Axion dark matter with low-scale inflation

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A (partial) list of open questions in particle physics and cosmology

- Why do strong interactions not violate CP?
- What is the nature of dark matter?
- At what scale did inflation take place?

The Strong CP problem

$$\mathcal{L}_{\rm QCD} \supset \theta \frac{g_s^2}{32\pi^2} G_a^{\mu\nu} \tilde{G}_{a\mu\nu} \qquad \tilde{G}_{a\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G_a^{\rho\sigma}$$

The gauge invariant combination $G\tilde{G}$ is odd under parity (P), but even under charge conjugation (C), so odd under CP. This is like in EM:

$$F\tilde{F} = \vec{E}\cdot\vec{B} \xrightarrow[\text{P}]{} -\vec{E}\cdot\vec{B}$$

One can show that $G\tilde{G}$ is a total derivative. As such it cannot have any effect in perturbation theory. It does have, however, physical non-perturbative effects, due to non-trivial gauge field configurations (instantons).

The Strong CP problem

Including also the quarks

Performing a global U(1) chiral rotation of the quark fields $q \to e^{i\theta\gamma_5} q$

introduces the term $\theta \frac{g_s^2}{32\pi^2} G_a^{\mu\nu} \tilde{G}_{a\mu\nu}$ in the lagrangian.

From $$\mathcal{L}_{\rm QCD} \supset \theta \frac{g_s^2}{32\pi^2} G_a^{\mu\nu} \tilde{G}_{a\mu\nu} + \bar{q}_{Ri} M_{ij} q_{Lj} + \text{h.c.}$$

we then have that the physical CP violating angle in QCD is

$$\bar{\theta} = \theta + \arg\, \det M$$

Using chiral lagrangian techniques, to study QCD below the confinement scale, one can compute the dependence of the neutron electric dipole moment (EDM) on the parameter $\bar{\theta}$.
The experimental non observation of a neutron EDM implies: $\bar{\theta} < 10^{-10}$

The Strong CP problem

In the Standard Model the quark masses are proportional to their yukawas, and

$$\arg\det M = \arg\det(Y_d Y_u)$$

In the weak interactions, starting from the yukawas one defines the CKM matrix, where the CP violation is parametrized by the phase δ, which is experimentally measured to be of order one.

So, we have evidence for a CP violating phase of order one in the weak interactions, no evidence for a CP violating phase in the strong interactions.
We call this the strong CP problem.

Anthropic solution?

What would be different in nature if theta were of order one?

How does theta affect nuclear physics?

$$\mathcal{L}_{\chi\mathrm{PT}} \supset g_{\pi NN} \bar{N} \pi^a \tau^a \gamma_5 N + g_{\bar{\theta}} \bar{\theta} \bar{N} \pi^a \tau^a N$$

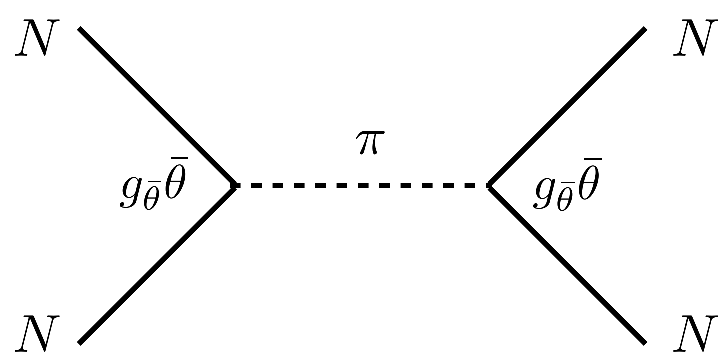
Effect of theta on the deuteron binding energy

Big Bang Nucleosynthesis (BBN) is sensitive to the deuteron binding energy (2.2 MeV)

$$n + p \leftrightarrow D + \gamma$$
$$D + D \leftrightarrow T + p$$
$$D + T \leftrightarrow {}^4He + n$$

$$\frac{1}{\eta} e^{-\frac{E_D}{T}} = \frac{n_\gamma}{n_b} e^{-\frac{E_D}{T}} < 1$$

$$T \simeq 0.1 \text{ MeV}$$
$$t \simeq 3 \text{ min}$$



$$V_1 = \frac{3}{4\pi} g_{\bar\theta}^2 \bar\theta^2 \frac{e^{-m_\pi r}}{r}$$

repulsive

$$\Delta E_D < 0.1 \text{ MeV}$$

for $\bar\theta \sim 1$

LU 2010

No reason from nuclear physics and BBN why theta should be small!

NO anthropic solution to strong CP

The Axion solution

Augment the Standard Model (SM) by a global chiral $U(1)_{\rm PQ}$ symmetry, anomalous under the QCD gauge group. This can be achieved by adding at least one complex scalar field to the SM

$$\Phi = \rho e^{i\sigma/f} \xrightarrow[U(1)_{\rm PQ}]{} \rho e^{i(\sigma/f+\alpha)}$$

PQ = Peccei-Quinn

The anomaly of the $U(1)_{\rm PQ}$ generates the term $\frac{\sigma}{f}\frac{g_s^2}{32\pi^2}G\tilde{G}$ in the QCD lagrangian.

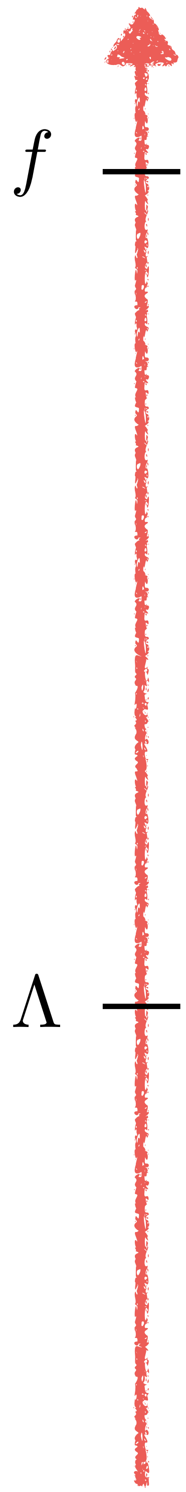
At scales below confinement, $\Lambda \approx 200\ \mathrm{MeV}$, we get the potential

$$V(\bar{\theta},\sigma) \approx \Lambda^4\left[1-\cos\left(\frac{\sigma}{f}+\bar{\theta}\right)\right]$$

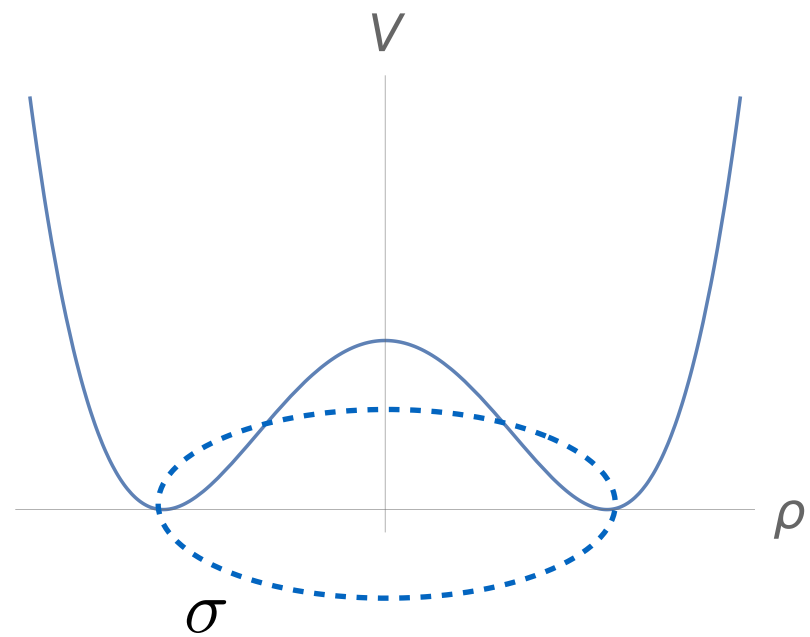
minimized at $\langle\sigma\rangle = -f\bar{\theta}$.

