Outcomes of repeated measurements

on non-replicable Unruh-DeWitt detectors

N. Pranzini, G. García-Pérez, E. Keski-Vakkuri, S. Maniscalco

27th October 2022 - QFC 2022

Univeristy of Helsinki - QTF Centre of Excellence
InstituteQ - The Finnish Quantum Institute
Università degli studi di Pisa

Table of contents

AIMS:

1. extend the Born rule to non-replicable systems
2. provide an example of this procedure

What is the Born rule?

Given

- a quantum system in the state $|\psi\rangle$;
- a measurement described by a POVM $\{\hat{E}_m\}$;

What is the Born rule?

Given

- a quantum system in the state $|\psi\rangle$;
- a measurement described by a POVM $\{\hat{E}_m\}$;

Born rule: the probability finding the outcome m is

$$p_m = \langle\psi| \hat{E}_m |\psi\rangle \ .$$

What is the Born rule?

Given

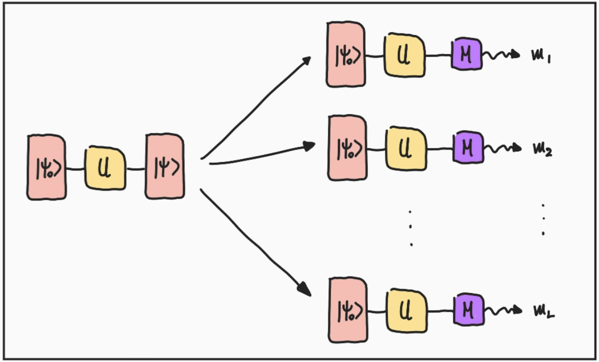
- a quantum system in the state $|\psi\rangle$;
- a measurement described by a POVM $\{\hat{E}_m\}$;

Born rule: the probability finding the outcome m is

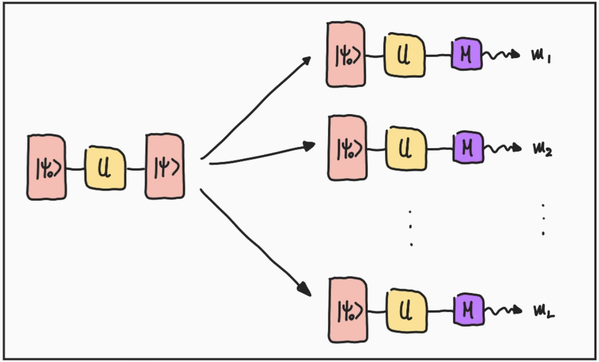
$$p_m = \langle\psi| \hat{E}_m |\psi\rangle \; .$$

$\Rightarrow$ intrinsic probabilistic & frequentist meaning

What is domain of validity of the Born rule?

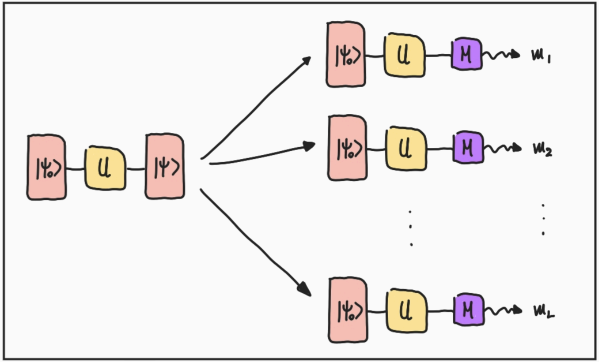


What is domain of validity of the Born rule?



Basic assumption: we can replicate without error any number of times $\Rightarrow$ *i.i.d.*

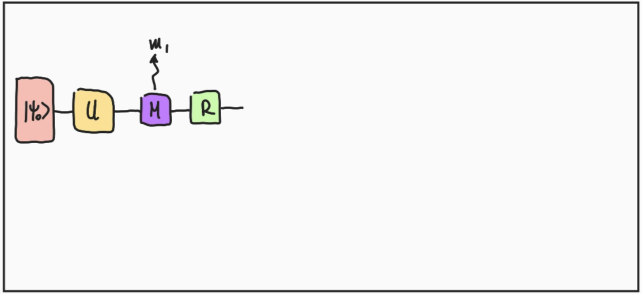
What is domain of validity of the Born rule?



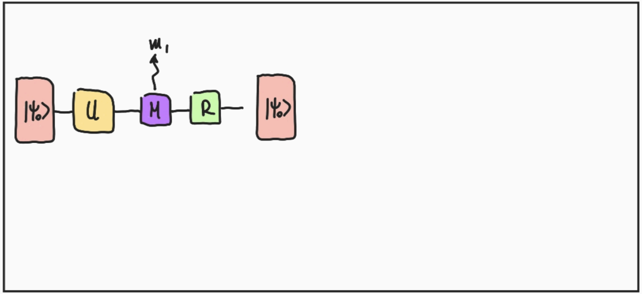
Basic assumption: we can replicate without error any number of times $\Rightarrow$ *i.i.d.*

Let us rephrase this procedure in a slightly different way

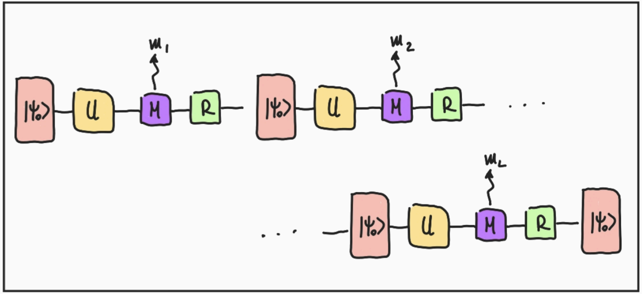
A different take on the Born rule



A different take on the Born rule



A different take on the Born rule



Identical distribution but with **one** system (*i.i.d.*)

It can be a non-replicable system

Non-replicable systems

Non-replicable system cannot be copied.

