Mitigation of the onset of hosing in the linear regime through plasma frequency detuning

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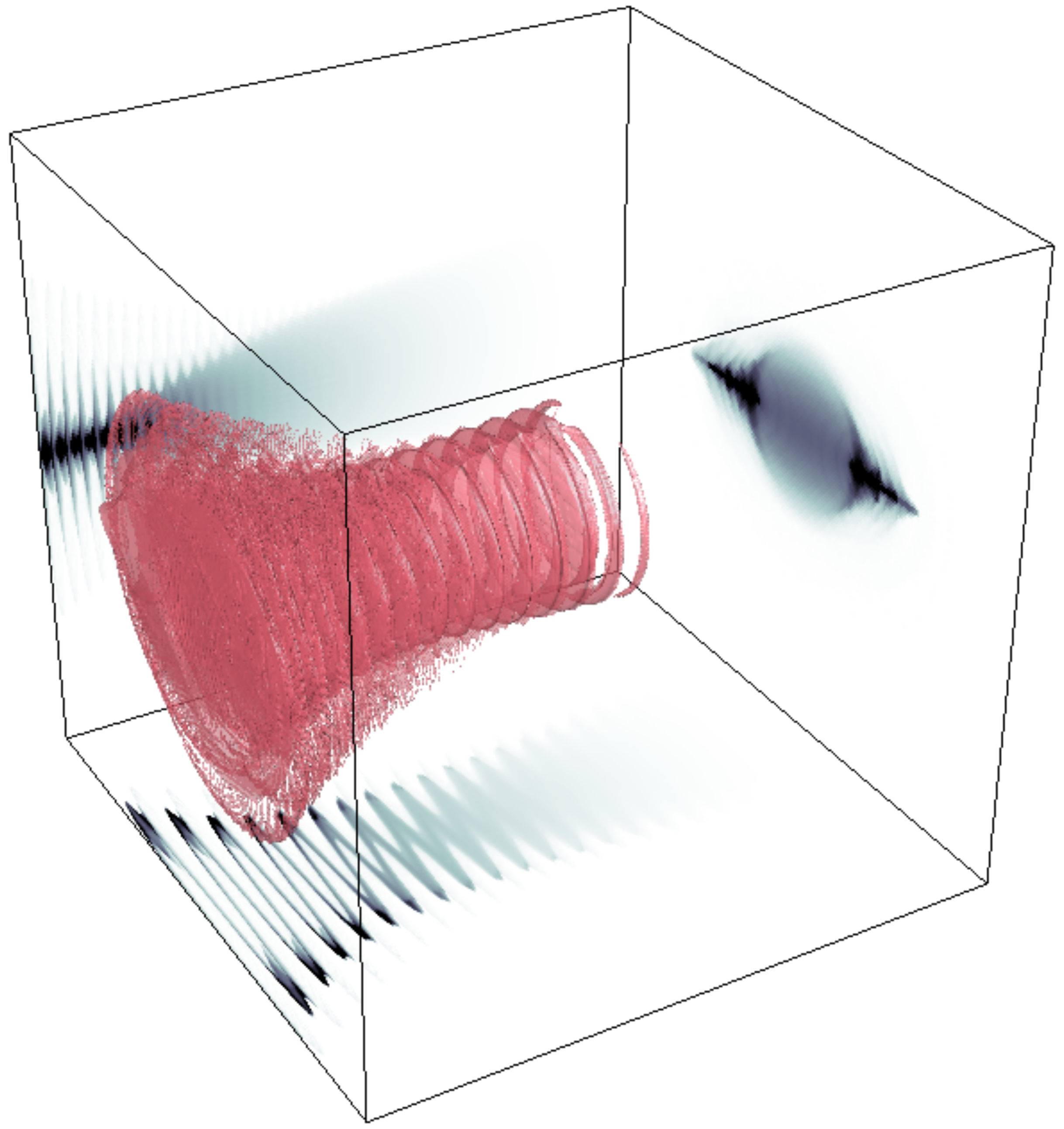

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Acknowledgments

The AWAKE Collaboration, in particular the AWAKE team based at CERN

B. Holzer

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PRACE

An instability with many faces

The bogeyman of wakefield acceleration

- disruptive instability that modulates the **bunch centroid** at the plasma wavelength
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Suppressing hosing in particle drivers

- a lot of research towards mitigation has focused on the short-bunch, nonlinear regime*
- fewer options for mitigation in the **long-beam, linear-wakefield regime**** exist (relevant for single-stage TeV-level PWFA schemes)

* T. J. Mehrling, et al., Phys. Rev. Accel. Beams 22, 031302 (2019)
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Growth rate - it's a spectrum