At the minimum of this potential, QCD is CP conserving: strong CP problem solved!

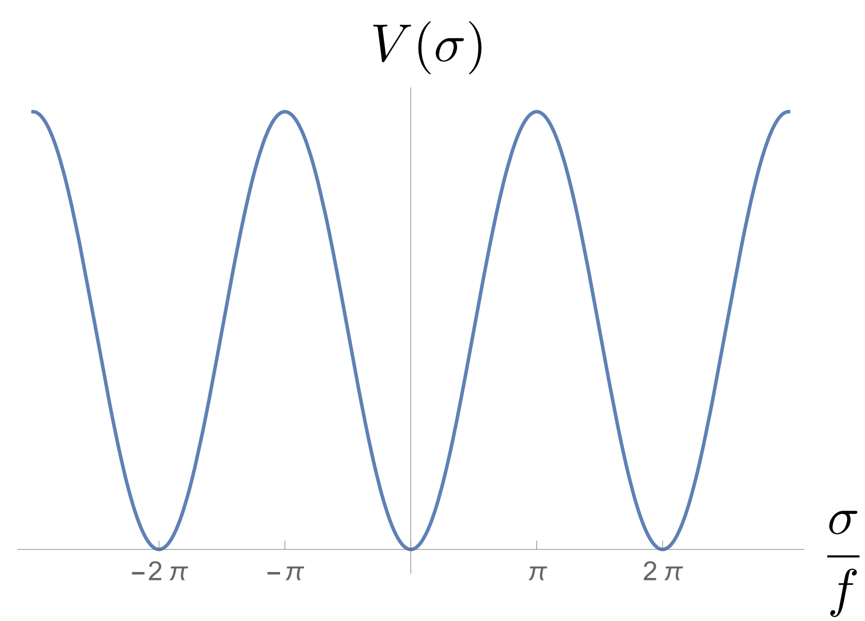
Spontaneous vs explicit breaking of PQ symmetry



$$V(\Phi) = (|\Phi|^2 - f^2)^2$$
$$\Phi = \rho e^{i\sigma/f}$$



$$V(\sigma) \approx \Lambda^4 \left[1 - \cos\frac{\sigma}{f}\right]$$
$$\approx \frac{1}{2}\frac{\Lambda^4}{f^2}\sigma^2 - \frac{1}{24}\frac{\Lambda^4}{f^4}\sigma^4$$



Temperature dependence of the axion mass

Λ : confinement scale of a strong gauge group

At zero temperature $\quad m_{\sigma 0} = \xi \dfrac{\Lambda^2}{f}$

At finite temperature $\quad m_\sigma(T) \simeq \begin{cases} \lambda\, m_{\sigma 0} \left(\dfrac{\Lambda}{T}\right)^p & \text{for } T \gg \Lambda, \\ m_{\sigma 0} & \text{for } T \ll \Lambda, \end{cases}$

For the QCD axion: $\quad \Lambda \approx 200 \text{ MeV}, \quad p \approx 4, \quad \lambda \approx 0.1, \quad \xi \approx 0.1$

Due piccioni con una fava

On top of providing a solution to the strong CP problem,
the axion can also provide a good cold dark matter candidate.

The Axion in the expanding universe

FRW metric $ds^2 = -dt^2 + a^2(t) d\vec{x}^2$

Hubble parameter $H \equiv \frac{\dot{a}}{a} = \frac{\rho}{3M_P^2}$

$$S = -\int d^4x \sqrt{-g}\; g^{\mu\nu} \left(\frac{1}{2} \partial_\mu \sigma \partial_\nu \sigma + \frac{1}{2} m_\sigma^2 \sigma^2 \right)$$

From the action derive the equation of motion of the axion

$$\ddot{\sigma} + 3H\dot{\sigma} + m_\sigma^2 \sigma - \frac{\nabla^2}{a^2} \sigma = 0$$

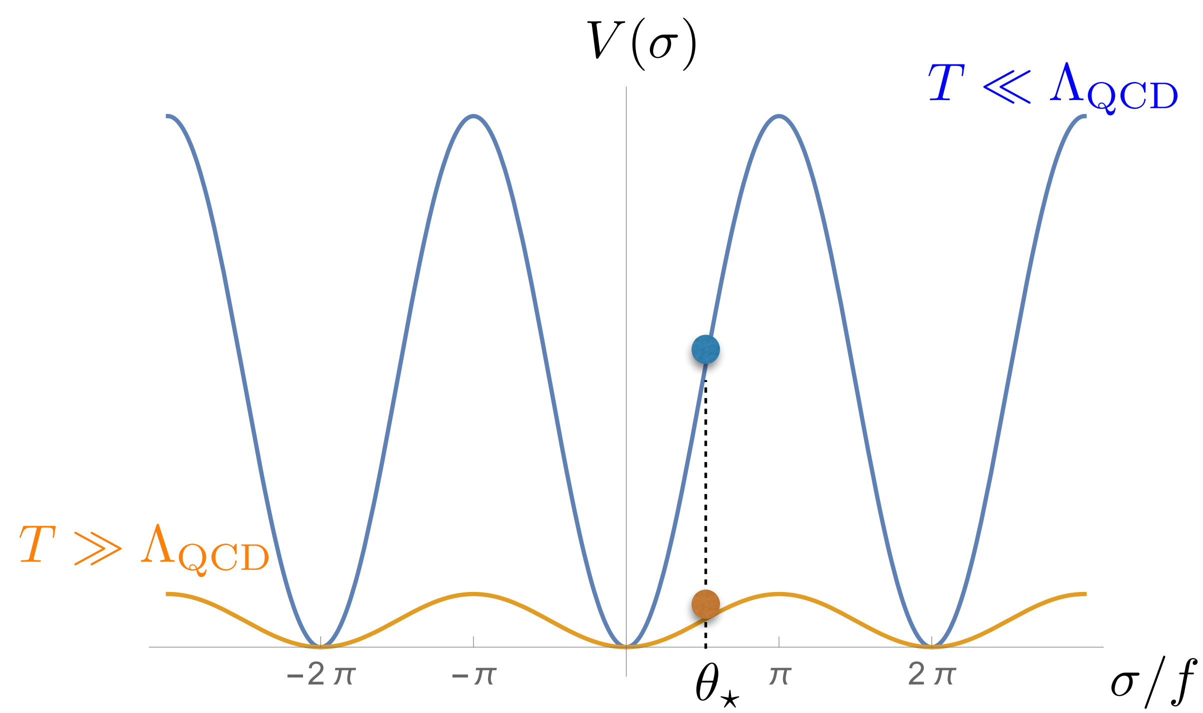
Take the axion to be homogeneous in space $\sigma(\vec{x}, t) \to \sigma(t)$

$$\ddot{\sigma} + 3H\dot{\sigma} + m_\sigma^2 \sigma = 0$$

Axion dark matter

$$\ddot{\sigma} + 3H\dot{\sigma} + m_\sigma(T)^2\sigma = 0$$

Consider the evolution during the radiation dominated era $\qquad H \approx \dfrac{T^2}{M_P}$



When $H \sim m_\sigma(T)$ the axion field starts to oscillate.
Soon after, the energy density of the oscillating field redshifts like non relativistic matter:

$$\rho_\sigma \propto a^{-3}$$

Light but very cold dark matter

For this calculation we need only CLASSICAL field theory.

