Nuclear Physics Mid Term Plan in Italy

LNS – Session

Catania, April 4th-5th 2022

Nuclear Matrix Elements towards $0\nu\beta\beta$: theoretical model development

Angela Gargano

INFN – Sezione di Napoli

Istituto Nazionale di Fisica Nucleare

- Jessica Bellone – INFN LNS
- Stefano Burrello –Technische Universität Darmstadt & Universidad de Sevilla
- Maria Colonna – INFN LNS
- Giovanni De Gregorio – Università degli Studi della Campania & INFN Napoli
- Danilo Gambacurta – INFN LNS
- Horst Lenske - Institut für Theoretische Physik Giessen
- Xavier Roca Maza – Università degli Studi di Milano & INFN Milano
- Elena Santopinto – INFN Genova
- Alessandro Spatafora – Università degli Studi di Catania & INFN LNS
- José Antonio Valera – Universidad de Sevilla
- Javier Valiente Dobon – INFN LNL

Outline

- Neutrinoless double beta ($0\nu\beta\beta$) decay

✓ $0\nu\beta\beta$ decay rate and the nuclear matrix element (NME)

✓ Nuclear structure models and $0\nu\beta\beta$ NME

✓ Quenching problem of axial coupling constant

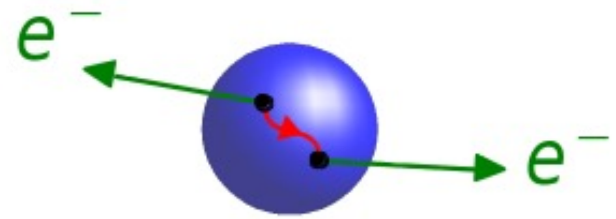
- How to access observable correlated to $0\nu\beta\beta$ decay

✓ Correlation between the $\gamma\gamma$ decay and $0\nu\nu\beta$ NMEs

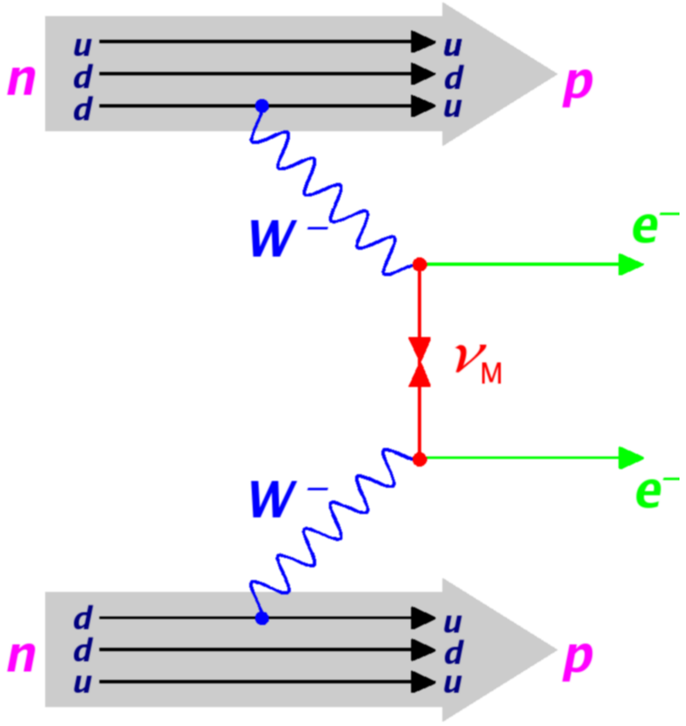
✓ Correlation between the DGT and $0\nu\nu\beta$ NMEs

✓ SCE and DCE reactions induced by heavy ions

- Summary and conclusions



$(A, Z) \rightarrow (A, Z+2) + 2e^-$



The 0νββ-decay is the most promising way to probe neutrino properties and search for physics beyond the standard model →

- Neutrinos are Majorana particles
- Unique information on the neutrino mass scale
- Total Lepton number is not conserved

GERDA	^{76}Ge	$T_{1/2} > 1.8 \times 10^{26}$ y	completed
KamLAND-Zen 400	^{136}Xe	$T_{1/2} > 1.07 \times 10^{26}$ y	completed
EXO-200	^{136}Xe	$T_{1/2} > 3.5 \times 10^{25}$ y	completed
MAIORANA dem	^{76}Ge	$T_{1/2} > 2.7 \times 10^{25}$ y	completed
CUORE	^{130}Te	$T_{1/2} > 2.2 \times 10^{25}$ y	data taking
CUPI-0	^{82}Se	$T_{1/2} > 4.7 \times 10^{24}$ y	completed
CUPID-Mo	^{100}Mo	$T_{1/2} > 1.8 \times 10^{24}$ y	completed
NEMO-3	^{76}Ge	$T_{1/2} > 1.1 \times 10^{24}$ y	completed

KamLAND-Zen 800	^{136}Xe	data taking
Amore-I	^{100}Mo	data taking
.		
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SNO+	^{130}Te	construction/commissioning
LEGENG-200	^{76}Ge	construction/commissioning
CUPID	^{100}Mo	R&D
AMORE-II	^{100}Mo	R&D
LEGEND-1000	^{76}Ge	R&D
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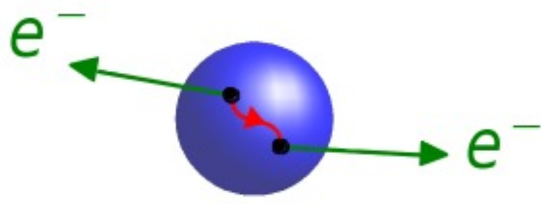
Rate for 0νββ decay

The rate for 0νββ decay - assuming that it mediated by the exchange of light Majorana neutrinos - is



$(A, Z) \rightarrow (A, Z+2) + 2e^-$

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu}\, g_A^4\, |M^{0\nu}|^2 \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

- $G^{0\nu}$ is the so-called phase-space factor, which can be **accurately evaluated** by atomic physics calculations
- g_A is the **axial coupling constant**
- $\langle m_\nu \rangle = \left|\sum_i m_i U_{ei}^2\right|$ is the **effective neutrino mass** which depends on the neutrino masses m_i and their mixing matrix matrix elements U_{ei}
- $M^{0\nu}$ is the NME relating the wave functions of the initial and final nuclei. It is the **most critical ingredient**

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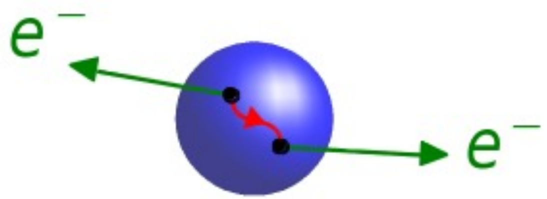

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NME

$$M^{0\nu} = \langle f|\hat{M}^{0\nu}|\, i\rangle$$

$$= \sum_k \sum_{j_p j_n j'_p j'_n} \langle f|a_p^+ a_n|\, k\rangle \left\langle k\left|a_{p'}^+ a_{n'}\right|\, i\right\rangle \langle j_p j_{p'}|\hat{M}^{0\nu}|j_n j_{n'}\rangle \Longrightarrow$$

$$\sum_{j_p j_n j'_p j'_n} \left\langle f\left|a_p^+ a_n a_{p'}^+ a_{n'}\right|\, i\right\rangle \langle j_p j_{p'}|\hat{M}^{0\nu}|j_n j_{n'}\rangle$$

(closure approximation)

$$\hat{M}^{0\nu} = \hat{M}^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 \hat{M}^{0\nu}_F + \hat{M}^{0\nu}_T$$

Solutions of the nuclear many-body problem

→ almost exact solutions up to A~12 → then for A>12 …

Ab initio methods

- No-core shell model
- Coupled cluster method
- In-medium Similarity renormalization group
- …

✓ All microscopic degrees of freedom are taken into account in a large basis space
✓ Computational techniques are used to solve the many-body problem within controlled approximation schemes
✓ Realistic nuclear forces
→ Very demanding calculations

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Although the considerable progress made in recent years, applications to heavy nuclei are out of reach
→ 0νββ candidate emitters, except ^{48}Ca, are still too computationally demanding to be studied with ab initio methods

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by reducing the number of active degrees of freedom
or invoking dynamical symmetries, characterized by definite underlying algebraic structures, in which only key configurations are isolated

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- **Shell Model**

provides the framework for a microscopic description of nuclei based essentially on the use of effective Hamiltonians, in which only a fraction of the A nucleons occupying a truncated space is considered as active

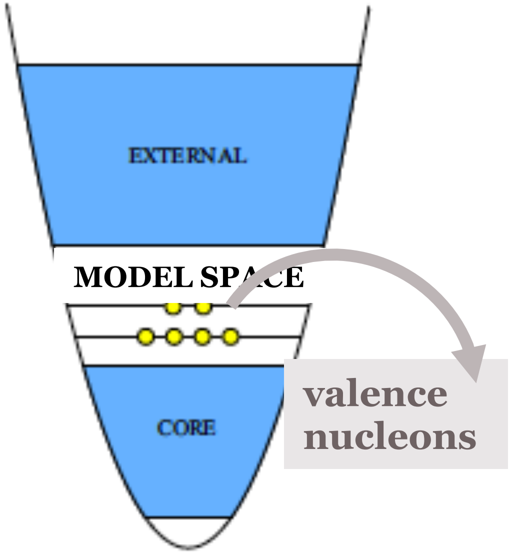
- **Approaches based on Energy Density Functional (i.e. RPA and extensions)**

includes all nucleons in a quite big space but the computational cost is reduced by considering only a specific class of excitations which are constructed starting from product states of independent particles

- **Interacting boson model**

is a phenomenological model where even-even nuclei are described in terms of bosons → due to the algebra of boson operators, nuclear degrees of freedom are significantly reduced

Suitable to describe low-energy spectroscopic properties



$$H_{\rm eff}|\Psi_\alpha\rangle = E_\alpha|\Psi_\alpha\rangle,$$ with $$H_{\rm eff} = H_0 + V_{\rm eff},$$

defined in the model space for only valence nucleons

$$|\Psi_\alpha\rangle = \sum_i C_i^\alpha |\phi_i\rangle$$ with $$|\phi_i\rangle = \sum_{abc\cdots\in model\ space} c^i_{abc\ldots}[\underbrace{a_a^\dagger a_b^\dagger a_c^\dagger \ldots}_{n}]_i|c\rangle,$$

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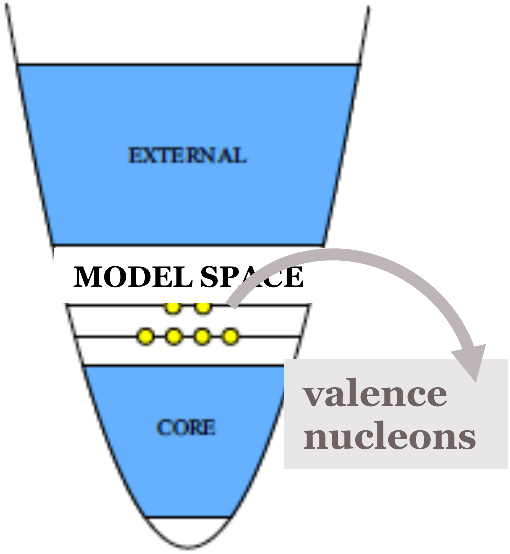

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H_{eff} should take into account in an effective way all the degrees of freedom not considered explicitly, namely excitations of core nucleons and valence nucleons in the external space

Use of effective operators is also required in the calculations of matrix elements of decay operators which are connected to measurable quantities, such as B(M1), B(E2), B(GT)… strengths

for a one-body operator $\langle\Psi_\alpha||O^\lambda_{\text{eff}}||\Psi_\beta\rangle = \sum_{ab} OBTD(ab\lambda, \alpha\beta)\langle a||O^\lambda_{\text{eff}}||b\rangle$

