Scrutinizing the pion-condensed phase

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References

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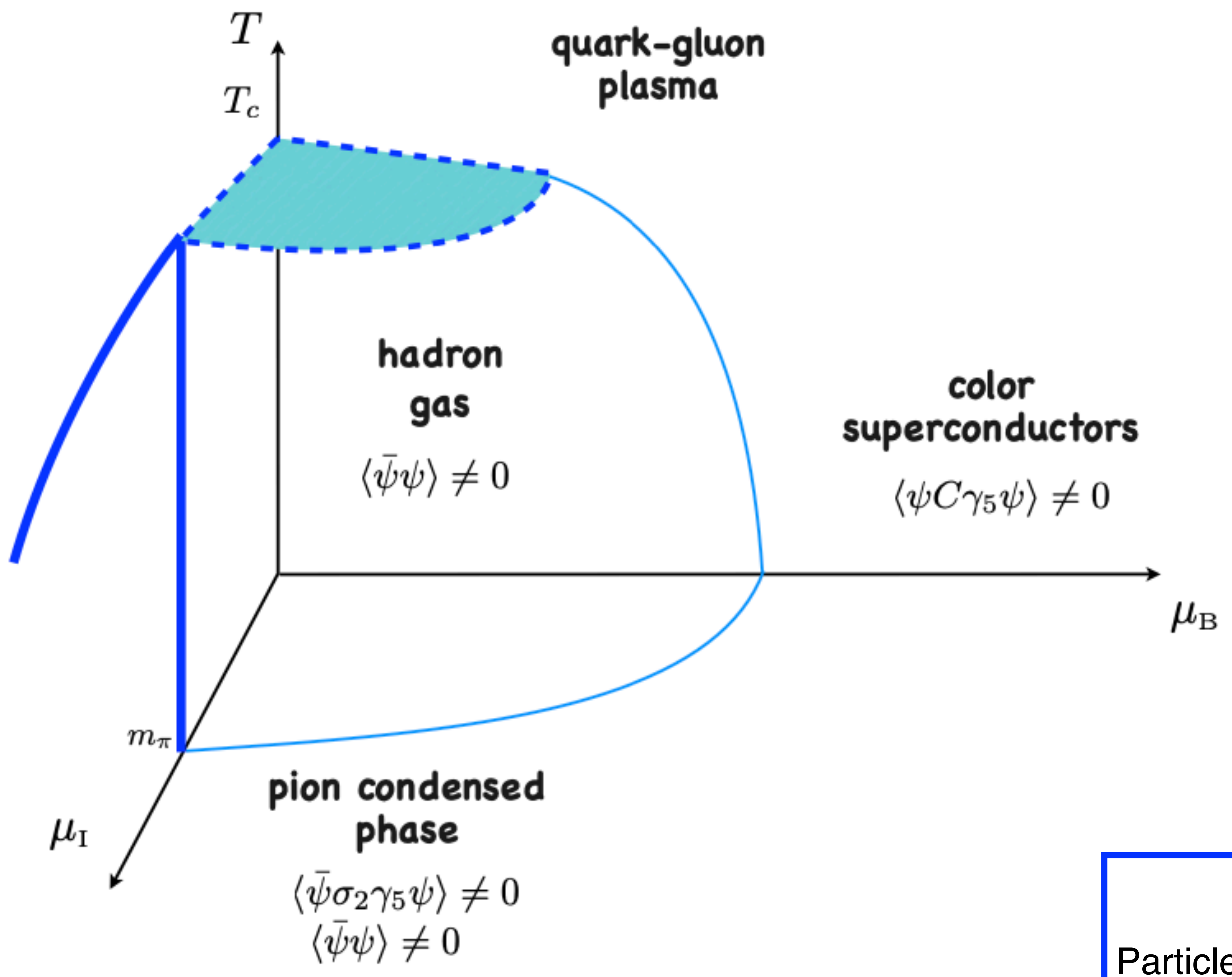
- A. Mammarella and M. Mannarelli, Phys. Rev. D 92, 085025 (2015).

- S. Carignano, A. Mammarella and M. Mannarelli, Phys. Rev. D 93, 051503(R) (2016).

- L. L. and M. Mannarelli, Phys. Rev. D 99, 096011 (2019).