In fourier space, a field homogeneous in space corresponds to a field with zero momentum.

The classical axion field then describes what in quantum field theory would be a very large number of scalar particles at zero momentum.

These are clearly non-relativistic, very cold indeed.

Axion relic abundance

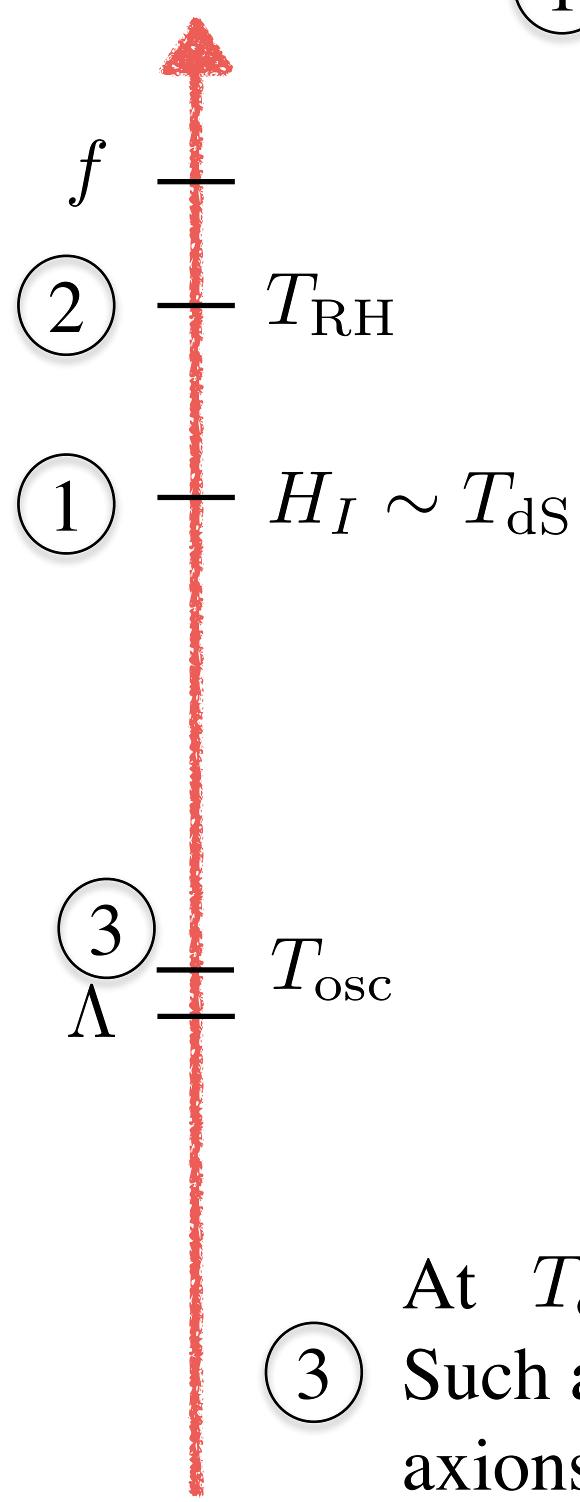
$$\Omega_\sigma h^2 = \kappa_p \, \theta_\star^2 \left(\frac{g_{s*}(T_{\rm osc})}{100}\right)^{-1} \left(\frac{g_*(T_{\rm osc})}{100}\right)^{\frac{p+3}{2p+4}} \left(\frac{\lambda}{0.1}\right)^{-\frac{1}{p+2}} \left(\frac{\xi}{0.1}\right)^{\frac{p+1}{p+2}} \left(\frac{\Lambda}{200\,{\rm MeV}}\right) \left(\frac{f}{10^{12}\,{\rm GeV}}\right)^{\frac{p+3}{p+2}}$$

Recall

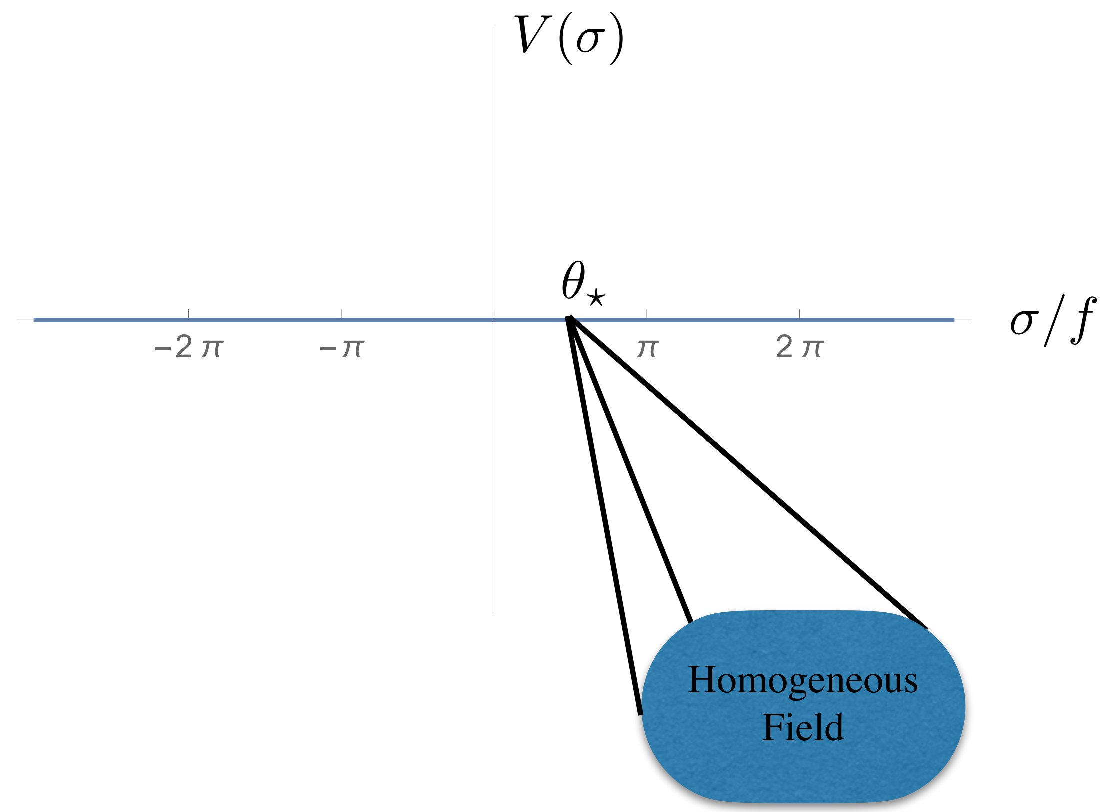
$$m_\sigma(T) \simeq \begin{cases} \lambda \, m_{\sigma 0} \left(\dfrac{\Lambda}{T}\right)^p & \text{for } T \gg \Lambda, \\ m_{\sigma 0} & \text{for } T \ll \Lambda, \end{cases} \qquad m_{\sigma 0} = \xi \frac{\Lambda^2}{f}$$

