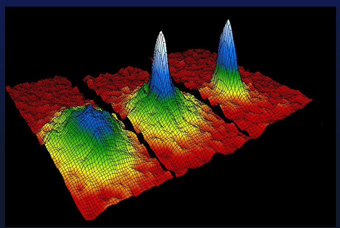
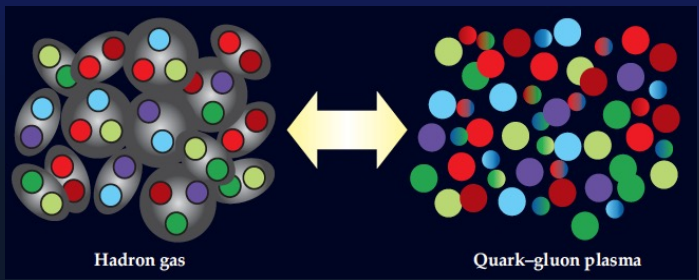
Matter under extreme conditions
-- from atoms to the cosmos

Gordon Baym

University of Illinois, Urbana





QFC2022, PISA

26 October 2022

Three main areas of this meeting:

Equation of state of compact objects

Tests of general relativity

Quantum simulators for fundamental interactions and cosmology

seen from the vantage points of

Laboratory experiment and theory

Cosmic observations and theory

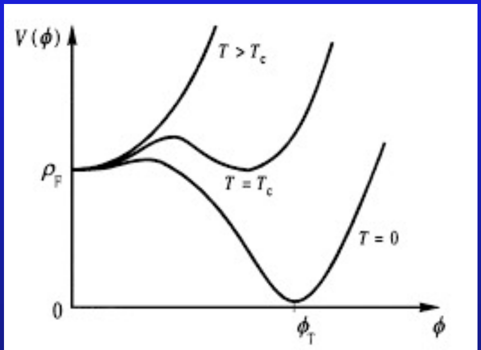
To paraphrase Stanisław Ulam's famous remark on physics and biology,

"Ask not what physics can do for biology— ask what biology can do for physics"

Ask what condensed matter physics (including cold atoms) can do for high energy and nuclear physics and cosmology

Focus on study of matter under extreme conditions

Various impacts of condensed matter physics:



Early universe
-- Phase transitions in the very early universe
-- quark deconfinement transition

Neutron stars – densest matter in the universe
-- Physics of the crust, vortices, glitches
-- Bardeen-Cooper-Schrieffer (BCS) pairing in nucleons and quarks
-- BEC-BCS crossover physics in quark matter

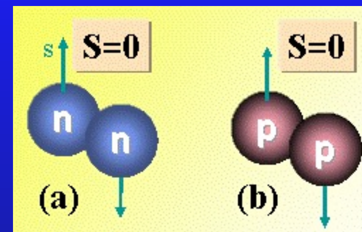
Elementary particle physics
-- symmetry breaking and condensates, from chiral phase transitions to BCS pairing, to gluon condensates, to Higgs physics

Cold atom simulations of high energy phenomena
-- lattice gauge theory via cold atoms
-- acoustic analogs of Hawking radiation. Information "paradox"

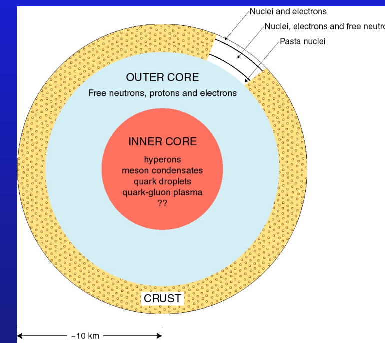
BCS everywhere else

BCS beyond lab superconductors

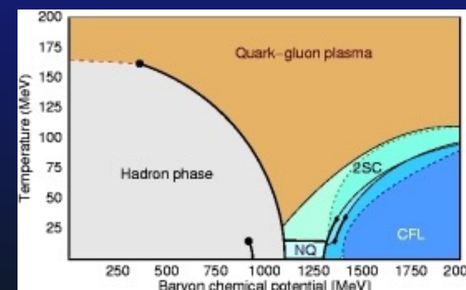
Pairing of nucleons in nuclei



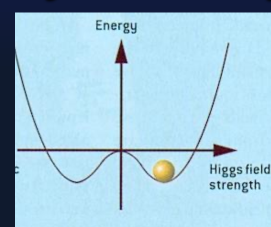
Neutron stars: pairing in neutron star matter



Pairing of quarks in degenerate quark-gluon plasmas

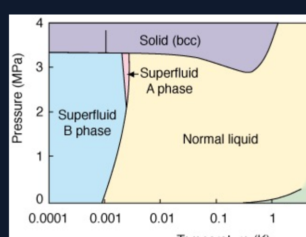


Elementary particle physics – broken symmetry

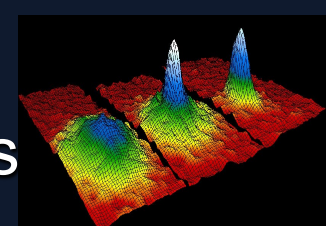


and even on Earth:

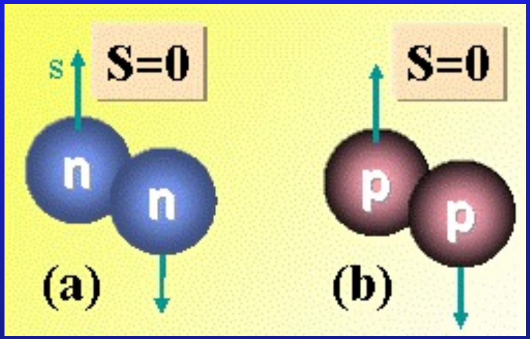
Helium-3



Cold fermionic atoms



BCS applied to nuclear systems - 1957



Pairing of even numbers of neutrons or protons outside closed shells

David Pines brings BCS to Niels Bohr Institute in Copenhagen, Summer 1957, as BCS was being finished in Urbana.

Aage Bohr, Ben Mottelson and Pines (57) suggest BCS pairing in nuclei to explain energy gap in single particle spectrum – odd-even mass differences

Pairing gaps deduced from odd-even mass differences:

$\Delta \sim 12\ A^{-1/2}$ MeV for both protons and neutrons

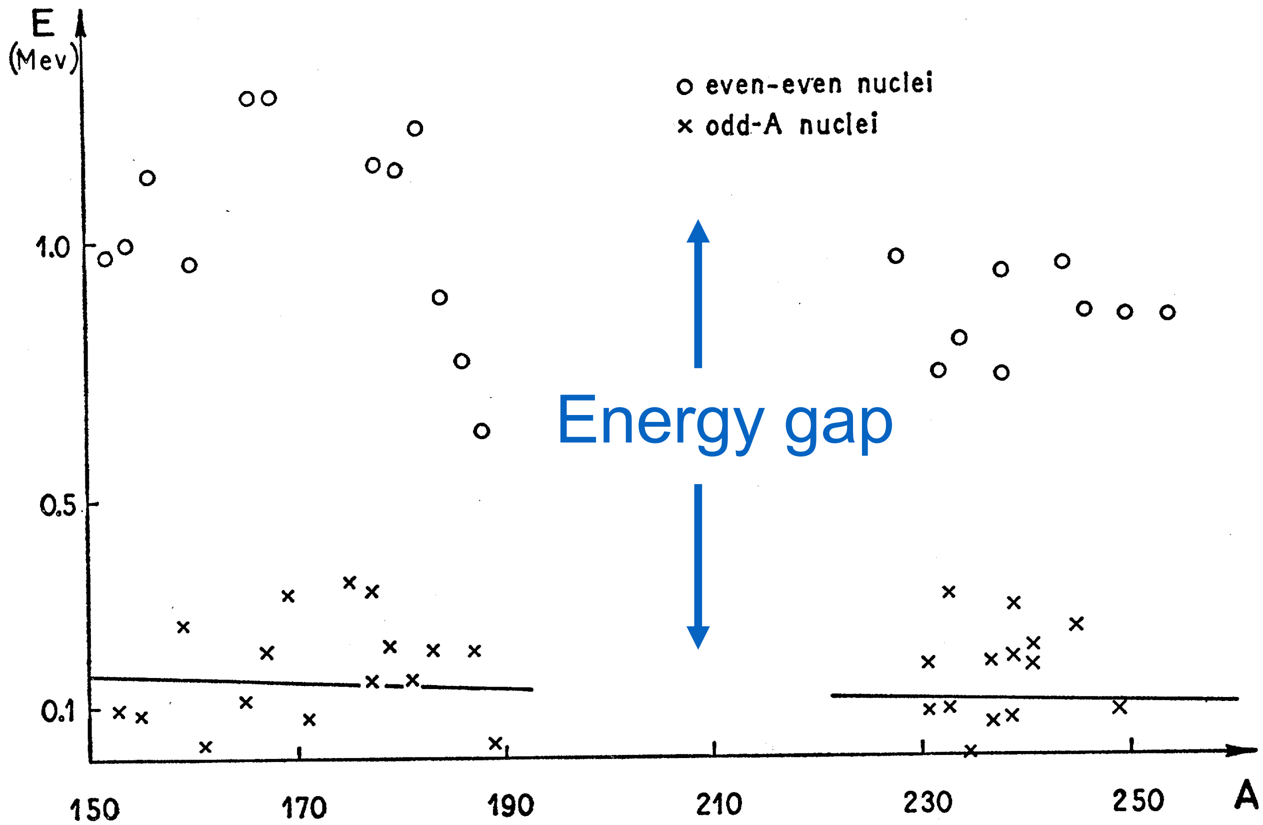
Energies of first excited states:

even no. of neutrons – even no. of protons (BCS paired) vs. odd A (unpaired) nuclei

FIG. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in *Nuclear Data Cards* [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\delta/2$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd-A nuclei (see reference 1).

The figure contains all the available data for nuclei with $150<A<190$ and $228<A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A=25$; in this latter region the available data on odd-A nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.



And in addition BCS explained more widely space rotational spectra, $E = J(J+1)/2I$, of deformed **nuclei:**

moment of inertia, I, reduced from rigid body value, I_{cl} .

Reduction of moment of inertia due to BCS pairing = analog of Meissner effect. Detailed calculations by Migdal (1959).

Element	β [7]	x_p	x_n	$\left(\frac{\mathcal{J}}{\mathcal{J}_0}\right)_{rect.}$	$\left(\frac{\mathcal{J}}{\mathcal{J}_0}\right)_{osc.}$	$\left(\frac{\mathcal{J}}{\mathcal{J}_0}\right)_{exper.}$ [7]
Nd^{150}	0.26	0.54	0.94	0.15	0.38	0.35
Sm^{152}	0.24	0.65	1.02	0.17	0.43	0.38
Gd^{154}	0.26	0.52	0.88	0.13	0.35	0.36
Gd^{156}	0.33	0.87	1.37	0.22	0.57	0.48
Gd^{157}	0.29	0.93	1.60	0.22	0.64	0.60
Dy^{162}	0.30	0.84	1.43	0.23	0.57	0.50
Hf^{179}	0.20	0.99	1.75	0.27	0.66	0.52
Os^{186}	0.18	0.44	0.69	0.09	0.26	0.28
Th^{230}	0.22	0.63	0.95	0.15	0.40	0.43
Th^{232}	0.22	0.84	1.42	0.24	0.60	0.44
U^{238}	0.24	0.83	1.29	0.22	0.54	0.43

1.C:
1.E.6

Nuclear Physics **13** (1959) 655—674; *North-Holland Publishing Co., A*

SUPERFLUIDITY AND THE MOMENTS OF INERTIA OF NUCLEI

A. B. MIGDAL

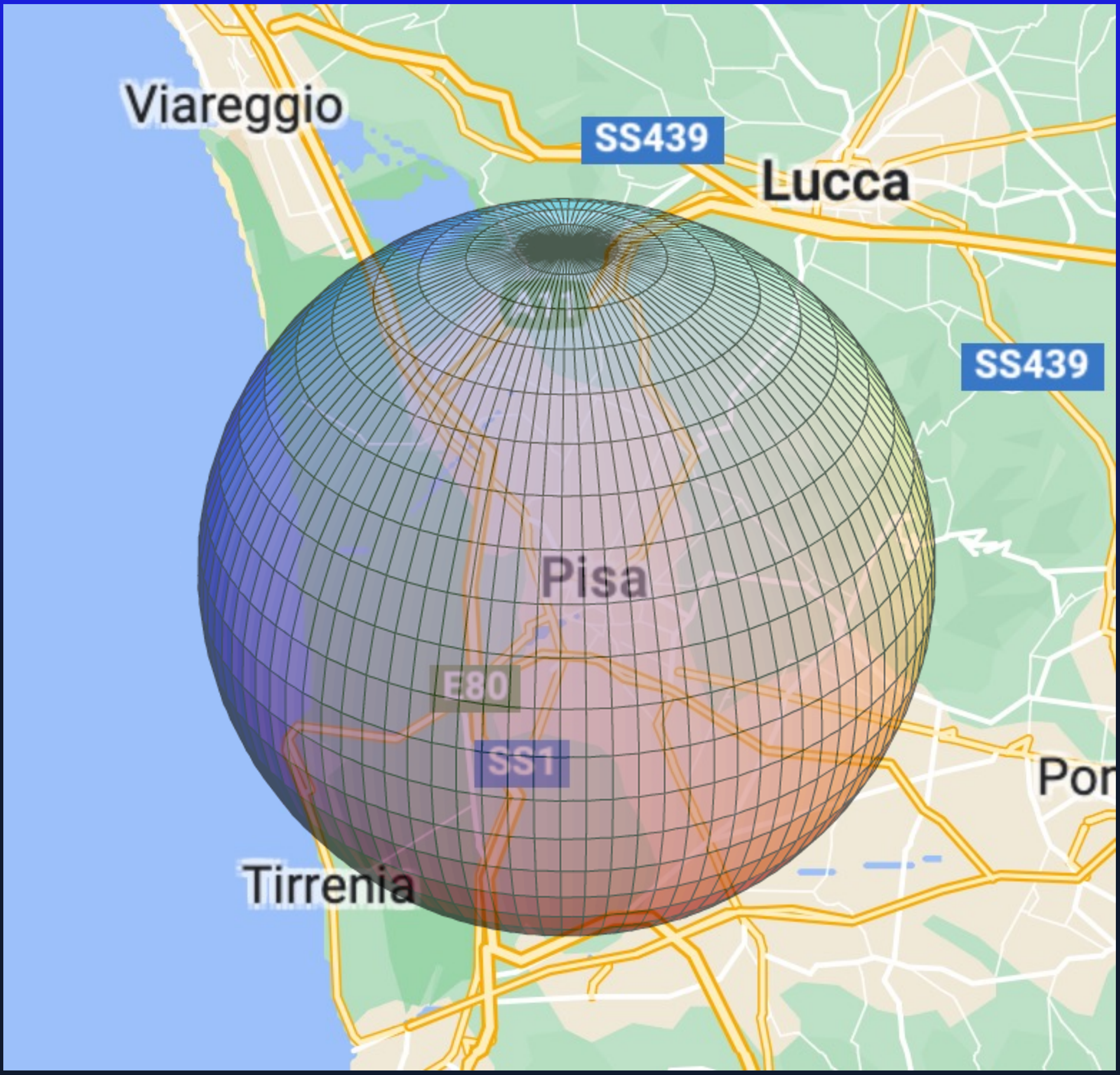
Atomic Energy Institute of USSR, Academy of Sciences, Moscow

Received 11 April 1959

Abstract: A method is presented which permits one to study superfluidity in finite size systems. Moments of inertia are computed by this method in the quasi-classical approximation and satisfactory agreement with the observed values is obtained. The calculated increase of the moment of inertia upon transition from even to odd-mass nuclei and also the gyromagnetic ratio for rotating nuclei are in agreement with the experiments. These results thus confirm the assumption of superfluidity of nuclear matter.