Motivations

- To characterize further the phase diagram of QCD



M. Mannarelli,
Particles 2019, 2(3), 411-443

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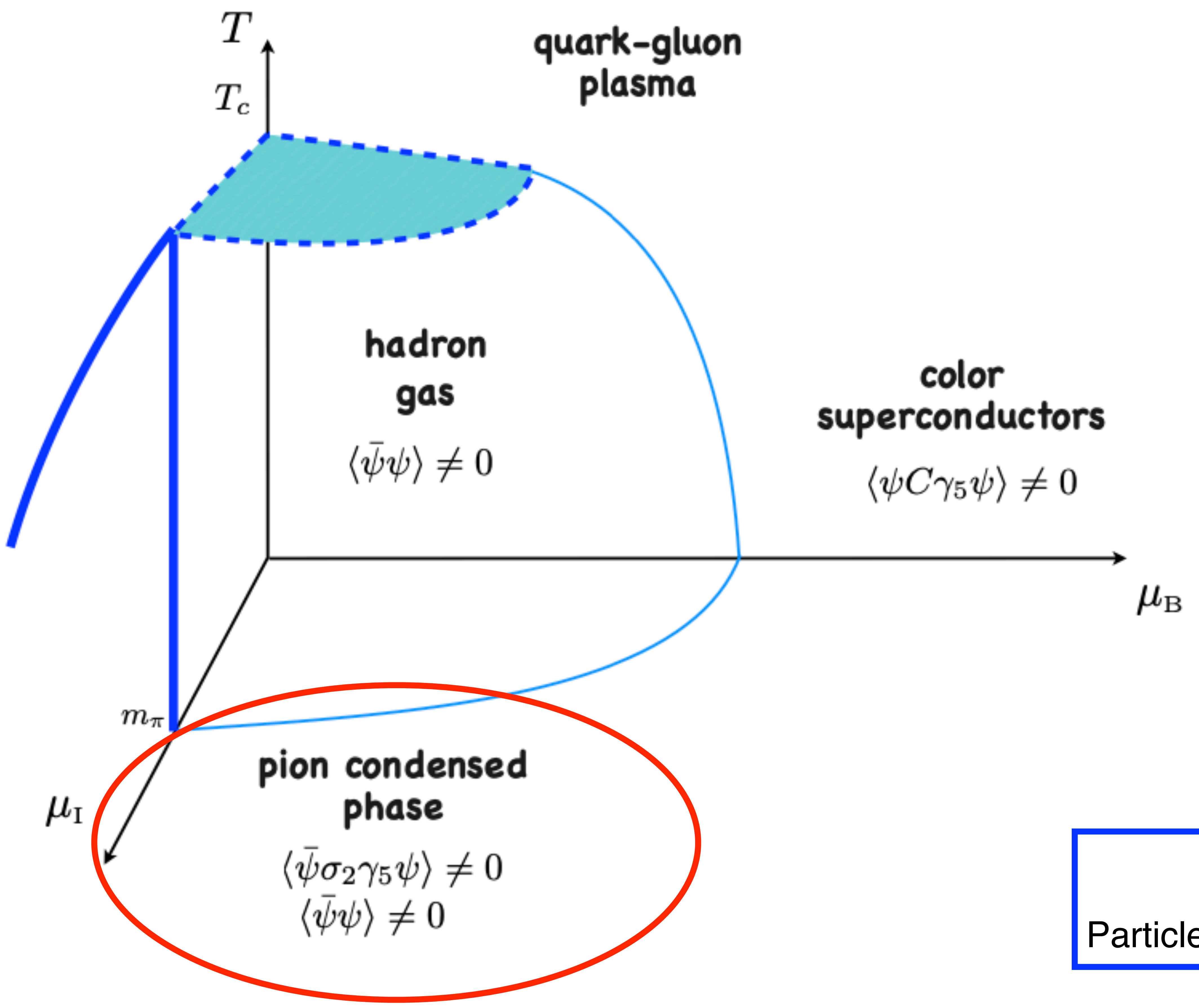
- To characterize further the phase diagram of QCD



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Motivations

- To characterize further the phase diagram of QCD
- For the pion-condensed phase, to characterize the low-energy excitations
- For the pion-condensed phase, to characterize the phase transition(s)
- Accessible to lattice-QCD simulations

