Coherent and Incoherent Structures in Fuzzy Dark Matter Halos

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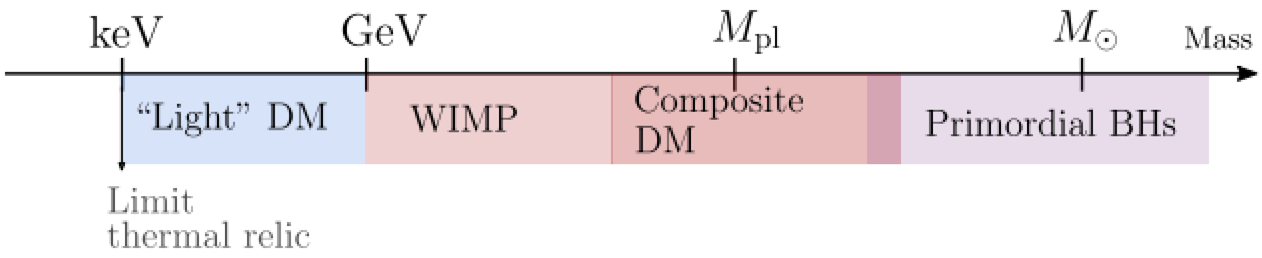
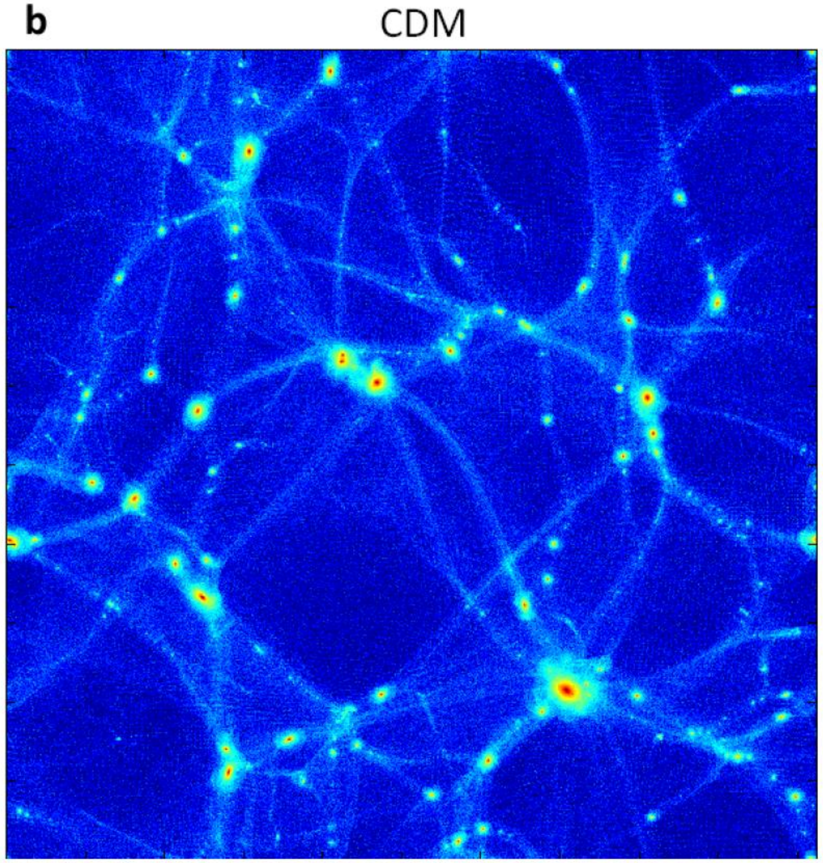
N. Proukakis


M. Indjin

QFC2022, Pisa, 26.10.2022

Fuzzy Dark Matter





Schive et al., Nat.Phys. 10, 496 (2014)

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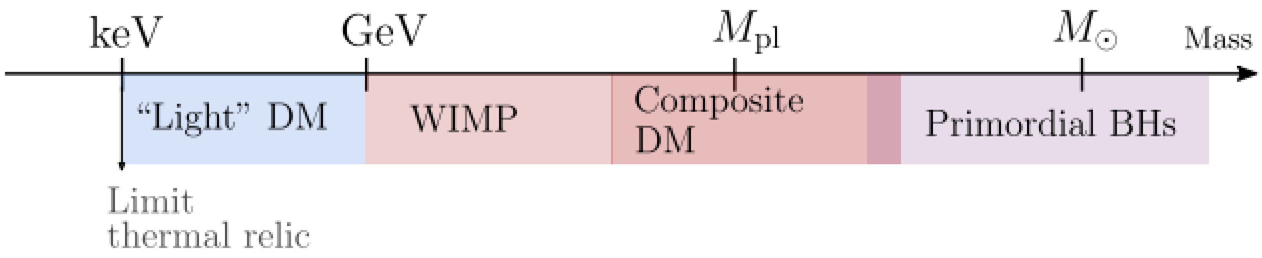
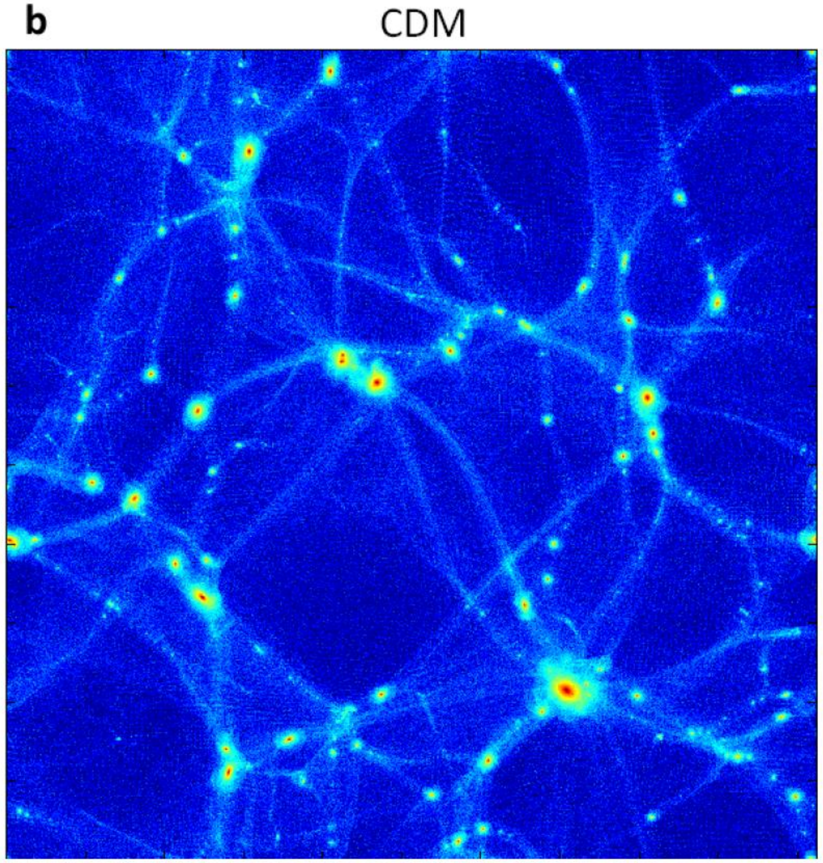


Small scale crises

- Missing satellites problem
- Too-big-to-fail problem
- Cusp-core problem

$$\rho_{\rm NFW}(r) = \rho_h \left(\frac{r}{r_h}\right)^{-1} \left(1 + \frac{r}{r_h}\right)^{-2}$$



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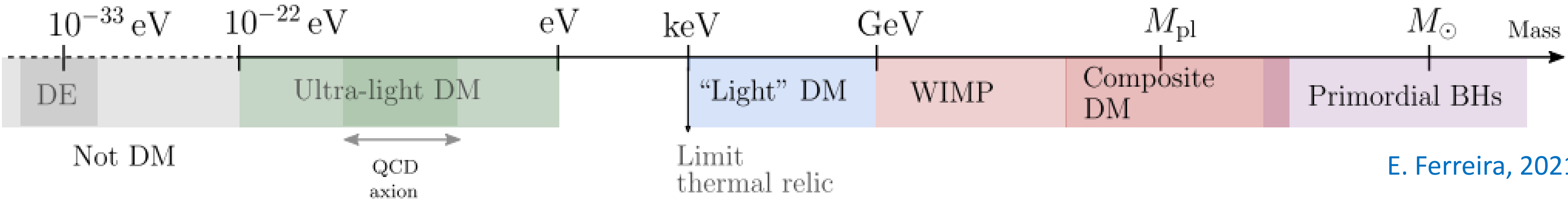
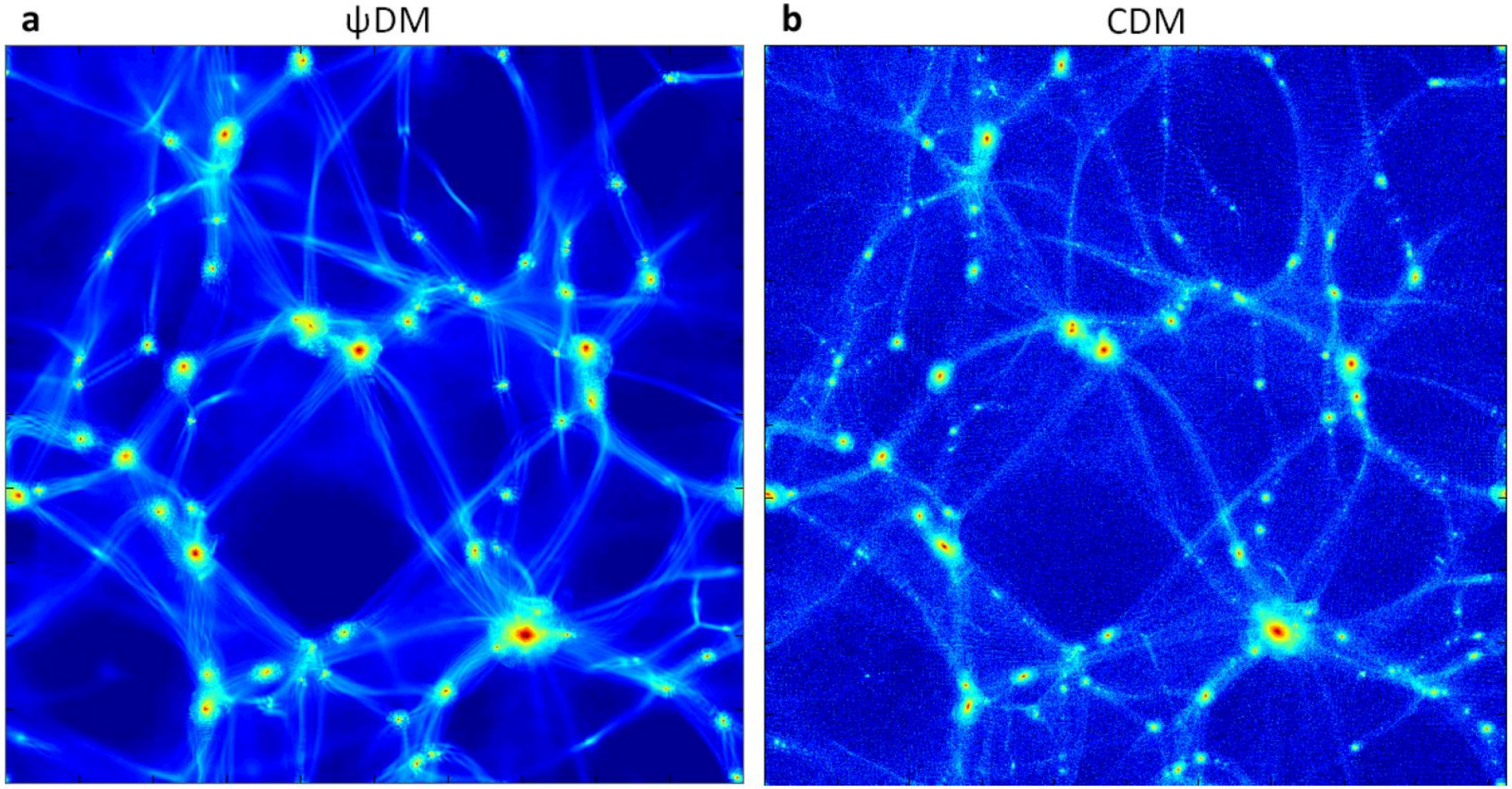


E. Ferreira, 2021

$$\lambda_{dB} = \frac{h}{mv}$$



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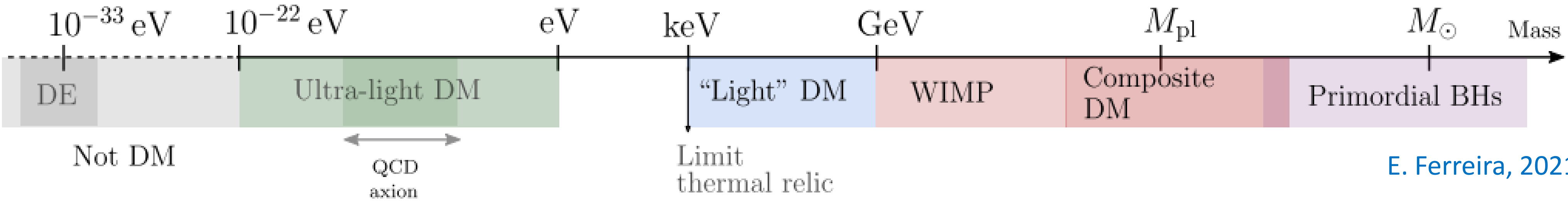

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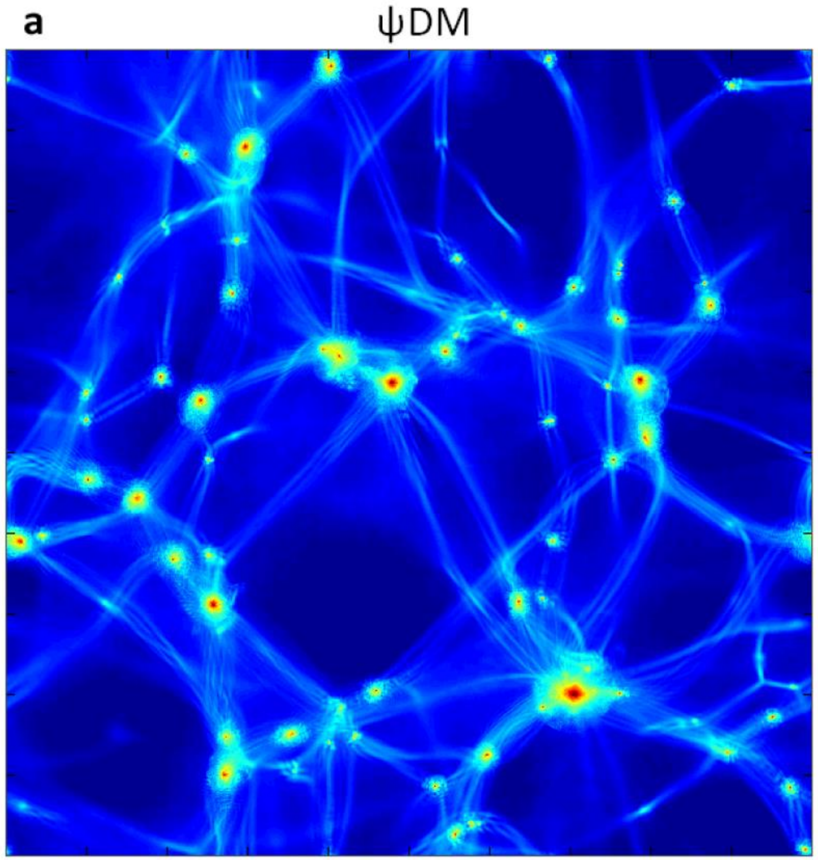

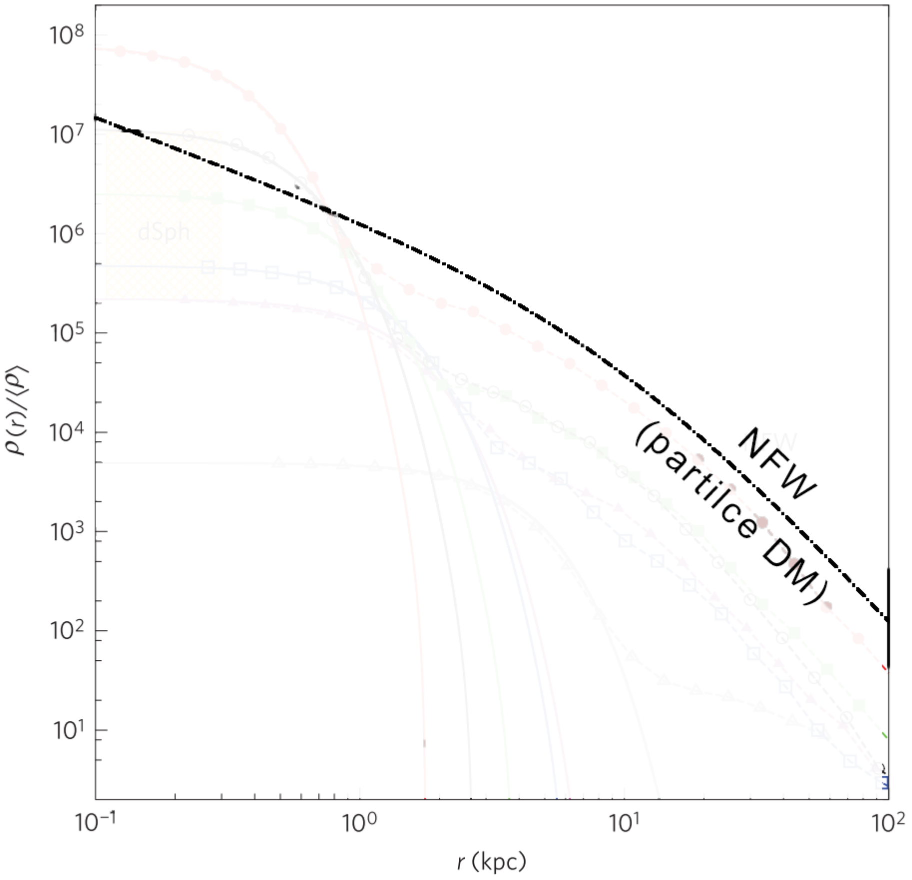

Schive et al., Nat.Phys. 10, 496 (2014)
Navarro, Frenk &White, ApJ, 462, 563 (1996)

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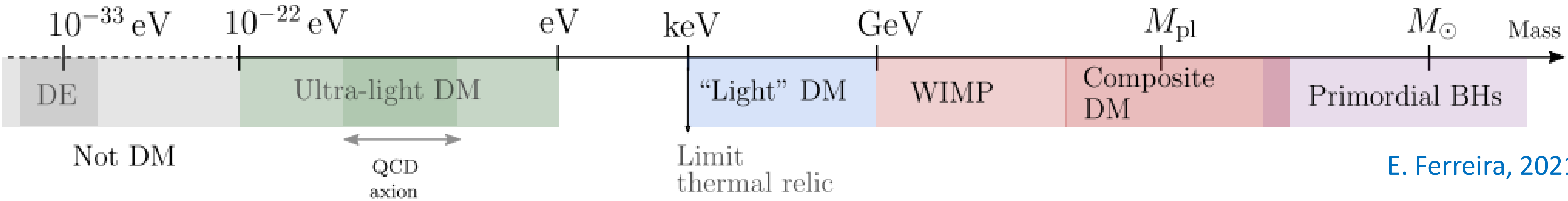

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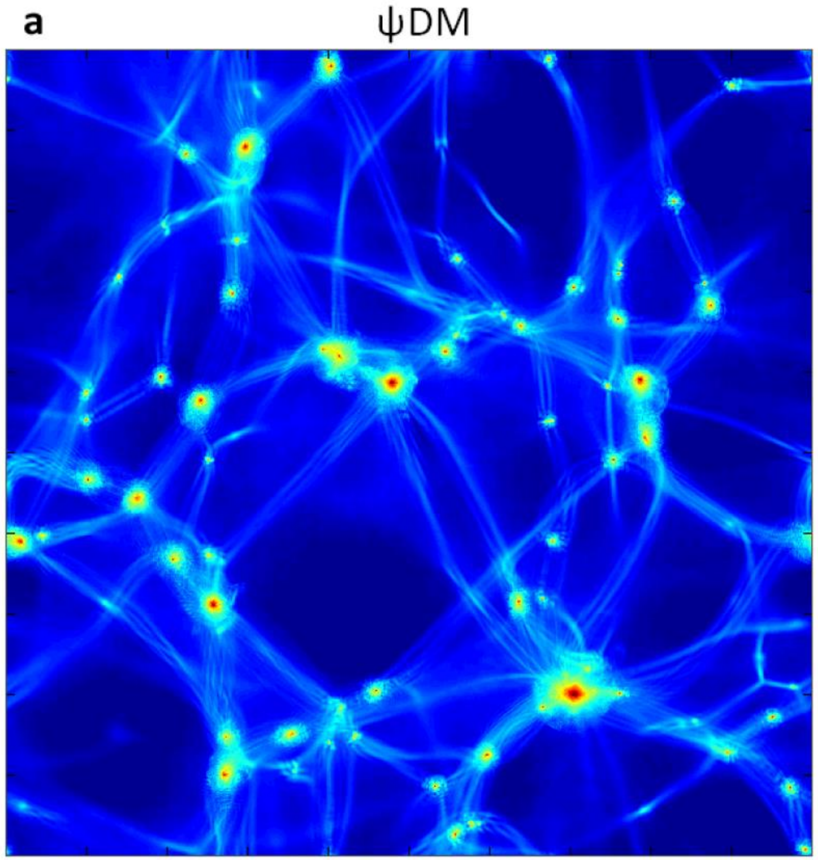

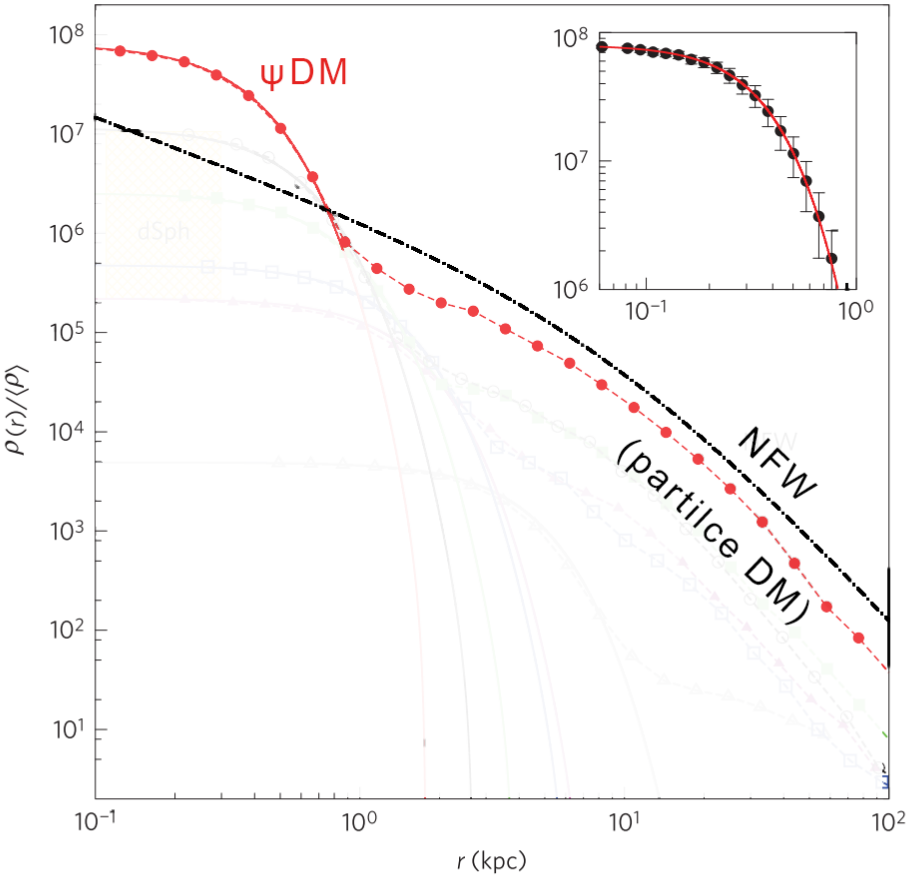

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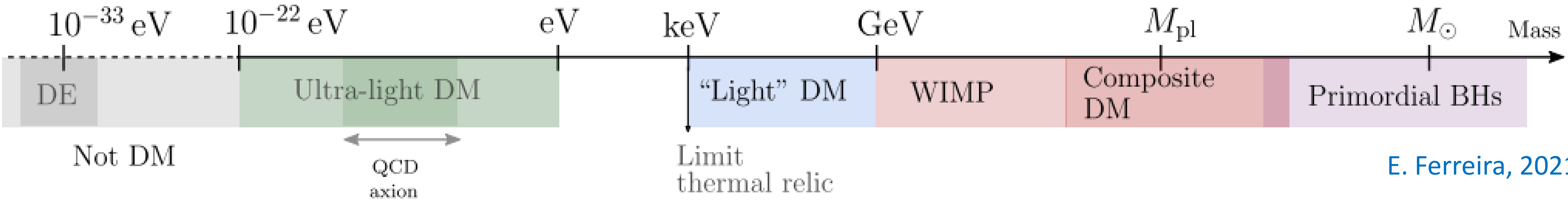

