Università di Pisa

Probing dynamical signatures in continuously monitored systems

Beyond measurement-induced phase transitions

Jorge Yago Malo, QFC2022, University of Pisa, 26th October 2022

University of Strathclyde Glasgow

Motivation

Usefulness of open quantum systems

- Open quantum systems are essential for our understanding of:

1. Condensed matter and Quantum Optics.
2. Current quantum technologies.
3. High energy physics, fundamental interactions, cosmology: via q. metrology or simulated models.
4. Biological systems.

Motivation

Usefulness of open quantum systems

- Open quantum systems are essential for our understanding of:

1. Condensed matter and Quantum Optics.
2. Current quantum technologies.
3. High energy physics, fundamental interactions, cosmology: via q. metrology or simulated models.
4. Biological systems.

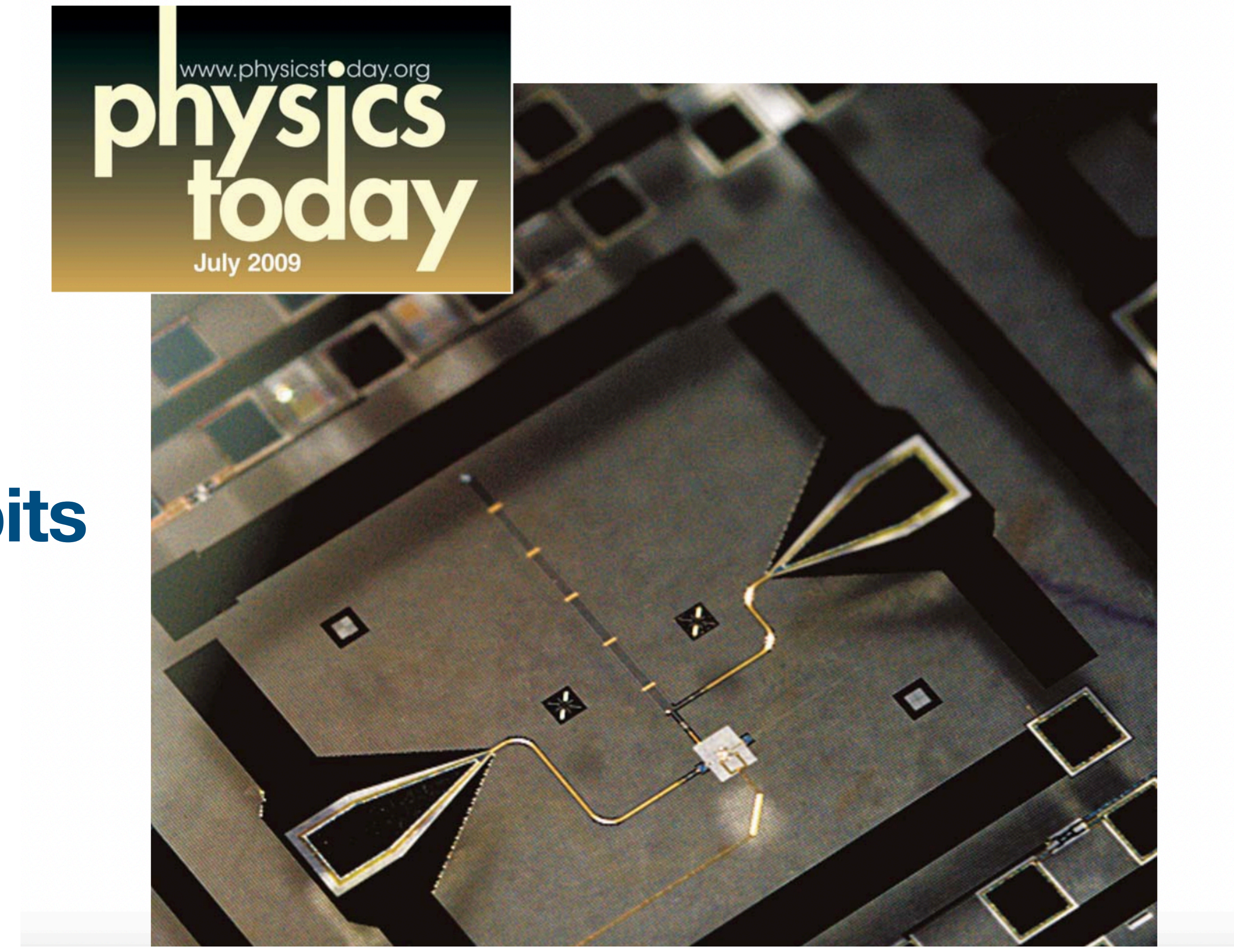
- Why?

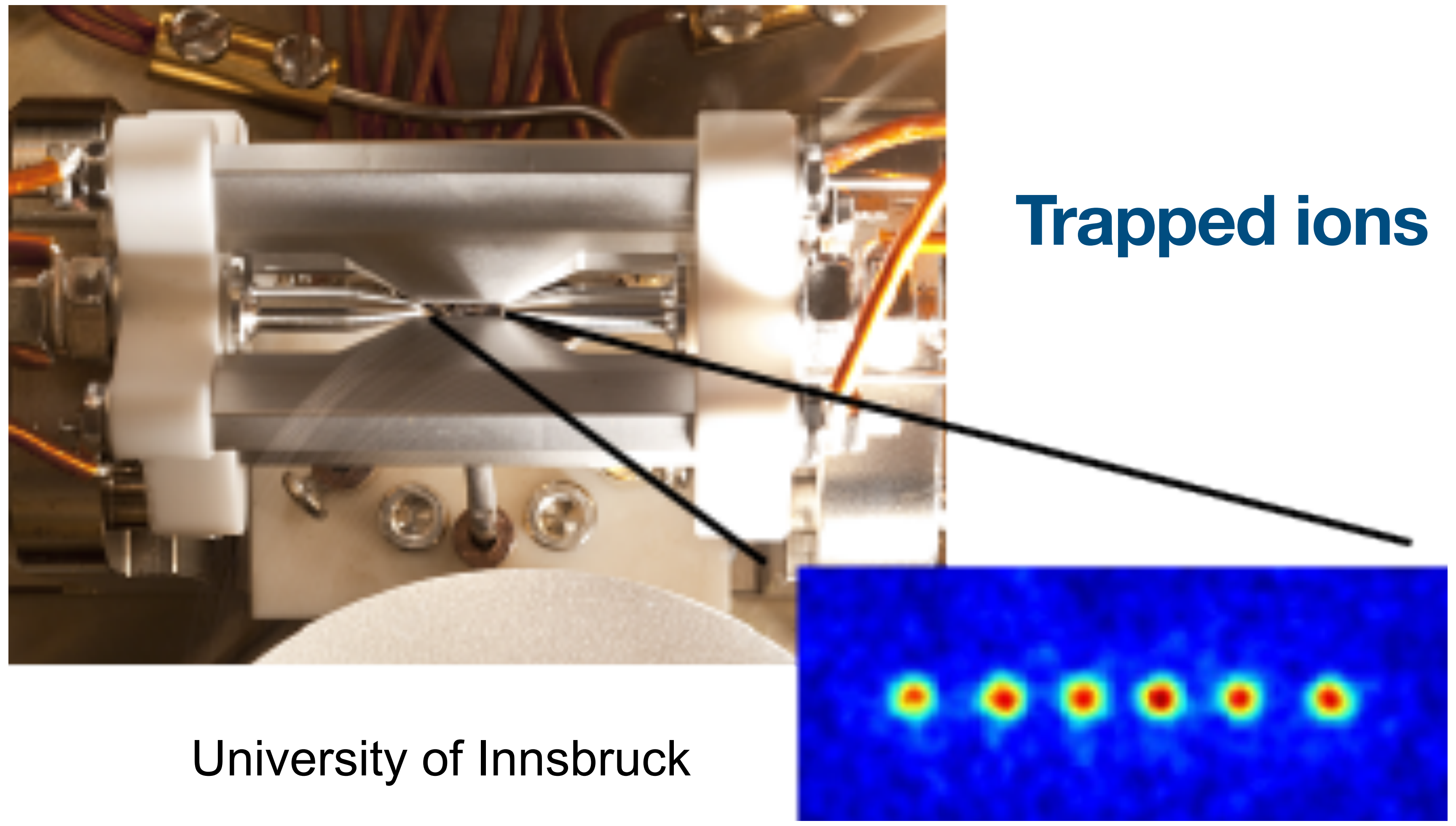
A. Every system is inherently open.
B. Need to measure and probe a system.
C. Quantum/classical crossover.

QT platforms

Different sources of dissipation but a common *strategy*

Superconducting qubits

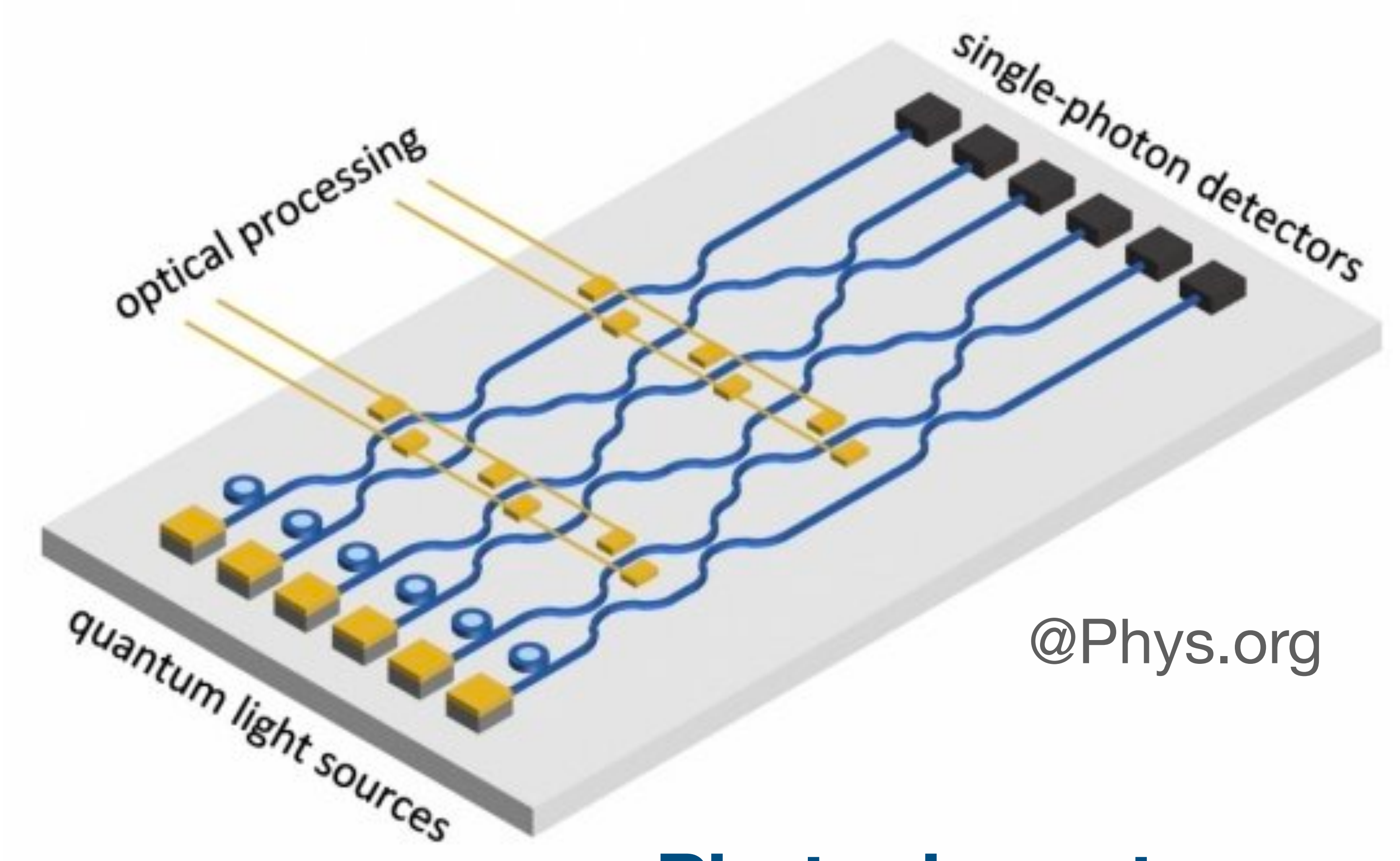




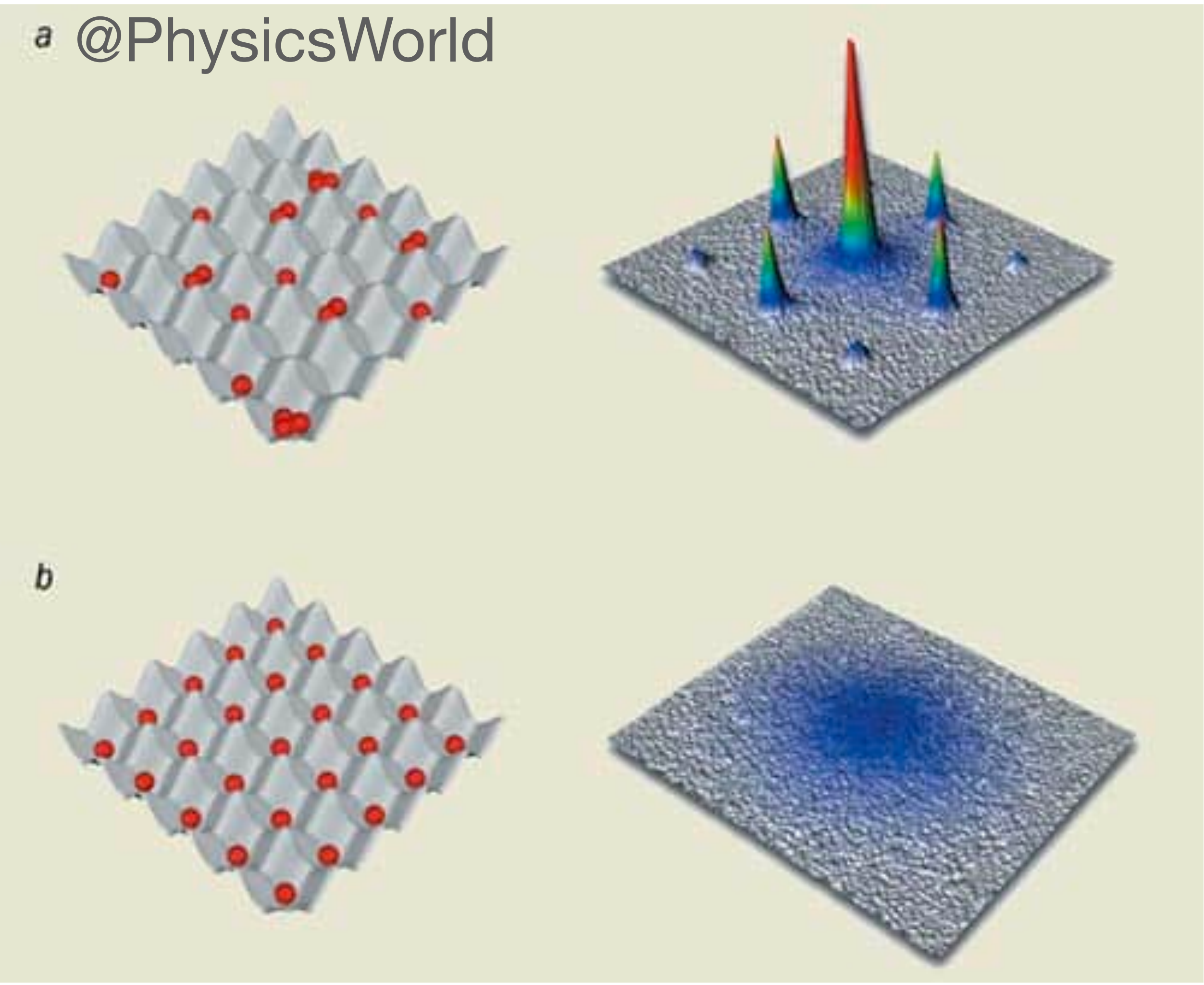
Trapped ions

University of Innsbruck

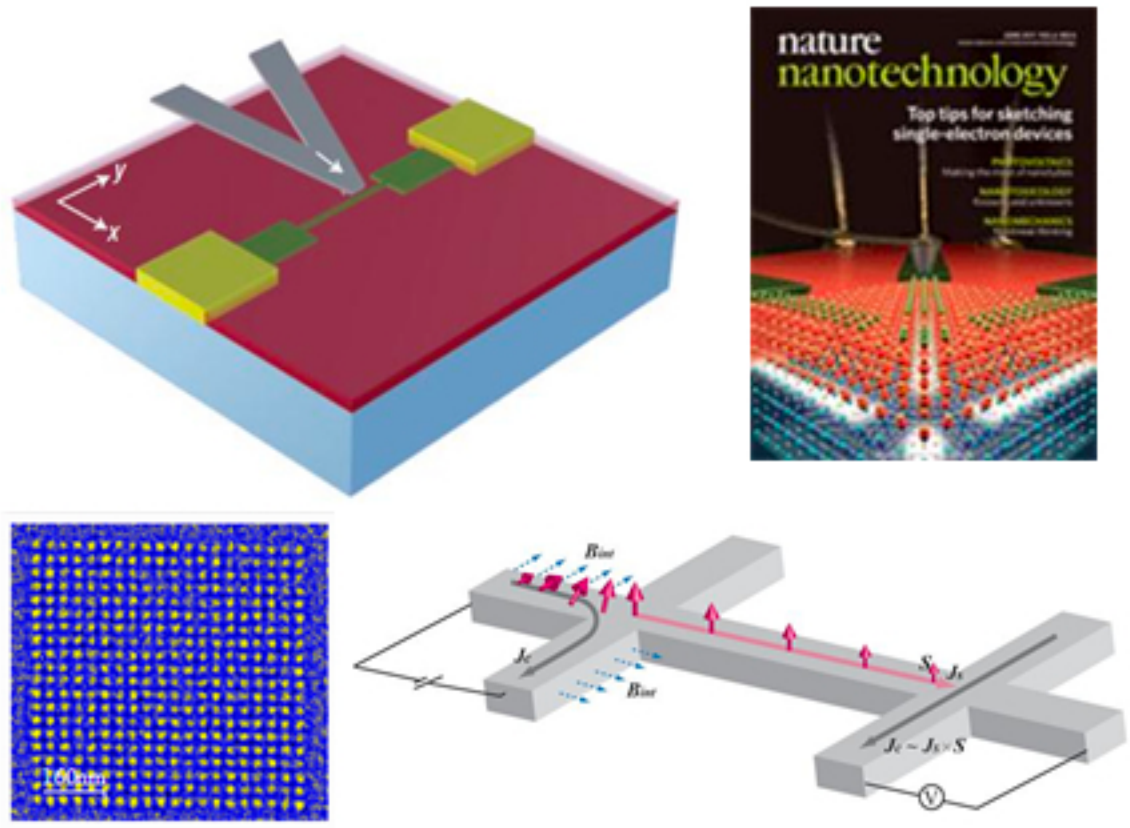
Solid state nano devices



Photonic systems



Neutral atoms in optical lattices



Levy's group, Pittsburgh

+ N-V centers, cold molecules...