$$\Omega_{\rm DM} h^2 = 0.12$$

Summary so far

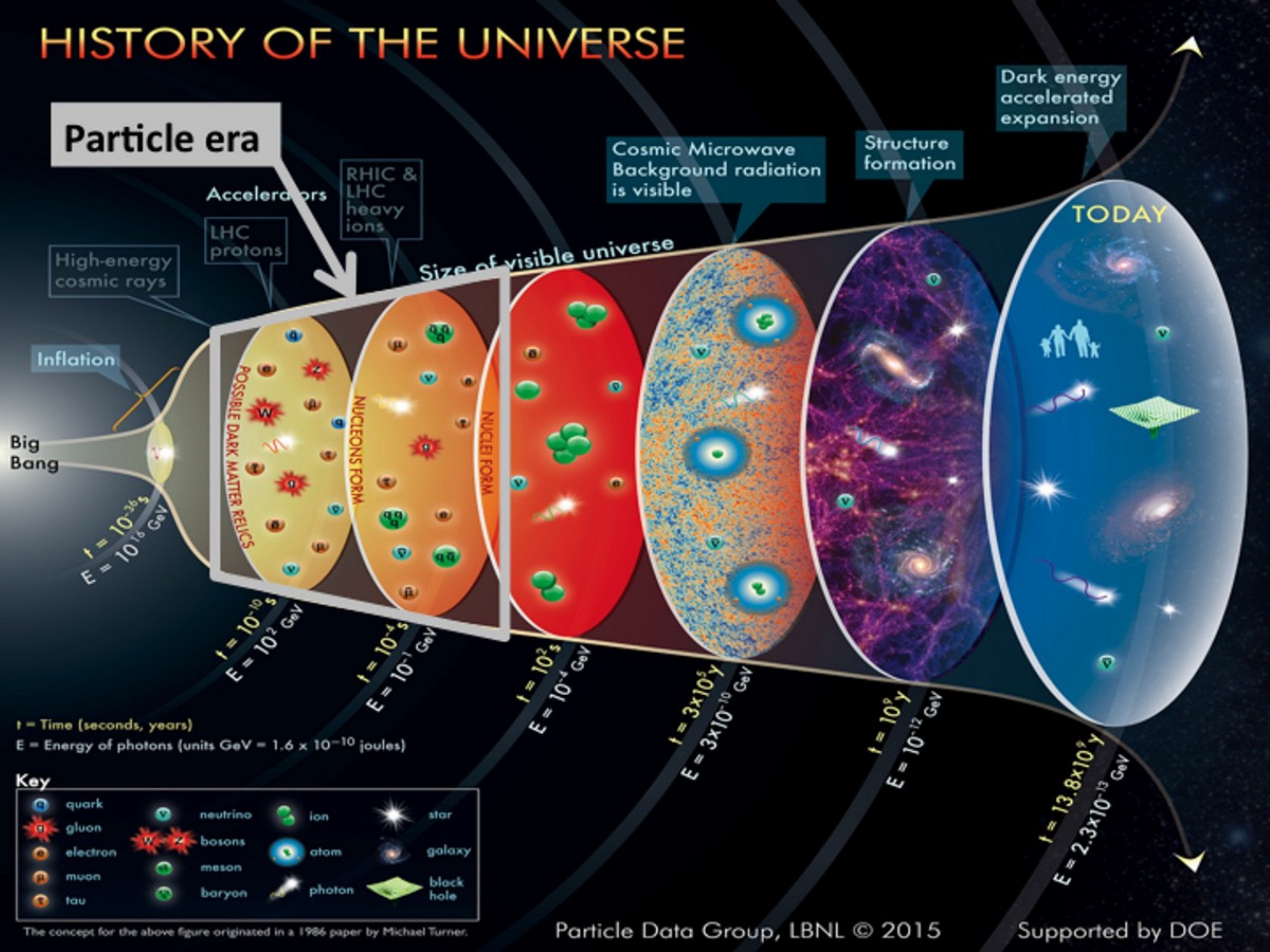


1 The Universe undergoes a phase of exponentially accelerated expansion: inflation.



2 The Universe reheats to a temperature $T_{\rm RH}$, and as it expands it cools down.

3 At $T_{\rm osc}$, when $m_\sigma(T_{\rm osc}) \approx H(T_{\rm osc})$, the axion field starts oscillating. Such an oscillating field describes a collection of non-relativistic axions, which survive until today.



HISTORY OF THE UNIVERSE
Particle era
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
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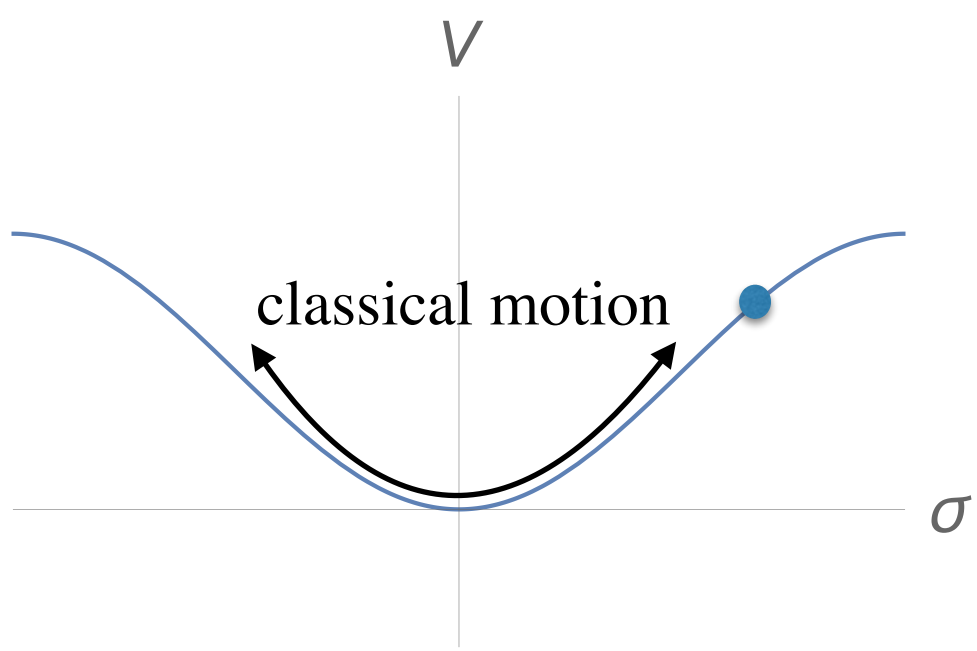
Axion dark matter with low scale inflation

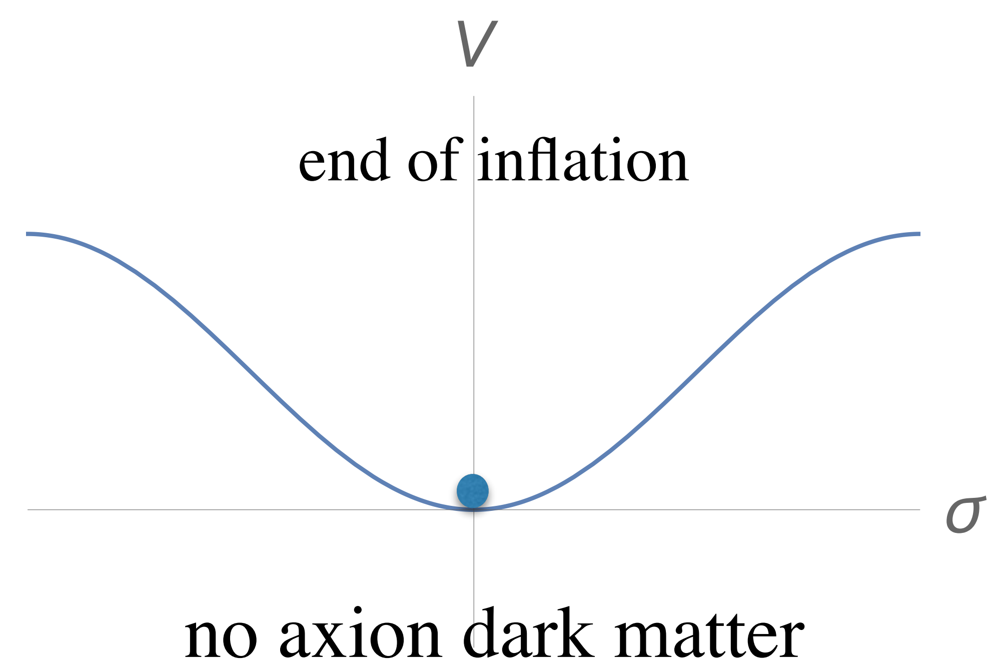
What if the scale of inflation is as low as