Pion dynamics

- described efficiently by chiral perturbation theory $\mathcal{O}(p^2)$

$$\mathcal{L}_2 = \frac{f_\pi^2}{4}\,\mathrm{Tr}(D_\nu \Sigma D^\nu \Sigma^\dagger) + \frac{f_\pi^2 m_\pi^2}{4}\,\mathrm{Tr}(\Sigma + \Sigma^\dagger)$$

$$\frac{p}{\Lambda_\chi} \ll 1$$

$$\Lambda_\chi \sim 1\,\mathrm{GeV}$$

$$D_\mu \Sigma = \partial_\mu \Sigma - \frac{i}{2}[v_\mu, \Sigma] \qquad v_\mu = \mu_I \delta_{\mu 0} \sigma_3$$

$\Sigma = u\bar{\Sigma}u \quad \text{with } u = e^{i\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}/2}$ $\qquad \varphi_i\ (i = 1, 2, 3)$ real scalars

$\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha \qquad SU(2)$ vacuum

$(\alpha, \mathbf{n})$ variational parameters, maximizing $\mathcal{L}_2$

Normal phase

$$\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha \qquad \mu_I < m_\pi$$

$$\alpha = 0 \quad \rightarrow \quad \bar{\Sigma} = \mathbf{I}$$

- charged pion fields $\pi_\pm = \dfrac{\varphi_1 \mp i\varphi_2}{\sqrt{2}}$ $\qquad \pi_+ \sim u\bar{d}$

with effective masses $m_{\pi_\pm} = m_\pi \mp \mu_I$ as Stark effect

If $\mu_I > m_\pi - m_e$ no weak decay to leptons

Pion condensation

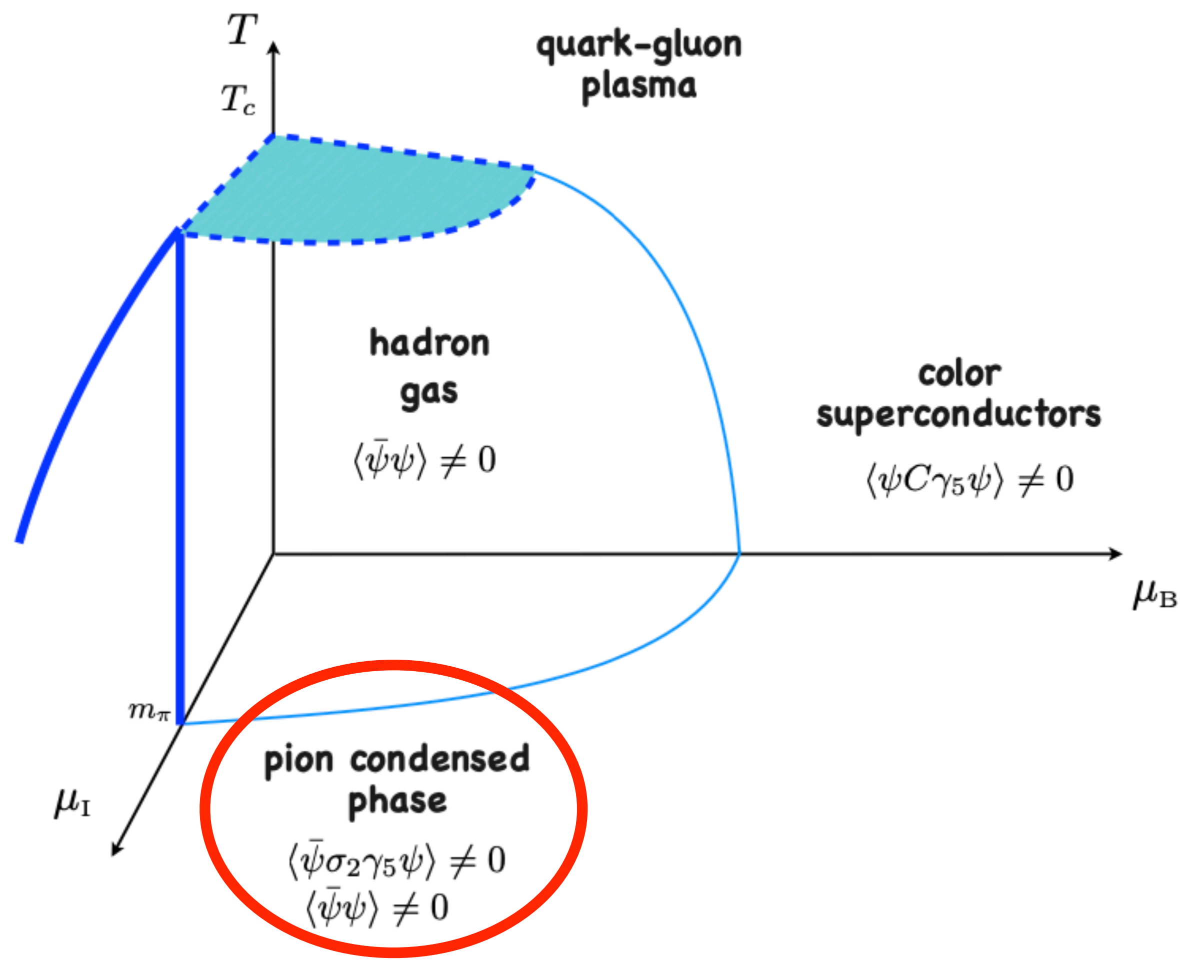
- if $\mu_I \geq m_\pi$ BEC condensation for π_+