Open quantum systems in QT

Not just improving our description

Current quantum technologies:

- Problem: Even with the degree of technological advances in QT we cannot prevent these systems from coupling dissipatively to their environment and that can be the limiting factor in their operational time.

- Opportunities: Controlled dissipation has emerged as a new tool for engineering quantum matter into new exciting phases.

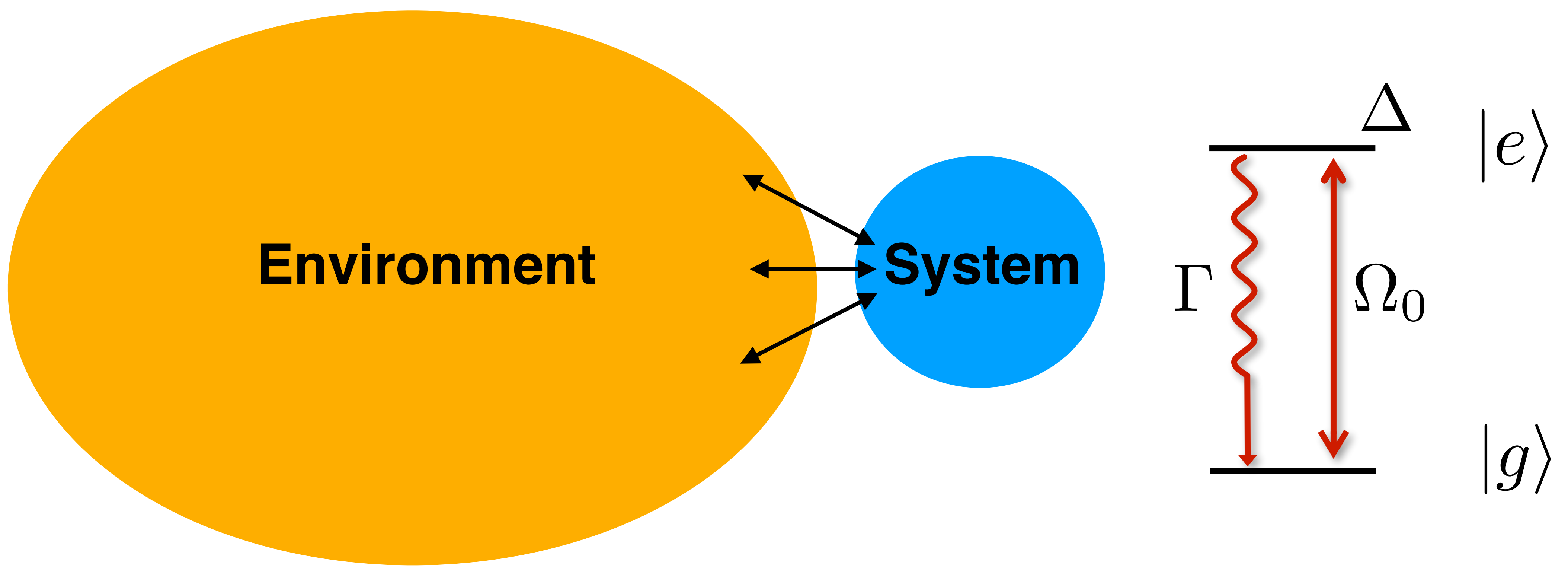
  - Quantum optics description: well-known microscopic models and controlled approximations, master equations…

  - Quantum Optics tools: laser cooling, optical pumping…

  - NMR & Quantum Chemistry tools: dissipative quantum control…

  - …

Recap on GSKL ME approximations



$$\dot{\rho} = -i[H,\rho] - \frac{1}{2}\sum_m \left[c_m^\dagger c_m \rho + \rho c_m^\dagger c_m - 2 c_m \rho c_m^\dagger\right]$$

- ***Born:*** The coupling is weak compared with system and environment energy scales.
- ***Secular:*** energy non-conserving terms in the interacting Hamiltonian can be neglected.
- ***Markov:*** the part of the environment that couples to our system rapidly returns to equilibrium. The environment is unchanged in time. This also implies that no information returns to the system from the environment, so the state of the system is independent from its history.

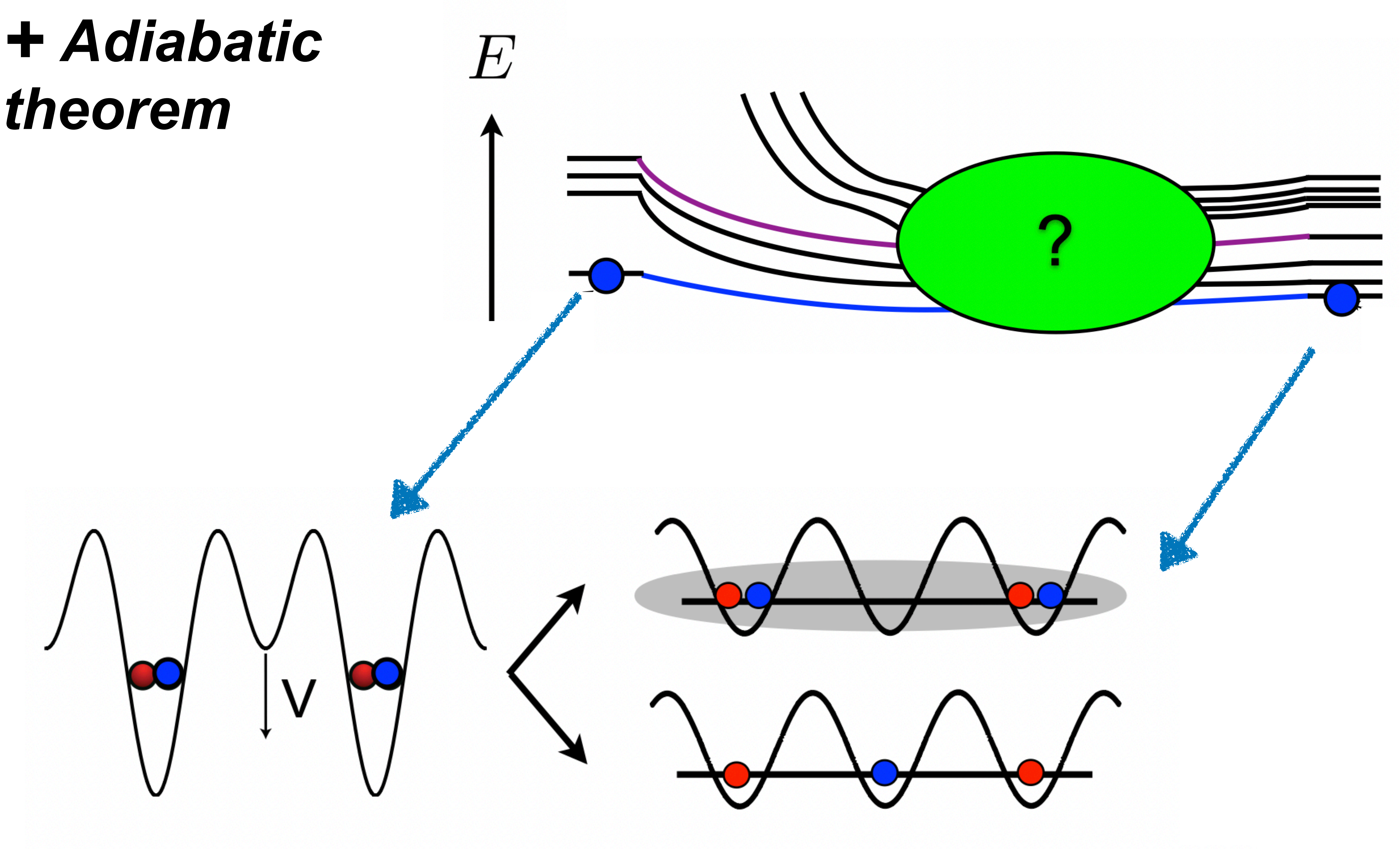
Hamiltonian engineering

- Typical preparation schemes in quantum simulation are based on cooling the system into their ground state.

$$\hat{H}|\psi_0\rangle = E_0|\psi_0\rangle$$

- By tuning the terms of our Hamiltonian we can prepare relevant initial states for QT.

+ Adiabatic theorem



P. Rabl et al, PRL **91**, 110403 (2003)
A. Kantian et al., PRL **104**, 240406 (2010)
M. Lubasch et al., PRL **107**, 165301 (2011)

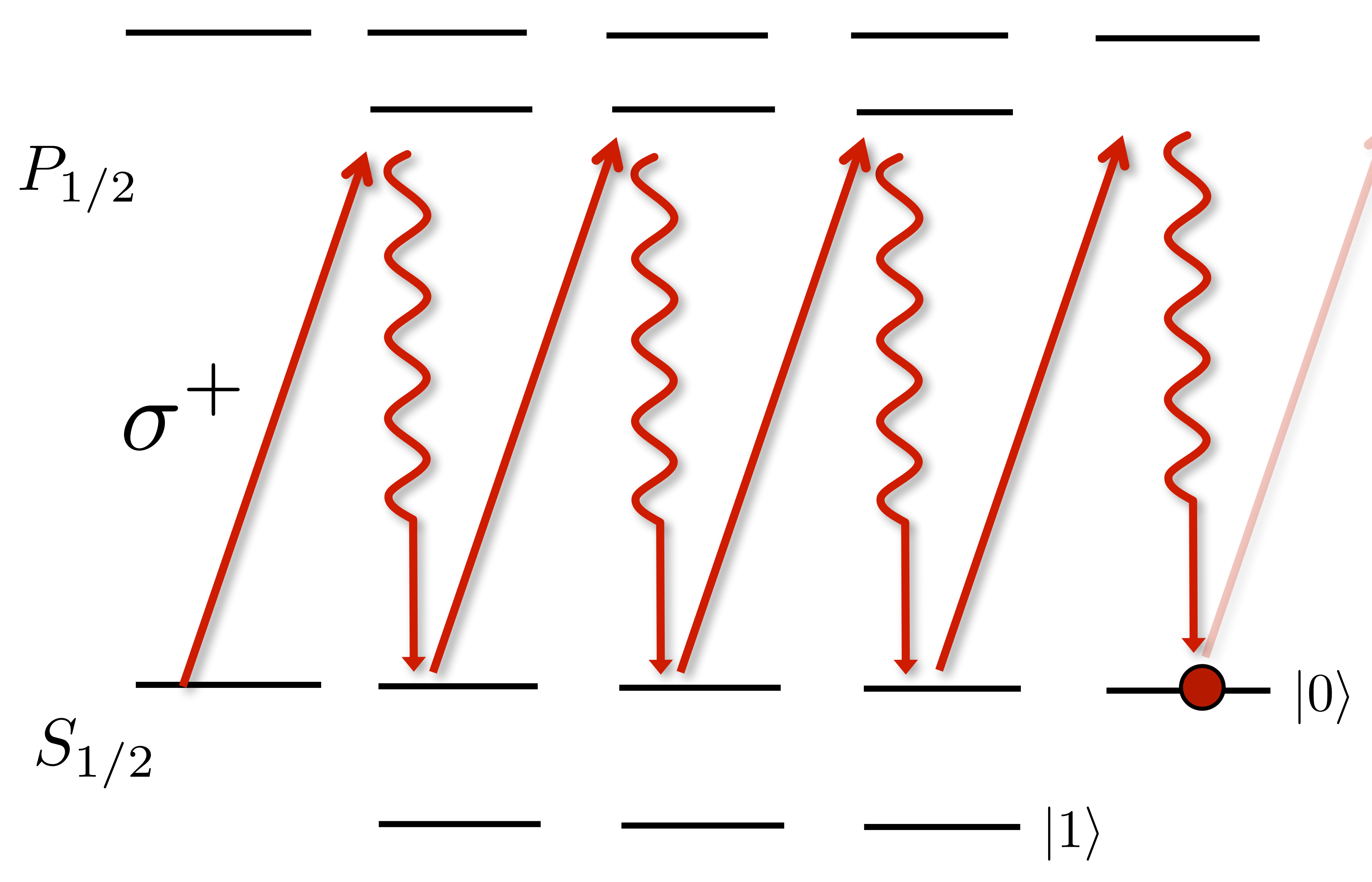
Reservoir engineering

$$\rho \rightarrow \rho_{ss} = |\psi_{ss}\rangle\langle\psi_{ss}|$$

A **pure steady state** should be a unique eigenstate of *H* and an eigenstate of all *c* operators with zero eigenvalue, i.e. a **dark state**

$$H|\psi_{ss}\rangle = \alpha_{ss}|\psi_{ss}\rangle \quad \forall c_m, \quad c_m|\psi_{ss}\rangle = 0$$

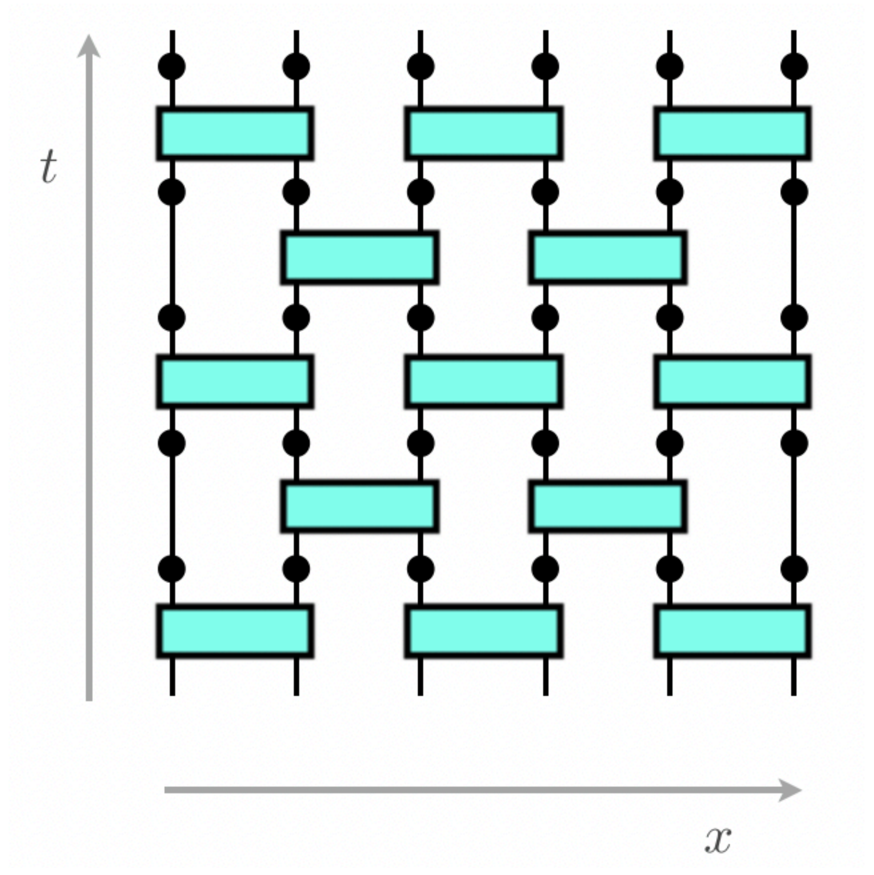
E.g., optical pumping (driving to other hyperfine levels by photon absorption)

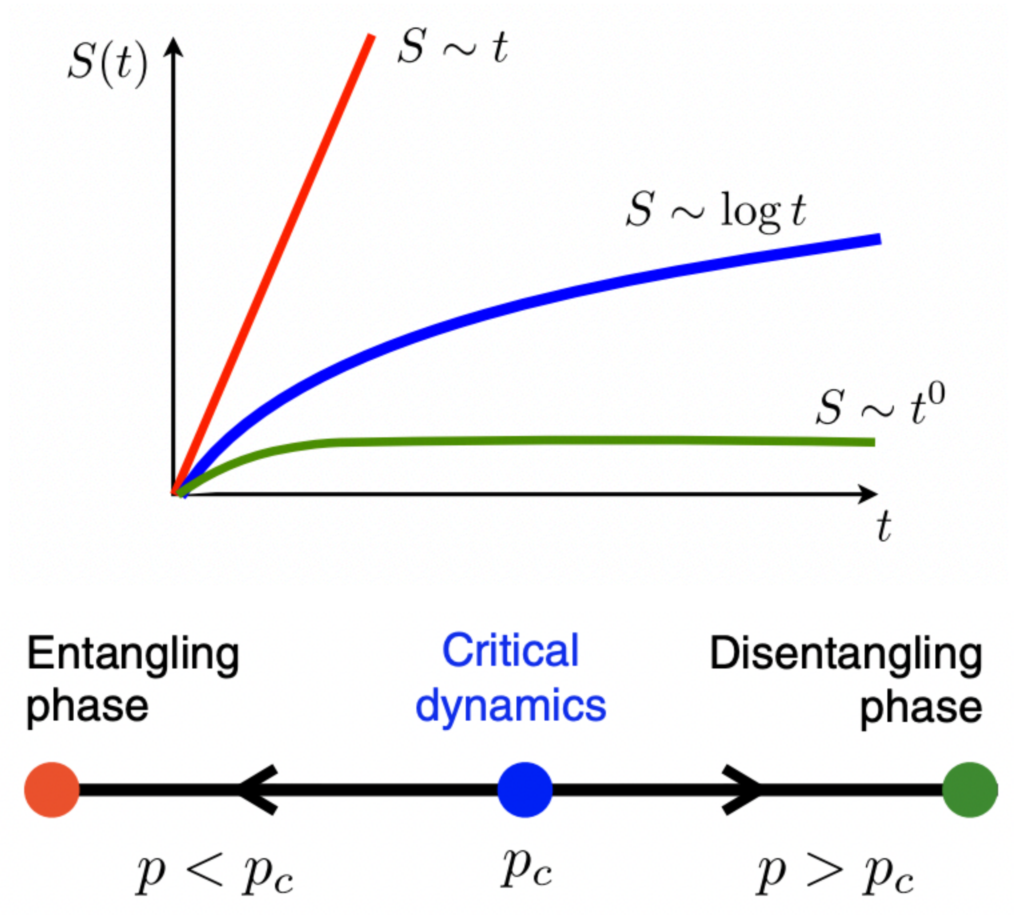


Dissipative phase transitions

Dynamical phase transitions in monitored systems

The interplay between coherent and dissipative dynamics can generate relevant phases of matter.