$$H_I \ll m_\sigma$$

$$\ddot{\sigma} + 3H_I\dot{\sigma} + m_\sigma^2\sigma = 0$$

$$\sigma(t) = \sigma_* e^{-\frac{3}{2}H_I t}\cos(m_\sigma t) \to 0$$

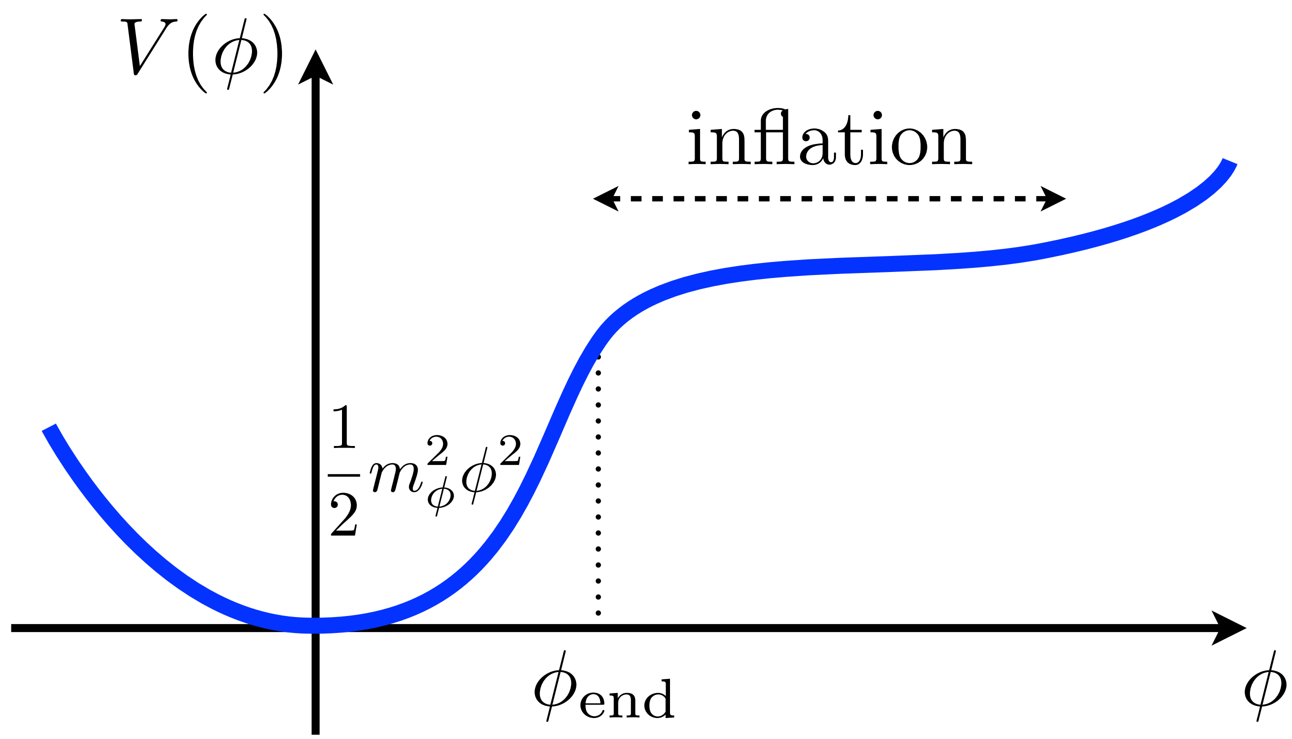




Are there ways to obtain a relic abundance of axions in this scenario?

σ : axion

ϕ : inflaton



What if the inflaton and the axion are coupled?

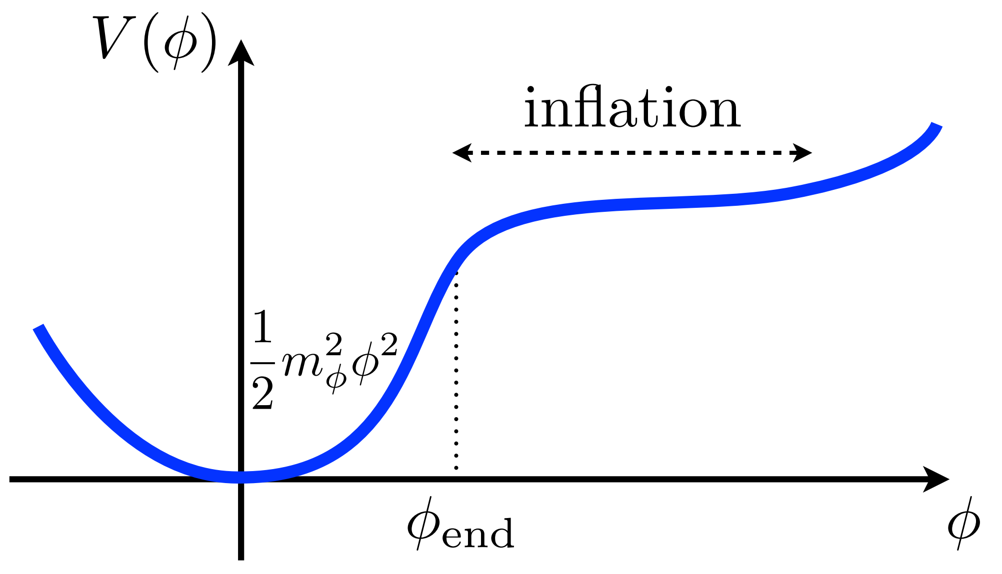
$$-\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}g^{\mu\nu}(\partial_\mu\sigma\partial_\nu\sigma + \partial_\mu\phi\partial_\nu\phi + 2\alpha\partial_\mu\sigma\partial_\nu\phi) + \frac{1}{2}m_\sigma^2\sigma^2 + V(\phi)$$

Kinetic mixing

Via the kinetic mixing, when the inflaton starts oscillating around the minimum of its potential (reheating) it will kick the axion away from its minimum.

The axion displacement, in turn, allows for the re-alignment mechanism of dark matter production to take place.

