# SMEFT and Anomalies

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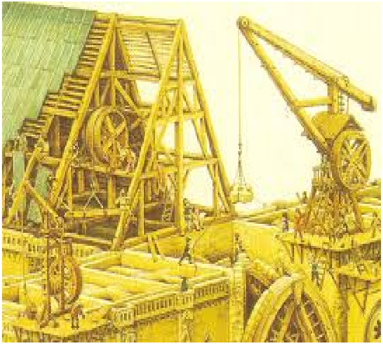
Despite the striking fact that a large number of scientists are working , the vast stretches of the unknown and the unanswered and the unfinished still far outstrip our collective comprehension.

*OK, what do we do in the meantime?*

Friends, lend me your ears I come to praise EFT, not a model (or approximation), but a sequence of low-energy effective actions $\mathrm{S}_{\mathsf{eff}}(\Lambda)$, for all $\Lambda < \infty$, from SMEFT to GRSMEFT *

> The problem is not how to imagine wild scenarios, the problem is how to arrive to the correct scenario by making only small steps, without having to make unreasonable assumptions.

*A theory is aimed at a generalized statement aimed at explaining a phenomenon. A model, on the other hand, is a purposeful representation of reality.



When people say ***by construction the SMEFT is always a valid QFT*** what do they mean?

Among other things, we need to make the SMEFT $\mathrm{S}$-matrix UV (and IR) finite, including $\mathrm{dim} = 6$ operators and, at least, $\mathrm{dim} = 8$ operators (truncation uncertainty). The verification of any **claim** with **explicit computations** is of importance.

We will not discuss IR/coll, unitarity, stability and Ostrogradsky ghosts. We will discuss EFT gauge anomalies and anomaly cancellation; perhaps, a deeper understanding of SMEFT is required, is SMEFT a low-energy limit of an underlying anomaly-free theory?


I do not make any warranties about the completeness of this information

- Ward:1950xp, Takahashi:1957xn, Taylor:1971ff, Slavnov:1972fg,

  Veltman:1988au, Sterling:1981za, Preskill:1990fr, Hartmann:2001zz,

  Grzadkowski:2010es, Costello2011,
  Cata:2020crs,Bonnefoy:2020tyv,Feruglio:2020kfq

- We briefly review the **SMEFT Lagrangian**: consider the standard model, described by a Lagrangian $\mathscr{L}^{(4)}_{\mathrm{SM}}$ with a symmetry group $\mathrm{G} = SU(3) \times SU(2) \times U(1)$. The SMEFT extension is described by a Lagrangian

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}^{(4)}_{\mathrm{SM}} + \sum_{d>4} \sum_{i} \frac{\mathrm{a}^{d}_{i}}{\Lambda^{4-d}} \mathscr{O}^{(d)}_{i} ,$$

- where $\Lambda$ is the cutoff of the effective theory, $\mathrm{a}^{d}_{i}$ are Wilson coefficients and $\mathscr{O}^{(d)}_{i}$ are $\mathrm{G}$-invariant operators of mass-dimension $d$ involving the $\mathscr{L}^{(4)}_{\mathrm{SM}}$ fields. In this talk we will use the so-called "Warsaw basis".

# SMEFT and renormalization

- With SMEFT we have lost **strict renormalizability** but this should not come at the price of loosing computability; whether the predictions of a theory are matched by Nature is a completely different matter and can be decided only by comparing the predictions with experiment (calculability).
- A **renormalizable theory** is determined by a fixed number of parameters; once these are determined (after finite renormalization) we can make definite predictions at a fixed accuracy. An **EFT theory** requires at higher and higher energies **more and more** counterterms; the asymptotic expansion in $\mathrm{E}/\Lambda$ may break down completely above some scale. Given a truncated expansion, we still have a large family of UV-complete theories with these low order terms, which have different behavior at higher energies.
- Therefore, it is crucial to prove that our EFT is closed under renormalization, order-by-order in the asymptotic expansion, although the number of counterterms will grow with the order (the predictive power is lost at scales approaching the cutoff).