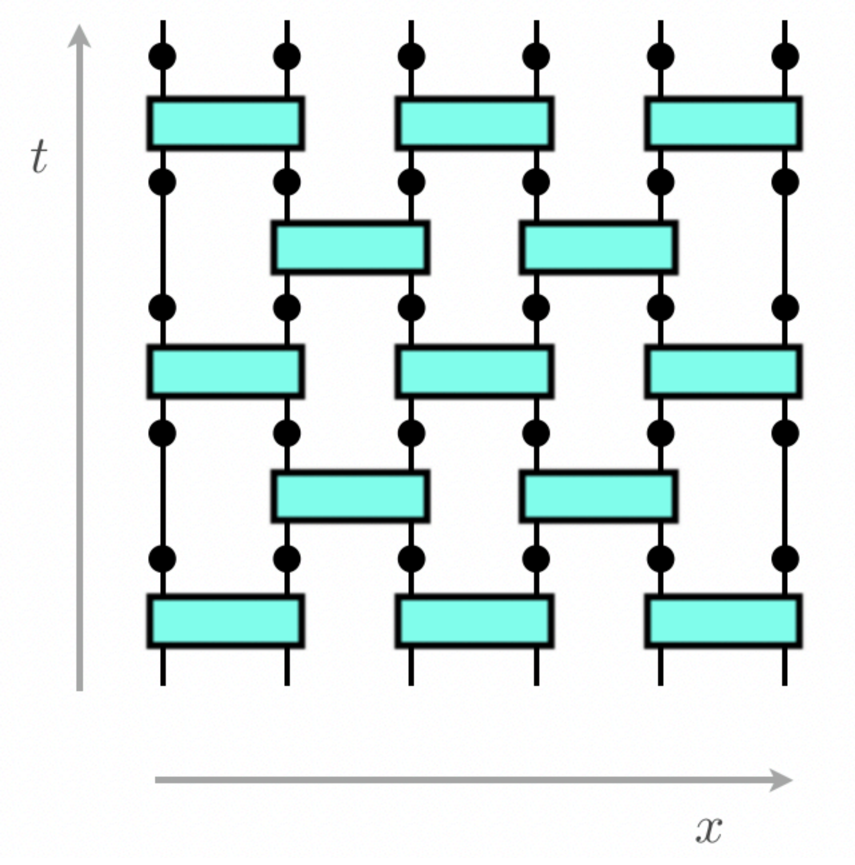
First observed in random circuits, when combining entangling and measuring gates.

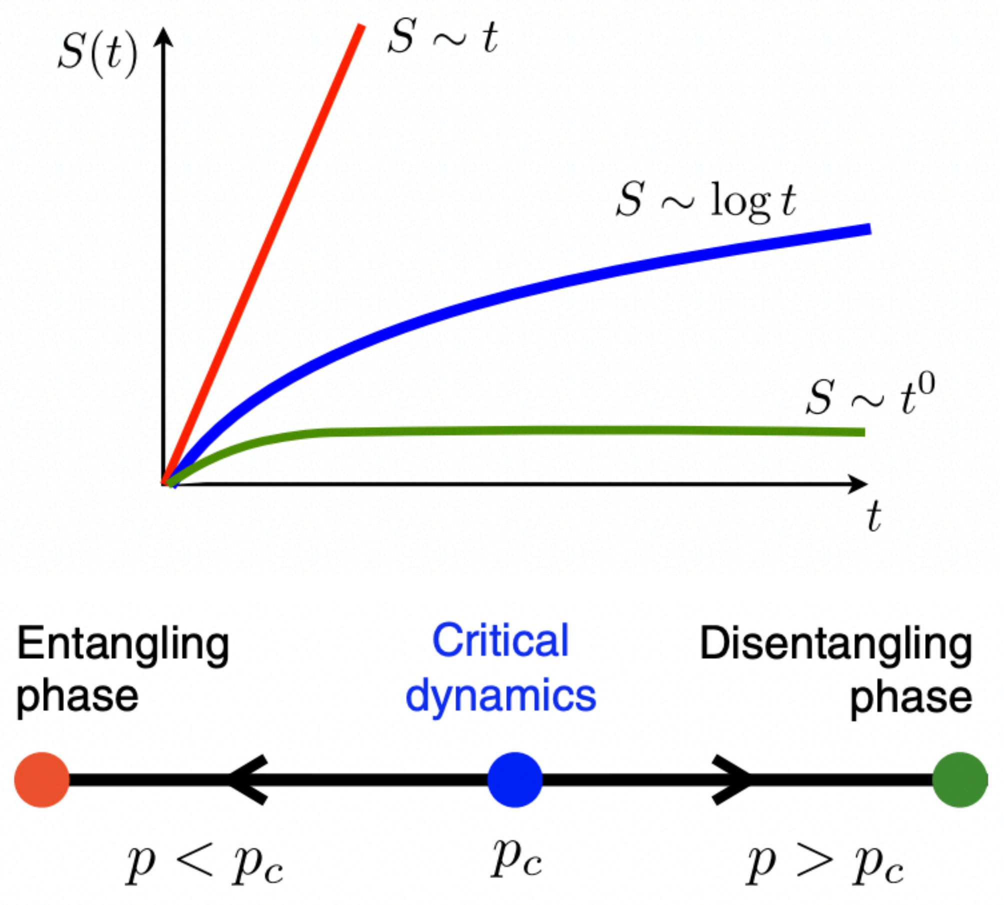
Even though increasing the number of measurements does not change the steady state, it gives rise to different entanglement properties in the individual trajectories.

Y. Li, X. Chen, and M. P. A. Fisher, Phys. Rev.B 98, 205136 (2018).
B. Skinner, J. Ruhman, and A. Nahum, Phys. Rev. X, 9, 031009 (2019).
Y. Li, X. Chen, and M. P. A. Fisher, Phys. Rev. B, 100, 134306 (2019)

Dynamical phase transitions in monitored systems

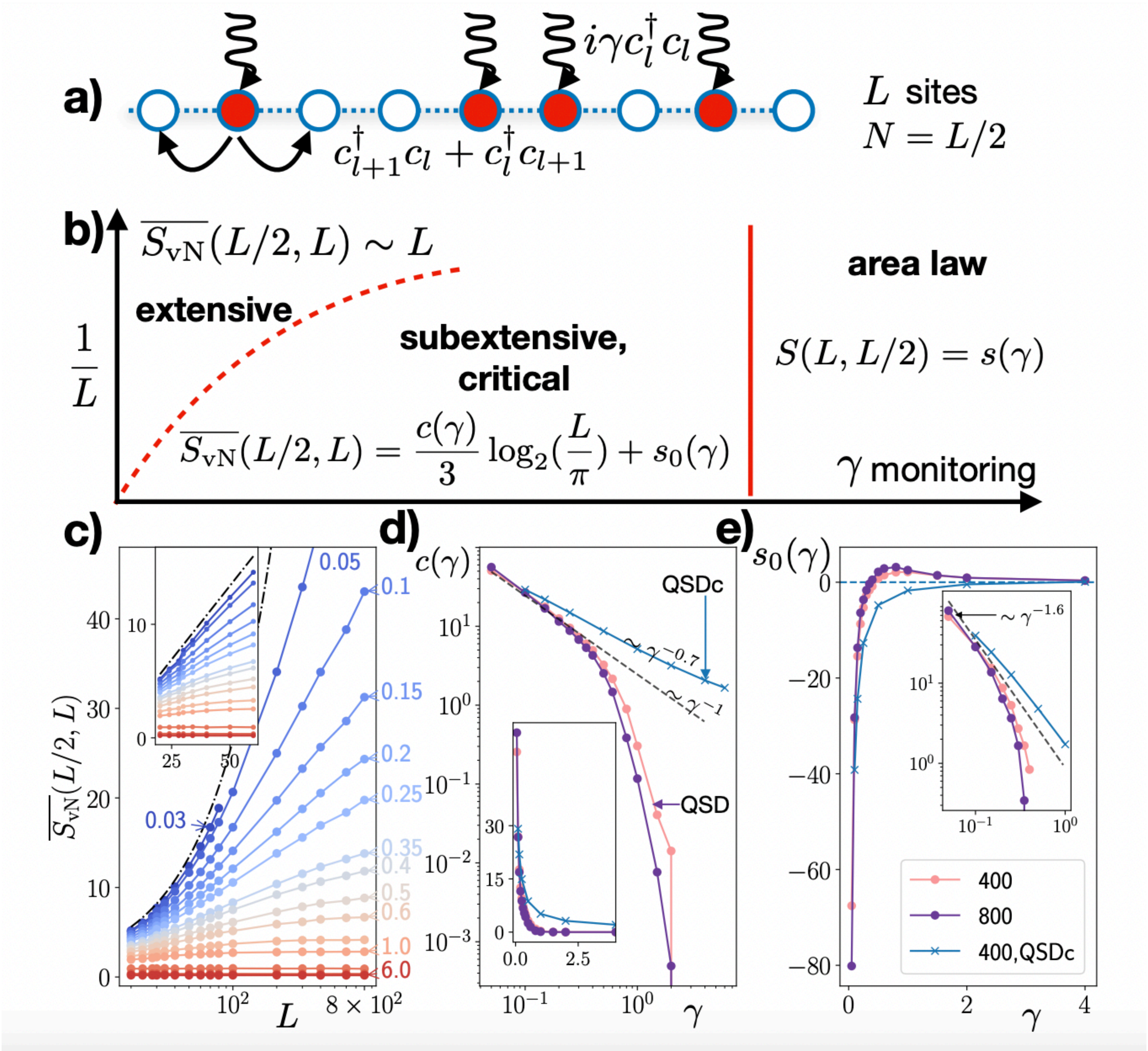
The interplay between coherent and dissipative dynamics can generate relevant phases of matter.





First observed in random circuits, when combining entangling and measuring gates.

Even though increasing the number of measurements does not change the steady state, it gives rise to different entanglement properties in the individual trajectories.



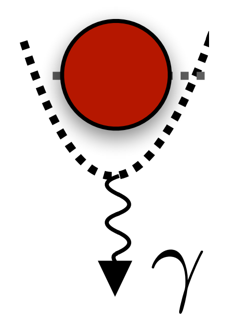
More recently, these have also been predicted in continuous systems in both fermions and bosons.

Y. Li, X. Chen, and M. P. A. Fisher, Phys. Rev.B 98, 205136 (2018).
B. Skinner, J. Ruhman, and A. Nahum, Phys. Rev. X, 9, 031009 (2019).
Y. Li, X. Chen, and M. P. A. Fisher, Phys. Rev. B, 100, 134306 (2019).

Alberton, et al, Phys. Rev. Lett. 126, 170602 (2021).

Why are these dissipative PT interesting?

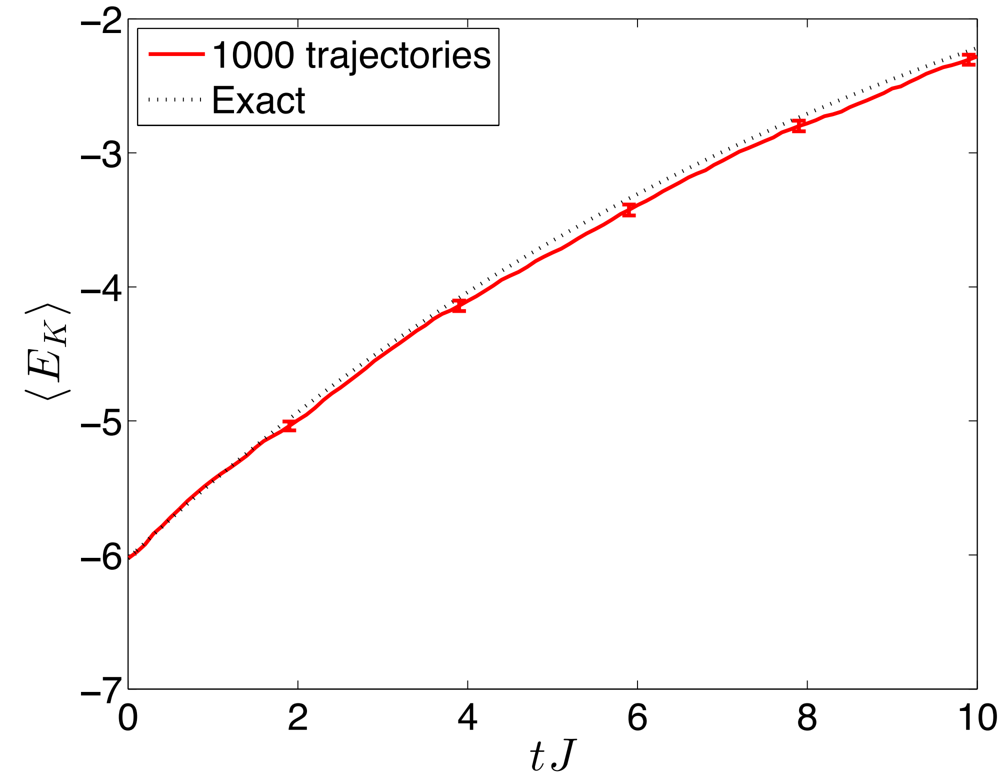
- The continuous measurement of the system induces heating that leads to a trivial steady state in the average over realizations:



$c_i \propto n_i$

$$\dot{\rho} = -i[H,\rho] - \frac{1}{2}\sum_m \left[c_m^\dagger c_m \rho + \rho c_m^\dagger c_m - 2 c_m \rho c_m^\dagger\right]$$

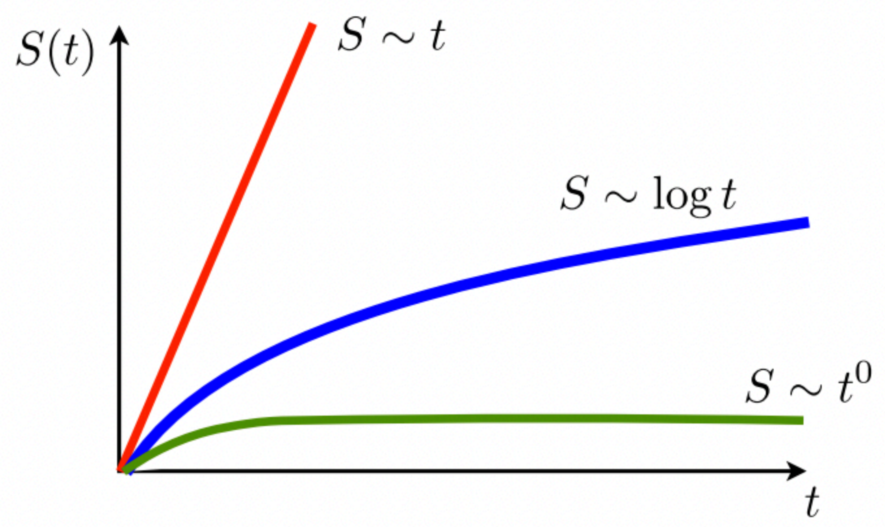
$$\rho(t \to \infty) = \mathbb{I}/N$$

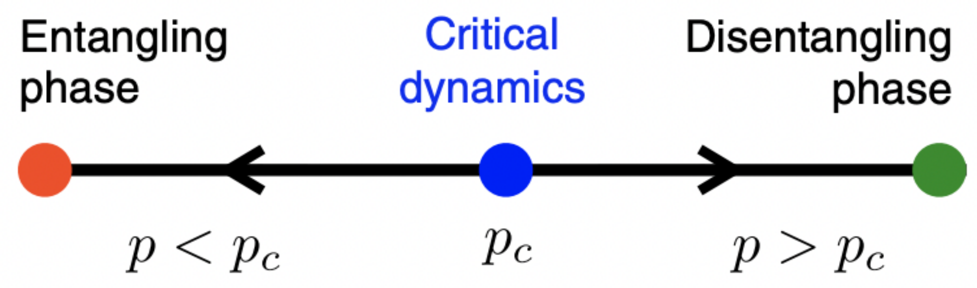


A. J. Daley, Advances in Physics 63, 77 (2014)

- The nature of these phases is only revealed by measuring quantities that are **nonlinear** in the density operator, requiring in principle to probe a state twice.

E.g.: entanglement entropy, two-point correlation functions.





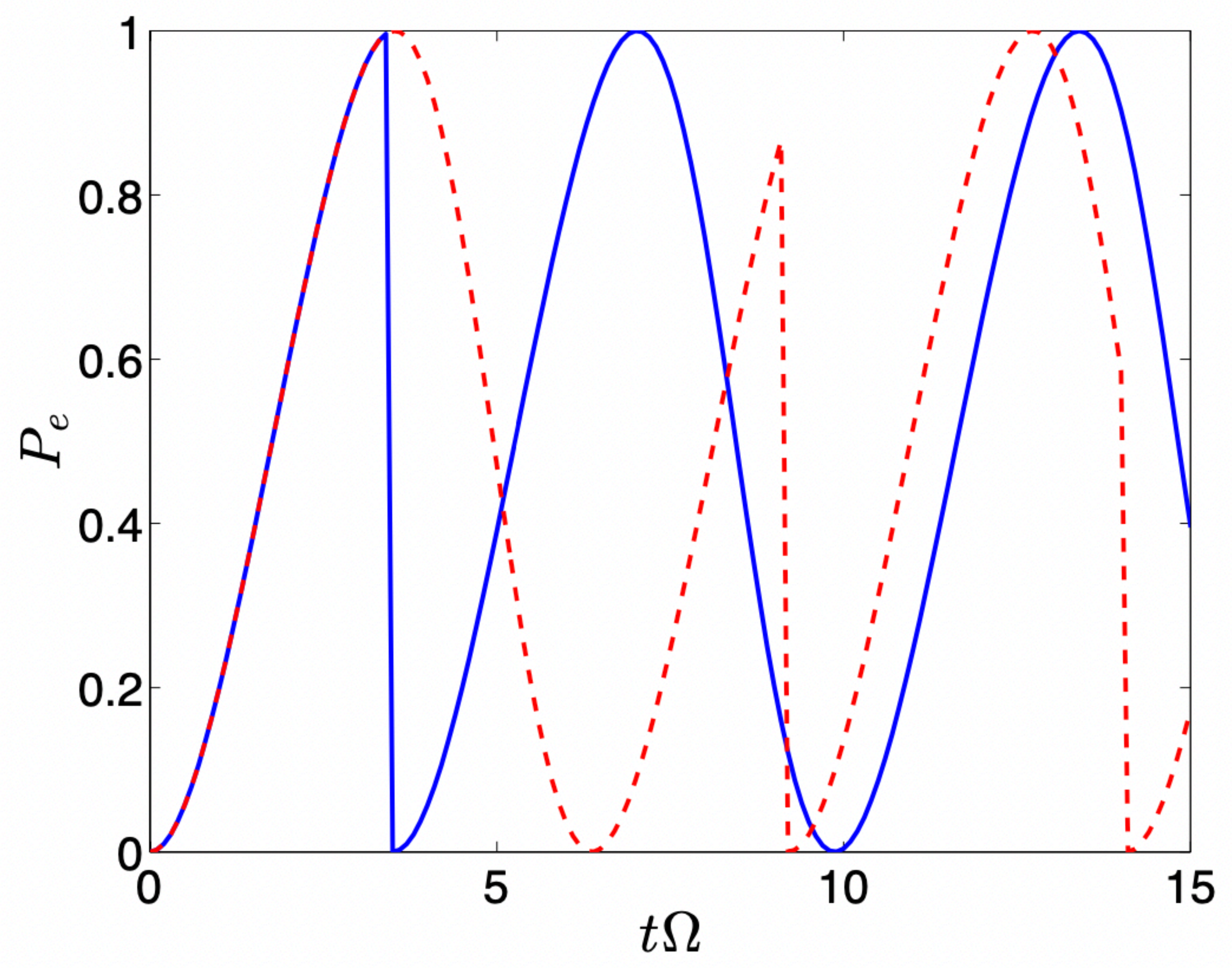
- In an experiment, an individual measurement will occur in an **unraveled picture**: single random statistical instance and not the density matrix. This makes some of these quantities inaccessible as we will show.