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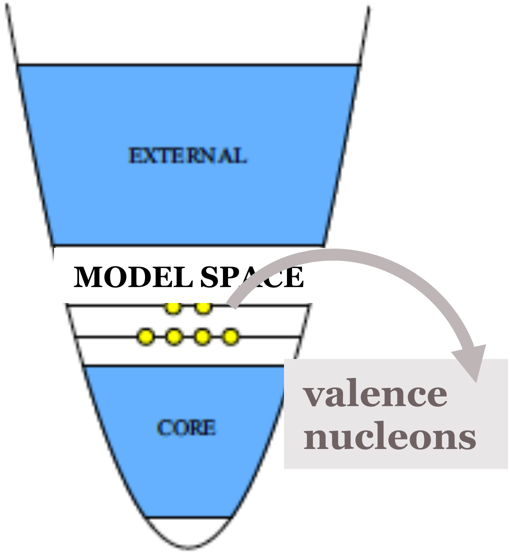

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1. **empirical approach** – matrix elements of $H_{\rm eff}$ and $O^\lambda_{\rm eff}$ are determined by introducing parameters which are adjusted on the experimental data

2. **microscopic approach** – matrix elements of $H_{\rm eff}$ and $O^\lambda_{\rm eff}$ are derived from realistic bare nuclear potentials and bare operators by means of a well-suited many-body theory

Suitable to describe high collective modes and high-energy excitations

Starting from a ground state described within the Energy Density Functional framework nuclear excitations are described by phonons

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$$Q_\nu^\dagger = \sum_{ph}(X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p)$$

RPA/QRPA: configuration space 1p-1h/2 quasiparticles states

$$Q_\nu^\dagger = \sum_{ph}(X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p) + \sum_{p_1<p_2;h_1<h_2}(X_{p_1h_1p_2h_2}^{(\nu)} a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2} - Y_{p_1h_1p_2h_2}^{(\nu)} a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2})$$

Second Random Phase Approximation (SRPA) configuration space 1p-1h, 2p-2h

$$\tilde{Q}_N^\dagger = \sum_{ph,n}(\tilde{X}_{ph,n}^{(N)} a_p^\dagger a_h Q_n^\dagger - \tilde{Y}_{ph,n}^{(N)} Q_n a_h^\dagger a_p)$$

RPA + PVC (particle-vibration coupling) configuration space (1p-1h, 1p-1h) ⊗ phonon

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Usually, adopted interactions are fitted to reproduce global properties such as radii and masses across large regions of the nuclear chart

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even-even nuclei

$$H^B = H^B_\nu + H^B_\pi + V^B_{\nu\pi}$$

odd-even nuclei

$$H = H^B + H^F_\rho + V^{BF}_\rho$$

$\rho = \nu, \pi$

odd-odd nuclei

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odd-odd nuclei

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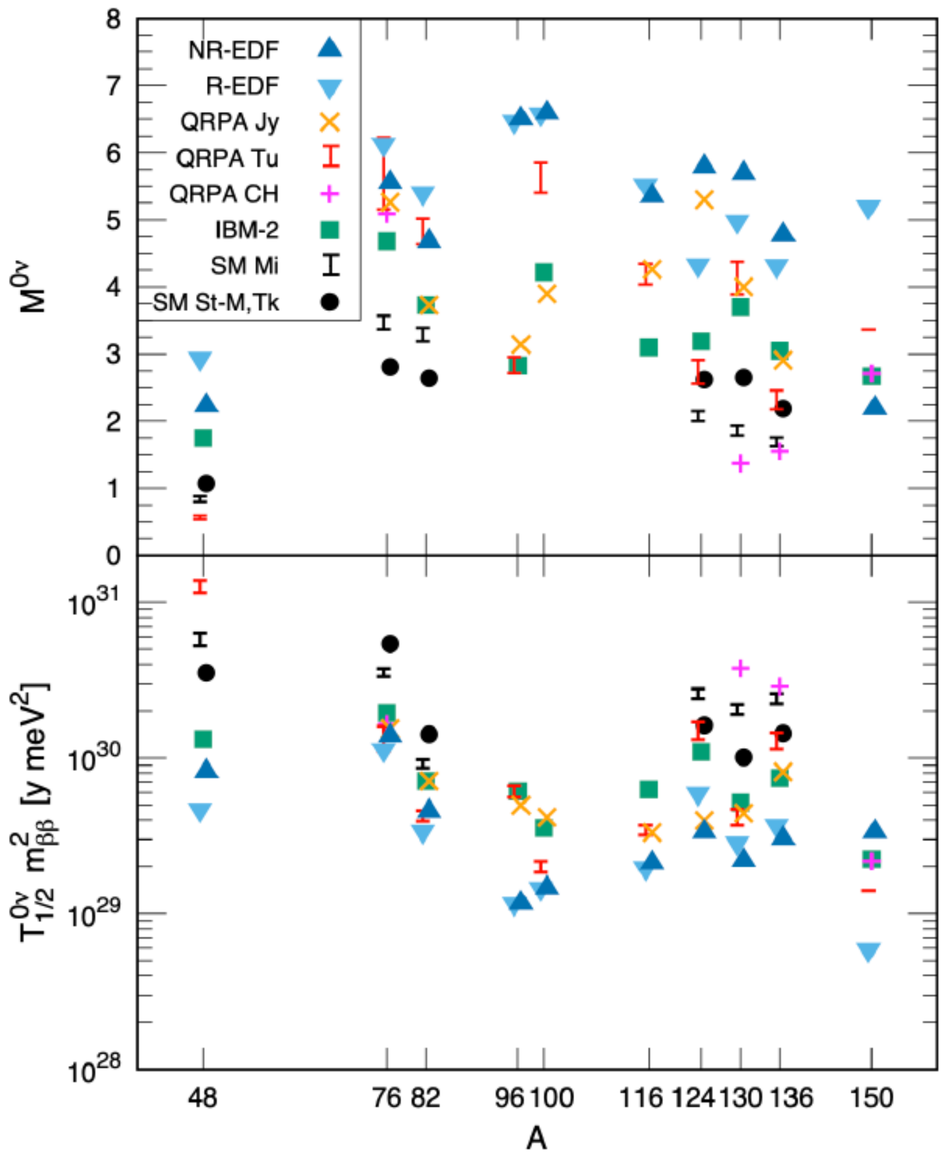
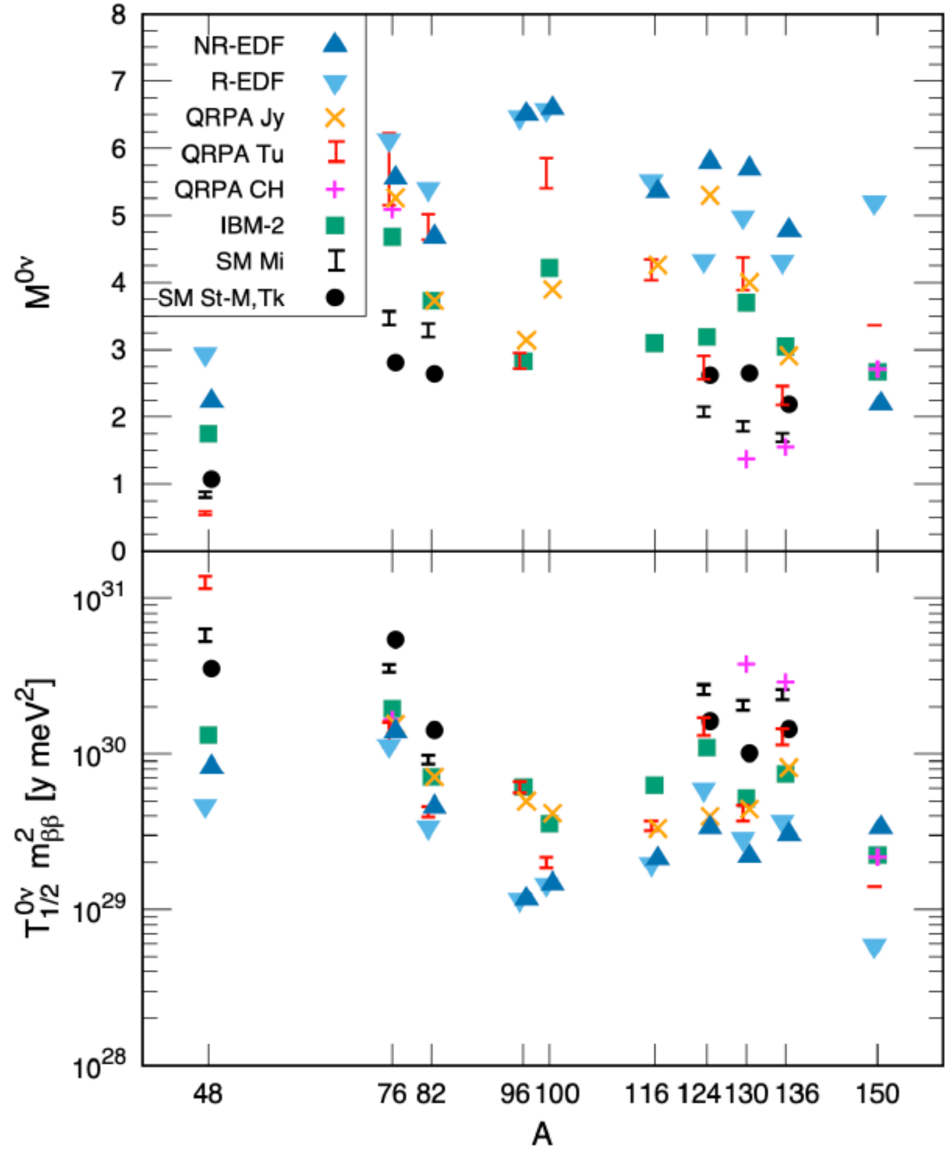
Empirical Hamiltonians are used with parameters fitted on experimental data



Results produced by different models show a large spread, a factor of about three

J. Engel and J. Menendez, Rep. Prog. Phys. **80,** 046301 (2017)



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1 The uncertainty affects the choice of material to be used in experimental devices for 0νββ decay searchs, as well as its amount.

An uncertainty of a factor three in the NME corresponds to nearly an order of magnitude uncertainty in the amount of material required

2 A reduction in the uncertainty of the calculations will be crucial if we wish to fully exploit an eventual measurement of the decay half-life to obtain information about the neutrino absolute masse scale

Predicted single β and 2νββ decay lifetimes are almost always shorter than measured lifetimes, *i.e.* corresponding matrix elements are too large

$$\beta^- \quad {}^{A}Z \rightarrow {}^{A}(Z+1) + e^- + \bar{\nu}_e$$

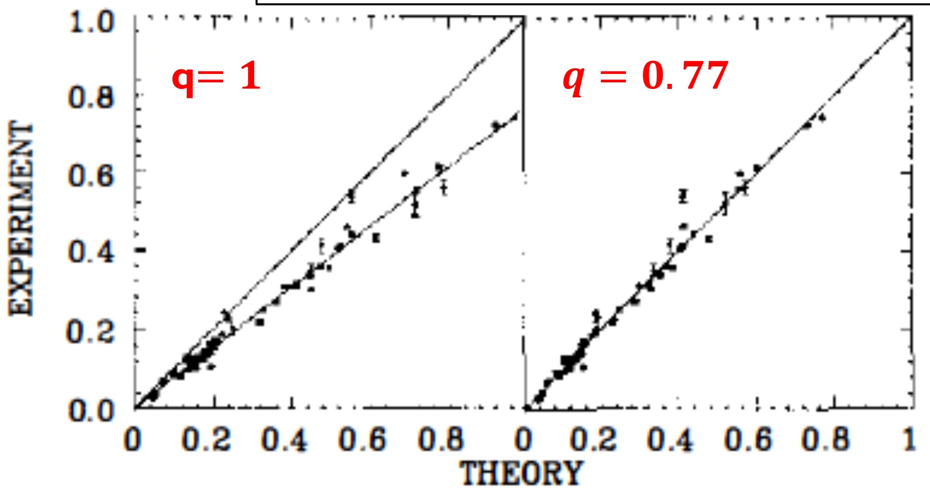
$$\beta^+ \quad {}^{A}Z \rightarrow {}^{A}(Z-1) + e^+ + \nu_e$$