The *i.i.d.* often fails:

Non-replicable systems

Non-replicable system cannot be copied.
The *i.i.d.* often fails:

- A system with parts we cannot control
  (e.g. a qubit interacting with a large environment)

Non-replicable systems

Non-replicable system cannot be copied.
The *i.i.d.* often fails:

- A system with parts we cannot control
(e.g. a qubit interacting with a large environment)
- An evolving system we can observe only partially
(e.g. a *quantum* universe)

Non-replicable systems

Non-replicable system cannot be copied.

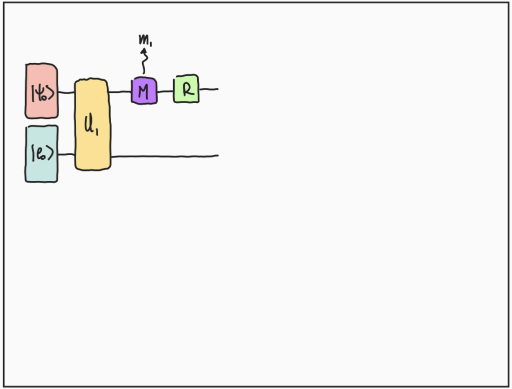
The *i.i.d.* often fails:

- A system with parts we cannot control
  (e.g. a qubit interacting with a large environment)
- An evolving system we can observe only partially
  (e.g. a *quantum* universe)

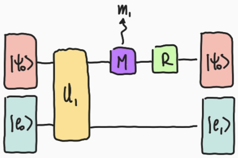
Can we extend the Born rule to these systems?

$\Rightarrow$ Repeated Measurement (RM) framework

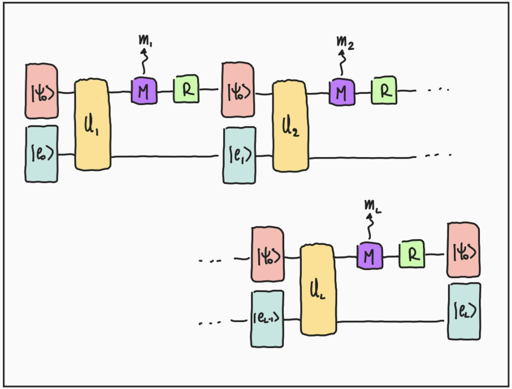
RM scenario



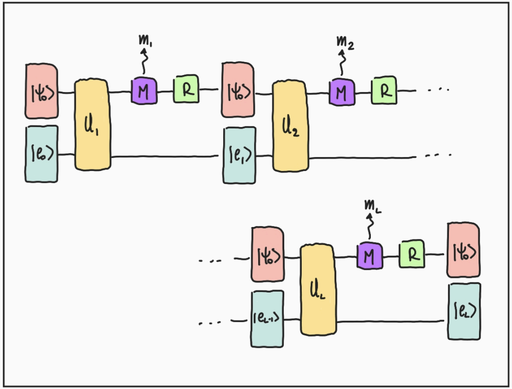
RM scenario



RM scenario



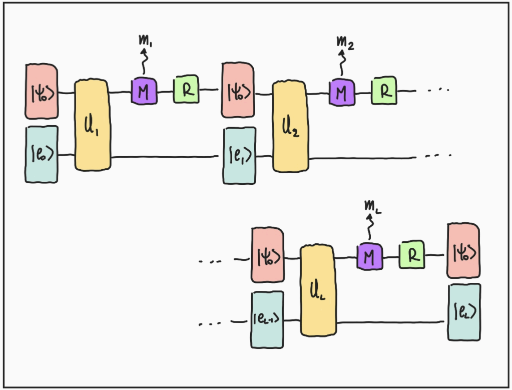
RM scenario



Born case: $P(m_i)$ $\longrightarrow$ RM case: $P(m_i|m_1, \ldots, m_{i-1})$

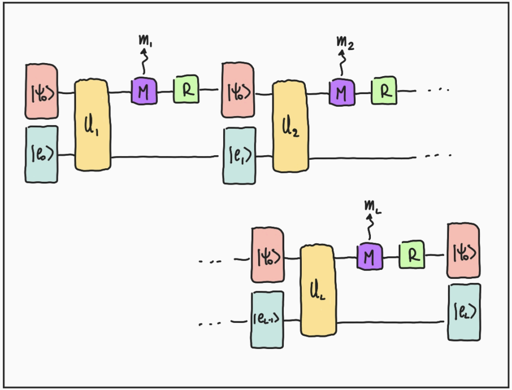
Hard to evaluate!

RM scenario - Weak interaction

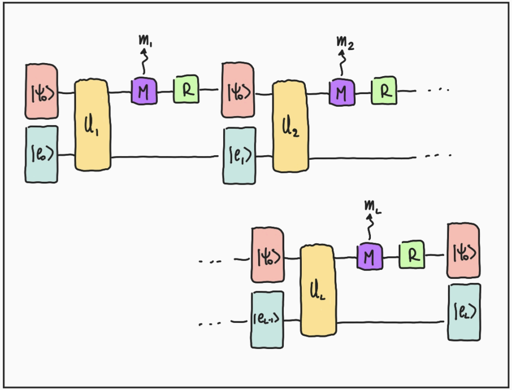


$$\hat{U}_k = \hat{U} \otimes \mathbb{I}_{\mathcal{E}} + \epsilon \sum_l \hat{A}_l \otimes \hat{B}_l(k) + O(\epsilon^2)$$