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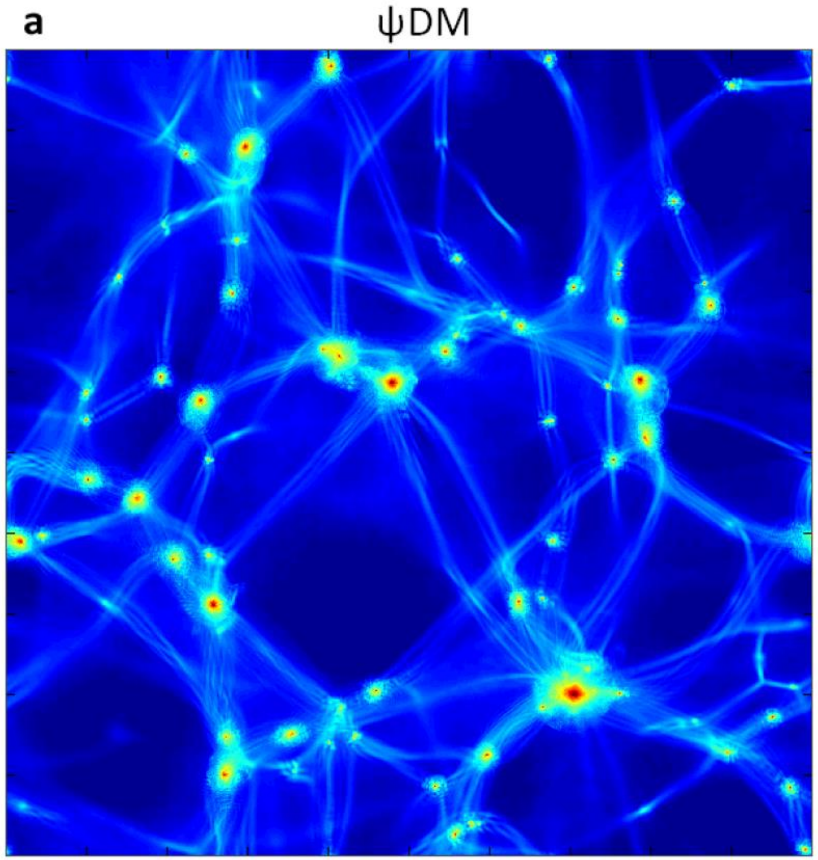
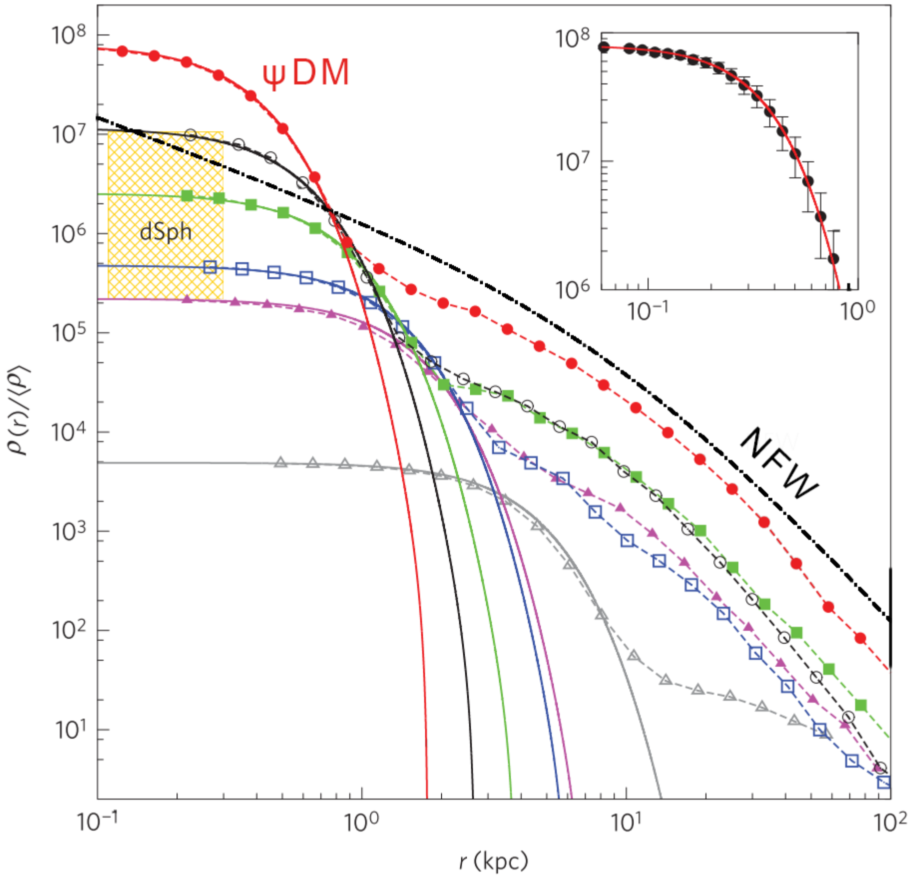




Schive et al., Nat.Phys. 10, 496 (2014)
PRL 113, 261302 (2014)

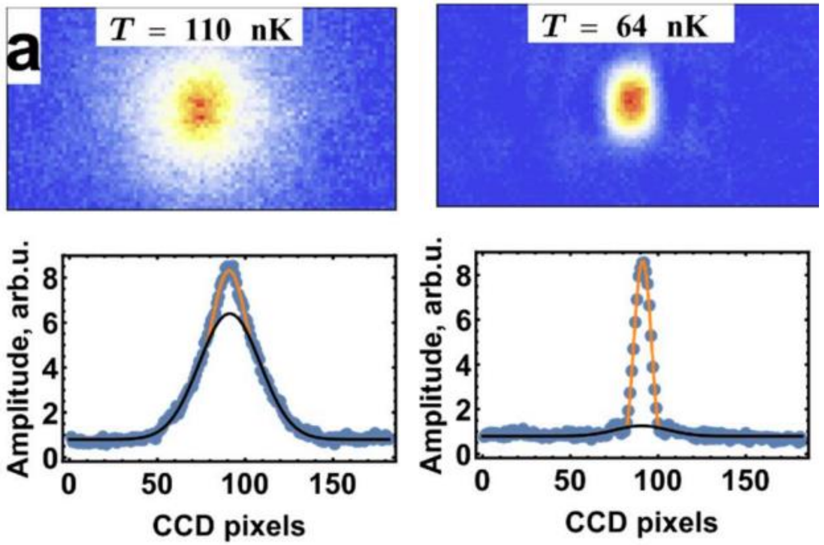
Halo structures

- Cored-halo profile (Bimodal fit)

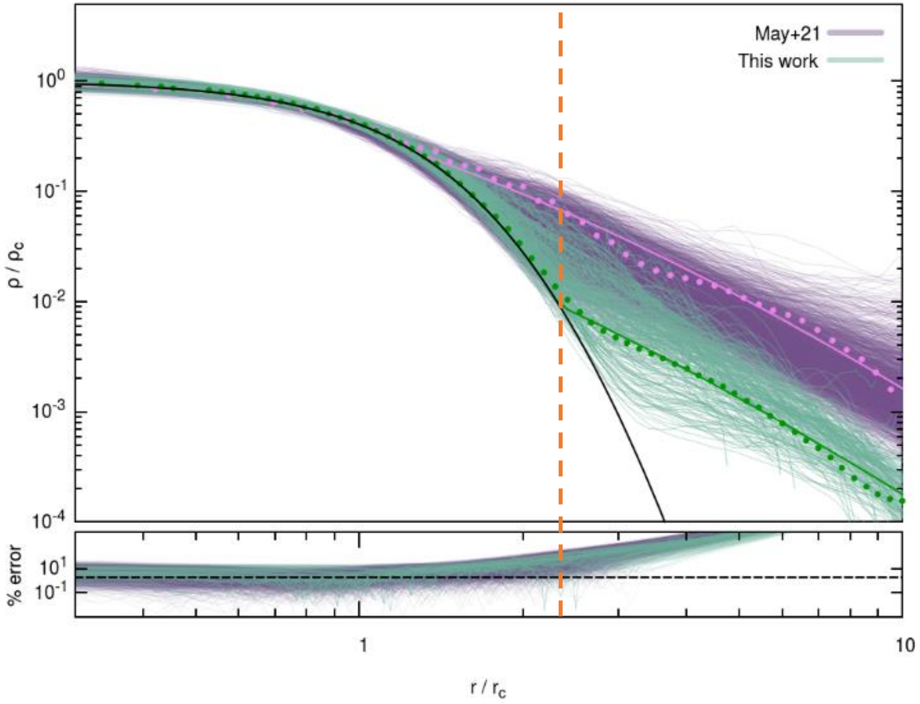
$$\rho_{\mathrm{cNFW}}(r) = \begin{cases} \rho_{\mathrm{soliton}}(r) & , r \le r_t \\ \rho_{\mathrm{NFW}}(r) & , r > r_t \end{cases}$$

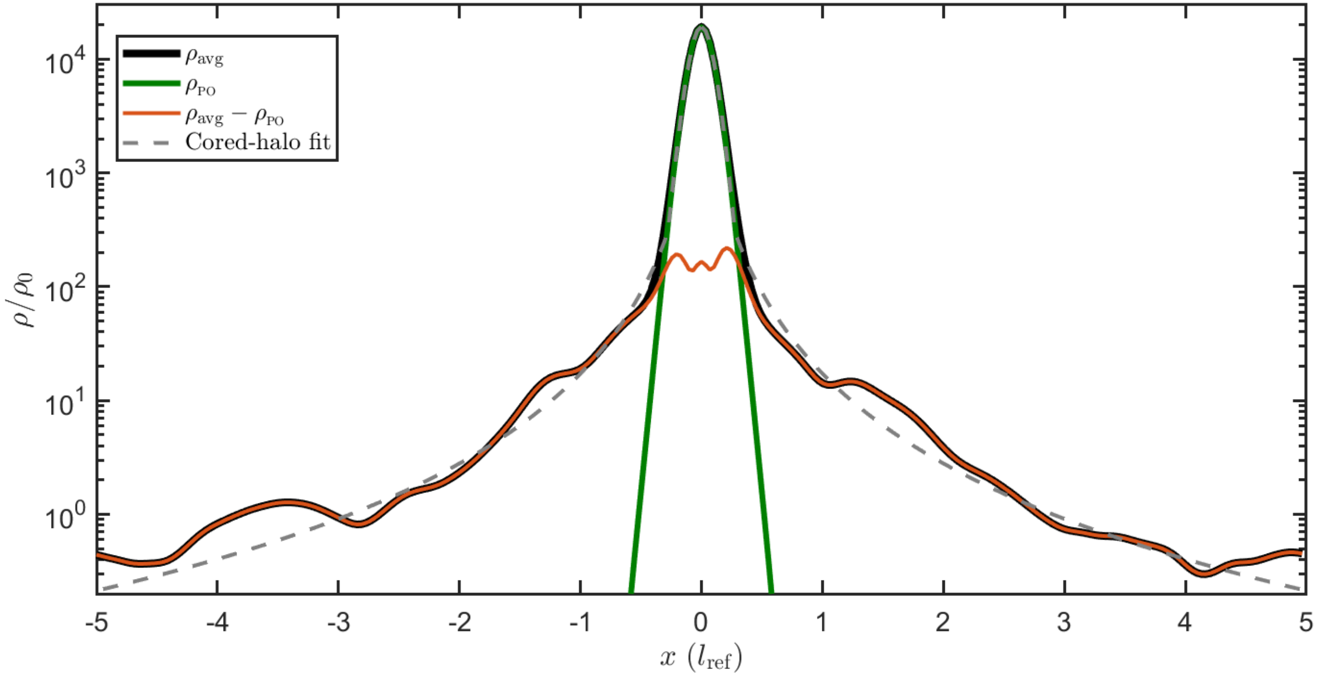
$$\rho_{\mathrm{soliton}}(r) = \rho_c \left[1 - \lambda \left(\frac{r}{r_c}\right)^2\right]^{-8} \quad \rho_{\mathrm{NFW}}(r) = \rho_h \left(\frac{r}{r_h}\right)^{-1} \left(1 + \frac{r}{r_h}\right)^{-2}$$

E. T. Davletov et al., PRA 102, 011302(R) (2020)



Chan et al., arXiv:2110.11882





Fuzzy Dark Matter

- Ultralight mass, $m \approx [10^{-22}, 10]$ eV

L. Hui, arXiv:2101.11735
Dentler et al., arXiv:2111.01199

Schrödinger-Poison equations (SPE)

$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + \mathrm{m}\Phi(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$

$$\nabla^2\Phi(\mathbf{r},t) = 4\pi G[\rho(\mathbf{r},t) - \bar{\rho}]$$

$$\Psi(\mathbf{r},t) = \sqrt{\rho(\mathbf{r},t)}e^{i\varphi(\mathbf{r},t)}$$

$$\mathbf{v}(\mathbf{r},t) = \frac{\hbar}{m}\nabla\varphi(\mathbf{r},t)$$

Hydrodynamic description

$$\frac{\partial}{\partial t}\rho + \nabla\cdot(\rho\mathbf{v}) = 0$$

$$\frac{\partial}{\partial t}\mathbf{v} + \nabla\left[\frac{|\mathbf{v}|^2}{2} + \Phi - \frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right] = 0$$

Scaling invariant/symmetry

$$\{t, x, \Phi, \Psi, \rho\} \to \{\Lambda^2 t', \Lambda x', \Lambda^{-2}\Phi', \Lambda^{-2}\Psi', \Lambda^{-4}\rho'\}$$

$$\{t, x, \Phi, \Psi, \rho\} \to \{\alpha^{-1} t', x', \alpha^2\Phi', \alpha\,\Psi', \alpha^2\rho'\}$$

Ji & Sin, PRD, 50, 3655 (1994); Mocz et al., MNRAS 471, 4559 (2017)

Wavy/quantum features

- Wave interferences (fuzzy DM/ψDM/BECDM)
- Quantum pressure

Superfluidity

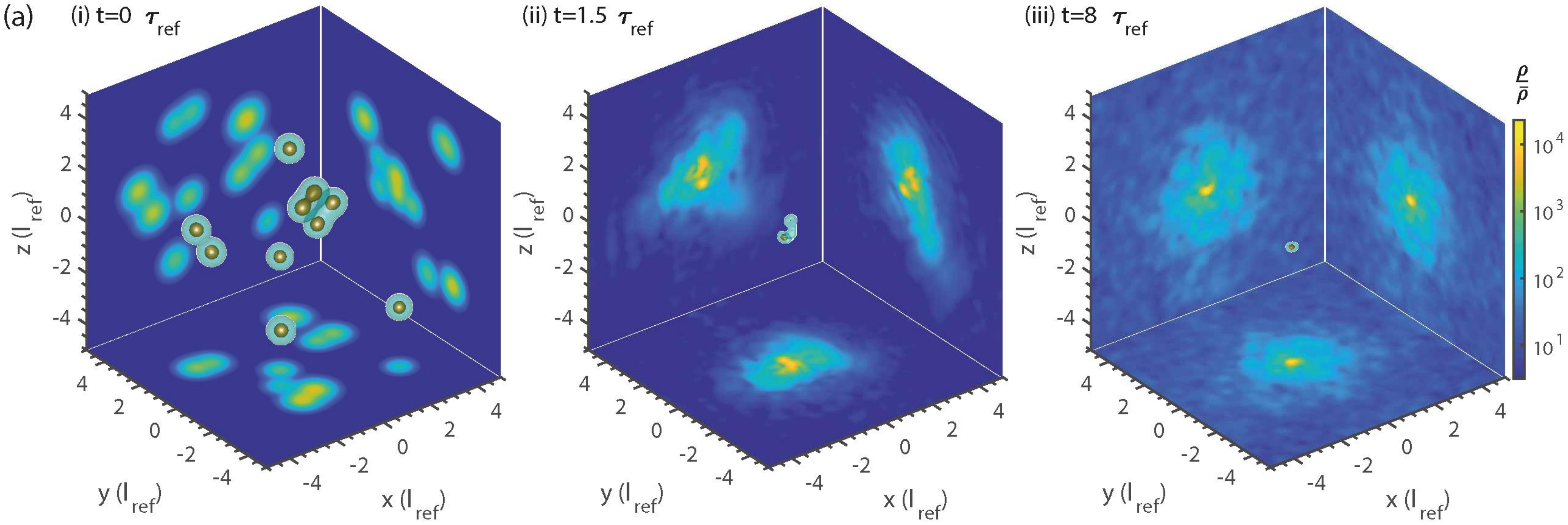
- Viscosity free
- Critical velocity (m-dependent Jeans scale)
- Irrotational

Halo structure

Soliton merger simulation

Mocz et al., MNRAS 471, 4559 (2017)
Chan et al., arXiv:2110.11882

Primary sample $(M = 100 M_{\text{ref}})$



$$E_{\text{ref}} = \hbar\sqrt{G\rho_{\text{ref}}}$$

$$\tau_{\text{ref}} = (G\rho_{\text{ref}})^{-1/2}$$

$$l_{\text{ref}} = \left(\frac{\hbar^2}{m^2 G\rho_{\text{ref}}}\right)^{1/4}$$

$$\rho_{\text{ref}} = 10^3 M_\odot \text{kpc}^{-3}$$

$$m_{\text{ref}} = 2.5 \times 10^{-23}\ \text{eV}$$

$$M_{\text{ref}} \approx 1.26 \times 10^6 M_\odot$$

$$\tau_{\text{ref}} \approx 14.91\ \text{Gyr}$$

$$l_{\text{ref}} \approx 10.81\ \text{kpc}$$

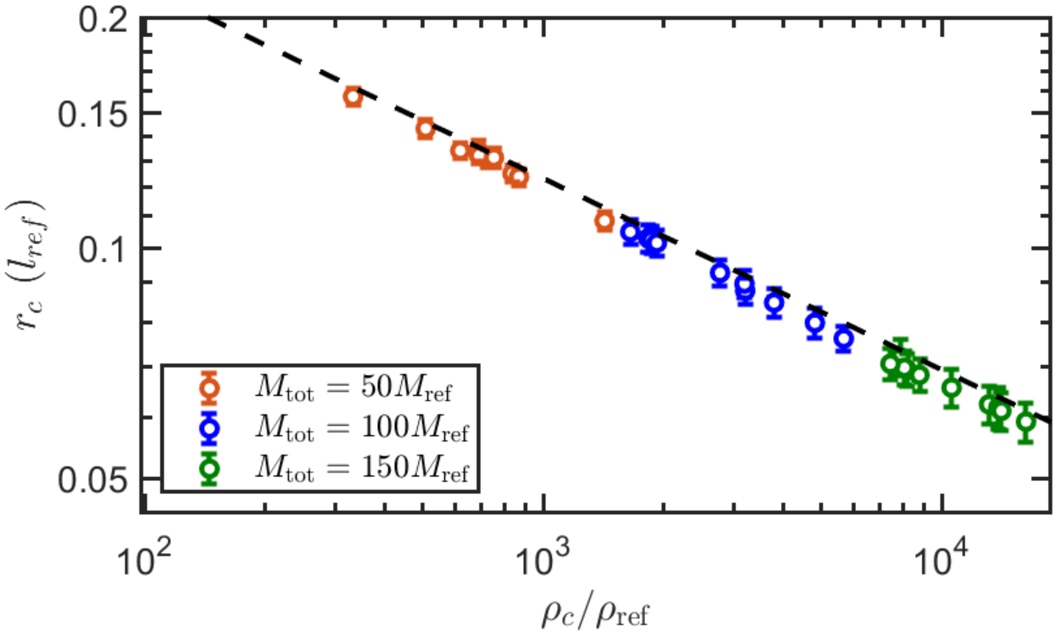
Halo structure

Energy functional of the SP system

$$E = E[\Psi^*, \Psi] = \frac{1}{m}\int d\mathbf{r}\,\Psi^*\left[-\frac{\hbar^2\nabla^2}{2m} + \frac{1}{2}m\Phi\right]\Psi$$

Energetically preferred soliton solution, $\Psi(r) = \sqrt{\rho_{\text{soliton}}(r)}$, for a fixed M_{soliton}

$$r_c = \left(\frac{11303\lambda^2}{128}\frac{\hbar^2}{m^2 G\pi}\right)^{\frac{1}{4}}\rho_c^{-\frac{1}{4}}$$



Mocz et al., MNRAS 471, 4559 (2017); Chiang et al., PRD 103, 103019 (2021)

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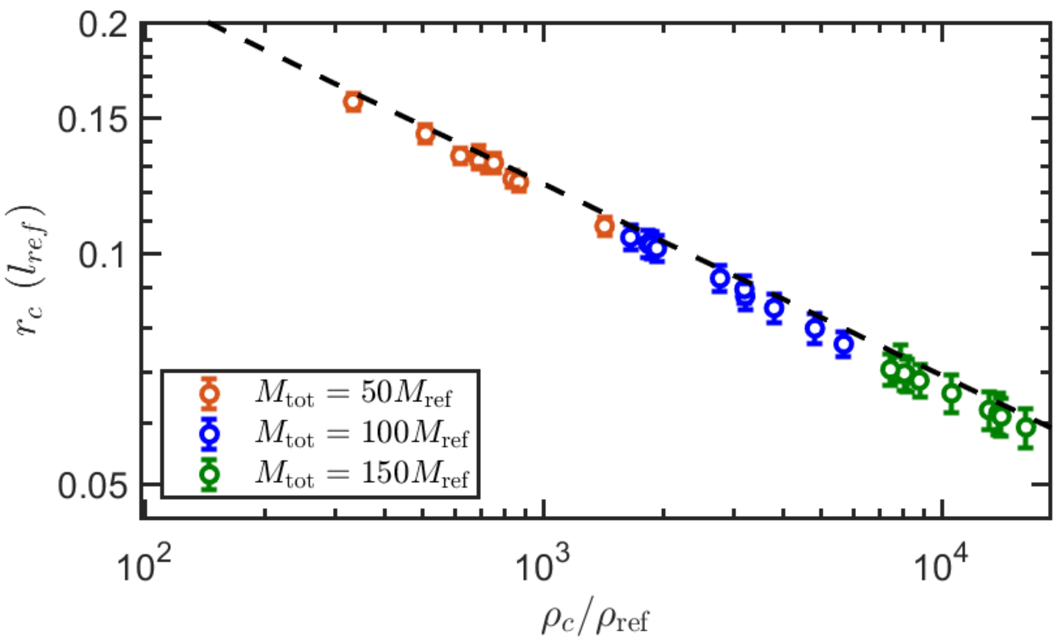
$$r_c = \left(\frac{11303\lambda^2}{128}\frac{\hbar^2}{m^2 G\pi}\right)^{\frac{1}{4}} \rho_c^{-\frac{1}{4}} \approx 23.686\left(\frac{2.5\times 10^{-23}\text{eV}}{m}\right)^{\frac{1}{2}}\left(\frac{10^3 M_\odot \text{kpc}^{-3}}{\rho_c}\right)^{\frac{1}{4}} \text{kpc}$$