End of inflation

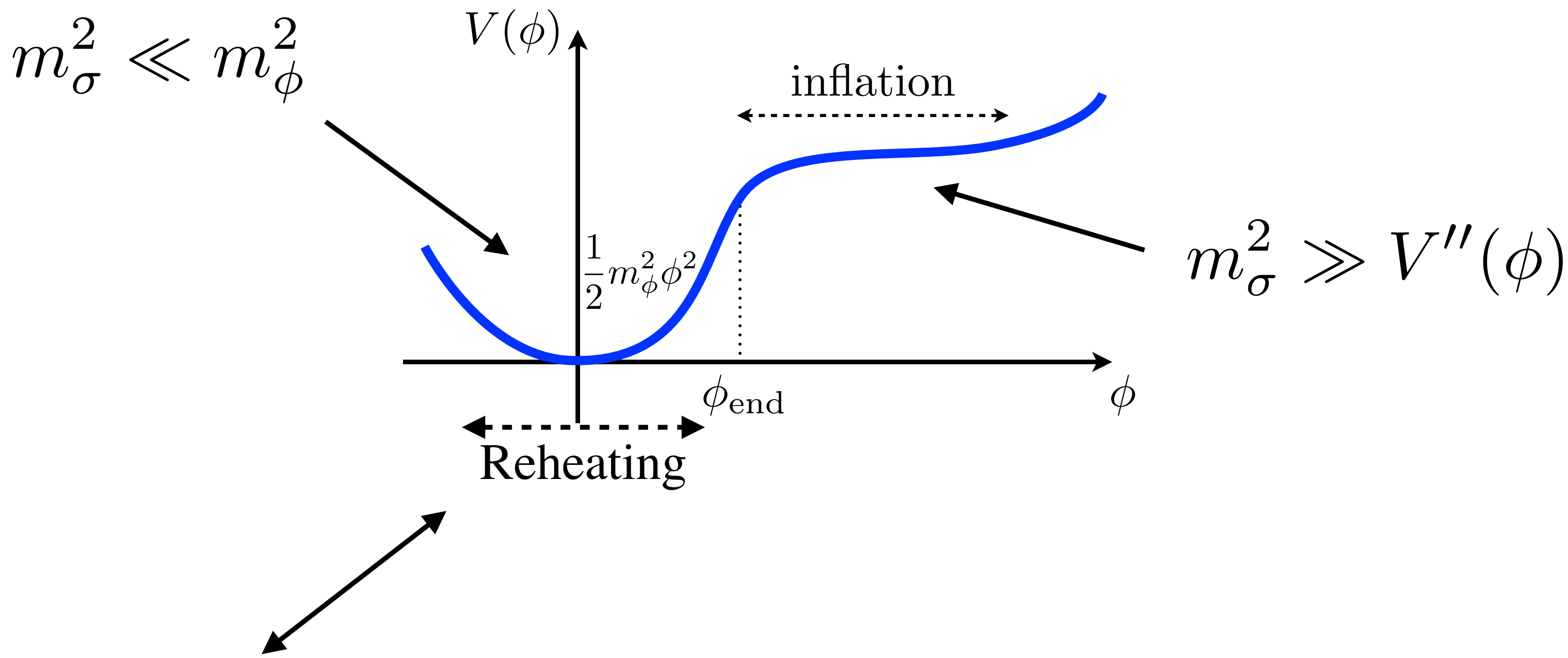


$$-\frac{\dot{H}}{H^2} = 1$$

$$\dot{\phi}^2 = V(\phi) = 2M_P^2 H^2$$

$$\phi_{\rm end} \simeq \frac{2M_P H_{\rm end}}{m_\phi}$$

Diagonal basis at reheating



Diagonal basis during reheating

$$\varphi_{\rm DM} \simeq \alpha\,\phi + \sigma, \quad \varphi_{\rm RH} \simeq \sqrt{1-\alpha^2}\left(\phi - \alpha\frac{m_\sigma^2}{m_\phi^2}\,\sigma\right)$$

$$m_{\rm DM}^2 \simeq m_\sigma^2, \quad m_{\rm RH}^2 \simeq \frac{m_\phi^2}{1-\alpha^2}$$

At the end of inflation

$$\phi_{\rm end} \simeq \frac{2 M_P H_{\rm end}}{m_\phi}$$

$$\varphi_{\rm DMend} = \alpha\phi_{\rm end} + \sigma_{\rm end} \simeq \alpha\phi_{\rm end}$$

$$= \frac{C\alpha M_P H_{\rm end}}{m_\phi}$$

Reheating

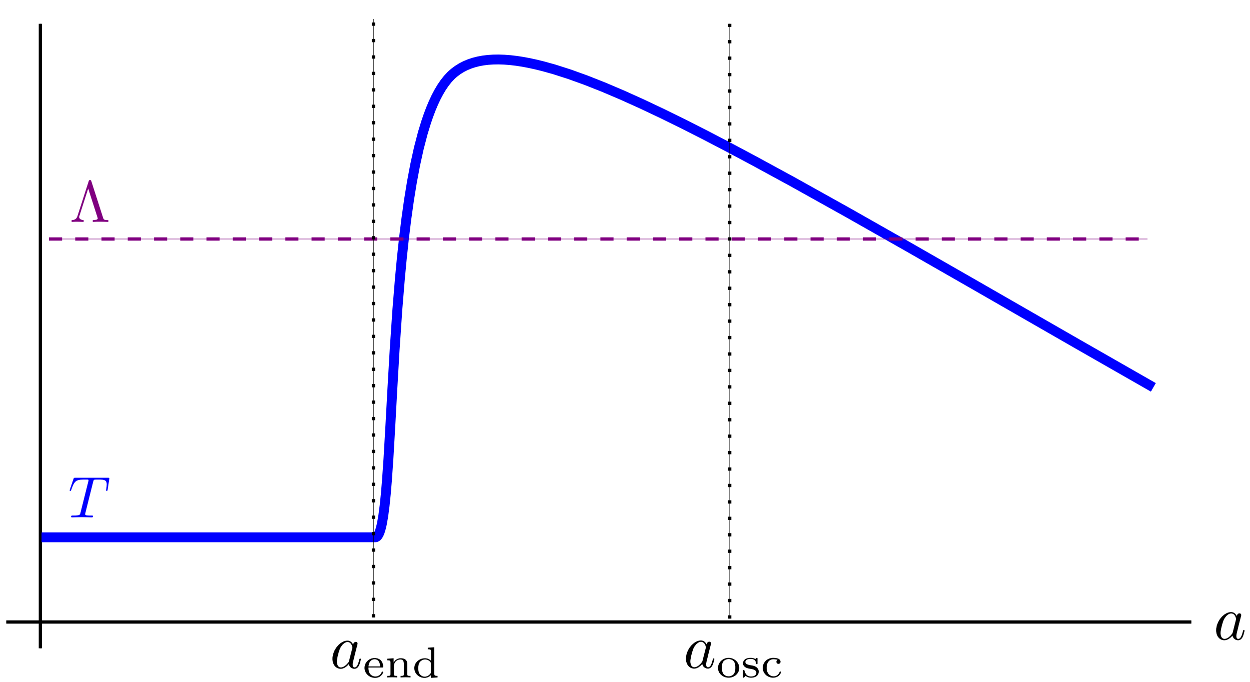
$$\frac{\alpha_\gamma}{8\pi f}\sigma F_{\mu\nu}\tilde{F}^{\mu\nu} + g_{\phi f f}\,\phi\bar{\psi}i\gamma^5\psi$$

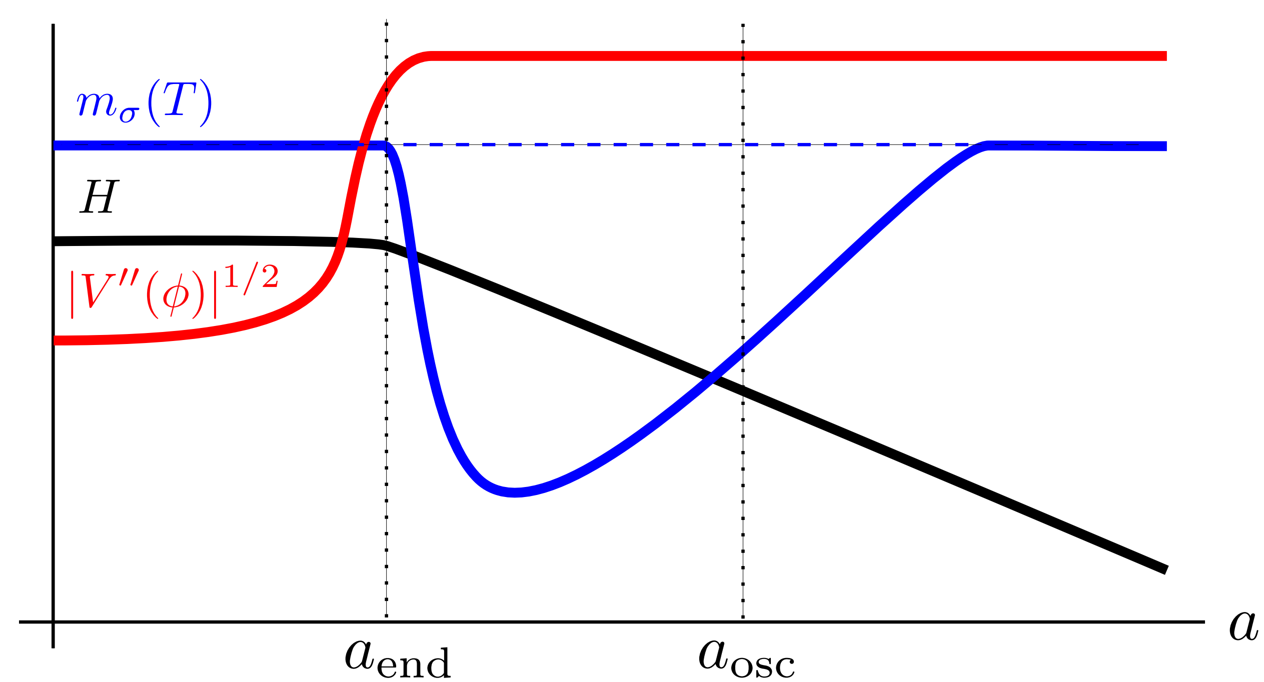
$$\Gamma(\varphi_{\rm DM}\to\gamma\gamma)\simeq\frac{\alpha_\gamma^2}{256\pi^3 f^2}m_\sigma^3$$