Neutron stars

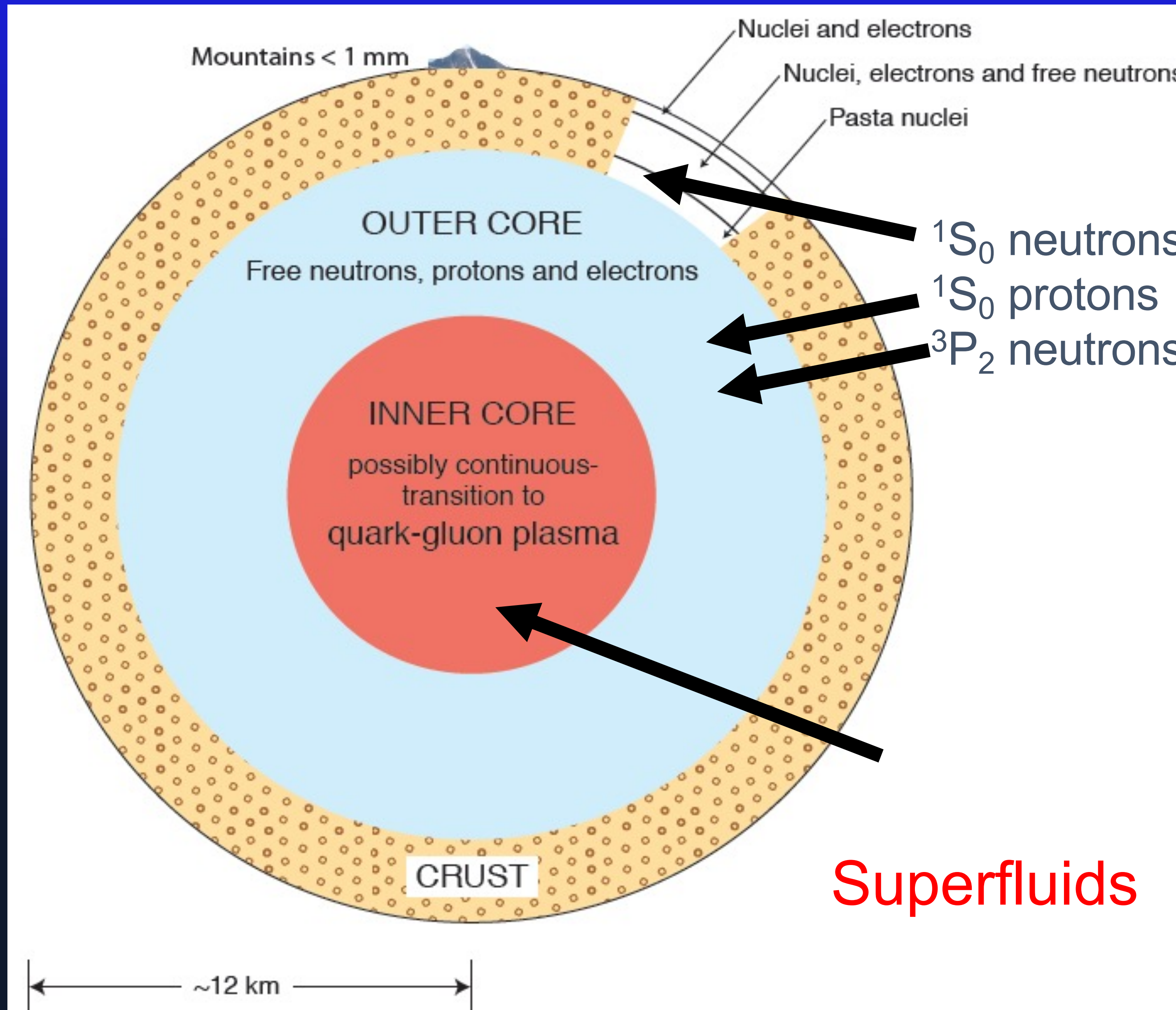
Neutron star over Pisa

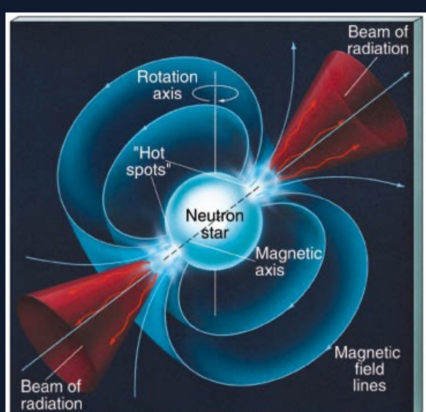


Neutron star interior

Mass ~ 1.2-2 M_{sun}
Radius ~ 10-12 km
Temperature
~ 10^6-10^9 K

Surface gravity
~10^{11} that of Earth
Surface binding
~ 1/10 mc^2



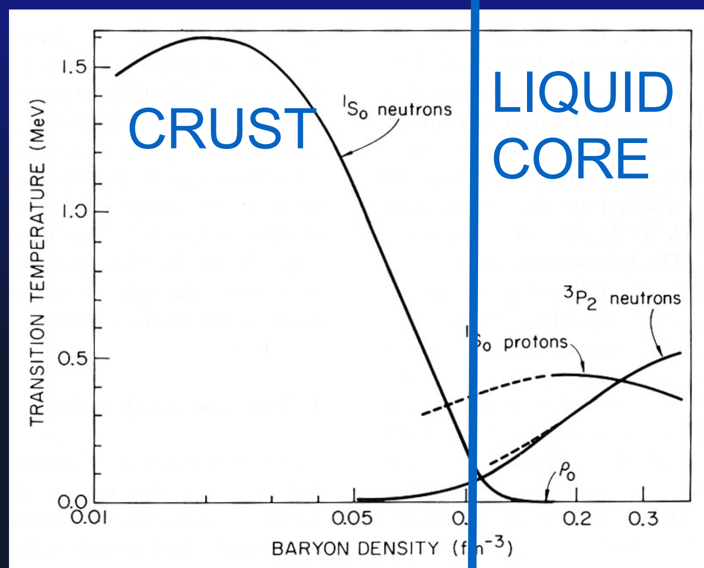


Superfluidity of nuclear matter in neutrons stars

Migdal 1959, Ginzburg & Kirshnits 1964; Ruderman 1967; GB, Pines & Pethick, 1969

Neutron stars (very big Dewars) have the preponderance of superfluids in the universe, and with the highest T_c's ~ 10^{10-11} K

Estimated pairing gaps and T_c's from scattering phase shifts:



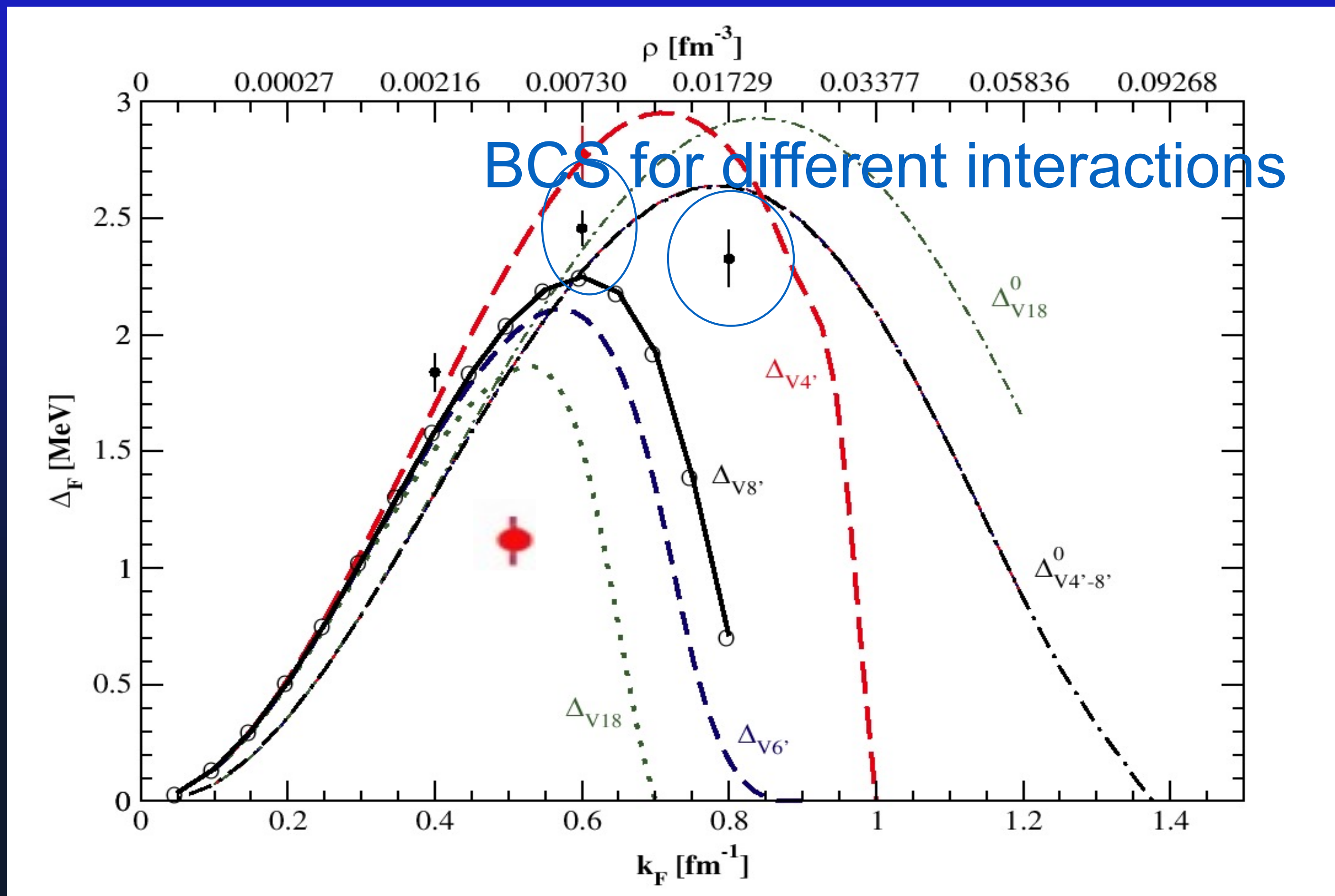
Neutron fluid in crust BCS-paired in relative $^1S_0$ states (singlet spin)

Neutron fluid in core $^3P_2$ paired (triplet spin)

Proton fluid $^1S_0$ paired

Quantum Monte Carlo (AFDMC) $^1S_0$ nn gap in crust:

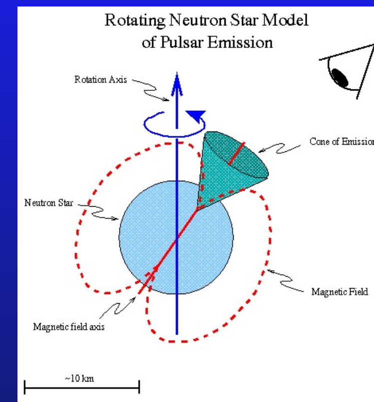
Fabrocini et al, PRL 95, 192501 (2005)



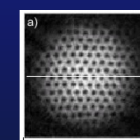
QMC (black points) close to standard BCS (upper curves)
Green's function Monte Carlo (Gezerlis 2007)

Superconducting protons in neutron star magnetic fields, $\sim 10^{12-16}$G

Even though superconductors expel magnetic flux, for magnetic field below critical value, flux diffusion times in neutron stars are >> age of universe.
Electric conductivity >>> Cu at room temp.
Proton superconductivity forms with field present.



Proton fluid threaded by triangular (Abrikosov) lattice of vortices parallel to magnetic field (for Type II superconductor)



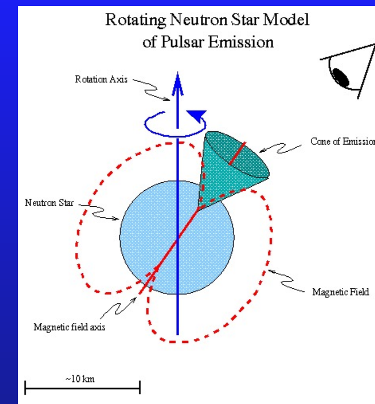
Quantized magnetic flux per vortex:

$$\oint_C \mathbf{B} \cdot d\ell = \frac{2\pi\hbar c}{2e} = \phi_0 = 2 \text{ X } 10^{-7} \text{ G cm}^2$$

Vortex core ~ 10 fm,

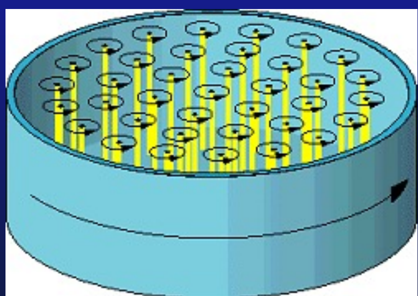
$n_{vort} = B/\phi_0$ => spacing ~ 5 x 10^{-10} cm $(B/10^{12}\text{G})^{-1/2}$

Rotating superfluid neutrons

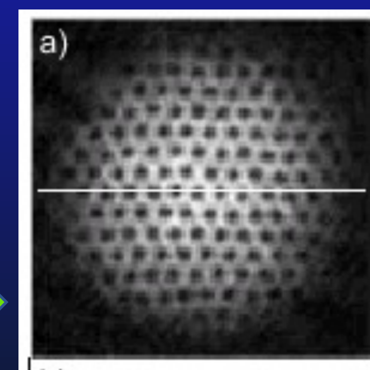


(Rotation periods from few seconds to > msec.)

Rotating superfluid threaded by triangular lattice of vortices parallel to stellar rotation axis



Bose-condensed ^{87}Rb atoms
Schweikhard et al., PRL92 040404 (2004)



Quantized circulation of superfluid velocity about vortex:

$$\oint_C \mathbf{v_s} \cdot d\ell = \frac{2\pi\hbar}{2m_n}$$

Vortex core ~ 10 fm. Vortex separation ~ $0.01P(s)^{1/2}$cm. P=89 ms
Vela pulsar (PSR0833-45) ~ 10^{17} vortices

Pulsar glitches

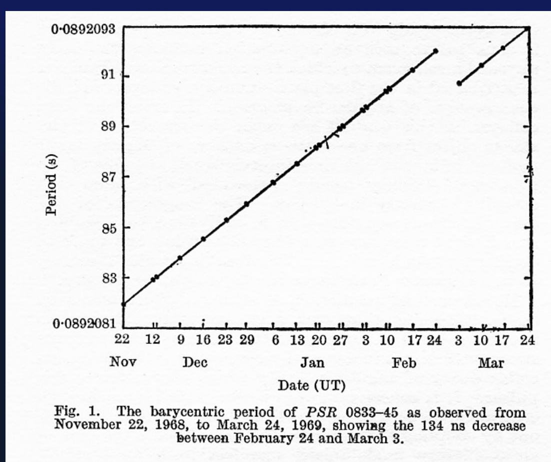
Sudden speedups in rotation period, relaxing back in days to years, with no significant change in pulsed electromagnetic emission: ~500 glitches detected in > 100 pulsars

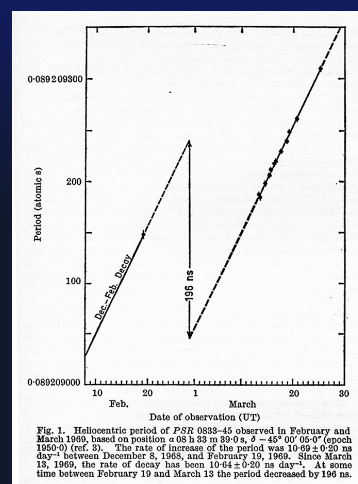
Vela (PSR0833-45) Period=1/Ω =0.089sec

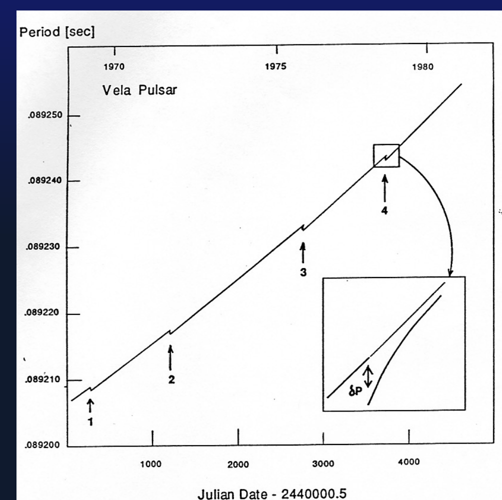
>15 glitches since discovery in 1969

$\Delta\Omega/\Omega \sim 10^{-6}$ Largest = 3.14 X 10^{-6} on Jan. 16, 2000

Moment of inertia ~ 10^{45} g-cm^2 => $\Delta E_{rot} \sim 10^{43}$ erg







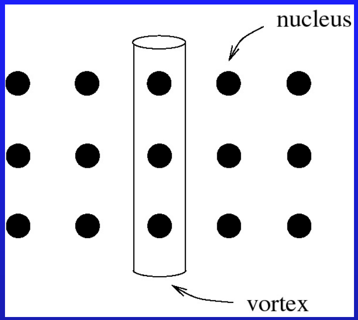
(2018)

Reichley and Downs, Nature 1969 *Radhakrishnan and Manchester, Nature 1969*

Crab (PSR0531+21) P = 0.033sec 25 glitches since 1969

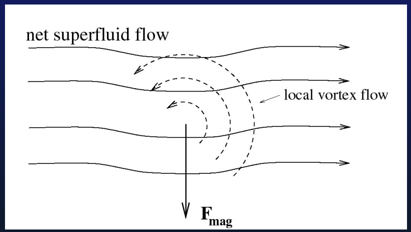
$\Delta\Omega/\Omega \sim 10^{-9}$ to 0.5 X 10^{-6} (in 2018)

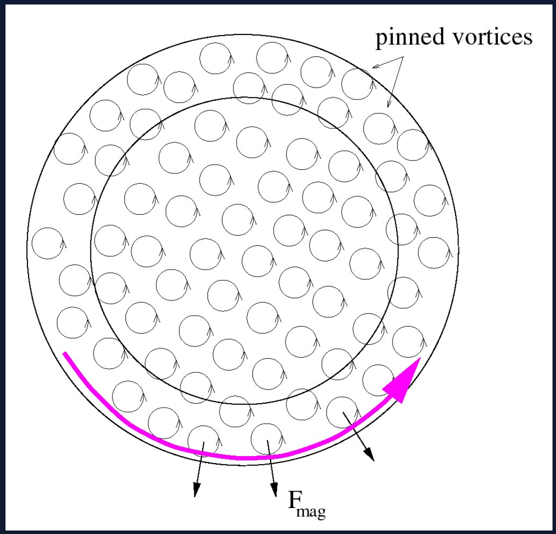
Vortex model of glitches



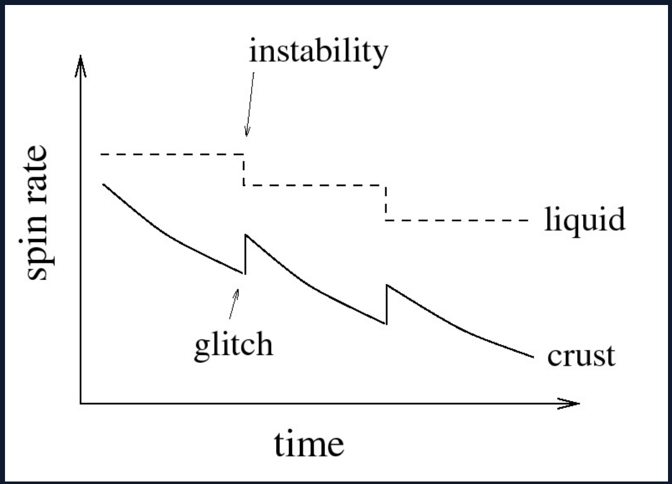
Pin vortices on nuclei in inner crust.
E ~ few Mev/nucleus.
(Bogoliubov- de Gennes calculations suggest pinning between nuclei)

$n_{vortices}$ fixed => $\Omega_{superfluid}$ fixed; Ω_{crust} decreases as star radiates.
As $\Omega_{sf} - \Omega_{crust}$ grows, Magnus force = $\rho_s\ \Omega \times (\mathbf{v}_{vortex}-\mathbf{v}_{superfl})$
drives unpinning (glitch) and outward relaxation.



