Non-perturbative effects and resurgence in Jackiw–Teitelboim gravity at finite cutoff

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JT gravity a crash course

[Teitelboim 83, Jackiw 85]

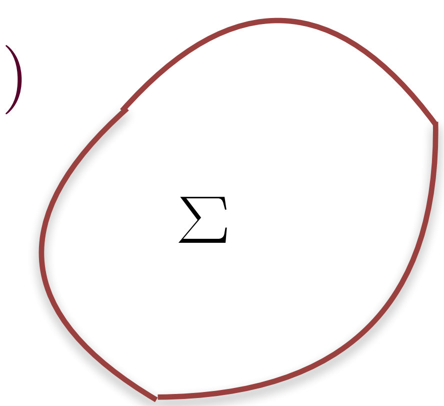
In two dimensions the Einstein-Hilbert action

$$I_{\rm EH} = \frac{1}{4\pi}\int_{\Sigma} {\rm d}x^2 \sqrt{g}\, R + \frac{1}{2\pi}\int_{\partial\Sigma} {\rm d}x \,\sqrt{h}\,\kappa$$

computes the Euler characteristic $\chi(\Sigma)=2-2g-n$ and thus does not provide any e.o.m for the metric.

To obtain non-trivial equation of motion we have to include a scalar d.o.f. (dilation field Φ)

$$I_{\rm JT} = -S_0\, I_{\rm EH} - \frac{1}{2}\int_{\Sigma} {\rm d}x^2 \sqrt{g}\,\phi(R+2) - \int_{\partial\Sigma} {\rm d}x\,\sqrt{h}\,\phi(\kappa-1)$$



$\partial\Sigma =$ boundary of Σ

κ is the extrinsic curvature of the boundary

S_0 topological coupling similar to g_s

Motivations:

- Dimensional reduction of s-wave 3d gravity ($\Lambda < 0$)
- Near-horizon limit of near-extremal higher-dimensional black holes
- Low-energy dynamics of the SYK model
- Solvable example of quantum gravity!

JT gravity a crash course

The eqs. of motion of the theory:

Φ: $$R + 2 = 0$$

$g_{\mu\nu}$: $$(\nabla_\mu \nabla_\nu - g_{\mu\nu})\phi = 0$$

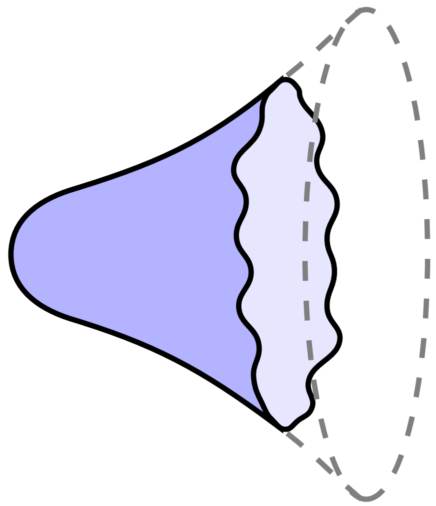
The dilaton field acts as a Lagrange multiplier fixing R = −2. In 2d fixing the scalar curvature is equivalent to fix the local geometry. In our case the local geometry is (E)AdS_2.

The solution can be anyway non-trivial, for instance we can have BH geometries:

$$ds^2 = (r^2 - r_h^2)dt^2 - \frac{dr^2}{r^2 - r_h^2}$$

Dirichlet boundary conditions: The path integral is performed over field configurations that obey two constraints:

1. We fix the length of the boundary of Σ to be l (fixing the metric along the boundary)
2. The dilation is taken to be constant along the boundary $\phi|_{\partial\Sigma} = \phi_b$



Quantizing the (euclidean) theory: We consider the simplest case the disk (g=0, n=1)

Our euclidean path integral runs over all the distinct way of embedding a non self intersecting circle (the boundary) in (E)AdS_2

Quantizing JT gravity

The case of infinite cut-off:.

When we perform the integration over the dilaton, we fix the metric to be (E)AdS$_2$.

$$ds^2 = \frac{dt^2 + dz^2}{z^2}$$

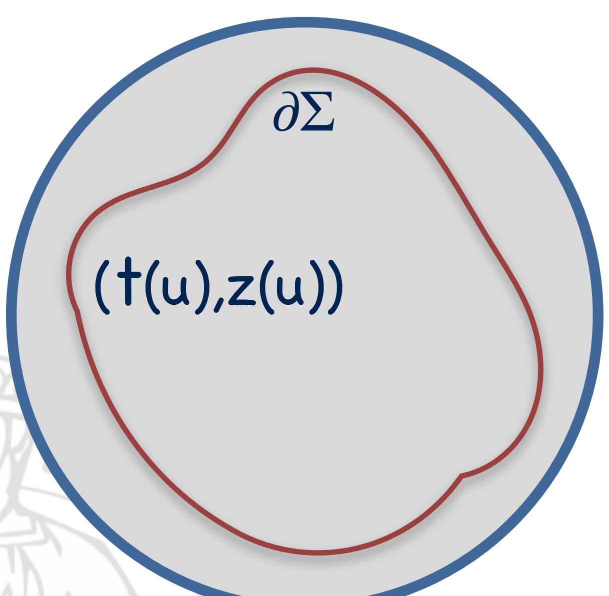
The AdS-conformal boundary of (E)AdS$_2$ is at z=0. We fix our boundary $\partial\Sigma$ along the curve (t(u),z(u)). We fix the induced metric along $\partial\Sigma$ as:

$$g_{uu} = \frac{1}{\epsilon^2} \quad \Longrightarrow \quad \ell = \frac{\beta}{\epsilon} \qquad (0 \leq u \leq \beta)$$

Next we fix the value of the dilation along $\partial\Sigma$:

$$\phi_\Sigma = \frac{\phi_r}{\epsilon}$$

The parameter ε controls the position Σ with respect to the asymptotic boundary of AdS$_2$



For ε approaching 0 the boundary $\partial\Sigma$ becomes closer and closer to the conformal boundary (blue circle).

