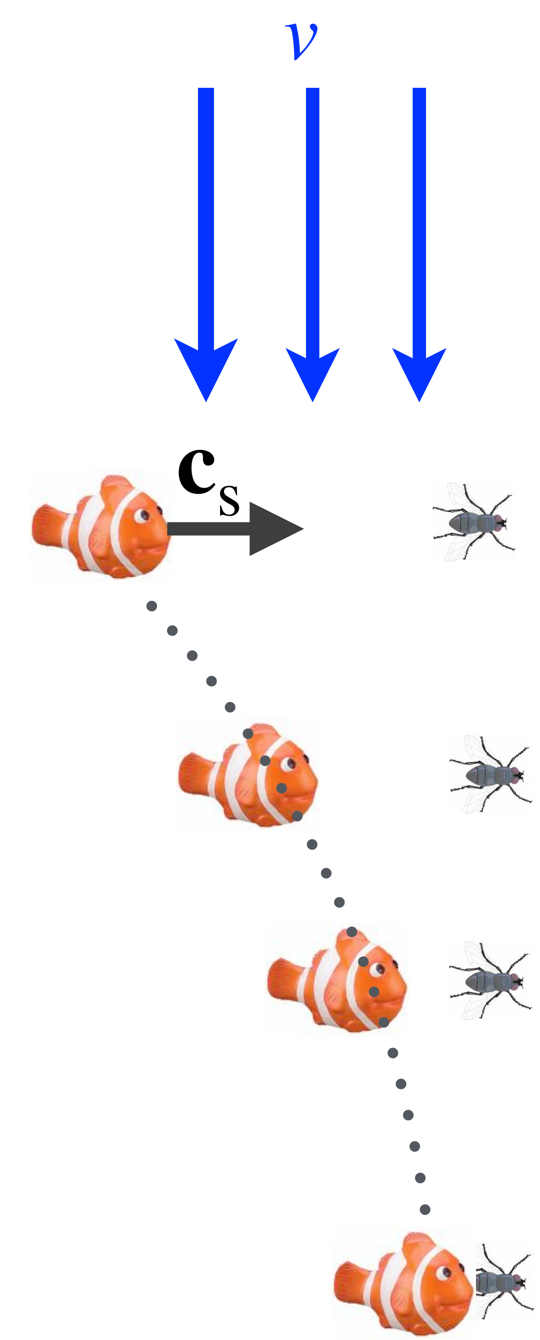


A bent trajectory

$$\frac{dy}{dx} = \frac{v}{c_s}$$



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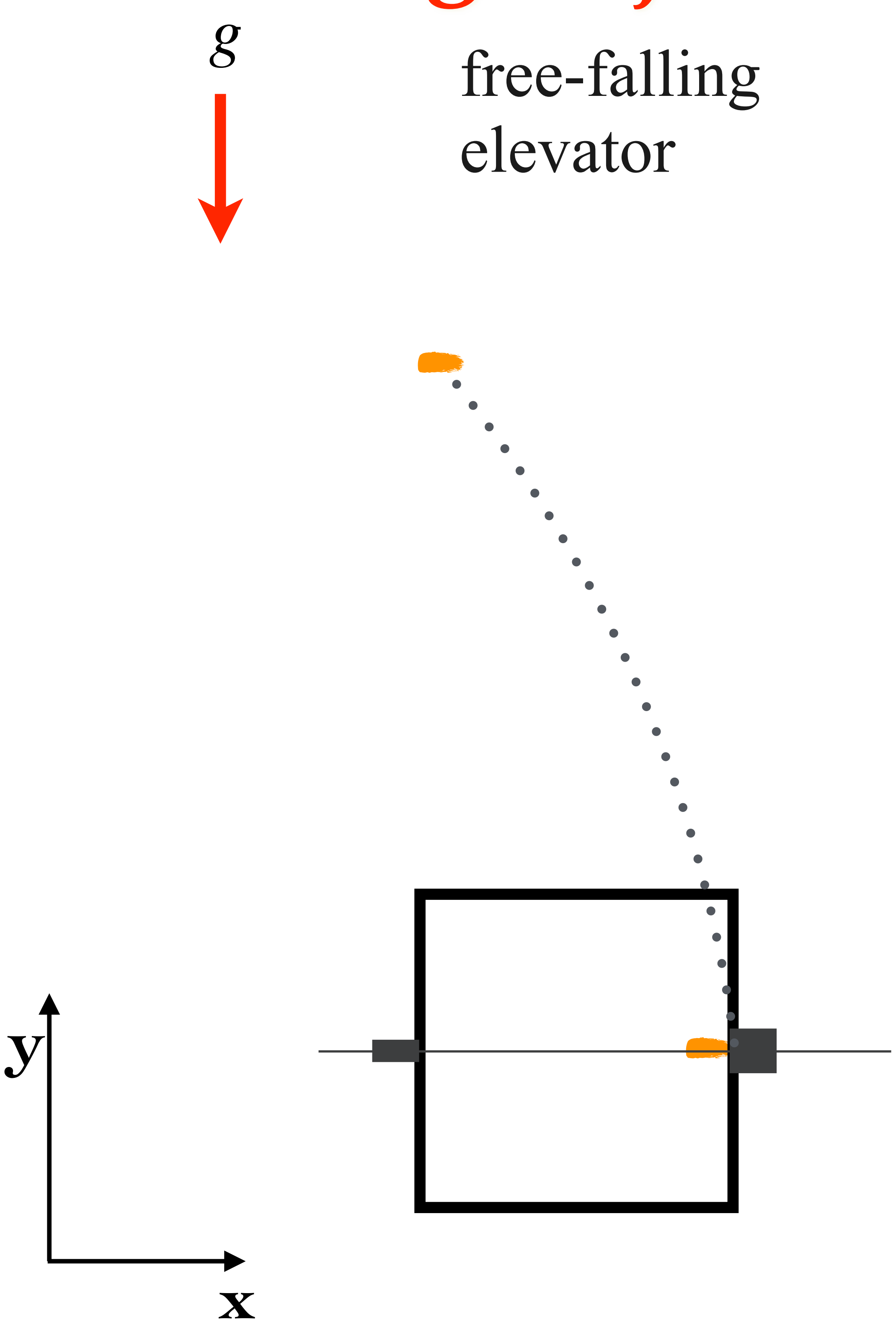