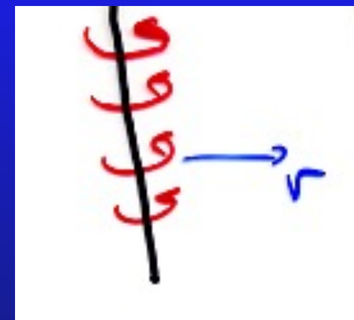


Collective outward motion of many (~10^{14}) Vortices produces large glitch



Vortices in superfluids: quantized circulation

Order parameter $\Psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})}$



p = mv

Superfluid velocity $\vec{v}(\vec{r}) = (\hbar/m)\nabla\phi$

$$\oint \nabla\phi \cdot dl = 2\pi n$$

Quantized circulation $\oint_C \vec{v} \cdot \vec{d\ell} = \frac{2\pi\hbar n}{m}$

n = integer

Singly quantized (n=1) vortex flow: $v_\varphi(r) = \hbar/mr$

But what m should one use in an interacting system?

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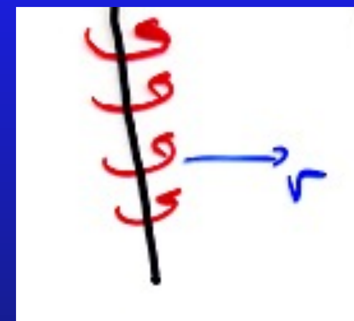

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How is the superfluid velocity related to the momentum?

$$\phi = \vec{p} \cdot \vec{r} - \mu t \Rightarrow \vec{v} = \vec{p}/\mu$$

μ = chemical potential including rest mass

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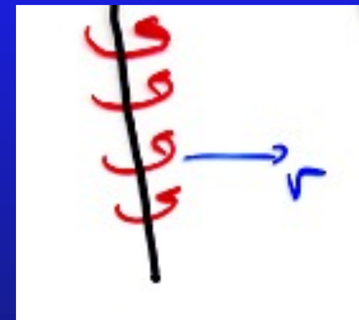

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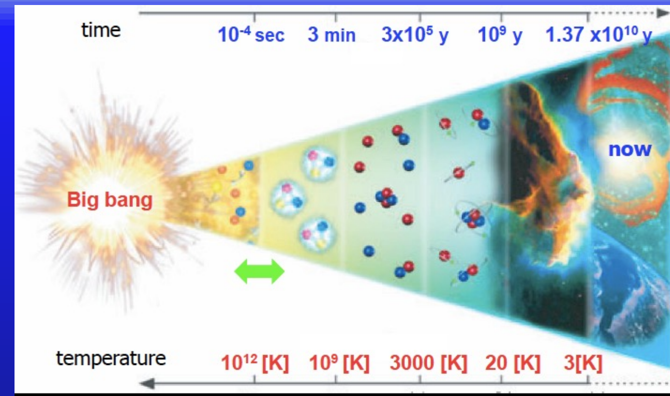
In circulation, replace m by μ more correctly
and specific enthalpy at finite temperature

$$\oint_C \vec{v}\cdot d\vec{\ell} = \frac{2\pi\hbar n}{\mu}$$

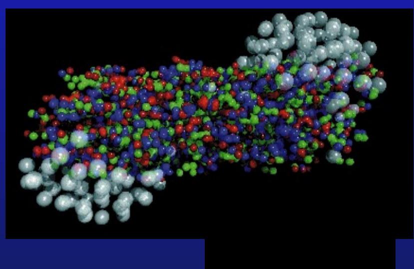
In superfluid ^{4}He, -7.17K correction to m_4 is only ~ 1 : 6 X10^{12}

Quark matter

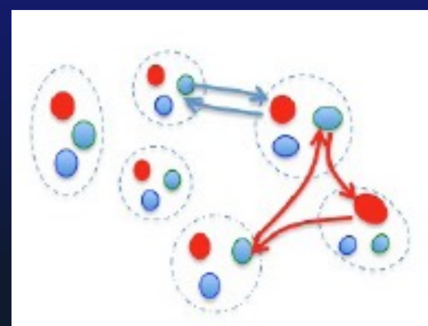
Quarks in dense matter



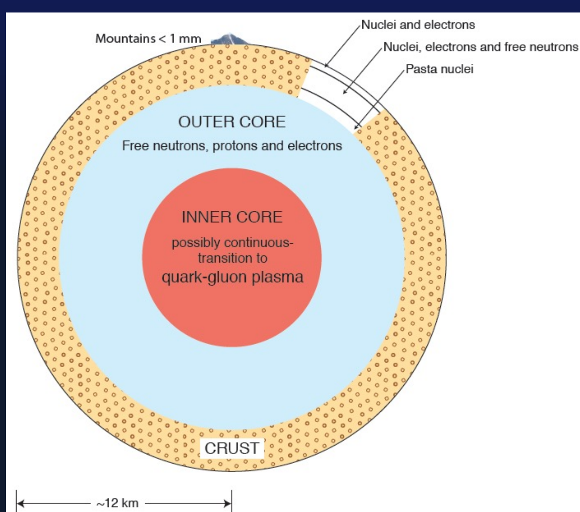
The early universe before one microsecond after the big bang -- hot quark gluon plasma



and created in ultrarelativistic heavy ion collisions



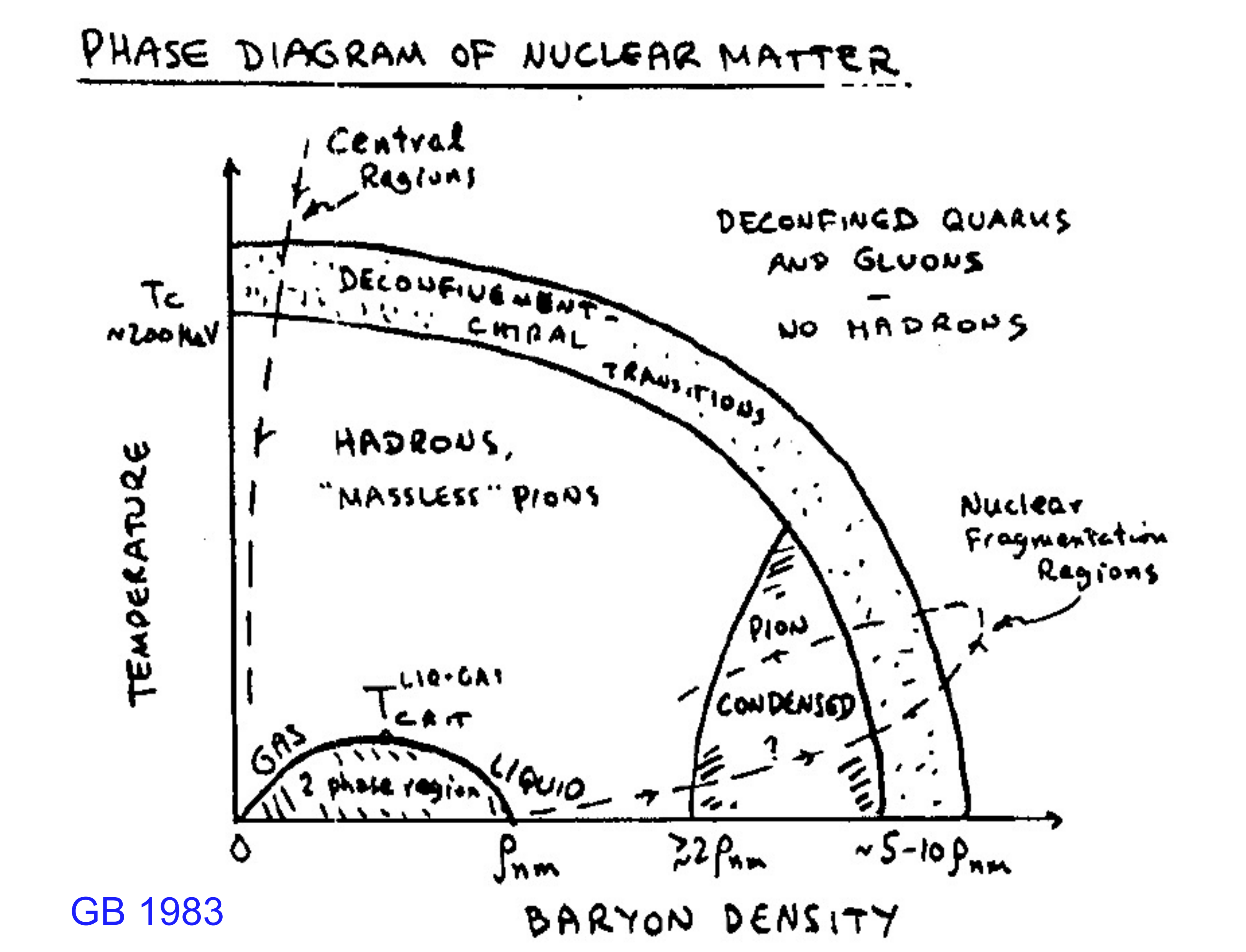
Quarks (and gluons) in nuclei will be mapped by future Electron-Ion Collider



Cold quark matter cores of high mass neutron stars –

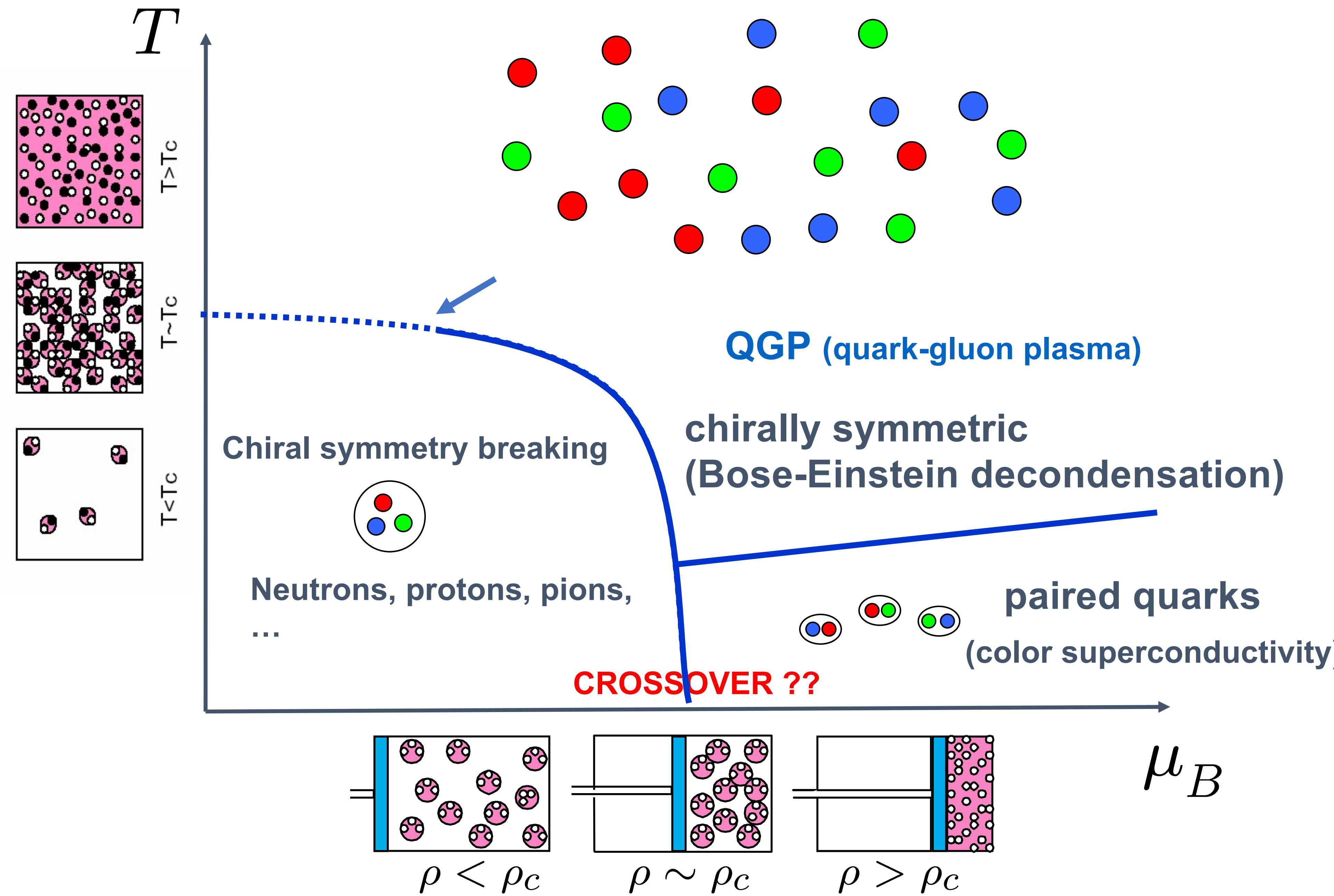
Strongly interacting system: cannot do lattice QCD simulations at finite density, zero temperature, owing to fermion sign problem.

Phase diagram of dense matter



GB 1983

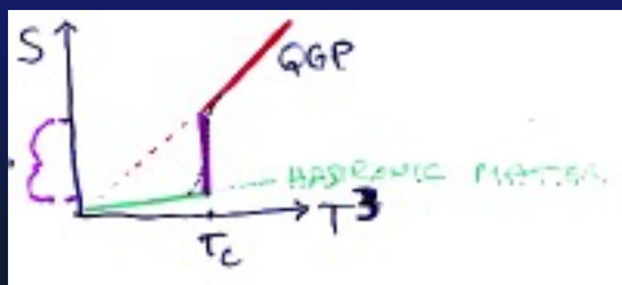
Phase diagram of quark-gluon plasma



Quark-gluon plasma state

Degrees of freedom are deconfined quarks and gluons
Theory is quantum chromodynamics - $SU_C(3)$ gauge symmetry

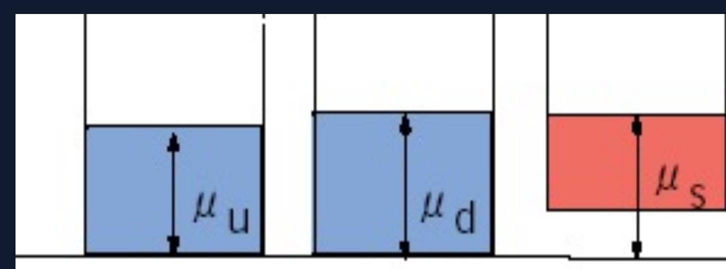
Many more degrees of freedom than hadronic matter
(color, spin, particle-antiparticle, & flavor);
much larger entropy at given temperature.