We interpret 1/ε as a cut-off and **study first the theory in the limit ε→0**

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Quantizing JT gravity

In this limit we can easily solve the boundary condition: [Maldacena,Stanford '16]

$$g_{uu} = \frac{[t'(u)]^2 + [z'(u)]^2}{z^2(u)} = \frac{1}{\epsilon^2}$$

and find that

$$z(u) = \epsilon t'(u) + O(\epsilon^3)$$

If we substitute back into the original action the bulk term vanishes (the metric is locally AdS_2) and we left just with the boundary term:

$$\int_{\partial\Sigma} \mathrm{d}u \sqrt{h}\, \phi(\kappa - 1) = \int_0^\beta \mathrm{d}u \frac{\phi_r}{\epsilon^2}(1 + \epsilon^2 \mathrm{Sch}(t,u) + O(\epsilon^3) - 1) = \phi_r \int_0^\beta du\ \mathrm{Sch}(t,u) + O(\epsilon)$$

All the dynamics of the theory is carried by t(u) which is governed by a Schwarzian action. We can understand this result in terms of symmetries:

Asymptotic (E)AdS_2 geometries:
asymptotic symmetries
Diff(S^1)

Spontaneous breaking

t(u) Goldstone mode

(E)AdS_2 geometry:
Exact symmetry
SL(2,R)

[Mignemi, Cadoni '99]

$$t(u) \sim \frac{at(u) + b}{ct(u) + d}$$

Quantizing JT gravity

We remain with a path-integral over t(u) and each t(u) is weighted by the value of the Schwarzian action. Can we compute the partition function? (Disk partition function)

Stanford Witten [2017] have shown that the Schwarzian quantum mechanics is one-loop exact, namely localise. The partition function is

$$Z^{\text{disk}} = \sqrt{\frac{\phi_r^3}{2\pi\beta^3}} \exp\left(\frac{2\pi^2\phi_r}{\beta}\right)$$

This expression also admits the following integral representation:

$$Z = \int_0^\infty \mathrm{d}E\, \frac{\sinh(2\pi\sqrt{E})}{4\pi^2}\, e^{-\beta E}\,.$$

We have Φ_r=1/2 for convenience. This representation view the partition function as a thermal partition function where the temperature is β and the density of states is

$$\rho(E) = \frac{\sinh(2\pi\sqrt{E})}{4\pi^2}$$

Positive but not normalizable.

The holographic puzzle

Let us try to read this result in the language of AdS/CFT:

Naively the boundary theory is a quantum mechanics (Schwarzian theory). The partition function of the gravity on the disk should be the partition function of the quantum mechanics at finite temperature:

$$Z = \int_0^\infty \mathrm{d}E \, \frac{\sinh(2\pi\sqrt{E})}{4\pi^2} \, e^{-\beta E} = \mathrm{Tr}\left(e^{-\beta H_{QM}}\right) \; .$$

Hovewer a quantum mechanics where the $\mathrm{Tr}\left(e^{-\beta H_{QM}}\right)$ exists and it is finite must have a discrete spectrum! and thus cannot have a continuous density of states.

In other words, we cannot interpret the partition function of JT gravity over the disk as the partition function of a quantum mechanics. So what is the dual of JT?

JT is not dual to a specific quantum mechanics but to an ensemble of QM over which we have averaged! (For instance a large N limit of a matrix model)

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Going beyond the disk

Can we compute the partition function/correlators on generic surface of genus g with n boundaries.

We can consider:

- **Dirichlet boundaries:** they possess nontrivial dynamics described by the boundary Schwarzian action
- **Geodesic boundaries:** vanishing extrinsic curvature; the boundary action is trivial.

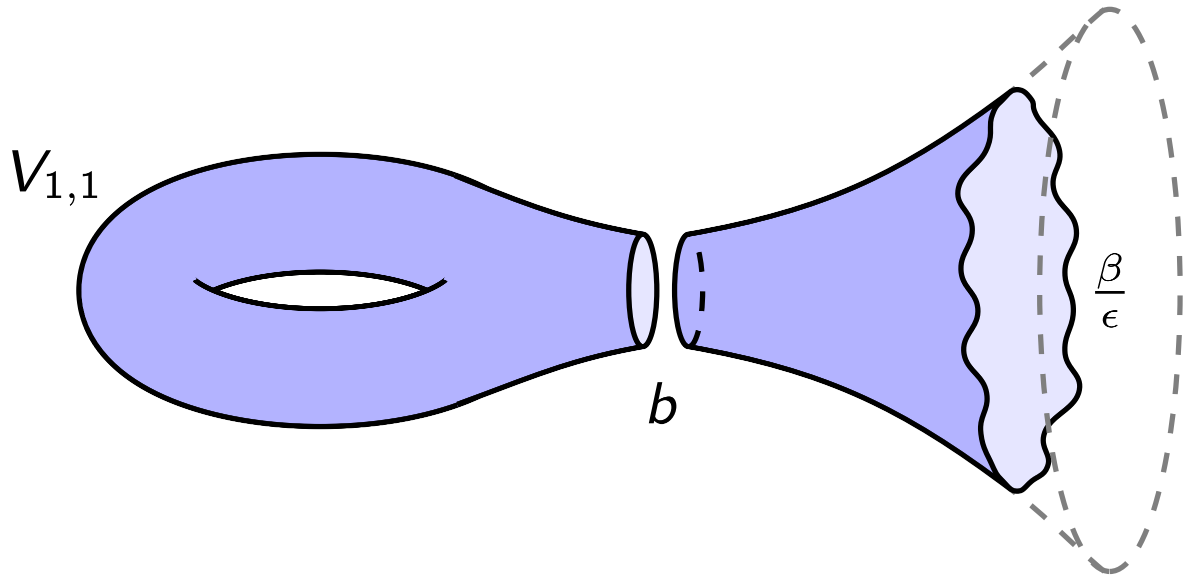
$$Z_{g,n} = \int \frac{[dg_{\mu\nu}]}{\mathrm{Vol(Diff)}} [d\phi] e^{-I_{JT}} = \int \frac{[dg_{\mu\nu}]}{\mathrm{Vol(Diff)}} \delta(R+2) e^{-I_{bdry}}.$$

For a given topology, if we have only geodesics boundaries, this integral counts the number of inequivalent hyperbolic Riemann surfavces, namely the volume of the moduli space of Riemann surfaces with Hyperbolice metric. [Weyl-Petersson volume]

The Weyl-Petersson volume $V_{g,n}$ $(b_1,\dots,b_n)$ is a polynomial of degree 3g-3+n in $b_1^2,\dots, b_n^2$. A general method, via recursion relation, was provided by [Mirzakhani]. So we can consider their expression as known.