Our goal is to consider the SMEFT with its WTST [†] identities

- in these calculations a certain amount of $\gamma$-matrix manipulation is unavoidable and we must specify the regularization scheme to be used, in particular, we must specify how to treat $\gamma^5$. We will use the scheme developed by Veltman. Therefore, $\gamma^\mu, \gamma^5$, and $\varepsilon^{\mu\nu\alpha\beta}$ are formal objects where

$$
\begin{aligned}
\{\gamma^\mu\,,\gamma^\nu\} = 2\,\delta^{\mu\nu}\,\mathrm{I} &\qquad \mathrm{Tr}\,\mathrm{I} = 4\,, \\
\delta^{\mu\nu} = \delta^{\bar\mu\bar\nu} + \delta^{\hat\mu\hat\nu},\ \ \delta^{\bar\mu\bar\mu} = 4, &\qquad \delta^{\hat\mu\hat\mu} = \mathrm{d} - 4, \\
\delta^{\mu\alpha}\,\delta^{\alpha\nu} &= \delta^{\bar\mu\bar\alpha}\,\delta^{\bar\alpha\bar\nu} + \delta^{\hat\mu\hat\alpha}\,\delta^{\hat\alpha\hat\nu}\,,
\end{aligned}
\tag{1}
$$

where $\mathrm{d}$ is the space-time dimension.

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[†] Ward, Takahashi, Slavnov, Taylor

We have the following relations:

$$\mathrm{Tr}\left(\cdots\gamma^{\alpha}\cdots\gamma^{\beta}\cdots\right)=\mathrm{Tr}\left(\cdots\gamma^{\bar{\alpha}}\cdots\gamma^{\bar{\beta}}\cdots\right)+\mathrm{Tr}\left(\cdots\gamma^{\hat{\alpha}}\cdots\gamma^{\hat{\beta}}\cdots\right),$$

- where the dots indicate strings of four-dimensional gamma matrices and also $\gamma^5$. The second trace in the r.h.s. is computed according to the following rules:
  1. move all the $\gamma^{\hat{\mu}}$ matrices to the right using $\gamma^{\hat{\mu}}\gamma^{\bar{\nu}}=-\gamma^{\bar{\nu}}\gamma^{\hat{\mu}}$,
  2. for a trace containing an odd number of $\gamma^5$ matrices use $\gamma^{\hat{\mu}}\gamma^5=\gamma^5\gamma^{\hat{\mu}}$,
  3. for a trace containing an even number of $\gamma^5$ matrices use $\gamma^{\hat{\mu}}\gamma^5=-\gamma^5\gamma^{\hat{\mu}}$.

$$\boxed{\mathrm{Tr}\left(\cdots\gamma^{\hat{\alpha}}\gamma^{\hat{\beta}}\cdots\right)=\mathrm{Tr}\left(\cdots\right)\mathrm{Tr}\left(\gamma^{\hat{\alpha}}\gamma^{\hat{\beta}}\right)}$$

$\mathscr{G}$

Consider the SMEFT (not strictly renormalizable); if there were no anomalies, all the UV divergences could be cancelled in $\mathrm{S}^{\text{eff}}(\Lambda)$, order-by-order in $1/\Lambda$.

1. Due to the beakdown of WTST identities at the one-loop level, the symmetry will be broken and the mechanism of cancelling divergences is disturbed.
2. For example one relevant identity concerns the amplitude for $\mathrm{Z} \to \gamma\gamma$: if the WTST identity is violated we can still restore it by introducing a UV-finite counterterm as it would be the case in the SM with only an electron and a neutrino.
3. However, this new term is of non-renormalizable type giving rise to infinities at higher orders

The "complete" SM is an anomaly-free theory and we want to investigate the SMEFT from the following point of view: are there **anomalies in the SMEFT**?