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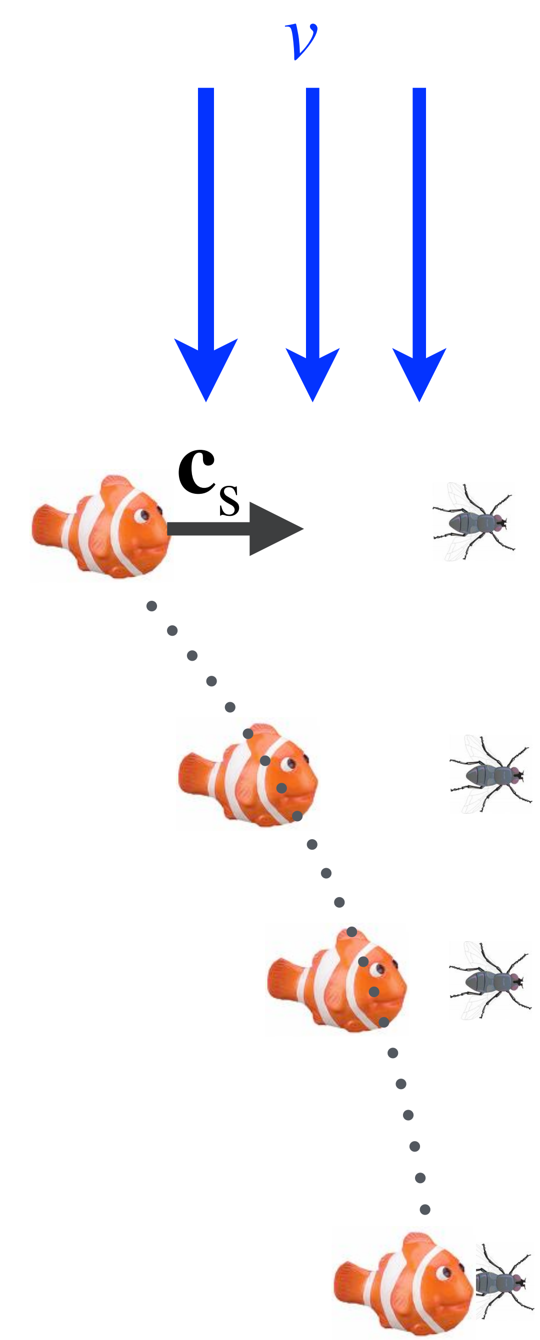

A **velocity space gradient** produces the analog of light bending

Quick recap

To have an horizon we need a transonic flow

$$v < c_s \qquad v = c_s \qquad v > c_s$$

It cannot be 3D

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Quick recap

To have an horizon we need a transonic flow

$$v < c_s \qquad v = c_s \qquad v > c_s$$

It cannot be 3D

- We need to embed quantum effects
- Measure a dim phonon emission
- **How to avoid turbulence?** Use a Bose-Einstein condensate!

Bose-Einstein condensate (BEC)

It is a **coherent state of matter** with a "thermodynamically" large number of particles in the same quantum state

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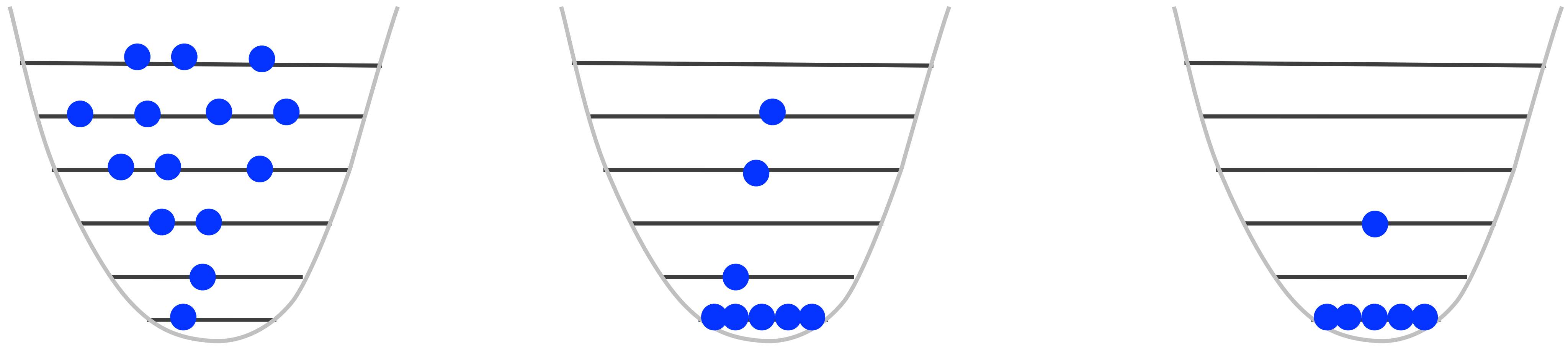