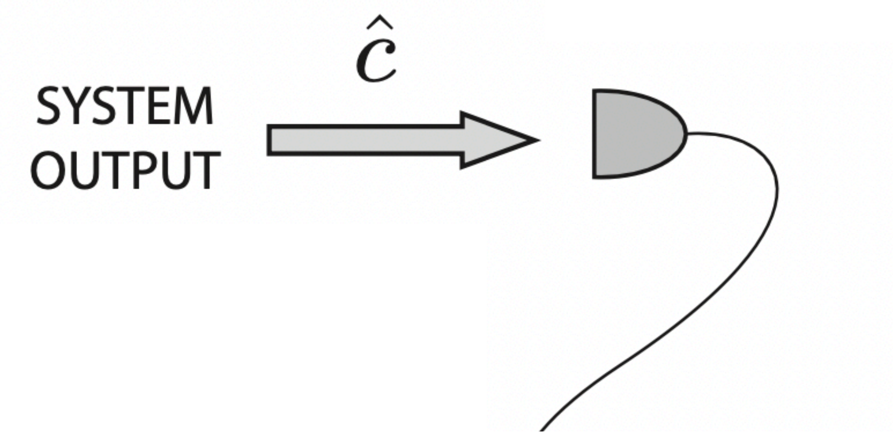
- So far the only proposals for their observation require the use of replica methods or the use preselection to distinguish both phases.

Jian et al, Phys. Rev. B **101**, 104302 (2020), Buchhold et al, arXiv:2208.10506.

Different forms of unraveling

Photon counting

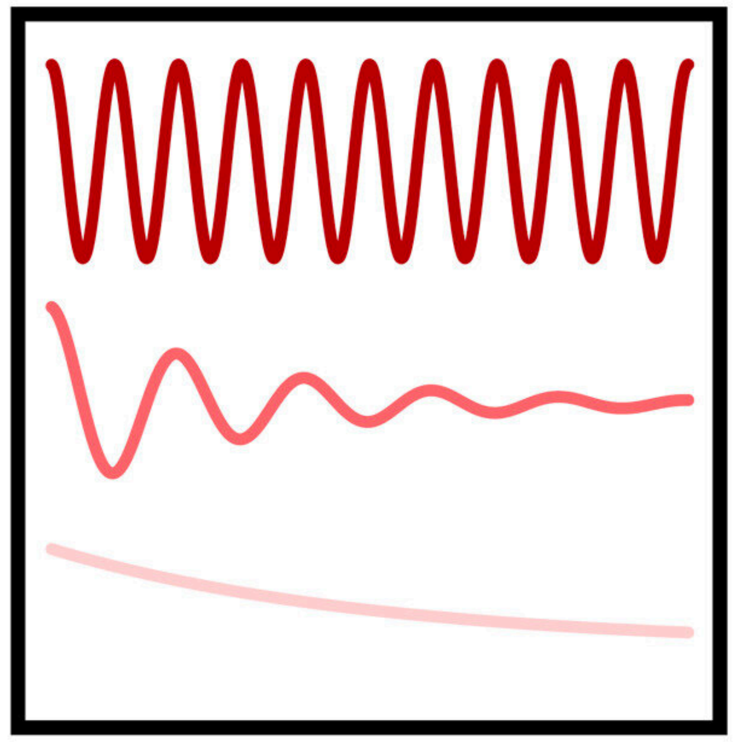


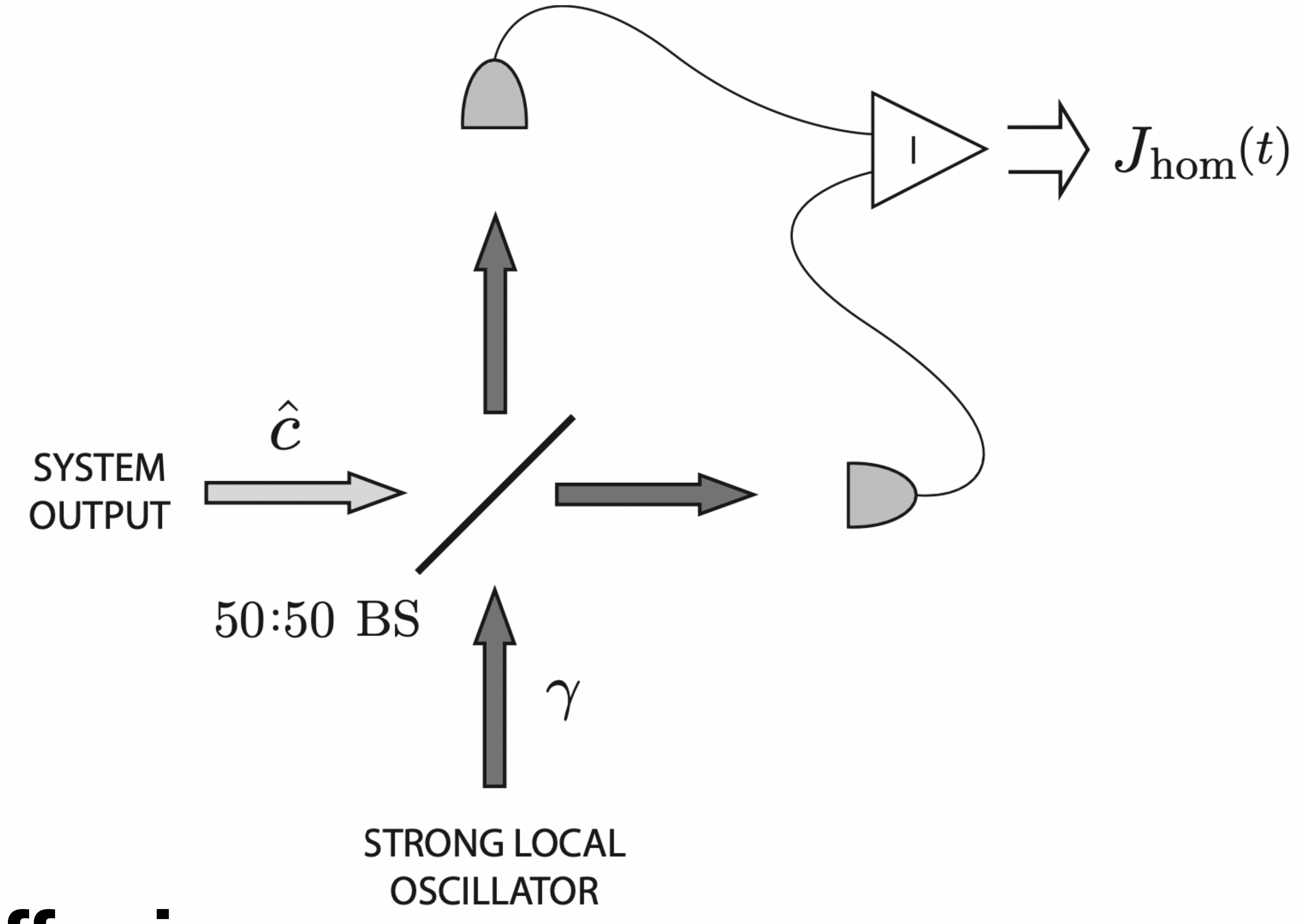


Quantum jumps

- Measuring a photon in the detector implies that the state of the system has changed.
- We can describe the evolution of the system in every run of the experiment, as coherent evolution plus a set of *jumps*.

Homodyne detection





Quantum state diffusion

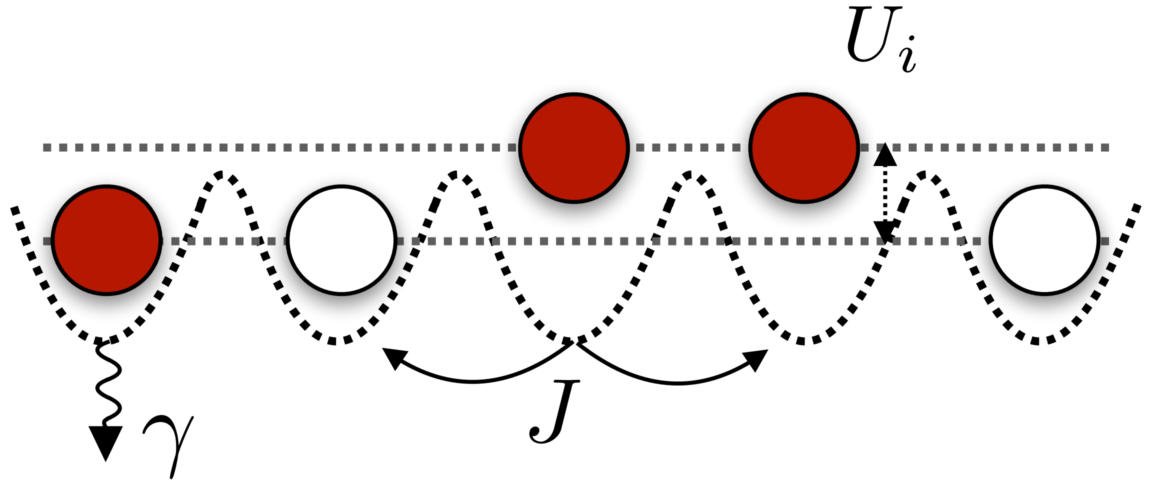
- We couple the system to an oscillator field to not directly perturb the state and produce a weak measurement.
- This leads to an evolution under the presence of a noise term, whose properties depend on the bath.

A. J. Daley, Advances in Physics 63, 77 (2014)

H Wiseman & G Milburn (2009), *Quantum Measurement and Control*

Our system

We consider a 1D chain of interacting hard-core boson atoms under continuous monitoring:



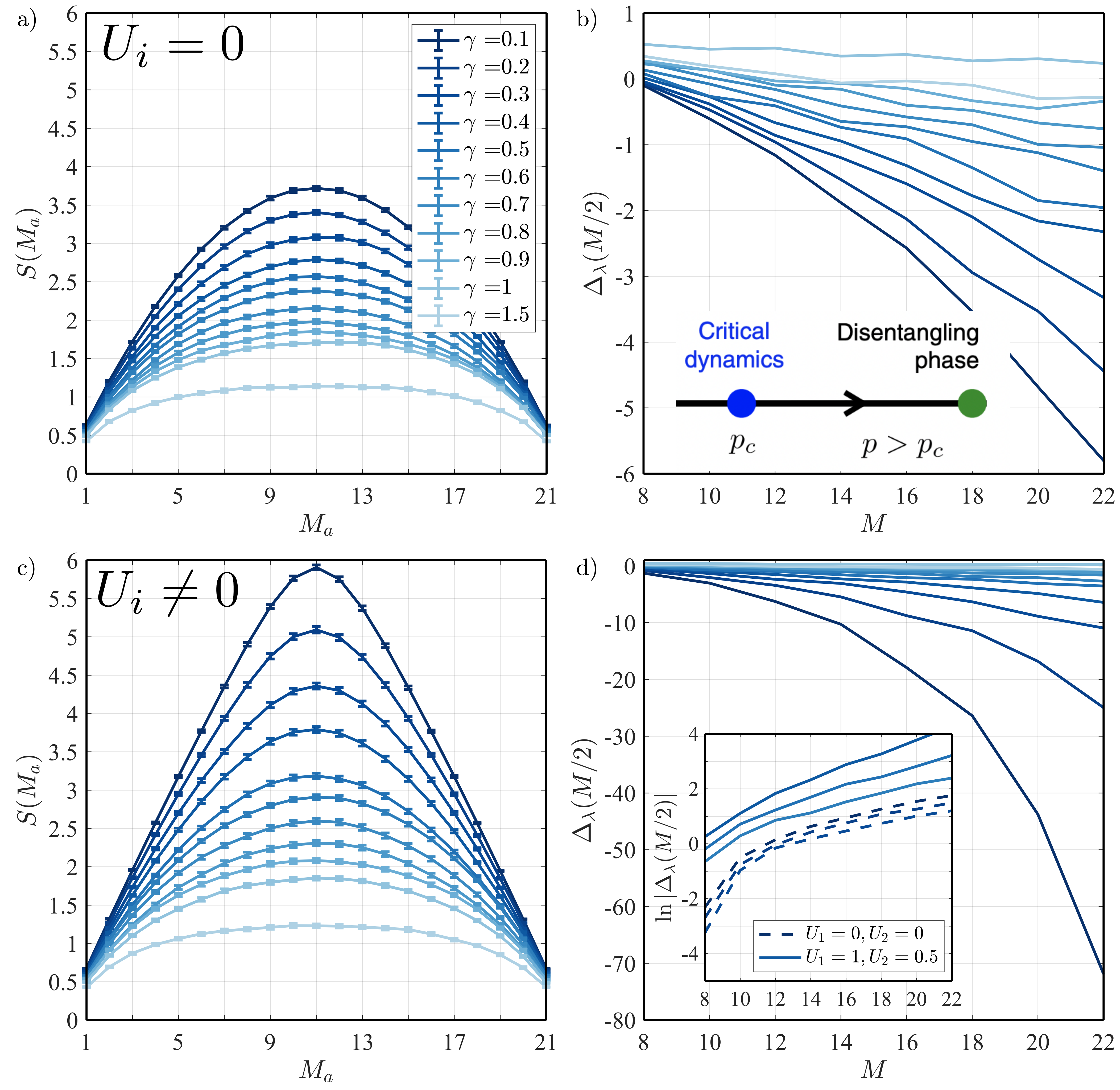
$$c_i = \sqrt{\gamma} n_i$$

$$H_{\text{hop}} = -J \sum_{i=1}^{M} (a_i^\dagger a_{i+1} + \text{h.c.}),$$

$$H_{\text{int}} = U_1 \sum_{i=1}^{M} n_i n_{i+1} + U_2 \sum_{i=1}^{M} n_i n_{i+2}$$

$$\dot{\rho} = -i[H,\rho] - \frac{1}{2} \sum_m \left[c_m^\dagger c_m \rho + \rho c_m^\dagger c_m - 2 c_m \rho c_m^\dagger \right]$$

$$S(M_a) = -\sum_{i=1} \lambda_i \ln(\lambda_i),$$



The system undergoes a PT as a function of the measurement rate, varying from:

- a volume-law phase, where entanglement generated by coherent dynamics dominates.
- an area-law phase where the projective measurement prevent the build-up of entanglement.

T. Bintener, **JYM**, M. Buchhold, P. Kirton, S. Diehl, A. Daley, in preparation (2022)

Our search

We cannot make use of:

- Entanglement entropy: poor scaling.

PRL 109, 020505 (2012), Nature 528, 77-83 (2015) , PRL 120, 050406 (2018).

- Two-point correlations: vanishing non-linear term.

Recent studies state that standard dynamics might just render chaotic evolution and might be blind to the different phases.

arXiv:2210.07256

T. Bintener, **JYM**, M. Buchhold, P. Kirton, S. Diehl, A. Daley, in preparation (2022)

Our search

We cannot make use of:

- Entanglement entropy: poor scaling.

PRL 109, 020505 (2012), Nature 528, 77-83 (2015) , PRL 120, 050406 (2018).

- Two-point correlations: vanishing non-linear term.

Recent studies state that standard dynamics might just render chaotic evolution and might be blind to the different phases.

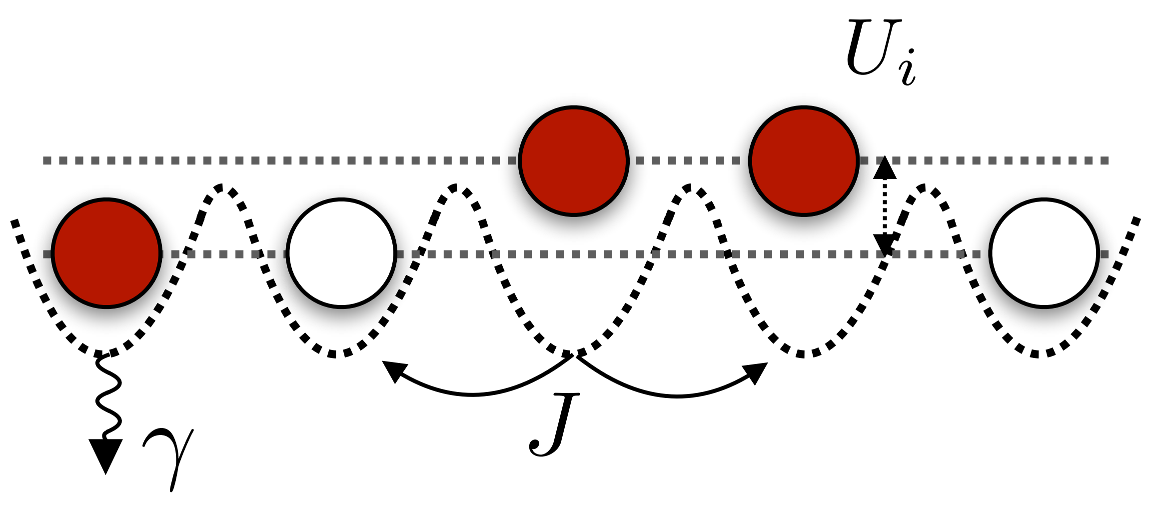
arXiv:2210.07256

- Measuring MIPT is a difficult and interesting question and might not be solved in general.
- However, the underlying competition between coherent and dissipative dynamics occurs also at short times and in conditions where we could observe it in an experiment.
- Our focus is on trying to identify signatures that can characterize this interplay and the information scrambling that occurs in the system and are experimentally measurable.