$$\Gamma(\varphi_{\rm RH}\to\gamma\gamma)\simeq\frac{\alpha^2}{(1-\alpha^2)^{5/2}}\frac{\alpha_\gamma^2}{256\pi^3 f^2}m_\phi^3$$

$$\Gamma(\varphi_{\rm DM}\to f\bar{f})\simeq\alpha^2\frac{g_{\phi f f}^2}{8\pi}\frac{m_\sigma^5}{m_\phi^4}$$

$$\Gamma(\varphi_{\rm RH}\to f\bar{f})\simeq\frac{1}{(1-\alpha^2)^{3/2}}\frac{g_{\phi f f}^2}{8\pi}m_\phi$$



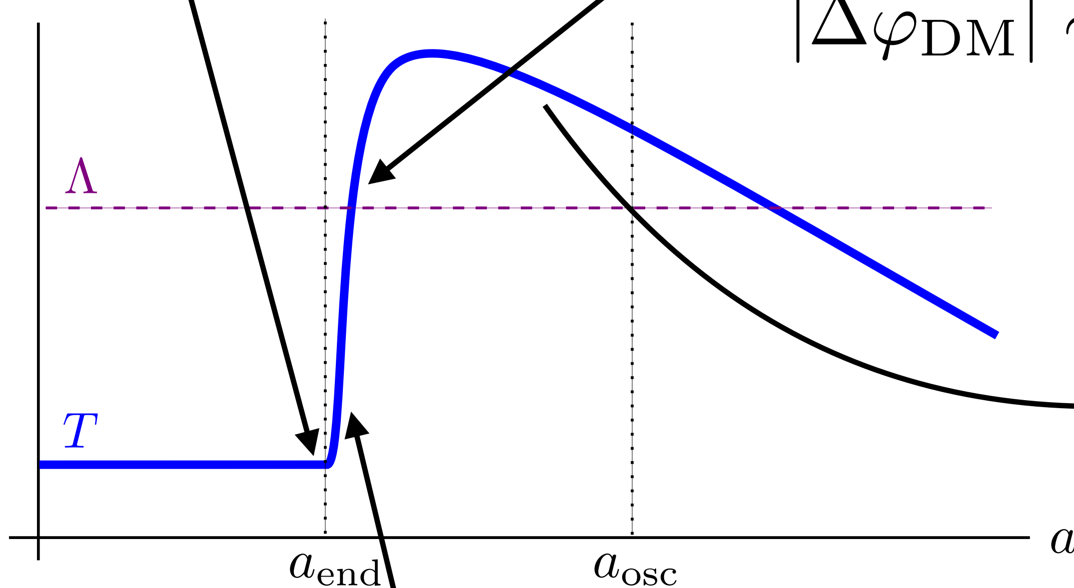


$$\varphi_{\rm DMend} = \frac{C\alpha M_P H_{\rm end}}{m_\phi}$$

The field drifts on a flat potential and stops within a few Hubble times

$$\ddot{\varphi}_{\rm DM} + 3H\dot{\varphi}_{\rm DM} \approx 0$$

$$|\Delta\varphi_{\rm DM}| \sim \left|\frac{\dot{\varphi}_{\rm DMend}}{H_{\rm end}}\right| \sim \frac{m_{\sigma 0}}{H_{\rm end}}|\varphi_{\rm DMend}| > |\varphi_{\rm DMend}|$$



$$\varphi_{\rm DM\star} = \frac{B\alpha M_P m_{\sigma 0}}{m_\phi}$$

B is a number of order 1

$$\varphi_{\rm DM} = a^{-3/2}\varphi_{\rm DMend}\cos(m_{\sigma 0}t)$$

$$|\dot{\varphi}_{\rm DMend}| \approx m_{\sigma 0}|\varphi_{\rm DMend}|$$

Summary so far

The dark matter field

$$\varphi_{\rm DM} \simeq \alpha\phi + \sigma$$

is a linear combination of axion and inflaton, hence we dub it inflaxion dark matter.

After the dynamics that happen during reheating, it is stuck at

$$\varphi_{\rm DM\star} = \frac{B\alpha M_P m_{\sigma 0}}{m_\phi}$$

away from the minimum of its potential.

That is the misalignment. From that point on, the calculation of the relic abundance proceeds in the same way as for scenario 2.