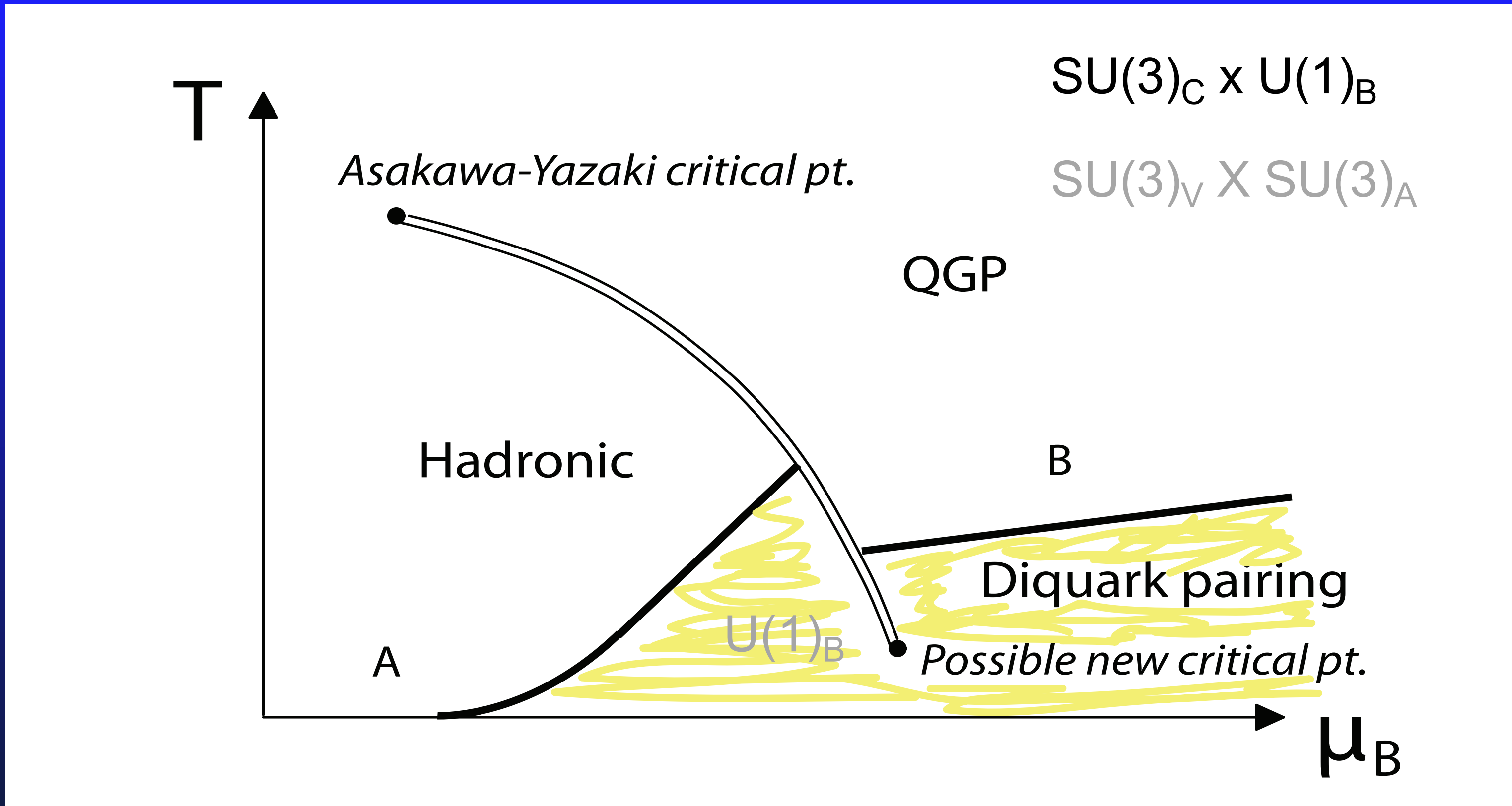


<= Large latent heat
(or sharp rise at least)

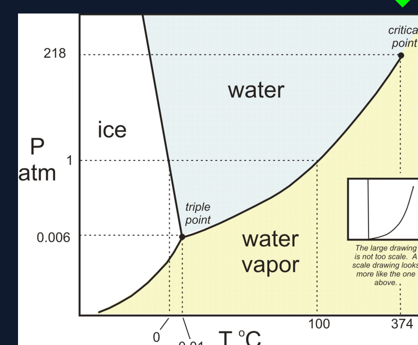
neutron = udd, proton = uud

At low temperatures form Fermi
seas of degenerate u,d, and s quarks:
(e.g., in neutron stars)



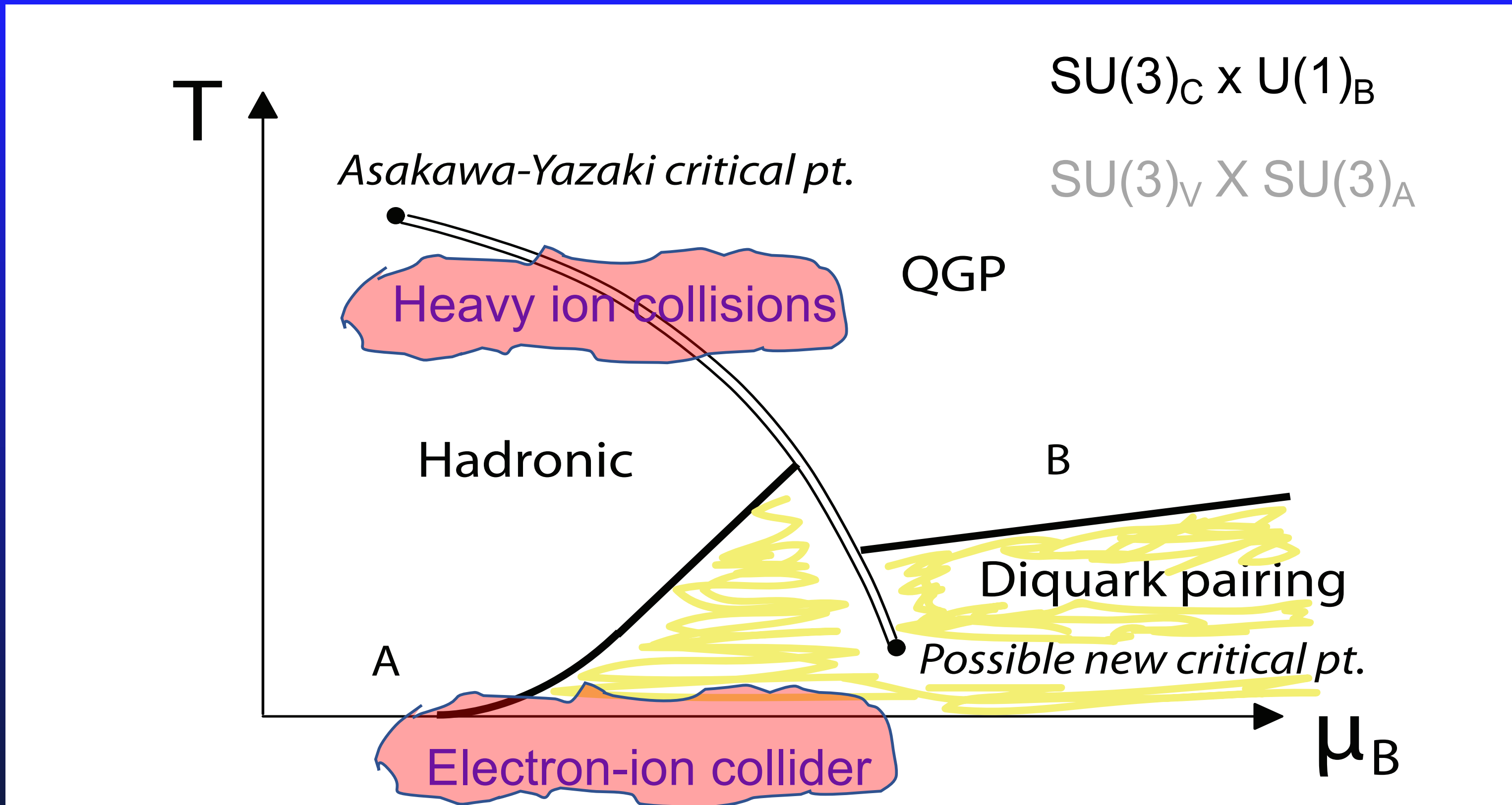


Critical points similar to those in liquid-gas phase diagram (H_2O). Neither critical point necessary!!



Can go continuously from A to B around the upper critical point. Liquid-gas phase transition.

In lower shaded region have BCS pairing of nucleons, of quarks, and possibly other states (meson condensates, quarkyonic). Different symmetry structure than at higher T.



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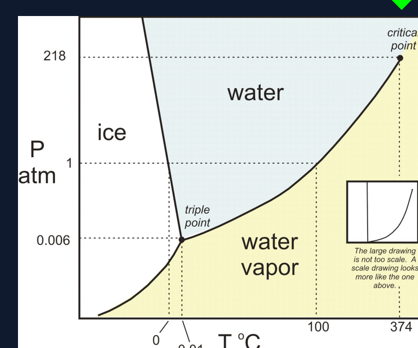

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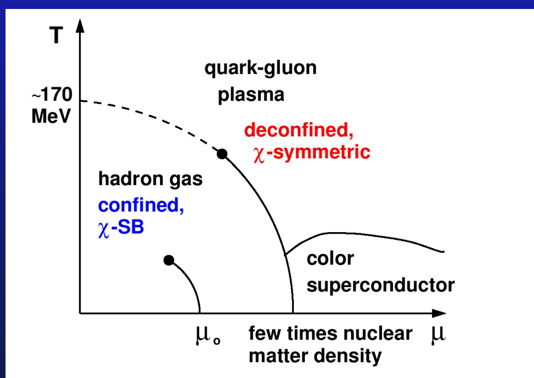
In lower shaded region have BCS pairing of nucleons, of quarks, and possibly other states (meson condensates, quarkyonic). Different symmetry structure than at higher T.

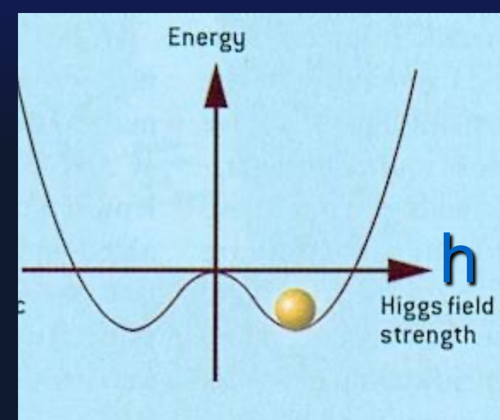
Pairing in high energy physics

Vacuum condensates: quark-antiquark pairing underlies

chiral SU(3) x SU(3) breaking of vacuum=> $\langle \bar{q}q \rangle_{\rm vacuum} \neq 0$



Experimental Bose-Einstein decondensation



Broken symmetry –

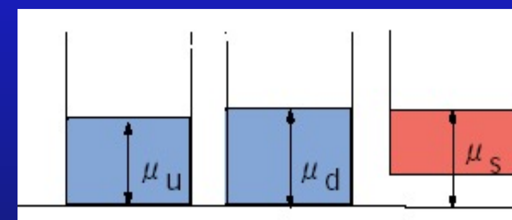
Particle masses via Higgs field

$L_m = gh\psi^y \gamma^0 \psi \Rightarrow g \langle h \rangle \psi^y \gamma^0 \psi \Rightarrow m = g\langle h \rangle$

Color pairing in quark matter

In quark matter have “free quarks” = spin ½ with *flavor* u,d,s and *color* = internal degree of freedom for SU(3) gauge symmetry.

Two interesting pairing states:

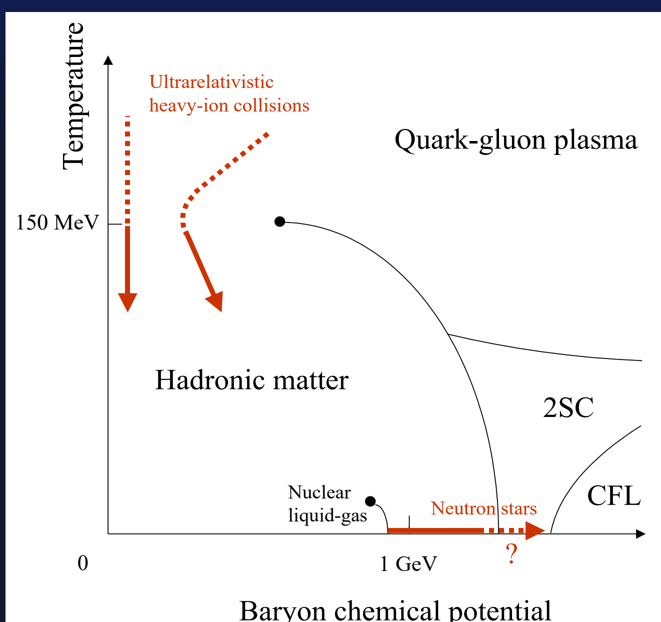


2SC (u,d)

$\langle u_R d_B \rangle$

Color-flavor locked (CFL) ($m_u = m_d = m_s$)

$\langle u\, d \rangle = \langle d\, s \rangle = \langle s\, u \rangle$



2SC similar to superconducting protons:

e.m. vortices in magnetic field.

London moment under rotation

CFL similar to superfluid neutrons:

$U(1)_B$ vortices under rotation

(Partial screening of magnetic fields.)

BCS pairing in Color Flavor Locked (CFL) phase

In free equally populated up, down, and strange quark matter have $SU(3)_F$ symmetry in flavor (uds) and $SU(3)_C$ symmetry in color (rgb)

Most favored BCS pairing state is anti-symmetric in spin, flavor (i), and color (α):

$$\Phi_{\alpha i} \propto \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\langle q_{\beta j} C\gamma_5 q_{\gamma k}\rangle \chi_{\mathrm{spin-singlet}}$$

$$\Phi = \begin{pmatrix} \Phi^{\bar{r}\bar{u}} & 0 & 0 \\ 0 & \Phi^{\bar{g}\bar{d}} & 0 \\ 0 & 0 & \Phi^{\bar{b}\bar{s}} \end{pmatrix} \chi \rightarrow \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{pmatrix} \chi$$

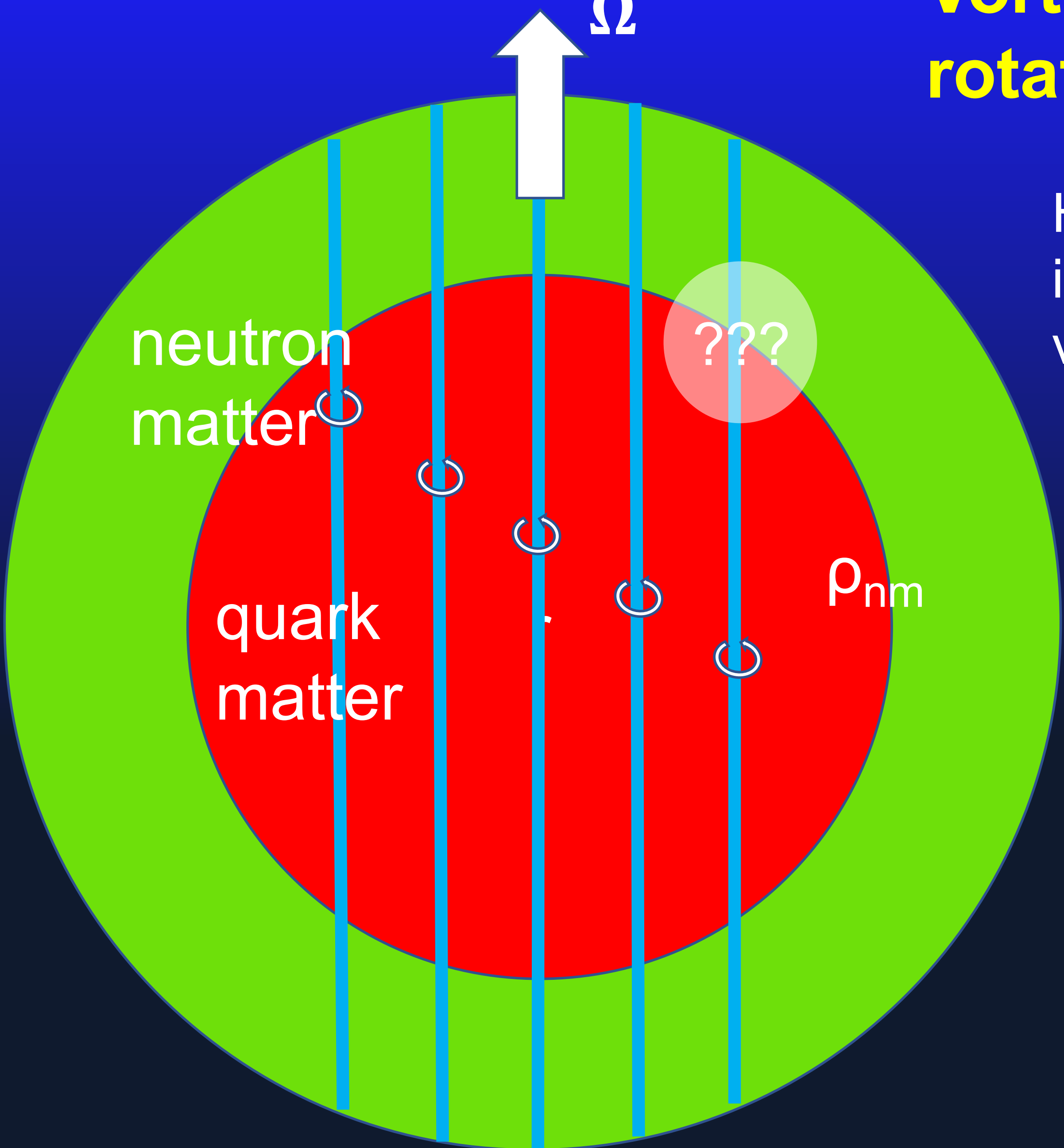
flavor ->

color ->

CFL order parameter in ground state

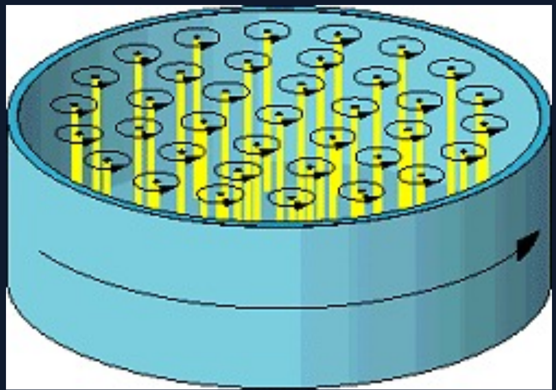
Pairing with correlation of color and flavor reduces symmetry from $SU(3)_C \times SU(3)_F \times U(1)_B$ to $SU(3)_{C+F}$

Vortices threading rotating neutron star



How do neutron vortices interface with quark (CFL) vortices??