RM scenario - Weak interaction



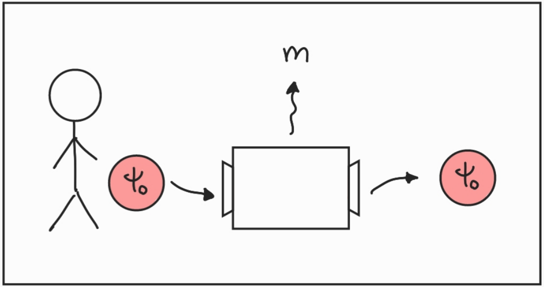
$$\hat{U}_k = \hat{U} \otimes \mathbb{I}_{\mathcal{E}} + \epsilon \sum_l \hat{A}_l \otimes \hat{B}_l(k) + O(\epsilon^2) \Rightarrow \begin{cases} p_m(k)[e_{k-1}] = p_m + \epsilon Q_m^{(1)}(k)[e_{k-1}] + O(\epsilon^2) \\ p_m = \langle m | \hat{U} | 0 \rangle^2 \end{cases}$$



$$\hat{U}_k = \hat{U} \otimes \mathbb{I}_{\mathcal{E}} + \epsilon \sum_l \hat{A}_l \otimes \hat{B}_l(k) + O(\epsilon^2) \Rightarrow \begin{cases} p_m(k)[e_{k-1}] = p_m + \epsilon Q_m^{(1)}(k)[e_{k-1}] + O(\epsilon^2) \\ p_m = \langle m | \hat{U} | 0 \rangle^2 \end{cases}$$

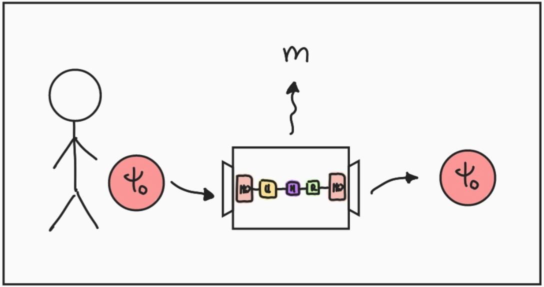
How to decide between using Born and RM?

Effective Born rule from RM - Setup



$$M_L = (m_1, \dots, m_L)$$

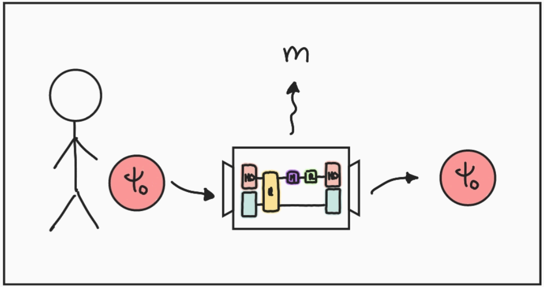
Effective Born rule from RM - Setup



$$M_L = (m_1, \ldots, m_L)$$

$$P(M_L) = P(m_1)P(m_2)\ldots P(m_L)$$

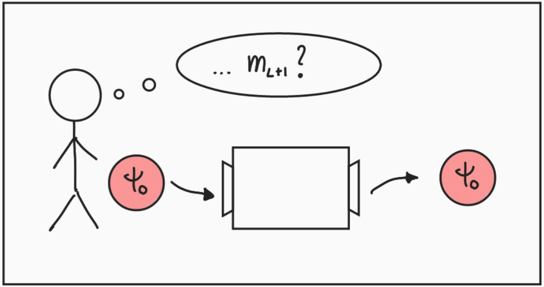
Effective Born rule from RM - Setup



$$M_L = (m_1, \ldots, m_L)$$

$$P(M_L) = P(m_1)P(m_2)\ldots P(m_L)$$
$$\tilde{P}(M_L) = P(m_1)P(m_2)\ldots P(m_L) + \epsilon \Delta P(M_L)$$

Effective Born rule from RM - Setup



$$M_L = (m_1, \ldots, m_L)$$

$$P(M_L) = P(m_1)P(m_2)\ldots P(m_L)$$
$$\tilde{P}(M_L) = P(m_1)P(m_2)\ldots P(m_L) + \epsilon \Delta P(M_L)$$

Which one Alice must use to predict future outcomes?

Effective Born rule from RM - Details in the general case

Alice formulates two hypotheses:

- $\mathcal{H}_1$: Born rule holds strictly: $p(m_k) = \langle m_k | \hat{U} | 0 \rangle^2$
- $\mathcal{H}_2$: Born rule holds approximately: $p(m_k | M_{k-1}) = p(m_k) + \epsilon \Delta p(m_k | M_{k-1})$

Effective Born rule from RM - Details in the general case

Alice formulates two hypotheses:

- $\mathcal{H}_1$: Born rule holds strictly: $p(m_k) = \langle m_k | \hat{U} | 0 \rangle^2$
- $\mathcal{H}_2$: Born rule holds approximately: $p(m_k|M_{k-1}) = p(m_k) + \epsilon \Delta p(m_k|M_{k-1})$

To select $\mathcal{H}_2$ (or vice-versa) it must be

$$\frac{P(\mathcal{H}_2|M_L)}{P(\mathcal{H}_1|M_L)} \gg 1$$

Effective Born rule from RM - Details in the general case

Alice formulates two hypotheses:

- $\mathcal{H}_1$: Born rule holds strictly: $p(m_k) = \langle m_k| \hat{U} |0\rangle^2$
- $\mathcal{H}_2$: Born rule holds approximately: $p(m_k|M_{k-1}) = p(m_k) + \epsilon \Delta p(m_k|M_{k-1})$

To select $\mathcal{H}_2$ (or vice-versa) it must be

$$\frac{P(\mathcal{H}_2|M_L)}{P(\mathcal{H}_1|M_L)} \gg 1 \Rightarrow \frac{\Delta P(M_L)}{P(M_L)} \gg \frac{1}{\epsilon}$$

Effective Born rule from RM - Details in the general case

Alice formulates two hypotheses:

- $\mathcal{H}_1$: Born rule holds strictly: $p(m_k) = \langle m_k | \hat{U} | 0 \rangle^2$
- $\mathcal{H}_2$: Born rule holds approximately: $p(m_k | M_{k-1}) = p(m_k) + \epsilon \Delta p(m_k | M_{k-1})$