$T > T_c$ $T \simeq T_c$ $T < T_c$

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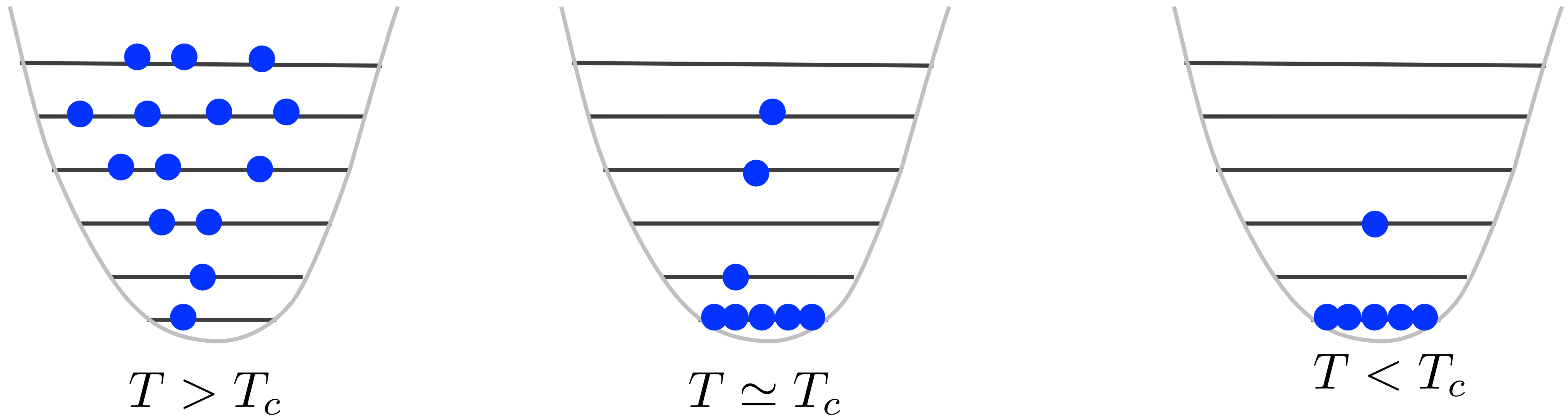
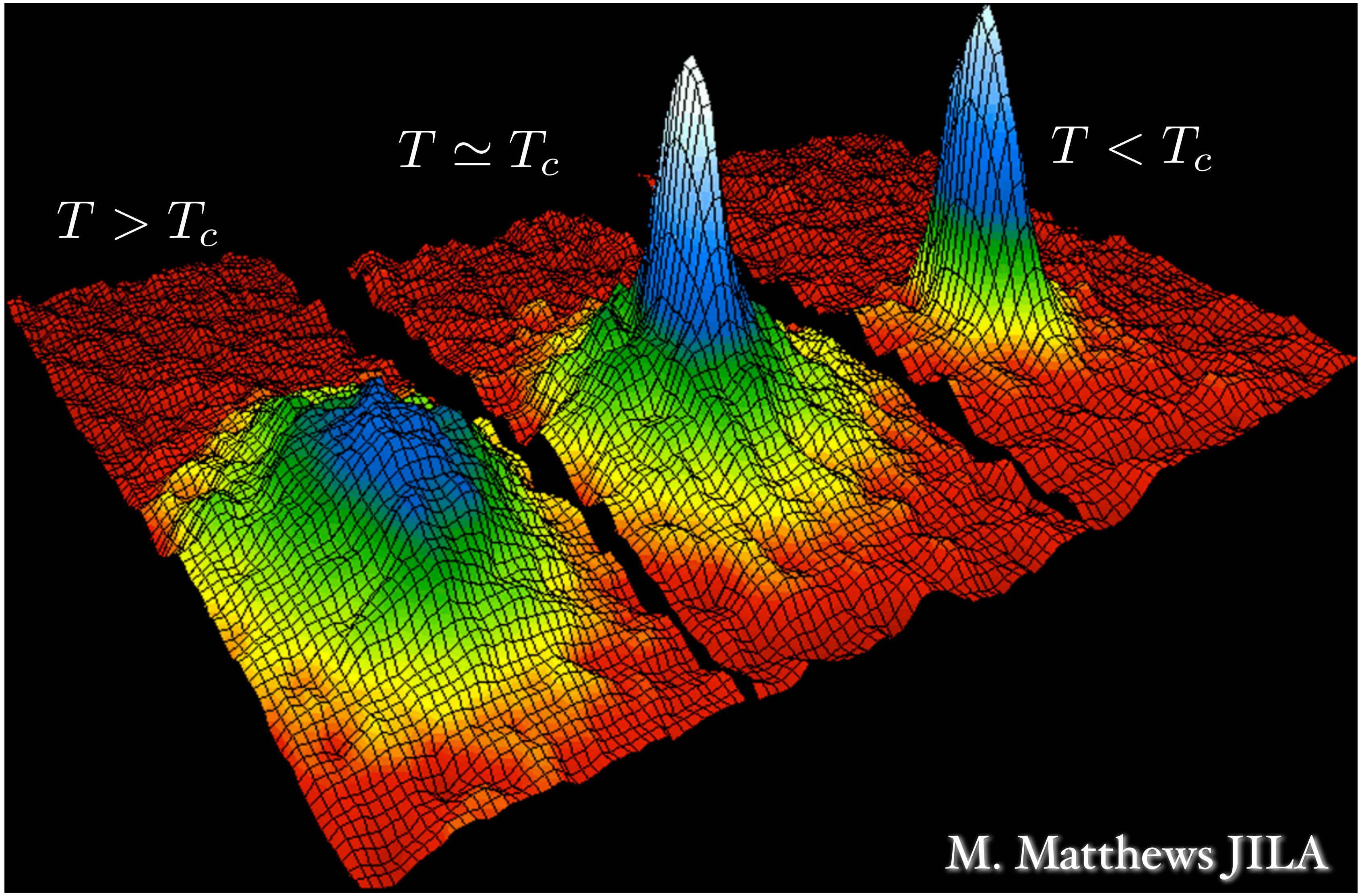


Requirements:

1. Particles must be **bosons**
2. **Cold system**: A fight between thermal disorder and quantum coherence
3. Particles must be **stable**