# LEGENDA

- Consider the following amplitudes:

$$\mathrm{A}^{\mathrm{Z}}_{\mu\,;\,\nu_1,\dots,\nu_n}(p\,;\,q_1\,\dots,q_n\,;\,k_1\,,\dots,k_m)\,,$$
$$\mathrm{A}^{\phi}_{\nu_1,\dots,\nu_n}(p\,;\,q_1\,\dots,q_n\,;\,k_1\,,\dots,k_m)\,, \tag{2}$$

  involving a $\mathrm{Z}$-boson (or a $\phi^0$ Higss-Kibble ghost) of momentum $p$, $n$ gauge bosons $(\mathrm{A}, \mathrm{Z}, \mathrm{W})$ and $m$ Higgs bosons (all momenta are flowing inwards).
- The corresponding **WTST identity** is

$$\boxed{\Gamma_{\nu_1,\dots,\nu_n} \;=\; i\,p^{\mu}\,\mathrm{A}^{\mathrm{Z}}_{\mu\,;\,\nu_1,\dots,\nu_n} + \mathrm{M_Z}\,\mathrm{A}^{\phi}_{\nu_1,\dots,\nu_n} \;=\; 0\,.}$$

- We will study 2 different schemes:

  **S1)** $\mathrm{Z}$ and $\phi^0$ off-shell, remaining gauge bosons coupled to physical sources, i.e. $\partial_\mu\,\mathrm{J}_\mu = 0$, anti-commuting $\gamma^5$;

  **S2)** $\mathrm{Z}$ and $\phi^0$ off-shell, remaining gauge bosons coupled to physical sources, i.e. $\partial_\mu\,\mathrm{J}_\mu = 0$, Veltman-prescription for $\gamma^5$, four-dimensional, on-shell, external momenta.

FINE POINTS

- Each one-loop amplitude can be decomposed as follows:

$$\mathrm{A} = \frac{\mathrm{S}}{\mathrm{d}-4} + \mathrm{R} + \sum_{\mathrm{a}} \mathrm{f}_{\mathrm{a}}\, \mathrm{A}_0^{\mathsf{fin}}(\mathrm{a}) + \sum_{\mathrm{b}} \mathrm{f}_{\mathrm{b}}\, \mathrm{B}_0^{\mathsf{fin}}(\mathrm{b}) + \sum_{\mathrm{c}} \mathrm{f}_{\mathrm{c}}\, \mathrm{C}_0(\mathrm{c}) + \sum_{\mathrm{d}} \mathrm{f}_{\mathrm{d}}\, \mathrm{D}_0(\mathrm{d}),$$

  where $\mathrm{A}_0, \ldots \mathrm{D}_0$ are scalar one, ... four point functions, $\mathrm{d}$ is the space-time dimension and “fin” denotes the UV finite part.

- It is worth noting that anomalies can be cancelled by adding counterterms if and only if the **violation of WTST identities** is of $\mathrm{R}$ **type**, i.e. UV finite and local. Locality of the UV-finite counterterms is related to the unitarity of the theory.


- Anomalies and anomalous terms, although correlated, are not the same thing. Anomalies have to do with WTST identities, perhaps the best example is given by

$$\Gamma_{\alpha\beta} = i p^{\lambda}\, \mathrm{A}^{\mathrm{Z}}_{\lambda\,;\,\alpha\,,\beta} + \mathrm{M}_{\mathrm{Z}}\, \mathrm{A}^{\phi}_{\alpha\,,\beta}\,,$$

  the $\mathrm{ZAA}$ WTST identity. If this identity is violated, say

$$\Gamma_{\alpha\beta} = \mathrm{X}\, \varepsilon_{\mu\nu\alpha\beta}\, p_1^{\mu}\, p_2^{\nu}\,,$$

- with $\mathrm{X}$ UV-finite and local, we can restore it by adding to the Lagrangian a term $\mathrm{X}\, \varepsilon_{\mu\nu\alpha\beta}\, \phi^0\, (\partial_\mu\, \mathrm{A}_\alpha)\, (\partial_\nu\, \mathrm{A}_\beta)$.
- One could think of introducing two counterterms, $\mathrm{ZAA}$ and $\phi^0\mathrm{AA}$, in order to restore the identity and to cancel, at the same time, the anomalous $\mathrm{ZAA}$ and $\phi^0\mathrm{AA}$ couplings. However, these counterterms are not local.
- Having loop-induced anomalous couplings is not a surprise, even in the SM. The important thing is the **absence of anomalies**, not the **presence of anomalous couplings**.