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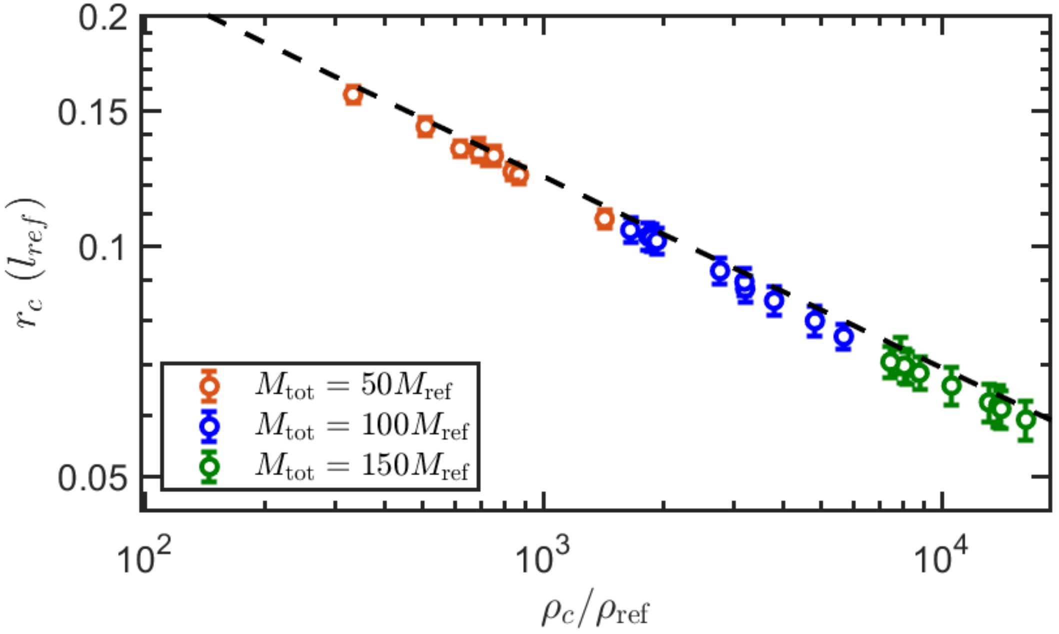

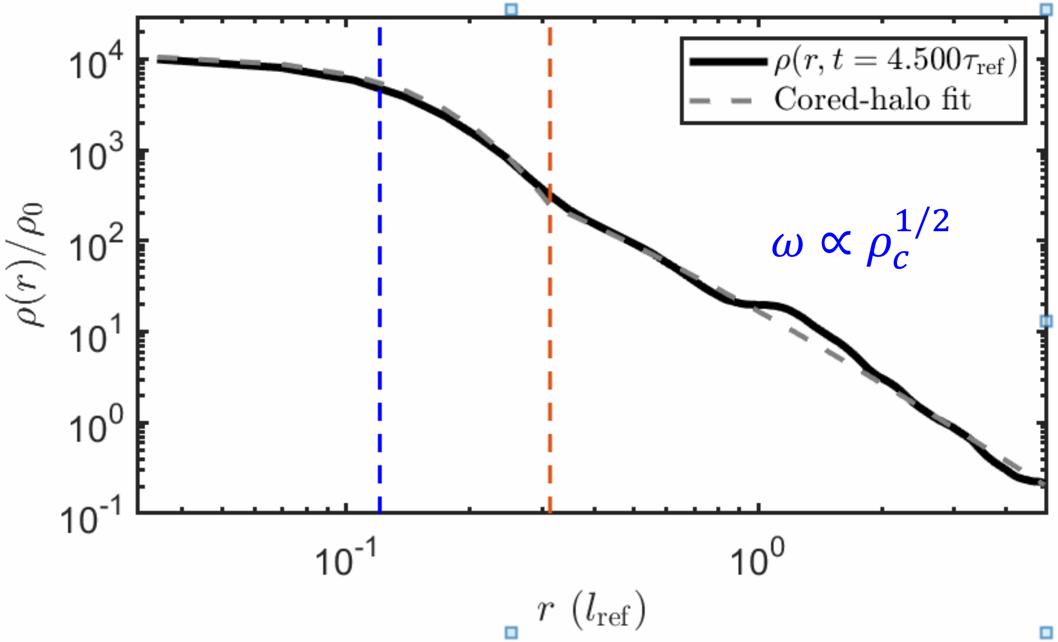

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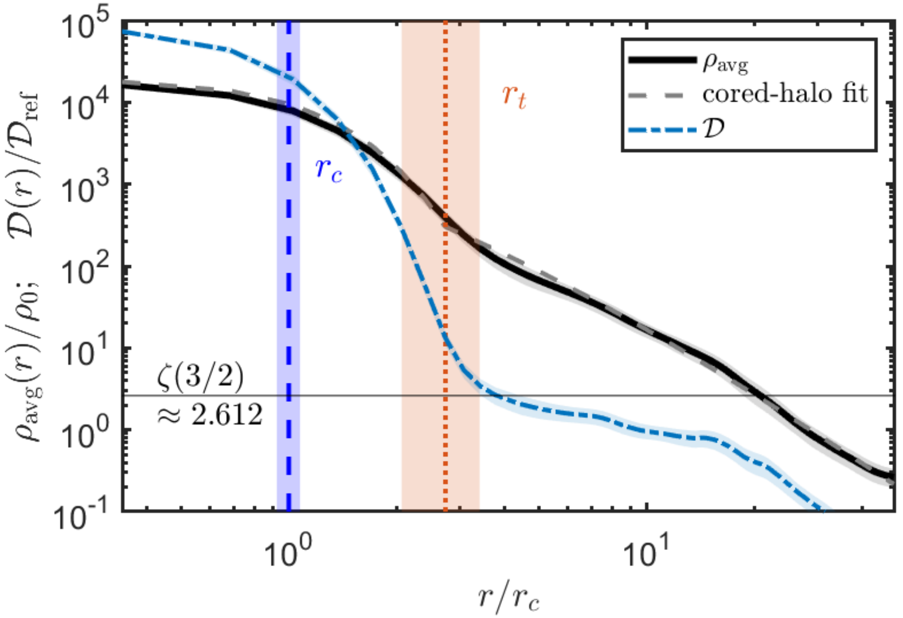
Halo structures

Coherence measures

Phase space density

$$\mathcal{D}(r) = \lambda_{\mathrm{dB}}^3 n = \frac{\langle \rho'(r) \rangle}{\langle v'(r) \rangle^3} \left(\frac{\hbar^3 \rho_{\mathrm{ref}}}{m^4 v_{\mathrm{ref}}^3} \right)$$



F. Dalfovo et al., Review of Modern Physics 71, 463 (1999); P. B. Blakie et al., Adv. Phys. 57, 363 (2008); Liu et al., Comm.Phys. 1, 24 (2018) & PRR 2, 0333183 (2020)

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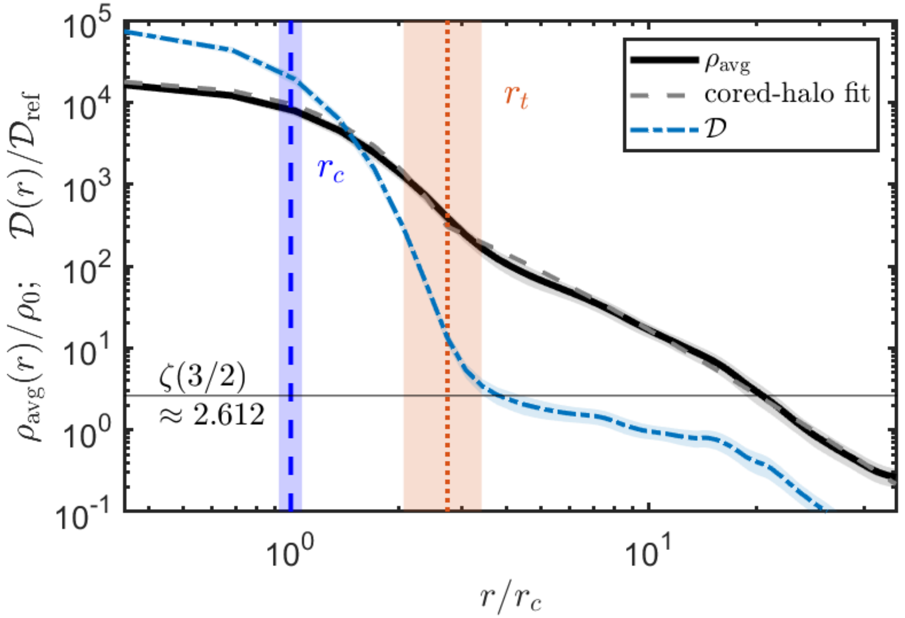
$\Rightarrow$ $\mathcal{D}_{\mathrm{ref}} \approx 2.1 \times 10^{106}$

High phase density!



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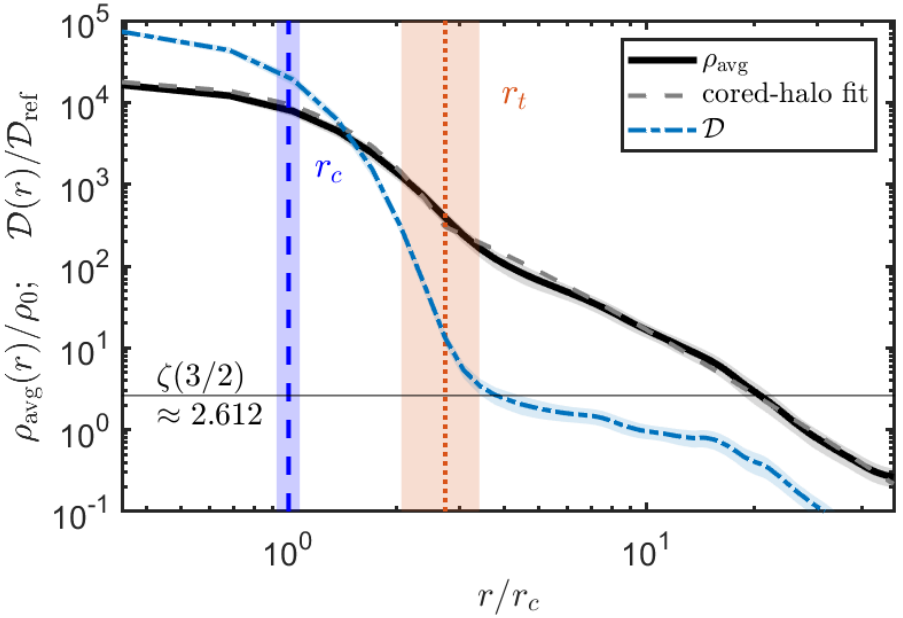
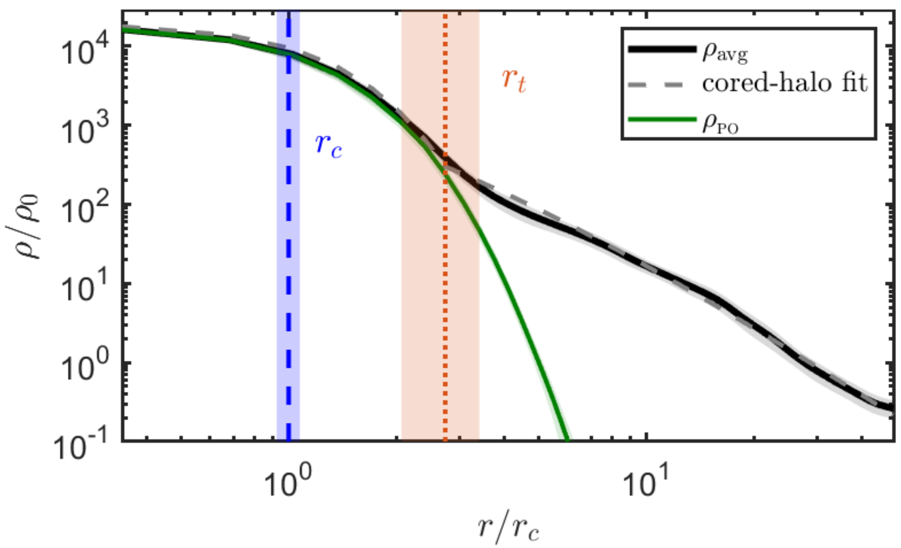
Penrose-Onsager condensate mode

$$\hat{\varrho}(\mathbf{r}, \mathbf{r}') = \langle \Psi^*(\mathbf{r}') \Psi(\mathbf{r}) \rangle$$

$$\int \hat{\varrho}(\mathbf{r}, \mathbf{r}') \Psi_{\mathrm{PO}}(\mathbf{r}) = N_{\mathrm{PO}} \Psi_{\mathrm{PO}}(\mathbf{r})$$





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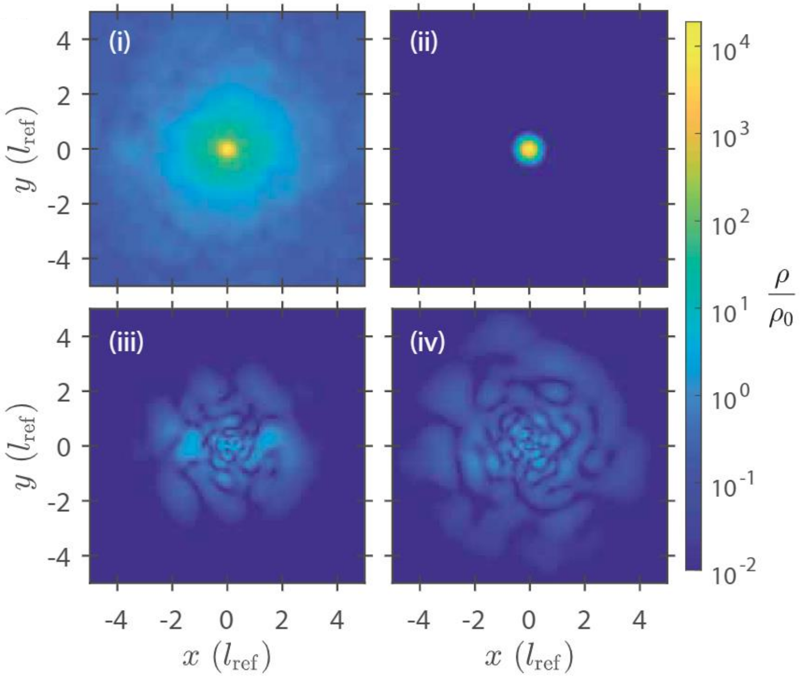

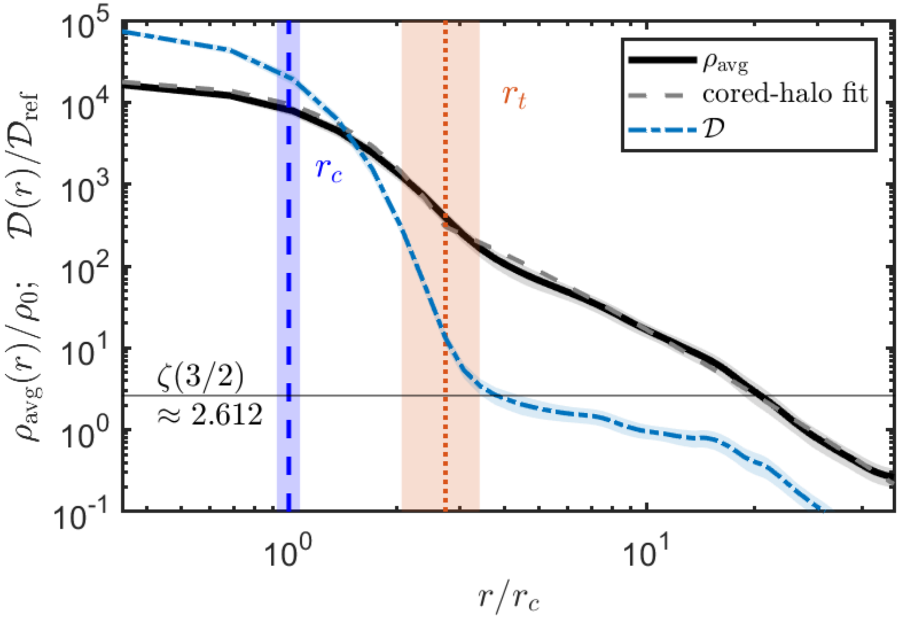

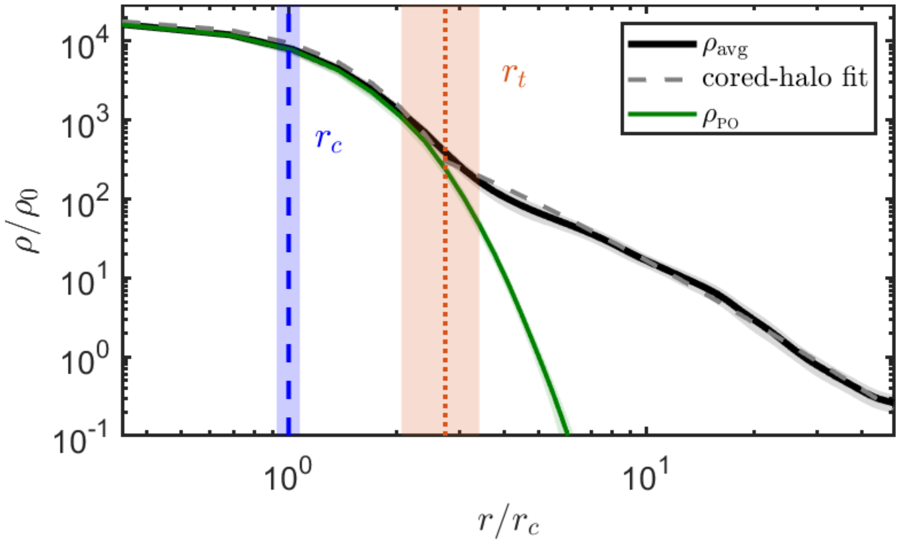

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High phase density!

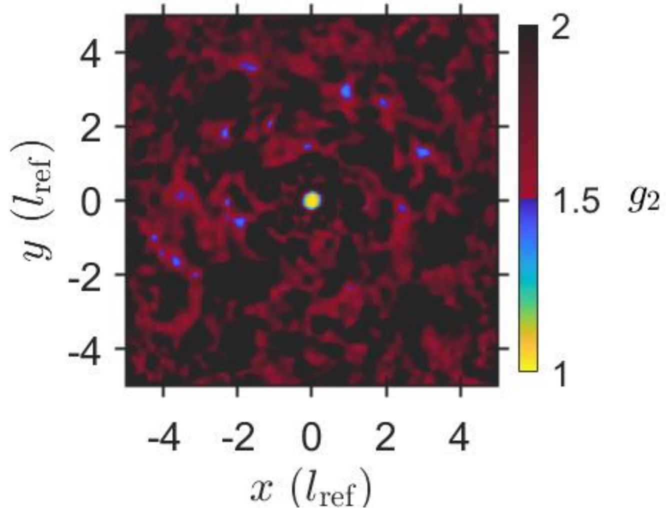
First-order spatial correlation function

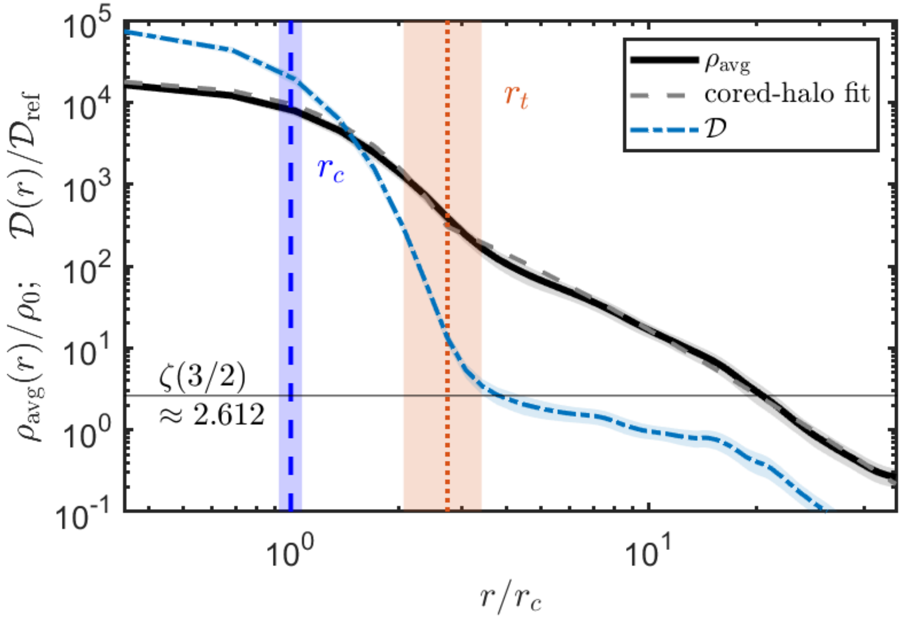
$$\mathrm{g}_1(\mathbf{r}, \mathbf{r}') = \frac{\langle \Psi^*(\mathbf{r}')\Psi(\mathbf{r})\rangle}{\sqrt{\langle|\Psi(\mathbf{r}')|^2\rangle\langle|\Psi(\mathbf{r})|^2\rangle}} \qquad \mathbf{r}' = \mathbf{0}$$

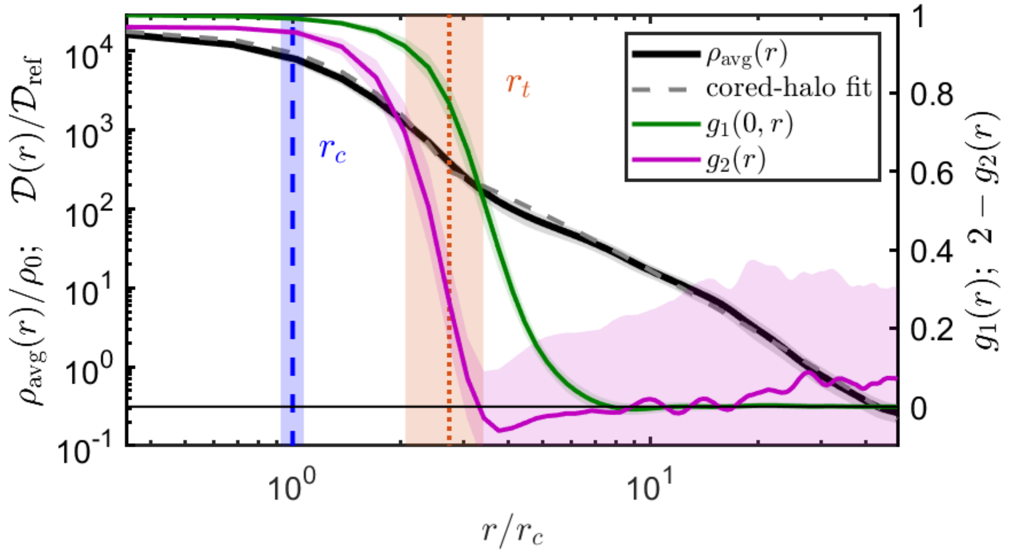
Second-order spatial auto correlation function

$$\mathrm{g}_2(\mathbf{r}) = \frac{\langle \rho^2(\mathbf{r})\rangle}{\langle \rho(\mathbf{r})\rangle^2}$$

- $\mathrm{g}_2 = 1$ for coherent density
- $\mathrm{g}_2 = 2$ for strong density fluctuation







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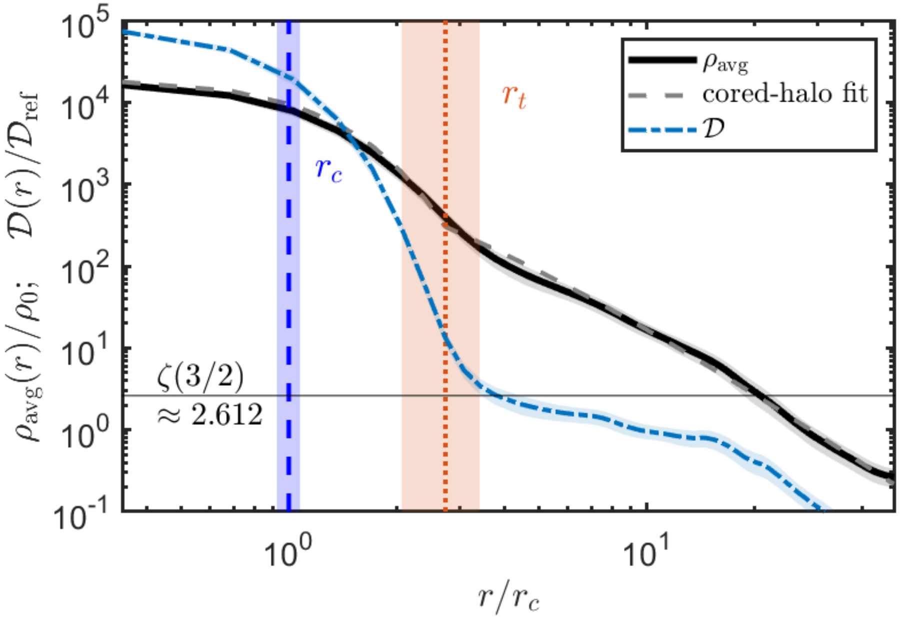