Ultracold atoms in an optical trap



Velocity distribution of ^{87}Rb atoms $\qquad T_c \simeq 200$ nK

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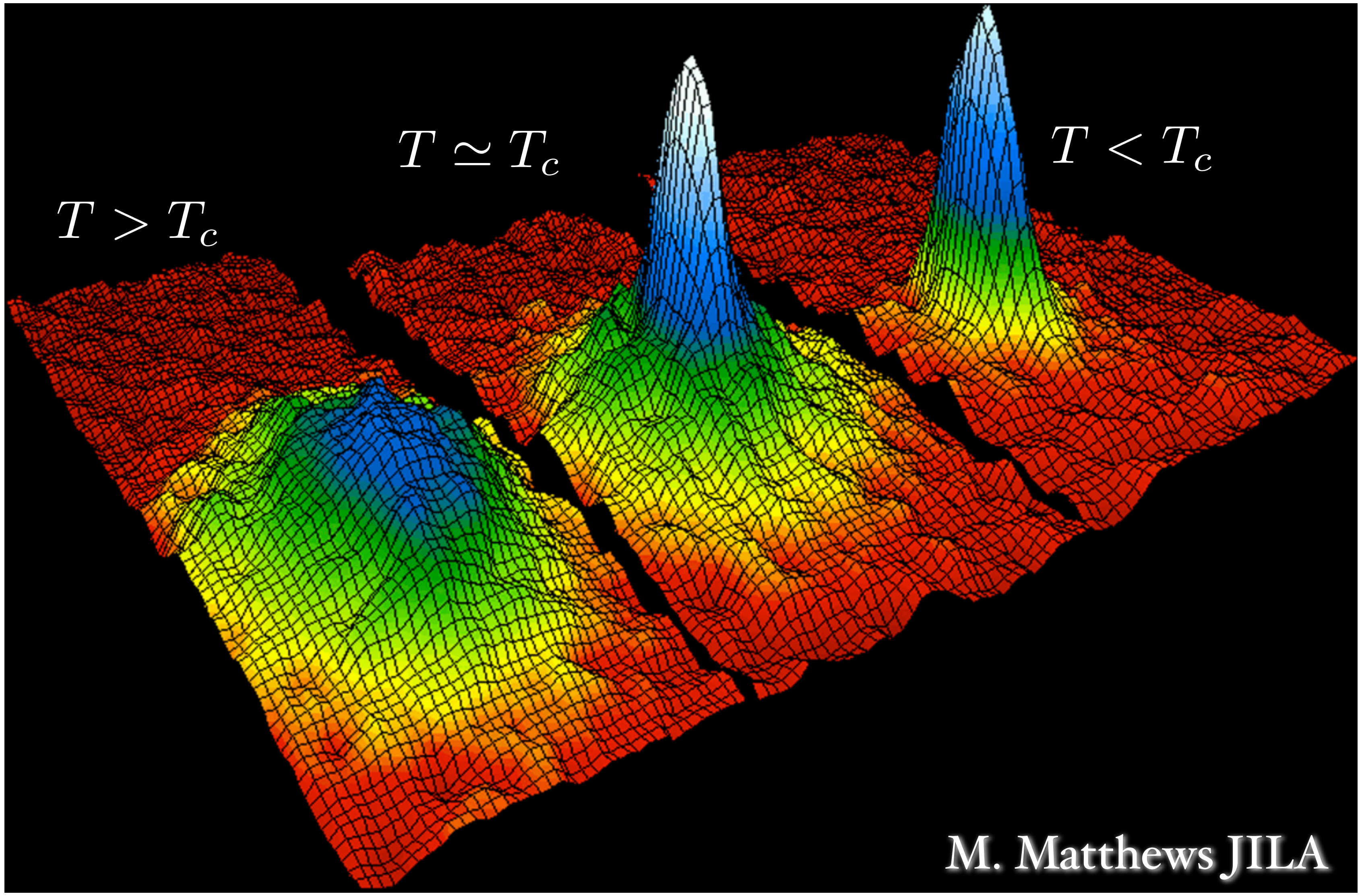

Velocity distribution of ^{87}Rb atoms $\qquad T_c \simeq 200$ nK

1. ^{87}Rb is **bosonic**
2. can be **cooled**
3. has a lifetime of about $\mathbf{10^{10}}$ **years** (the experiment lasts $\sim 10^3$s)

A simple geometrical picture

In medium phonon $\dfrac{d\mathbf{x}}{dt} = c_s \hat{\mathbf{n}} + \mathbf{v}$ as $c_s \hat{\mathbf{n}}\, dt = d\mathbf{x} - \mathbf{v}\, dt$

Square it $$c_s^2 dt^2 - (d\mathbf{x} - \mathbf{v} dt)^2 = 0$$

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Null geodesic $g_{\mu\nu} dx^\mu dx^\nu = 0$

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acoustic metric $$g_{\mu\nu} = \left(\begin{array}{c|c} c_s^2 - v^2 & \mathbf{v}^t \\ \hline \mathbf{v} & -I \end{array}\right)$$

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Note that $\sqrt{-g} = \sqrt{-\det g} = c_s$

Acoustic metric

Promoting to special relativity we have that

$$g_{\mu\nu} = \eta_{\mu\nu} + \left(c_s^2 - 1\right) v_\mu v_\nu \qquad \text{where} \qquad v_\mu = \gamma(1, -\mathbf{v})$$