## NOTATIONS

- $g$ is the $SU(2)$ coupling constant, $c_\theta = \mathrm{M_W}/\mathrm{M_Z}$ is the cosine of the weak-mixing angle. Furthermore, we introduce the following combinations,

$$
\begin{aligned}
a_{\mathrm{l\,W}} &= s_\theta\, a_{\mathrm{l\,WB}} + c_\theta\, a_{\mathrm{l\,BW}}\,, & a_{\mathrm{l\,B}} &= -c_\theta\, a_{\mathrm{l\,WB}} + s_\theta\, a_{\mathrm{l\,BW}}\,,\\
a_{\mathrm{d\,W}} &= s_\theta\, a_{\mathrm{d\,WB}} + c_\theta\, a_{\mathrm{d\,BW}}\,, & a_{\mathrm{d\,B}} &= -c_\theta\, a_{\mathrm{d\,WB}} + s_\theta\, a_{\mathrm{d\,BW}}\,,\\
a_{\mathrm{u\,W}} &= s_\theta\, a_{\mathrm{u\,WB}} + c_\theta\, a_{\mathrm{u\,BW}}\,, & a_{\mathrm{u\,B}} &= c_\theta\, a_{\mathrm{u\,WB}} - s_\theta\, a_{\mathrm{u\,BW}}\,,
\end{aligned}
$$

  where $a_{\mathrm{l\,W}}$, etc. are Wilson coefficients in the Warsaw basis.

- It is worth noting that in SMEFT we have both triangles and bubbles due to four-point vertices like $\mathrm{A}\phi^0\bar{\mathrm{f}}\mathrm{f}$ etc.. If they are not included, the anomaly contains an UV-divergent term.

$\mathcal{G}$


$$\mathrm{C}^{\mathrm{A}} = -\partial_\mu \mathrm{A}_\mu \,, \;\; \mathrm{C}^{\mathrm{Z}} = -\partial_\mu \mathrm{Z}_\mu + \mathrm{M}_{\mathrm{Z}} \, \phi^0 \,, \;\; \mathrm{C}^{\pm} = -\partial_\mu \mathrm{W}^{\pm}{}_{\mu} + \mathrm{M}_{\mathrm{W}} \, \phi^{\pm} \,.$$

- In the **diagrammatic** language the validity of the WTST identities is equivalent to the statement that the $\mathrm{C}$ are free fields and any Green's function with one or more external $\mathrm{C}$-sources is zero.

- In the **functional** language, consider the effective action $\mathrm{S}$, observe that diagrams determine $\mathrm{S}$ only up to an arbitrary choice of local counterterms and we are free to redefine $\mathrm{S}$ by adding to it $\mathrm{S}_{\mathrm{ct}}$ with an arbitrary coefficent.


## ❍ WTST identity for $\mathrm{C^Z AA}$

- In this case we have $\mathrm{C^Z}(P) \to \mathrm{A}_\alpha(p_1) + \mathrm{A}_\beta(p_2)$. Summing over fermion generations we obtain

$$\Gamma_{\alpha\beta} = \frac{g^3}{8\pi^2}\,\frac{s_\theta}{c_\theta^3}\,g_6\,\varepsilon_{\mu\nu\alpha\beta}\,p_1^\mu\,p_2^\nu \sum_{\{\mathrm{g}\}} \left( \frac{m_{\mathrm{l}}^2}{M_{\mathrm{Z}}^2}\,a_{\mathrm{l\,WB}} + \frac{m_{\mathrm{d}}^2}{M_{\mathrm{Z}}^2}\,a_{\mathrm{d\,WB}} + 2\,\frac{m_{\mathrm{u}}^2}{M_{\mathrm{Z}}^2}\,a_{\mathrm{u\,WB}} \right),$$

  for schemes 1 and 2.

- As expected there is no anomaly in $\dim = 4$ but **there is one** in $\dim = 6$ which is mass dependent. The standard treatment is that **the anomaly can be removed** by adding to the Lagrangian a term proportional to $\varepsilon_{\mu\nu\alpha\beta}\,\phi^0\,\partial^\mu \mathrm{A}^\alpha\,\partial^\nu\,\mathrm{A}^\beta$ .