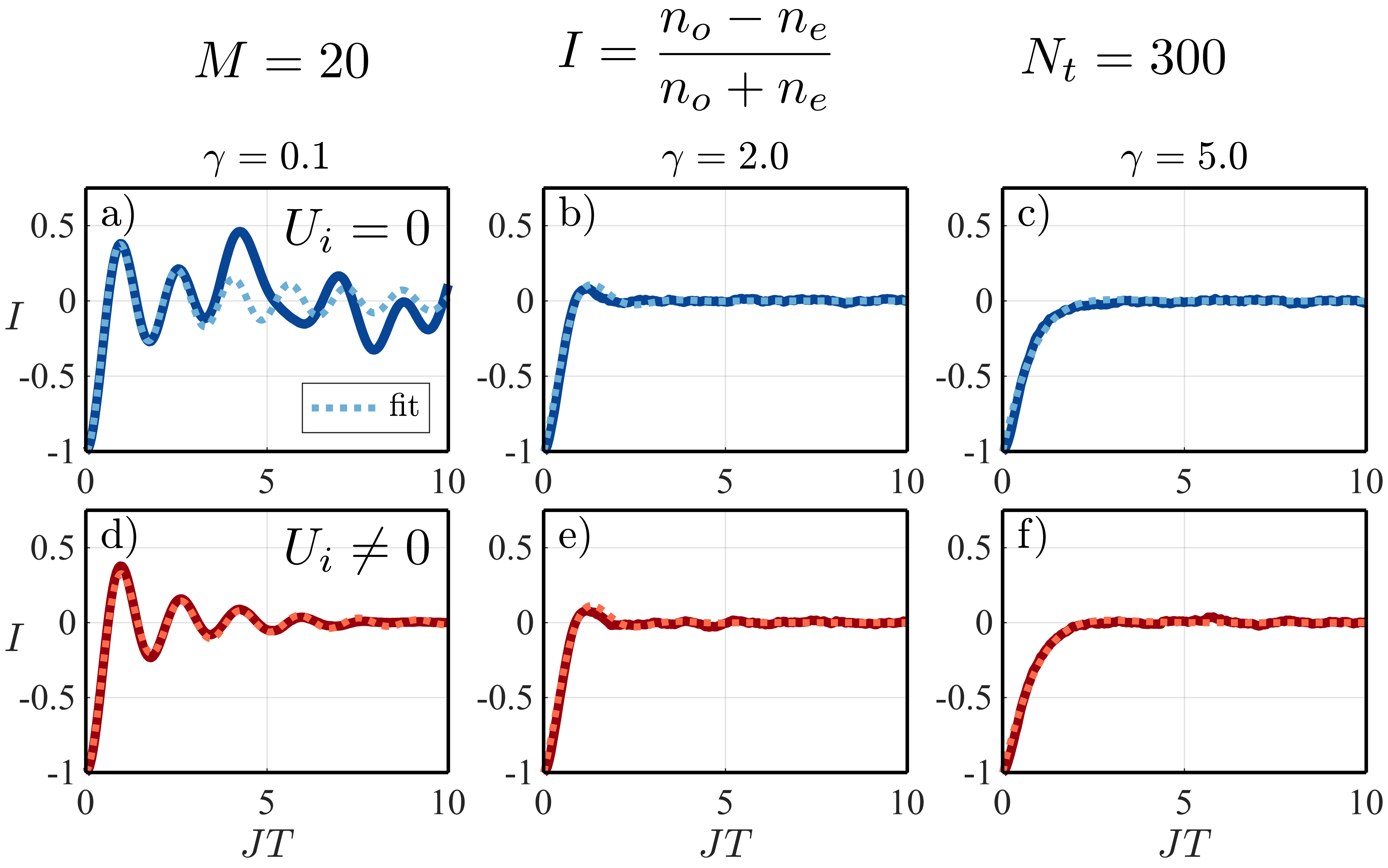
T. Bintener, **JYM**, M. Buchhold, P. Kirton, S. Diehl, A. Daley, in preparation (2022)

Our results

We consider a 1D chain of interacting hard-core boson atoms under continuous monitoring:



- We analyze the short-time dynamics of the system starting from a CDW state.
- The imbalance, a simple lattice-averaged quantity, exhibits clear differences between the coherent phase and the disentangled one.



Our results are consistent with studies on free fermions based on the transport properties in the presence of dissipation.

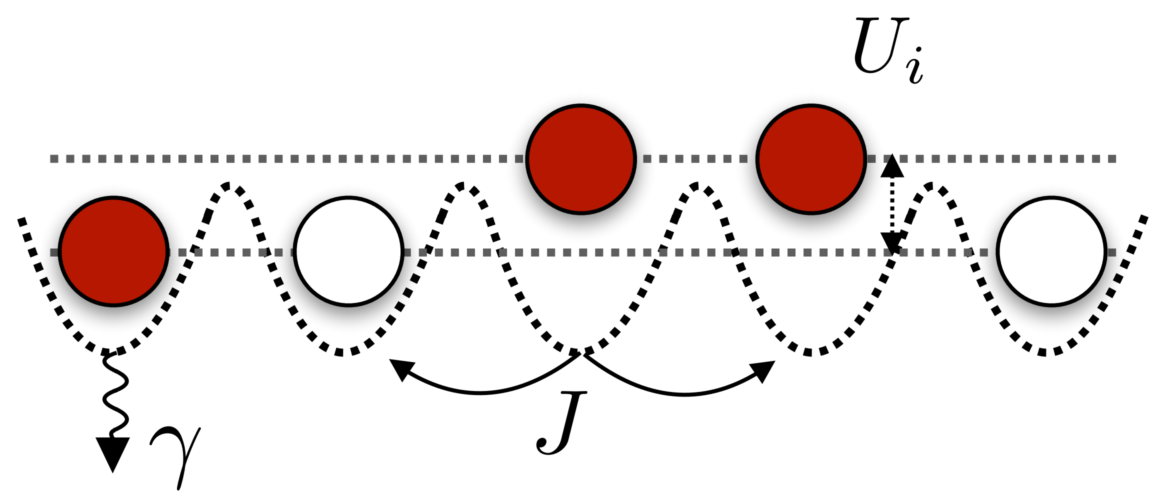
SciPost Phys. 7, 024 (2019).

T. Bintener, **JYM**, M. Buchhold, P. Kirton, S. Diehl, A. Daley, in preparation (2022)

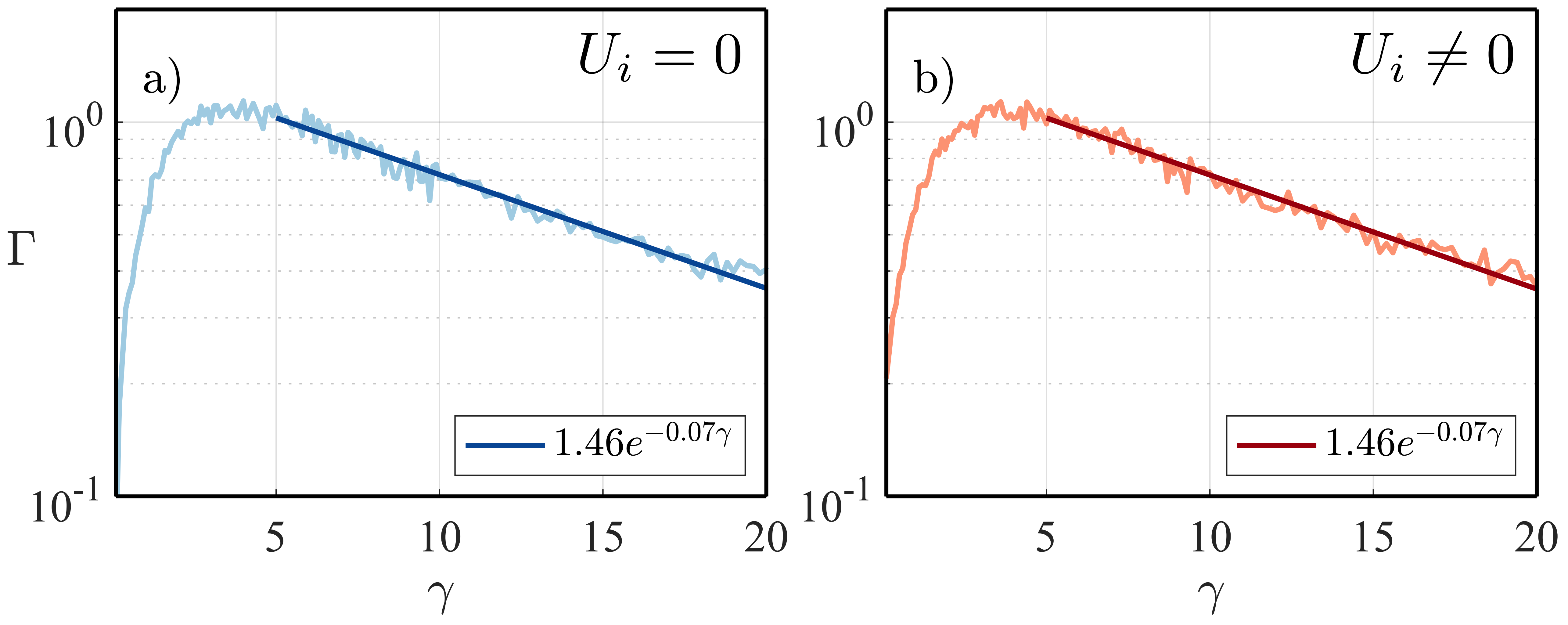
Our results

We can relate the damping in the oscillatory coherent signal with the role of dissipation that dephases the system.

We can obtain a functional form by fitting the signal to a damped exponential.



$M = 20 \qquad f(\gamma) = -J_0(at)e^{-\Gamma\gamma} \qquad N_t = 300$



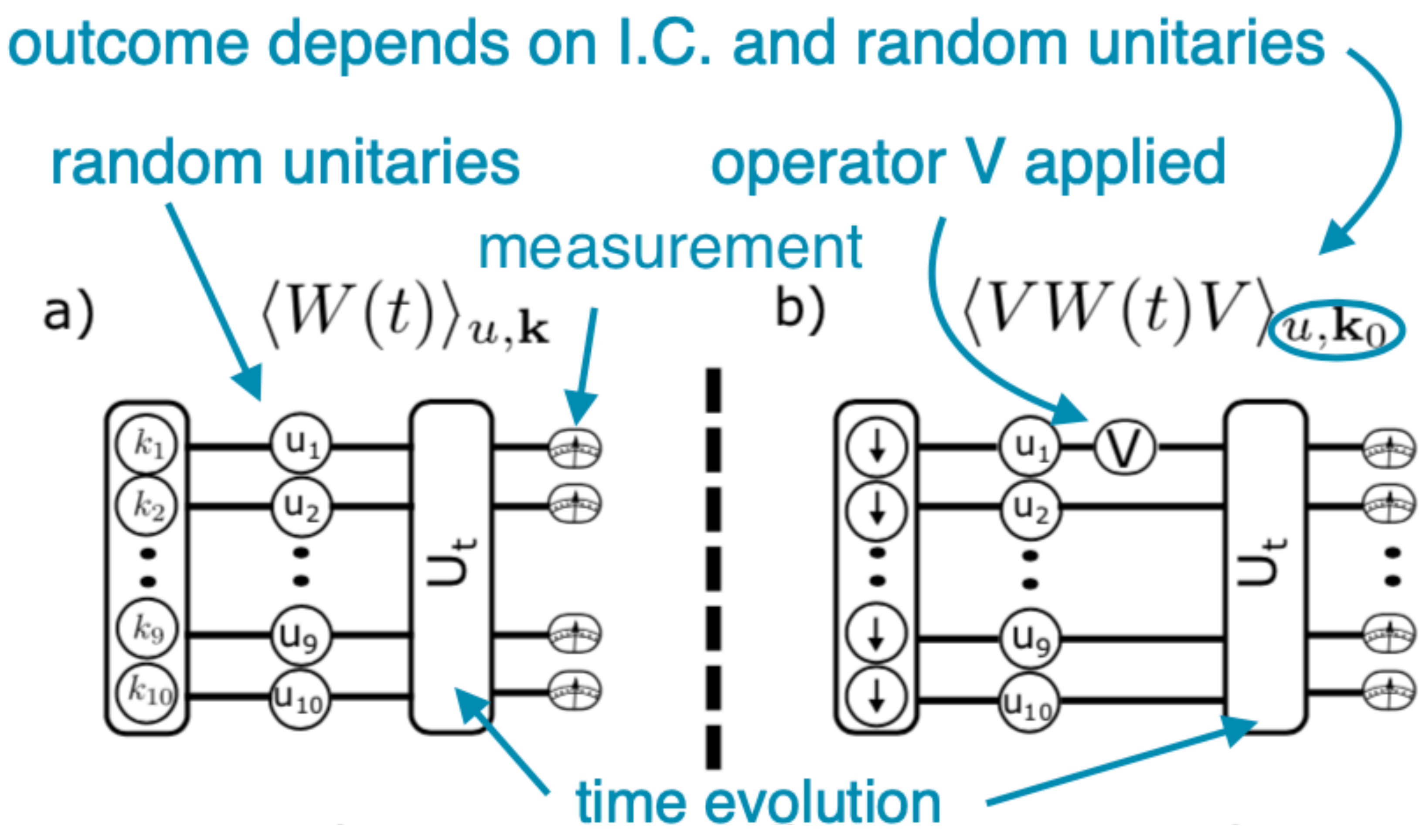
- We observe that the damping rate is independent of the strength of the interaction.
- The system's coherent oscillations are initially damped up to a maximum rate where the contribution from quantum Zeno effect becomes relevant leading to the freezing of dynamics.

T. Bintener, **JYM**, M. Buchhold, P. Kirton, S. Diehl, A. Daley, in preparation (2022)

Our results

An alternative approach is to consider proving short-time information spreading in the steady state.

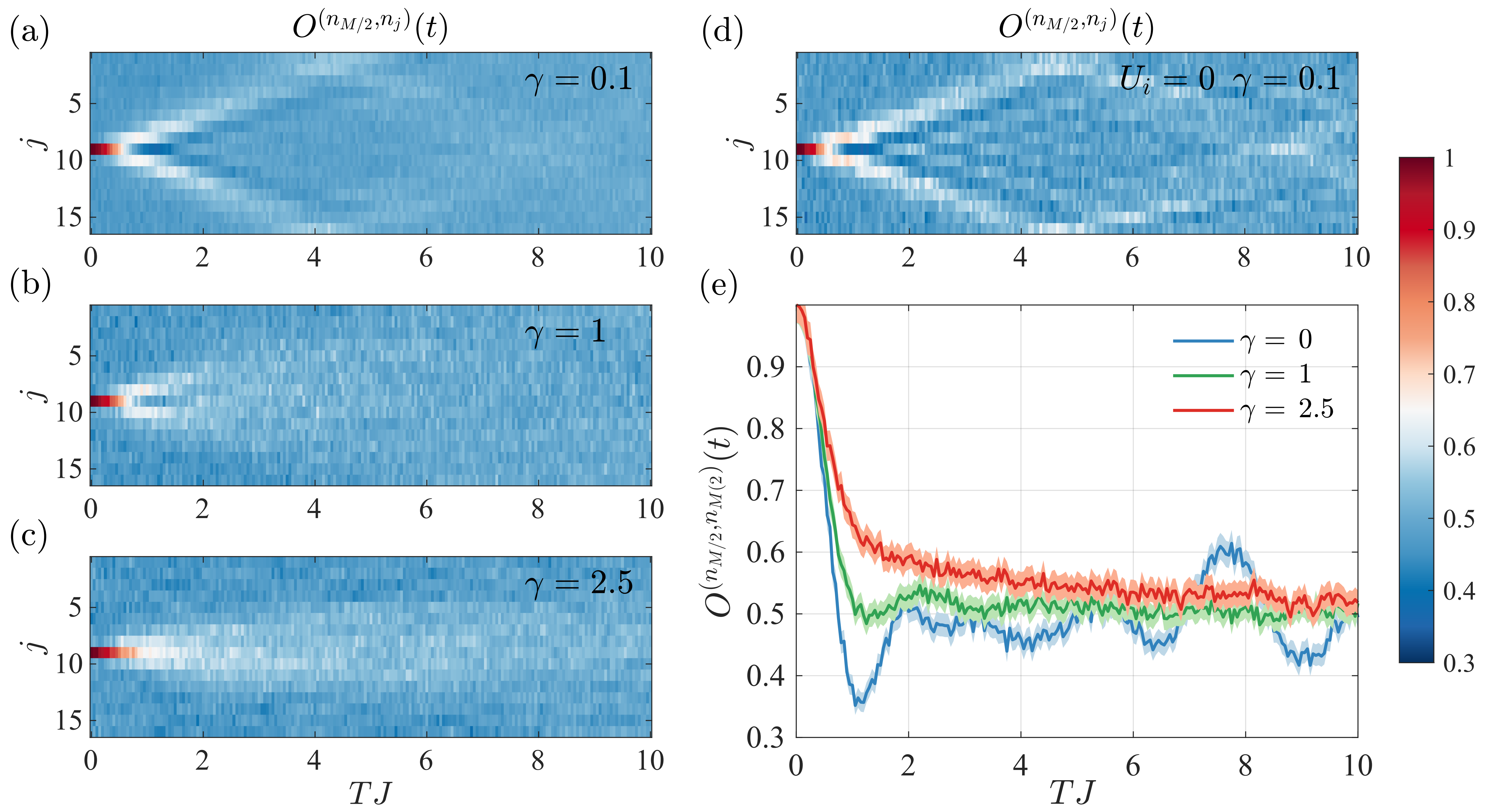
We can probe the infinite T state by using randomly rotated states sampled from the CUE.



Phys. Rev. X 9, 021061 (2019).

(Preliminary)

$$O_n(t) = \frac{\overline{\sum_{k\in E_n} c_k \langle W(t)\rangle_{u,|k\rangle}\langle VW(t)V\rangle_{u,|k_0\rangle}}}{\overline{\sum_{k\in E_n} c_k \langle W(t)\rangle_{u,|k\rangle}\langle W(t)\rangle_{u,|k_0\rangle}}}$$



We observe qualitative differences in the information spreading between both regimes.

T. Bintener, **JYM**, M. Buchhold, P. Kirton, S. Diehl, A. Daley, in preparation (2022)

Conclusions

- We have analyzed the underlying competition between coherent and dissipative dynamics in continuously monitored systems.
- We have found signatures in the early time dynamics of certain observables that can distinguish different regimes linked to MIPTs.
- More importantly these signatures help characterize this dynamical interplay and the way in which information spreads in these systems.
- This offers new insight in fundamental quantum mechanics and in the systems that QT can simulate.

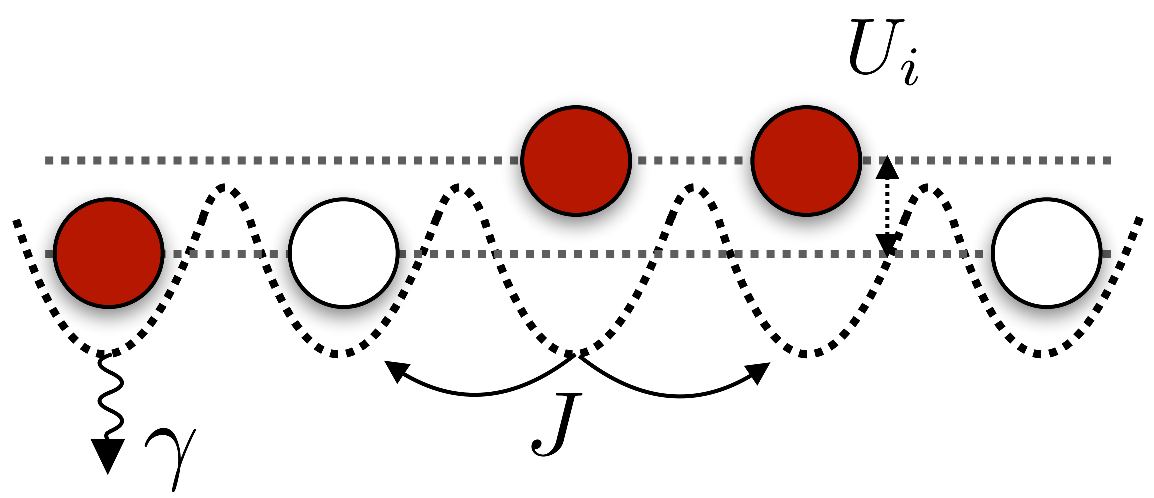
Open questions and beyond:

- Introducing feedback in the system, can allow us to harness the information we obtain from probing the system and potentially stirring it into specific distinguishable phases.