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flat spacetime

Acoustic metric

Promoting to special relativity we have that

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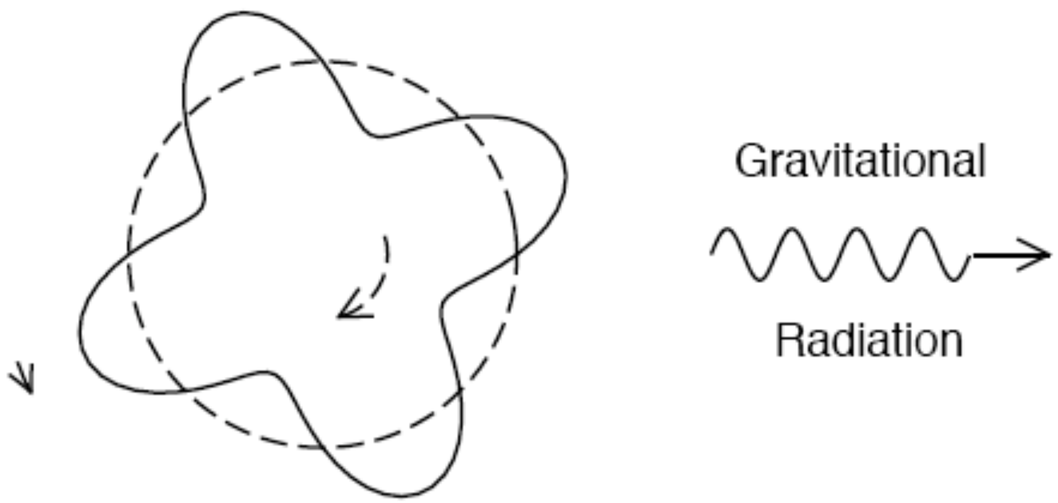
in-medium effects

Description of the motion of point particles in a moving medium.

The gravity analog at work

R-mode instability of rotating stars



Quick spin down of pulsars

Lindblom, astro-ph/0101136
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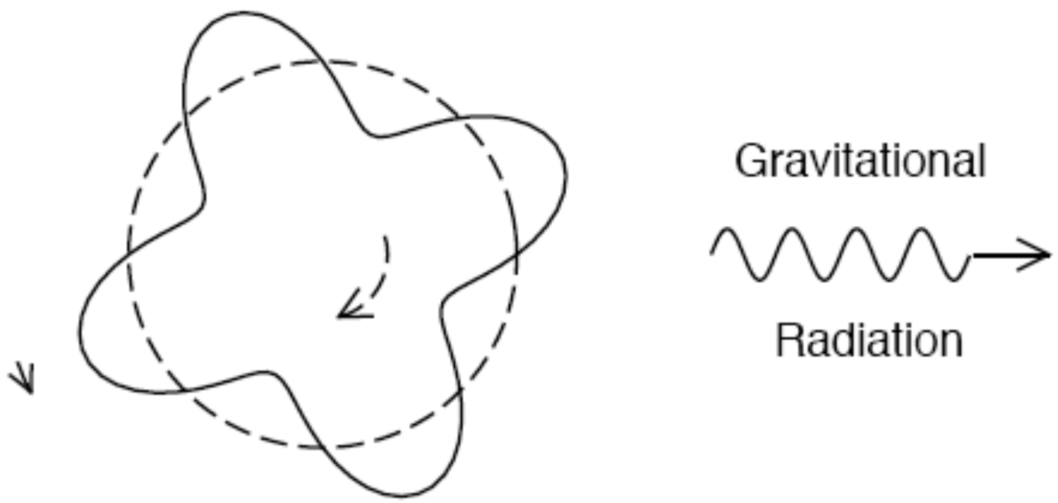

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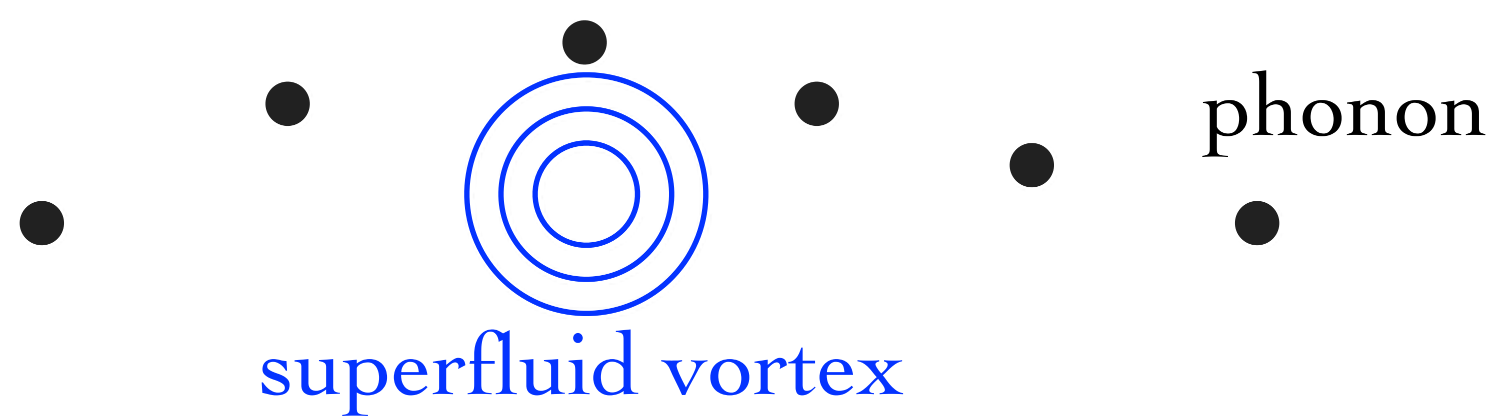