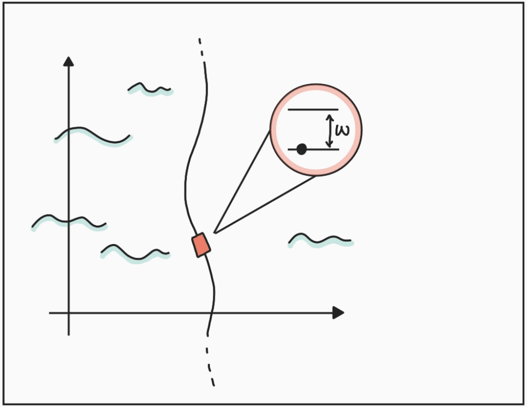
To select $\mathcal{H}_2$ (or vice-versa) it must be

$$\frac{P(\mathcal{H}_2 | M_L)}{P(\mathcal{H}_1 | M_L)} \gg 1 \;\Rightarrow\; \frac{\Delta P(M_L)}{P(M_L)} \gg \frac{1}{\epsilon}$$

$\Delta P(M_L) \ll P(M_L) \;\Rightarrow\;$ Inability to select $\;\Rightarrow\;$ FAPP, RM $\simeq$ Born

Unruh-DeWitt detectors

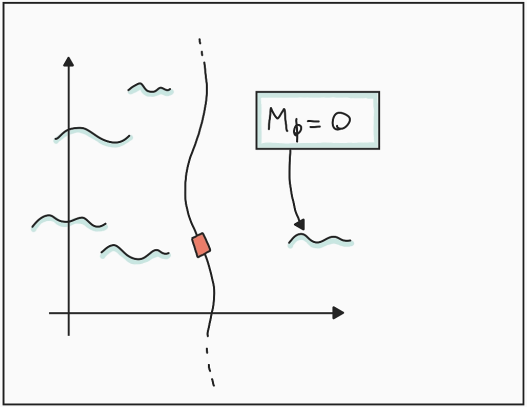
W. G. Unruh (1976) & **B. S. DeWitt** (1980)



$$X(\tau) = (t(\tau), \mathbf{x}(\tau)) \quad \text{and} \quad \hat{H}_D = \omega \left|1\right\rangle \left\langle 1\right| \quad \text{initially in} \quad \left|0\right\rangle$$

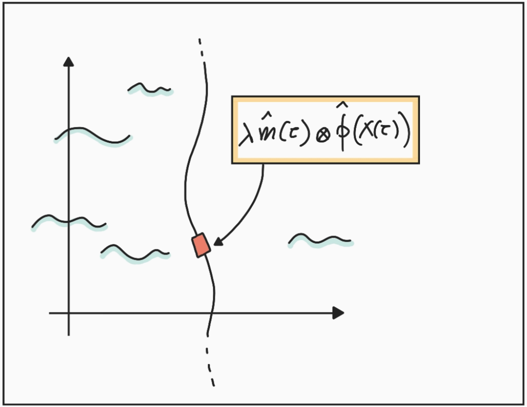
Unruh-DeWitt detectors

W. G. Unruh (1976) & **B. S. DeWitt** (1980)



$$\hat{H}_\phi = \frac{1}{2} \int \partial_\mu \phi(x) \partial^\mu \phi(x) d^4x \quad \text{initially in} \quad |0_M\rangle$$

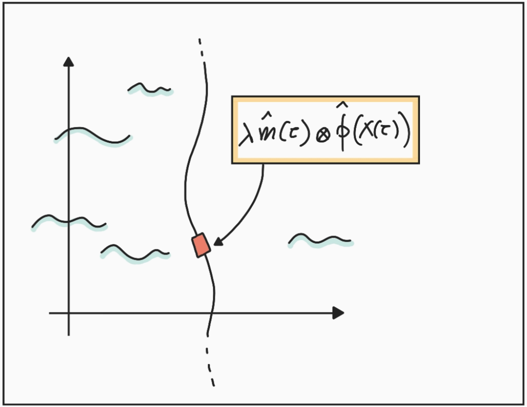
W. G. Unruh (1976) & **B. S. DeWitt** (1980)



$$\hat{H}_{int}(\tau) = \chi(\tau)\hat{m}(\tau) \otimes \hat{\phi}(X(\tau))$$

Unruh-DeWitt detectors

W. G. Unruh (1976) & **B. S. DeWitt** (1980)



$$\hat{H}_{tot}(\tau) = \hat{H}_D + \hat{H}_\phi + \lambda \hat{H}_{int}(\tau)$$

Unruh-DeWitt detectors - The switching function

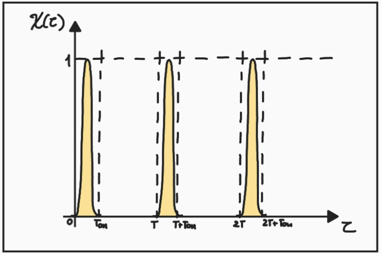
Switching function $\chi(\tau)$ describes interaction times

- must be smooth
- has compact support (at least FAPP)

Unruh-DeWitt detectors - The switching function

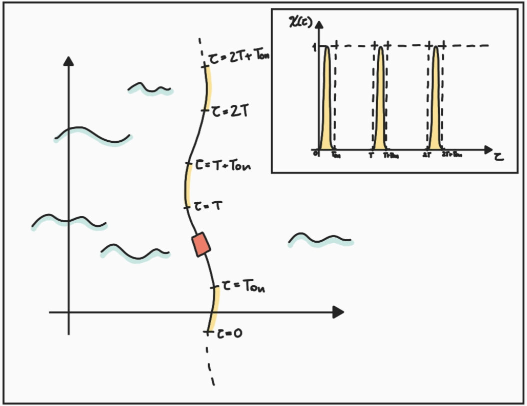
Switching function $\chi(\tau)$ describes interaction times

- must be smooth
- has compact support (at least FAPP)



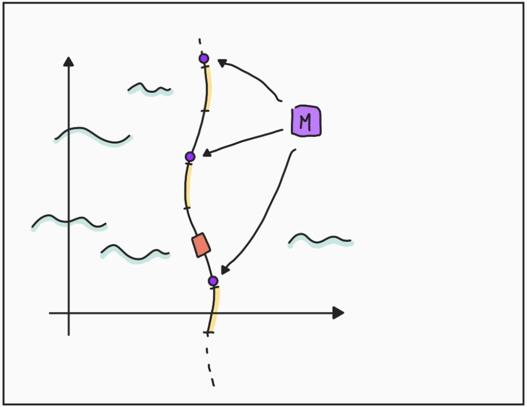
$$\hat{H}_{tot}(\tau) = \hat{H}_D + \hat{H}_\phi + \lambda \hat{H}_{\text{int}}(\tau) \quad \longrightarrow \quad \hat{H}_{tot}(\tau) = \hat{H}_D + \hat{H}_\phi + \lambda \sum_{k=0} \chi_k(\tau) \hat{m}(\tau) \otimes \hat{\phi}(X(\tau))$$