m_{π_+} as μ in BEC

Possibility of superfluid if π_+ interact at "T = 0"….

Expected to coexist since at "T = 0" $\frac{n_s}{n_{\rm tot}} \to 1$

Pion superfluid



- characterized by a condensate

J. Kogut, D. Toublan,
Phys. Rev. D 64, 034007 (2001).

$$\cos\alpha = 1/\gamma^2 \qquad \gamma = \frac{\mu_I}{m_\pi}$$

$$\langle\bar{u}u\rangle = \langle\bar{d}d\rangle \propto \cos\alpha$$

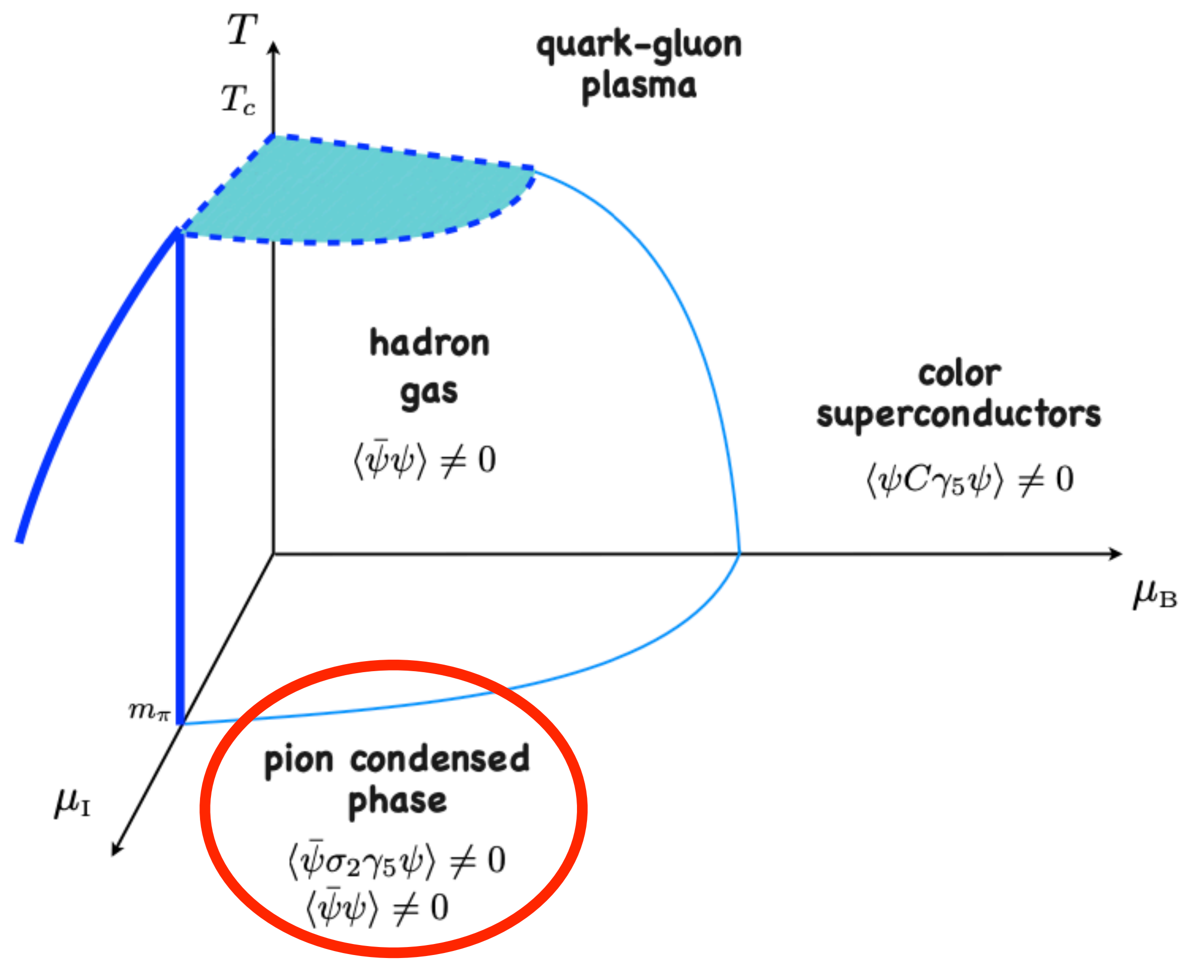
$$\langle\bar{d}\gamma_5 u + \mathrm{h.c.}\rangle \propto \sin\alpha$$