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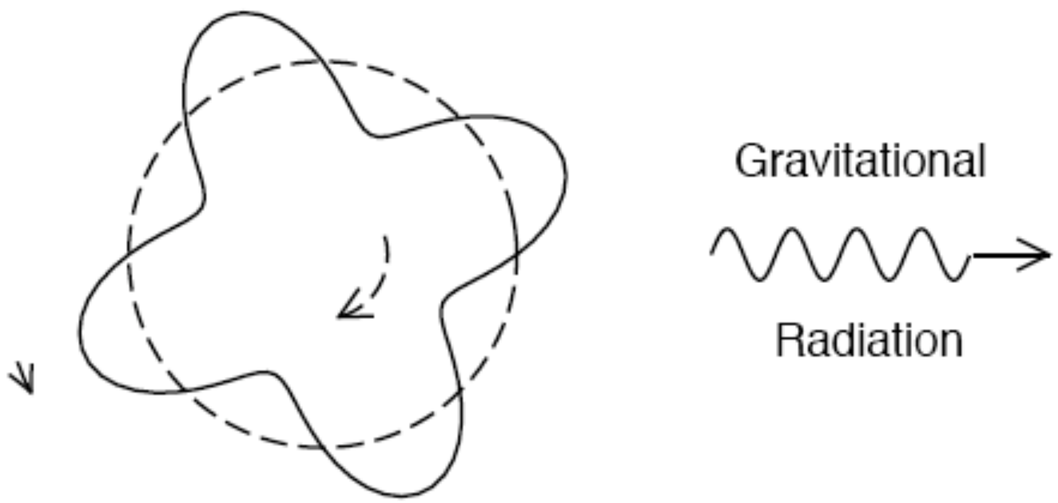

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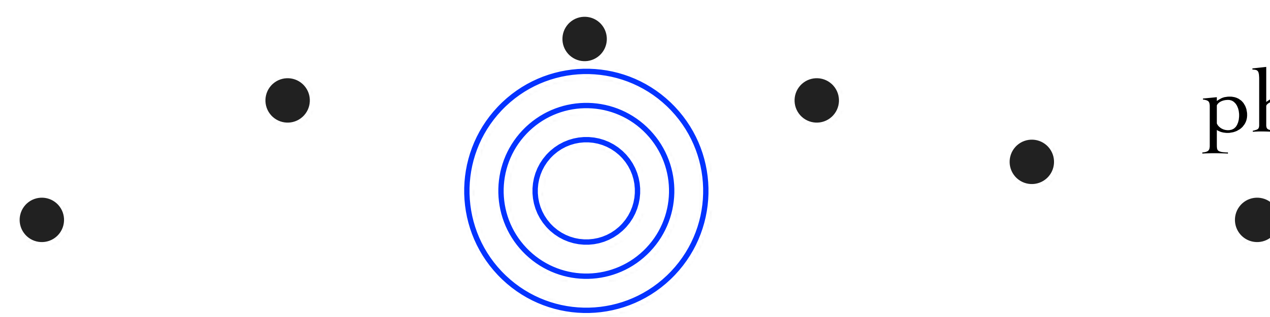

phonon

superfluid vortex

Analytic cross section

$$\frac{d\sigma}{d\theta} = \frac{c_s}{2\pi E}\frac{\cos^2\theta}{\tan^2\frac{\theta}{2}}\sin^2\frac{\pi E}{\Lambda}$$

MM, C. Manuel and B. A. Sa'd, Phys.Rev.Lett. 101 (2008) 241101

The realm of the analogy II

- ***Particle-wave duality***

Propagation of **massless bosons**

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wave propagation in hydrodynamics

$$\frac{\partial^2 \psi}{\partial t^2} - c^2 \nabla^2 \psi = 0$$

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Propagation of **massless bosons** | **analogy** | **wave propagation** in hydrodynamics

$$\frac{\partial^2\psi}{\partial t^2} - c^2\nabla^2\psi = 0$$

This analogy is valid in the absence of interactions.

Including interactions the particle behavior is different: scattering, quantum corrections etc.

Gravity analogs

If we can rephrase a given problem as a geometrical problem we can look for a solution using the analogy with general relativity (GR)

When should we use/look for analogies?

- Dealing with **difficult problems**: they may be rephrased (mapped) as different solvable problems.

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- When we are lost in a forest of many different models **analogies can provide a guidance**

Acoustic vs GR

- Sound wave propagation as a **scalar field** propagation in an **emerging GR background**

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One can certainly calculate the **Ricci and Einstein tensors** of the fluid using the acoustic metric.
However, they **do not satisfy the Hilbert-Einstein equation**.

Detection strategy

The phonon emission perturbs the system producing long-range density correlations