## WTST identity for $\mathrm{C^{Z}ZA}$

- In this case we have $\mathrm{C^{Z}}(P) \to \mathrm{Z}_{\alpha}(p_1) + \mathrm{A}_{\beta}(p_2)$ and obtain

$$
\begin{aligned}
\Gamma^{\mathrm{S1}}_{\alpha\beta} \quad &= \quad \frac{g^3}{32\,\pi^2}\,g_6\,\varepsilon_{\mu\nu\alpha\beta}\,p_1^{\mu}\,p_2^{\nu}\,\sum_{\{\mathrm{g}\}}\Big[2\,\frac{s_\theta}{c_\theta^3}\,\Big(\frac{m_{\mathrm{l}}^2}{M_{\mathrm{Z}}^2}\,a_{\mathrm{l\,BW}} + \frac{m_{\mathrm{d}}^2}{M_{\mathrm{Z}}^2}\,a_{\mathrm{d\,BW}} + 2\,\frac{m_{\mathrm{u}}^2}{M_{\mathrm{Z}}^2}\,a_{\mathrm{u\,BW}}\Big) \\
&+ \quad \frac{1}{c_\theta^4}\,\Big(\frac{m_{\mathrm{l}}^2}{M_{\mathrm{Z}}^2}\,\mathrm{v_l}\,a_{\mathrm{l\,WB}} + 3\,\frac{m_{\mathrm{d}}^2}{M_{\mathrm{Z}}^2}\,\mathrm{v_d}\,a_{\mathrm{d\,WB}} + 3\,\frac{m_{\mathrm{u}}^2}{M_{\mathrm{Z}}^2}\,\mathrm{v_u}\,a_{\mathrm{u\,WB}}\Big) \\
&+ \quad \frac{4}{3}\,\frac{s_\theta}{c_\theta^2}\,\Big(3\,a^{(3)}_{\phi\mathrm{q}} + a^{(1)}_{\phi\mathrm{q}} - 8\,a_{\phi\mathrm{u}} - 2\,a_{\phi\mathrm{d}} - 3\,a^{(3)}_{\phi\mathrm{l}} + 3\,a^{(1)}_{\phi\mathrm{l}} - 6\,a_{\phi\mathrm{l}}\Big) \\
&- \quad \frac{8}{3}\,s_\theta\,\Big(3\,a^{(3)}_{\phi\mathrm{q}} + 5\,a^{(1)}_{\phi\mathrm{q}} - 4\,a_{\phi\mathrm{u}} - a_{\phi\mathrm{d}} - 3\,a^{(3)}_{\phi\mathrm{l}} + 3\,a^{(1)}_{\phi\mathrm{l}} - 3\,a_{\phi\mathrm{l}}\Big)\Big]\,,
\end{aligned}
$$

$\mathscr{G}$

$$
\begin{aligned}
\Gamma^{\mathrm{S2}}_{\alpha\beta} \quad &= \quad \frac{g^3}{64\,\pi^2}\,g_6\,\varepsilon_{\mu\nu\alpha\beta}\,p_1^{\mu}\,p_2^{\nu}\,\sum_{\{\mathrm{g}\}}\Big[4\,\frac{s_\theta}{c_\theta^3}\,\big(\frac{m_{\mathrm{l}}^2}{M_{\mathrm{Z}}^2}\,a_{\mathrm{l\,BW}} + \frac{m_{\mathrm{d}}^2}{M_{\mathrm{Z}}^2}\,a_{\mathrm{d\,BW}} + 2\,\frac{m_{\mathrm{u}}^2}{M_{\mathrm{Z}}^2}\,a_{\mathrm{u\,BW}}\big) \\
&+ \quad \frac{1}{c_\theta^4}\,\big(\frac{m_{\mathrm{l}}^2}{M_{\mathrm{Z}}^2}\,\mathrm{v_l}\,a_{\mathrm{l\,WB}} + 3\,\frac{m_{\mathrm{d}}^2}{M_{\mathrm{Z}}^2}\,\mathrm{v_d}\,a_{\mathrm{d\,WB}} + 3\,\frac{m_{\mathrm{u}}^2}{M_{\mathrm{Z}}^2}\,\mathrm{v_u}\,a_{\mathrm{u\,WB}}\big) \\
&- \quad 4\,\frac{s_\theta}{c_\theta^2}\,\big(a^{(3)}_{\phi\mathrm{q}} + 3\,a^{(1)}_{\phi\mathrm{q}} + 2\,a_{\phi\mathrm{u}} + a_{\phi\mathrm{d}} - a^{(3)}_{\phi\mathrm{l}} + a^{(1)}_{\phi\mathrm{l}} + a_{\phi\mathrm{l}}\big)\Big]\,,
\end{aligned}
$$