RM on UDW detectors



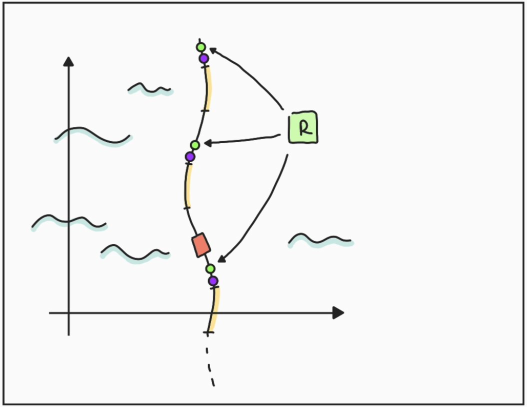
RM on UDW detectors

J. Polo-Gómez, Et. Al., Phys. Rev. D 105 (2022)



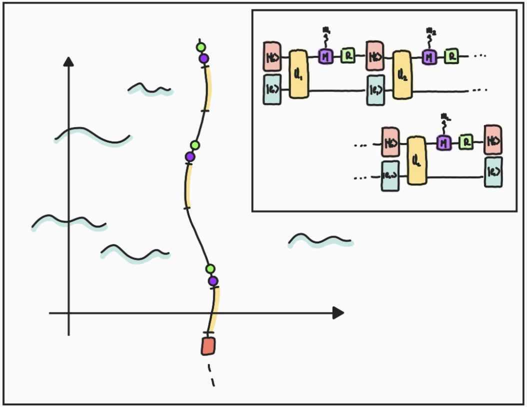
$$\hat{M}_0 = |0\rangle\langle 0| \quad , \quad \hat{M}_1 = |1\rangle\langle 1|$$

RM on UDW detectors

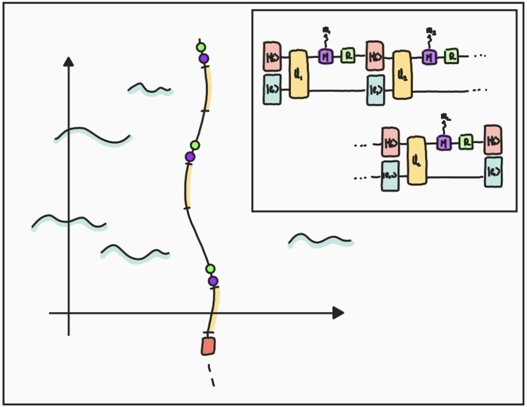


$$\hat{R} |\psi\rangle \longmapsto |0\rangle$$

RM on UDW detectors



RM on UDW detectors



$$\Rightarrow \quad M_L = (m_1, \dots, m_L) \mapsto B_L = (b_1, \dots, b_L)$$

We need $P(b_{L+1} = 1 | B_L)$ and $P(b_{L+1} = 0 | B_L)$

History-dependent transition probabilities

$$P(b_{L+1} = 1 | B_L) = \frac{\lambda^2}{\prod \mathcal{P}_j} \iint_{N_1,\ldots,N_n,L+1} \mathcal{W}_{2(n+1)}(X_{L+1}, X'_{L+1}, \ldots, X_{N_1}, X'_{N_1})$$

History-dependent transition probabilities

$$P(b_{L+1} = 1|B_L) = \frac{\lambda^2}{\prod \mathcal{P}_j} \iint_{N_1,\ldots,N_n,L+1} \mathcal{W}_{2(n+1)}(X_{L+1}, X'_{L+1}, \ldots, X_{N_1}, X'_{N_1})$$

We need **3** necessary and **2** auxiliary assumptions...

...amongst the others

History-dependent transition probabilities

$$P(b_{L+1} = 1 | B_L) = \frac{\lambda^2}{\prod \mathcal{P}_j} \iint_{N_1,\ldots,N_n,L+1} \mathcal{W}_{2(n+1)}(X_{L+1}, X'_{L+1}, \ldots, X_{N_1}, X'_{N_1})$$

We need **3** necessary and **2** auxiliary assumptions...

...amongst the others on trajectory,

A0: $\mathcal{W}(\tau', \tau) = \mathcal{W}(\tau' - \tau)$, ok for **Inertial** and **Accelerated**

History-dependent transition probabilities

$$P(b_{L+1} = 1 | B_L) = \frac{\lambda^2}{\prod \mathcal{P}_j} \iint_{N_1,\dots,N_n,L+1} \mathcal{W}_{2(n+1)}(X_{L+1}, X'_{L+1}, \dots, X_{N_1}, X'_{N_1})$$

We need **3** necessary and **2** auxiliary assumptions...