GT strength

$$B(GT^{\pm}) = q^2 \frac{|\langle \Phi_f || \sum_j \vec{\sigma}_j \tau_j^{\pm} | \Phi_i \rangle|^2}{2J_i + 1}$$

$$q = \frac{g_A^{\mathrm{eff}}}{g_A^{\mathrm{free}}} \qquad g_A^{\mathrm{free}} = 1.27$$

Experimental versus theoretical strengths for GT transitions in nuclei with A=16-40
B. A, Brown, B. H. Wildenthal, Ann. Rev. Nucl. Part. Sci. 38, 29 (1988)



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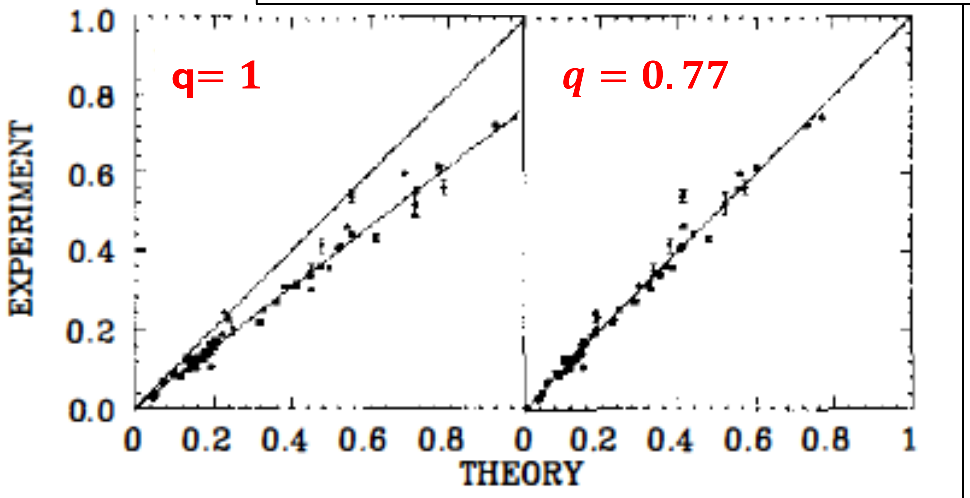

Possible sources of the quenching:

- Renormalizations needed to account for the missing many-body correlations
- Renormalizations needed to account for the many-nucleon weak currents due to the sub-nucleonic structure of nucleons

Quenching of the axial coupling constant g_A

Usually the strength of the axial coupling constant appearing in the $0\nu\beta\beta$ NME is reduced by introducing a quenching factor deduced from the observed $2\nu\beta\beta$ decays.
But it cannot be the good choice since there are marked differences between the $0\nu\beta\beta$ and $2\nu\beta\beta$ processes in both the momentum transfer and multipolarity of the intermediate states

Usually the strength of the axial coupling constant appearing in the 0νββ NME is reduced by introducing a quenching factor deduced from the observed 2νββ decays.
But it cannot be the good choice since there are marked differences between the 0νββ and 2νββ processes in both the momentum transfer and multipolarity of the intermediate states

Realistic shell-model calculations

Both effective Hamiltonians and decay operators are microscopically derived by means of many-body perturbation theory → renormalization effects due to the missing correlations are microscopically introduced without resorting to empirical quenching factors

Decay	$M^{0\nu}_{bare}$	$M^{0\nu}_{\text{eff}}$	$M^{0\nu}_{g_A-2\nu\beta\beta}$
$^{48}Ca \rightarrow {}^{48}Ti$	0.53	0.30	0.40
$^{76}Ge \rightarrow {}^{76}Se$	3.41	2.66	1.41
$^{82}Se \rightarrow {}^{82}Kr$	3.30	2.73	1.31
$^{130}Te \rightarrow {}^{130}Xe$	3.19	3.19	1.78
$^{136}Xe \rightarrow {}^{136}Ba$	2.30	2.34	1.15

Weaker renormalization effects are found in 0νββ with respect to the 2νββ NME
→ a larger value of q value is needed

L. Coraggio et al, Universe, **6** 233 (2020)
L. Coraggio et al, PRC **101**, 044315 (2020)

- $M^{0\nu}_{g_A-2\nu\beta\beta}$ with quenching factors deduced for the 2νββ decay

1 Spectroscopy well described

- ✓ Masses
- ✓ Spectra
- ✓ Electromagnetic properties
- ✓ 1or 2 particle separation energies
- ✓ Occupation numbers
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Can we reduce the uncertainties on $0\nu\beta\beta$ NME by studying nuclear properties more directly related to this process?

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1. B. Romeo, J. Menéndez, C. Peña Garay, Physics Letters B 827 (2022) 136965

→ **good linear correlation between the double gamma and 0νββ decay NME**

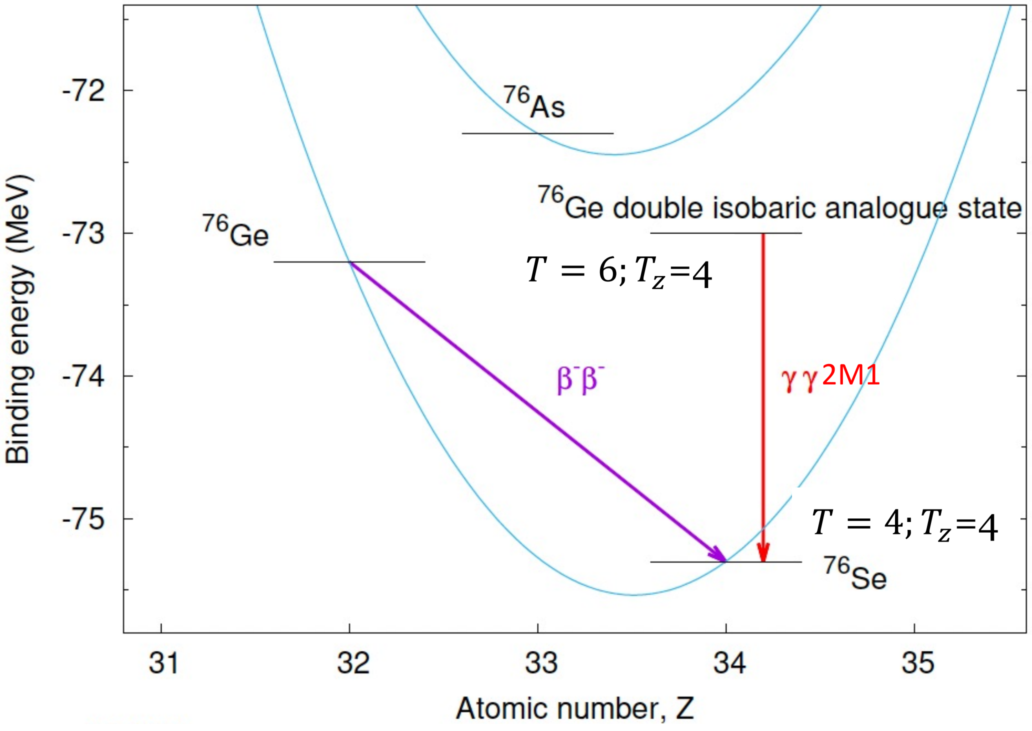
Experimental steps in this direction may be undertaken @ LNS by using the MAGNEX spectrometer (J.J.Valiente-Dobon et al.)

2. J. M Shimizu, J. Menéndez, K. Yako, Pysics Review letter 120 142502 (2018)

→ **good linear colleration between Double Gamow-Teller (DGT) and 0νββ decay NME**

Modern searches of the DGT GR are based on novel heavy-ion double charge-exchange reactions (RNCP Osaka, RIBF RIKEN, LNS within the NUMEN project)

$$ {}^A_Z Y^*_N \longrightarrow {}^A_Z Y_N + 2\gamma \Leftrightarrow {}_{Z-2}^{A} X_{N+2} \longrightarrow {}^A_Z Y_N + 2e^- $$



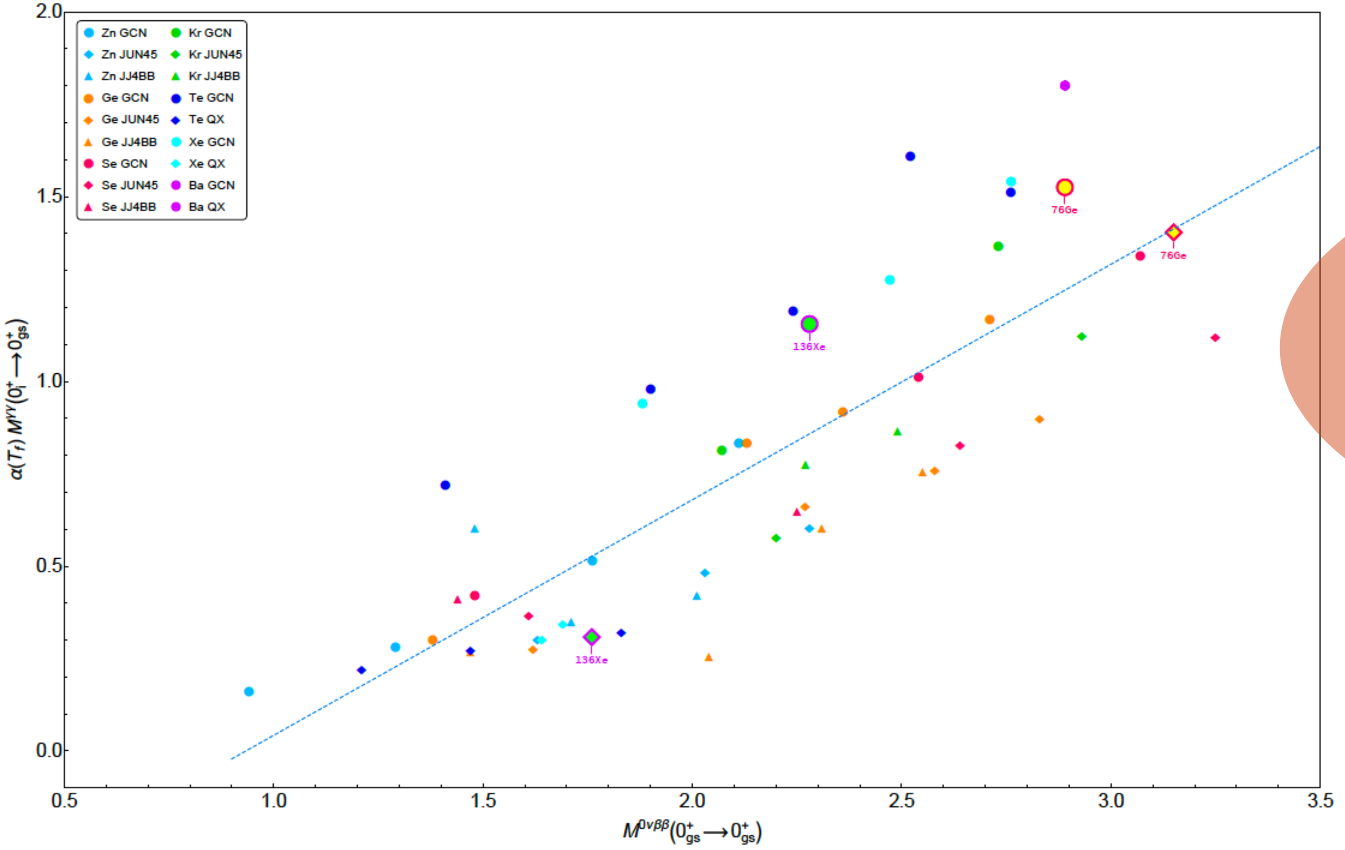
- Decay of the double isobaric analogue state (DIAS) of the initial ββ state (an excited state of the final ββ nucleus) into the final ββ state
- Isospin symmetry assures a good correspondence between the DIAS and the initial ββ state
- Focus on EM double-M1decay, which depends, like the 0νββ operator, on the nuclear spin

$$ M^{\gamma\gamma}(M1M1) = \sum_n \frac{\langle 0_f^+ || M1 || 1_n^+ \rangle \langle 1_n^+ || M1 || 0_i^+ \rangle}{\varepsilon_n} $$

$$ \varepsilon_n = E_n - (E_i + E_f)/2 $$

B. Romeo, J. Menéndez, C. Peña Garay, Physics Letters B 827 (2022) 136965

$M^{\gamma\gamma}$ and $M^{0\nu}$ from shell-model calculations for nuclei from A=46 to 136 using different effective interactions