The case of Dirichlet Boundaary

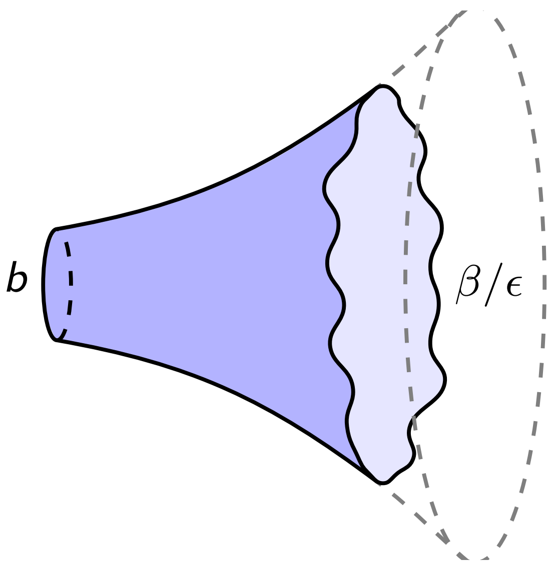
We consider an explicit example $Z_{1,1}$. La funzione di partizione a genus 1 con un boundary di Dirichlet. We can cut the surface in two parts by inserting a geodesic boundary of length b.



The first piece is a Riemann surface of genus 1 with a geodesic boundary of length b. Its contribution to the partition function is simply $V_{1,1}(b)$.

The second contribution is a surface with two boundaries one geodesic and one Dirichlet. We usually refer to this surface as **the trumpet.** The partition function of this surface can be computed similarly to the case of the disk:

$$Z^{\text{trumpet}} = \int_0^\infty \mathrm{d}E \, \frac{\cos(b\sqrt{E})}{2\pi\sqrt{E}} \, e^{-\beta E}$$



[Saad, Shenker, Stanford '19][Mertens, Turiaci '19]

The case of Dirichlet Boundary

We can glue together the two contribution to get $Z_{1,1}$ by integrating over the geodesic length b with the right measure

$$Z_{1,1}(\beta) = \int_0^\infty db\; b\; V_{1,1}(b) Z^{tumpet}(\beta, b)$$

In general with n Dirichlet boundaries

$$Z_{g,n}(\beta_1, \ldots, \beta_n) = \int_0^\infty \mathrm{d}b_1\; b_1 \ldots \int_0^\infty \mathrm{d}b_n\; b_n\, V_{g,n}(b_1, \ldots, b_n) \times Z^{\mathrm{trumpet}}(\beta_1, b_1) \ldots Z^{\mathrm{trumpet}}(\beta_n, b_n)\ ,$$

The amplitudes obtained in this way are fully quantum. None of these topologies are associated to classical solutions of eq. of motion (except disk and trumpet). Eq. of motion JT gravities implies the existence of a Killing vector; this surface in general do not have Killing vectors!

Summing over topologies:

Remember, there is a topological action (EH), that weights different topologies.

$$\langle Z(\beta_1, \ldots, \beta_n)\rangle = e^{(2-n)S_0} \sum_{g=0}^{\infty} e^{-2gS_0}\; Z_{g,n}(\beta_1, \ldots, \beta_n)$$

Matrix Model Origin of the Picture

Consider an integral over N x N Hermitian matrices

$$Z(V,N) = \int \frac{\mathrm{d}H}{\mathrm{vol}(\mathrm{U}(N))} \exp(-N \,\mathrm{tr}\, V(H)) \ .$$

Any matrix model admits a large N expansion of the form

$$\log Z(V,N) = \sum_{g=0}^{\infty} N^{2-2g} \, F_g(V) \ .$$

The fundamental quantities governing the matrix model are the resolvent and its Laplace transform:

$$R(E) = \mathrm{tr} \, \frac{1}{E-H} \ , \qquad Z(\beta) = \mathrm{tr}\, e^{-\beta H} \ .$$

from which we can define the density of eigenvalues:

$$\mathrm{Disc}\, R(E) = -2\pi i \rho(E) \ , \qquad \rho(E) = \sum_{i=i}^{N} \delta(E-\lambda_i) \ .$$

Double Scaling

We would like to view H as the putative Hamiltonian of our dual theory and Z(β) (the Laplace transform of the resolvent) as our gravitational partition function.

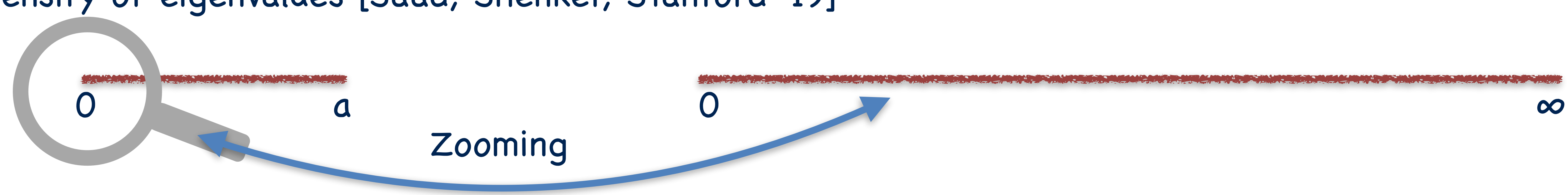
We are both averaging over the Hamiltonian with a potential V(H) and taking the large N limit to get a continuous spectrum.

We would like to reinterpret the disk partition function as

$$\langle Z(\beta)\rangle_0 = \int \mathrm{d}E \, \langle \rho(E)\rangle_0 \, e^{-\beta E}$$

But $\langle \rho_0(E)\rangle = e^{S_0} \sinh(2\pi\sqrt{E})/4\pi^2$ cannot be viewed as a usual density

Double scaling: We have to zoom around one of the endpoint of the interval supporting the density of eigenvalues [Saad, Shenker, Stanford '19]



We also scale the normalisation e^{S_0} in the same way as N.