M. Alford, GB, K. Fukushima, T. Hatsuda, & M. Tachibana, PR D 99, 036004 (2019).



Try to match circulations

Circulation: $C = \oint_C \vec{v} \cdot \vec{d\ell} = \frac{2\pi\hbar n}{\mu}$ v = superfluid velocity p/μ

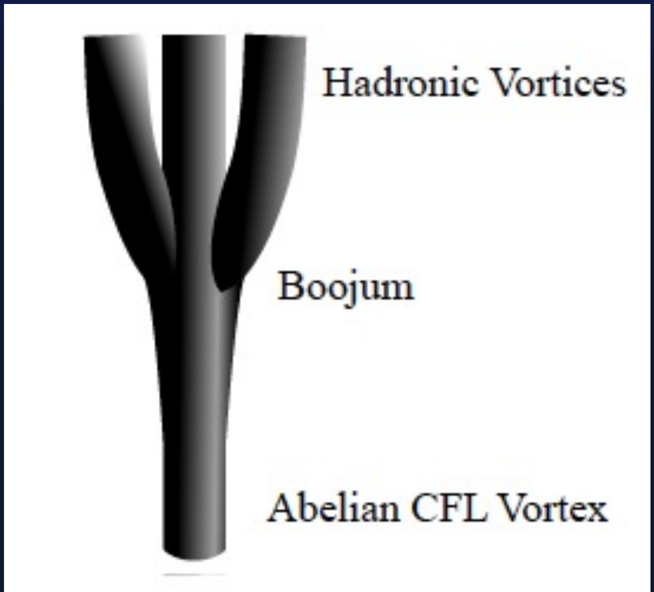
In paired hadronic phase $\mu = 2\mu_n$ (μ_n=neutron chemical potential).

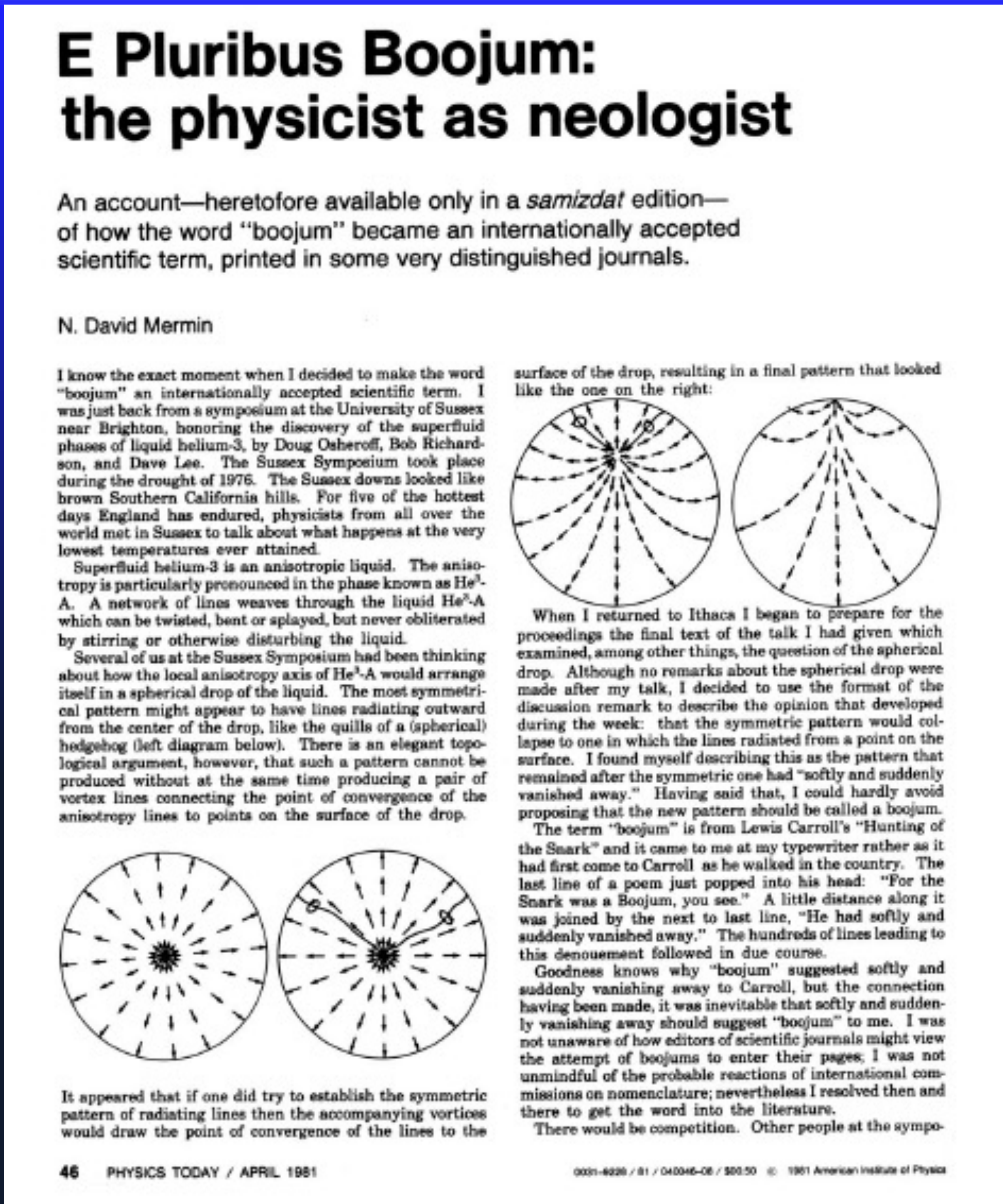
In paired quark phase $\mu = 2\mu_q = 2\mu_n/3$ (μ_q= quark chemical pot.), since nucleon is made of 3 quarks, $\mu_n = 3\mu_q$

=> quark phase superfluid velocity = 3X velocity in hadronic phase.

Continuity in flow states in neutron star would require 3 hadron vortices merging into a single quark vortex.

A boojum!





E Pluribus Boojum: the physicist as neologist

An account—heretofore available only in a *samizdat* edition—of how the word "boojum" became an internationally accepted scientific term, printed in some very distinguished journals.

N. David Mermin

I know the exact moment when I decided to make the word "boojum" an internationally accepted scientific term. I was just back from a symposium at the University of Sussex near Brighton, honoring the discovery of the superfluid phases of liquid helium-3, by Doug Osheroff, Bob Richardson, and Dave Lee. The Sussex Symposium took place during the drought of 1976. The Sussex downs looked like brown Southern California hills. For five of the hottest days England has endured, physicists from all over the world met in Sussex to talk about what happens at the very lowest temperatures ever attained.

Superfluid helium-3 is an anisotropic liquid. The anisotropy is particularly pronounced in the phase known as He^3-A. A network of lines weaves through the liquid He^3-A which can be twisted, bent or splayed, but never obliterated by stirring or otherwise disturbing the liquid.

Several of us at the Sussex Symposium had been thinking about how the local anisotropy axis of He^3-A would arrange itself in a spherical drop of the liquid. The most symmetrical pattern might appear to have lines radiating outward from the center of the drop, like the quills of a (spherical) hedgehog (left diagram below). There is an elegant topological argument, however, that such a pattern cannot be produced without at the same time producing a pair of vortex lines connecting the point of convergence of the anisotropy lines to points on the surface of the drop.

It appeared that if one did try to establish the symmetric pattern of radiating lines then the accompanying vortices would draw the point of convergence of the lines to the surface of the drop, resulting in a final pattern that looked like the one on the right:

When I returned to Ithaca I began to prepare for the proceedings the final text of the talk I had given which examined, among other things, the question of the spherical drop. Although no remarks about the spherical drop were made after my talk, I decided to use the format of the discussion remark to describe the opinion that developed during the week: that the symmetric pattern would collapse to one in which the lines radiated from a point on the surface. I found myself describing this as the pattern that remained after the symmetric one had "softly and suddenly vanished away." Having said that, I could hardly avoid proposing that the new pattern should be called a boojum.

The term "boojum" is from Lewis Carroll's "Hunting of the Snark" and it came to me at my typewriter rather as it had first come to Carroll as he walked in the country. The last line of a poem just popped into his head: "For the Snark was a Boojum, you see." A little distance along it was joined by the next to last line, "He had softly and suddenly vanished away." The hundreds of lines leading to this denouement followed in due course.

Goodness knows why "boojum" suggested softly and suddenly vanishing away to Carroll, but the connection having been made, it was inevitable that softly and suddenly vanishing away should suggest "boojum" to me. I was not unaware of how editors of scientific journals might view the attempt of boojums to enter their pages; I was not unmindful of the probable reactions of international commissions on nomenclature; nevertheless I resolved then and there to get the word into the literature.

There would be competition. Other people at the sympo-

David Mermin
Physics Today
April 1981

A boojum tree

Abelian vortex in CFL phase

Order parameter matrix of Abelian CFL vortex.

$$\Phi(r,\phi) = \Delta \cdot f(r) e^{i\varphi} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Circulation = $2\pi\hbar/2\mu_q = 3(2\pi\hbar/2\mu_n)$

But this vortex is unstable against decay into three color flux tubes with ~1/3 kinetic energy *(A. P. Balachandran et al. PR D 73 (2006); E. Nakano et al., PR D 78, 045002 (2008). Phys. Lett. B 672 (2009)):*

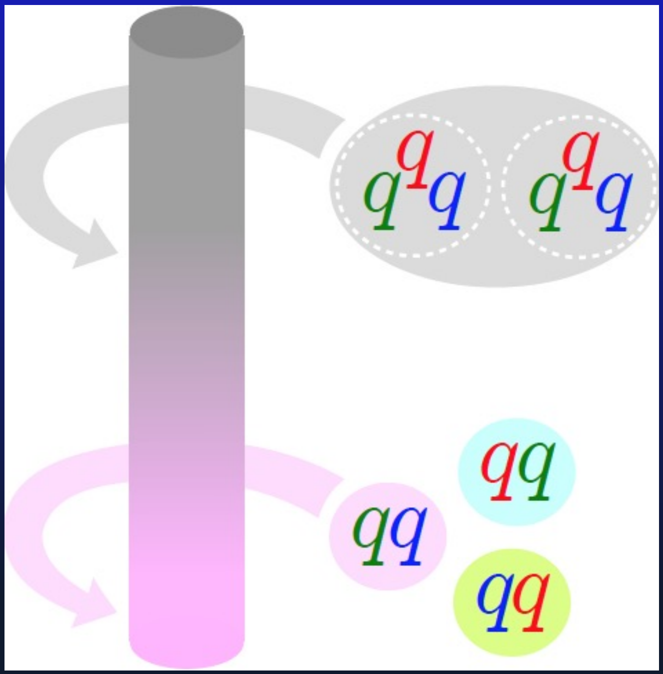
Color flux tube

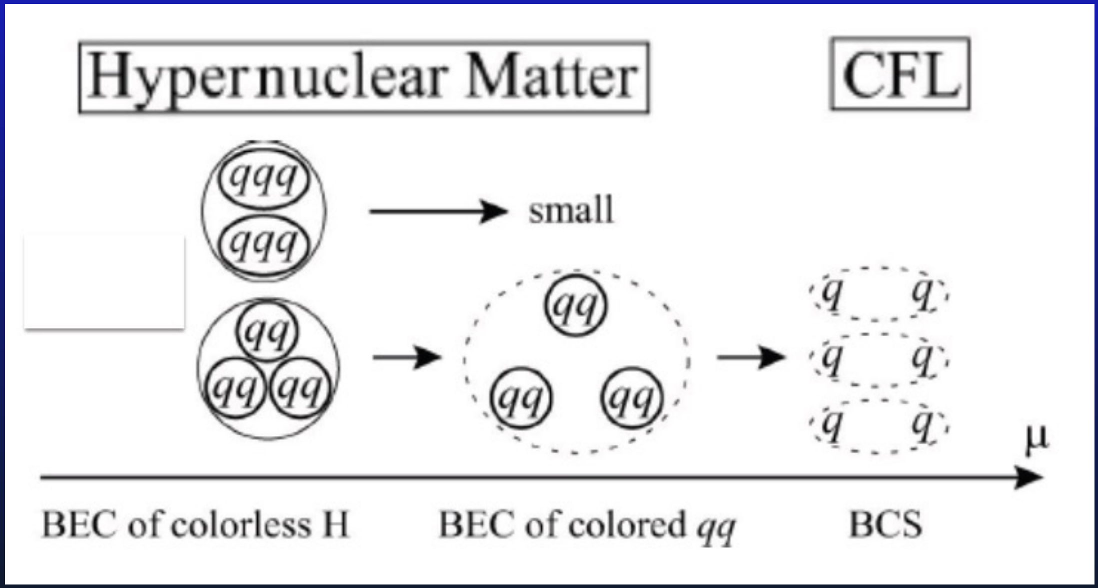
$$\Phi^R_{\alpha i} = \Delta \begin{pmatrix} e^{i\varphi} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix} = \Delta e^{\frac{i}{3}\varphi} \begin{pmatrix} e^{\frac{2i}{3}\varphi} f(r) & 0 & 0 \\ 0 & e^{-\frac{i}{3}\varphi} g(r) & 0 \\ 0 & 0 & e^{-\frac{i}{3}\varphi} g(r) \end{pmatrix}$$

"red" flux tube order parameter

Leading phase (1/3) is $U(1)_B$. Phases within => color rotation; do not contribute to circulation = $\frac{1}{3} 2\pi\hbar/2\mu_q = 2\pi\hbar/2\mu_n$

Single color flux tube has circulation 1/3 that of initial (unstable) Abelian CFL vortex – same as a single original hadronic vortex.





Pairing continuity

K. Fukushima, PRD (2004)

Conclude that three hadronic vortices can turn into three non-Abelian CFL vortices, with no discontinuity in circulation. But: gauge invariance???

Gauge invariant description of flux tubes

$$\Phi^R_{\alpha i} = \Delta \begin{pmatrix} e^{i\varphi} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix} = \Delta e^{\frac{i}{3}\varphi} \begin{pmatrix} e^{\frac{2i}{3}\varphi} f(r) & 0 & 0 \\ 0 & e^{-\frac{i}{3}\varphi} g(r) & 0 \\ 0 & 0 & e^{-\frac{i}{3}\varphi} g(r) \end{pmatrix}$$

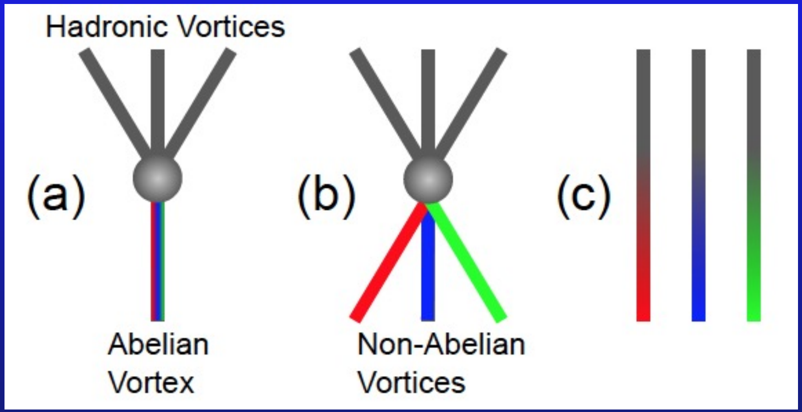
red flux tube order parameter

Then

$$\Upsilon(\vec{r}) = \frac{1}{6}\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\Phi_{\alpha i}\Phi_{\beta j}\Phi_{\gamma k} \qquad = e^{i\varphi}\Delta^3 f(r) g^2(r)$$

is gauge invariant order parameter, independent of choice of color of the gauge fixed $\Phi^R_{\alpha i}$. Only one gauge invariant physical object.

Quark-hadron continuity



Can envision continuous evolution of vortices from nuclear (hadronic) phase to quark phase provided order parameter in hadronic phase is anti-symmetric in flavor.