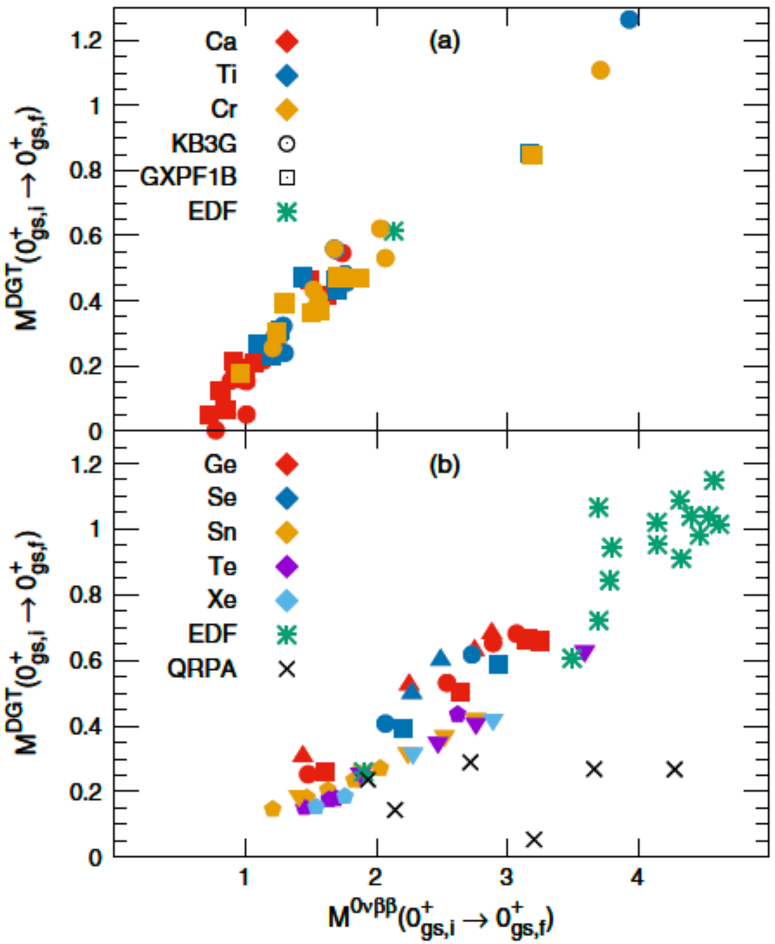


Good linear correlation between $M^{\gamma\gamma}$ and $M^{0\nu}$ across the nuclear chart, independently on the nuclear interaction used

B. Romeo, J. Menéndez, C. Peña Garay, Physics Letters B 827 (2022) 136965

Correlation between M^{DGT} and $M^{0\nu}$ with $M^{DGT} = \sqrt{B(DGT^{\pm};\, \lambda;\, 0^+_{gs,i} \rightarrow 0^+_{gs,f})}$

$$\left\{ \begin{array}{l} B(DGT^{\pm};\, \lambda;\, i \rightarrow f) = \dfrac{1}{2J_i + 1} \left| \left\langle f \right| \left| \mathcal{O}^{(\lambda)}_{\pm} \right| \left| i \right\rangle \right|^2 \\ \mathcal{O}^{(\lambda)}_{\pm} = \left[\sum_j \sigma_j \tau_j^{\pm} \times \sum_j \sigma_j \tau_j^{\pm} \right]^{(\lambda)};\quad \lambda = 0,2 \end{array} \right.$$



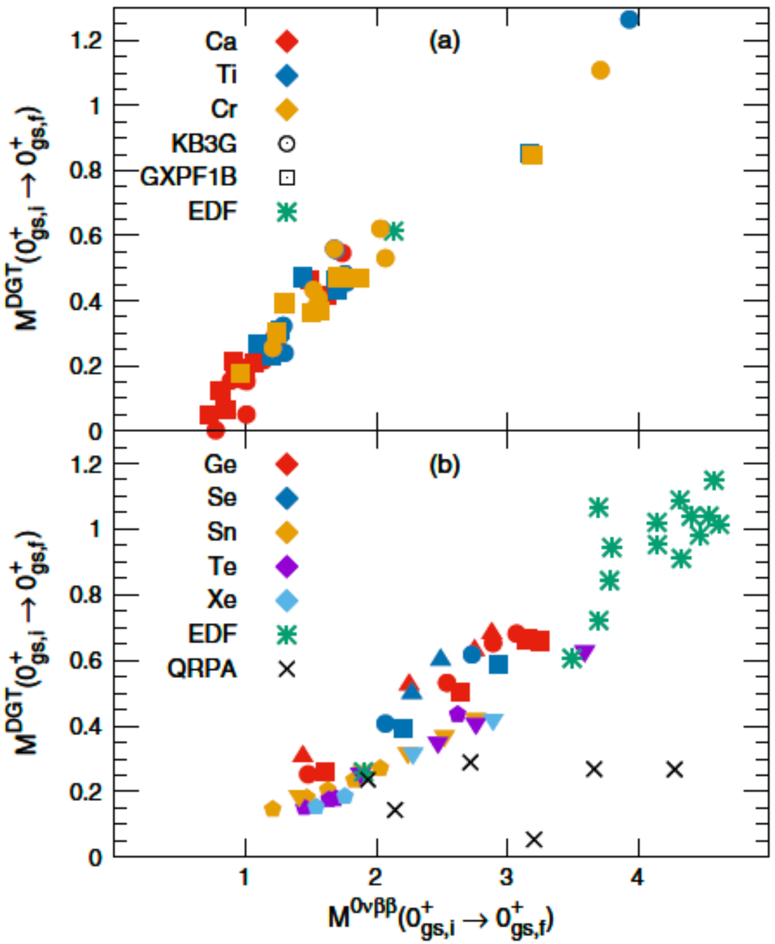
$42 \leq A \leq 60$
SM results
EDF result for ^{48}Ca

$76 \leq A \leq 136$
including 6 ββ emitters
SM results
EDF results for cadmium isotopes
QRPA results for ββ emitters

J. M Shimizu, J. Menéndez, K. Yako, Physics Review Letter 120 142502 (2018)

Correlation between M^{DGT} and $M^{0\nu}$ with $M^{DGT} = \sqrt{B(DGT^{\pm};\, \lambda; 0^+_{gs,i} \to 0^+_{gs,f})}$

$$\begin{cases} B(DGT^{\pm};\, \lambda; i \to f) = \dfrac{1}{2J_i + 1} \left| \left\langle f \right| \left| \mathcal{O}^{(\lambda)}_{\pm} \right| \left| i \right\rangle \right|^2 \\ \mathcal{O}^{(\lambda)}_{\pm} = \left[\sum_j \sigma_j \tau_j^{\pm} \times \sum_j \sigma_j \tau_j^{\pm} \right]^{(\lambda)}; \quad \lambda = 0,2 \end{cases}$$



$42 \leq A \leq 60$
SM results
EDF result for ^{48}Ca

$76 \leq A \leq 136$
including 6 ββ emitters
SM results
EDF results for cadmium isotopes
QRPA results for ββ emitters

- A simple linear relation exists between M^{DGT} and $M^{0\nu}$ within the SM
- A quite similar correlation is found with EDF
- QRPA gives small $M^{DGT} \lesssim 0.4$ matrix elements independently of the associated 0νββ decay NME values

J. M Shimizu, J. Menéndez, K. Yako, Physics Review Letter 120 142502 (2018)

How to extract the DGT matrix elements from heavy-ion double-charge exchange (DCE) reactions?

How to extract the DGT matrix elements from heavy-ion double-charge exchange (DCE) reactions?

- **DCE cross section can be factorized in terms of reaction and nuclear structure parts**
- **Nuclear structure part can be factorized in terms of target and projectile matrix elements**

by means of Chiral Effective Field Theory within the closure approximation and the low-momentum-transfer limit, corresponding to very forward angles

$$\frac{d\sigma}{d\Omega} \rightarrow F(\theta)\left(\frac{\mathcal{M}^{\mathrm{DGT}}_{\mathrm{T}\rightarrow\mathrm{T}'}\mathcal{M}^{\mathrm{DGT}}_{\mathrm{P}\rightarrow\mathrm{P}'}}{\bar{E}^{\mathrm{GT}}_p+\bar{E}^{\mathrm{GT}}_t} + \frac{\mathcal{M}^{\mathrm{DF}}_{\mathrm{T}\rightarrow\mathrm{T}'}\mathcal{M}^{\mathrm{DF}}_{\mathrm{P}\rightarrow\mathrm{P}'}}{\bar{E}^{\mathrm{F}}_p+\bar{E}^{\mathrm{F}}_t}\right)\Bigg|^2_{[q_1,q_2\approx 0]}$$

$$\boxed{\mathcal{M}^{\mathrm{DGT}}_{\mathrm{A}\rightarrow\mathrm{A}'} = c_{\mathrm{GT}}\langle\Phi^{(\mathrm{A}')}_{J'}|\sum_{n,n'}[\vec{\sigma}_n\times\vec{\sigma}_{n'}]^{(0)}\vec{\tau}_n\vec{\tau}_{n'}|\Phi^{(\mathrm{A})}_J\rangle}$$

$$\boxed{\mathcal{M}^{\mathrm{DF}}_{\mathrm{A}\rightarrow\mathrm{A}'} = c_{\mathrm{T}}\langle\Phi^{(\mathrm{A}')}_{J'}|\sum_{n,n'}\vec{\tau}_n\vec{\tau}_{n'}|\Phi^{(\mathrm{A})}_J\rangle}$$

E. Santopinto, H. García Tecocoazti, R.I. Magaña Vsevolodovna, J. Ferretti, Phys. Rev. C98 (2018)(R) 061601

How to extract the DGT matrix elements from heavy-ion double-charge exchange (DCE) reactions?

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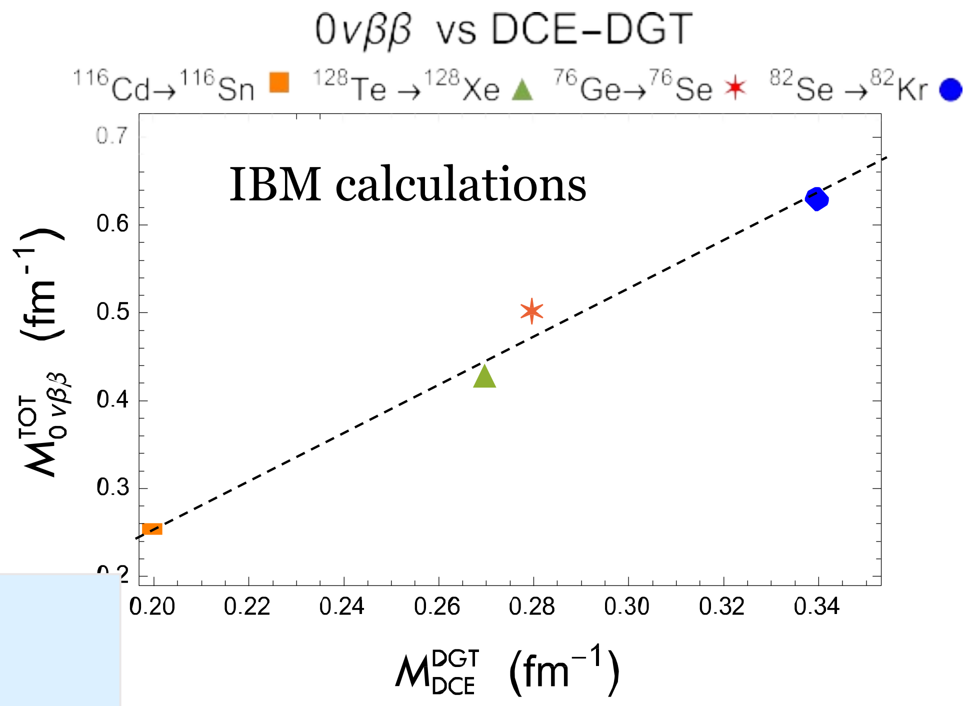
$$\frac{d\sigma}{d\Omega} \to F(\theta)\left(\frac{\mathcal{M}^{\rm DGT}_{\rm T\to T'}\mathcal{M}^{\rm DGT}_{\rm P\to P'}}{\bar{E}^{\rm GT}_p+\bar{E}^{\rm GT}_t}+\frac{\mathcal{M}^{\rm DF}_{\rm T\to T'}\,\mathcal{M}^{\rm DF}_{\rm P\to P'}}{\bar{E}^{\rm F}_p+\bar{E}^{\rm F}_t}\right)\Bigg|^2_{[q_1,q_2\approx 0]}$$

$$\mathcal{M}^{\rm DGT}_{\rm A\to A'} = c_{\rm GT}\left\langle\Phi^{(\rm A')}_{J'}\right|\sum_{n,n'}[\vec{\sigma}_n\times\vec{\sigma}_{n'}]^{(0)}\vec{\tau}_n\vec{\tau}_{n'}\left|\Phi^{(\rm A)}_J\right\rangle$$

$$\mathcal{M}^{\rm DF}_{\rm A\to A'} = c_{\rm T}\left\langle\Phi^{(\rm A')}_{J'}\right|\sum_{n,n'}\vec{\tau}_n\vec{\tau}_{n'}\left|\Phi^{(\rm A)}_J\right\rangle$$



