THERE IS ONLY ONE TIME

time and classical equations of motion from quantum entanglement via the Page and Wootters mechanism with generalized coherent states

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IS THERE A PROBLEM ?

- why should there be any
  spacetime is cool
  (and QFT is amazing)

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- why should there be any

  spacetime is cool

  (and QFT is amazing)

  ... uhm ...

- time is not a quantum observable

  $\Delta E \Delta t \geq \hbar$ does not make sense

  (and QFT does not help when it comes to GR $\longleftrightarrow$ QM)

?

GENERALIZED COHERENT STATES (GCS)

$$\mathcal{H}, \ \mathfrak{g}, \ |G\rangle$$

$$|\Omega\rangle \in \mathcal{H} \longleftrightarrow \Omega \in \mathcal{M}$$

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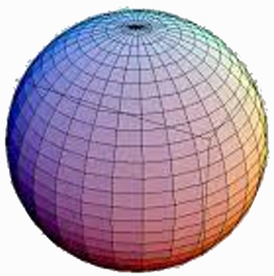
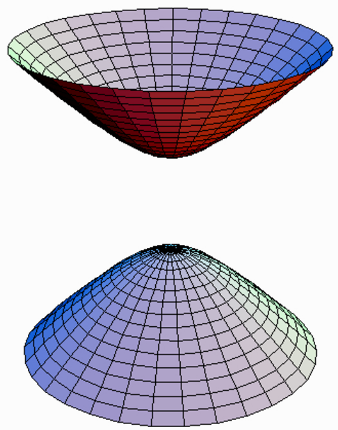
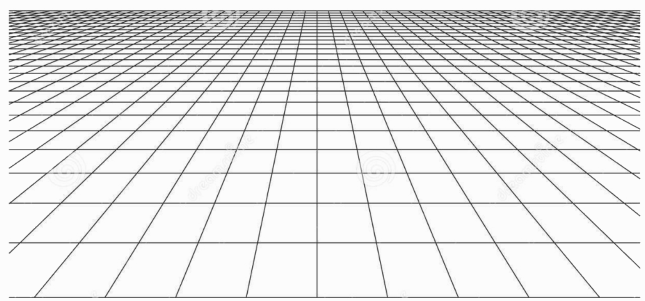
$\mathcal{M}$ is symplectic and depends on $\mathfrak{g}$



$SU(2)$



$SU(1,1)$



h_4

GCS AND THE LARGE-N LIMIT

QUANTUM-TO-CLASSICAL CROSSOVER

conditions: $Q_N \xrightarrow[N\to\infty]{} Cl$, in terms of GCS

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⇓

states $|\psi\rangle$ that survive: $\langle\Omega|\Omega'\rangle \xrightarrow[N\to\infty]{} \delta(\Omega-\Omega')$

$|\Omega\rangle \in \mathcal{H}$ $\qquad$ $\Omega \in \mathcal{M}$

observables $\hat{A}$ that survive: $\dfrac{\langle\Omega|\hat{A}|\Omega'\rangle}{\langle\Omega|\Omega'\rangle} \xrightarrow[N\to\infty]{} < \infty$

$\langle\Omega|\hat{A}|\Omega\rangle$ $\qquad$ $A(\Omega)$

symbol

GCS AND COMPOSITE SYSTEMS $\Psi = e + \Gamma$

A PARAMETRIC REPRESENTATION

$$|\Psi\rangle\rangle = \sum_{\gamma\xi} e_{\gamma\xi} |\xi\rangle \otimes |\gamma\rangle \in \mathcal{H}_\Psi = \mathcal{H}_e \otimes \mathcal{H}_\Gamma$$

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$$\underbrace{\qquad\qquad\qquad}_{\int d\mu(\hat{\Omega}) |\Omega\rangle\langle\Omega|}$$

$$= \int_{\mathcal{M}} d\mu(\Omega)\, \chi(\Omega) |\Omega\rangle \otimes |\phi(\Omega)\rangle\rangle$$