BCS pairs in neutron gas have 6 quarks: ddu + ddu.

$$\langle nn \rangle \to \langle ud \rangle \langle ud \rangle \langle dd \rangle$$

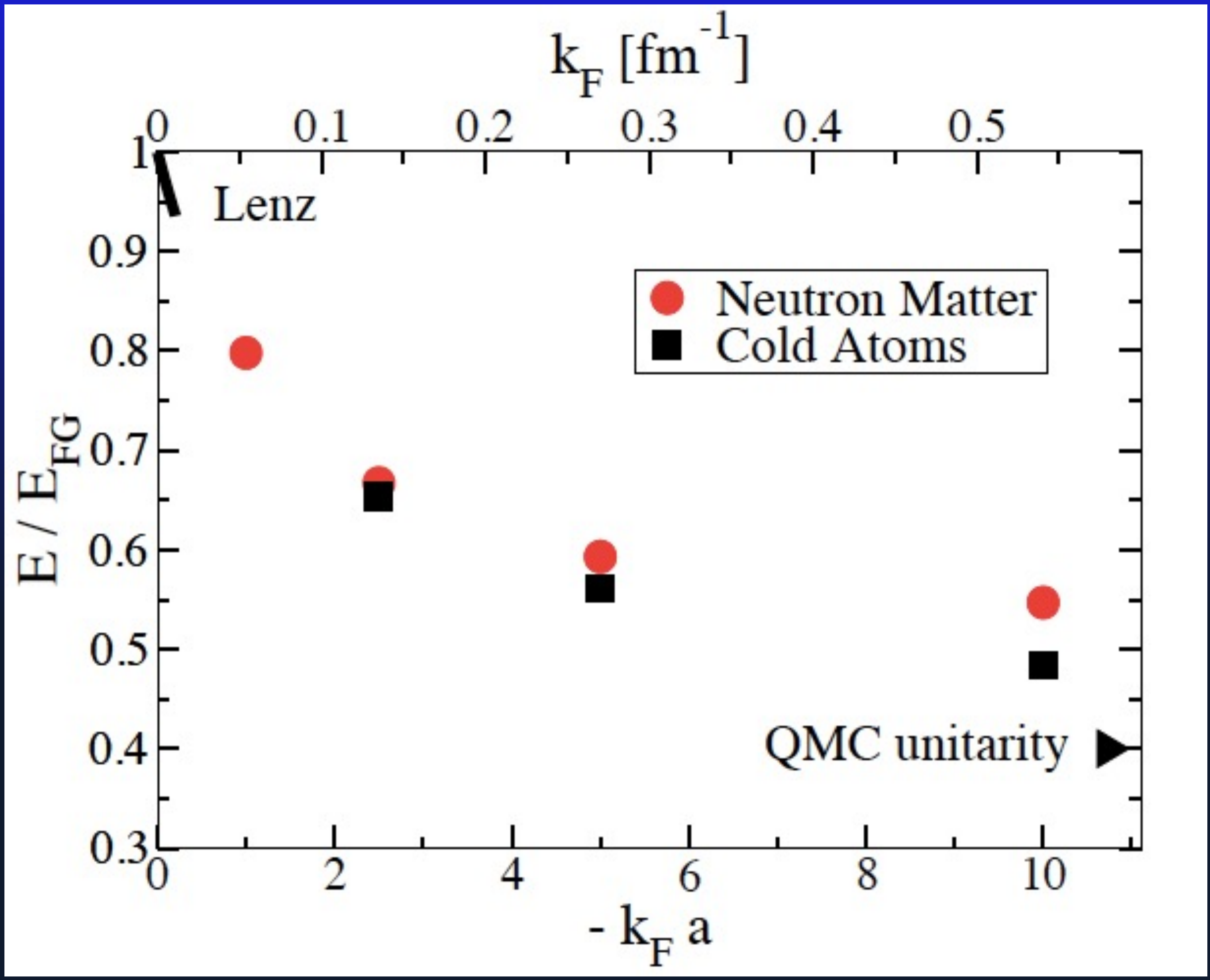
Cannot arrange into flavor anti-symmetric quark pairing.
But in $SU(3)_{flavor}$ invariant hadronic matter with equal mass n, p, Λ, Σ, and Ξ baryons can have flavor antisymmetric pairings

$$\langle -\sqrt{\frac{1}{8}}[\Lambda\Lambda] + \sqrt{\frac{3}{8}}[\Sigma\Sigma] + \sqrt{\frac{4}{8}}[N\Xi] \rangle$$

Connecting neutron matter to usual CFL quarks requires transition.
Other quark matter pairings, e.g., $^3P_2$, pairing could work.

Cold atoms and high density matter

Remarkably similar behavior of ultracold fermionic atoms and low density neutron matter (a_{nn}= -18.5 fm)



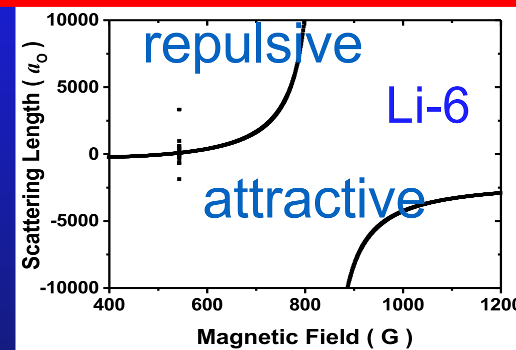
A. Gezerlis and J. Carlson, Phys. Rev. C 77, 032801(R) (2008)

Similarities of cold fermionic atomic clouds & quark matter

-- In lab both are small clouds with ~ $10^4 - 10^7$ degrees of freedom

-- Strongly interacting:

atomic clouds via Feshbach resonances

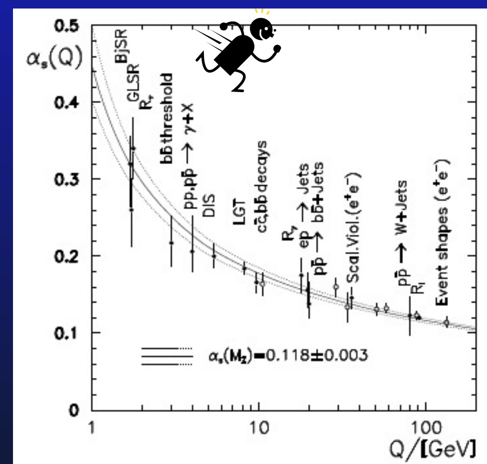


Resonance at B= 830 G

quark-gluon plasmas
always strongly interacting

$$\alpha_s(p) = \frac{g_s^2}{4\pi} = \frac{6\pi}{(33 - 2N_f)\ln(p/\Lambda)}$$

Running coupling constant



-- Scale free in strongly coupled regime

$F_{qgp} \sim \text{const}\ n_{exc}^{4/3}$ $\qquad E_{\text{cold atoms}} \sim \text{const}\ n^{2/3}/m$

In cold atoms near resonance only length-scale is density.
No microscopic parameters enter equation of state:

$$\frac{E}{N} = \frac{3}{5} E_F^0 (1 + \beta)$$

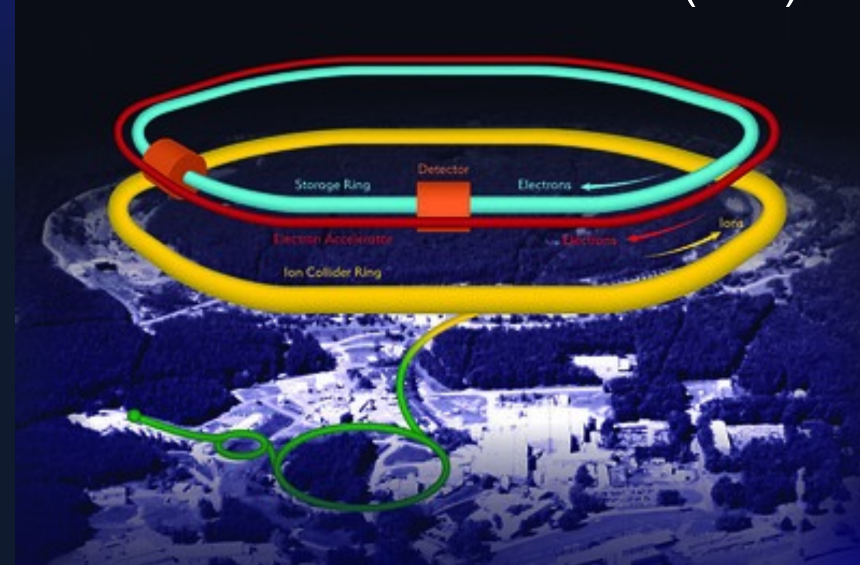
Green's Function Monte Carlo -- β = -0.60

Experiment: -0.61

Creating high energy density matter in the lab

Relativistic Heavy Ion Collider (Brookhaven) since 2000	Beams 100 GeV/A
Large Hadron Collider (CERN) since 2010	now 2760 GeV/A
HADES at GSI	~1.25 GeV/A
FAIR (GSI) ca. 2025+	to 45 GeV/A

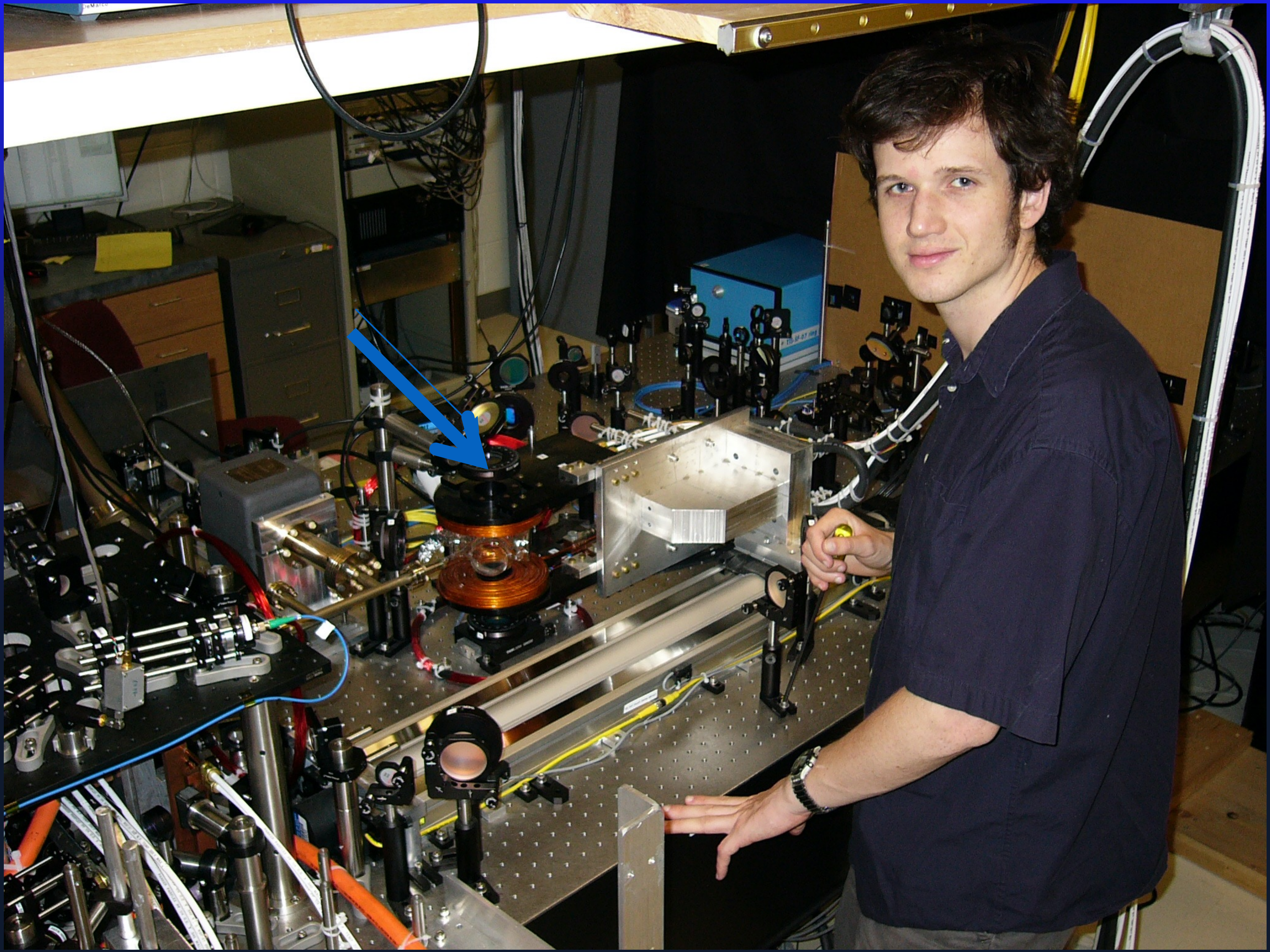
Future electron-ion collider (EIC)



Au(197×100GeV)+Au(197×100GeV)

Energy scale ~ 10^{20} times cold atom scale

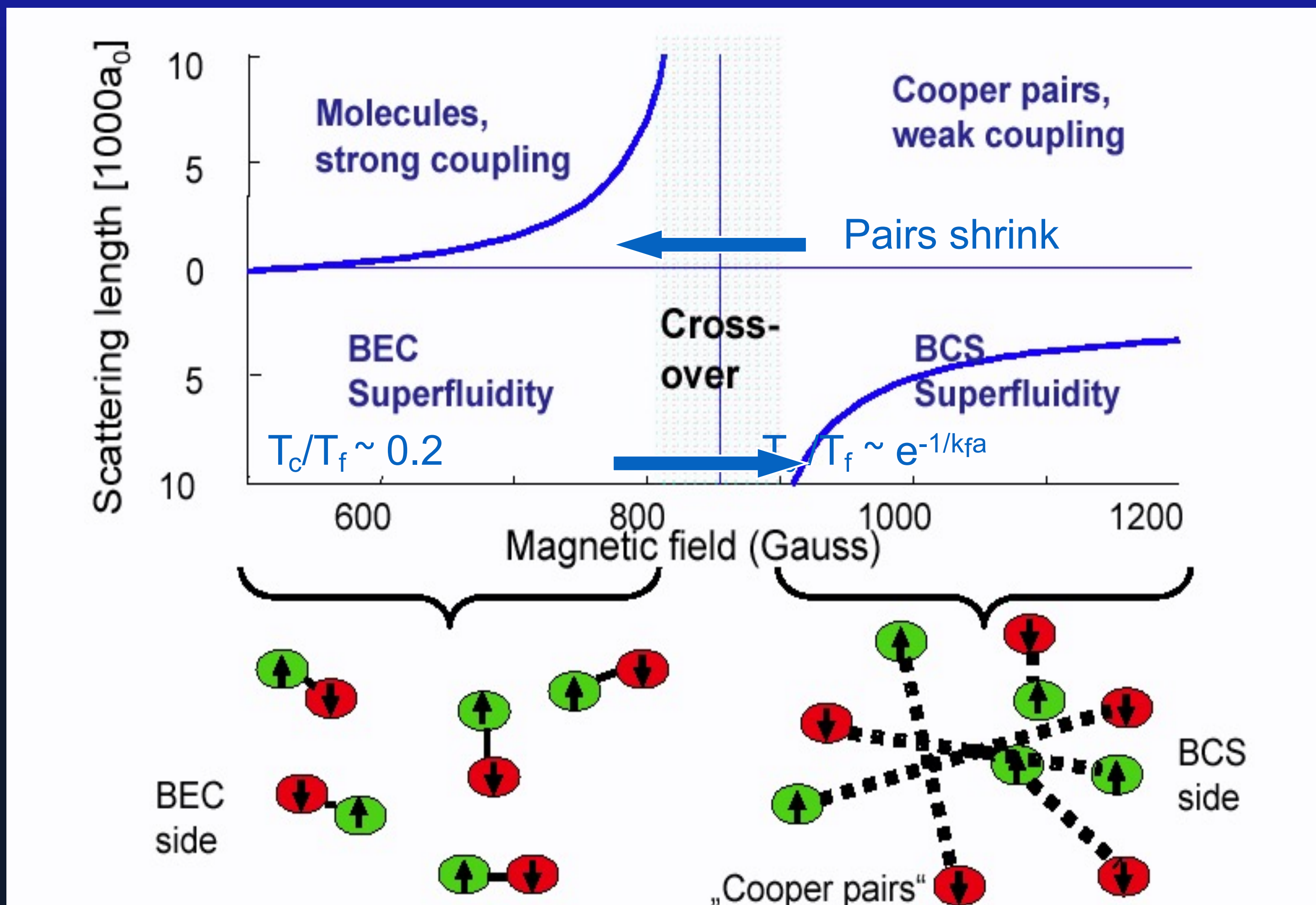
Trapped atom experiments done on table tops



Former grad student David McKay in Brian DeMarco's lab in Urbana

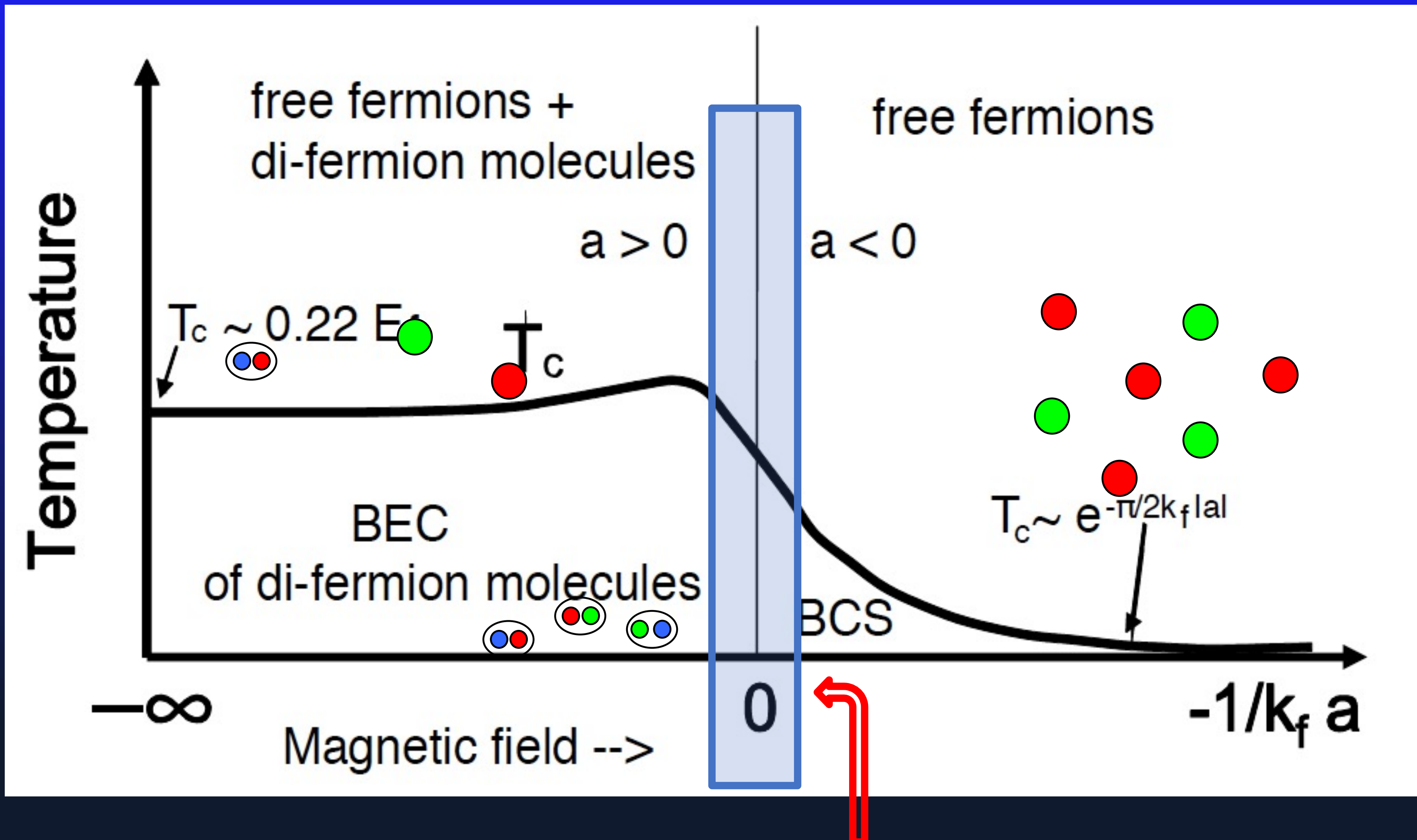
BEC-BCS crossover in Fermi systems

Continuously transform from molecules to Cooper pairs:
D.M. Eagles (1969)
A.J. Leggett, J. Phys. (Paris) C7, 19 (1980)
P. Nozières and S. Schmitt-Rink, J. Low Temp Phys. 59, 195 (1985)