The existence of this linear correlation combined with the possibility of carrying out a factorization of the HI-DCE cross section emphasize the importance of future experiments within the NUMEN project @ LNS to acquire information on the 0νββ NME and to clarify the quenching problem

E. Santopinto, H. García Tecocoazti, R.I. Magaña Vsevolodovna, J. Ferretti, Phys. Rev. C98 (2018)(R) 061601

Charge exchange (CE) reactions represent a useful tool to investigate isospin and spin-isospin modes of excitation in nuclei

The connection between single CE reaction and β decay has been widely explored in the past with light nuclei

β^- direction → (p, n), (^{3}He, t), (^{6}Li, ^{6}He)
β^+ direction → (n, p), (d,^{2}He),(t,^{3}He), (^{7}Li,^{7}Be)

There exists a close proportionality between the CE cross-section, in the limit of vanishing momentum transfer and ΔL=0, and the B(GT) values

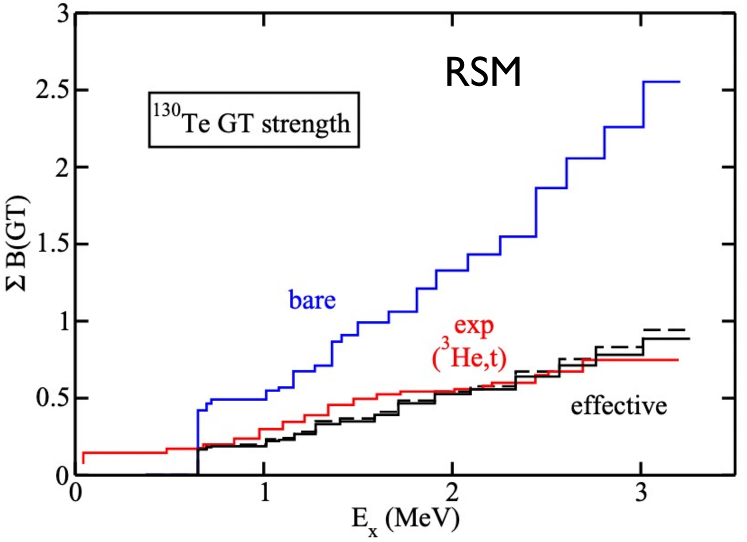
$$\frac{d\sigma_{GT}}{d\Omega}(q,\omega) = \hat{\sigma}_{GT}(q,\omega)B(GT), \qquad q \to 0 \;\; \& \;\; B(GT) = \frac{\left|\langle \Phi_f || \sum_j \vec{\sigma}_j \tau_j^{\pm} | \Phi_i \rangle\right|^2}{2J_i + 1}$$

B(GT) strengths can be extracted from CE reaction cross sections also when β decay studies are limited because of the *Q* value and provide a good test for nuclear models

D. Frekers and M. Alanssari, Eur. Phys. J. A 54, (2018) 177
T. N. Taddeucci, et al., Nuclear Phys. A 469 (1987) 125

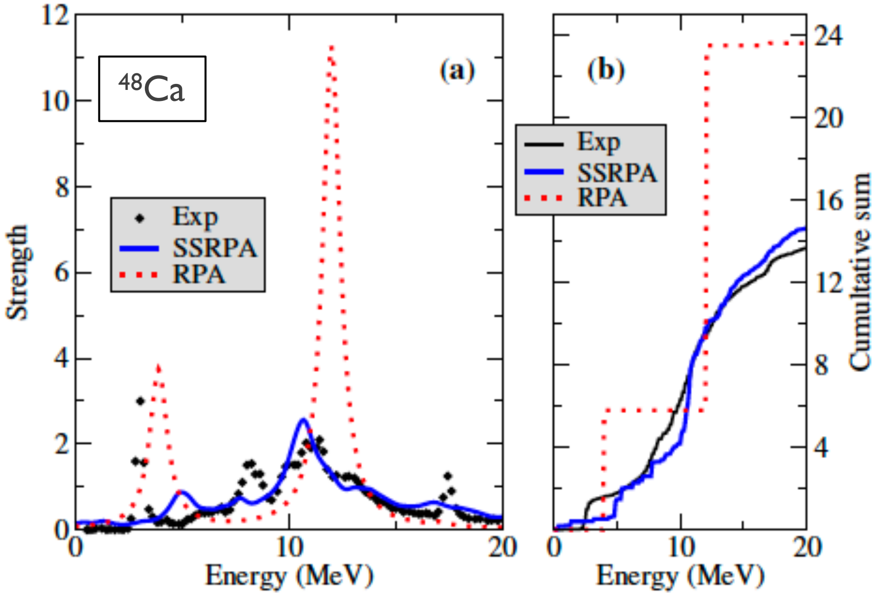
Comparison between calculated GT strengths/β decay half-lives and experimental values extracted from SCE reactions with light nuclei

Experimental values from
(^{3}He,t) (p,n)

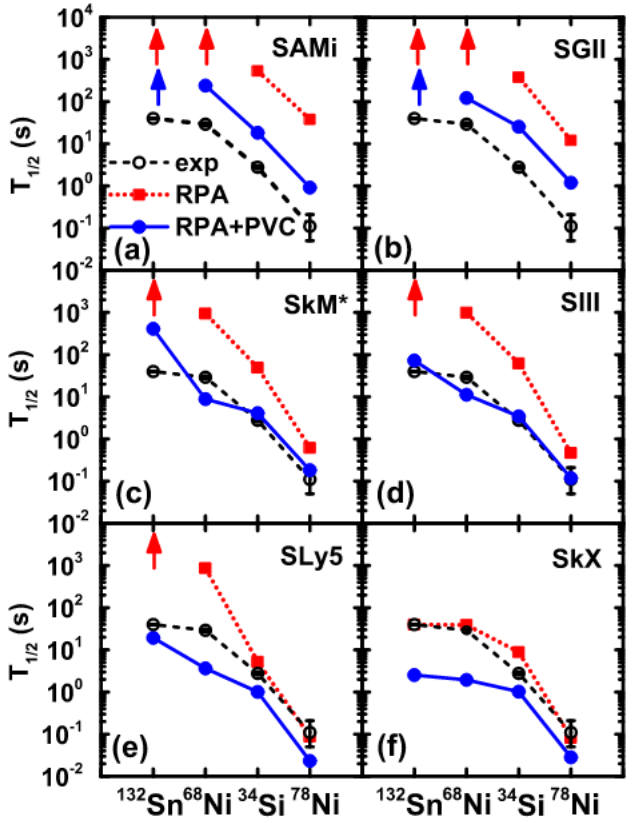


L. Coraggio et al, PRC **100** (2019) 014316

D. Gambacurta, et al, PRL **125**, 212501 (2020)



Y. F. Niu et al, PLB **114**, 142501 (2019)



SCE $A(Z_A, N_A) + a(Z_a, N_a) \rightarrow B(Z_A \pm 1, N_A \mp 1) + b(Z_a \mp 1, N_a \pm 1)$

DCE $A(Z_A, N_A) + a(Z_a, N_a) \rightarrow B(Z_A \pm 2, N_A \mp 2) + b(Z_a \mp 2, N_a \pm 2)$

- ✓ A broad range of projectile-target combinations can be used which allows to project out selectively specific features
- ✓ SCE reactions may populate high-spin states which are expected to play a relevant role in intermediate virtual states of 2νββ decay virtual states
- ✓ DCE reactions involve the same nuclear configurations of the 0νββ decay, and in both weak and strong processes the transition operators are a superposition of short-range isospin, spin-isospin and rank-two tensor components with a relevant available momentum (~100 MeV)

SCE $A(Z_A,N_A)+a(Z_a,N_a)\rightarrow B(Z_A\pm1,N_A\mp1)+b(Z_a\mp1,N_a\pm1)$

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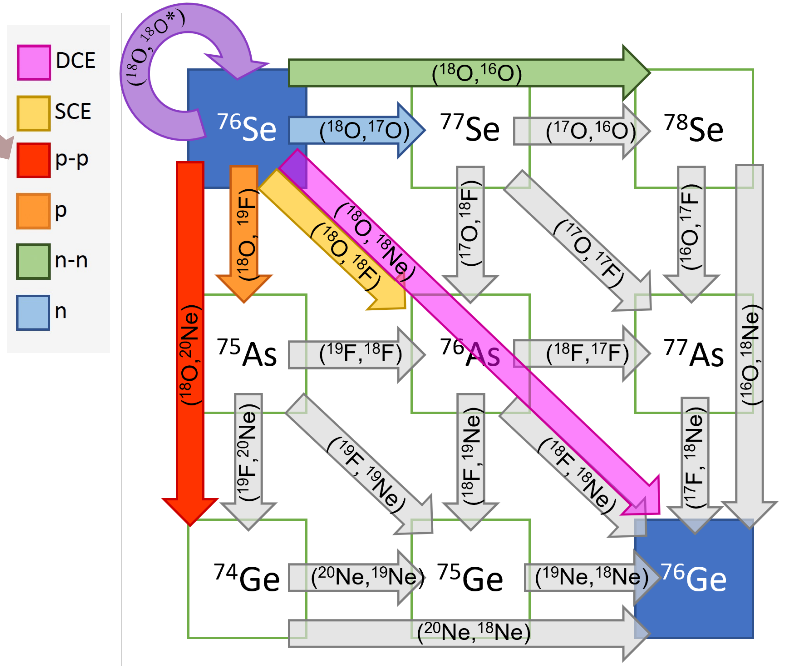
- ✓ HI CE reactions proceed by two competing reaction channels:
- ➢ direct channel which can directly probe the isospin structure of the ions
- ➢ transfer channel, namely the sequential exchange of protons and neutrons, which can supply spectroscopic information