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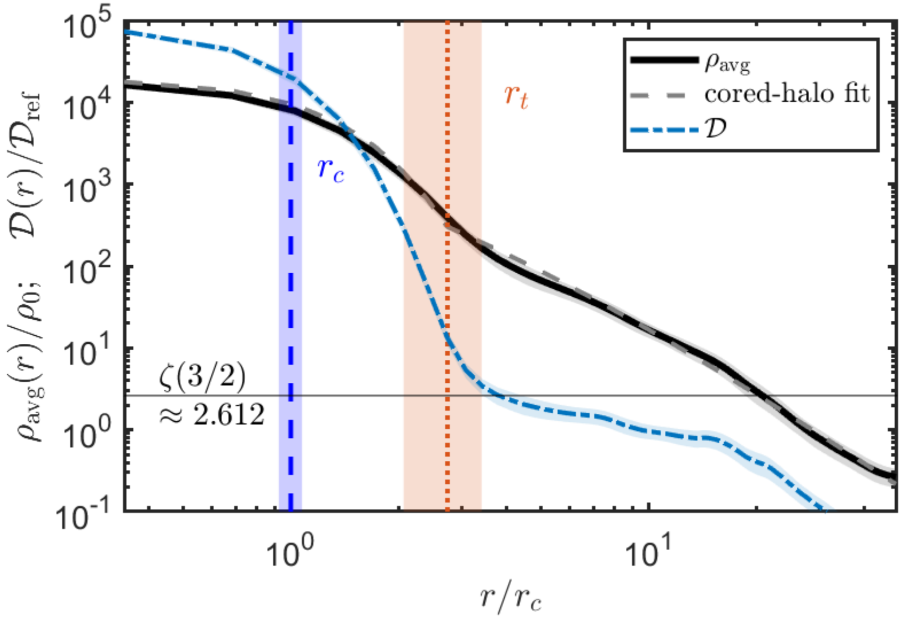

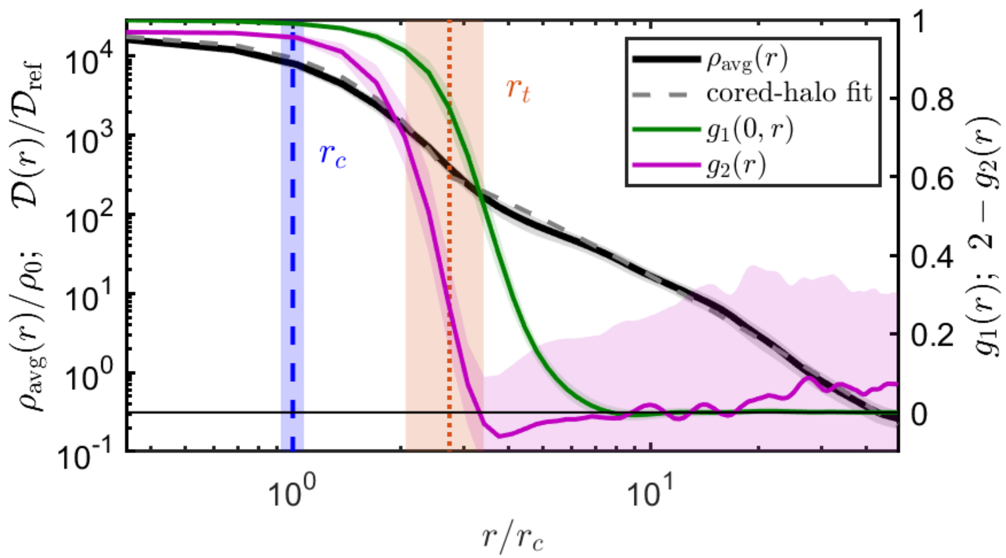

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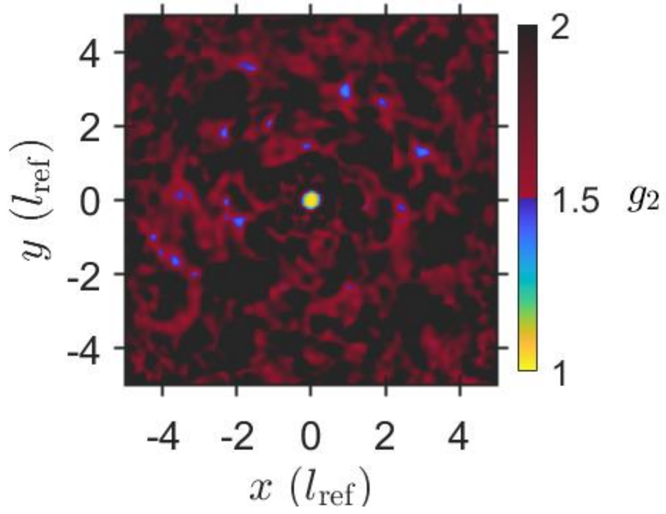
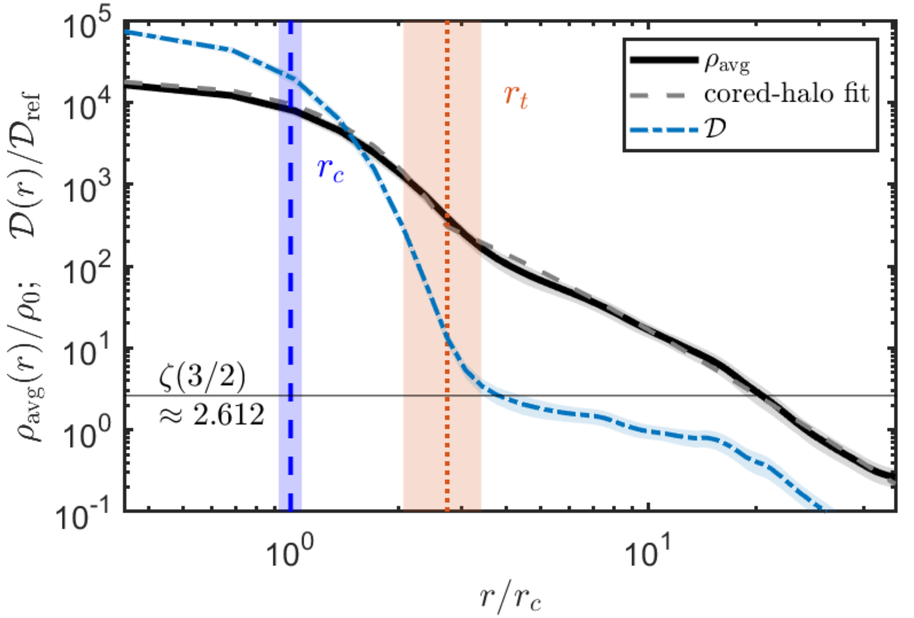
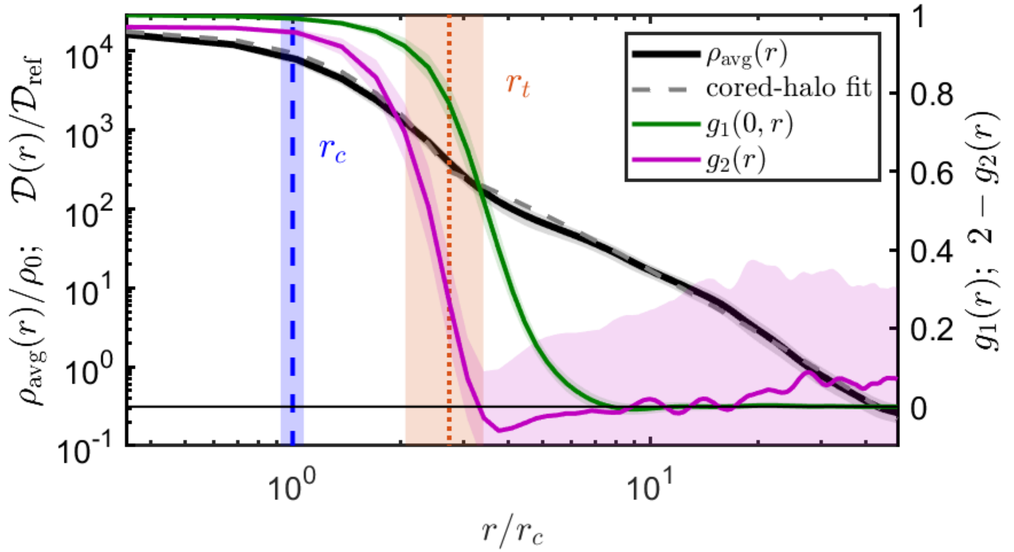






F. Dalfovo et al., Review of Modern Physics 71, 463 (1999); P. B. Blakie et al., Adv. Phys. 57, 363 (2008); Liu et al., Comm.Phys. 1, 24 (2018) & PRR 2, 0333183 (2020)

Halo structures

Energy profiles

$$E = \int d\mathbf{r} \left\{ \left[\underbrace{\varepsilon_{\mathrm{ke}}(\mathbf{r}) + \varepsilon_{\mathrm{qp}}(\mathbf{r})}_{-\hbar^2\nabla^2/2\mathrm{m}^2} \right] + \underbrace{\varepsilon_{\Phi}(\mathbf{r})}_{\Phi(\mathbf{r})\rho(\mathbf{r})/2} \right\} = \mathrm{E}_{\mathrm{ke}} + \mathrm{E}_{\mathrm{qp}}$$

$$\varepsilon_{\mathrm{ke}}(\mathbf{r}) = \frac{1}{2}\rho(\mathbf{r})|\mathbf{v}(\mathbf{r})|^2 \qquad \varepsilon_{\mathrm{qp}}(\mathbf{r}) = \frac{\hbar^2}{2m^2}\left|\nabla\sqrt{\rho(\mathbf{r})}\right|^2$$

$$\boxed{\mathrm{E}_{\mathrm{qp}} \gtrsim 2\mathrm{E}_{\mathrm{ke}}}$$

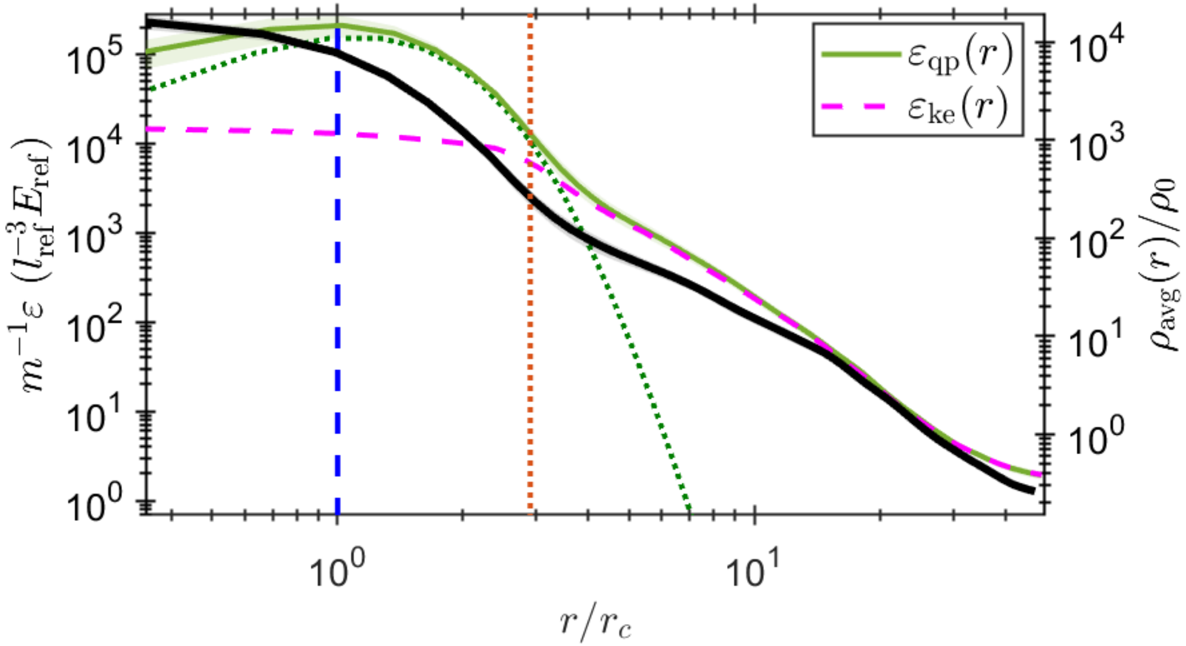
Halo structures

Energy profiles

$$E = \int d\mathbf{r}\left\{\left[\varepsilon_{\rm ke}(\mathbf{r}) + \underbrace{\varepsilon_{\rm qp}(\mathbf{r})}_{}\right]^{-\hbar^2\nabla^2/2\mathrm{m}^2} + \varepsilon_{\Phi}(\mathbf{r})^{\Phi(\mathbf{r})\rho(\mathbf{r})/2}\right\} = \mathrm{E}_{\rm ke} + \mathrm{E}_{\rm qp}$$

$$\varepsilon_{\rm ke}(\mathbf{r}) = \frac{1}{2}\rho(\mathbf{r})|\mathbf{v}(\mathbf{r})|^2 \qquad \varepsilon_{\rm qp}(\mathbf{r}) = \frac{\hbar^2}{2m^2}\left|\nabla\sqrt{\rho(\mathbf{r})}\right|^2$$

$$\boxed{\mathrm{E}_{\rm qp} \gtrsim 2\mathrm{E}_{\rm ke}}$$



Halo structures

Energy profiles

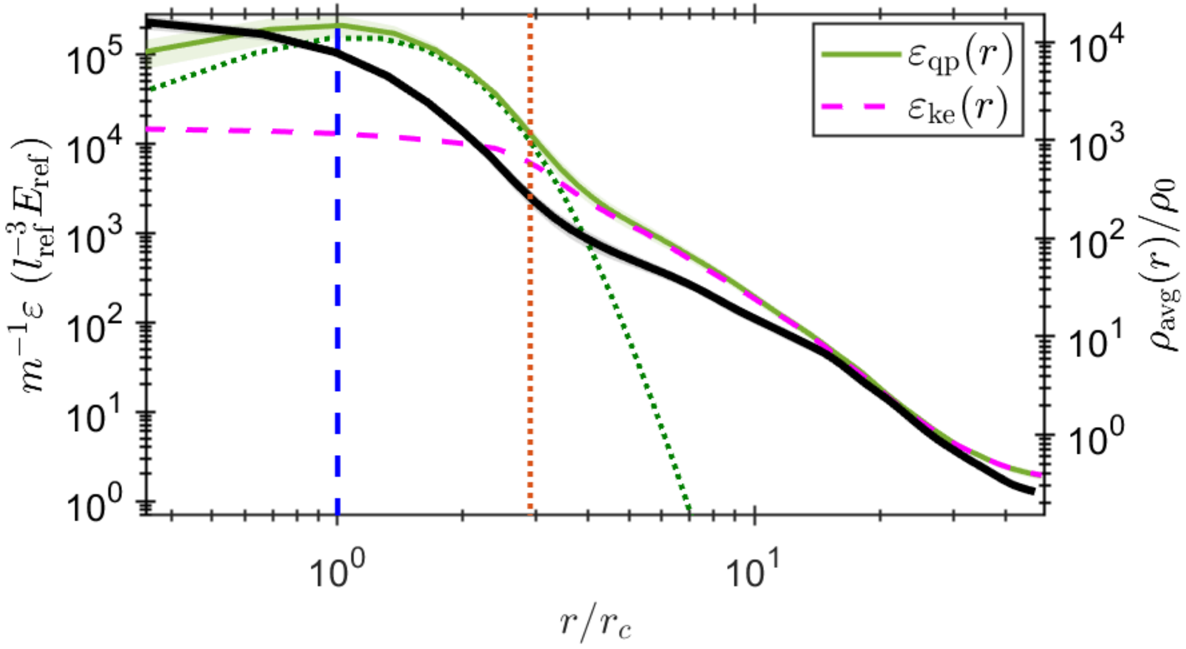
$$E = \int d\mathbf{r}\left\{\left[\varepsilon_{\rm ke}(\mathbf{r}) + \varepsilon_{\rm qp}(\mathbf{r})\right] + \varepsilon_\Phi(\mathbf{r})\right\} = \mathrm{E_{ke}} + \mathrm{E_{qp}}$$

$-\hbar^2\nabla^2/2\mathrm{m}^2$

$\Phi(\mathbf{r})\rho(\mathbf{r})/2$

$$\varepsilon_{\rm ke}(\mathbf{r}) = \frac{1}{2}\rho(\mathbf{r})|\mathbf{v}(\mathbf{r})|^2 \qquad \varepsilon_{\rm qp}(\mathbf{r}) = \frac{\hbar^2}{2m^2}\left|\nabla\sqrt{\rho(\mathbf{r})}\right|^2$$

$$\boxed{\mathrm{E_{qp}} \gtrsim 2\mathrm{E_{ke}}}$$



$$\mathrm{E_{ke}} = \mathrm{E^c_{ke}} + \mathrm{E^i_{ke}}$$

$$\varepsilon_{\rm ke}(\mathbf{r}) = \frac{1}{2}\left|\mathbf{F}^c(\mathbf{r}) + F^i(\mathbf{r})\right|^2$$

$$\nabla \times \mathbf{F}^c(\mathbf{r}) = 0$$

$$\nabla \cdot \mathbf{F}^i(\mathbf{r}) = 0$$

Halo structures

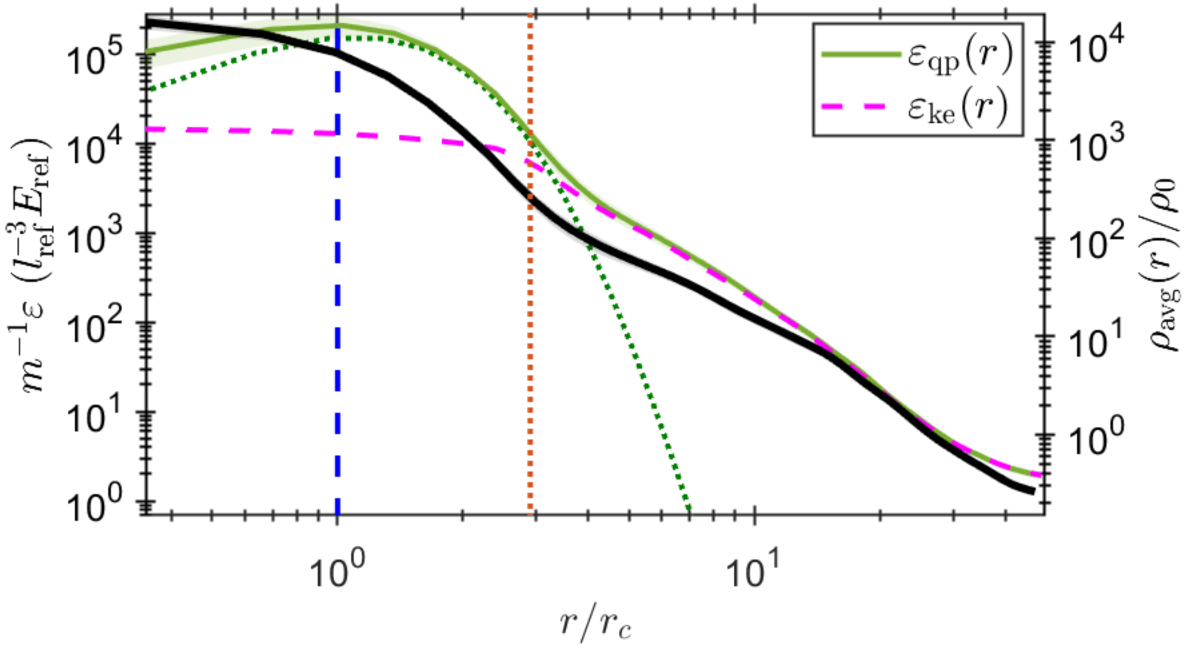
Energy profiles

$$E = \int d\mathbf{r}\left\{\left[\varepsilon_{\rm ke}(\mathbf{r}) + \varepsilon_{\rm qp}(\mathbf{r})\right] + \varepsilon_{\Phi}(\mathbf{r})\right\} = \mathrm{E}_{\rm ke} + \mathrm{E}_{\rm qp}$$

$-\hbar^2\nabla^2/2\mathrm{m}^2$ $\quad\Phi(\mathbf{r})\rho(\mathbf{r})/2$

$$\varepsilon_{\rm ke}(\mathbf{r}) = \frac{1}{2}\rho(\mathbf{r})|\mathbf{v}(\mathbf{r})|^2 \qquad \varepsilon_{\rm qp}(\mathbf{r}) = \frac{\hbar^2}{2m^2}\left|\nabla\sqrt{\rho(\mathbf{r})}\right|^2$$

$$\boxed{\mathrm{E}_{\rm qp} \gtrsim 2\mathrm{E}_{\rm ke}}$$



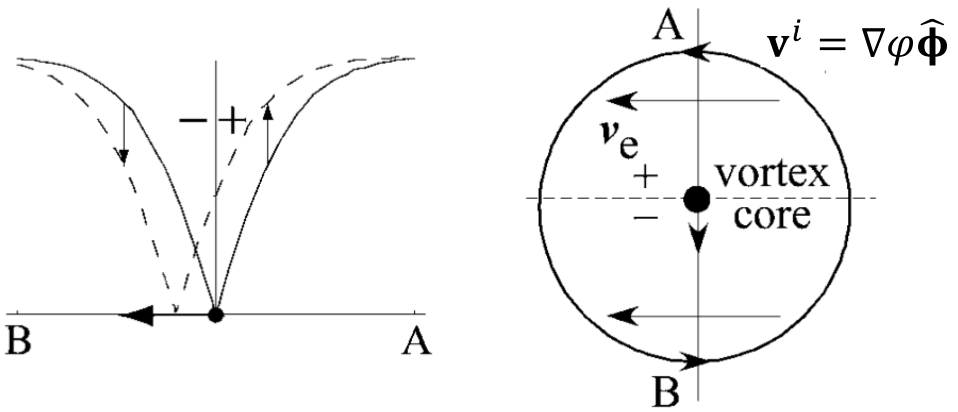
$$\mathrm{E}_{\rm ke} = \mathrm{E}^c_{\rm ke} + \mathrm{E}^i_{\rm ke}$$

$$\varepsilon_{\rm ke}(\mathbf{r}) = \frac{1}{2}\left|\mathbf{F}^c(\mathbf{r}) + F^i(\mathbf{r})\right|^2$$

$$\boxed{\begin{array}{c}\nabla\times\mathbf{F}^c(\mathbf{r}) = 0\\ \nabla\cdot\mathbf{F}^i(\mathbf{r}) = 0\end{array}}$$

Quantized vortex

The incompressible/rotational component corresponds to the topological defect



Halo structures

Energy profiles

$$E = \int d\mathbf{r} \{ [\underbrace{\varepsilon_{\rm ke}(\mathbf{r})}_{} + \overbrace{\varepsilon_{\rm qp}(\mathbf{r})}^{-\hbar^2\nabla^2/2m^2}] + \overbrace{\varepsilon_\Phi(\mathbf{r})}^{\Phi(\mathbf{r})\rho(\mathbf{r})/2} \} = \mathrm{E_{ke}} + \mathrm{E_{qp}}$$

$$\varepsilon_{\rm ke}(\mathbf{r}) = \frac{1}{2}\rho(\mathbf{r})|\mathbf{v}(\mathbf{r})|^2 \qquad \varepsilon_{\rm qp}(\mathbf{r}) = \frac{\hbar^2}{2m^2}\left|\nabla\sqrt{\rho(\mathbf{r})}\right|^2$$