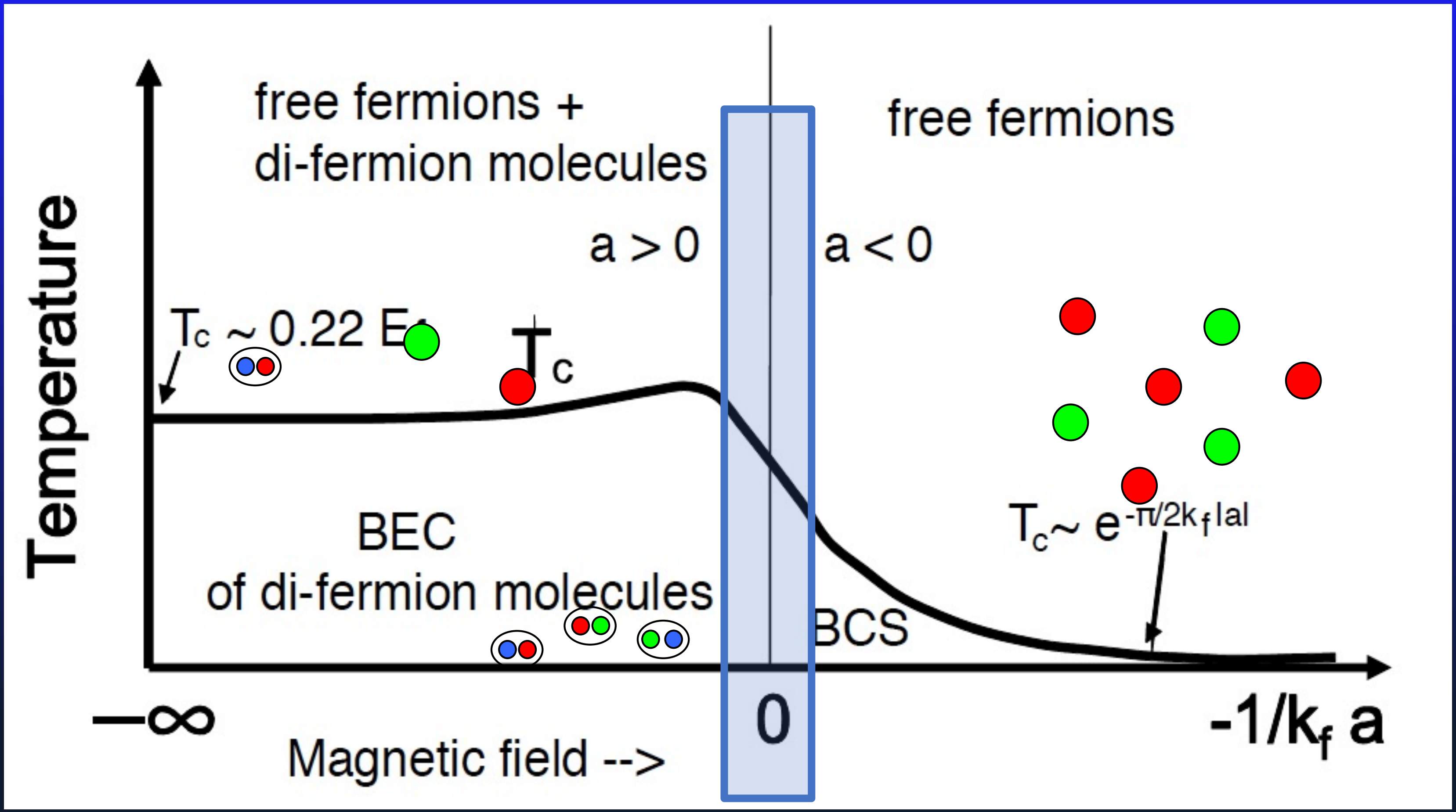
^{6}Li

Phase diagram of cold fermions vs. interaction strength



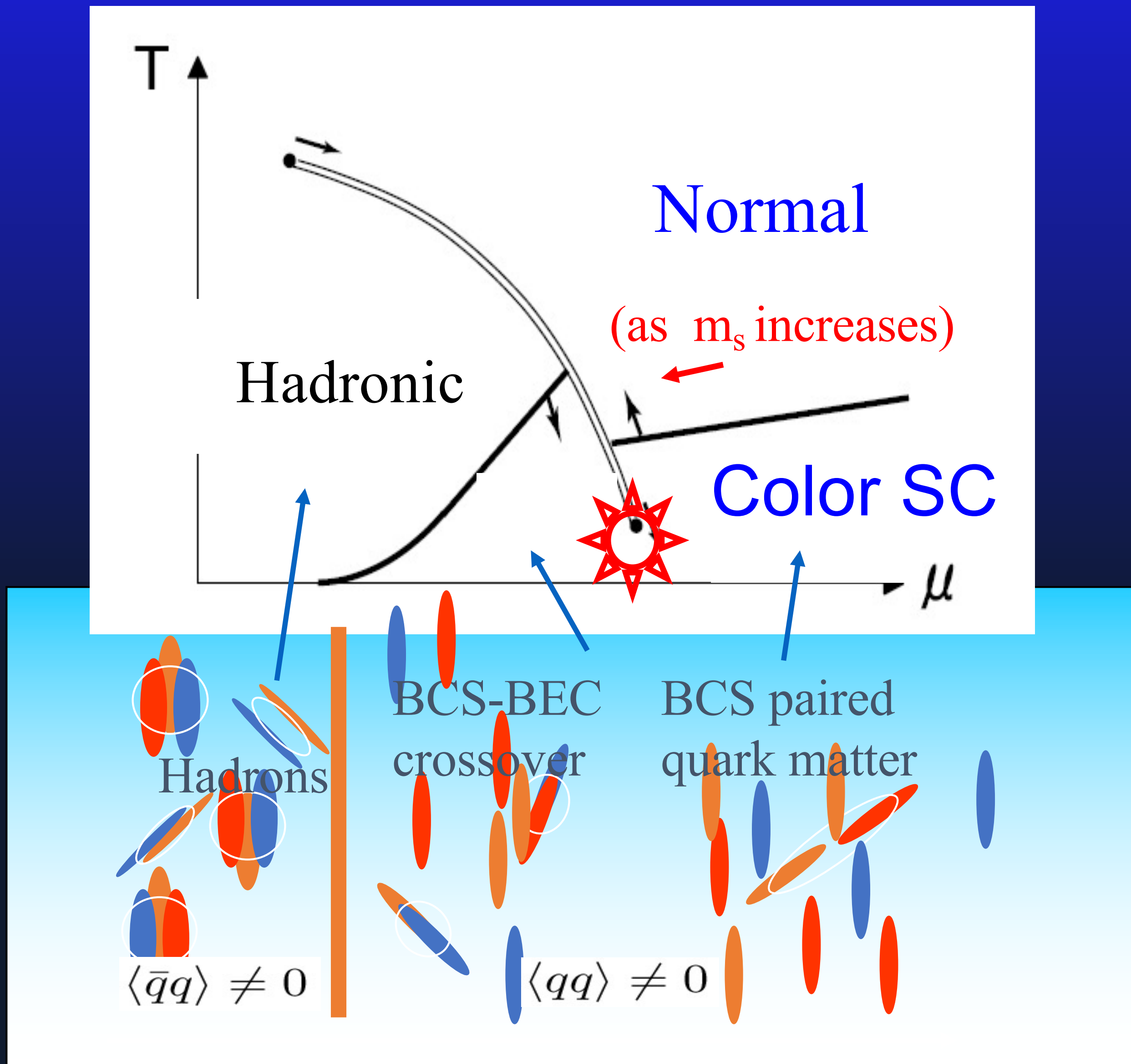
Unitary regime (Feshbach resonance) -- crossover.
No phase transition through crossover

Phase diagram of cold fermions vs. interaction strength



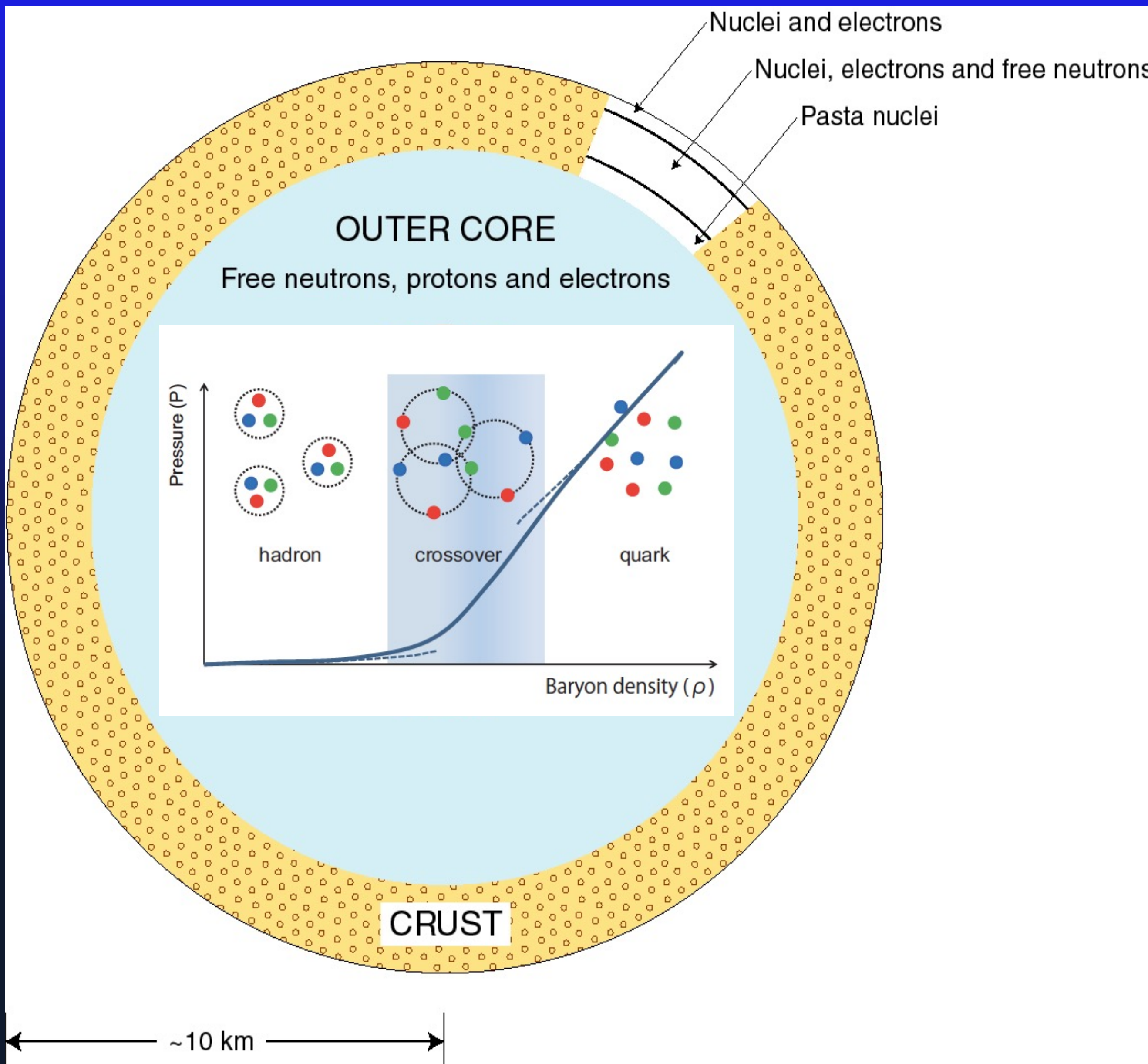
BEC-BCS crossover in quark matter phase diagram

GB, T. Hatsuda, M. Tachibana, N. Yamamoto J. Phys.G: Nucl. Part. Phys. 35 (2008)



Small quark pairs are diquarks

New view of neutron star interiors



Gradual transition in the core from hadrons (neutrons, protons) to diquarks to quark matter

T. Kojo, GB, & T. Hatsuda, Ap. J. 934:46 (2022)

Using cold atoms

Cold atom simulations of QCD

1) Analog models, e.g., three hyperfine states ⇔ quarks of 3 colors

2) Can simulate external magnetic fields. Major challenge is to induce electromagnetic-like interactions between atoms! (Dipolar atoms)

3) Cold atoms as analog computer. ex. Hubbard model.
To eventually do simulations of lattice gauge theory will need to address fields on each link of lattice, or at least more locally

Eventual goal:
Simulate SU(3) quantum chromodynamics with quarks.

Recent review: *M. Aidelsburger et al. Phil Trans (2021)*
https://doi.org/10.1098/rsta.2021.0064

"Confinement" of three atomic fermions on lattice – formation of "nucleons"

A. Rapp, G. Zarand, C. Honerkamp, & W. Hofstetter , PRL 98 (2007), PRB 77 (2008)

Hubbard model with 3 internal degrees of freedom

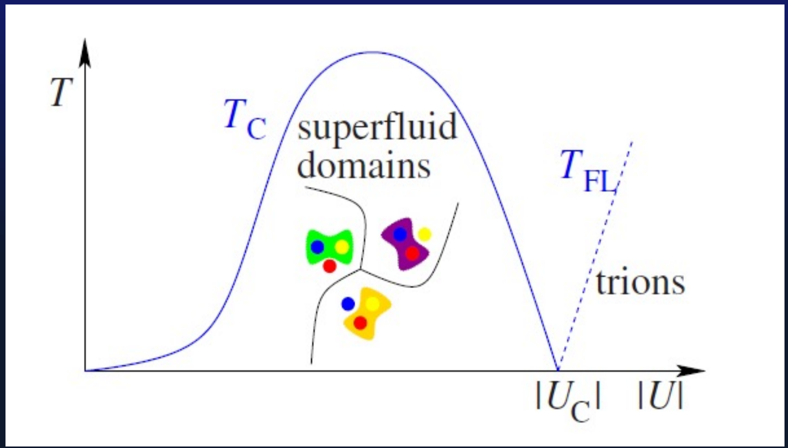
Red Green Blue

$$\hat{H} = -t \sum_{\langle i,j \rangle, \alpha} \hat{c}^{+}_{i\alpha} \hat{c}_{j\alpha} + \sum_{\alpha \neq \beta} \sum_{i} \frac{U_{\alpha\beta}}{2} (\hat{n}_{i\alpha} \hat{n}_{i\beta}),$$

3 lowest hyperfine states of ^{6}Li

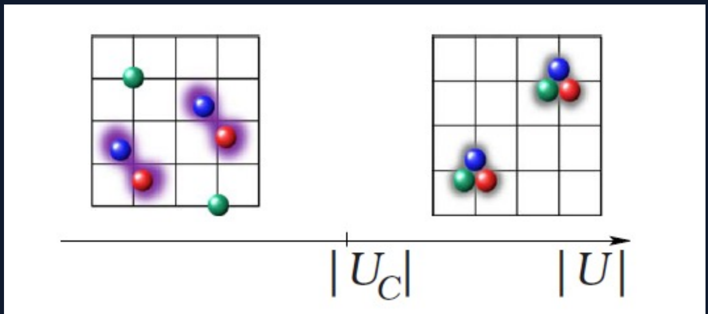
Small |U|: (U < 0)

Superfluid of two species
("color superfluid")



Large |U|:

Three particles bind together
(formation of trions = "baryons")

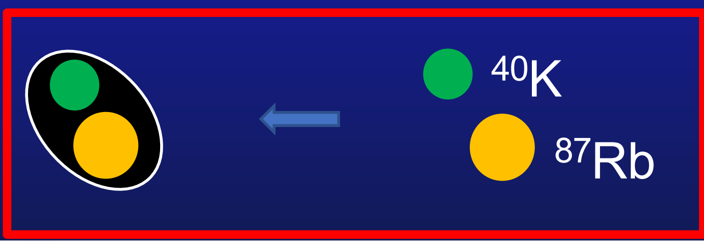


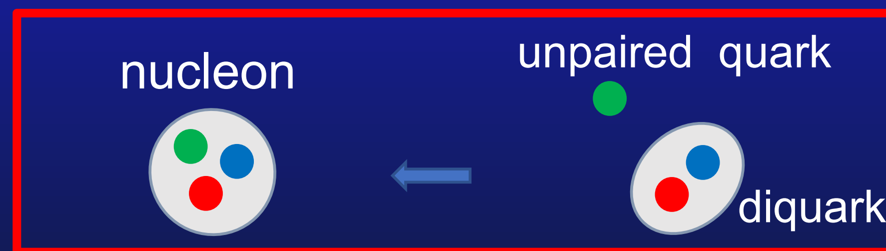
Boson-fermion mixtures of ultracold atoms -- and dense QCD -- forming superfluid of composite fermions