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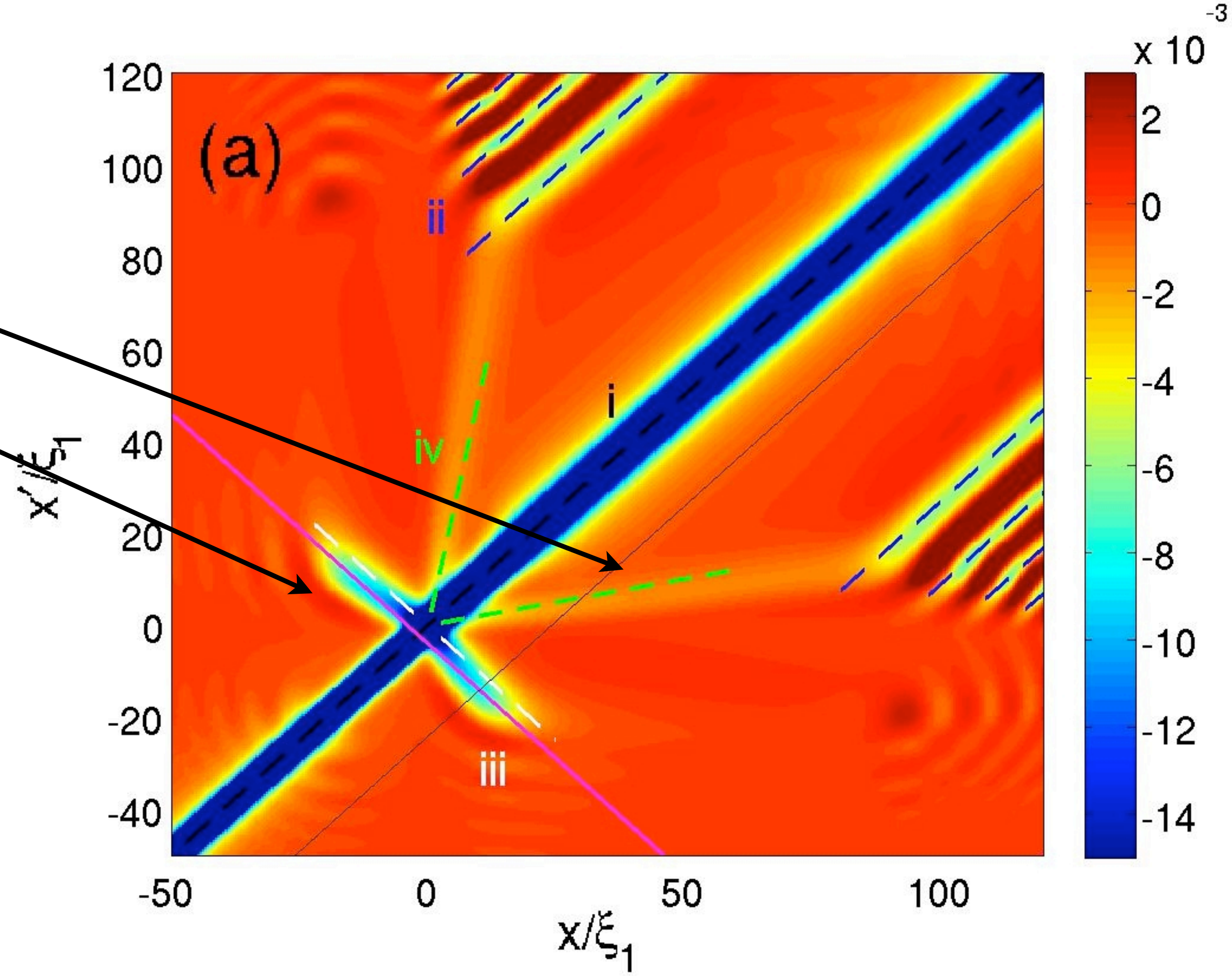
Parametric plot of the density-density correlation function

emitted phonons



Carusotto *et al New J. Phys.* 10 103001 (2008)

A dim emission

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The speed of sound is small $c_s \sim \mathrm{mm\,s^{-1}}$

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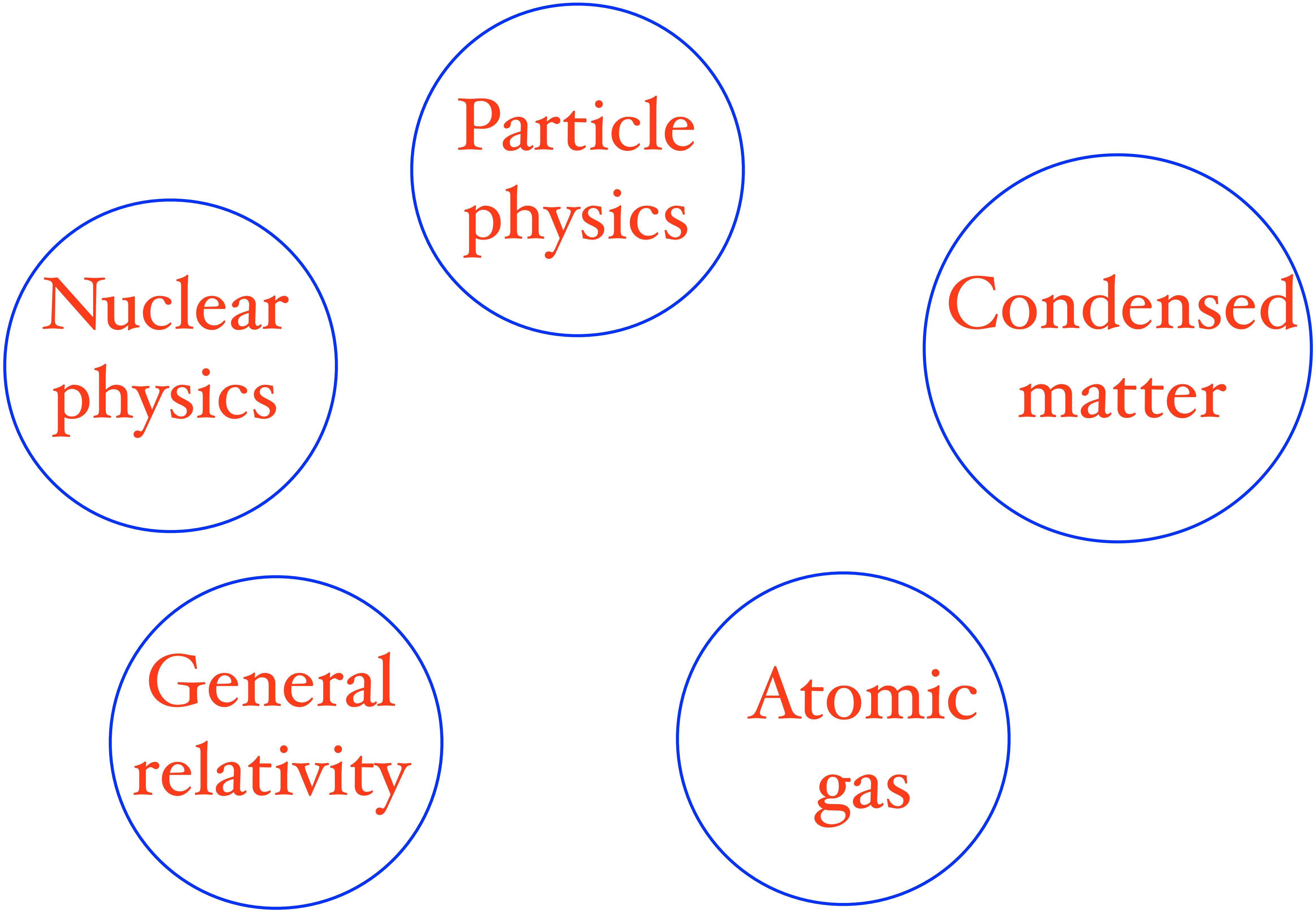
The richness of physics



• Recap of the Higgs-Anderson mechanism

Physical process

Sponantenous breaking of a local symmetry

$\longrightarrow$

Phenomenon

Gauge field acquires mass M

Range of the gauge field propagation $\sim 1/M$

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The analogy is about **kinematics not dynamics**

The analogy works in ***restricted energy regions:*** at high energies one sees the microphysics.