...amongst the others on trajectory, and switching

A0: $\mathcal{W}(\tau', \tau) = \mathcal{W}(\tau' - \tau)$, ok for **Inertial** and **Accelerated**

B2: $T_{off} \gg T_{on}$

History-dependent transition probabilities

$$P(b_{L+1} = 1 | B_L) = \frac{\lambda^2}{\prod \mathcal{P}_j} \iint_{N_1,\ldots,N_n,L+1} \mathcal{W}_{2(n+1)}(X_{L+1}, X'_{L+1}, \ldots, X_{N_1}, X'_{N_1})$$

We need **3** necessary and **2** auxiliary assumptions...
...amongst the others on trajectory,

A0: $\mathcal{W}(\tau', \tau) = \mathcal{W}(\tau' - \tau)$, ok for **Inertial** and **Accelerated**

B2: $T_{off} \gg T_{on}$

To obtain result, we need a specific switching

Gaussian switching and its consequences

- $\chi(\tau)$ is collection of **well spaces** gaussian peaks as switching function

Gaussian switching and its consequences

L. Sriramkumar and T. Padmanabhan, Class Quantum Gravity 13, (1996)

- $\chi(\tau)$ is collection of **well spaces** gaussian peaks as switching function
- Born probabilities for a single gaussian peak are known: q_I and q_A

Gaussian switching and its consequences

L. Sriramkumar and T. Padmanabhan, Class Quantum Gravity 13, (1996)

- $\chi(\tau)$ is collection of **well spaces** gaussian peaks as switching function
- Born probabilities for a single gaussian peak are known: q_I and q_A

Hence:

- Born case: $P_{q_\iota}(B_L) = q_\iota^n (1 - q_\iota)^{(L-n)}$

Gaussian switching and its consequences

L. Sriramkumar and T. Padmanabhan, Class Quantum Gravity 13, (1996)

- $\chi(\tau)$ is collection of **well spaces** gaussian peaks as switching function
- Born probabilities for a single gaussian peak are known: q_I and q_A

Hence:

- Born case: $P_{q_\iota}(B_L) = q_\iota^n (1 - q_\iota)^{(L-n)}$
- RM case: $\tilde{P}_{q_\iota}(B_L) = q_\iota^n (1 - q_\iota)^{(L-n)} + corrections$

Gaussian switching and its consequences

L. Sriramkumar and T. Padmanabhan, Class Quantum Gravity 13, (1996)

- $\chi(\tau)$ is collection of **well spaces** gaussian peaks as switching function
- Born probabilities for a single gaussian peak are known: q_I and q_A

Hence:

- Born case: $P_{q_\iota}(B_L) = q_\iota^n(1 - q_\iota)^{(L-n)}$
- RM case: $\tilde{P}_{q_\iota}(B_L) = q_\iota^n(1 - q_\iota)^{(L-n)} + corrections$

In the Born case, what results should we expect?

Interlude - Standard results from UDW theory

$$B_L = (b_1, \ldots, b_L) \quad \Rightarrow \quad \mathcal{R}^{sampled} = \frac{n}{L-n}$$

Interlude - Standard results from UDW theory

$$B_L = (b_1, \ldots, b_L) \;\Rightarrow\; \mathcal{R}^{sampled} = \frac{n}{L-n}$$

For q_I and q_A:

$$\mathcal{R}_I^{sampled} \longrightarrow \frac{p_I}{1-p_I} = \mathcal{R}_I^{theo}$$

$$\mathcal{R}_A^{sampled} \longrightarrow \frac{p_A}{1-p_A} = \mathcal{R}_A^{theo}$$

Interlude - Standard results from UDW theory

L. Sriramkumar and T. Padmanabhan, Class Quantum Gravity 13, (1996)

$$B_L = (b_1, \ldots, b_L) \quad \Rightarrow \quad \mathcal{R}^{sampled} = \frac{n}{L-n}$$

For q_I and q_A:

$$\mathcal{R}_I^{sampled} \longrightarrow \frac{p_I}{1-p_I} = \mathcal{R}_I^{theo} \simeq \mathcal{R}_I^{\infty} = 0$$

$$\mathcal{R}_A^{sampled} \longrightarrow \frac{p_A}{1-p_A} = \mathcal{R}_A^{theo} \simeq \mathcal{R}_A^{\infty} = \exp(-\omega/T_U)$$

Interlude - Standard results from UDW theory

L. Sriramkumar and T. Padmanabhan, Class Quantum Gravity 13, (1996)

$$B_L = (b_1, \ldots, b_L) \;\Rightarrow\; \mathcal{R}^{sampled} = \frac{n}{L-n}$$

For q_I and q_A:

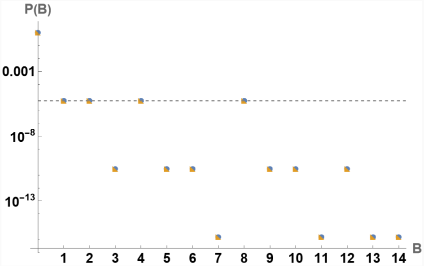
$$\mathcal{R}_I^{sampled} \longrightarrow \frac{p_I}{1-p_I} = \mathcal{R}_I^{theo} \simeq \mathcal{R}_I^{\infty} = 0$$

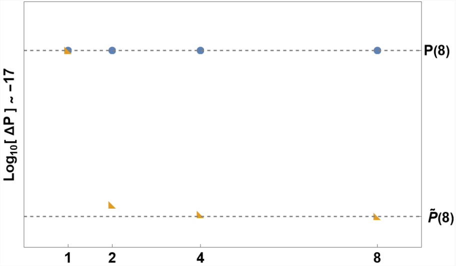
$$\mathcal{R}_A^{sampled} \longrightarrow \frac{p_A}{1-p_A} = \mathcal{R}_A^{theo} \simeq \mathcal{R}_A^{\infty} = \exp(-\omega/T_U)$$

$\mathcal{R}^{sampled} \longmapsto \mathcal{R}_{I/A}^{\infty}$ encoding the interesting results

Results - RM on inertial UDW

$$X(\tau) = (\tau, x_0)$$

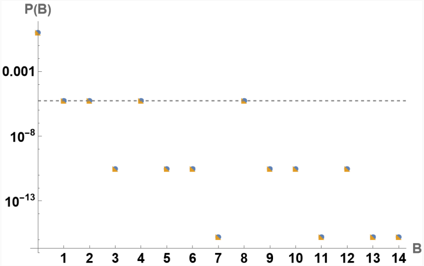


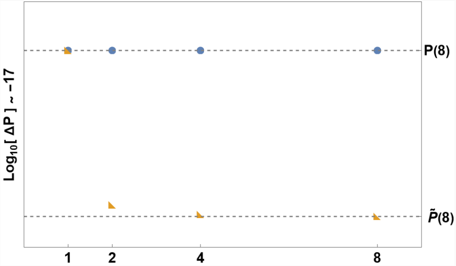


$$\omega = 0.2,\ \sigma = 1,\ T_{on} = T_{off}/10 = 8\sigma, \text{ and } \lambda = 10^{-2}$$