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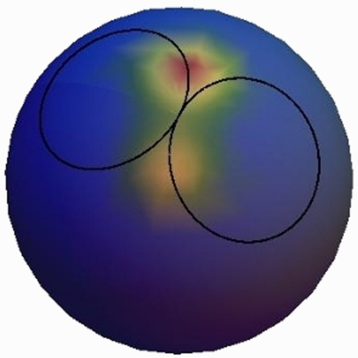
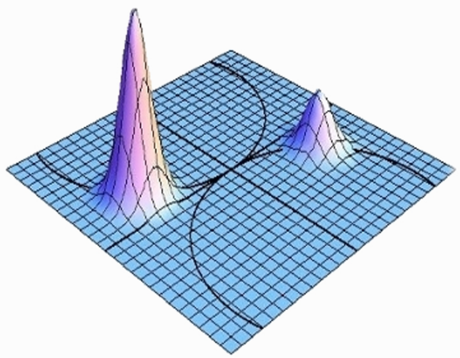
$$= \int_{\mathcal{M}} d\mu(\Omega) \, \chi(\Omega) \, |\Omega\rangle \otimes |\phi(\Omega)\rangle\rangle$$

$$\chi^2(\Omega) = \sum_\gamma \left| \sum_\xi c_{\gamma\xi} \langle\Omega|\xi\rangle \right|^2$$





THE PAW MECHANISM

@ a certain time t :

conditioned to a clock being in a state labeled by t

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↓

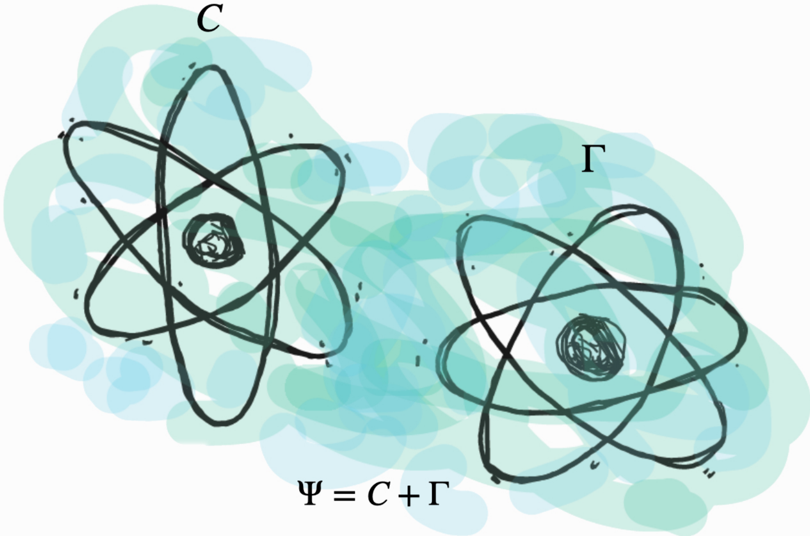
in a quantum setting: $\Psi = C + \Gamma$ and 3 assumptions

1. $\hat{H} = \hat{H}_C \otimes \hat{\mathbb{1}}_\Gamma - \hat{\mathbb{1}}_C \otimes \hat{H}_\Gamma$ non interacting

2. $|\Psi\rangle\rangle = \int_M d\mu(\Omega)\, \chi^2(\Omega)\, |\Omega\rangle \otimes |\phi(\Omega)\rangle$ entangled

3. $\hat{H}|\Psi\rangle\rangle = E\,|\Psi\rangle\rangle$ $E = 0$ if you like

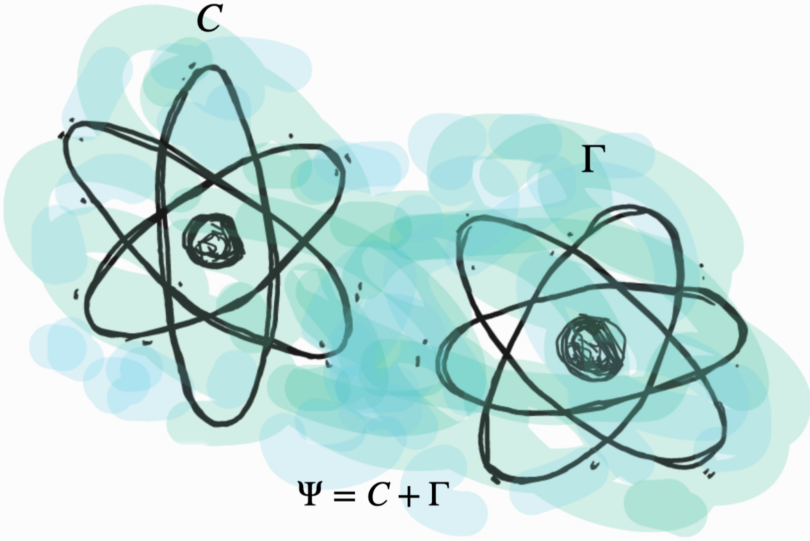
A QUANTUM CLOCK FOR A QUANTUM SYSTEM



$\lambda = \rho e^{-i\varphi} \in \mathbb{C}$

$\hat{R} = (\hat{R}\hat{R}^{+})^{1/2} e^{-i\hat{\phi}}$

A QUANTUM CLOCK FOR A QUANTUM SYSTEM



$$\langle \lambda | \hat{H} | \psi \rangle\rangle = 0$$

↓

$$i\varepsilon \frac{d}{d\varphi} |\Phi_e(\varphi)\rangle = \hat{H}_\Gamma |\Phi_e(\varphi)\rangle$$