Moreover, we have also to tune the potential in order to obtain the correct functional form of the eigenvalue density

Double scaling

On general ground, we can state that the correlator of the double scaled matrix model organised as series in e^{S_0}

$$\langle Z(\beta_1)\dots Z(\beta_n)\rangle_{\text{conn.}} = e^{(2-n)S_0}\sum_{g=0}^{\infty} e^{-2gS_0}\; Z_{g,n}(\beta_1,\dots,\beta_n)$$

$$\langle R(E_1)\dots R(E_n)\rangle_{\text{conn.}} = e^{(2-n)S_0}\sum_{g=0}^{\infty} e^{-2gS_0}\; R_{g,n}(E_1,\dots,E_n)$$

We don't need to worry about the precise details of the limit and of the form of the potential to compute these quantities in the matrix model because we can use Schwinger-Dyson-like identities (loop equations) to recursively compute any correlator starting from $R_{01}(E_1)$and $R_{02}(E_1,E_2)$.

Topological recursion formula: (Eynard,Orantin)

Define $E=-z^2$ and introduce the following objects:

Independent of the potential. Miracle of one-cut matrix model!

$$W_{0,1}(z_1) = -2i\pi\, z_1\, \rho(-z_1^2)\;,$$

$$W_{0,2}(z_1,z_2) = 4z_1 z_2\left(R_{0,2}(-z_1^2,-z_2^2) - \frac{1}{(z_1^2-z_2^2)^2}\right) = \frac{1}{(z_1-z_2)^2}$$

$$W_{g,n}(z_1,\dots,z_n) = (-2)^n z_1\dots z_n\, R_{g,n}(-z_1^2,\dots,-z_n^2)$$

Topological recursion relations

All other W 's can be obtained with this recursion formula

$$W_{g,n}(z_1,\ldots,z_n) = \operatorname*{Res}_{z\to 0} K(z_1,z)\left[W_{g-1,n+1}(z,-z,z_2,\ldots,z_n) + \sum_{\substack{h_1+h_2=g \\ I_1\cup I_2=J}}^{*} W_{h_1,1+|I_1|}(z,I_1)\, W_{h_2,1+|I_2|}(-z,I_2)\right]$$

The recursion kernel K that appears above is defined as

$$K(z_1,z) = \frac{1}{2[W_{0,1}(z)+W_{0,1}(-z)]}\int_{-z}^{z} \mathrm{d}z_2\, W_{0,2}(z_1,z_2)$$

This amplitudes computed starting from the matrix model coincide, at the perturbative level, with those compute from the path-integral with the topological Feynman-rule discussed previously

This is not a coincidence! As already stressed by Eynard and Orantin, the recursion relation computing the volume of Petersson moduli space obtained by Mirzakhani can be viewed as recursion in a suitable matrix model

The matrix model picture

The matrix model picture making sense also for finite N suggests a "possible ultraviolet completion" of JT gravity (non-unique). [Saad, Shenker, Stanford '19]

- **one-eigenvalue instanton:** This corresponds to configuration where one out of the N eigenvalues has been displaced away from the rest. [Similar to ZZ brane in Liouville gravity]. Geometrically, the spacetime is allowed to end at a new type of boundary associated to the location of the eigenvalue.

- The second type of effect corresponds to adding a "probe brane". This corresponds to consider the insertion of det(E-H) in the matrix integral. Geometrically, it corresponds to consider an infinite set of disconnected space-time. [Similar to FZZT brane in Liouville gravity]

JT gravity: TT deformation

- TT-deformation (integrable) of 2D CFTs is dual to a sharp radial cut-off in AdS_3 [McGough, Mezei, Verlinde '16]
- A dimensional reduction of the above duality relates a deformed Schwarzian theory to JT gravity at finite cutoff, at the classical level. [Gross, Kruthoff, Rolph, Shaghoulian '19]

Therefore investigating the TT-deformation of the Schwarzian theory provides a controlled framework where to explore finite cut-off effect in JT gravity. (Why is it interesting?)

The effect of the deformation is modify the Hamiltonian and thus the spectrum of the theory according to the equation

$$2\partial_t H = \frac{H^2}{2 - tH}$$

The solutions has two branches

$$H_\pm(t) = \frac{2}{t}\left(1 \mp \sqrt{1 - tE}\right)$$

however, only $H_+(t)$ reproduces the expected undeformed limit for $t \to 0$

$$Z = \int_0^\infty \mathrm{d}E \, \frac{\sinh(2\pi\sqrt{E})}{4\pi^2} \, \exp\left(\frac{2\beta}{t}\left(1 - \sqrt{1 - tE}\right)\right)$$

[Iliesiu, Kruthoff, Turiaci, Verlinde '20] argued that the connection with finite cut-off also holds at quantum level if we set $t=4\varepsilon^2$.

JT gravity: disk transeries

The integral

$$Z = \int_0^\infty \mathrm{d}E \, \frac{\sinh(2\pi\sqrt{E})}{4\pi^2} \, \exp\left(\frac{2\beta}{t}\left(1-\sqrt{1-tE}\right)\right)$$

is ill-defined diverges and the integrand becomes complex! The spectrum is not real! However if we formally expand in t the integrand around t=0, we obtained a real perturbative series in t

$$Z = \sum_{n=0}^{\infty} \omega_n(\beta)\, t^n + \sum_i e^{-S_i(\beta)/t} \sum_{n=0}^{\infty} \omega_n{}^{(i)}(\beta)\, t^n \,,$$

$$\omega_n(\beta) = \frac{(4\beta)^{-n-1}}{n!\sqrt{\pi^3\beta}}\, \Gamma\left(n-\frac{3}{2}\right) \Gamma\left(n+\frac{5}{2}\right) {}_1F_1\left(n+\frac{5}{2}; \frac{5}{2}-n; \frac{\pi^2}{\beta}\right)$$

The result carries instant corrections in t that appear as trans-series terms

Resurgence theory: the information on the instantonic corrections is encoded in the perturbative expansion.