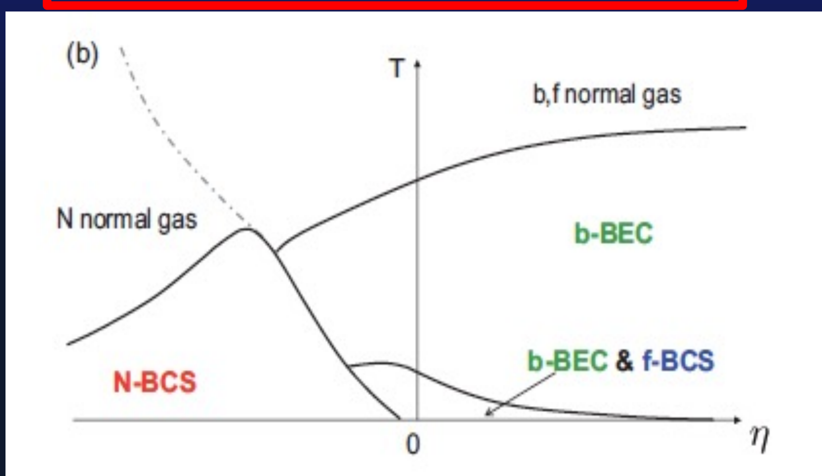
Bosons (^{87}Rb) ⇔ diquarks, fermions (^{40}K) ⇔ unpaired quarks

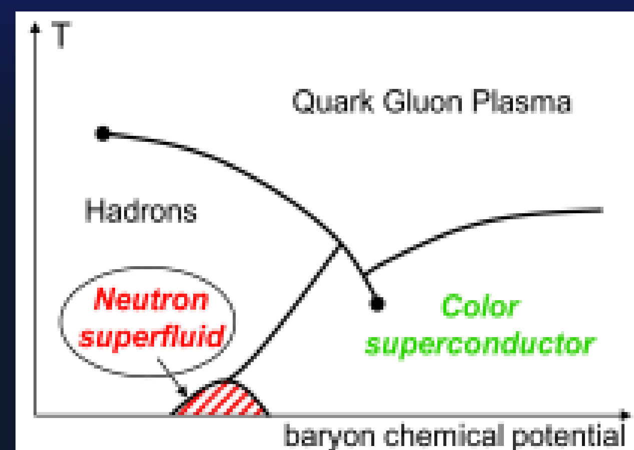
Formation of b-f molecules ⇔ transition to nucleons

K. Maeda, G.B, T. Hatsuda, PRL 103, 085301 (2009)









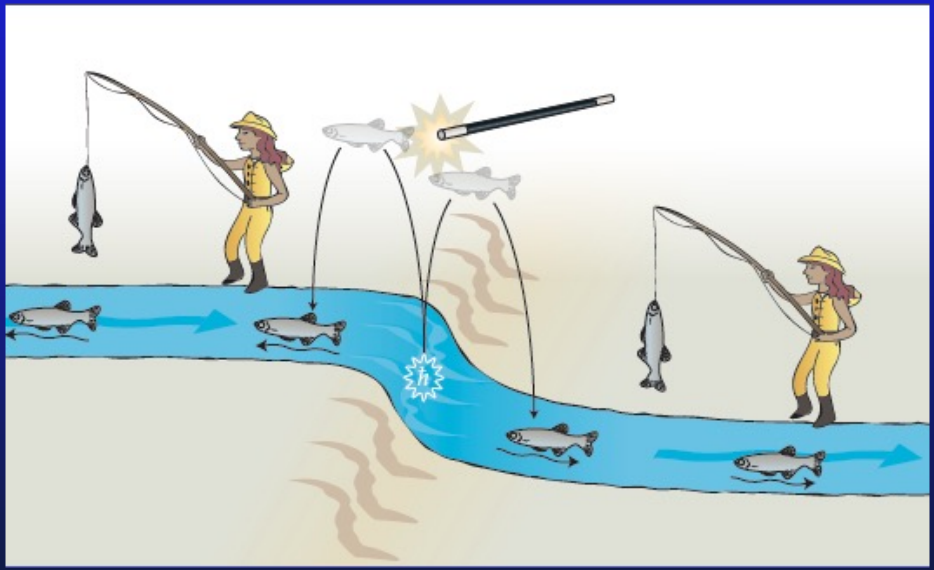
$$\eta \equiv -1/n^{1/3}a_{\mathrm{bf}}$$

n = b & f densities, a_{bf} = b-f scattering length

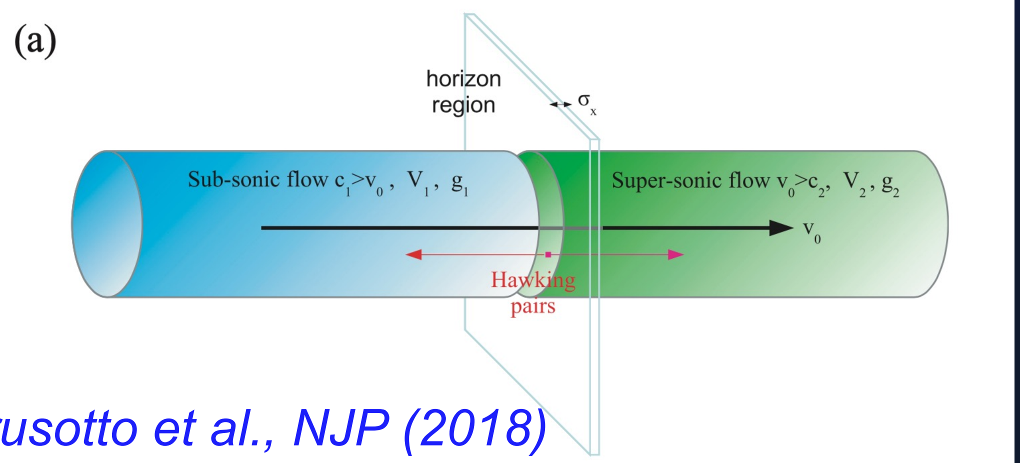
Acoustic "black holes" with cold atoms:
trapped BEC analogs of Hawking radiation

Pisa and Trento

Outside horizon

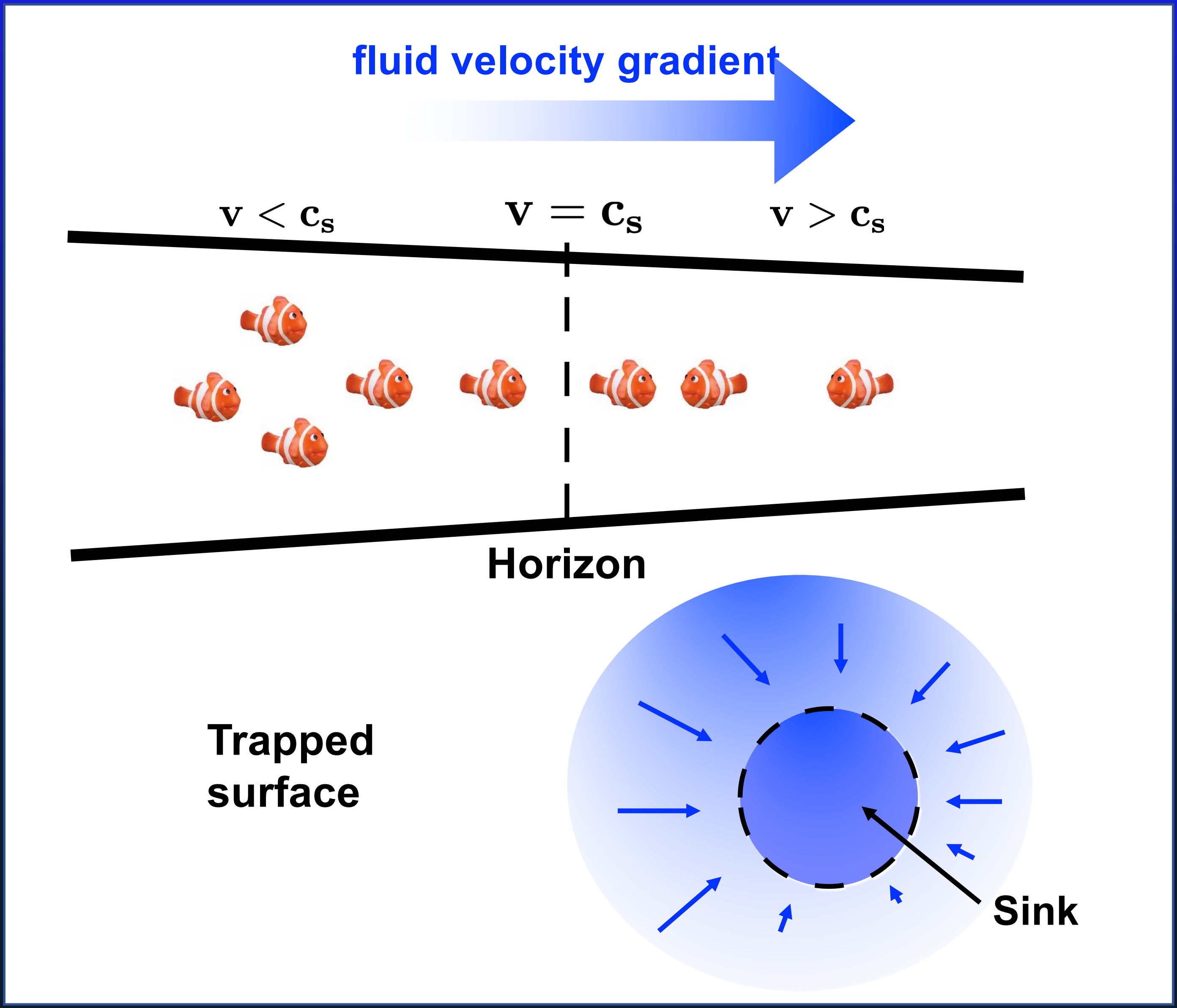


Inside horizon



Carusotto et al., NJP (2018)

Larger density "outside horizon" has larger sound velocity.
Subsonic particles outside become supersonic inside

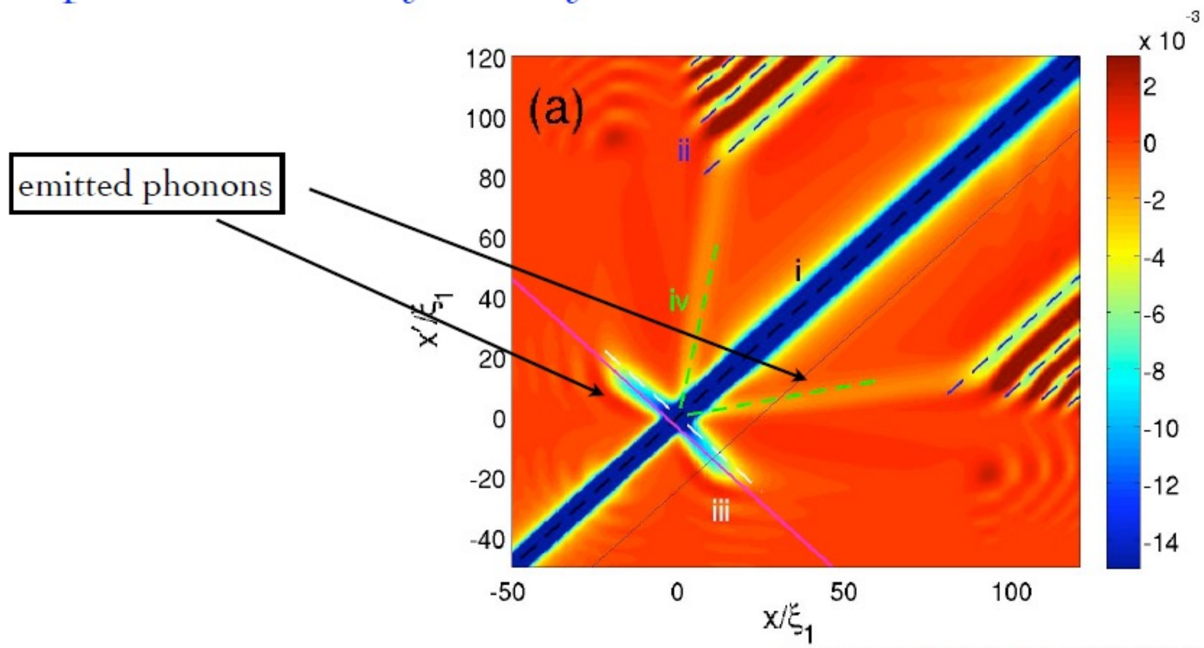


Phonon emission at “horizon” like Hawking radiation.
Look at correlations between phonons inside and outside horizon

Detection strategy

The phonon emission perturbs the system producing long-range density correlations

Parametric plot of the density-density correlation function

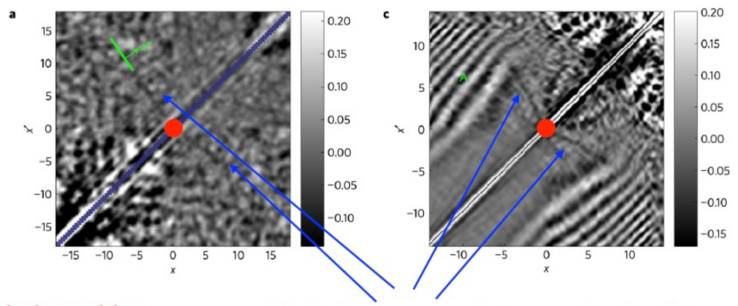


Carusotto *et al New J. Phys.* 10 103001 (2008)

Expts: *J. Steinhauer et al. (2016)*

Experimental observation

experiment numeric



black hole position

effect of phonon emission on the density

Image obtained by 4600 repetitions of the experiment

Fitted Hawking temperature $\sim 10^{-9} K$

Steinhauer, Nature Phys. 12 (2016) 959

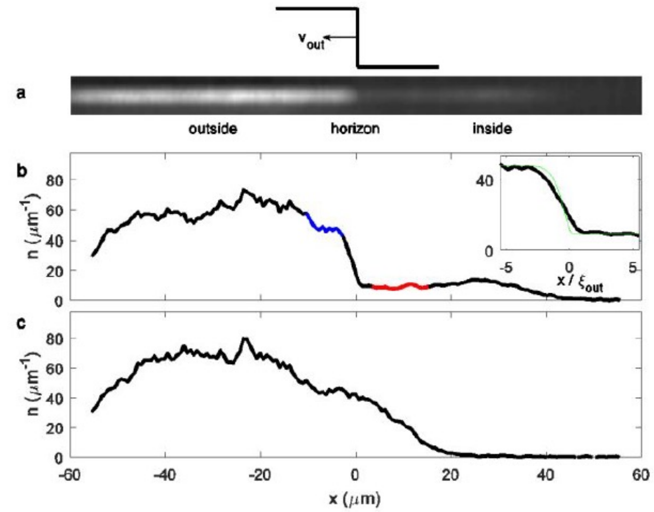


Figure 3.7: Analogue horizon. In this figure we show the experimental data for the acoustic hole horizon realized in the laboratory by Steinhauer. In figure (a) the 1D condensate, trapping the phonons in the right region: the averageover the ensamble is shown. In figure (b), blue and red regions correspond to outside and inside the black hole. Here $\xi_{out} = \xi_u$; the green line is a half a grey soliton [?].

density-density correlation function $\qquad G^{(2)}(x, x') \propto \dfrac{\langle n(x)n(x')\rangle}{\sqrt{n_x n_{x'}}}$

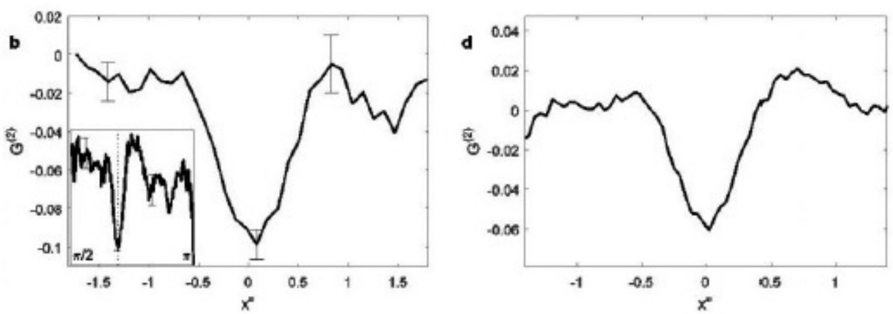


Figure 3.9: Hawking radiation. In (a) two-body correlation function: the horizon is at the origin. The dark bands emanating from the horizon are the correlations between the Hawking and partner particles. In (b) the profile of the Hawking-partner correlations: the Fourier transform of this curve measures the entanglement of the Hawking pairs via Δ. The error bars indicate the standard error of the mean. In (c,d) the corresponding numerical simulation of two-point correlation and profile: in (c) the fringes marked with A are artifact of the creation of fluctuations. From [55].

Wormholes?? *G. Pennington et al., N. Engelhardt et al. (ongoing)*

Thank you