Schroed - like

but $|\Phi_e(\varphi)\rangle = \langle \lambda | \psi \rangle\rangle$

unnormalized

?

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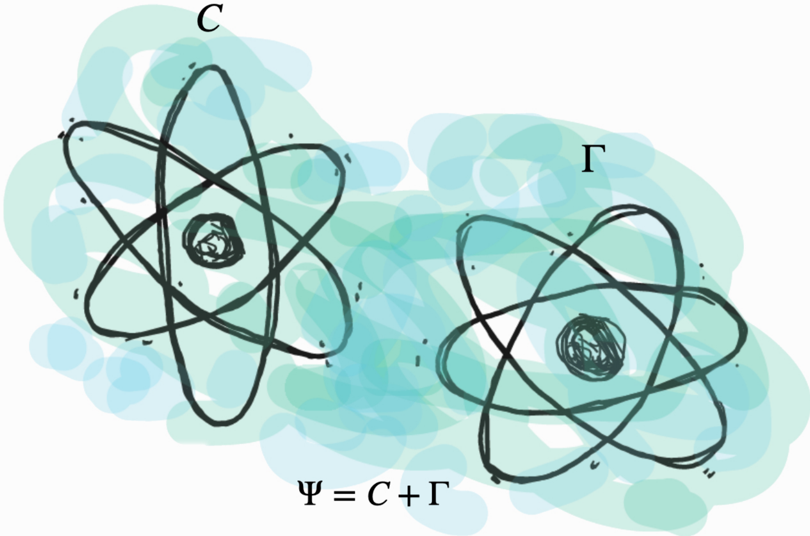

$$\langle \lambda | \hat{H} | \Psi \rangle\rangle = 0$$

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$$\hat{H}_c = \hat{H} + \hat{H}_\Gamma$$

↓

$$\Delta(\hat{H} + \hat{H}_\Gamma)\, \Delta \sin\hat{\phi} \geq \left| \frac{\varepsilon}{2} \langle \cos\hat{\phi} \rangle \right|$$

energy-time-like

but not yet

?

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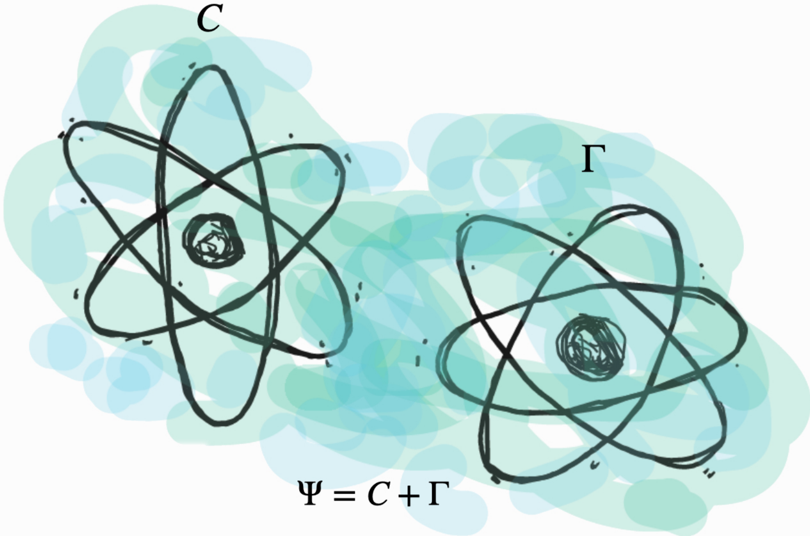