Heavy-ion SCE and DCE reactions

SCE $A(Z_A,N_A)+a(Z_a,N_a)\rightarrow B(Z_A\pm1,N_A\mp1)+b(Z_a\mp1,N_a\pm1)$

DCE $A(Z_A,N_A)+a(Z_a,N_a)\rightarrow B(Z_A\pm2,N_A\mp2)+b(Z_a\mp2,N_a\pm2)$

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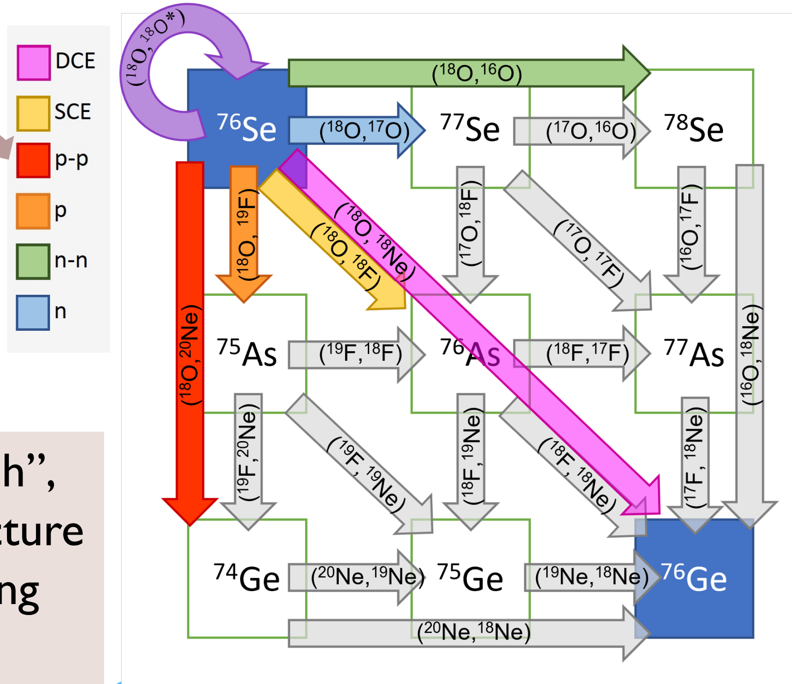
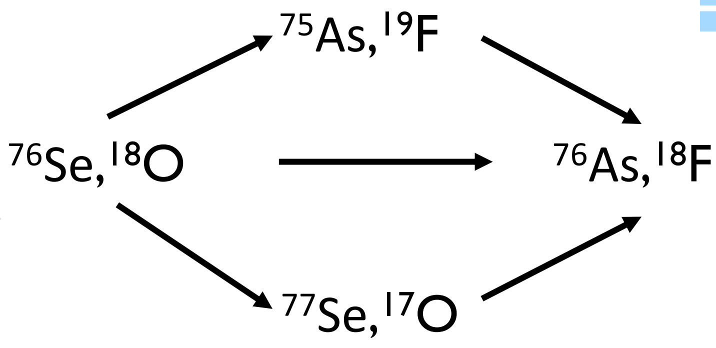


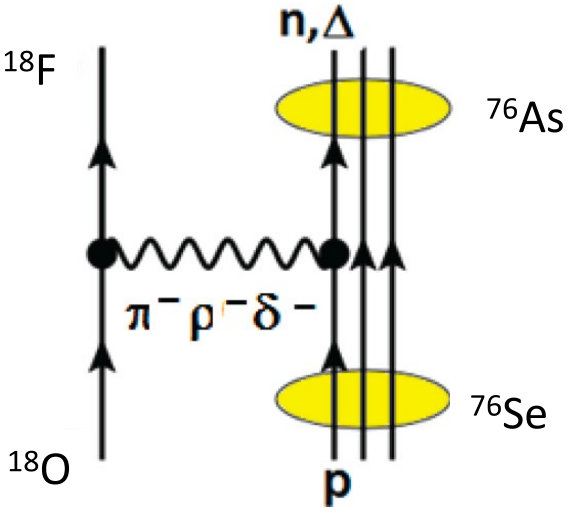
Accurate unified description of these reactions, through a so-called "multi-channel approach", is a demanding task for nuclear theory, for both reaction mechanism and nuclear structure input, but it may give the possibility to get information on the variety of observables coming from different channels

^{76}Se(^{18}O,^{18}F) ^{76}As

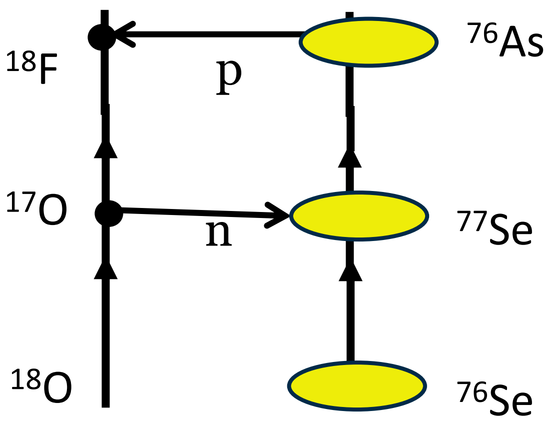


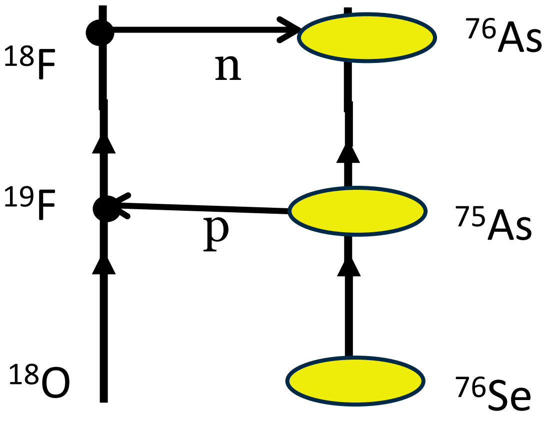
$T^{(2)}$: sequential transfer of neutron stripping + pr

$T^{(1)}$: direct mechanism (p, n)-type



$T^{(2)}$: sequential transfer of neutron stripping + proton pickup



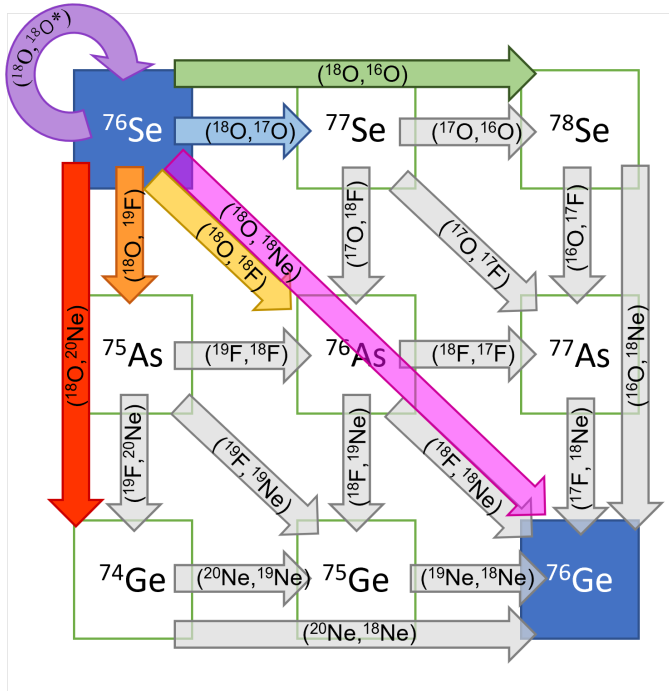


$$\sigma = \left(T^{(1)} + T^{(2)}\right)^2$$

$^{76}Se(^{18}O,^{18}Ne)\,^{76}Ge$

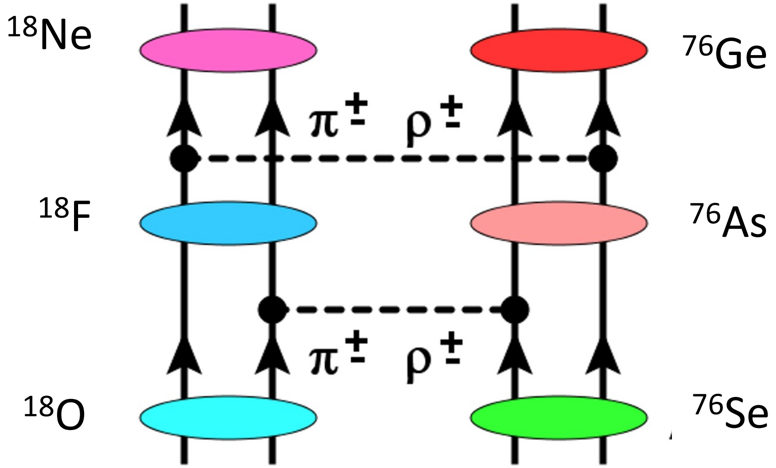
$^{76}Se,^{18}O \rightarrow \,^{76}Ge,^{18}Ne$

$^{76}Se,^{18}O \rightarrow \,^{76}As,^{18}F \rightarrow \,^{76}Ge,^{18}Ne$



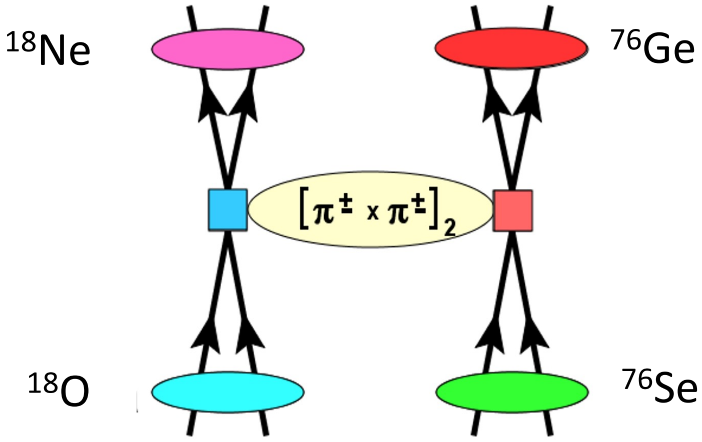
Sequential DCE (DSCE)

Two single CE **uncorrelated** processes



Majorana-like DCE (MDCE)

Single-step ***correlated*** exchange

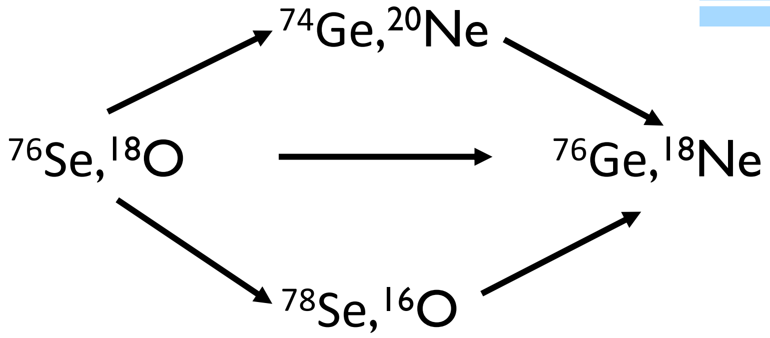


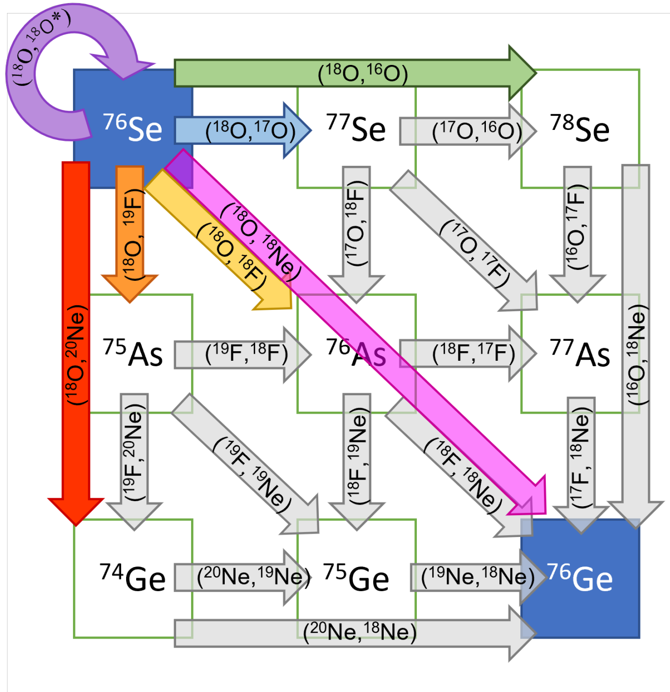
H. Lenske, IOP Conf. Series: Journal of Physics: Conf. Series **1056 (2018) 012030**