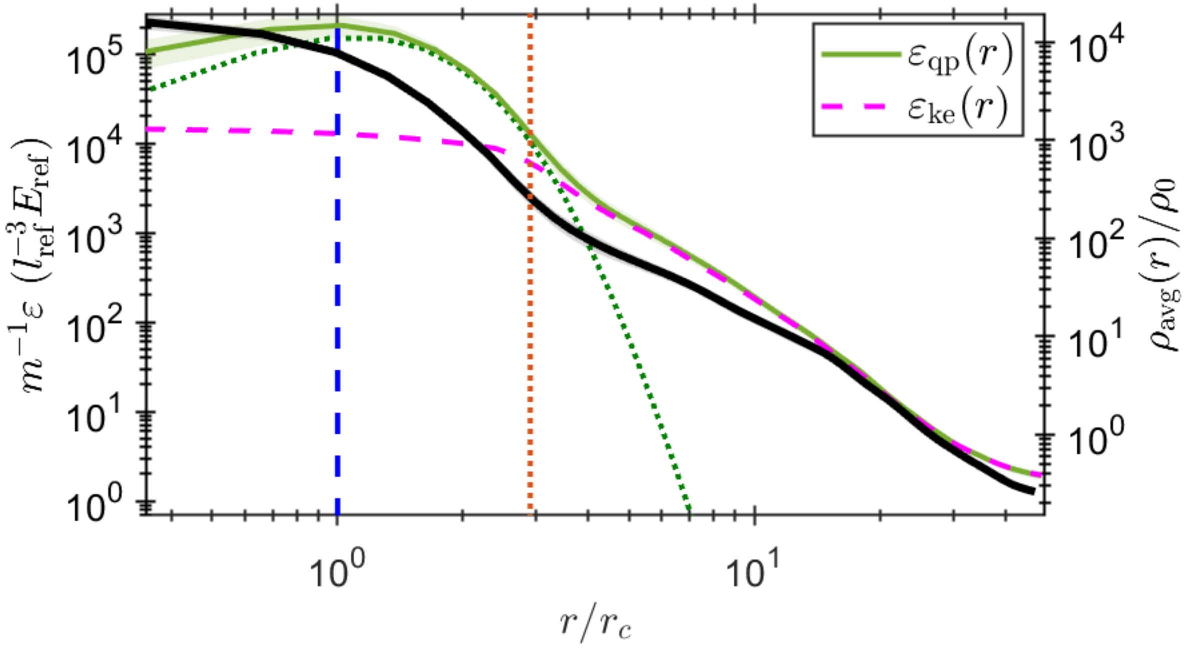
$$\mathrm{E_{ke}} = \mathrm{E^c_{ke}} + \mathrm{E^i_{ke}}$$

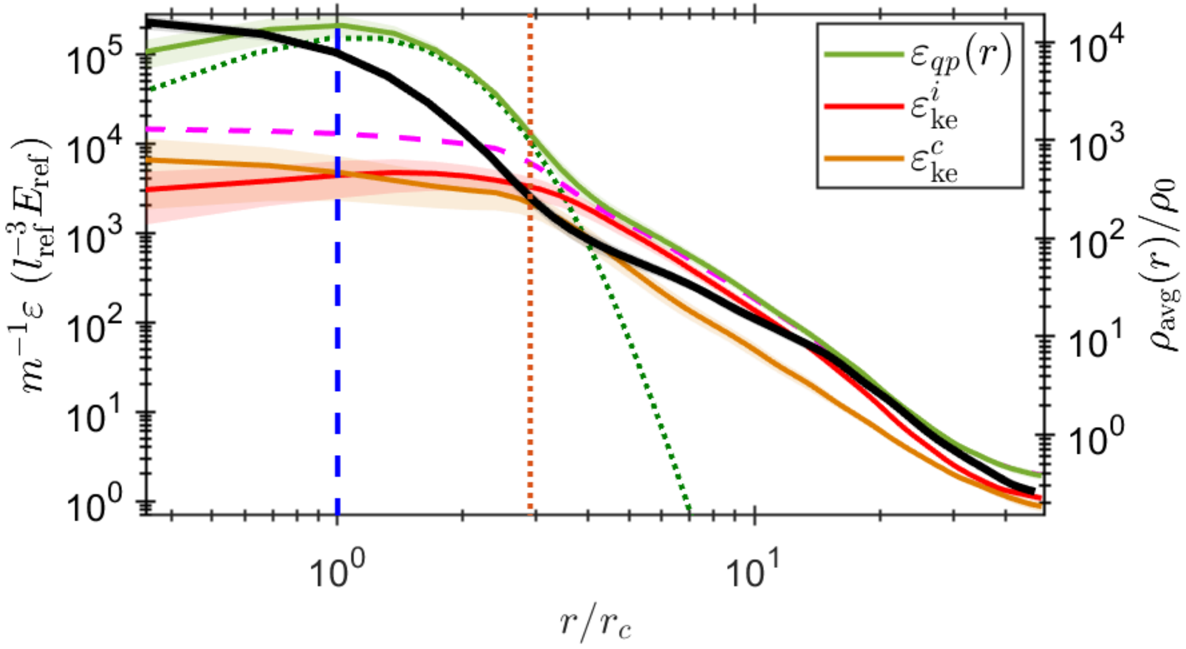
$$\varepsilon_{\rm ke}(\mathbf{r}) = \frac{1}{2}\left|\mathbf{F}^c(\mathbf{r}) + F^i(\mathbf{r})\right|^2$$

$$\boxed{\begin{aligned} \nabla \times \mathbf{F}^c(\mathbf{r}) &= 0 \\ \nabla \cdot \mathbf{F}^i(\mathbf{r}) &= 0 \end{aligned}}$$

$$\boxed{\mathrm{E_{qp}} \gtrsim 2\mathrm{E_{ke}}}$$

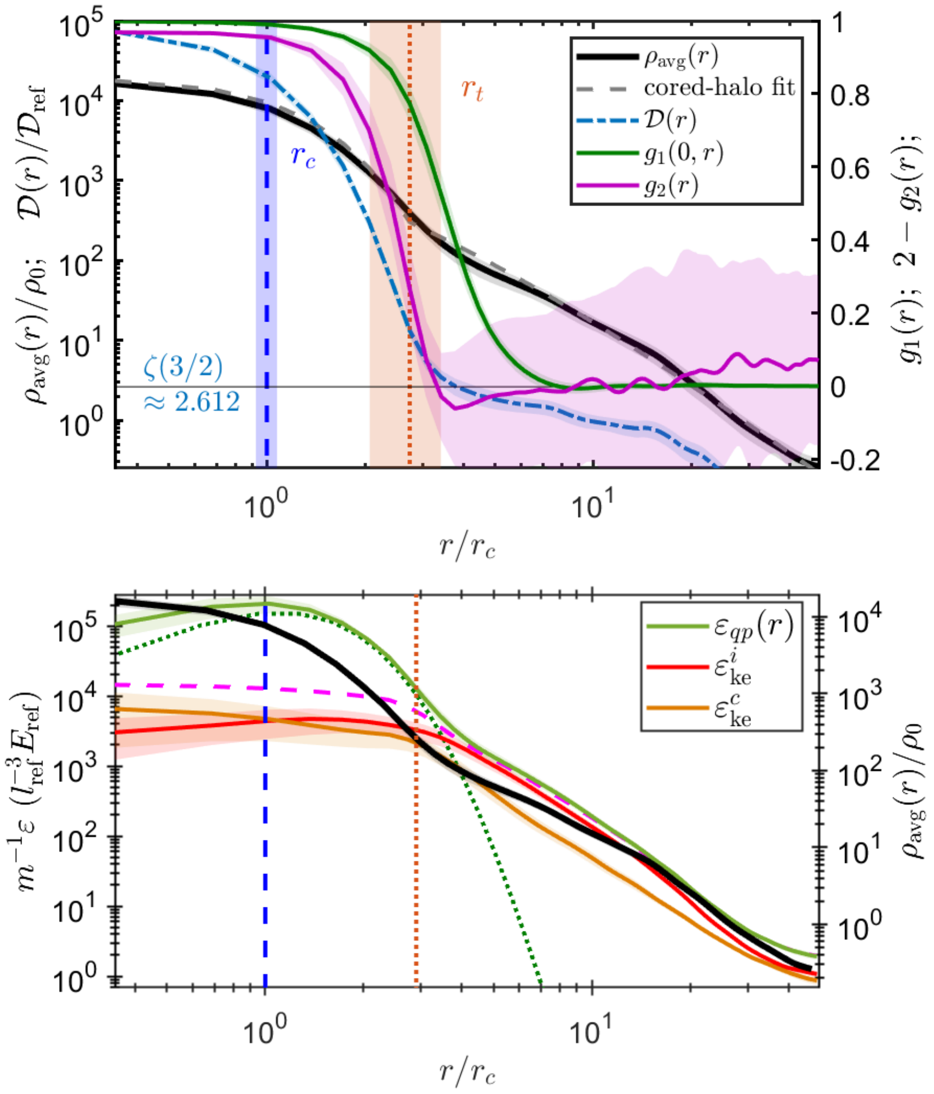
$$\boxed{\mathrm{E^i_{ke}} \gtrsim 1.5\mathrm{E^c_{ke}}}$$





Vortices in FDM halos and granule size

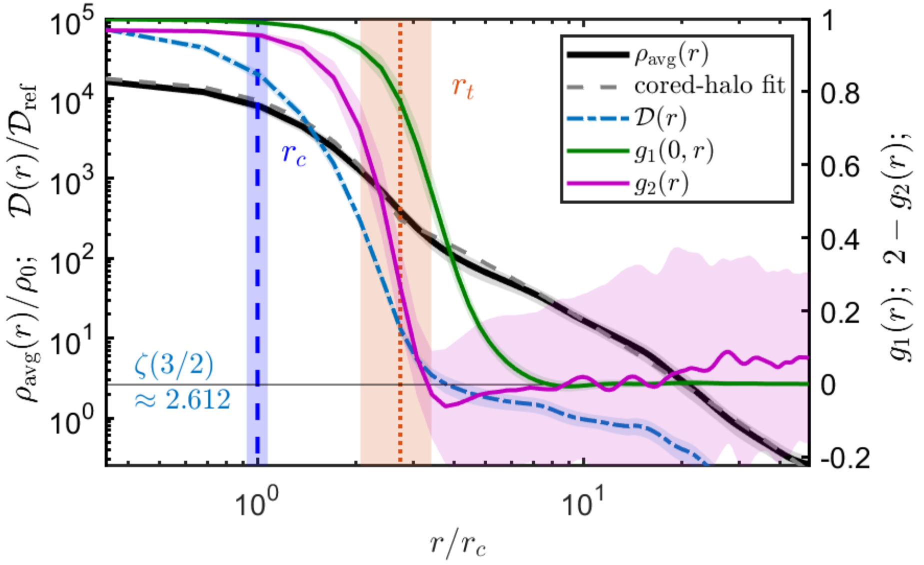
Vortical structure visualization

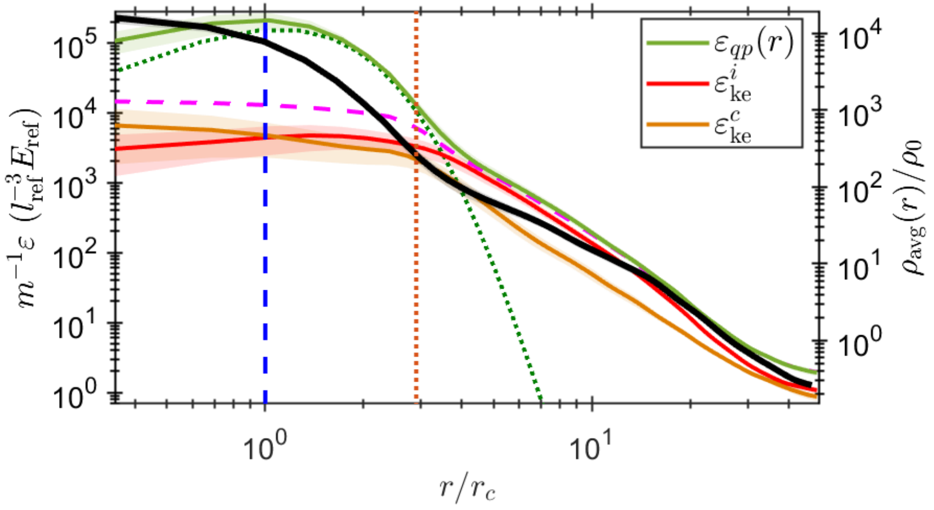


* Vortices in FDM have also been investigated in L. Hui et al., 2021 & Mocz et al., MNRAS 471, 4559 (2017)

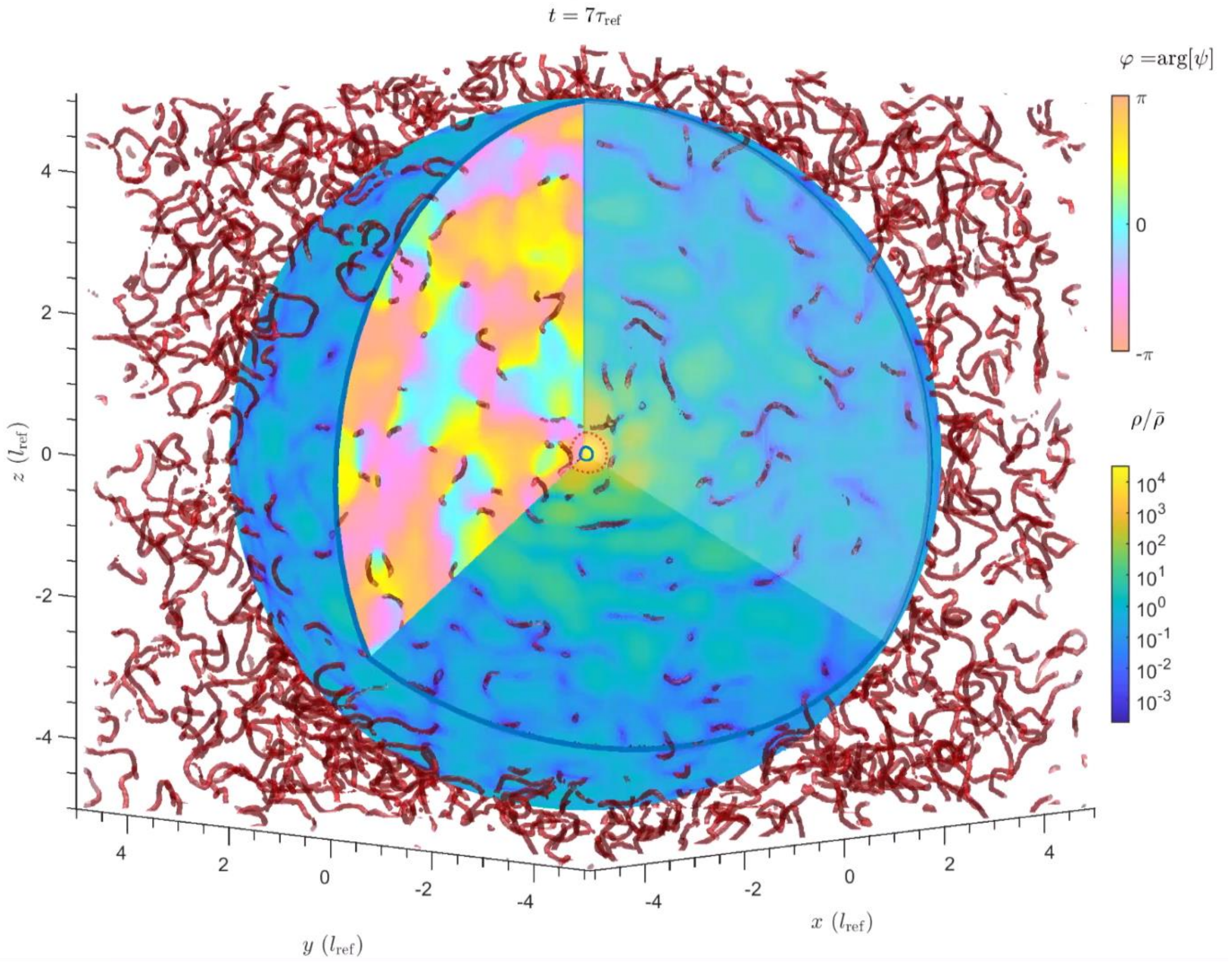
Vortices in FDM halos and granule size

Vortical structure visualization





$$|\mathbf{v}^i(\mathbf{r})|^2 \approx \frac{\hbar^2}{m^2\Delta x^2} \approx 800\,\frac{l_{ref}^2}{\tau_{ref}^2}$$



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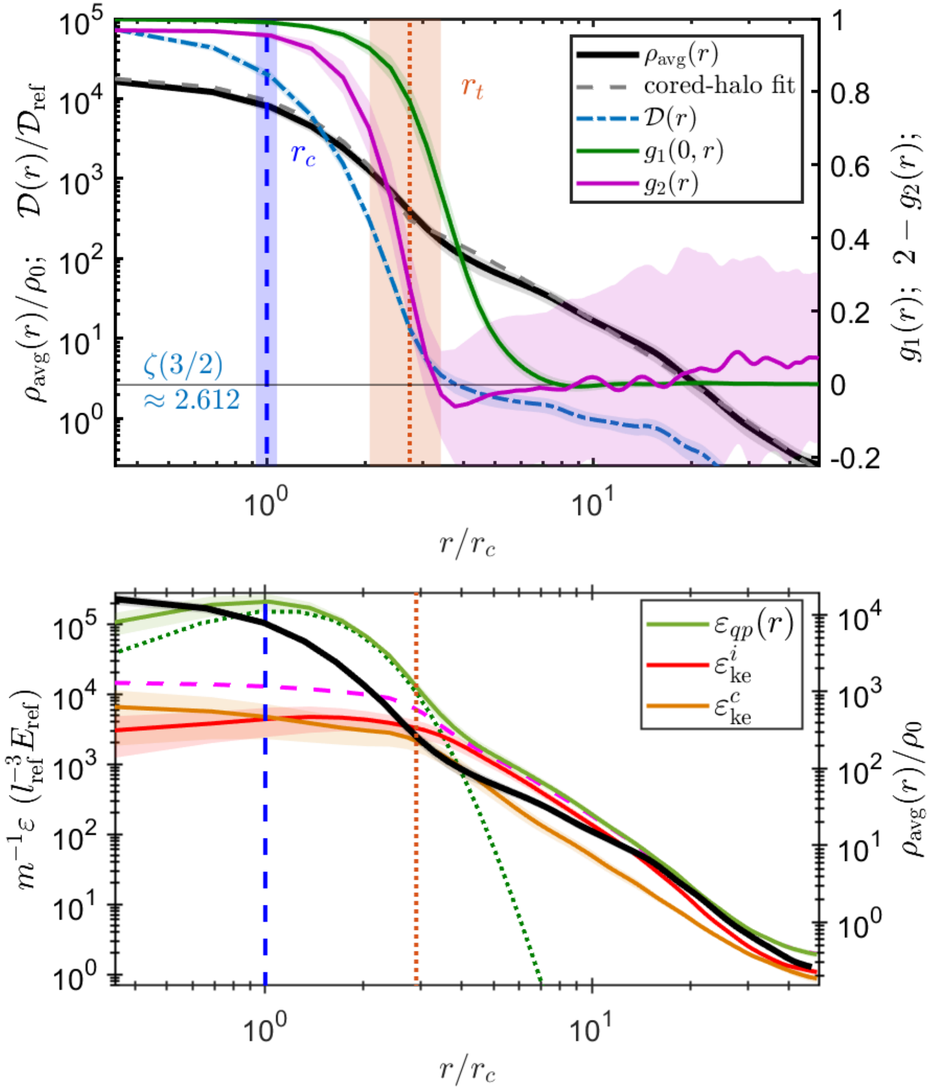