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energy-time-like

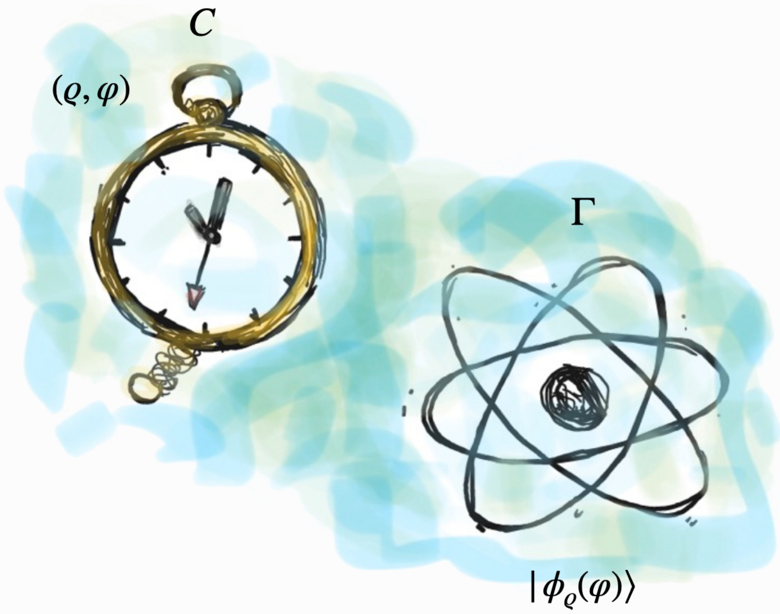
but not yet

↓

!

? $\frac{\hbar}{\varepsilon}\varphi$?

A CLASSICAL CLOCK FOR A QUANTUM SYSTEM



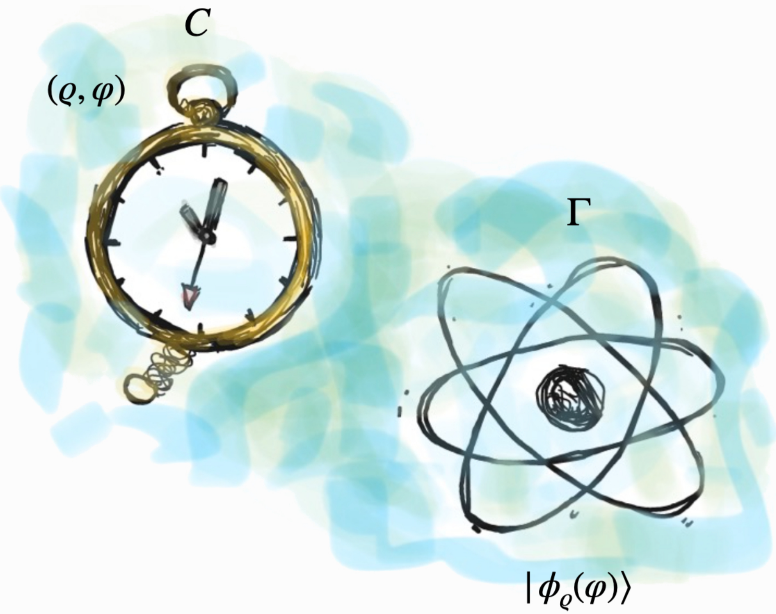
A CLASSICAL CLOCK FOR A QUANTUM SYSTEM

$$\lim_{N\to\infty} \langle \lambda | \Omega \rangle = \delta(\lambda - \Omega)$$

$$\langle \Phi_\varrho(\varphi) | \Phi_\varrho(\varphi) \rangle = \chi^2(\varrho)$$

$$|\phi_\varrho(\varphi)\rangle = \frac{|\Phi_\varrho(\varphi)\rangle}{\chi(\varrho)}$$

normalized



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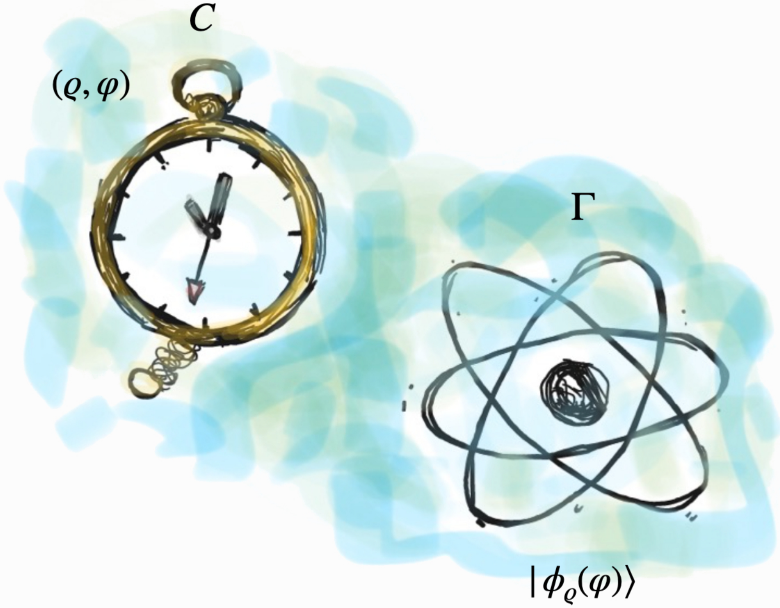