- "a **long-wavelength hosing instability** in laser-plasma interactions" has been studied some time ago***

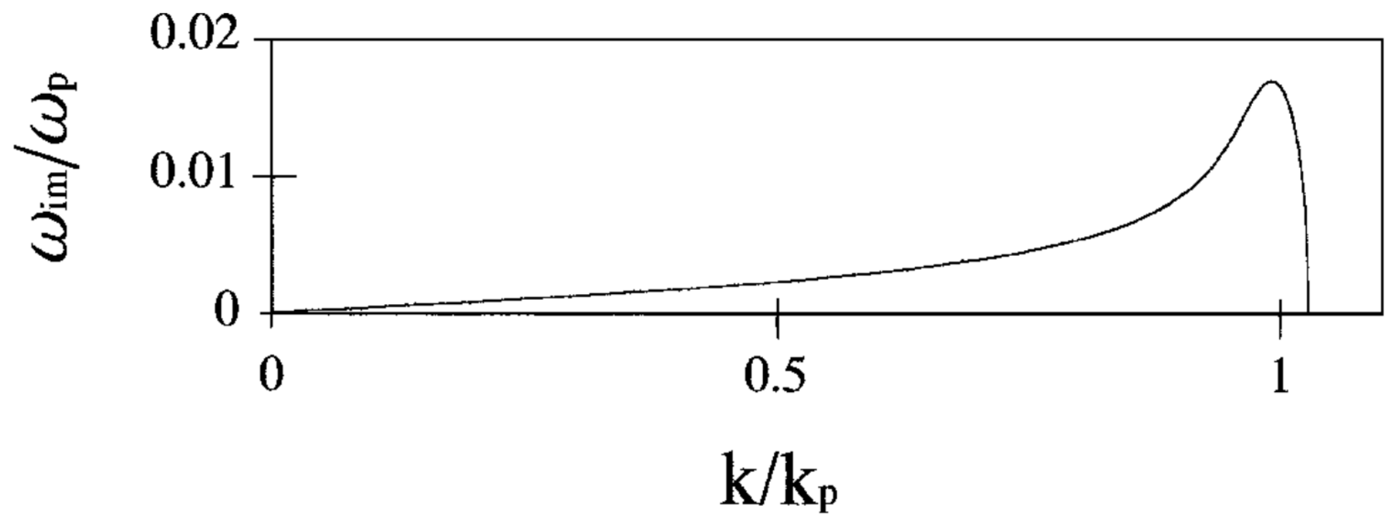

FIG. 2. The growth rate for hosing vs wave number for $\tilde{x}_R = 256$.

⇒ what does this **seed frequency dependence** look like for beam hosing?

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Methods and parameters

How?

1) initial **centroid perturbation**:
   $y_{c0}(\zeta) = 0.05\,\sin(k\,\zeta)$
2) obtain evolution of $y_c(\zeta, z)$
3) measure the **amplitude response**:

$$\Pi(z) = \frac{\int d\zeta\,|y_c(\zeta, z)|}{\int d\zeta\,|y_c(\zeta, 0)|}$$

with

- theoretical model
- simulations

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$$k_\beta^2 = \frac{1}{2\,\gamma_b}\left(\frac{\omega_b}{c}\right)^2 = \frac{1}{2\gamma_b}\,\frac{q_b^2\, n_{b0}}{\epsilon_0\, M_b}\,\frac{1}{c^2}$$

Theory

Bunch centroid equation:

$$\frac{d^2 y_c}{dz^2} = \frac{m_e}{\gamma M_b}\left\langle F_y \right\rangle = \mathrm{RHS}(y_c)$$

plasma response

First-order evolution of centroid (valid for $z \lesssim k_\beta^{-1}$):

$$y_c(\zeta, z) = y_{c0}(\zeta) + \mathrm{RHS}(y_{c0})\,\frac{1}{2}\,z^2$$

For a Gaussian transverse bunch profile (2D Cart.):

$$\left\langle F_y \right\rangle = \sqrt{\frac{\pi}{8}\frac{n_{b0}}{n_0}}\left(\frac{q_b}{e}\right)^2 \sigma_y \exp(\sigma_y^2) \int_\zeta^\infty d\zeta' \sin(\zeta - \zeta') f(\zeta') \Bigg\{ \exp\left[y_c(\zeta') - y_c(\zeta)\right] \mathrm{erfc}\left[\frac{y_c(\zeta') - y_c(\zeta) + 2\,\sigma_y^2}{2\,\sigma_y}\right] - \exp\left[y_c(\zeta) - y_c(\zeta')\right] \mathrm{erfc}\left[\frac{y_c(\zeta) - y_c(\zeta') + 2\,\sigma_y^2}{2\,\sigma_y}\right] \Bigg\}$$