Results - RM on inertial UDW

$$X(\tau) = (\tau, x_0)$$

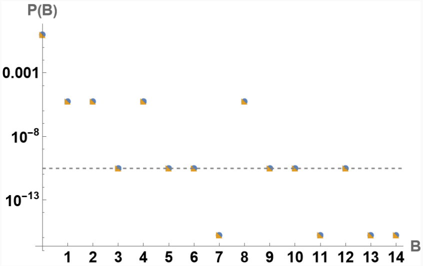


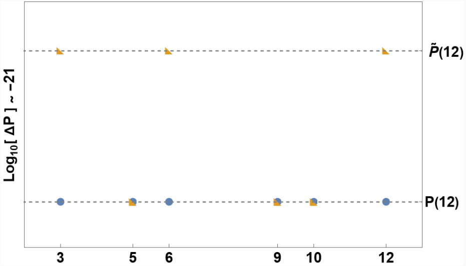


$$\omega = 0.2,\ \sigma = 1,\ T_{on} = T_{off}/10 = 8\sigma, \text{ and } \lambda = 10^{-2}$$

FAPP, RM gives same results as Born

$$X(\tau) = (\cosh(\tau/\alpha)/\alpha, x_0, y_0, \sinh(\tau/\alpha)\alpha)$$

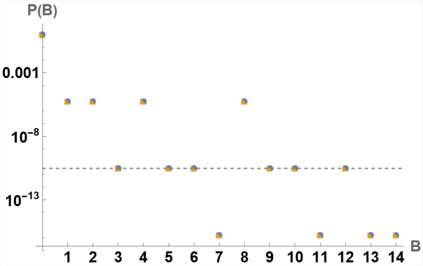


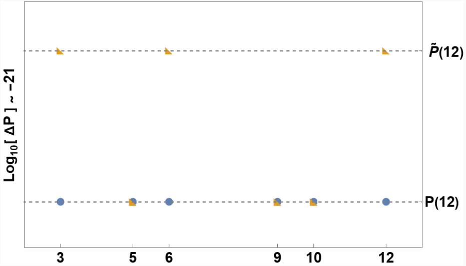


$\omega = 0.2$, $\sigma = 1$, $T_{on} = T_{off}/10 = 8\sigma$, $\lambda = 10^{-2}$, and $g = 0.1$.

Results - RM on accelerated UDW

$$X(\tau) = (\cosh(\tau/\alpha)/\alpha, x_0, y_0, \sinh(\tau/\alpha)\alpha)$$





$\omega = 0.2$, $\sigma = 1$, $T_{on} = T_{off}/10 = 8\sigma$, $\lambda = 10^{-2}$, and $g = 0.1$.

FAPP, RM gives same results as Born $\Rightarrow$ Unruh effect seen via RM

Summary

arxiv.org/2210.13347

In this work we:

- discussed **limits of the Born rule**
- defined **non-replicable systems**

Summary

arxiv.org/2210.13347

In this work we:

- discussed **limits of the Born rule**
- defined **non-replicable systems**
- provided a way to test non-replicable systems via **Repeated Measurements**...
- ... and shown that the **Born** rule can **hold FAPP**

Summary

arxiv.org/2210.13347

In this work we:

- discussed **limits of the Born rule**
- defined **non-replicable systems**
- provided a way to test non-replicable systems via **Repeated Measurements**...
- ... and shown that the **Born** rule can **hold FAPP**
- gave an application to **Unruh-DeWitt detectors**...
- ...showing that the **Unruh effect** can be **tested via RM**

Summary

arxiv.org/2210.13347

In this work we:

- discussed **limits of the Born rule**
- defined **non-replicable systems**
- provided a way to test non-replicable systems via **Repeated Measurements**...
- ... and shown that the **Born** rule can **hold FAPP**
- gave an application to **Unruh-DeWitt detectors**...
- ...showing that the **Unruh effect** can be **tested via RM**

Prospects: backreaction, QC experiment, measurement as quench on CFT boundary

Summary

arxiv.org/2210.13347

In this work we:

- discussed **limits of the Born rule**
- defined **non-replicable systems**
- provided a way to test non-replicable systems via **Repeated Measurements...**
- ... and shown that the **Born** rule can **hold FAPP**
- gave an application to **Unruh-DeWitt detectors...**
- ...showing that the **Unruh effect** can be **tested via RM**

Prospects: backreaction, QC experiment, measurement as quench on CFT boundary

Thank you for your attention!

History-dependent transition probabilities

$$P(b_{L+1} = 1|B_L) = \frac{\lambda^2}{\prod \mathcal{P}_j} \iint_{N_1,\ldots,N_n,L+1} \mathcal{W}_{2(n+1)}(X_{L+1}, X'_{L+1}, \ldots, X_{N_1}, X'_{N_1})$$

We need **3** necessary and **2** auxiliary assumptions...
...amongst the others

A0: $\mathcal{W}(\tau', \tau) = \mathcal{W}(\tau' - \tau)$

A1: the 2-point Wightman function is definite negative, monotonously increasing and such that $\lim_{s\to 0} \mathcal{W}(s) = -\infty$.

A2: $T_{\text{on}}\omega \leq \pi/2$.

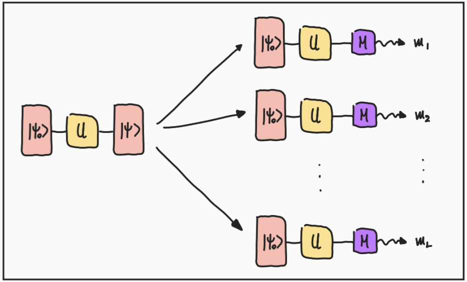
B1: $s \gg s' \;\Rightarrow\; \mathcal{W}(s) \gg \mathcal{W}(s')$.

B2: $T_{\text{off}} \gg T_{\text{on}}$, meaning that the detector rests long times between each measurement.