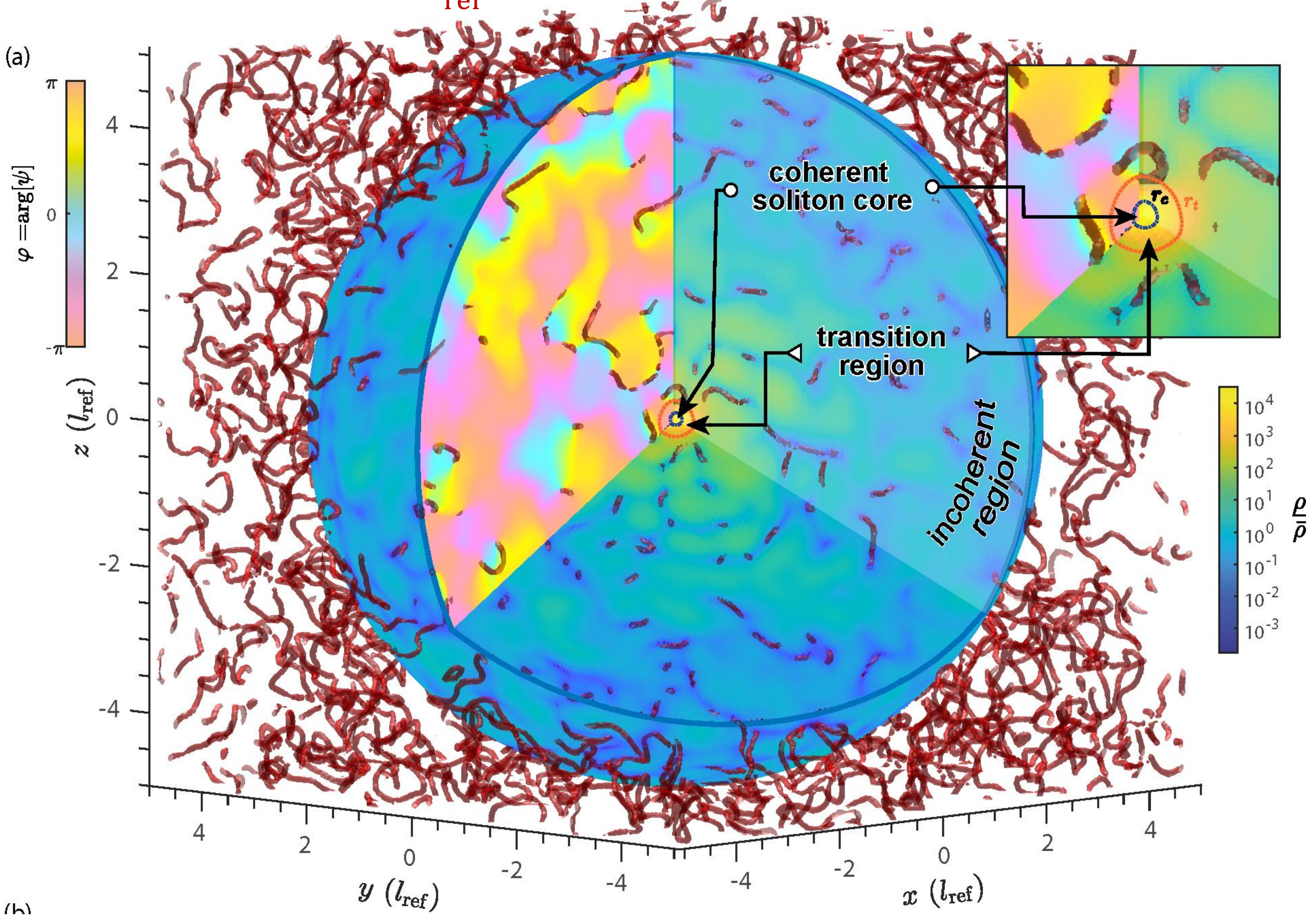

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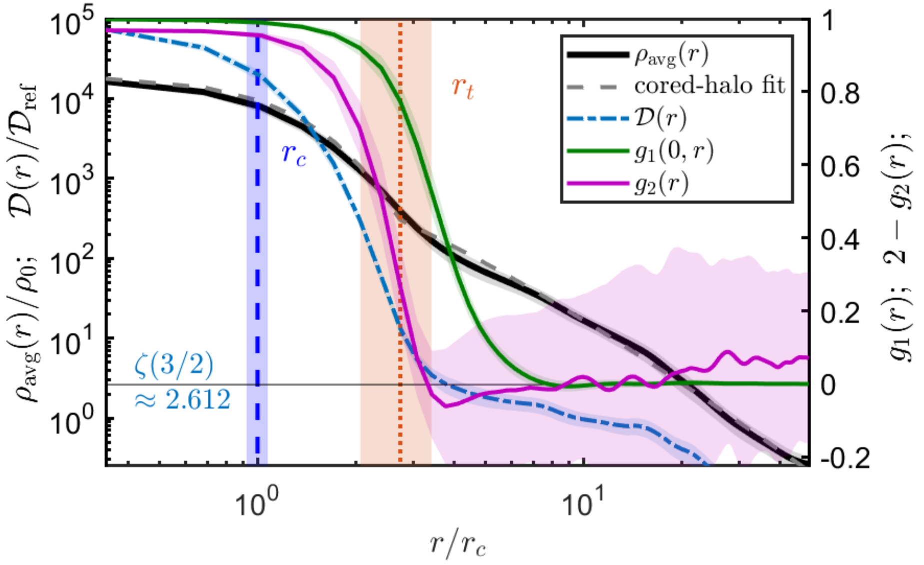
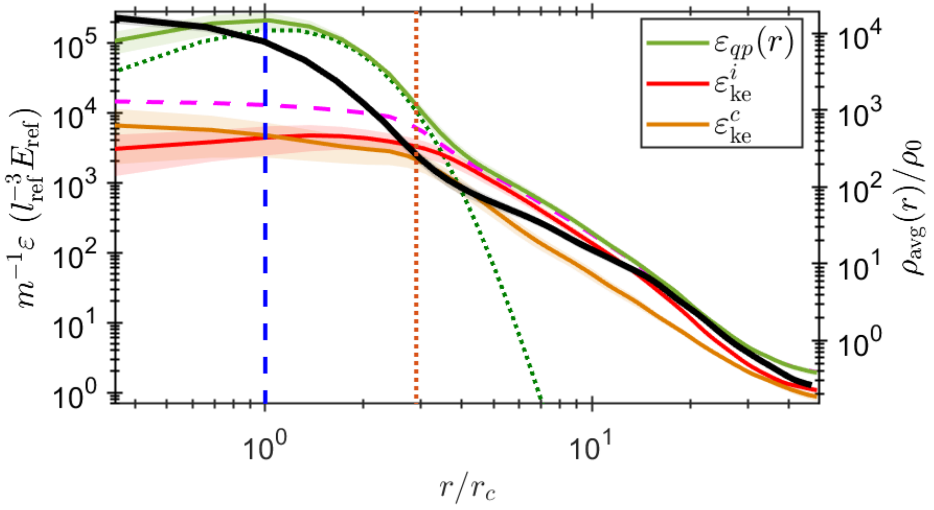
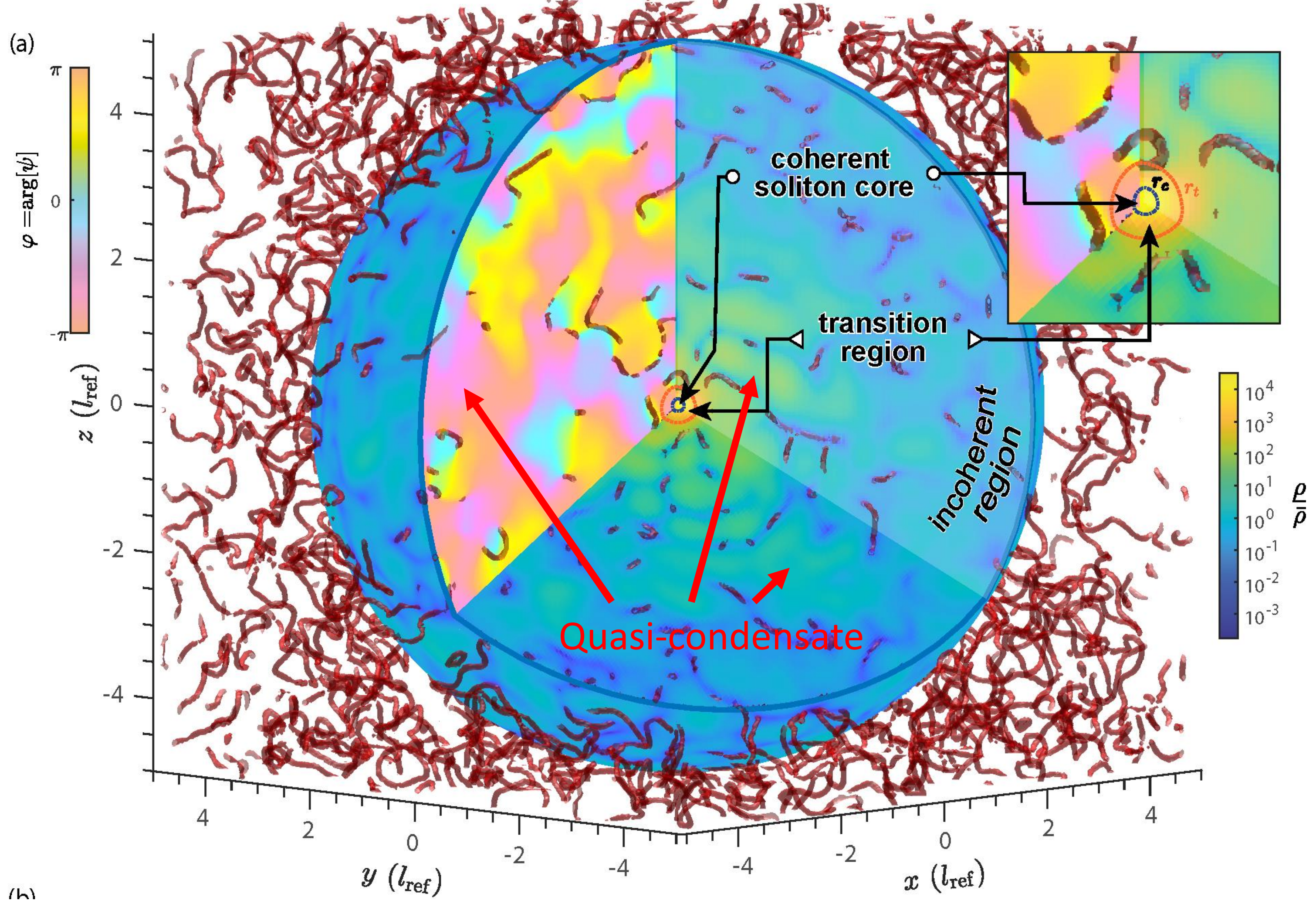






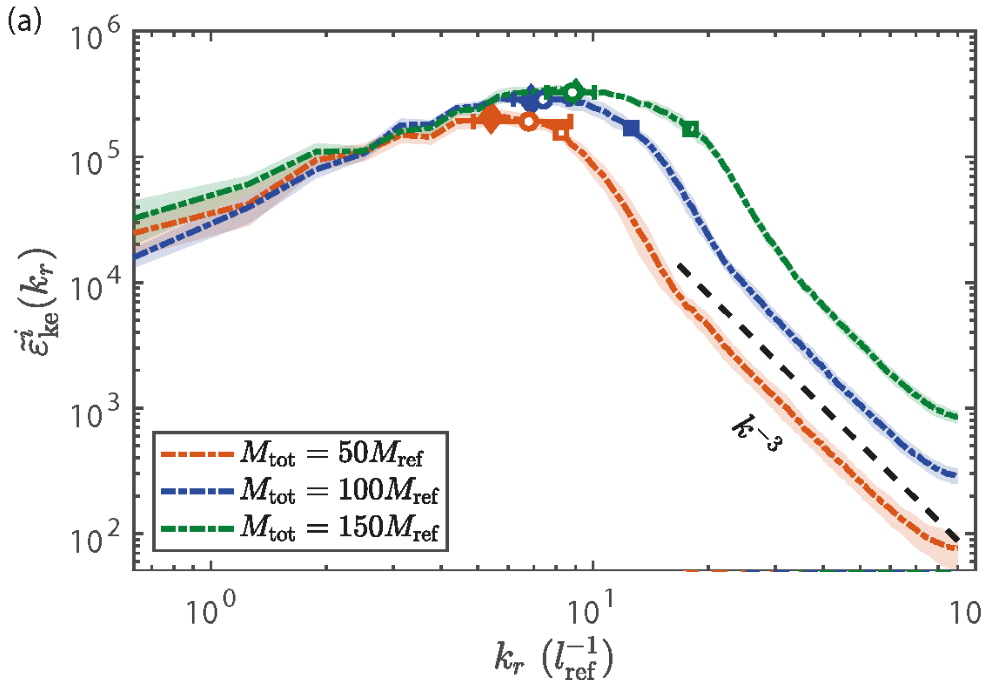
Vortices in FDM halos and granule size

Vortex energy and granule power spectra

Incompressible energy spectrum

$$\tilde{\varepsilon}^{i}_{\mathrm{ke}}(k_r) \equiv \int d\Omega_k \, k^2 \varepsilon^{i}_{\mathrm{ke}}(\mathbf{r}, t)$$

- No quasi-classical turbulence, $k^{-5/3}$
  - ❑required vortex bundles in SF
    A. Baggaley et al., PRL 109, 205304 (2012)
  - ❑FDM vortices are more chaotic
    Mocz et al., MNRAS 471, 4559 (2017)
- Vortex core structure, k^{-3}: $\rho\left|\mathbf{v}^i\right|^2 \approx$ Const. in large k
  Nore et al., Phys. of Fluids 9, 2644 (1997)
  Stagg et al., PRA 94, 053632 (2016)



Liu, Proukakis, Rigopoulos, 2207/08.XXXX

Vortices in FDM halos and granule size

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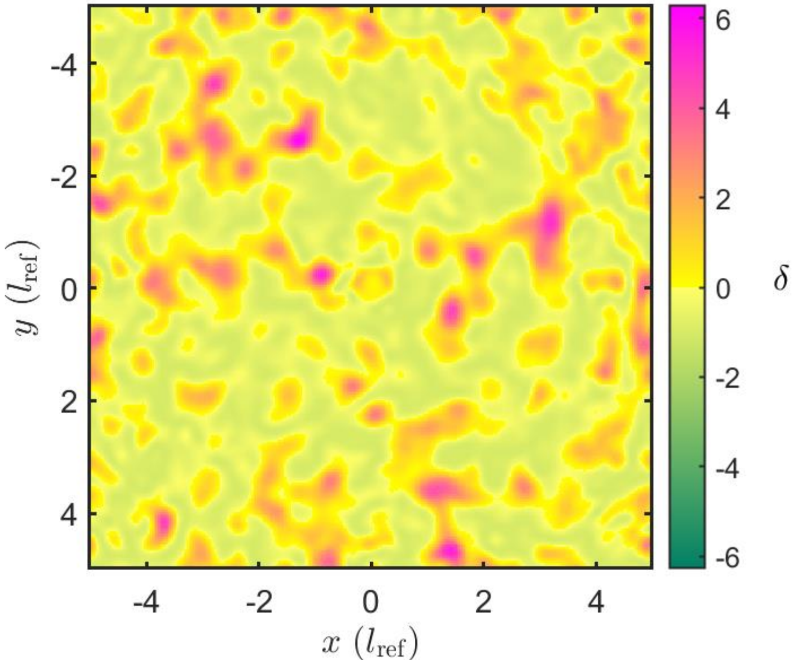
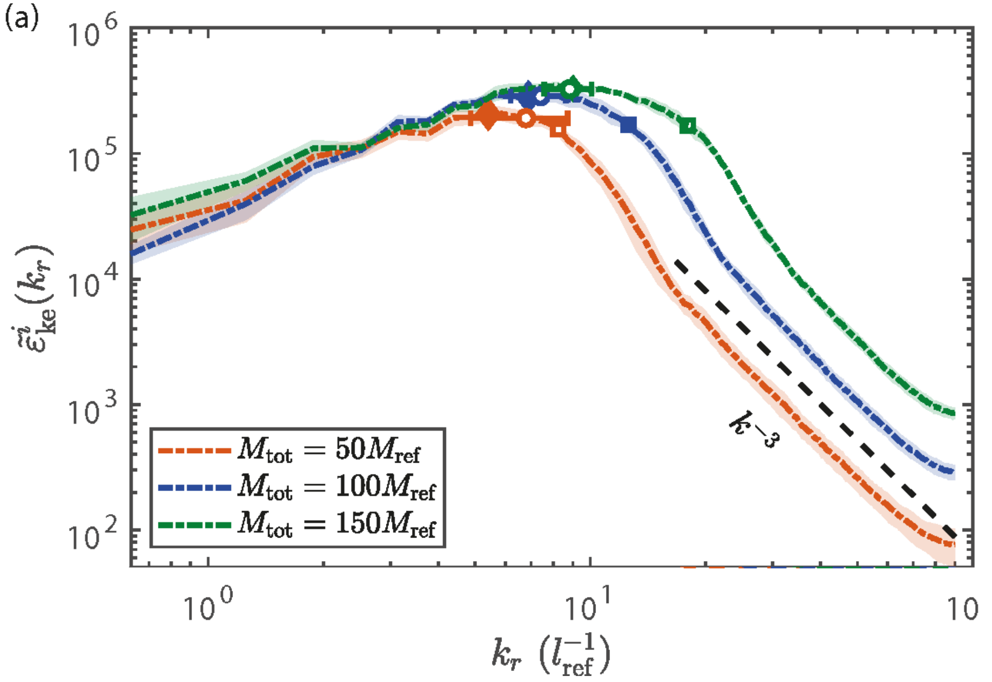
Granule power spectrum

$$\delta(\mathbf{r}, t) = \frac{\rho(\mathbf{r}, t) - \rho_{\mathrm{avg}}(\mathbf{r})}{\rho_{\mathrm{avg}}(\mathbf{r})}$$

Chan et al., MNRAS 478, 2686 (2018)
Lin et al. PRD 97, 103523 (2018)
Dutta Chowdhury et al. (2021)





Liu, Proukakis, Rigopoulos, 2207/08.XXXX

Vortices in FDM halos and granule size

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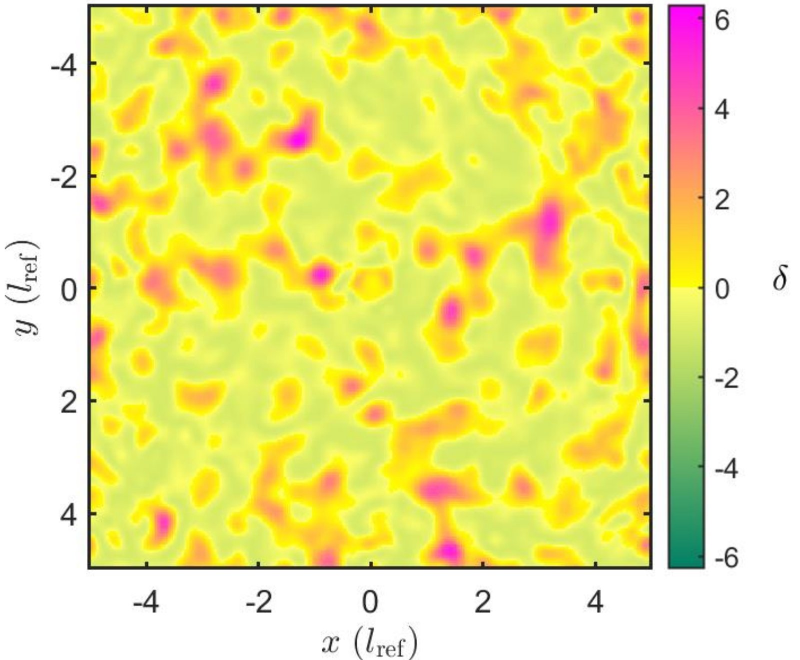

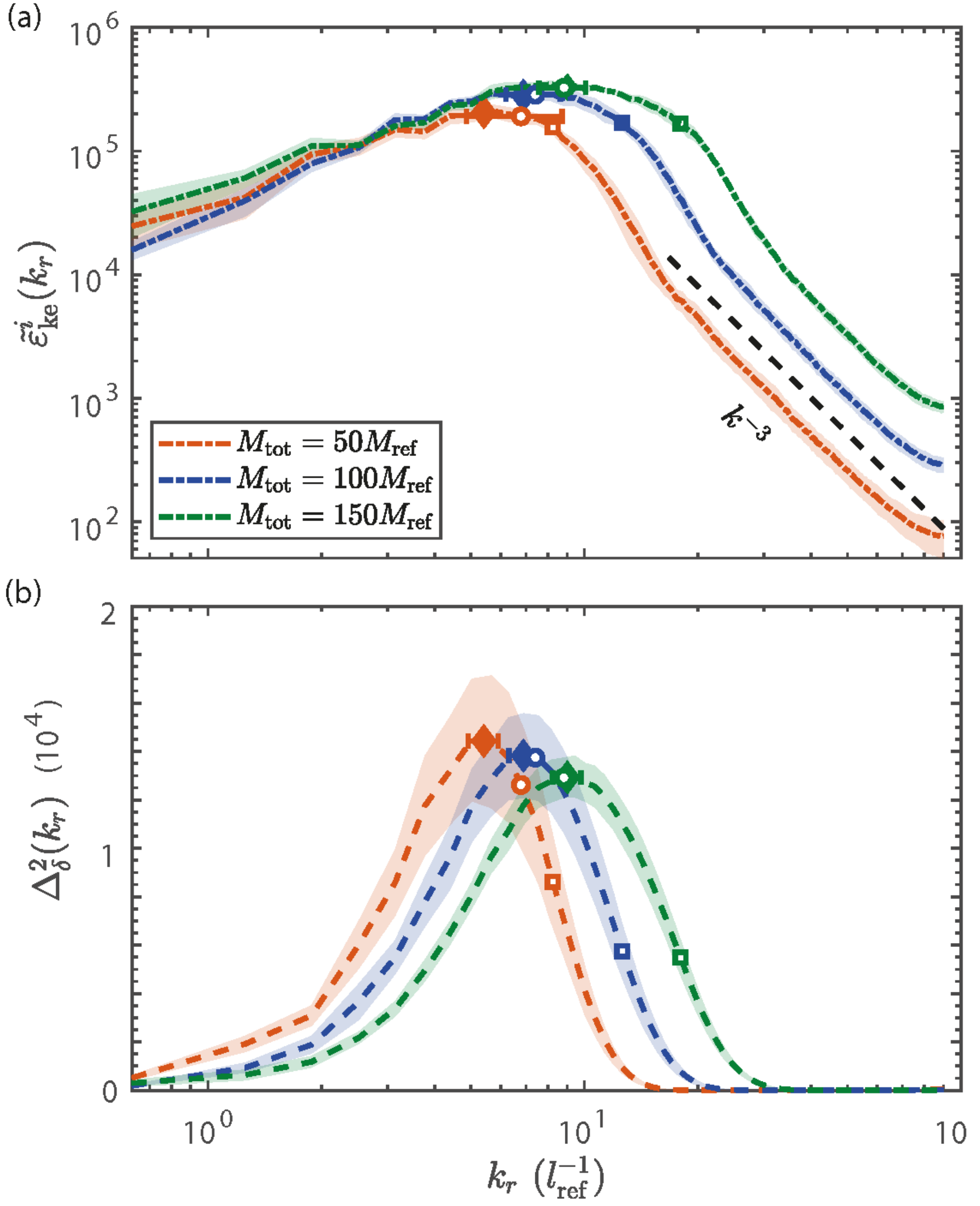

Liu, Proukakis, Rigopoulos, 2207/08.XXXX

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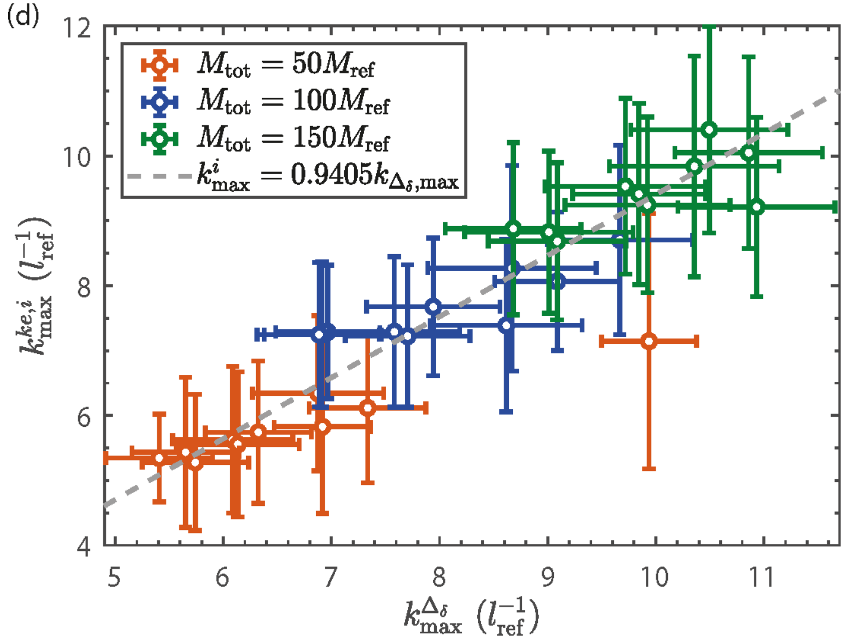

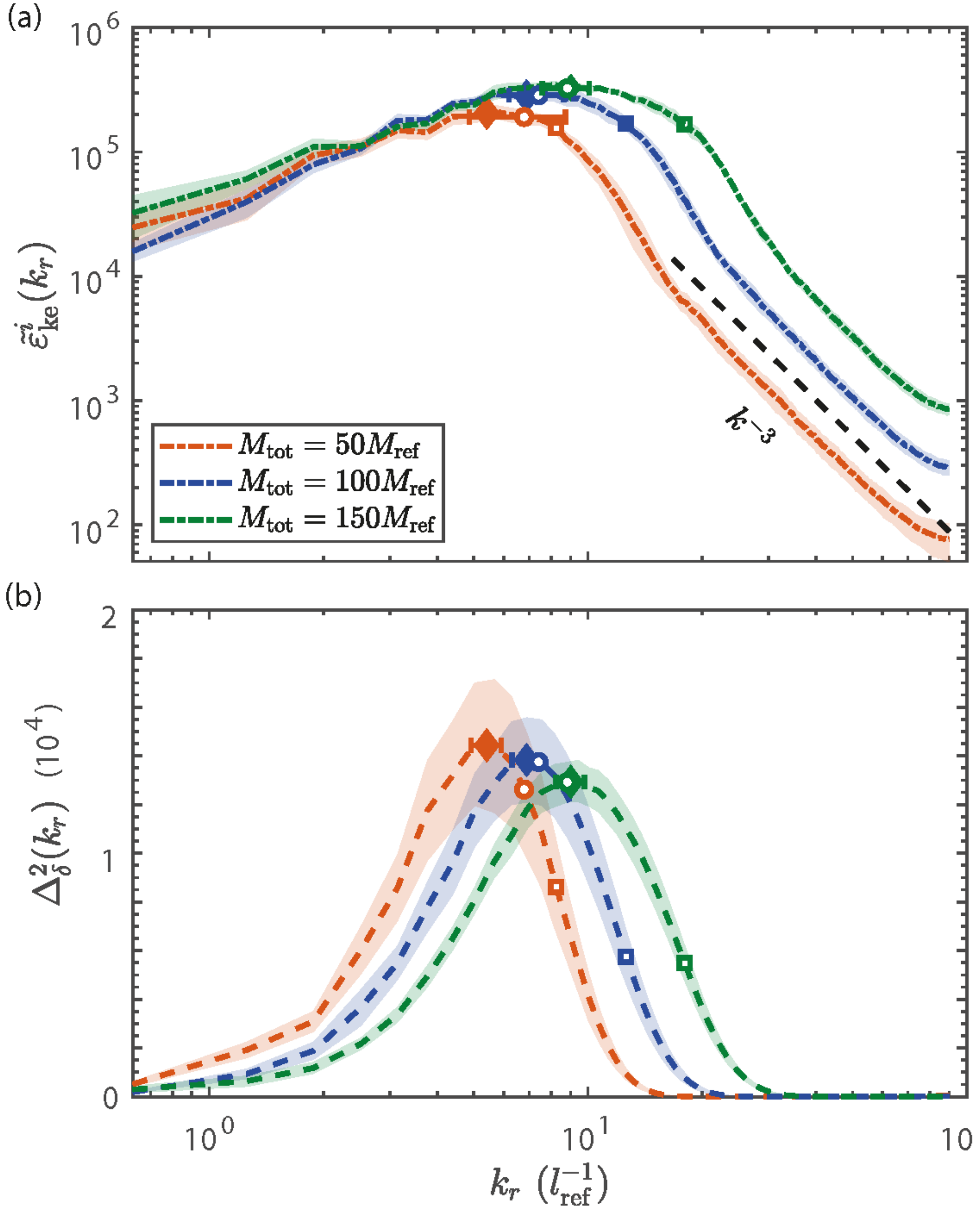

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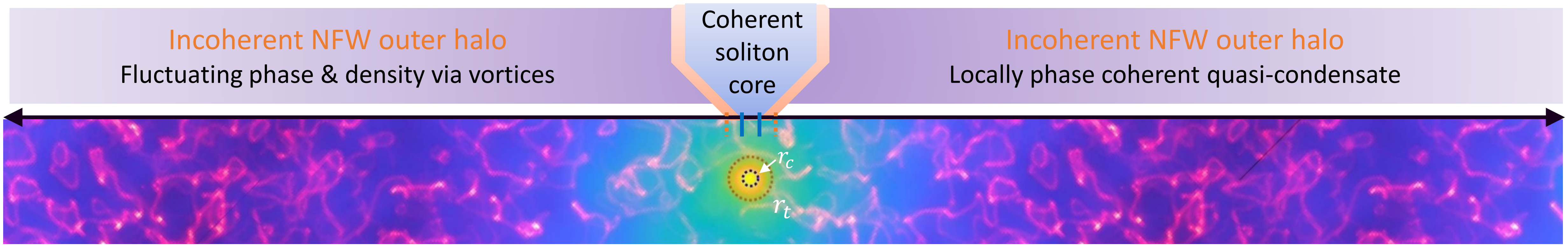
Conclusion



- The radial halo profile can be fairly caught by the cored-halo fit with the parameters r_c and r_t.
- The FDM mass can roughly by estimated by

$$\mathcal{D}_{\mathrm{ref}} = \left(\frac{\hbar^3 \rho_{\mathrm{ref}}}{m^4 v_{\mathrm{ref}}^3}\right) = 1.25 \times 10^{90} \left(\frac{10^{-22}\ \mathrm{eV}}{mc^2}\right)^4 \left(\frac{\rho_{\mathrm{ref}}}{10^3 M_\odot \mathrm{kpc}^{-3}}\right) \left(\frac{250\mathrm{km}/s}{v_{\mathrm{ref}}}\right)^3$$

  giving $m \approx [10^{-22}, 10]$ eV for FDM in our simulation.