$$\pi_+ \sim u\bar{d}$$

vacuum does not conserve isospin and N_{π_+}

N_{π_+} spontaneously broken ⟶ charged superfluid

Pion superfluid



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continuous (second-order) transition

Effective theory close the transition

$$\gamma = \frac{\mu_I}{m_\pi} \gtrsim 1$$

Gross-Pitaevskii effective Lagrangian

$(p_0 \sim m_\pi)$

$$\mathcal{L}_{\rm GP} = f_\pi^2 m_\pi^2 + i\psi^*\partial_0\psi + \mu_{\rm eff}\psi^*\psi - \frac{g}{2}|\psi^*\psi|^2 + \psi^*\frac{\nabla^2}{2M}\psi$$

$$\psi = \sqrt{2 f_\pi^2 \mu_I}\, \pi_+ \qquad \mu_{\rm eff} = \frac{\mu_I^2 - m_\pi^2}{2\mu_I}\,, \qquad g = \frac{4\mu_I^2 - m_\pi^2}{12 f_\pi^2 \mu_I^2}\,, \qquad M = \mu_I$$

chemical isospin potential strongly affects the effective theory

Effective theory for BEC close the transition

$$n = 6 f_\pi^2 m_\pi \gamma \frac{\gamma^2 - 1}{4\gamma^2 - 1} \qquad \gamma = 1 + \varepsilon \qquad \varepsilon \ll 1$$

$$\partial V / \partial n = 0$$

$$V(n) = \mu_{\rm eff}\, n - \frac{g}{2} n^2$$

$$n = 4\varepsilon f_\pi^2 m_\pi$$

number density of the condensate

controlled by μ_I

low-energy expansion $x = n\, a^3 \ll 1$ $a \propto g\, m_\pi$

beyond many more interactions

Effective theory for BEC close the transition

$$\mathcal{L}_{\mathrm{GP}} = f_\pi^2 m_\pi^2 + i\psi^* \partial_0 \psi + \mu_{\mathrm{eff}} \psi^* \psi - \frac{g}{2} |\psi^* \psi|^2 + \psi^* \frac{\nabla^2}{2M} \psi$$

$$\psi = \sqrt{2 f_\pi^2 \mu_I}\, \pi_+ \qquad \mu_{\mathrm{eff}} = \frac{\mu_I^2 - m_\pi^2}{2\mu_I}, \qquad g = \frac{4\mu_I^2 - m_\pi^2}{12 f_\pi^2 \mu_I^2}, \qquad M = \mu_I$$

$g > 0$ if $\mu_I \geq m_\pi$ when pions appear

from scattering $\pi_+ - \pi_+$, suggests superfluid at "T = 0"

Pion superfluid

superfluid suggests Goldstone mode (phonon)

Indeed, we find:

$$E = c_s p + \mathcal{O}(p)^3, \qquad \text{for } \gamma > 1 \qquad c_s = \sqrt{\frac{\gamma^4 - 1}{\gamma^4 + 3}}$$

(possibily eaten by Higgs mechanism)

Far from the transition

$$\Sigma = e^{i\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}} = \cos\rho + i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\varphi}}\sin\rho \qquad \boldsymbol{\varphi} = \rho\hat{\boldsymbol{\varphi}} \qquad \hat{\boldsymbol{\varphi}}\cdot\hat{\boldsymbol{\varphi}} = 1$$