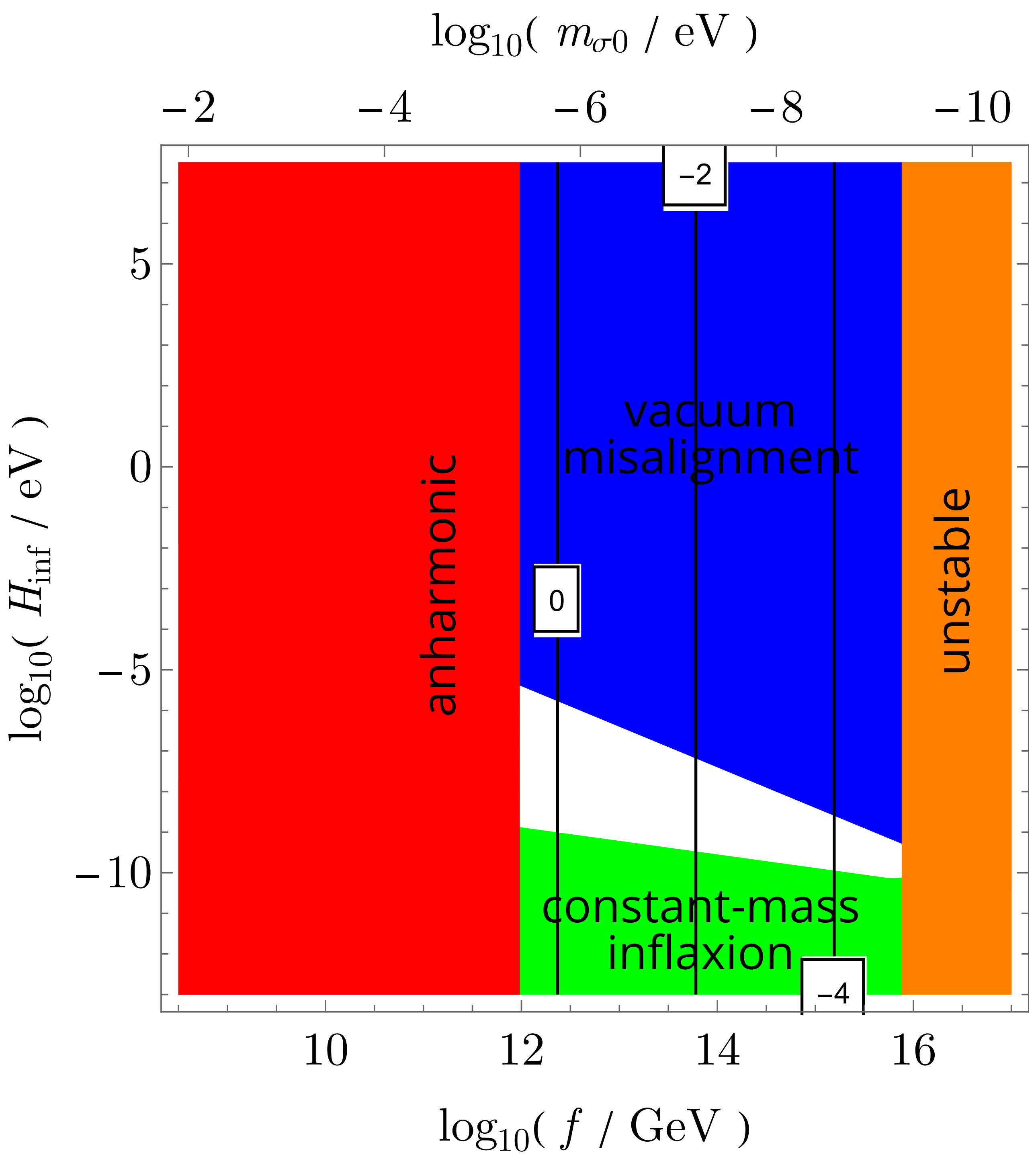
Final relic abundance

$$\Omega_\sigma h^2 = \kappa_p\, \theta_\star^2 \left(\frac{g_{s*}(T_{\rm osc})}{100}\right)^{-1} \left(\frac{g_*(T_{\rm osc})}{100}\right)^{\frac{p+3}{2p+4}} \left(\frac{\lambda}{0.1}\right)^{-\frac{1}{p+2}} \left(\frac{\xi}{0.1}\right)^{\frac{p+1}{p+2}} \left(\frac{\Lambda}{200\,{\rm MeV}}\right) \left(\frac{f}{10^{12}\,{\rm GeV}}\right)^{\frac{p+3}{p+2}}$$

$$\theta_\star = \frac{\varphi_{\rm DM\star}}{f} = \frac{B\alpha M_P m_{\sigma 0}}{f m_\phi}$$

Whereas in the first scenario I discussed the initial misalignment angle is not calculable, in the inflaxion scenario it is given in terms of the parameters in the Lagrangian.

Parameter space



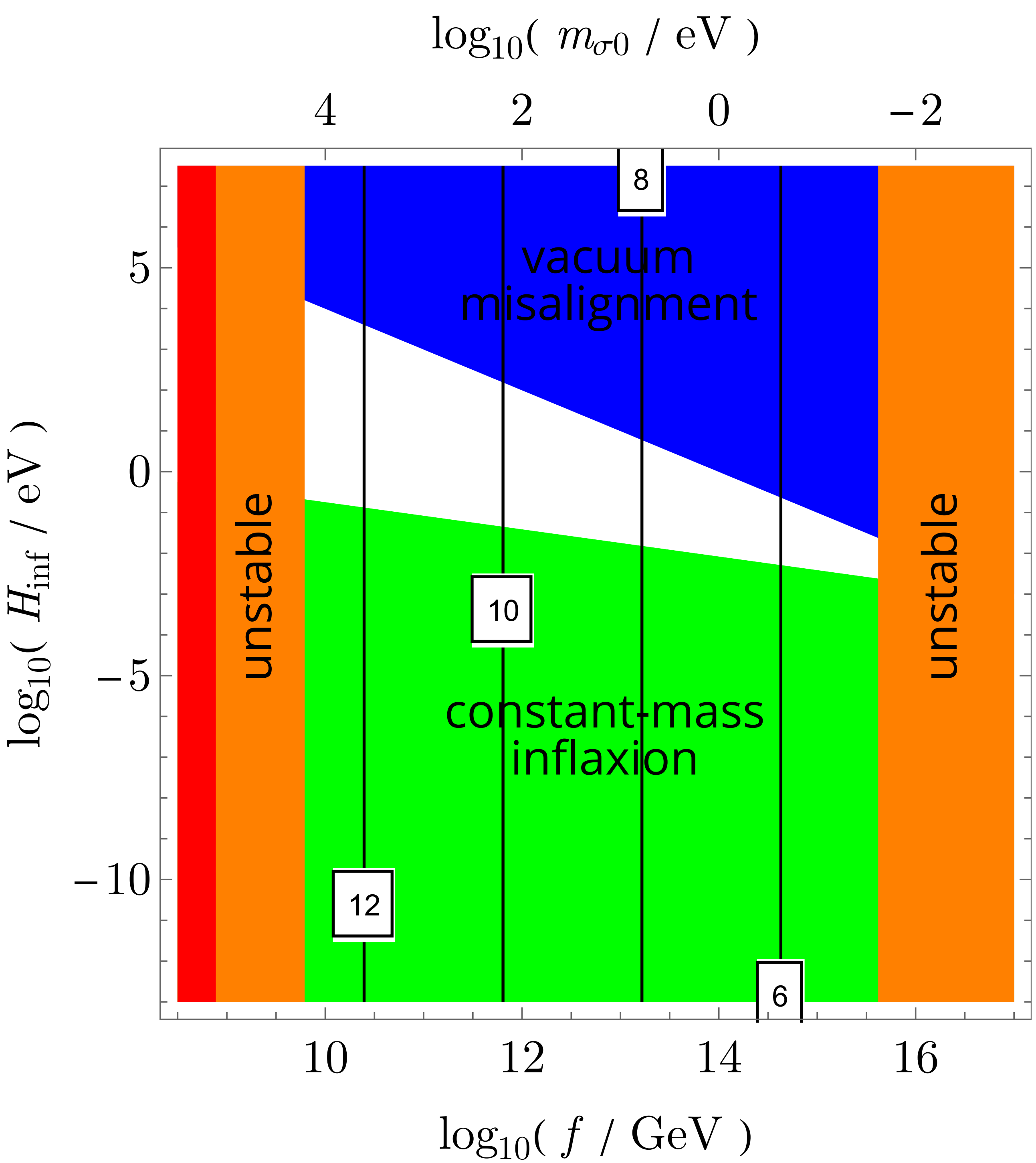
(a) $\Lambda = 200\,\text{MeV}$ (QCD axion)

Excluded regions, violation of:

$m_{\sigma 0} > H_{\rm inf}$	blue
$m_\sigma(T_{\rm max}) < H_{\rm inf}$	green
$\theta_\star < 1$	red
$\Gamma_{\rm DM} < H_0$	orange

$\alpha = 1/3$ $\quad g_{\phi f f} = 10^{-2}$

Parameter space



(c) $\Lambda = 10^3$ GeV

Excluded regions, violation of:

$m_{\sigma 0} > H_{\text{inf}}$	blue
$m_\sigma(T_{\text{max}}) < H_{\text{inf}}$	green
$\theta_\star < 1$	red
$\Gamma_{\text{DM}} < H_0$	orange

$\alpha = 1/3 \qquad g_{\phi f f} = 10^{-2}$

Conclusion

An axion (or any scalar) which mixes kinetically with the inflaton can provide a good dark matter candidate even if the scale of inflation is lower than its mass!