- The turbulent-like vortex dynamics results in the large density fluctuations in the outer halo.
- The inter-vortex distances and the granule length scale are comparable

Thank you for your attention

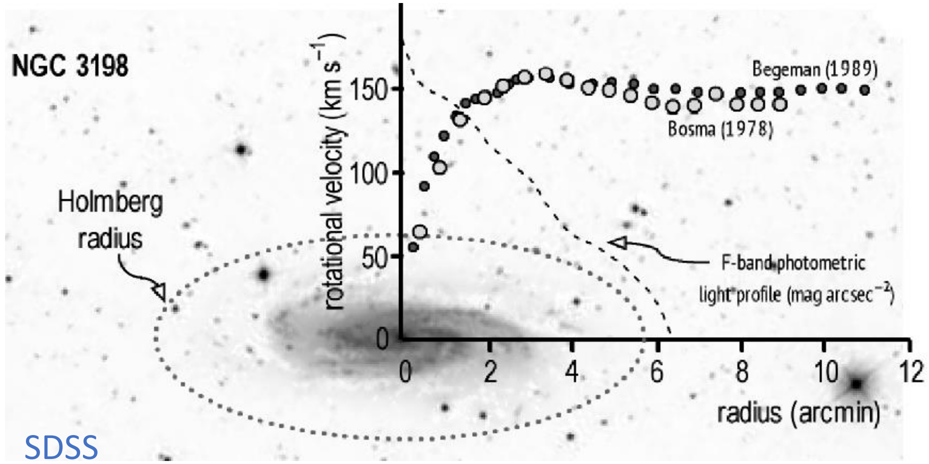
Newcastle University

Outline

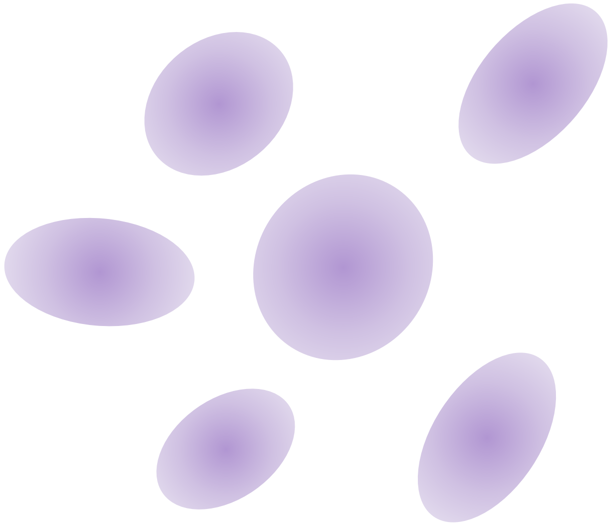
- Fuzzy dark matter
- Halo structures
- Vortices in FDM halos and granule size
- Conclusion

Fuzzy Dark Matter

- Rotational curve



- Galaxy alignment



- Lensing

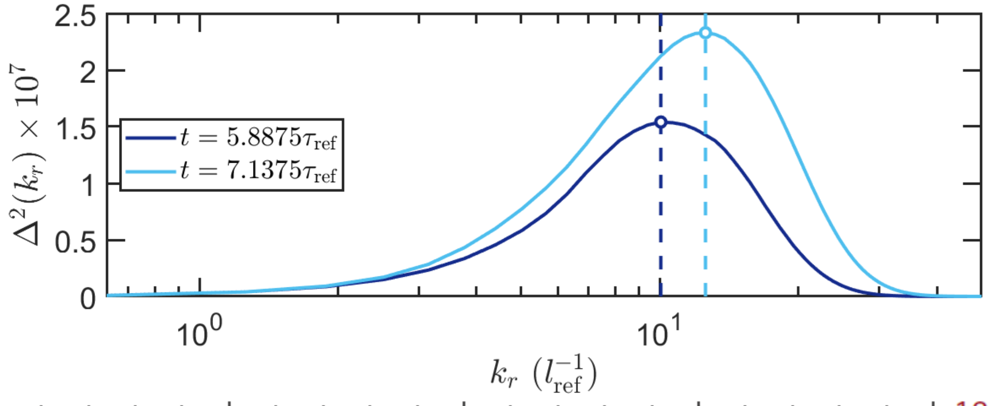
Halo structure: soliton core oscillations

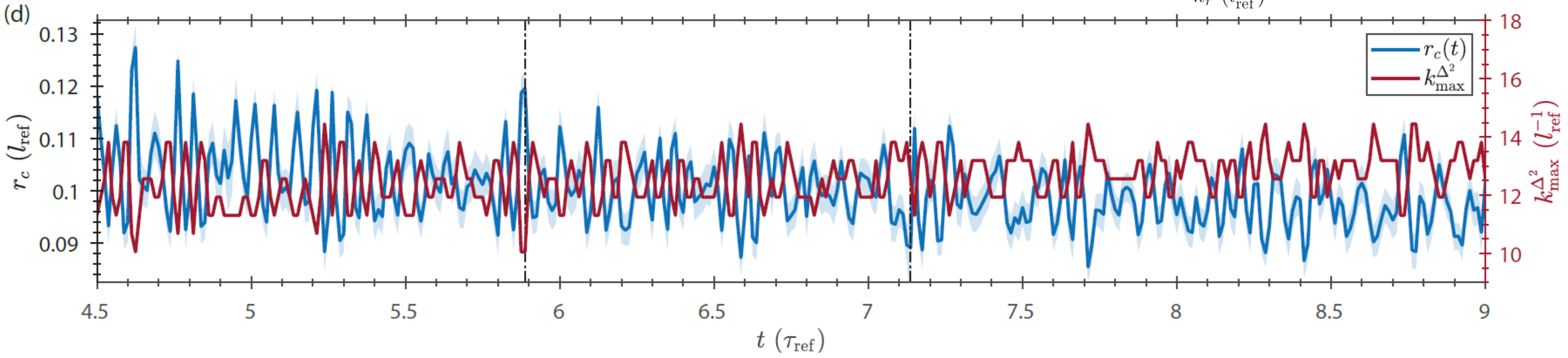
Power spectrum

$$\Delta^2(k_r,t) = \frac{k_r^3}{(2\pi)^2}\int \frac{d\mathbf{\Omega_k}}{4\pi k_r^2}P(\mathbf{k},t)$$

$$P(\mathbf{k},t) = |\tilde{\eta}(\mathbf{k},t)|^2$$

$$\eta(\mathbf{r},t) = \frac{\rho(\mathbf{r},t)-\bar{\rho}}{\bar{\rho}}$$





$r_c - k_{\rm max}^{\Delta^2}$ is anti correlated

Halo structures

Mode-mixing and nonlinear dynamic

Lin et al. PRD 97, 103523 (2018)

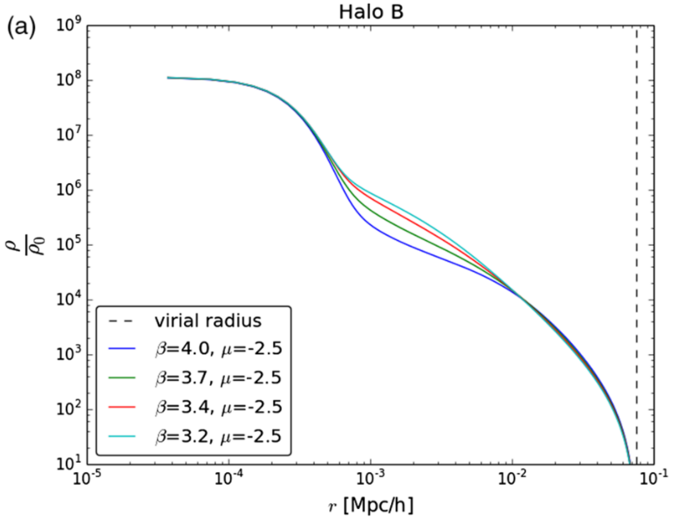
$$\Psi(\mathbf{r},t) = \sum_{nlm} a_{nlm} e^{iE_{nl}t/\hbar} \phi_{nlm}(\mathbf{r})$$

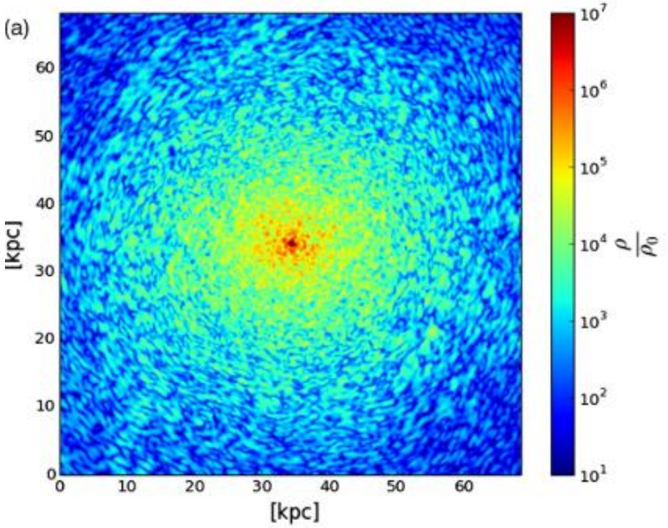
$$E_{nl}\phi_{nlm}(\mathbf{r}) = \left[-\frac{\hbar^2\nabla^2}{2m} + m\bar{\Phi}(\mathbf{r})\right]\phi_{nlm}(\mathbf{r})$$

$|\phi_{000}(\mathbf{r})|^2$ is the soliton profile

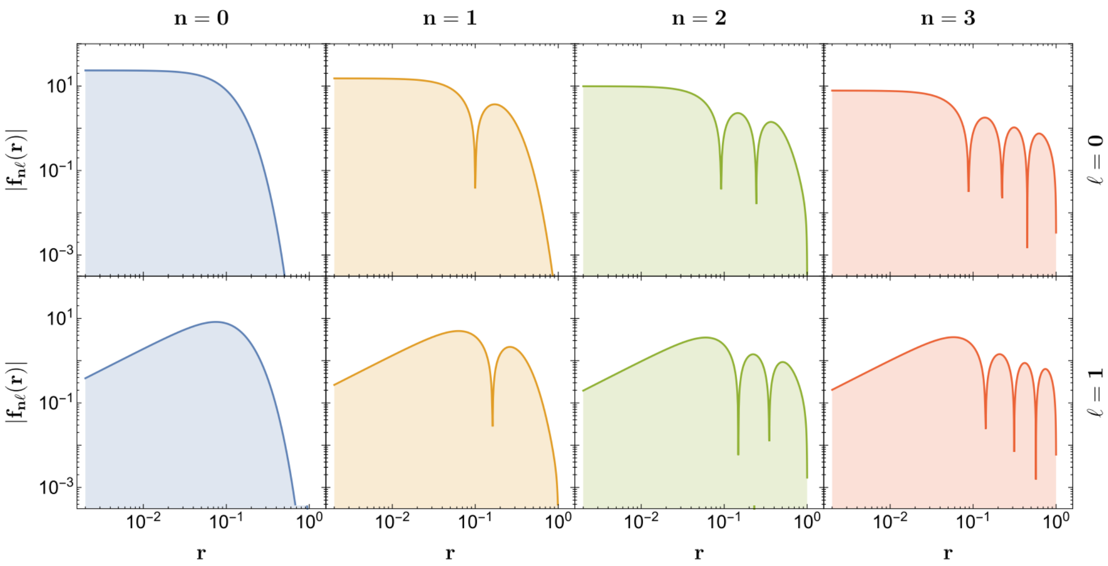
Random phase + Spherical-symmetry approximations

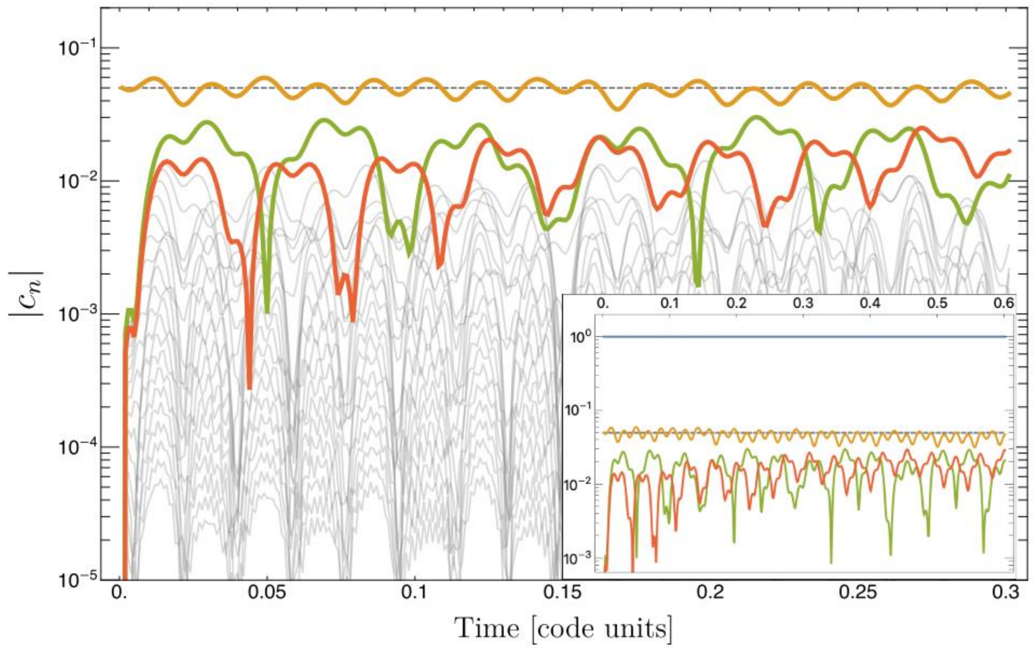
$$\nabla^2\bar{\Phi}(\mathbf{r}) = 4\pi G\bar{\rho}(r) = G\int d\mathbf{\Omega} \sum_{nlm} |a_{nlm}\phi_{nlm}(\mathbf{r})|^2$$





Zagorac et al., PRD 105, 103506



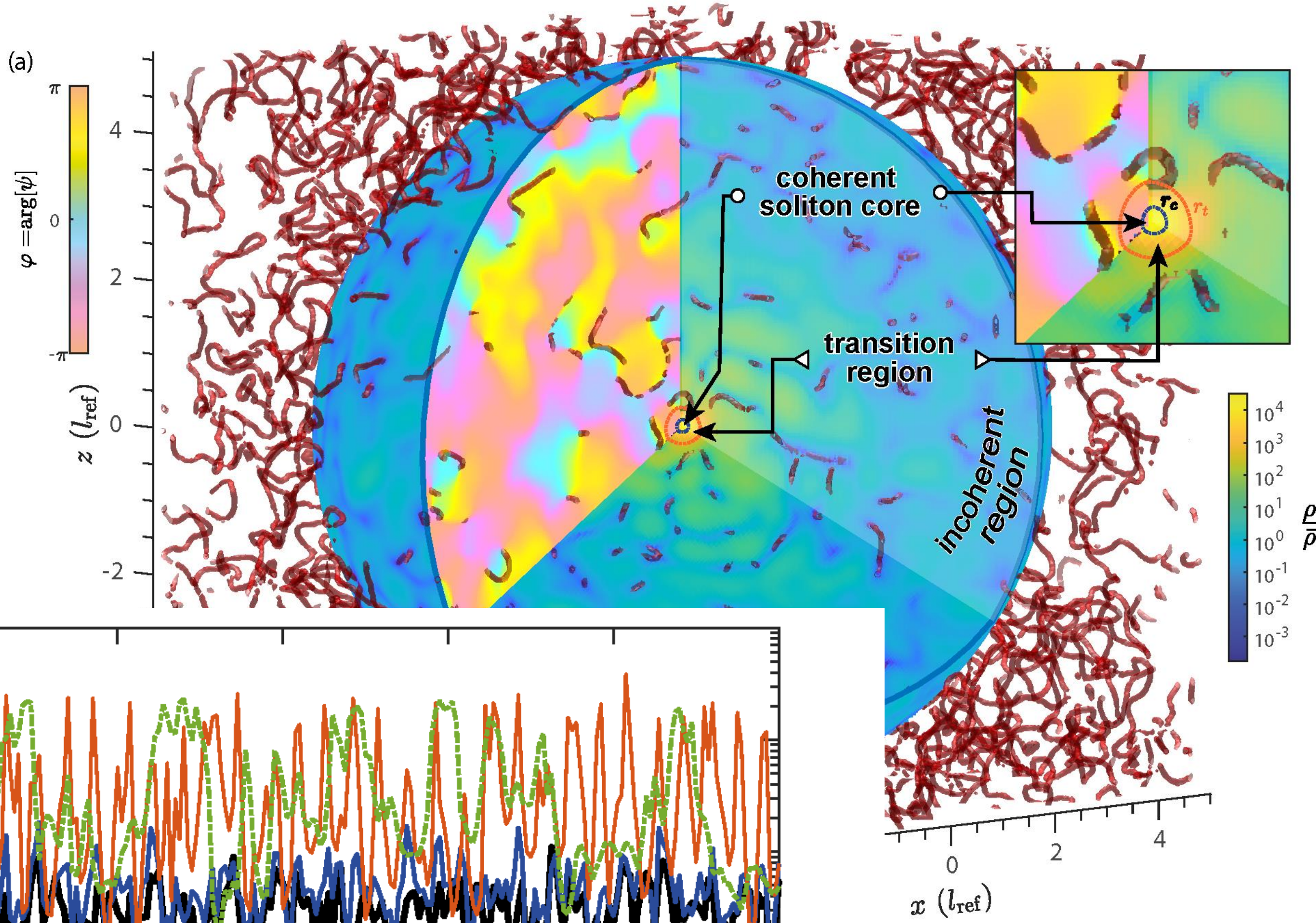


Vortices in FDM halos and granule size

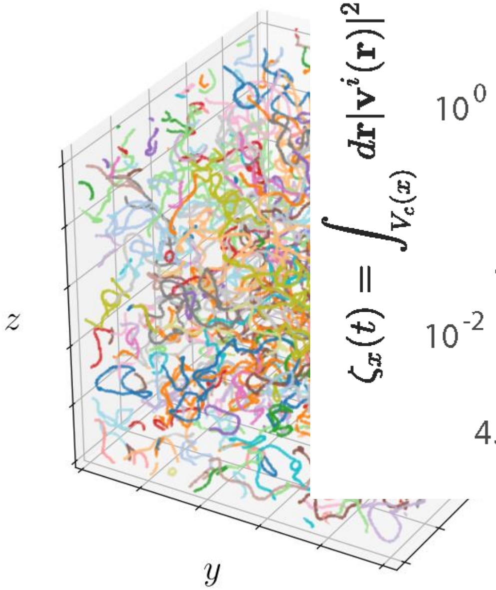
Vortical structure visualization

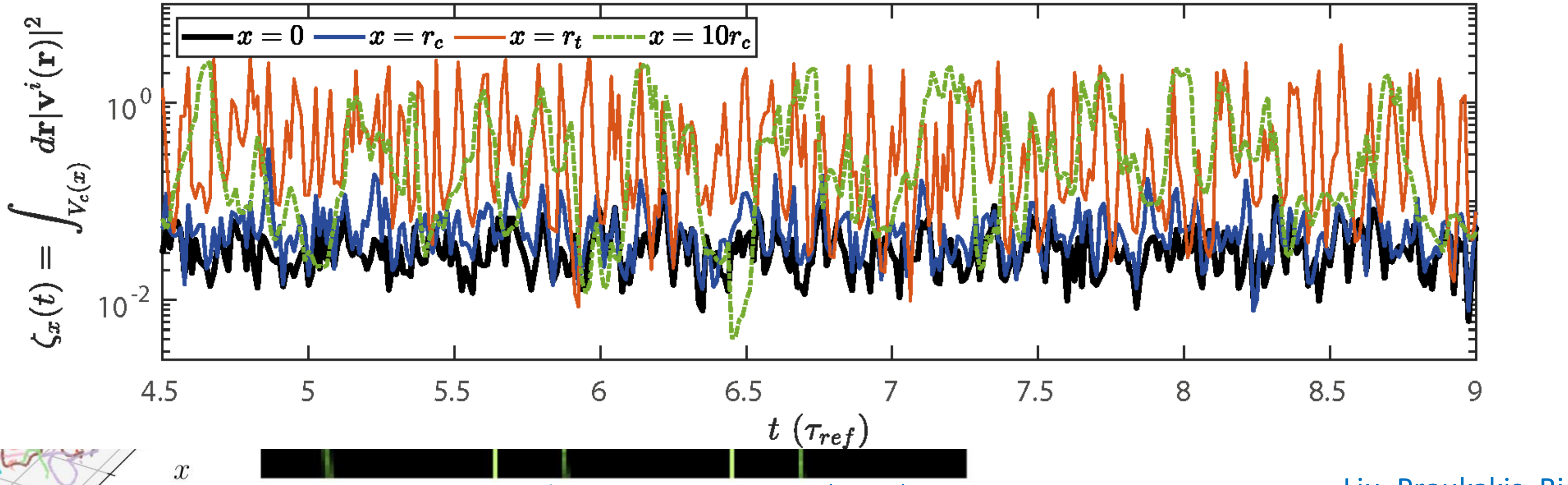
$$\left|\mathbf{v}^i(\mathbf{r})\right|^2 \approx \frac{\hbar^2}{m^2 \Delta x^2} \approx 800\, \frac{l_{\mathrm{ref}}^2}{\tau_{\mathrm{ref}}^2}$$

- Vortices are in closed loops.
- They are stretching, shrinking, reconnecting and tangling together



L. Hui et al., Journal of Cosmology and Astroparticle Physics 1, 011 (2021)





Mocz et al., MNRAS 471, 4559 (2017)

Liu, Proukakis, Rigopoulos, in preparation

Vortices in FDM halos and granule size

Vortex energy and granule power spectra

Incompressible energy spectrum

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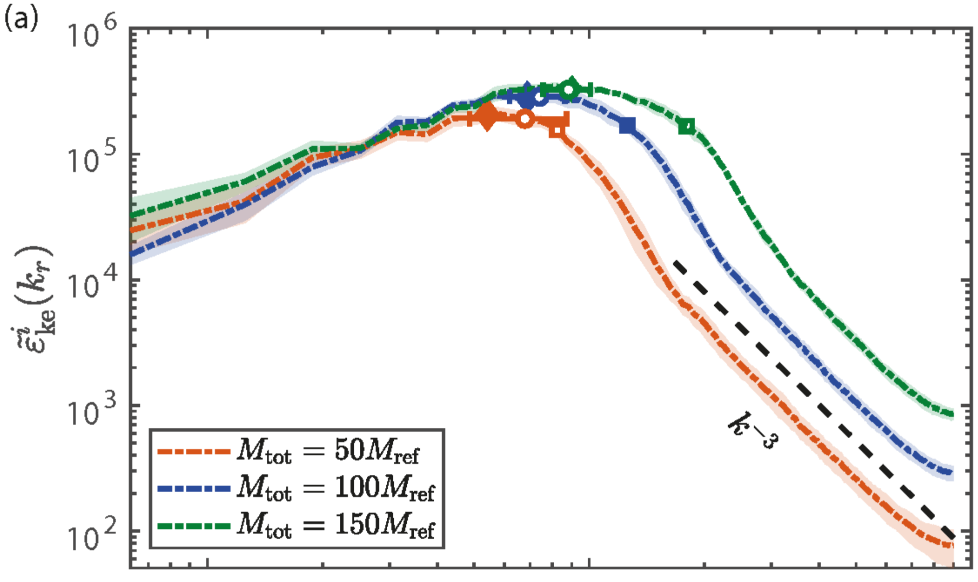
Nore et al., Phys. of Fluids 9, 2644 (1997)
Stagg et al., PRA 94, 053632 (2016)