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Borel resummation

We start from an asymptotic series

$$Z(t) = \sum_n \omega_n \, t^n \ , \qquad \qquad \omega_n \sim n!$$

1. Borel Transform

Define a new series with a finite radius of convergence

$$\mathcal{B}[Z](\zeta) = \sum_n \omega_n \, \frac{\zeta^n}{n!} \ .$$

This defines an analytic function around ζ=0.

2. Directional Laplace transform

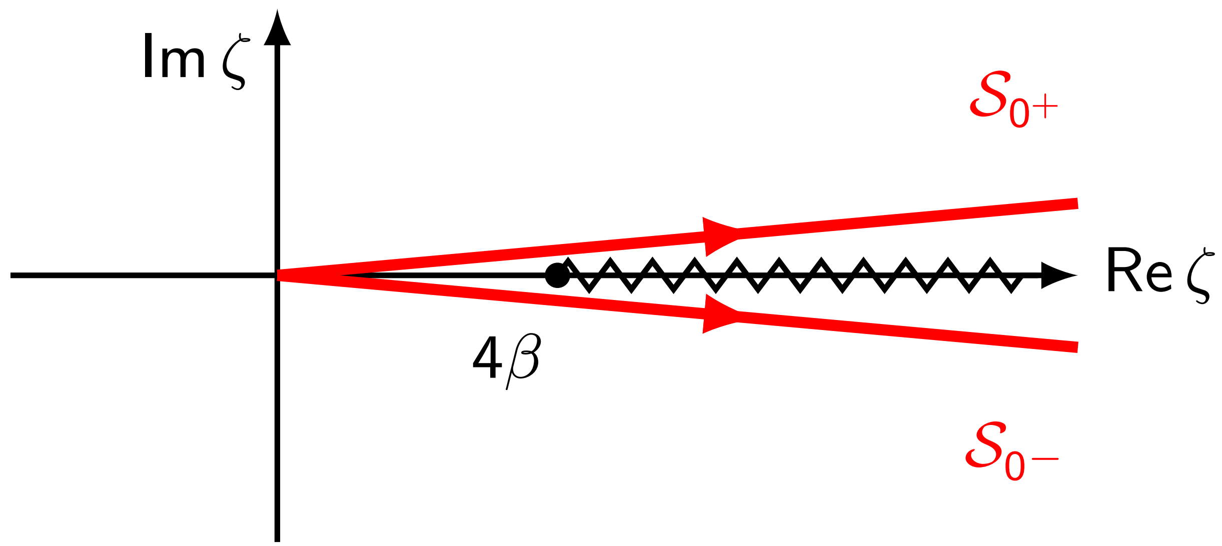
Take an integral transform along a chosen direction θ in the complex ζ-plane convergence

$$\mathcal{S}_\theta Z(t) = \frac{1}{t} \int_0^{e^{i\theta}\infty} \mathrm{d}\zeta \; e^{-\zeta/t} \, \mathcal{B}[Z](\zeta) \ ,$$

The directional resummation $S_\theta Z(t)$ defines an analytic function in the wedge $\mathrm{Re}(e^{-i\theta}t) > 0$ that, upon expansion in t, reproduces the asymptotic expansion of Z(t).

In our case, the Borel transform has a branch cut located on the positive real axis in the range $\zeta \in (4\beta, +\infty)$.

This is known as a Stokes line at $\theta = 0$. When taking a directional Laplace transform at $\theta = 0$, one runs into an ambiguity since the results obtained by approaching the Stokes line from above and below differ.



In the theory of resurgence, Stokes lines are associated with non-perturbative contributions, encoded by the discontinuity $(\mathcal{S}_{\theta+} - \mathcal{S}_{\theta-})Z(t)$

A real result is obtained by taking the median resummation

$$\mathcal{S}_{\text{med}} Z(t) = \frac{1}{2} \left(\mathcal{S}_{0+} + \mathcal{S}_{0-} \right) Z(t) \, .$$

Disk at finite t

Through resurgence, we have been able to fix the nonperturbative completions of the disk partition functions with just its perturbative expansions at t = 0 as input. The full result reads

$$Z = \frac{\beta}{2\sqrt{t}}\frac{e^{-2\beta/t}}{\beta^2+\pi^2 t}\, I_2\left(\frac{2}{t}\sqrt{\beta^2+\pi^2 t}\right) .$$

The trans-series has the following structure

$$Z = \sum_{n=0}^{\infty} \omega_n(\beta)\, t^n + e^{-4\beta/t}\sum_{n=0}^{\infty} \tilde{\omega}_n(\beta)\, t^n .$$

The non-perturbative corrections in t naturally carries information of the branch of the spectrum $H_-(t)$. [The branch not connected to the undeformed spectrum as t->0.]

The deformed spectrum

We can rewrite the expression for the disk partition function as

$$Z = \int_0^{1/t} \mathrm{d}E \, \frac{\sinh\left(2\pi\sqrt{E}\right)}{4\pi^2} \left(e^{-\beta H_+(t,E)} - e^{-\beta H_-(t,E)}\right)$$

The above differs from the naïve deformation in two ways:

1. the integration range is now capped at E = 1/t,
2. there is an additional term of instantonic origin.

We can recast the above as

$$Z = \int_0^{4/t} \mathrm{d}E \; \rho(E;t) \; e^{-\beta E} \; ,$$

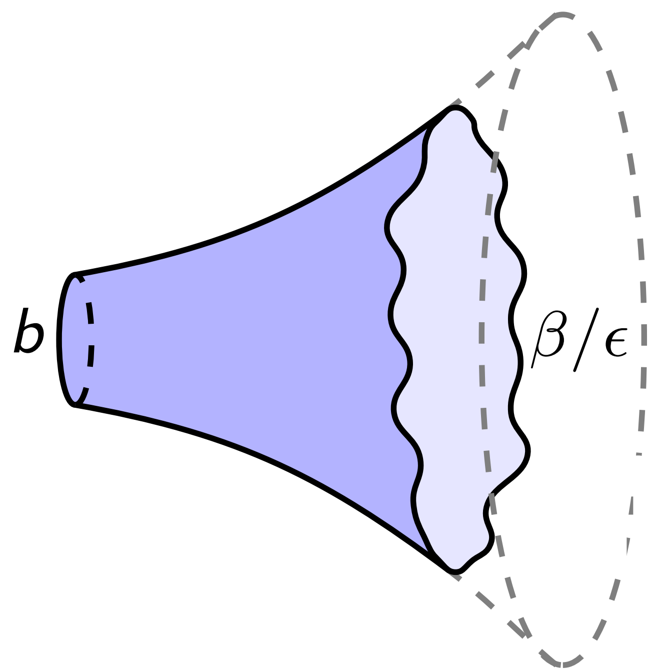
where the t-deformed density of states is given by

$$\rho(E;t) = \frac{1 - tE/2}{4\pi^2} \sinh\left(2\pi\sqrt{E(1 - tE/4)}\right)$$

The density is **negative** in the range (2/t,4/t).