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$$\hat{H}_\Gamma |\phi_\varrho(\varphi)\rangle = E_\Gamma(\varrho) |\phi_\varrho(\varphi)\rangle$$

$$E_\Gamma(\varrho) = \langle \lambda | \hat{H}_C | \lambda \rangle = \frac{\varepsilon k}{2J}\left(1 - \cos 2\varrho\right)$$

$$\Delta E_\Gamma(\varrho) \Delta\varphi \geq \frac{\varepsilon}{2}$$

A CLASSICAL CLOCK FOR A QUANTUM SYSTEM

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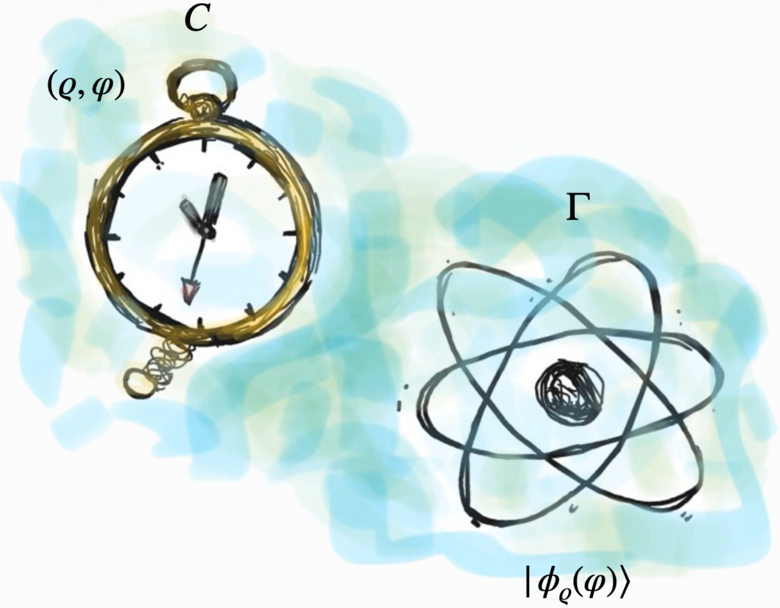

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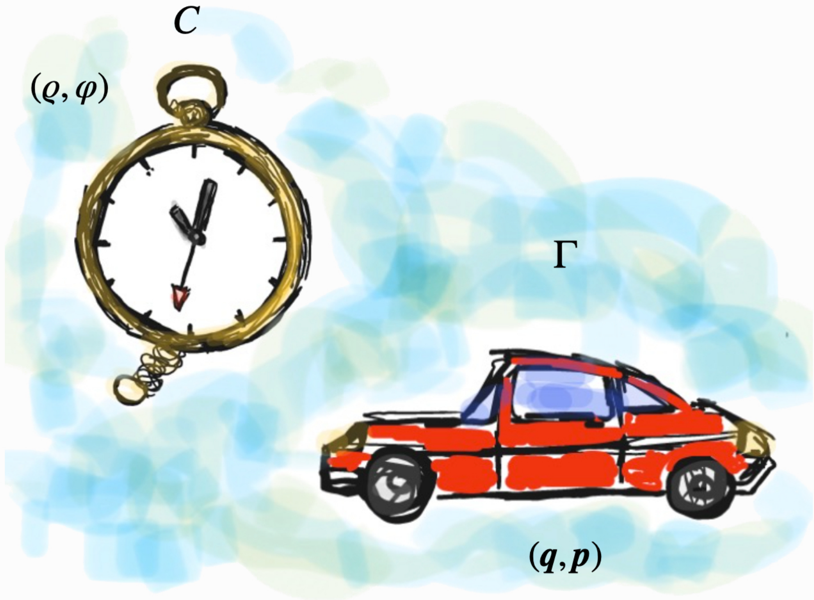
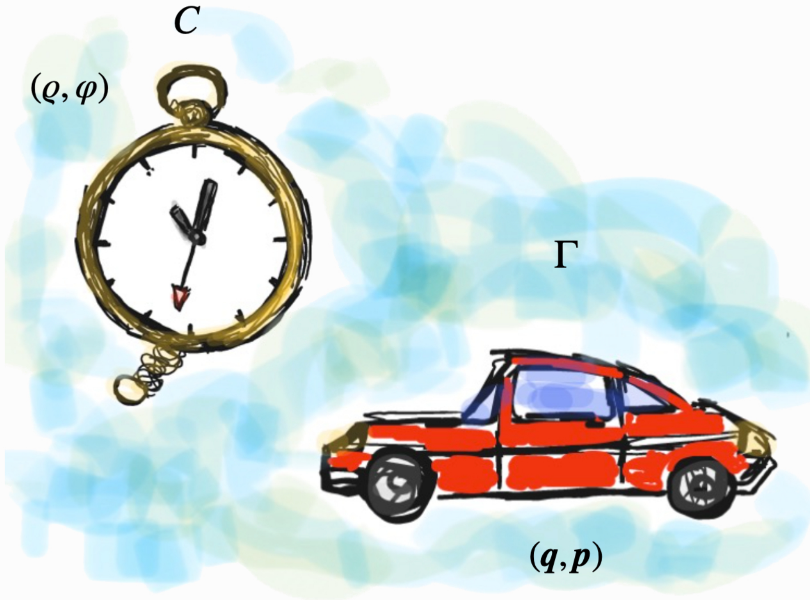
$$t^{QM} = \frac{\hbar}{\varepsilon}\varphi$$