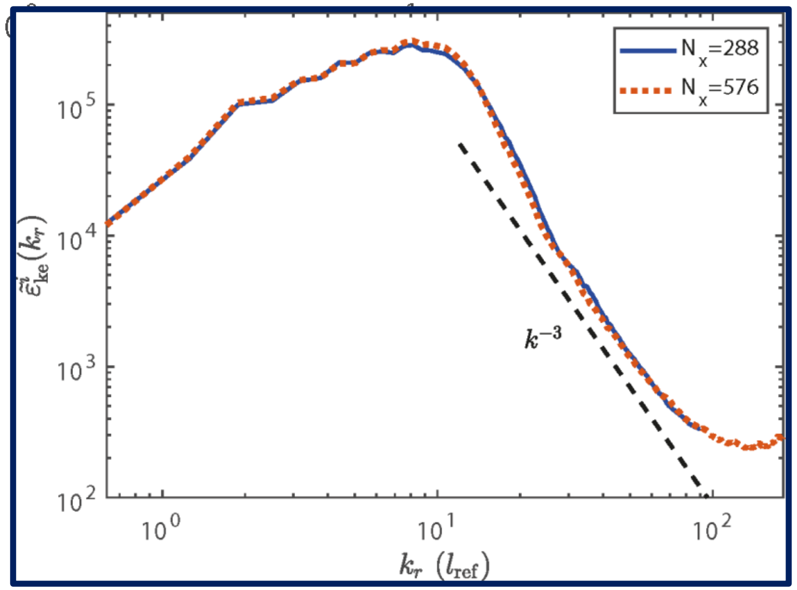
Quasi-classical turbulence, $k^{-5/3}$: Not seen

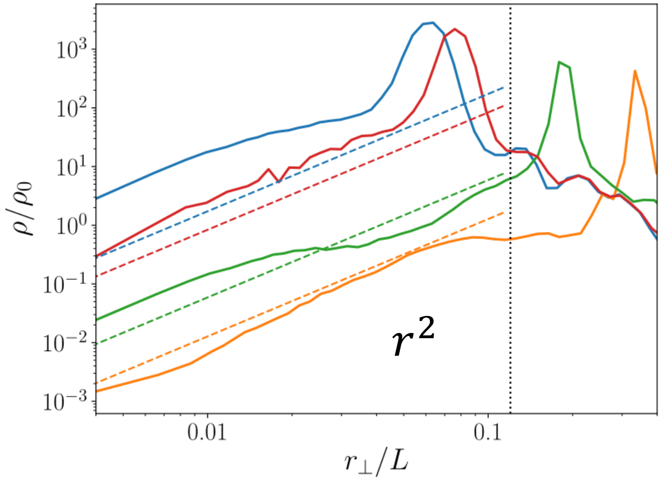
Mocz et al., MNRAS 471, 4559 (2017)

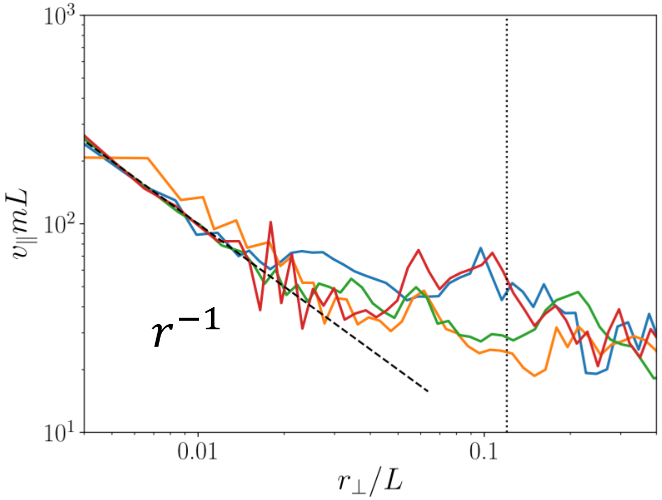
Vortex core structure, k^{-3}: $\rho|\mathbf{v}^i|^2 \approx$ Const. in large k

L. Hui et al., 2004.01188, JCOPAL (2021)
T. Chiueh et al., J. Phys. B 44, 115101 (2011)



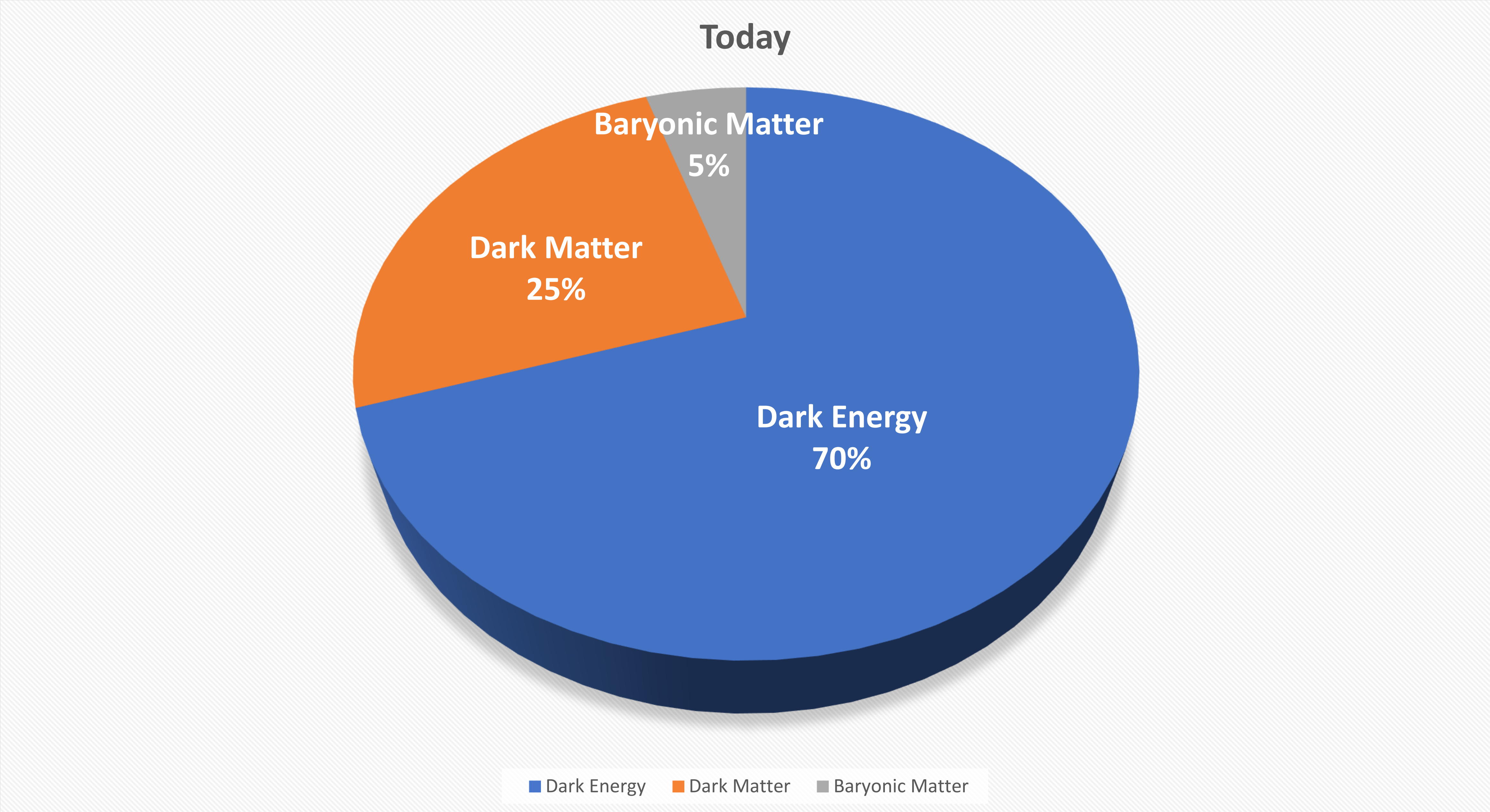






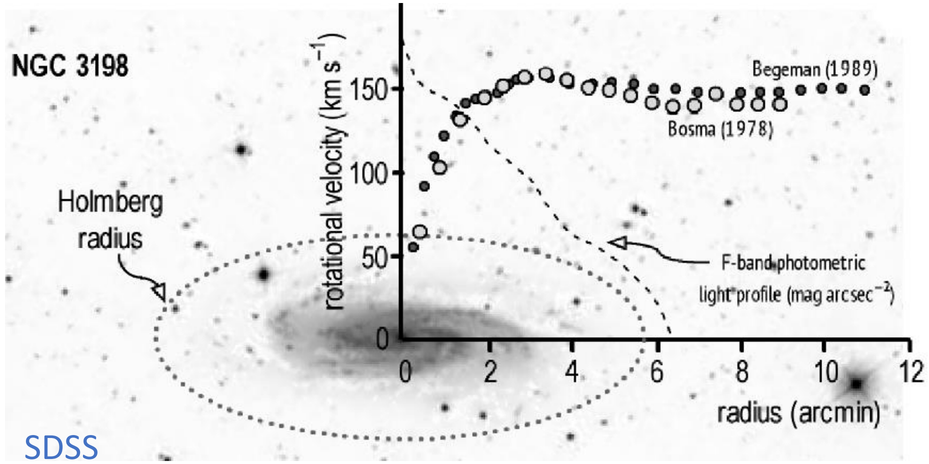
Liu, Proukakis, Rigopoulos, 2207/08.XXXX

Fuzzy Dark Matter



Fuzzy Dark Matter

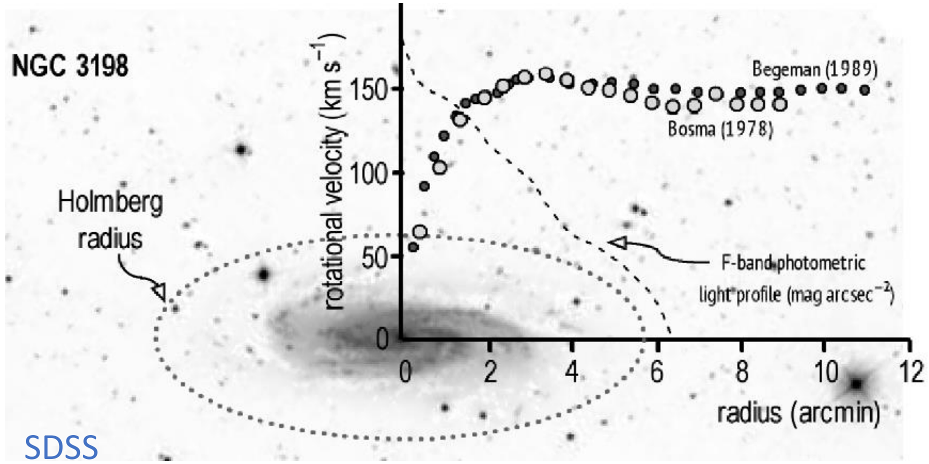
- Rotational curve



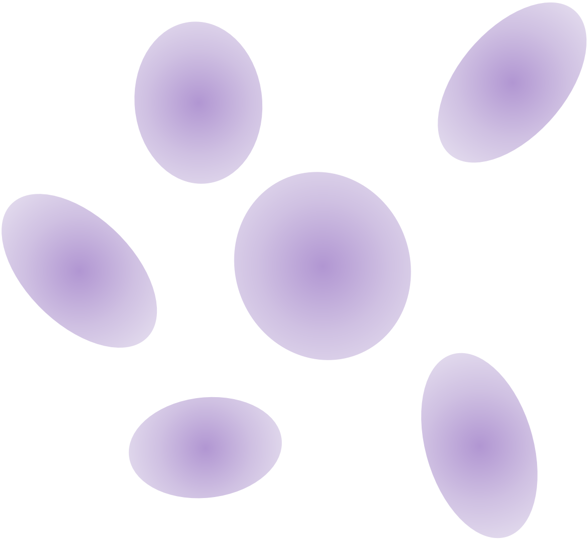
SDSS

Fuzzy Dark Matter

- Rotational curve



- Galaxy alignment



Fuzzy Dark Matter

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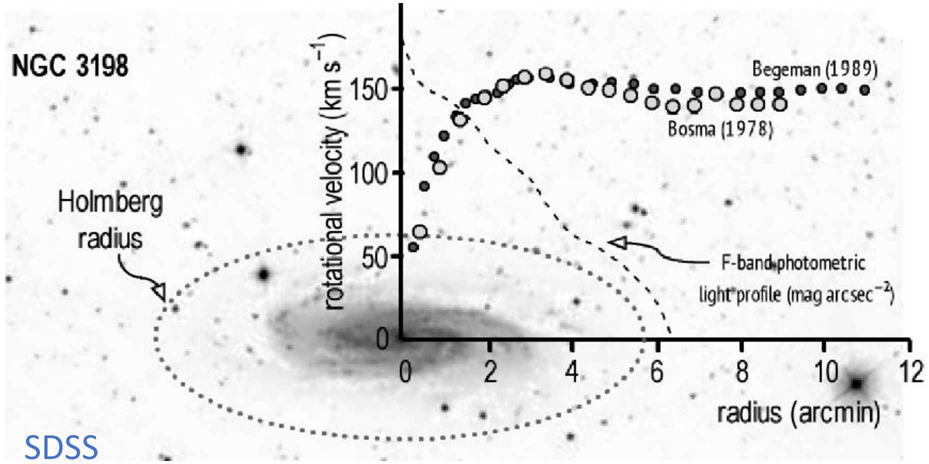
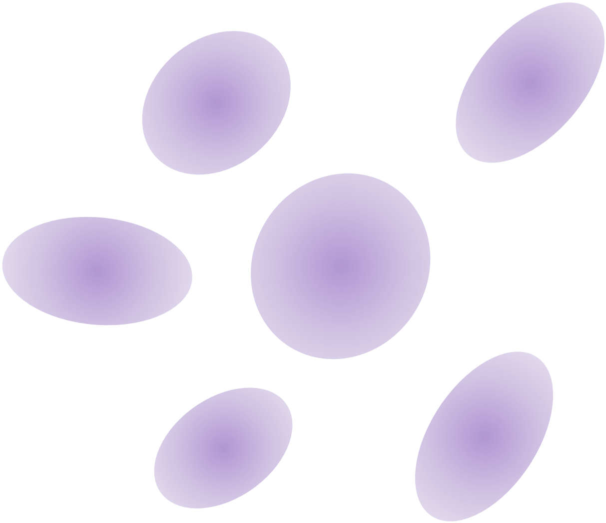


SDSS

- Galaxy alignment



Fuzzy Dark Matter

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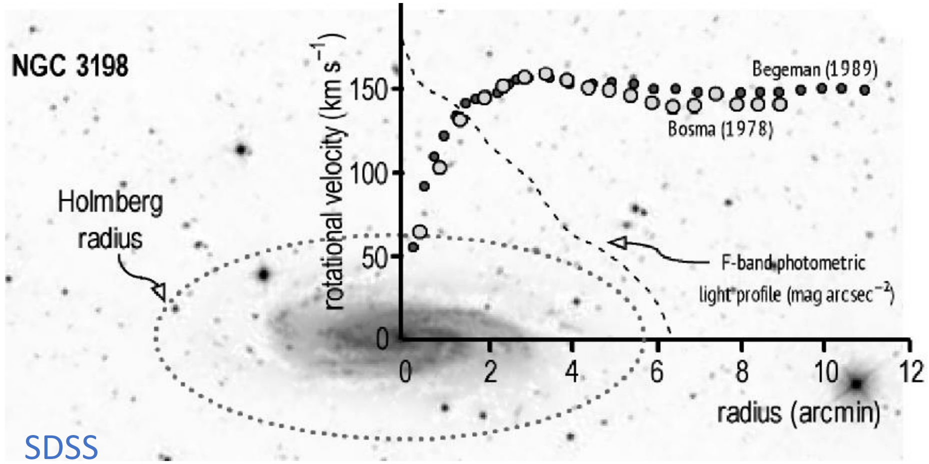
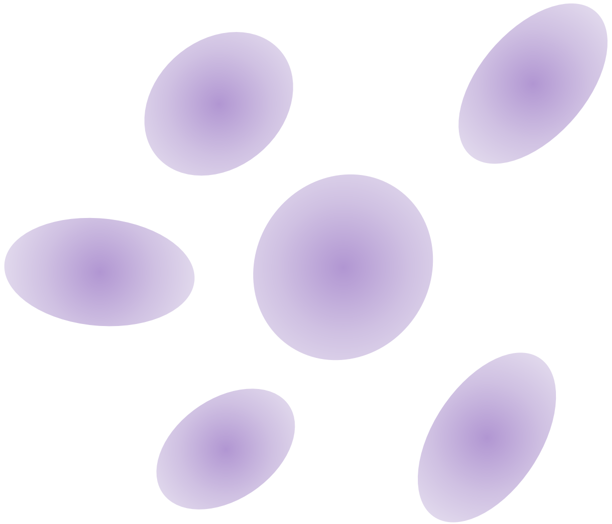
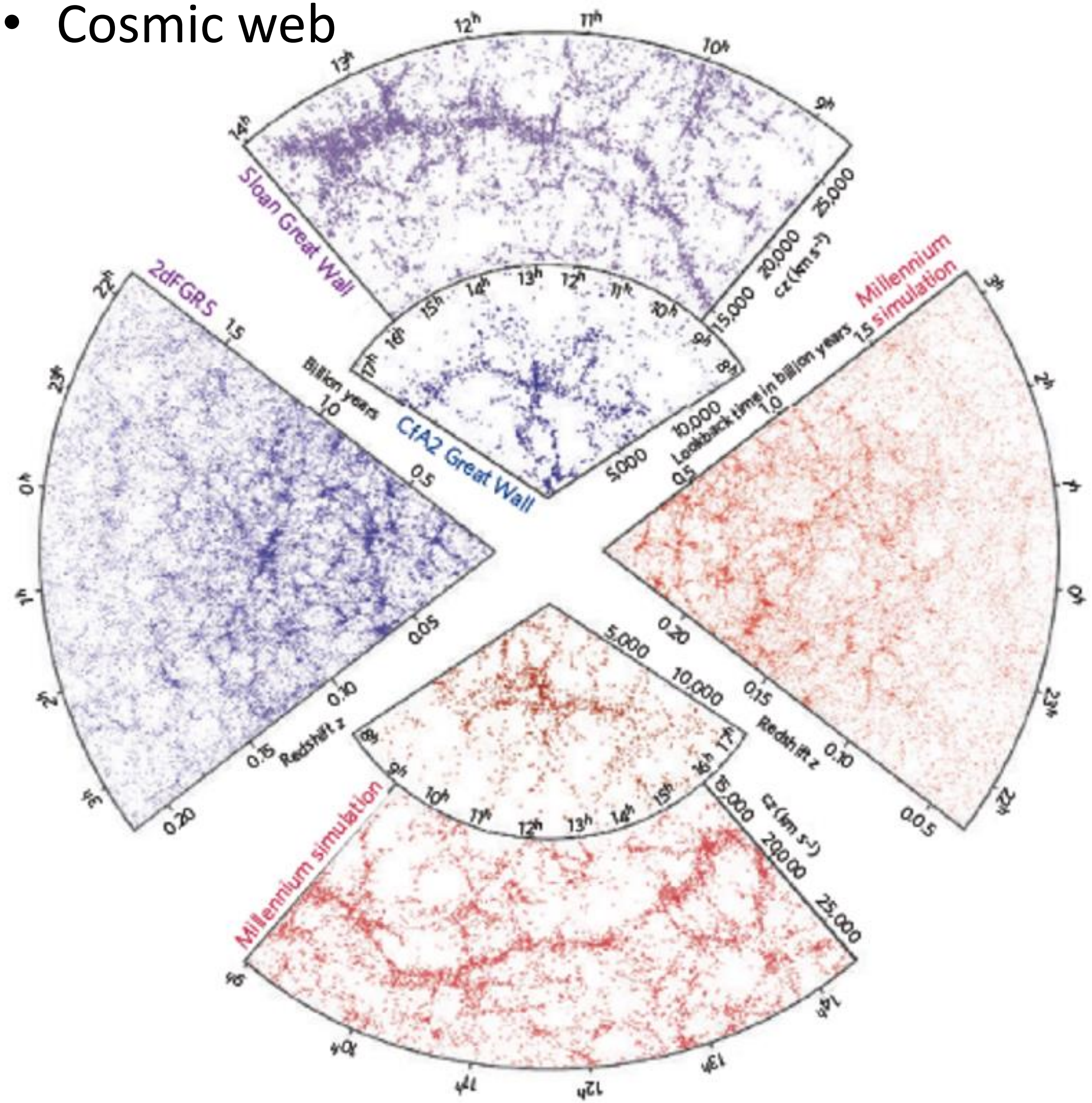


- Galaxy alignment



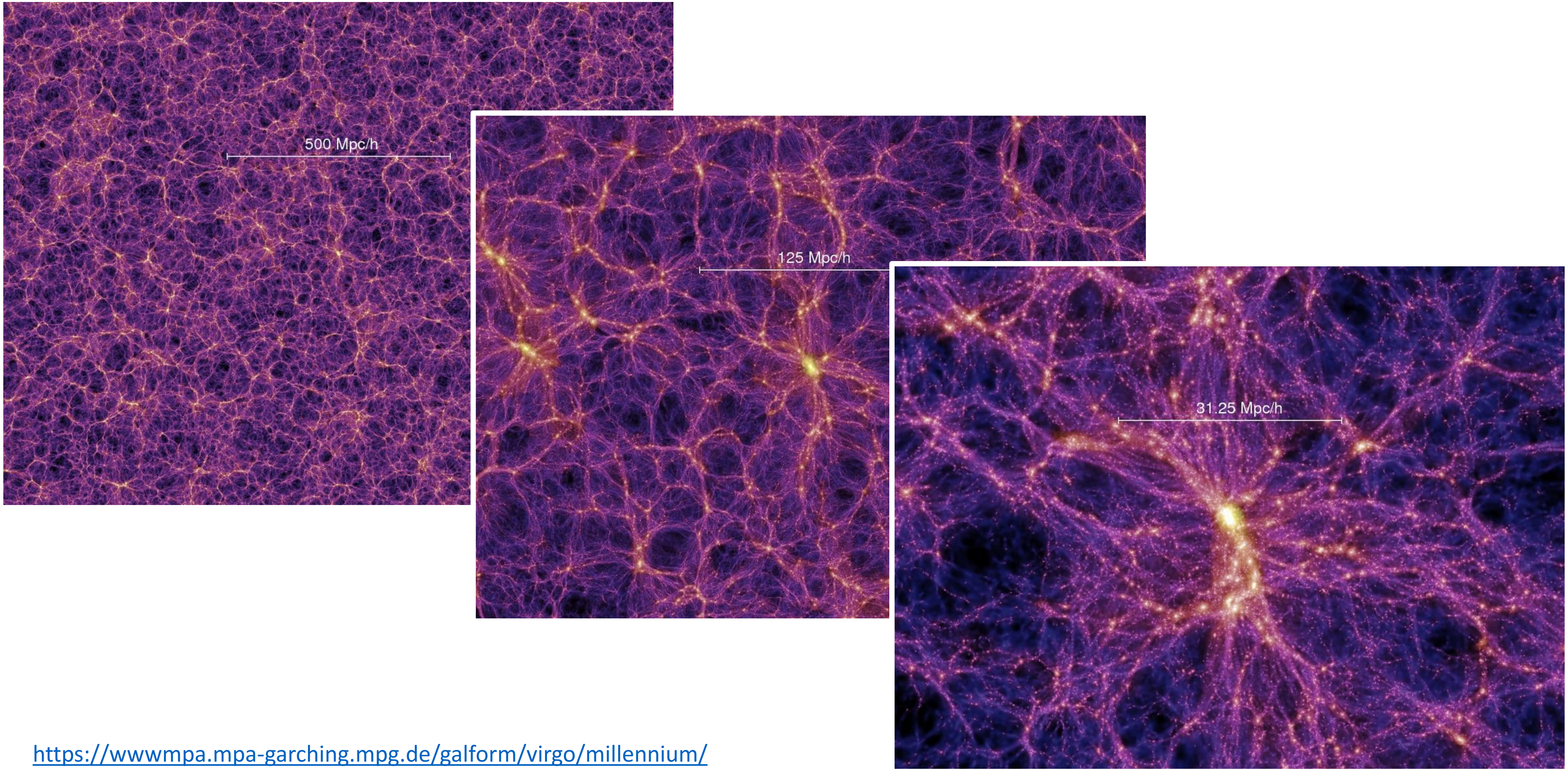
- Cosmic web



V. Springel et al., Nature 440, 1137 (2006)

Fuzzy Dark Matter

- Cosmic web



https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/