Deformed trumpet

We can repeat the same steps done for the disk and find the deformed trumpet partition function

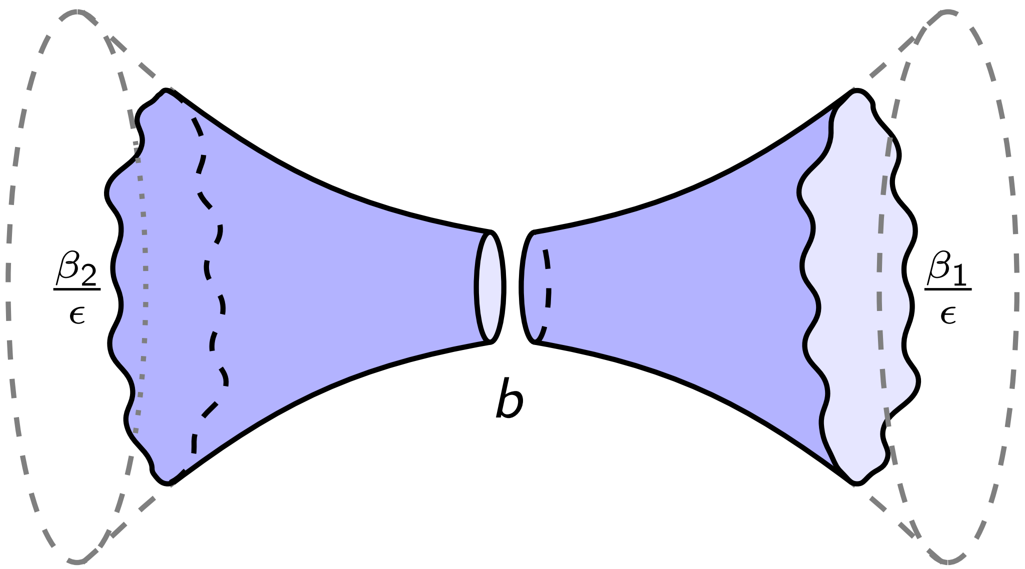


$$Z^{\text{trumpet}} = \frac{2\beta}{\sqrt{t}} \frac{e^{-2\beta/t}}{\sqrt{4\beta^2 - b^2 t}} I_1\left(\frac{1}{t}\sqrt{4\beta^2 - b^2 t}\right) .$$

We have reconstructed at finite t all the ingredient necessary at t=0 to build the perturbation expansion of the amplitudes via topological Feynman rules. **Can we consistently construct them?**

Caveat: The integration over the length b of the geodesic boundary also explores geodesic boundary which are bigger than the Dirichlet boundary!

Example: Cylinder at finite t



$$Z_{0,2}(\beta_1,\beta_2) = \int_0^\infty \mathrm{d}b\, b\, Z^{\text{trumpet}}(\beta_1, b)\, Z^{\text{trumpet}}(\beta_2, b)$$

By gluing two trumpets, we find the cylinder partition function:

$$\begin{aligned} Z_{0,2} &= \frac{\beta_1\beta_2\, e^{-2(\beta_1+\beta_2)/t}}{t(\beta_1^2-\beta_2^2)} \left[u_1\, I_0\left(\frac{2\beta_2}{t}\right)\, I_1\left(\frac{2\beta_1}{t}\right) - 2\beta_2\, I_0\left(\frac{2\beta_1}{t}\right)\, I_1\left(\frac{2\beta_2}{t}\right)\right] \\ &= \frac{1}{2\pi}\frac{\sqrt{\beta_1}\sqrt{\beta_2}}{(\beta_1+\beta_2)} + \frac{1}{52\pi}\frac{1}{\sqrt{\beta_1}\sqrt{\beta_2}}\, t + \frac{9}{1024\pi}\frac{\beta_1+\beta_2}{(\sqrt{\beta_1}\sqrt{\beta_2})^3}\, t^2 + O(t^3)\ . \end{aligned}$$

This amplitude was also studied in [Rosso '20] for $\beta_1=\beta_2$ By taking a Laplace transform we find the resolvent

$$\begin{aligned} R_{0,2} = &-\frac{t^2[(1-tE_1/2)^2+(1-tE_2/2)^2]}{4[(1-tE_1/2)^2-(1-tE_2/2)^2]^2} \\ &+\frac{t^2(1-tE_1/2)(1-tE_2/2)(tE_1^2/4+tE_2^2/4-E_1-E_2)}{4[(1-tE_1/2)^2-(1-tE_2/2)^2]^2\sqrt{-E_1(1-tE_1/4)}\sqrt{-E_2(1-tE_2/4)}}\,. \end{aligned}$$

Topological recursion relations at finite t

Again the amplitudes constructed at finite t can be derived by the topological recursion relations. The deformed ingredients are

$$W_{0,1}(z_1) = \frac{z_1(2+tz_1^2)}{4\pi}\,\sin\left(\pi z_1\sqrt{4+tz_1^2}\right)$$

$$W_{0,2}(z_1,z_2) = \frac{4(2+tz_1^2)\,(2+tz_2^2)}{(z_1^2-z_2^2)^2\,[4+t(z_1^2+z_2^2)]^2}\left(2z_1z_2+\frac{4(z_1^2+z_2^2)+t(z_1^4+z_2^4)}{\sqrt{4+tz_1^2}\sqrt{2+tz_2^2}}\right)$$

$$K(z_1,z) = \frac{(2+tz_1^2)\sqrt{4+tz^2}}{(2+tz^2)\sqrt{4+tz_1^2}}\,\frac{4\pi\csc(\pi z\sqrt{4+tz^2})}{(z_1^2-z^2)\,[4+t(z_1^2+z^2)]}$$

It is easy to show that this is a simple deformation of the Mirzakhani-Eynard-Orantin recursion formula.