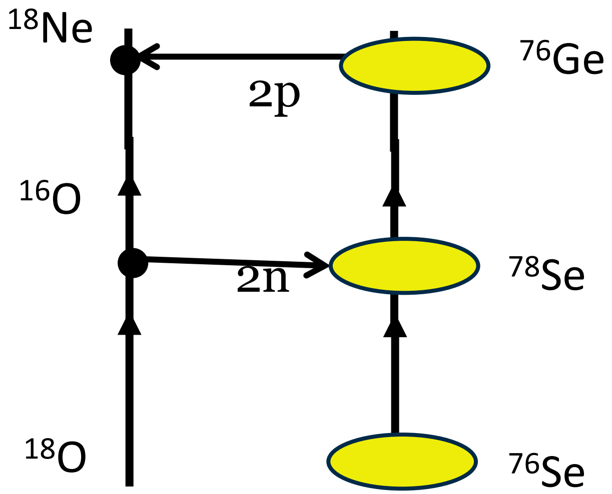
E. Santopinto *et al.*, Phys. Rev. C 98 061601 (R) (2018)

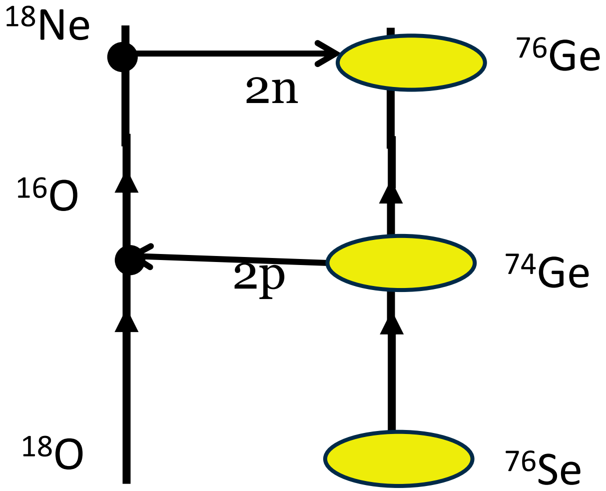
J.I. Bellone et al, Phys. Lett. B 807 (2020) 135528

$^{76}Se(^{18}O,^{18}Ne)\,^{76}Ge$









+ all other processes leading to $^{76}Ge,^{18}Ne$

H. Lenske, IOP Conf. Series: Journal of Physics: Conf. Series **1056 (2018) 012030**

E. Santopinto *et al.*, Phys. Rev. C 98 061601 (R) (2018)

J.I. Bellone et al, PLB 807 (2020) 135528

Large measurement campaing within the NUMEN project on transfer reactions

1. Transfer channels contribute to the measured CE cross sections → it is vital to know their weight and look for experimental conditions where they can be minimized

2. Transfer studies provide additional information on the nuclear states involved in β decays, and at the same time give access to specific spectroscopic information - single-particle orbitals and nucleon-nucleon pairing correlations can be explored for bound and resonant states

See presentation by Francesco Cappuzzello
for a detailed analysis
of all transfer channels

$$\underbrace{{}^{a}_{z}a + {}^{A}_{Z}A}_{\alpha} \rightarrow \underbrace{{}_{z\pm1}^{\;\;b}b + {}_{Z\mp1}^{\;\;B}B}_{\beta}$$

$$d^2\sigma_{\alpha\beta} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)} \times \sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta} \left|\mathcal{M}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\right|^2 d\Omega,$$

The reaction amplitude, $\mathcal{M}_{\alpha\beta}$,can separated into

✓ distortion coefficient for the reaction term

✓ projectile and target transition form factors

contain nuclear structure information

→ $\langle J_Y || T_{LSJ} || J_X \rangle$ transition densities induced by the operator $T_{LSJ} = \left(\frac{r}{R_d}\right)^L \, [\sigma^S \otimes Y_L]_J \, \tau_\pm$

H. Lenske, J. Bellone, M. Colonna, J.A. Lay, PRC 98, 044620 (2018)

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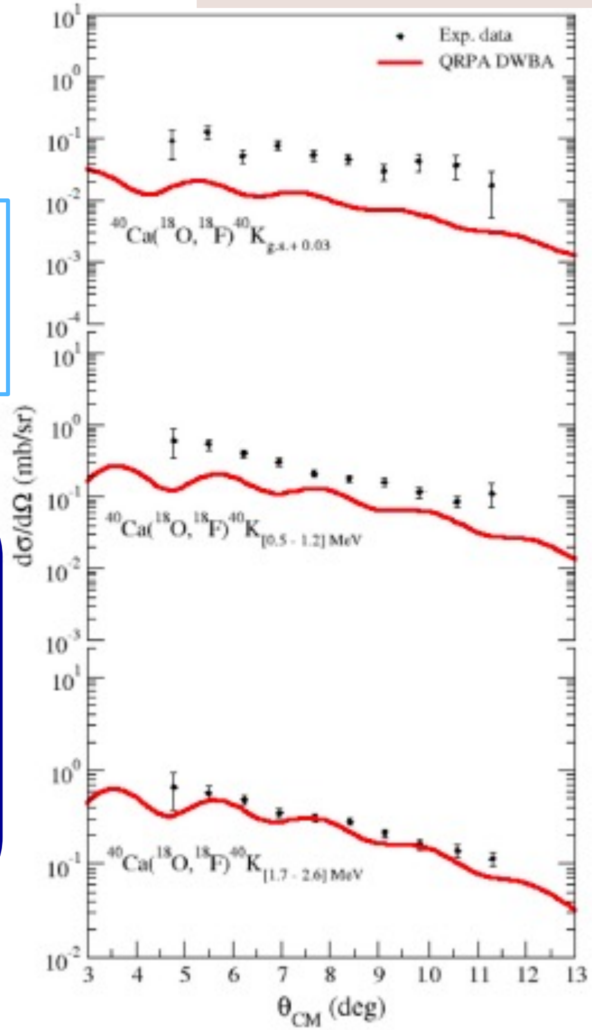
H. Lenske, J. Bellone, M. Colonna, J.A. Lay, PRC 98, 044620 (2018)

Applied to different systems
$^{40}Ca + ^{18}O$, $^{76}Se + ^{18}O$,
$^{76}Ge + ^{20}Ne$, $^{116}Cd + ^{20}Ne$

$^{40}Ca(^{18}O,^{18}F)^{40}K$ reaction

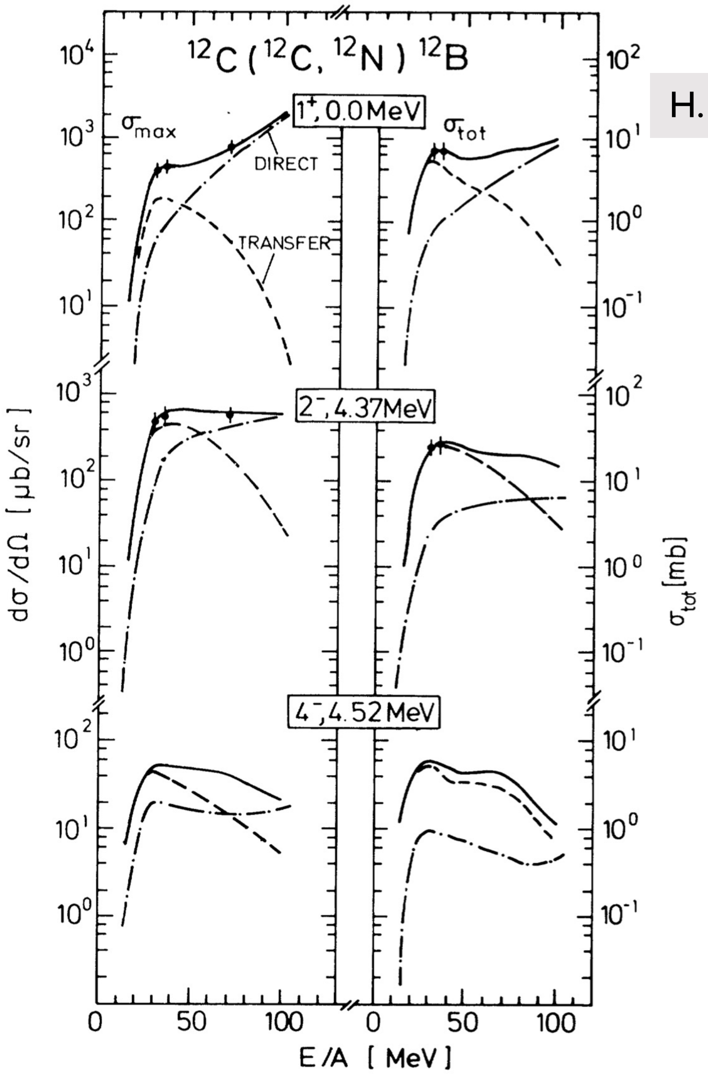
will be discussed by
Francesco Cappuzzello

QRPA calculations



M. Cavallaro et al, Front. Astron. Space Sci 8, 659815 (2021)

Direct + transfer mechanisms in heavy-ion SCE reaction



H. Lenske et al. PRL 62, 1457 (1989)

The reaction mechanism is changing from transfer to direct as the incident energy increases from E/A =10 MeV to 100 MeV and the transition point depends on the final state

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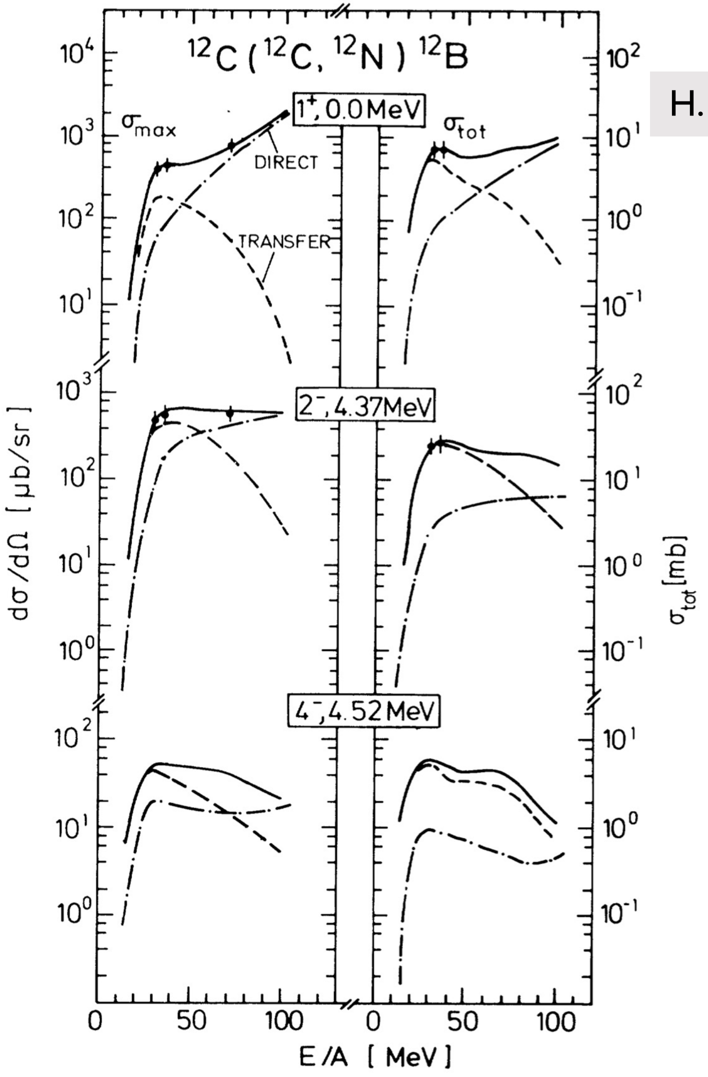
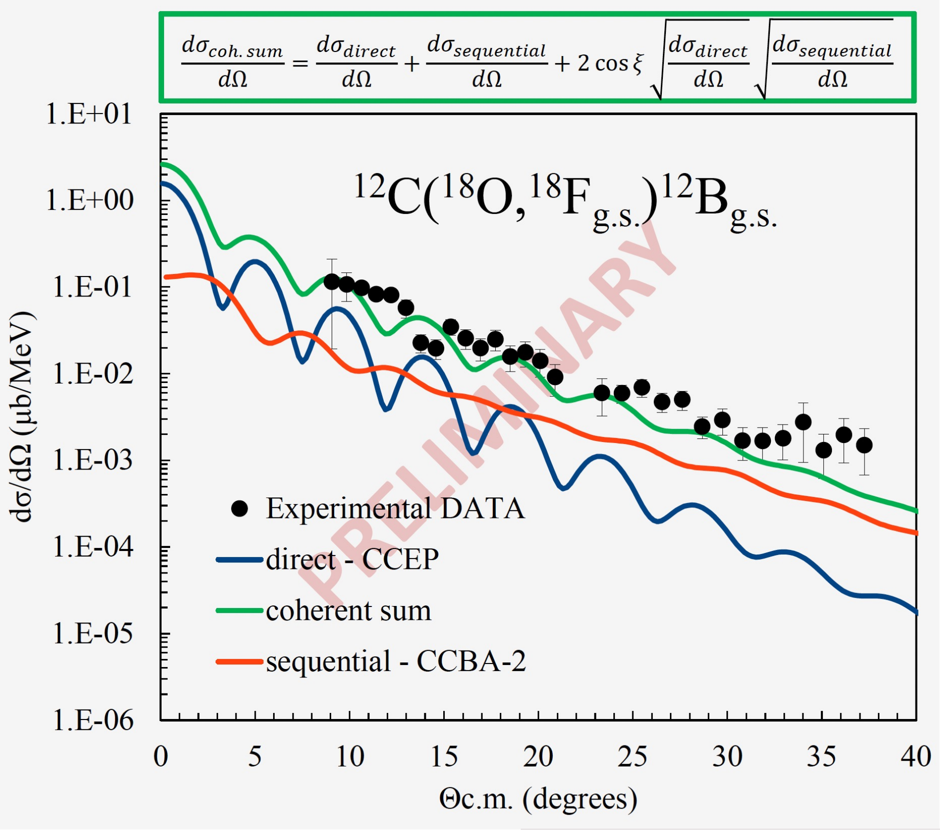