Extra material

Our system

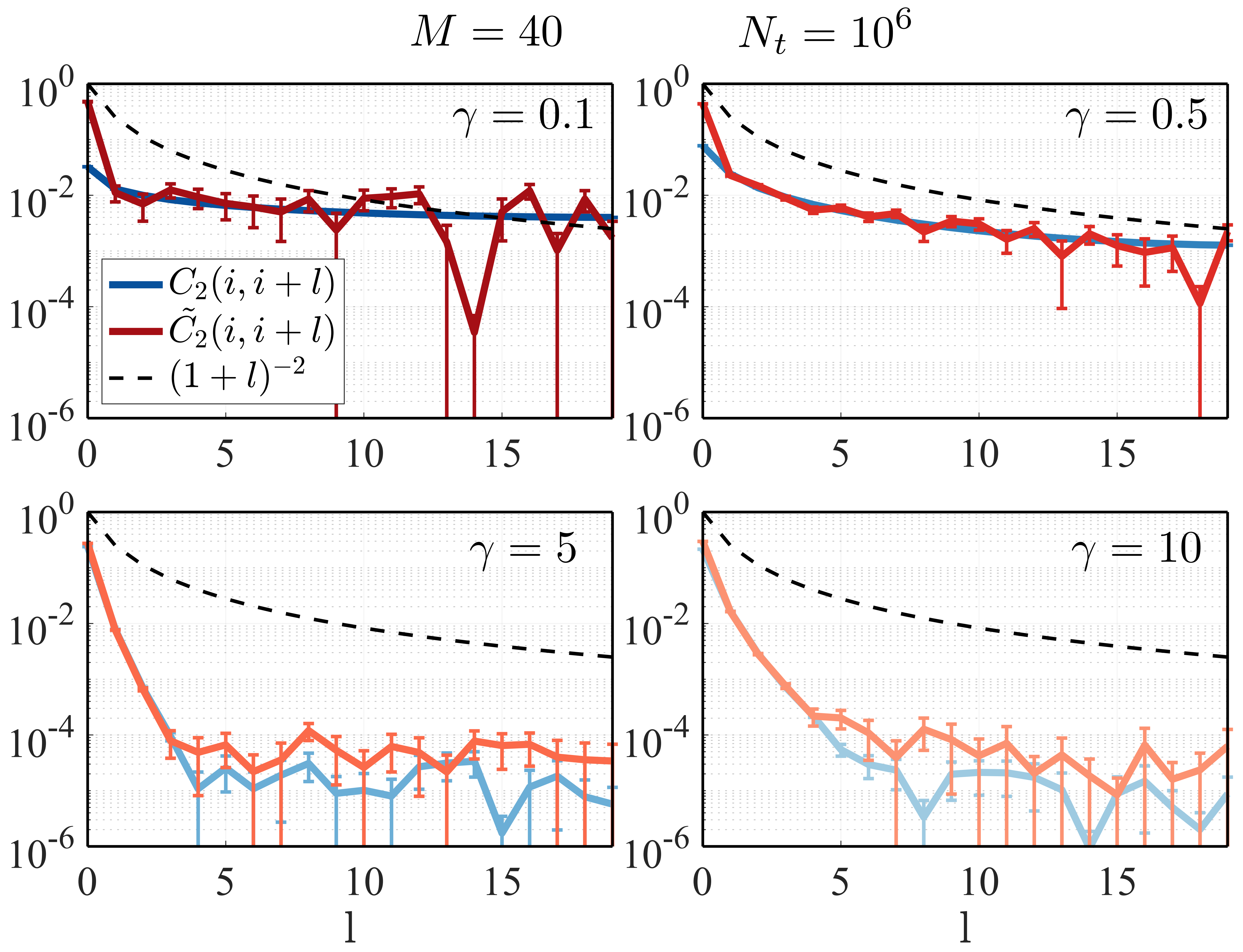
We consider a 1D chain of interacting hard-core boson atoms under continuous monitoring:



One could think, that it is possible to infer the correlations from the homodyne currents:

$$J_{hom} = \langle \hat{o} \rangle + \xi(t)$$

However, the nonlinear term that are the relevant ones to detect the transition, cancel out, as the current gets integrated over time those terms average to approx. zero.

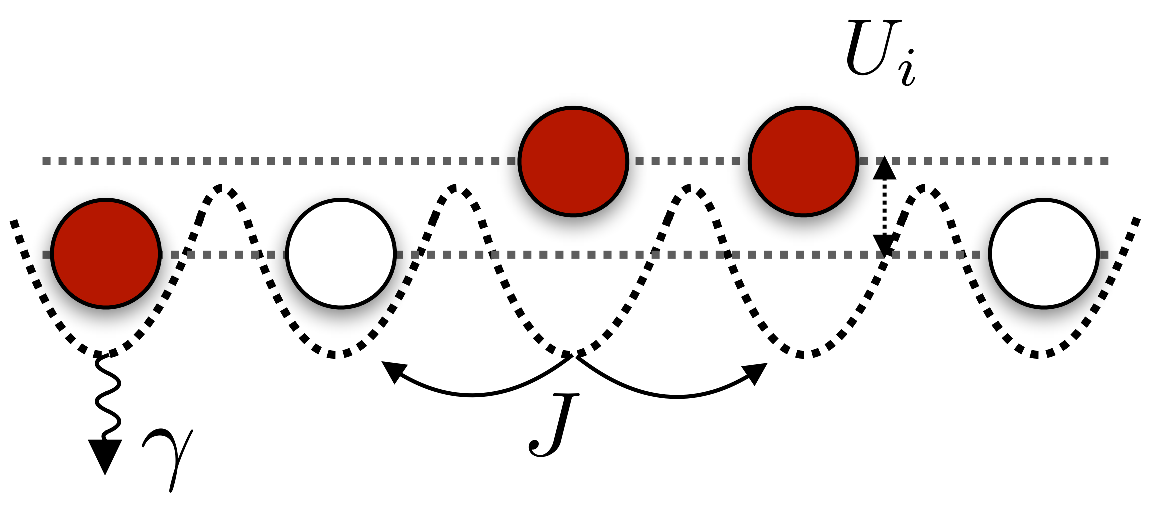


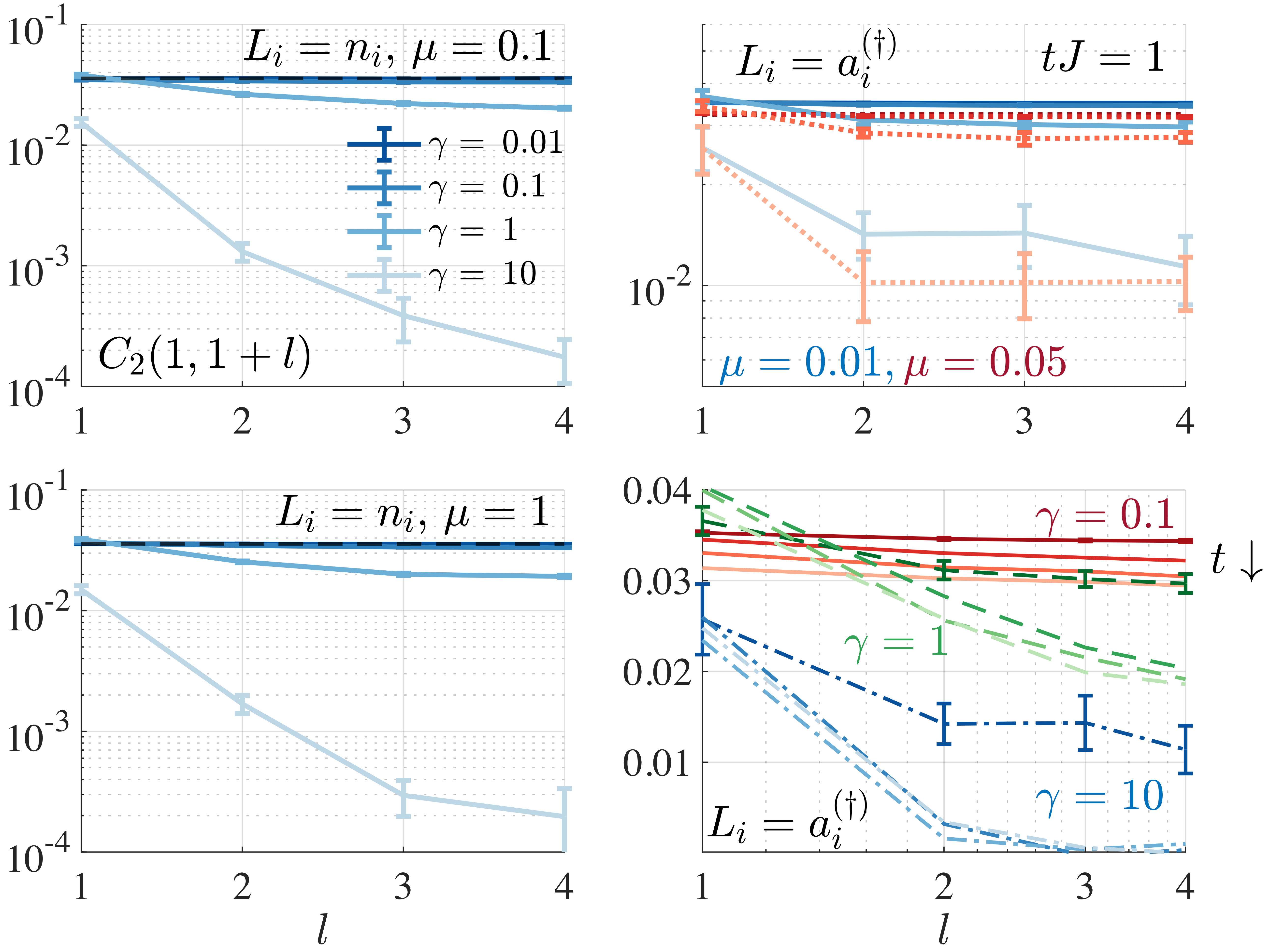
H Wiseman & G Milburn (2009), *Quantum Measurement and Control*

T. Bintener, **JYM**, M. Buchhold, P. Kirton, S. Diehl, A. Daley, in preparation (2022)

Our system

We consider a 1D chain of interacting hard-core boson atoms under continuous monitoring:





Are these correlations, even if experimentally hard to access, robust again other dissipation sources?

- Additional dephasing does affect the system.
- Other dissipation, e.g. particle loss, produces decay of corr. in moderate times.

T. Bintener, **JYM**, M. Buchhold, P. Kirton, S. Diehl, A. Daley, in preparation (2022)

Quantum trajectories: Stochastic unraveling

H. Carmichael, *An Open Systems Approach to Quantum Optics,* Springer, Berlin, 1993.
K. Mølmer, J. Dalibard, Y. Castin, JOSA B 10, 524 (1993).
R. Dum, A. S. Parkins, P. Zoller, and C. W. Gardiner, PRA 46, 4382 (1992).

What is stochastic unraveling

… and why it is important

- When describing theoretically quantum systems, we have to consider ways to cope with exponentially increasing Hilbert space dimensions.

- This problem gets exacerbated in the case of open quantum systems, when our description typically requires the use of the density matrix. This meaning that our mathematical objects grow as $\dim(\mathcal{H})^2$.

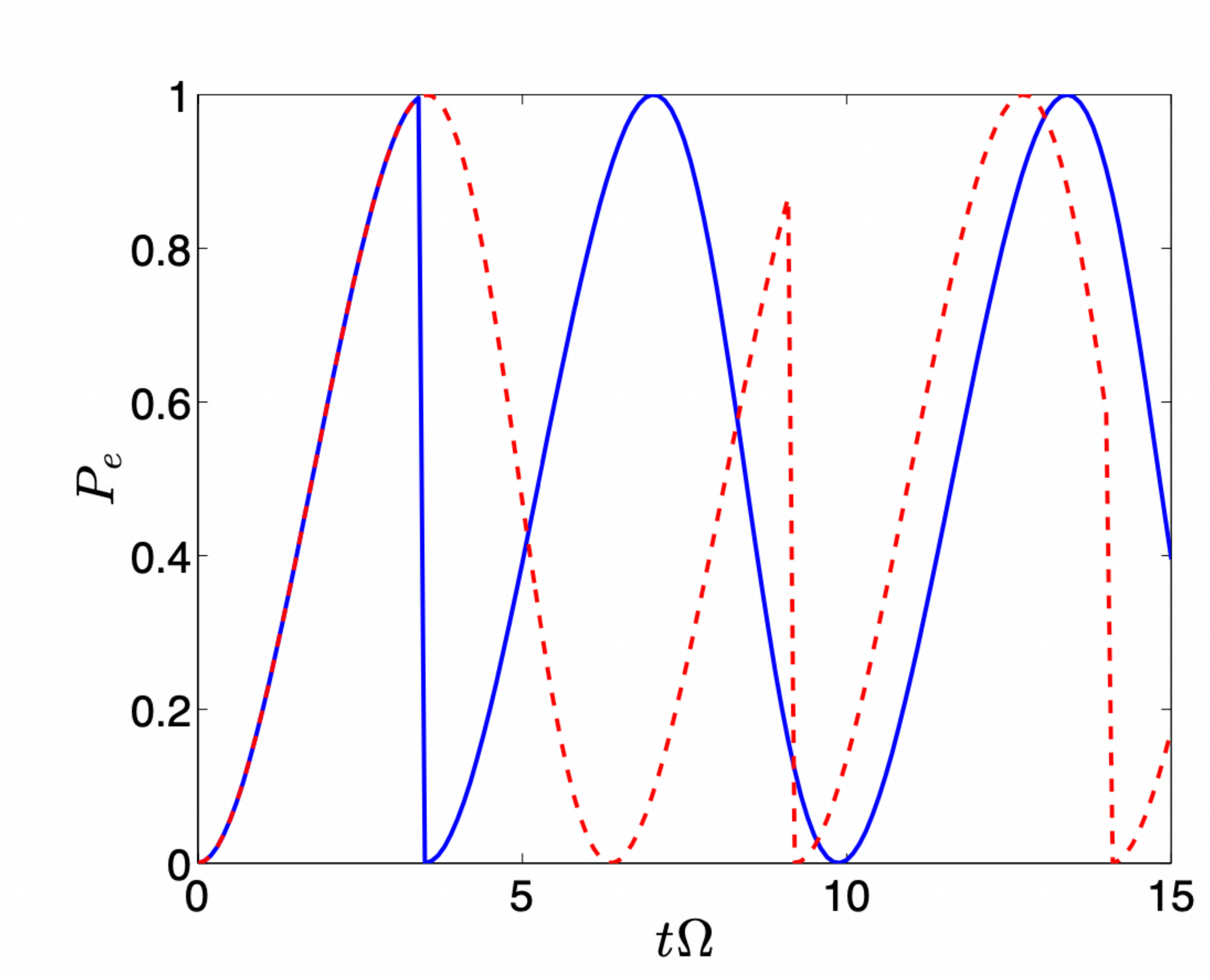
- However, we can take advantage of the way that the interaction between system and environment occurs to avoid using the density operator.

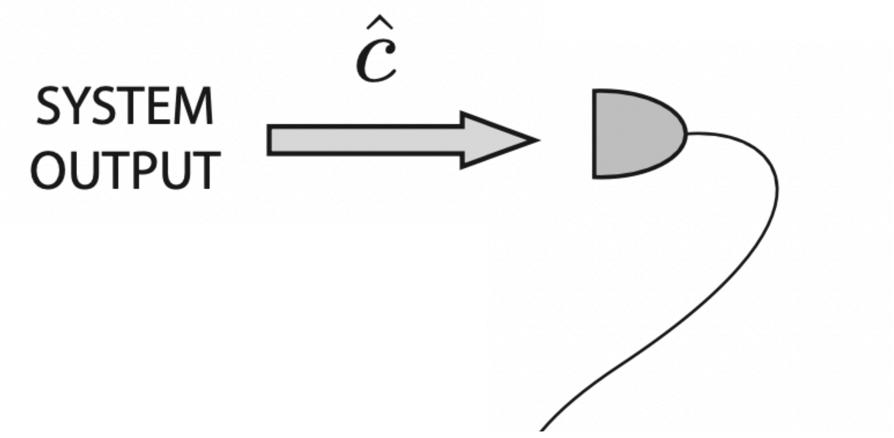
- ***Stochastic unraveling*** **allows to describe the evolution of our mixed state as a sum of pure state evolutions in the presence of random dissipative events.**

- This way, we can interpret the master equation as the average evolution over these events occurring at every possible time and location.