- where $\mathrm{v_f} = 1 - 8\,\mathrm{Q_f}\,\mathrm{I^3_f}\,s_\theta^2$. There is an anomaly in $\dim = 6$ which is mass dependent but UV finite and local; however **the anomaly is scheme dependent**.


For the full list of results see Acta Phys.Polon.B 52 (2021) 533
e-Print: 2104.13569 [hep-ph]

- A technical remark is needed: for amplitudes having a Born term we must take into account the relation between bare and renormalized parameters and also include Dyson-resummation of the propagators (when needed). As an example:

$$i\, p^{\lambda}\, \mathrm{A}^{\mathrm{W}}_{\lambda\alpha\beta} + \mathrm{M}\ \mathrm{A}^{\phi}_{\alpha\beta} = 0\,,$$

  - where $\mathrm{M}$ is the **bare** $\mathrm{W}$ mass and we use $\mathrm{M}_0 = \mathrm{M}/c_{\theta}$ for the **bare** $\mathrm{Z}$ mass.

    1. Inside and in front of loops we will use the **on-shell masses**, $\mathrm{M}_{\mathrm{W}}$ and $\mathrm{M}_{\mathrm{Z}}$.
    2. However, in this case there is a lowest order where the relation between **bare** and **on-shell** masses **must be corrected** at $\mathscr{O}(g^2)$, involving the corresponding self-energies.

- There are SMEFT anomalies, UV-finite, local, mass-dependent and **scheme-dependent**. There are different options, for instance use consistently a scheme, e.g. the (naive) anti-commuting $\gamma^5$ scheme or the Veltman scheme and introduce counterterms.
- Take the SM with one electron and one neutrino: due to the anomaly, the WTST indentities break down at the one-loop level. We then introduce **counterterms** and the identities are **restored** but the terms are of a non-renormalizable nature and they give rise to infinities, at the earliest at the two-loop level.
- "Alternatively", we could cancel the anomalies by imposing constraints. With the anti-commuting $\gamma^5$ scheme we obtain the following relations:

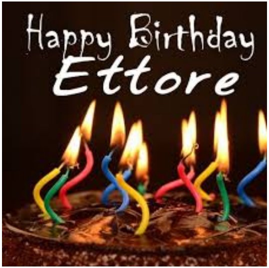
$$a_{\mathrm{f\,B}} = a_{\mathrm{f\,W}} = 0 \quad \forall \mathrm{f}\,, \quad a_{\mathrm{u\,g}} = a_{\mathrm{d\,g}} = 0\,,$$

$$a^{(3)}_{\phi \mathrm{l}} = a^{(3)}_{\phi \mathrm{q}}\,,\; a^{(1)}_{\phi \mathrm{l}} = -3\, a^{(1)}_{\phi \mathrm{q}}\,,\; a_{\phi \mathrm{l}} = -\frac{1}{3}\left(4\, a^{(1)}_{\phi \mathrm{q}} + 4\, a_{\phi \mathrm{u}} + a_{\phi \mathrm{d}}\right).$$

- Is it a good idea?
- No, it is not. Anomalies are defined as what's left after adding all possible counterterms. If a residue remains then we have a true anomaly.

***We choose to go beyond the SM straits. We choose to do that and do the other things, not because they are easy, but because they are hard, because that goal will serve to organize and measure the best of our energies and skills, because that challenge is one that we are willing to accept, one we are unwilling to postpone, and one which we intend to win, and the others, too.***

For alternative views ***If we don't see a deviation that's a much, much bigger gauntlet thrown down at the feet of theorists to try to figure out what is happening***. In any case, ***The questions raised by the Higgs discovery go to the heart of our understanding of space-time and QM***


Happy Birthday
Ettore

*Thank you for your attention*