Necessity of RM vs. Born

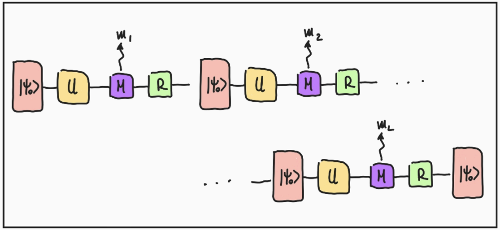
	i.i.d. outcomes	non *i.i.d.* outcomes
$\mathcal{S}$ replicable		
$\mathcal{S}$ non-replicable		

Necessity of RM vs. Born



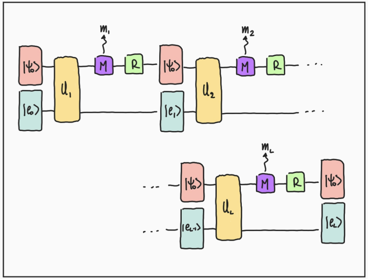
	i.i.d. outcomes	non *i.i.d.* outcomes
$\mathcal{S}$ replicable	$\{m_i\}$	
$\mathcal{S}$ non-replicable		

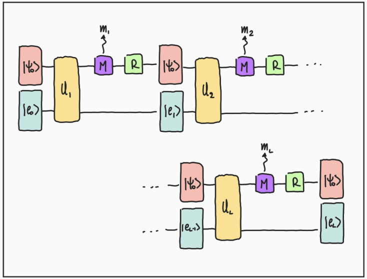
Necessity of RM vs. Born



	i.i.d. outcomes	non *i.i.d.* outcomes
$\mathcal{S}$ replicable	$\{m_i\}$	
$\mathcal{S}$ non-replicable	$\{m_i\}$	

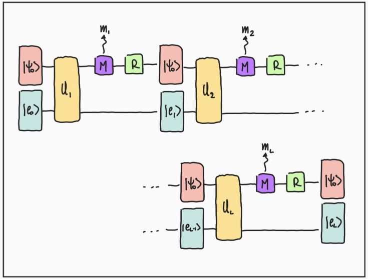
Necessity of RM vs. Born





	i.i.d. outcomes	non *i.i.d.* outcomes
$\mathcal{S}$ replicable	$\{m_i\}$	$\{(m_1, \ldots, m_L)\}$
$\mathcal{S}$ non-replicable	$\{m_i\}$	

Necessity of RM vs. Born



	i.i.d. outcomes	non *i.i.d.* outcomes
$\mathcal{S}$ replicable	$\{m_i\}$	$\{(m_1, \ldots, m_L)\}$
$\mathcal{S}$ non-replicable	$\{m_i\}$	~~Born~~, we need RM

Field's state update

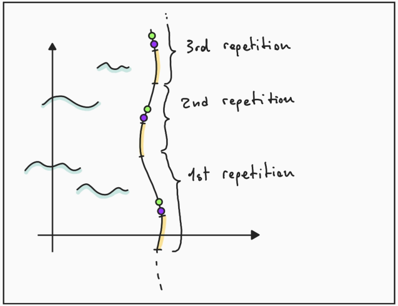
At first order

$$|\psi_0\rangle = |0\rangle \otimes |0_M\rangle$$

$$\xrightarrow{\text{int}} |0\rangle \otimes |0_M\rangle + \lambda |1\rangle \otimes |\phi_1\rangle + O(\lambda^2)$$

$$\xrightarrow{\text{M}} \begin{cases} |0\rangle \otimes |0_M\rangle \\ |1\rangle \otimes |\phi_1\rangle \end{cases}$$

$$\xrightarrow{R} \begin{cases} |0\rangle \otimes |0_M\rangle \\ |0\rangle \otimes |\phi_1\rangle \end{cases}$$



- The field state is contextual to the observer
- The collapse in the future lightcone $\mathcal{D}^+(M_1)$
- The detector never leaves $\mathcal{D}^+(M_1)$
- We can take the collapsed state

Effective Born rule from RM - Details for bit strings

Suppose $\{m_0, m_1\} = \{0, 1\}$ with $p_1 = q$ (and $p_0 = 1 - q$), then

Effective Born rule from RM - Details for bit strings

Suppose $\{m_0, m_1\} = \{0, 1\}$ with $p_1 = q$ (and $p_0 = 1 - q$), then

$$M_L = (m_1, \ldots, m_L) \mapsto (L; N_1, \ldots, N_n)$$

Effective Born rule from RM - Details for bit strings

Suppose $\{m_0, m_1\} = \{0,1\}$ with $p_1 = q$ (and $p_0 = 1-q$), then

$$M_L = (m_1, \ldots, m_L) \mapsto (L; N_1, \ldots, N_n)$$

$$P_q(M_L) = \prod_{\substack{j=1,\ldots,L \\ j\neq N_1,\ldots,N_n}} P(0|M_j) \prod_{j=N_1,\ldots,N_n} P(1|M_j) = q^n (1-q)^{L-n} \ .$$

Effective Born rule from RM - Details for bit strings

Suppose $\{m_0, m_1\} = \{0,1\}$ with $p_1 = q$ (and $p_0 = 1 - q$), then

$$M_L = (m_1, \ldots, m_L) \mapsto (L; N_1, \ldots, N_n)$$

$$P_q(M_L) = \prod_{\substack{j=1,\ldots,L \\ j\neq N_1,\ldots,N_n}} P(0|M_j) \prod_{j=N_1,\ldots,N_n} P(1|M_j) = q^n (1-q)^{L-n} \,.$$

$$\tilde{P}_q(M_L) = P_q(M_L) + \epsilon P_q(M_L) \left(\sum_{j=N_1,\ldots,N_n} \frac{Q^{(1)}_{b_j}(j)[f_j]}{q} + \sum_{\substack{j=1,\ldots,L \\ j\neq N_1,\ldots,N_n}} \frac{Q^{(1)}_{b_j}(j)[f_j]}{1-q} \right) \,,$$

References i

demo