Different forms of unraveling

Photon counting

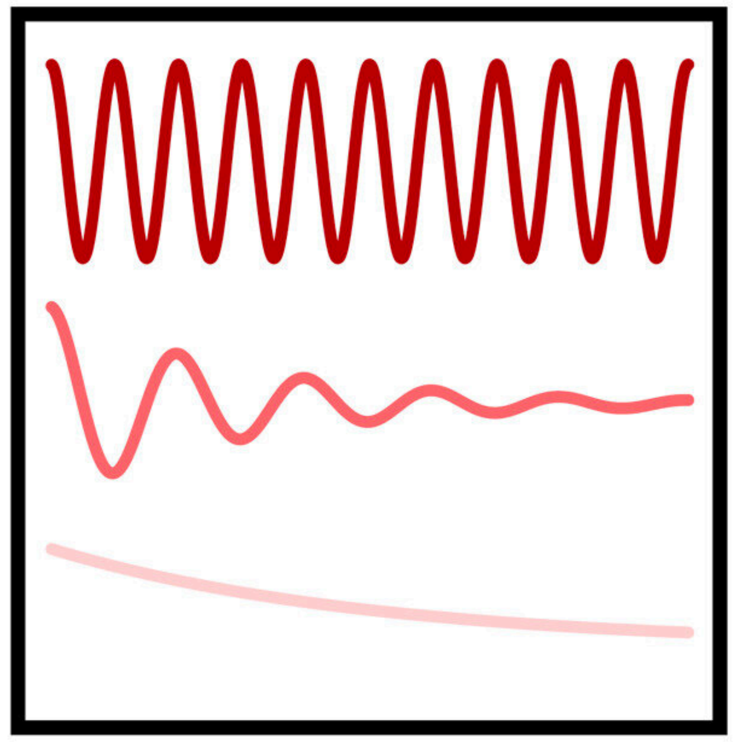


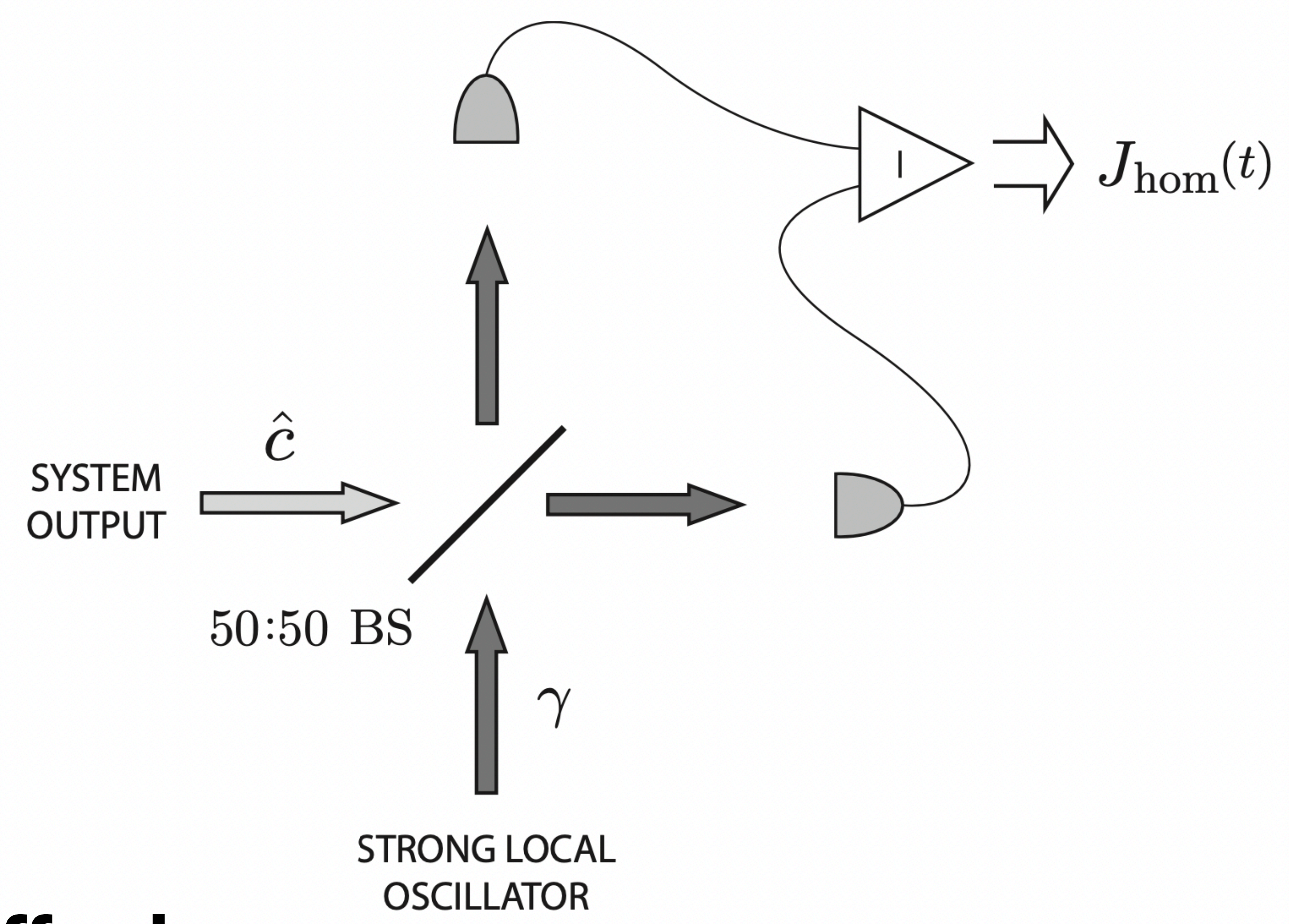


Quantum jumps

- Measuring a photon in the detector implies that the state of the system has changed.
- We can describe the evolution of the system in every run of the experiment, as coherent evolution plus a set of *jumps*.

Homodyne detection





Quantum state diffusion

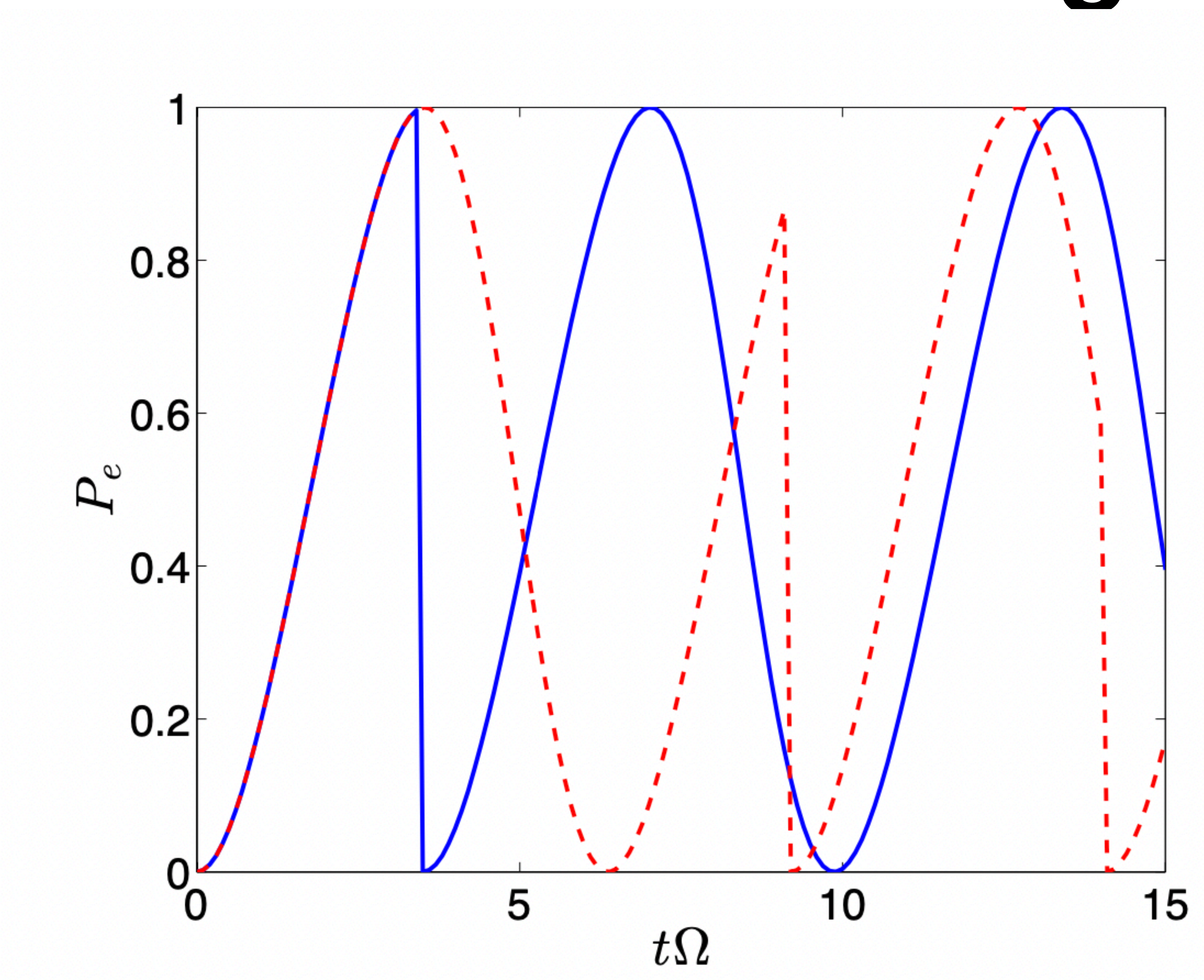
- We couple the system to an oscillator field to not directly perturb the state and produce a weak measurement.
- This leads to an evolution that under the presence of a noise term, whose properties depend on the bath.

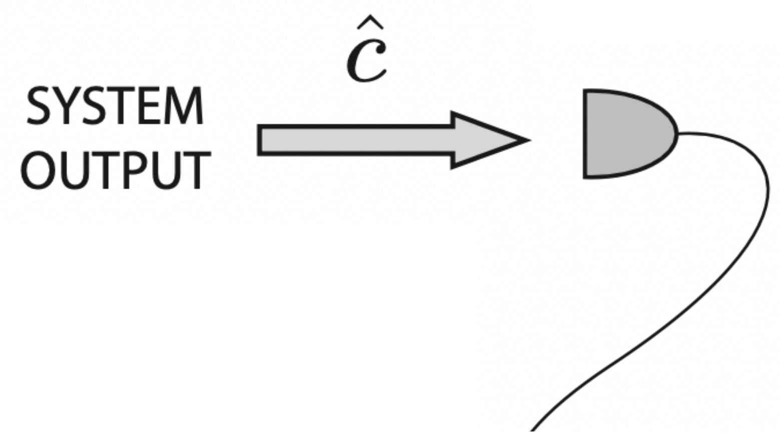
A. J. Daley, Advances in Physics 63, 77 (2014)

H Wiseman & G Milburn (2009), *Quantum Measurement and Control*

Different forms of unraveling

Photon counting



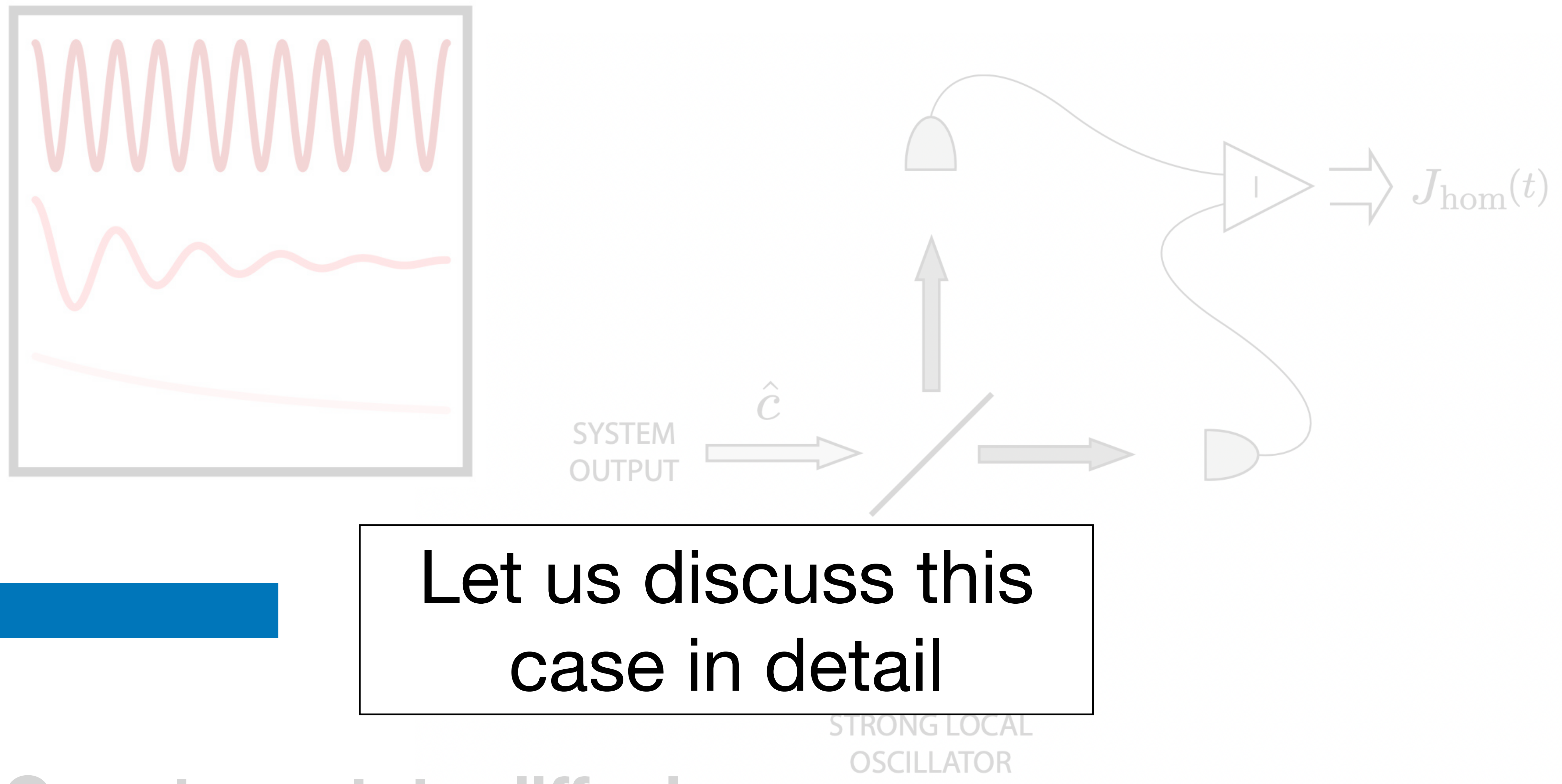


Quantum jumps

- Measuring a photon in the detector implies that the state of the system has changed.
- We can describe the evolution of the system in every run of the experiment, as coherent evolution plus a set of *jumps*.

A. J. Daley, Advances in Physics 63, 77 (2014)

Homodyne detection



Let us discuss this case in detail

Quantum state diffusion

- We couple the system to an oscillator field to not directly perturb the state and produce a weak measurement.
- This leads to an evolution that under the presence of a noise term, whose properties depend on the bath.

H Wiseman & G Milburn (2009), *Quantum Measurement and Control*

Quantum jumps: photodetection

In a time *t*, we have two options: I) No photon detected, II) We detect a photon and so the system has undergone a jump.

If we recall the GSKL master equation

$$\dot{\rho} = -i[H,\rho] - \frac{1}{2}\sum_m \left[c_m^\dagger c_m \rho + \rho c_m^\dagger c_m - 2 c_m \rho c_m^\dagger\right]$$

$$= -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger) + \sum_m c_m \rho c_m^\dagger \qquad H_{\text{eff}} = H - \frac{i}{2}\sum_m c_m^\dagger c_m$$

Then, a pure state $|\psi(t)\rangle$ sampled from ρ would evolve under (if no jump occurs):

$$|\phi^{(1)}(t+\delta t)\rangle = (1 - iH_{\text{eff}}\delta t)\,|\phi(t)\rangle$$

As this Hamiltonian is not hermitian, the norm of the state would decrease:

$$\langle \phi^{(1)}(t+\delta t)|\phi^{(1)}(t+\delta t)\rangle \quad = \quad \langle\phi(t)|\,(1 + iH_{\text{eff}}\delta t)\,(1 - iH_{\text{eff}}\delta t)\,|\phi(t)\rangle \quad = \quad 1 - \delta p$$

We can associate this norm decrease with the probability leakage to another pure state from our density matrix, i.e. the probability that a jump has happened.

Quantum jumps: photodetection

In a time *t*, we have two options: I) No photon detected, II) We detect a photon and so the system has undergone a jump.

If we recall the GSKL master equation

$$\dot{\rho} = -i[H,\rho] - \frac{1}{2}\sum_m \left[c_m^\dagger c_m \rho + \rho c_m^\dagger c_m - 2 c_m \rho c_m^\dagger\right]$$

$$= -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger) + \sum_m c_m \rho c_m^\dagger \qquad H_{\text{eff}} = H - \frac{i}{2}\sum_m c_m^\dagger c_m$$

Recycling term

Then, a pure state $|\psi(t)\rangle$ sampled from ρ would evolve under:

$$|\phi^{(1)}(t+\delta t)\rangle = (1 - iH_{\text{eff}}\delta t)\,|\phi(t)\rangle$$

As this Hamiltonian is not hermitian, the norm of the state would decrease:

$$\langle \phi^{(1)}(t+\delta t)|\phi^{(1)}(t+\delta t)\rangle = \langle \phi(t)|\,(1 + iH_{\text{eff}}\delta t)\,(1 - iH_{\text{eff}}\delta t)\,|\phi(t)\rangle = 1 - \delta p$$

We can associate this norm decrease with the probability leakage to another pure state from our density matrix, i.e. the probability that a jump has happened.