H. Lenske et al. PRL 62, 1457 (1989)



Private Communication by J.A Lay Valera

The reaction mechanism is changing from transfer to direct as the incident energy increases from E/A =10 MeV to 100 MeV and the transition point depends on the final state

- Transfer - transition amplitudes from SM
- SCE - transition densities from QRPA

No one of the two mechanisms reproduces the expt cross section → both processes should be considered

- Good agreement with expt is obtained when the two cross sections are coherently summed

but a consistent calculation is needed

Two single uncorrelated CE process (DSCE)

$$\underbrace{{}^{a}_{z}a + {}^{A}_{Z}A}_{\alpha} \rightarrow {}_{z\pm1}{}^{c}c + {}_{Z\mp1}{}^{C}C \rightarrow \underbrace{{}_{z\pm2}{}^{b}b + {}_{Z\mp2}{}^{B}B}_{\beta}$$

The reaction amplitude $\mathcal{M}_{\alpha\beta}$ can be factorized in reaction and structure terms
→ to single out the information on both projectile and target dSCE NME some approximations are needed in the treatment of the intermediate channels

Structure term for target →
(similar expression for projectile)

$$\boxed{\sum_{C} \frac{\langle J_B || T_{LSJ} || J_C \rangle \langle J_C || T_{LSJ} || J_A \rangle}{\omega - (E_C - E_A)}}$$

↕

similar to $2\nu\beta\beta$ NME

J. I. Bellone, S. Burrello, M. Colonna, J.A. Lay, H. Lenske, PLB 807 (2020) 135528

H. Lenske, J.I. B., M. Colonna, D. Gambacurta, Universe 7, 98 (2021)

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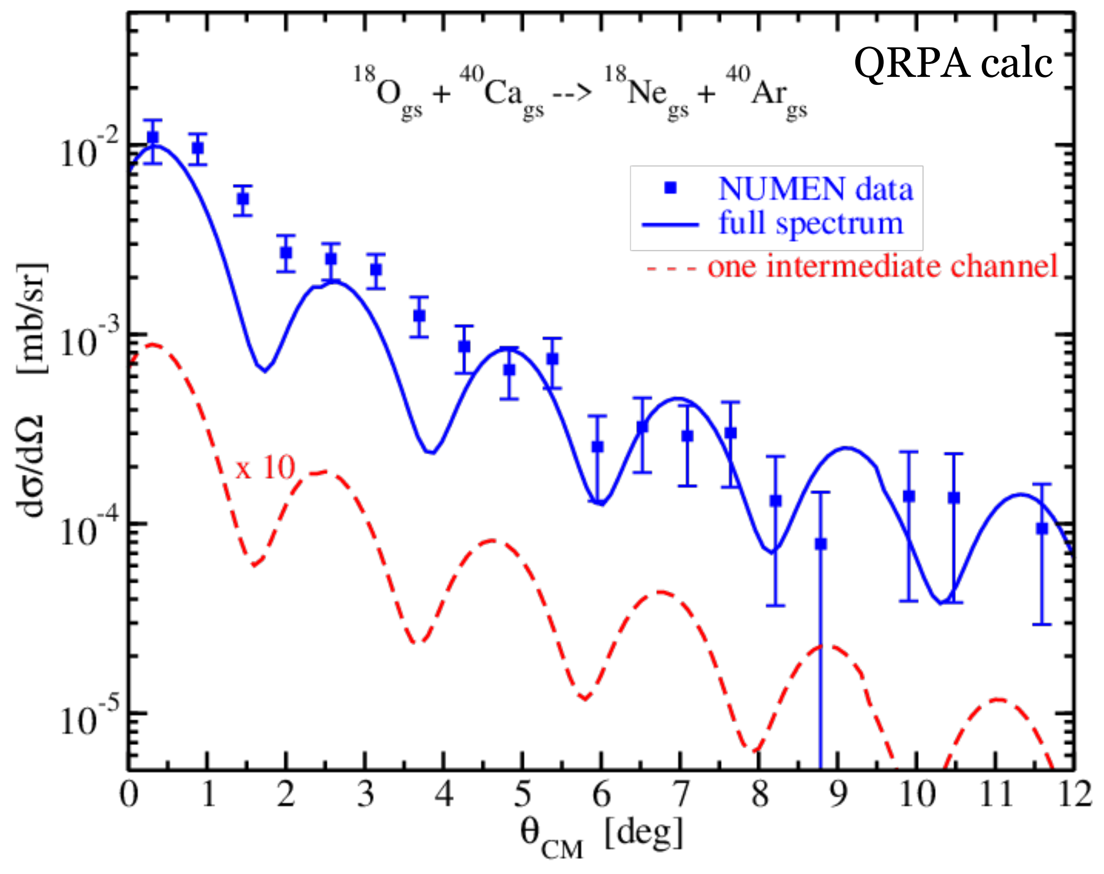
similar to 2νββ NME



- Order of magnitude recovered only with the full spectrum

However

- Experimental data are slightly underestimated especially at forward angles and the experimental diffraction structure is not reproduced

→ mDCE should be taken into account (since multi-nucleon transfer contribution is in this case safely negligible)

J. I. Bellone, S. Burrello, M. Colonna, J.A. Lay, H. Lenske, PLB 807 (2020) 135528
H. Lenske, J.I. B., M. Colonna, D. Gambacurta, Universe 7, 98 (2021)

Recent calculations including both DSCR and MSCE processes

- **One-step correlated exchange (MDCE)**

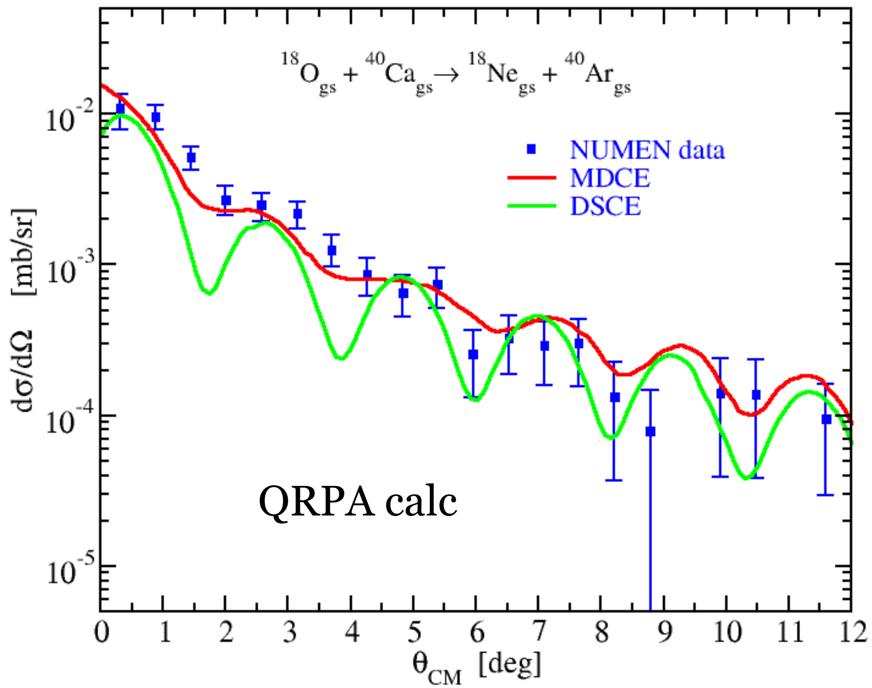
$$\underbrace{{}^{a}_{Z}a + {}^{A}_{Z}A}_{\alpha} \rightarrow \underbrace{{}^{b}_{Z\pm2}b + {}^{B}_{Z\mp2}B}_{\beta}$$

- **Two single uncorrelated CE process (DSCE)**

$$\underbrace{{}^{a}_{Z}a + {}^{A}_{Z}A}_{\alpha} \rightarrow {}^{c}_{Z\pm1}c + {}^{C}_{Z\mp1}C \rightarrow \underbrace{{}^{b}_{Z\pm2}b + {}^{B}_{Z\mp2}B}_{\beta}$$

$^{40}Ca(^{18}O,^{18}Ne)^{40}Ar$

MDCE with a scaling factor ~2



Private Communication by J.A lay Valera

✓ The inclusion of the MDCE channel gives contributions in the right direction

However further work is need to remove

- the scaling factor
- coherently sum the two processes

- NMEs are very relevant ingredients in $0\nu\beta\beta$ studies, but a factor 3 difference betwen predictions of different nuclear models leads to big uncercertainties
  - ✓ on the amount of material required in the experiments
  - ✓ on the effective neutrino mass in case $0\nu\beta\beta$ will be observed

→ Great efforts, in various directions, should be undertaken to improve the NME calculations, as for instance

- include higher order configurations
- overcome the limits of the closure approximation in the derivation of the $0\nu\beta\beta$ operator
- calculate renormalization of the $0\nu\beta\beta$ operator due to sub-nucleonic degrees of freedom from 2- and 3-body chiral potentials

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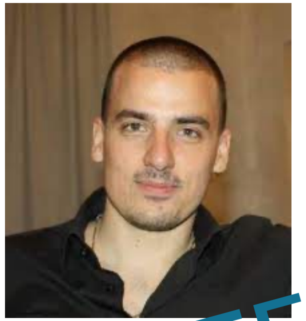
- On the other side, it is also very important to better constrain the models and in particular to find observables which are more directly correlated to the $0\nu\beta\beta$ NME

In this persective, $\gamma\gamma$ decay as well as CE reactions induced heavy ions may represent a very instrumental tool. However, to fully exploit this way it is needed to

- better investigate correlations between $\gamma\gamma$ decay/DCE and $0\nu\beta\beta$ NMEs with different improved models
- better understand reaction mechanism, in particular for DCE reactions
- accomplish a consistent description of all competing channels
- compare results of different models in the description of β decay NME and SCE/DEC form factor






THANKS FOR YOUR ATTENTION