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM



A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

GCS $|\Omega\rangle$
$\mathfrak{M}_c$



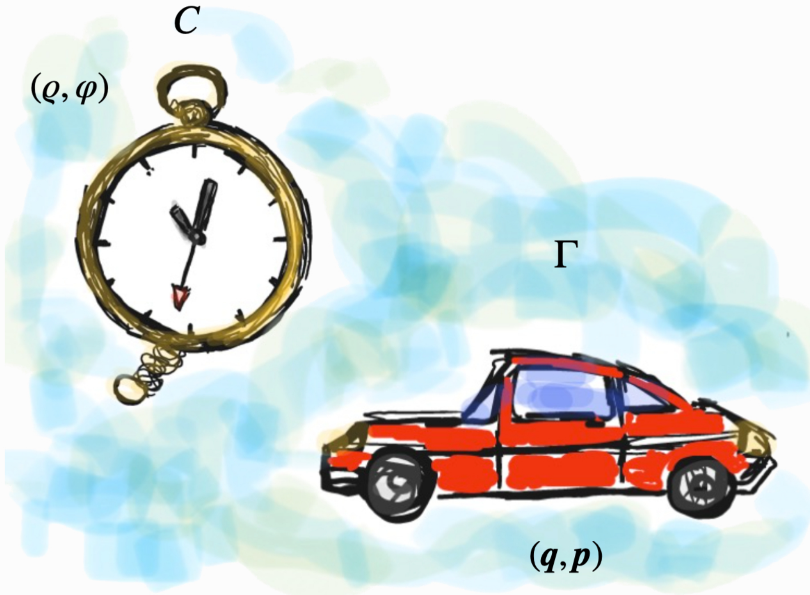
GCS $|\gamma\rangle$
$\mathfrak{M}_\Gamma$

$$|\Psi\rangle\rangle = \int_{\mathfrak{M}_c} d\mu(\Omega) \int_{\mathfrak{M}_\Gamma} d\mu(\gamma)\, \beta(\Omega, \gamma)\, |\Omega\rangle \otimes |\gamma\rangle$$

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

GCS $|\Omega\rangle$

$\mathcal{M}_c$



GCS $|\gamma\rangle$

$\mathcal{M}_S$

$$|\Psi\rangle\rangle = \int_{\mathcal{M}_c} d\mu(\Omega) \int_{\mathcal{M}_S} d\mu(\gamma)\, \beta(\Omega,\gamma)\, |\Omega\rangle \otimes |\gamma\rangle$$