Quantum jumps: photodetection

Then, we can simply evolve our system in a given timestep as:

No jump

- With probability $1 - \delta p$

$$|\phi(t+\delta t)\rangle = \frac{|\phi^{(1)}(t+\delta t)\rangle}{\sqrt{1-\delta p}}$$

Jump

- With probability δp

$$|\phi(t+\delta t)\rangle = \frac{c_m|\phi(t)\rangle}{\sqrt{\delta p_m/\delta t}}$$

choose *m* with probability:

$$\Pi_m = \delta p_m/\delta p$$

Quantum jumps: photodetection

Then, we can simply evolve our system in a given timestep as:

No jump

- With probability $1-\delta p$ $\quad |\phi(t+\delta t)\rangle = \frac{|\phi^{(1)}(t+\delta t)\rangle}{\sqrt{1-\delta p}}$

Jump

- With probability δp $\quad |\phi(t+\delta t)\rangle = \frac{c_m|\phi(t)\rangle}{\sqrt{\delta p_m/\delta t}}$

choose *m* with probability: $\Pi_m = \delta p_m/\delta p$

It is easy to see that if we take the average over pure states: $\sigma(t) = |\phi(t)\rangle\langle\phi(t)|$

$$\overline{\sigma(t+\delta t)} = (1-\delta p)\frac{|\phi^{(1)}(t+\delta t)\rangle}{\sqrt{1-\delta p}}\frac{\langle\phi^{(1)}(t+\delta t)|}{\sqrt{1-\delta p}} + \delta p \sum_m \Pi_m \frac{c_m|\phi(t)\rangle}{\sqrt{(\delta p_m/\delta t)}}\frac{\langle\phi(t)|c_m^\dagger}{\sqrt{(\delta p_m/\delta t)}}$$

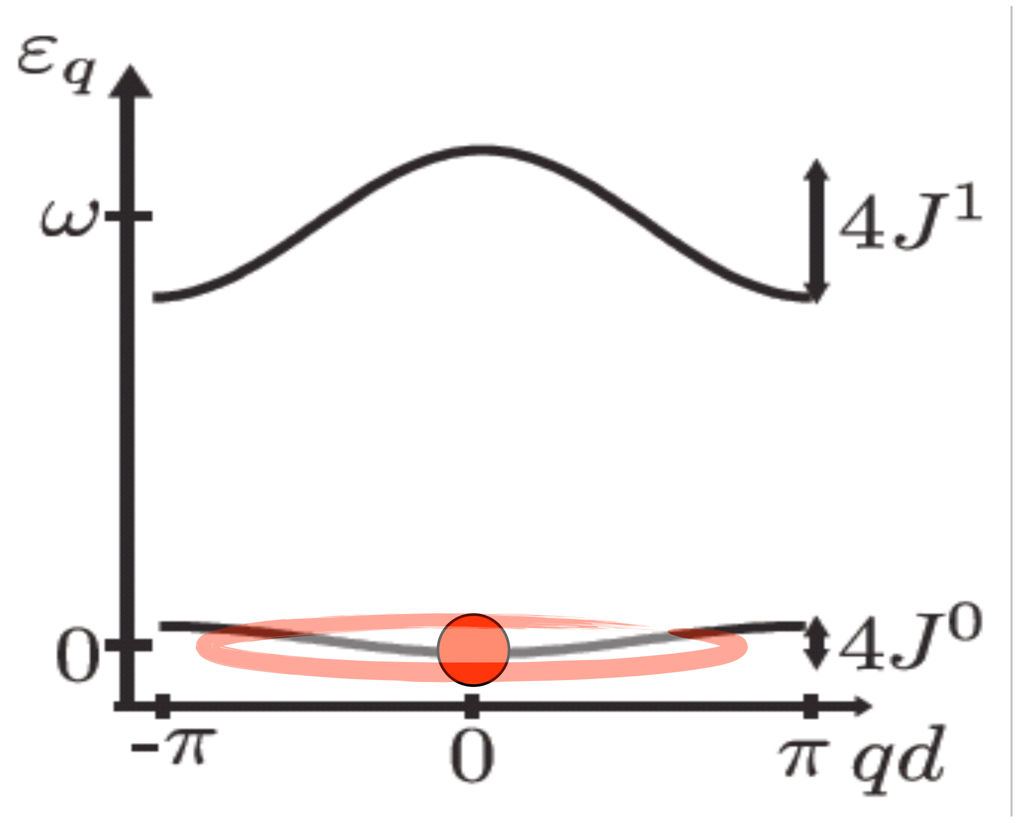
$$\overline{\sigma(t+\delta t)} = \sigma(t) - i\delta t(H_{\text{eff}}\sigma(t) - \sigma(t)H_{\text{eff}}^\dagger) + \delta t \sum_m c_m \sigma(t) c_m^\dagger$$

$$\dot{\rho} = -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger) + \sum_m c_m \rho c_m^\dagger$$

Example: heating of hard-core bosons with radiation field

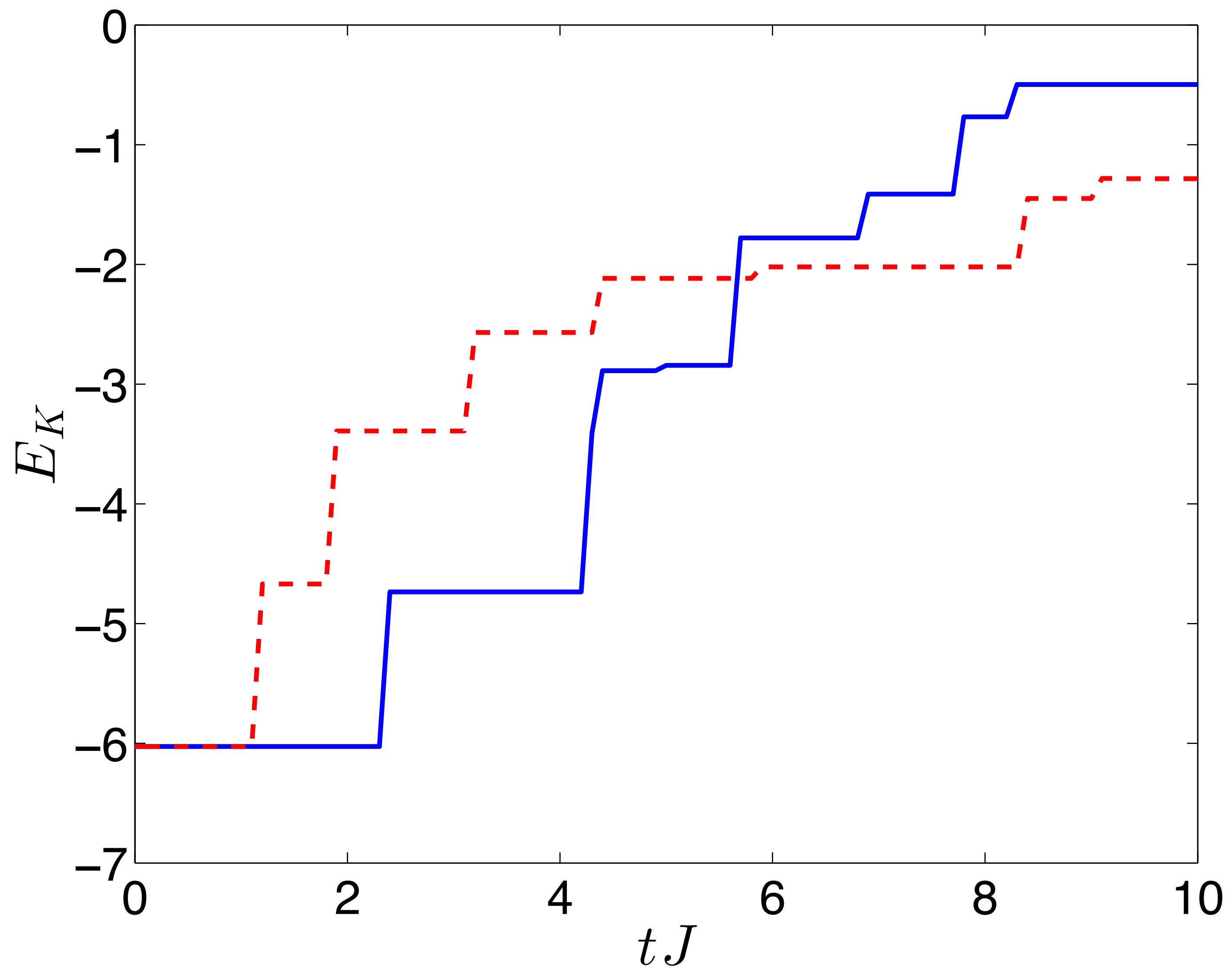
When a photon scatters from an atom, the environment learns about its position, thus, the jump operator is proportional to the number operator.

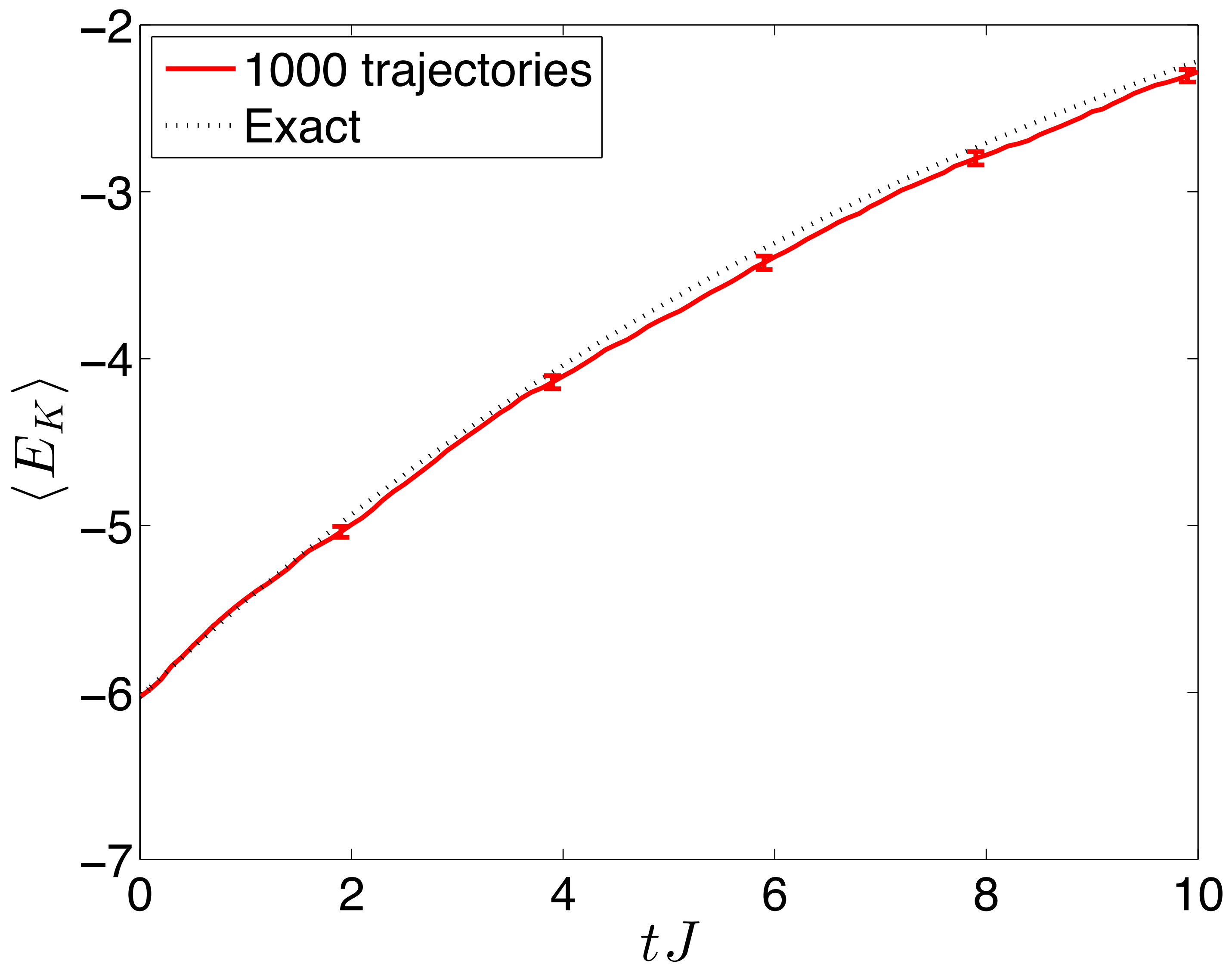
$U/J \to \infty$



$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}_1\rho$$

$$\mathcal{L}_1\rho = \frac{\gamma}{2}\sum_i (2n_i\rho n_i - n_i n_i \rho - \rho n_i n_i)$$



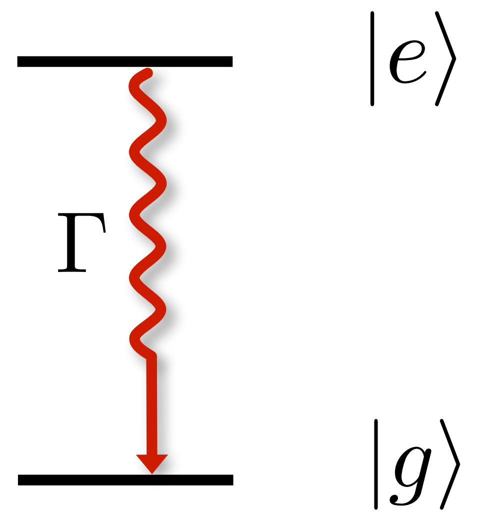


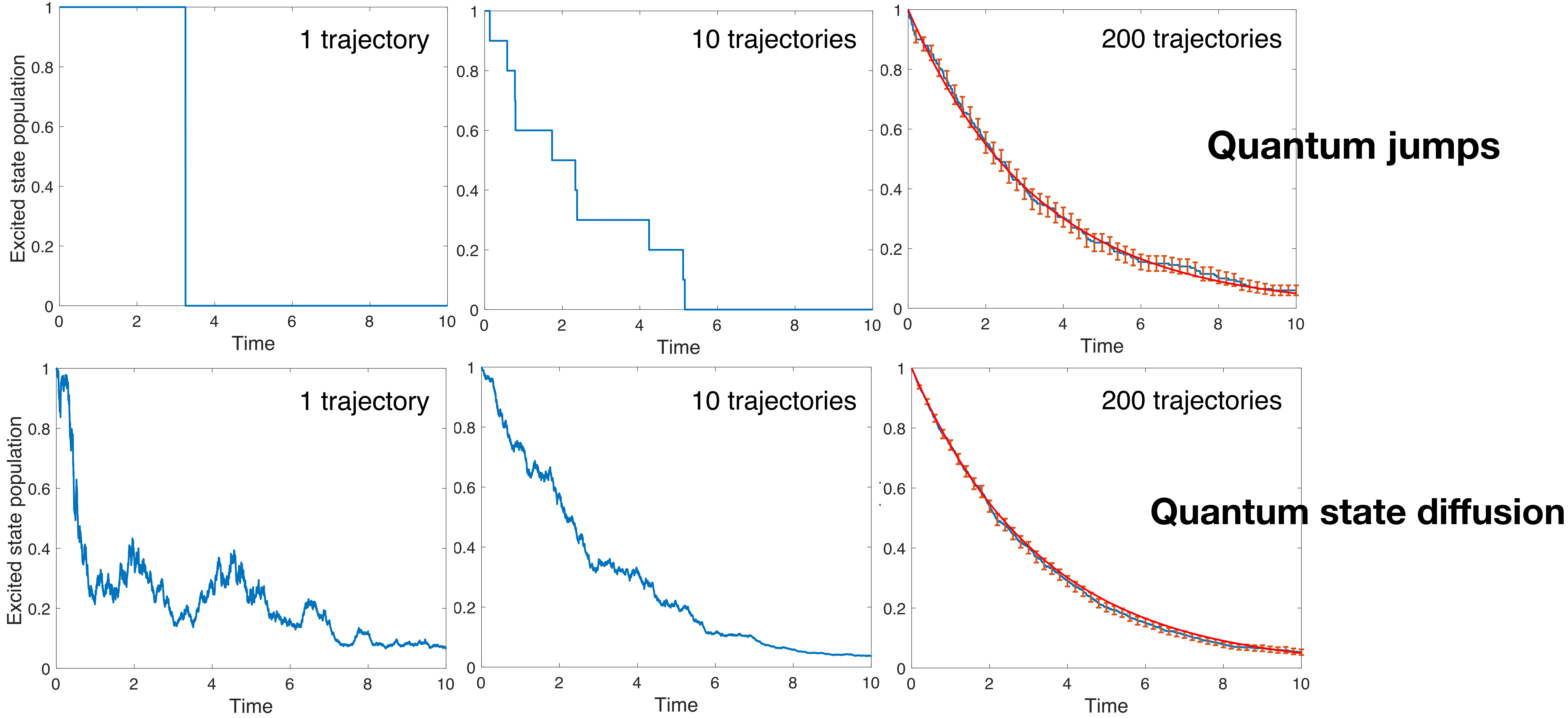
We observe that individual trajectories are noisy and exhibit numerous jumps but their average reproduce the exact results.

A. J. Daley, Advances in Physics 63, 77 (2014)

Example: two-level atom and spontaneous emission

$$\dot{\rho}(t) = -i[H, \rho(t)] + \frac{\Gamma}{2}\left[2\sigma_-\rho(t)\sigma_+ - \sigma_+\sigma_-\rho(t) - \rho(t)\sigma_+\sigma_-\right]$$

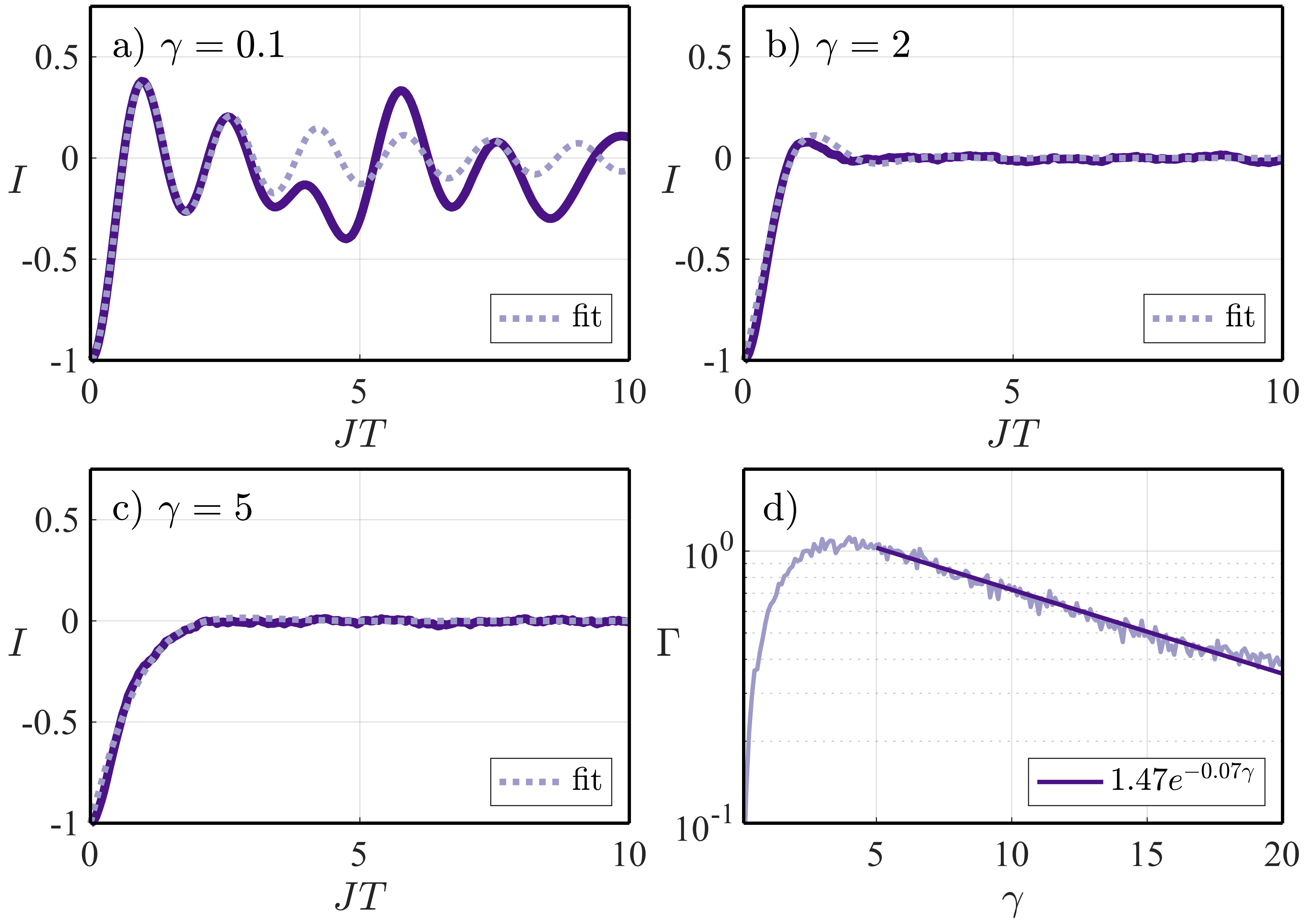




Results for fermions

Our system

We consider a 1D chain of interacting **fermionic** boson atoms under continuous monitoring:



T. Bintener, **JYM**, M. Buchhold, P. Kirton, S. Diehl, A. Daley, in preparation (2022)

Link to scrambling and black hole models