ρ radial field $\hat{\boldsymbol{\varphi}}$ angular field

from $\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha$: parameters to fields

$$\hat{\varphi}_1 = \sqrt{1-\hat{\varphi}_2^2} \qquad \theta = \arctan\left(\frac{\hat{\varphi}_2}{\hat{\varphi}_1}\right) = \hat{\varphi}_2 + \frac{2\hat{\varphi}_2^3}{3} + \mathcal{O}(\hat{\varphi}_2^5)$$

$$\hat{\varphi}_3 = 0 \sim \pi_0$$

Far from the transition

Integrating out the radial fluctuation $\rho = \bar{\rho} + \chi$

$$\left.\frac{\partial V}{\partial \rho}\right|_{\bar{\rho}} = 0 \to \bar{\rho} = \arccos \frac{1}{\gamma^2} \qquad V(\rho) = -f_\pi^2 m_\pi^2 \left(\cos\rho + \frac{\gamma^2}{2}\sin^2\rho\right)$$

yields $$\mathcal{L} = \frac{f_\pi^2}{2}\frac{\gamma^4+3}{\gamma^4} g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta\left(1 - 3\theta^2 + \mathcal{O}(\theta^4)\right)$$

$$g^{\mu\nu} = \mathrm{diag}(1, -c_s^2, -c_s^2, -c_s^2)$$ acustic metric $$c_s = \sqrt{\frac{\gamma^4 - 1}{\gamma^4 + 3}}$$

effective theory for the Goldstone mode

Further corrections in ChPT

$$\begin{aligned}\mathcal{L}_4 = L_1 &\left\{\mathrm{Tr}[D_\mu \Sigma (D^\mu \Sigma)^\dagger]\right\}^2 \\ &+ L_2\, \mathrm{Tr}\left[D_\mu \Sigma (D_\nu \Sigma)^\dagger\right] \mathrm{Tr}\left[D^\mu \Sigma (D^\nu \Sigma)^\dagger\right] \\ &+ L_3\, \mathrm{Tr}\left[D_\mu \Sigma (D^\mu \Sigma)^\dagger D_\nu \Sigma (D^\nu \Sigma)^\dagger\right] \\ &+ L_4\, \mathrm{Tr}\left[D_\mu \Sigma (D^\mu \Sigma)^\dagger\right] \mathrm{Tr}\left(\chi \Sigma^\dagger + \chi^\dagger \Sigma\right) \\ &+ L_5\, \mathrm{Tr}\left[D_\mu \Sigma (D^\mu \Sigma)^\dagger \left(\chi \Sigma^\dagger + \chi^\dagger \Sigma\right)\right] \\ &+ L_6 \left[\mathrm{Tr}\left(\chi \Sigma^\dagger + \chi^\dagger \Sigma\right)\right]^2 + L_7 \left[\mathrm{Tr}\left(\chi \Sigma^\dagger - \chi^\dagger \Sigma\right)\right]^2 \\ &+ L_8\, \mathrm{Tr}\left(\Sigma \chi^\dagger \Sigma \chi^\dagger + \chi \Sigma^\dagger \chi \Sigma^\dagger\right) + H_2\, \mathrm{Tr}(\chi \chi^\dagger)\end{aligned}$$

$\chi = m_\pi^2 \mathbf{1}_{2\times 2}$

$L_i\,,\quad H_2$ from fits

- stable $(1-10^{-1}) \lesssim \gamma \lesssim 1$
- first-order transition: seems to require largely different parameters, no lattice evidence in Δn

Conclusions and outlook

- Possibility of BEC of pions
- Related charged superfluidity of pions
- “Stable” second-order transition from normal state
- Our study suggests possibility of compact (condensed) pion stars $\mu_e > m_\pi$

Conclusions and outlook

- "BEC/BCS" crossover at larger $\mu_I \approx \sqrt{3}\, m_\pi$

- $K_+ = (u, \bar{s})$ condensation, increasing μ_I further

Thank you
for attention !

References

- S. Carignano, L. L., A. Mammarella, M. Mannarelli, and G. Pagliaroli, Eur. Phys. J. A 53, 35 (2017).

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Main goal

Study some aspects of the dynamics of pions in ultra-dense matter

Conclusions and outlook

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- “Stable” second-order transition from normal state
- Our study suggests possibility of compact (condensed) pion stars $\mu_e > m_\pi$
- $K_+ = (u, \bar{s})$ condensation, increasing μ_I