$+$

using $\hat{H}|\Psi\rangle\rangle = 0$ from PaW

$\downarrow$

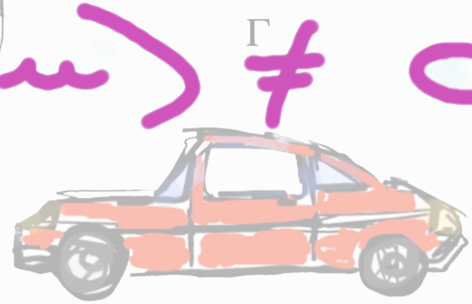
$\langle \Omega | H_c | \Omega \rangle = \langle \gamma | H_S | \gamma \rangle$ for $\Omega, \gamma : \beta(\Omega,\gamma) \neq 0$

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

$\Omega, \gamma : \wp(\Omega, \gamma) \neq 0$

λ, μ

$\hookrightarrow \quad \wp(\lambda, \mu) \neq 0$



$\lambda = \rho e^{-i\varphi}$

$H_c(\rho) = H_s(\mu)$ (q, p)

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

$\Omega, \gamma : \mathcal{B}(\Omega, \gamma) \neq 0$

λ, μ

$\mapsto \quad \mathcal{B}(\lambda, \mu) \neq 0$

$\lambda = \rho e^{-i\varphi}$

$H_c(\rho) = H_s(\mu)$

$\downarrow$

F

$:$

$H_c(\rho) = H_s(\mu = F(\lambda))$

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

C

(ϱ, φ)

$\Omega, \gamma : \beta(\Omega, \gamma) \neq 0$

λ, μ

$\hookrightarrow \quad \beta(\lambda, \mu) \neq 0$

Γ

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$\downarrow$

F

$:$

$\frac{k\varepsilon}{2}(\cos 2\varrho - 1) = \quad H_c(\varrho) = H_s(\mu = F(\lambda)) \quad = k\varepsilon(q^2 + p^2)$

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

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(ϱ, φ)

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$\downarrow$

F

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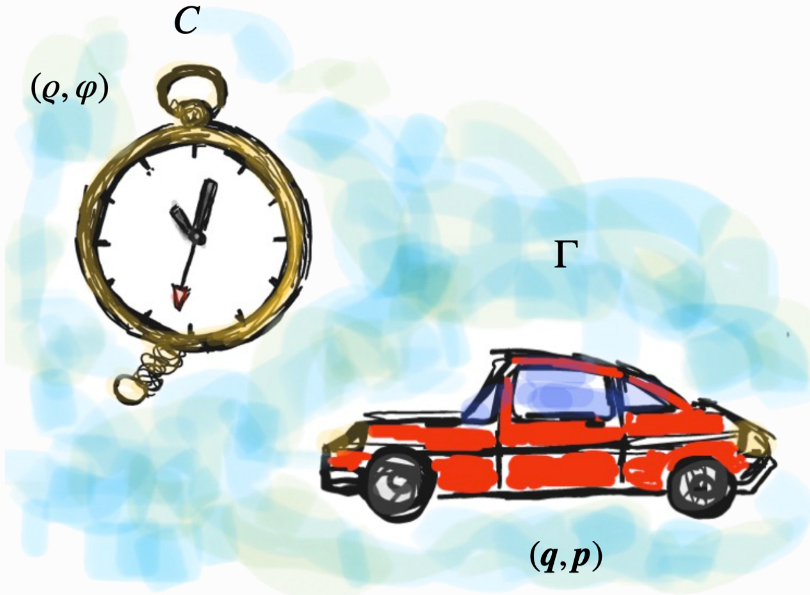
$\downarrow$

pullback-by-F $\quad F^*$

finally find

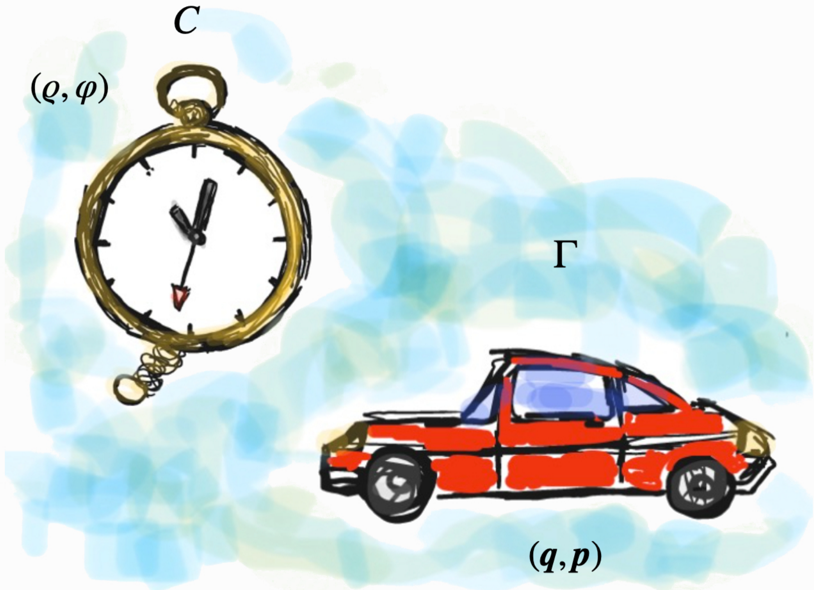
A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM



$$\{q, H_\varrho\}_\varrho = \frac{\varepsilon}{\hbar}\frac{dq}{d\varphi}$$

$$\{p, H_\varrho\}_\varrho = \frac{\varepsilon}{\hbar}\frac{dp}{d\varphi}$$

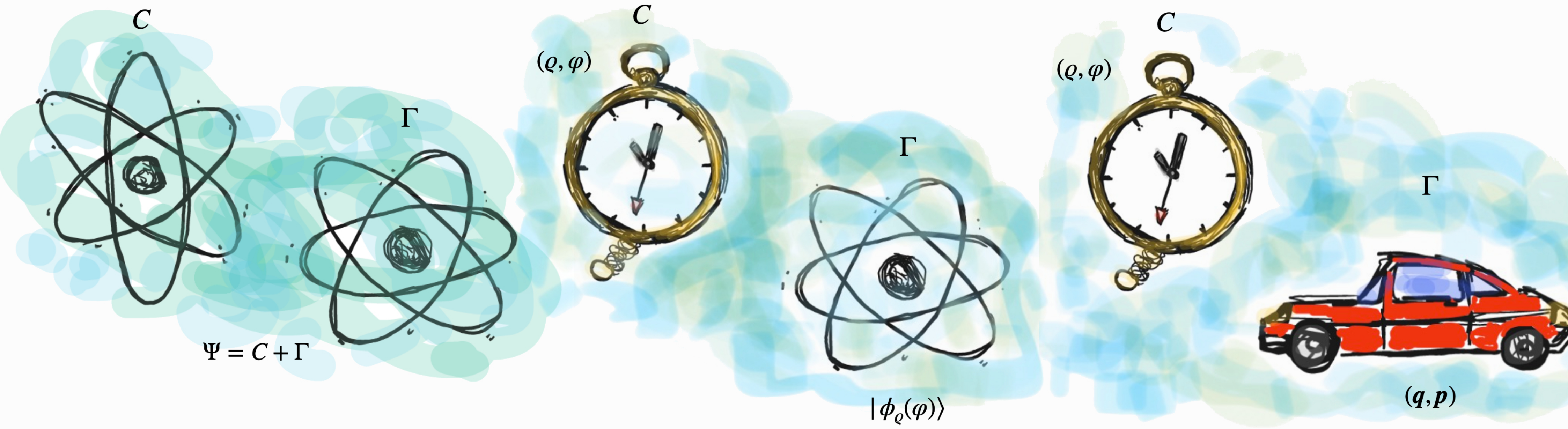
A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM



$$\{q, H_\Gamma\}_\Gamma = \frac{\varepsilon}{\hbar}\frac{dq}{d\varphi} \qquad \{p, H_\Gamma\}_\Gamma = \frac{\varepsilon}{\hbar}\frac{dp}{d\varphi}$$

setting $\hbar = k$

$$t^{CL} = \frac{k}{\varepsilon}\varphi$$



$$\frac{\hbar}{\varepsilon}\varphi = t^{QM} = t^{cl}$$

THERE IS ONLY ONE TIME

and it is a manifestation of entanglement

- energy-time uncertainty relation
- $\beta(\lambda,\mu) = \beta(\varrho,\varphi;q,p) \rightarrow \beta(E,\varphi;q,p) \rightarrow$ spacetime ...
- Schroedinger and geodesic . . .

Refer to

nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-021-21782-4 OPEN

Check for updates

Time and classical equations of motion from quantum entanglement via the Page and Wootters mechanism with generalized coherent states

Caterina Foti[1,2,3]✉, Alessandro Coppo[1,2], Giulio Barni[1], Alessandro Cuccoli[1,2] & Paola Verrucchi[1,2,4]

and

PAW	#	1, 4, 5
GCS	#	23, 25, 40
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PARAMETRIC REPRESENTATION WITH GCS	#	24, 41