Example:

$$W_{1,1}(z_1) = \operatorname*{Res}_{z\to 0} K(z_1,z)\,W_{0,2}(z,-z)$$
$$= \frac{(2+tz_1^2)\,[6+\pi^2 z_1^2\,(4+tz_1^2)]}{3z_1^4\,(4+tz_1^2)^{5/2}}\,.$$

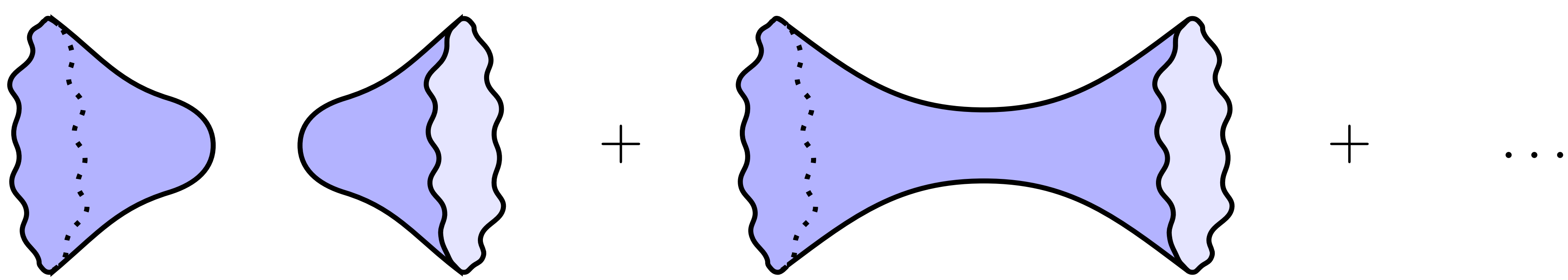
Spectral form factor

At t = 0, JT gravity was observed to reproduce the characteristic shape of a spectral form factor associated with an ensemble of Hamiltonians with random-matrix statistics. [Saad, Shenker, Stanford '19]

From a bulk perspective, the spectral form factor can be interpreted as a transition amplitude in the Hilbert space of two copies of JT gravity. [Saad '19]

It is computed by the analytic continuation of two boundaries, $\beta_1 \to \beta + \tau$, $\beta_2 \to \beta + \tau$, which introduces a timescale τ . The quantity includes terms coming from different topologies, each weighted by the usual topological factor,

$$F = e^{2S_0}$$

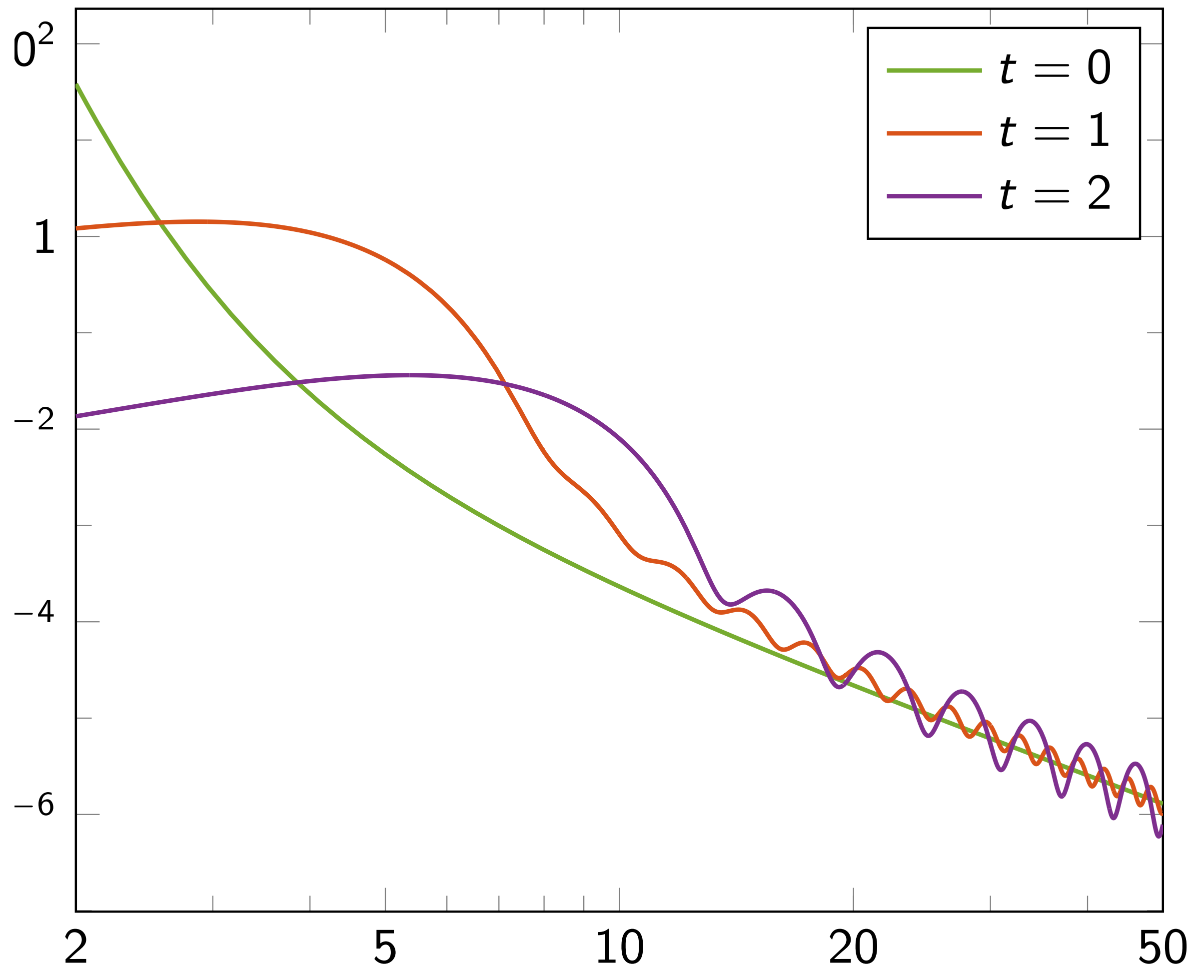


where the dots correspond to subleading terms associated with higher-genus topologies.