Schwarzschild acoustic metric?

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 \sin^2\theta d\phi^2)$$

Schwarzschid radius $R_s = 2M$

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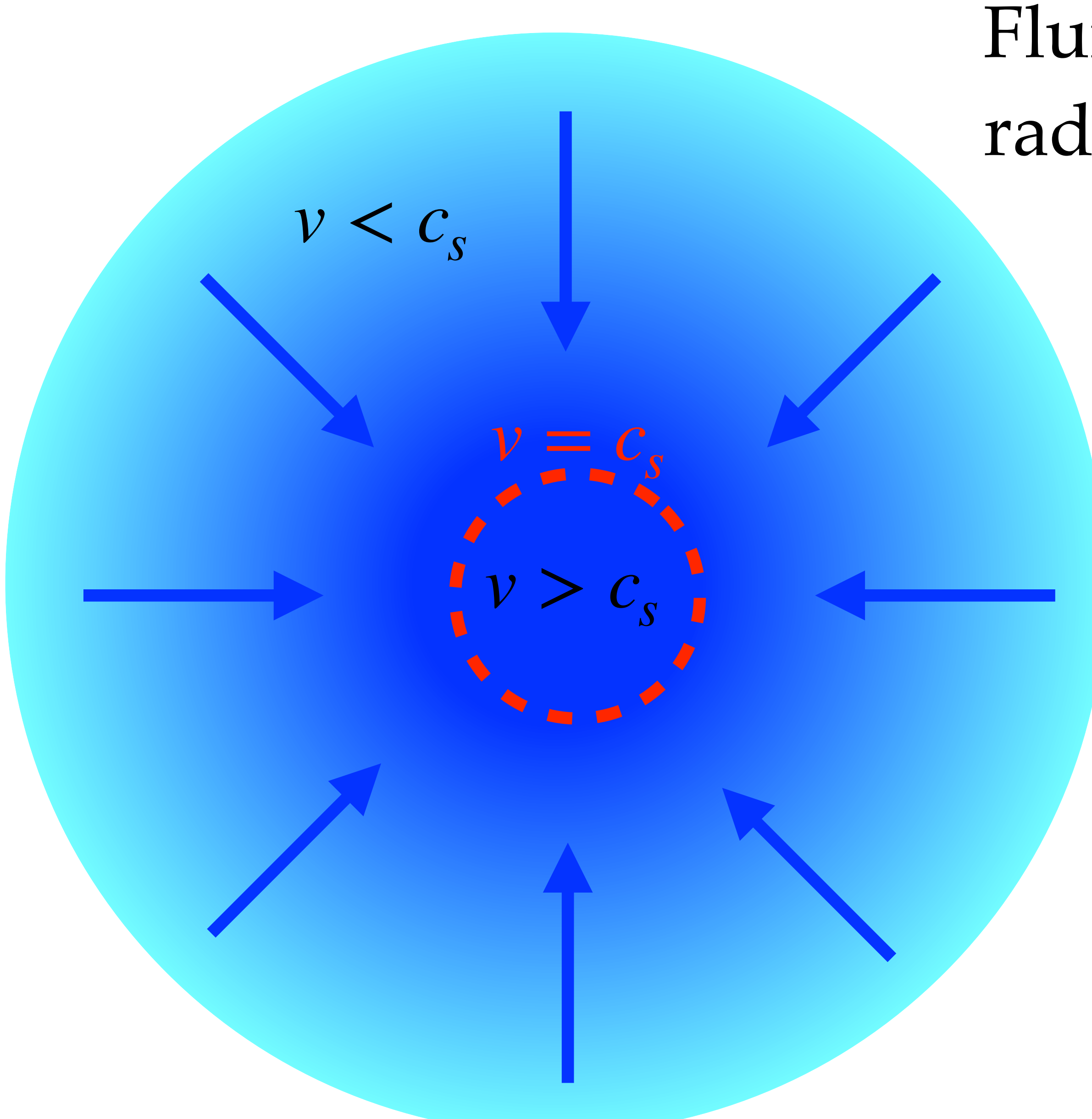
Fluid with radial flow

Does the fluid analog exist?



Lagrangian formulation

Consider the Lagrangian for a scalar field $\mathscr{L} \equiv \mathscr{L}(\phi, \partial_\mu \phi)$

Scale separation $\phi(x) = \phi_0(x) + \epsilon \phi_1(x)$

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background

"phonon"

$$\phi(x) = \phi_0(x) + \epsilon \phi_1(x)$$

long-wavelength

"short-wavelength"

Expand the action

$$S[\phi] = S[\phi_0] + \frac{\epsilon^2}{2} \int d^4x \left[\frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi_0)\partial(\partial_\nu \phi_0)} \partial_\mu \phi_1 \partial_\nu \phi_1 + \left(\frac{\partial^2 \mathcal{L}}{\partial \phi_0 \partial \phi_0} - \partial_\mu \frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi_0)\partial \phi_0} \right) \phi_1 \phi_1 \right]$$

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Phonon's action $$S[\phi_1] = \frac{1}{2}\int d^4x\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\,\phi_1\partial_\nu\phi_1 - M^2_{\phi_0}\phi_1\phi_1 \right)$$