Different features of the spectral form factor are associated with contributions coming from different topologies.

Slope

The initial "slope" region comes from considering two disjoint disks. Its characteristic shape can be observed by looking at its large-τ regime,

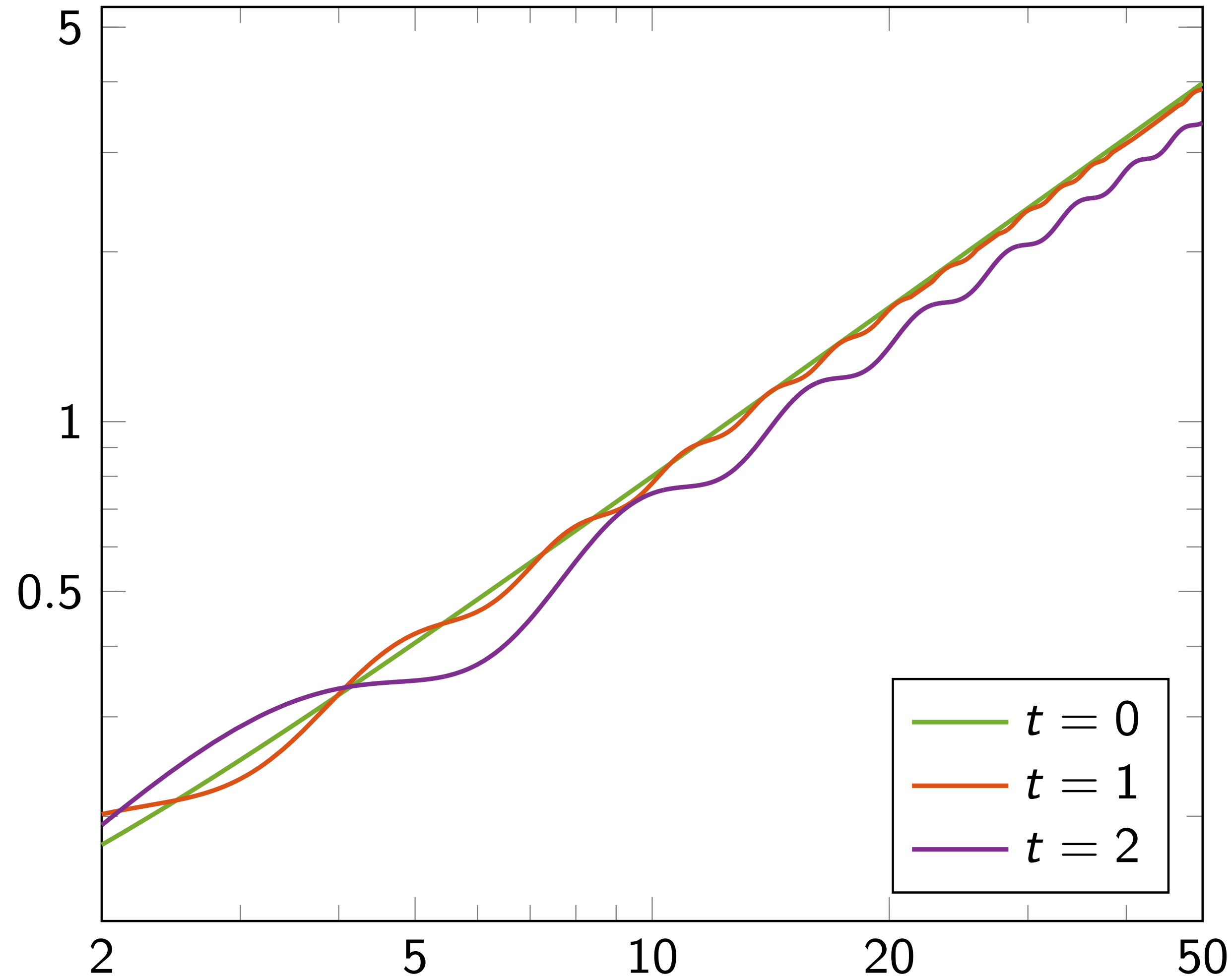


$$Z_{0,1}(\beta + \mathrm{i}\tau; t)\, Z_{0,1}(\beta - \mathrm{i}\tau; t) \sim \frac{1}{16\pi\tau^3}\left(1 - e^{-8\beta/t}\right) + \frac{e^{-4\beta/t}}{8\pi\tau^3}\,\sin\!\left(\frac{4\tau}{t}\right) .$$

The first term gives a cubic decay that reproduces the known $t \to 0$ limit, while the second term is an oscillation of period $\pi t/2$ whose amplitude is exponentially suppressed in $1/t$.

Ramp

Eventually, the slope phase will end, and other topologies will dominate the form factor. The characteristic "ramp" region comes from the connected topology, a Euclidean wormhole connecting the two boundaries.



$$Z_{0,2}(\beta + \mathrm{i}\tau, \beta - \mathrm{i}\tau; t) \sim \frac{\tau}{4\pi\beta}\left(1 - e^{-8\beta/t}\right) - \frac{e^{-4\beta/t}}{2\pi} \cos\left(\frac{4\tau}{t}\right) .$$

The transition time is

$$\tau \sim (\beta/4)^{1/4} e^{S_0/2}.$$

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Conclusions

Summary of the result at finite t:

- Resurgence gives a prescription to determine the nonperturbative corrections to the disk and the trumpet partition functions.
- The Weil–Petersson gluing leads to analytical results for any topology.
- The results satisfy a topological recursion formula which is a deformation of the Mirzakhani–Eynard–Orantin formula.

Outlooks:

- Derive the formulas for arbitrary topologies from a first-principles path-integral analysis, including the integration measure.
- Characterize the 1d side of the duality captured by the deformed topological recursion formula.
- Extend the finite-cutoff results to related models, e.g. JT supergravity. [Stanford, Witten '19]