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Parameters

$n_0 = 0.5 \cdot 10^{14}\ \mathrm{cm}^{-3}$

$\gamma_b = 480$

$\sigma_r = 200\ \mu\mathrm{m} \quad \approx 0.27\ k_p^{-1}$

$\sigma_z = 12\ \mathrm{cm} \quad \approx 160\ k_p^{-1}$

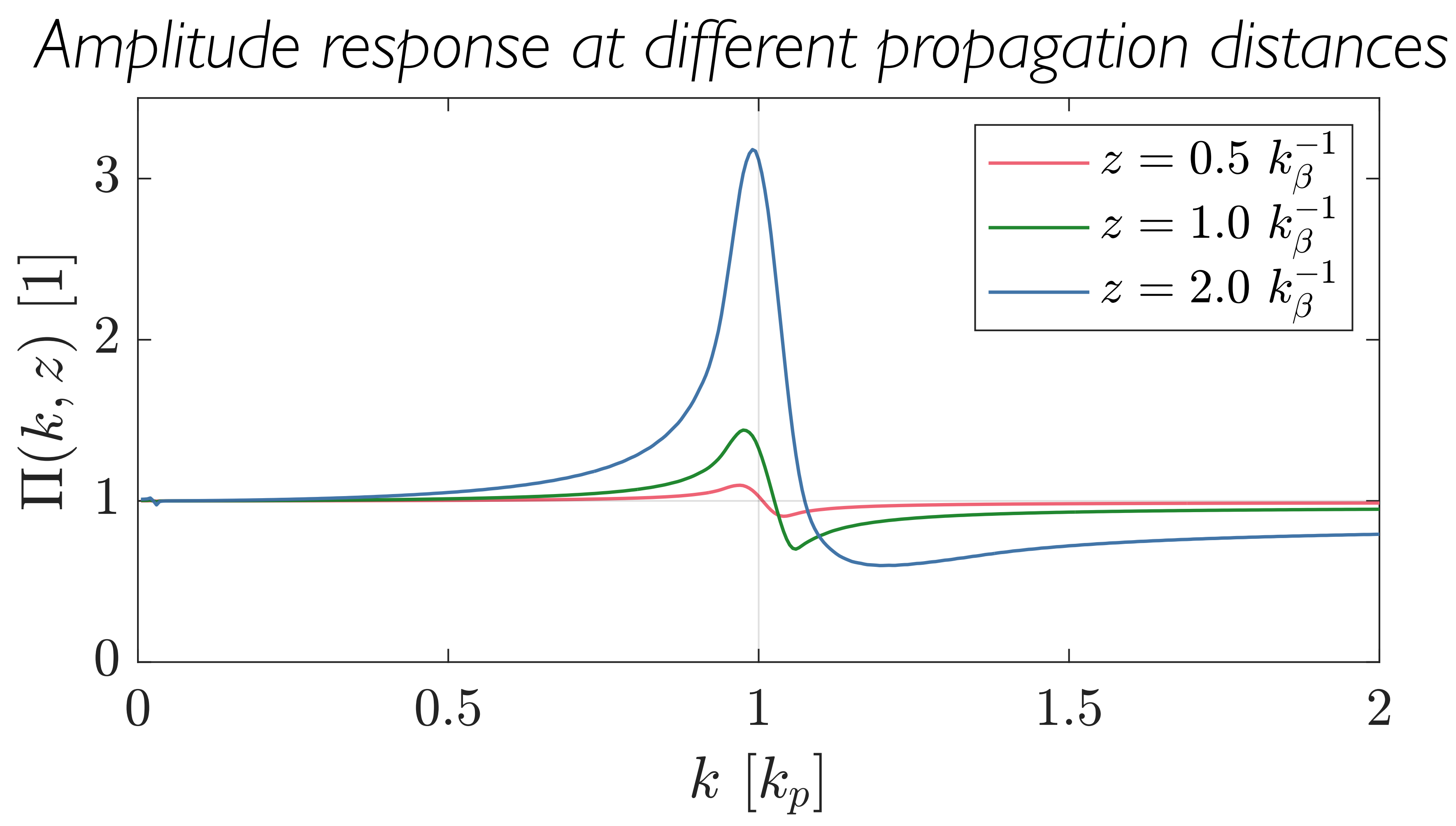
$M_b = m_e \quad \Rightarrow k_\beta^{-1}/k_p^{-1} \approx 980$

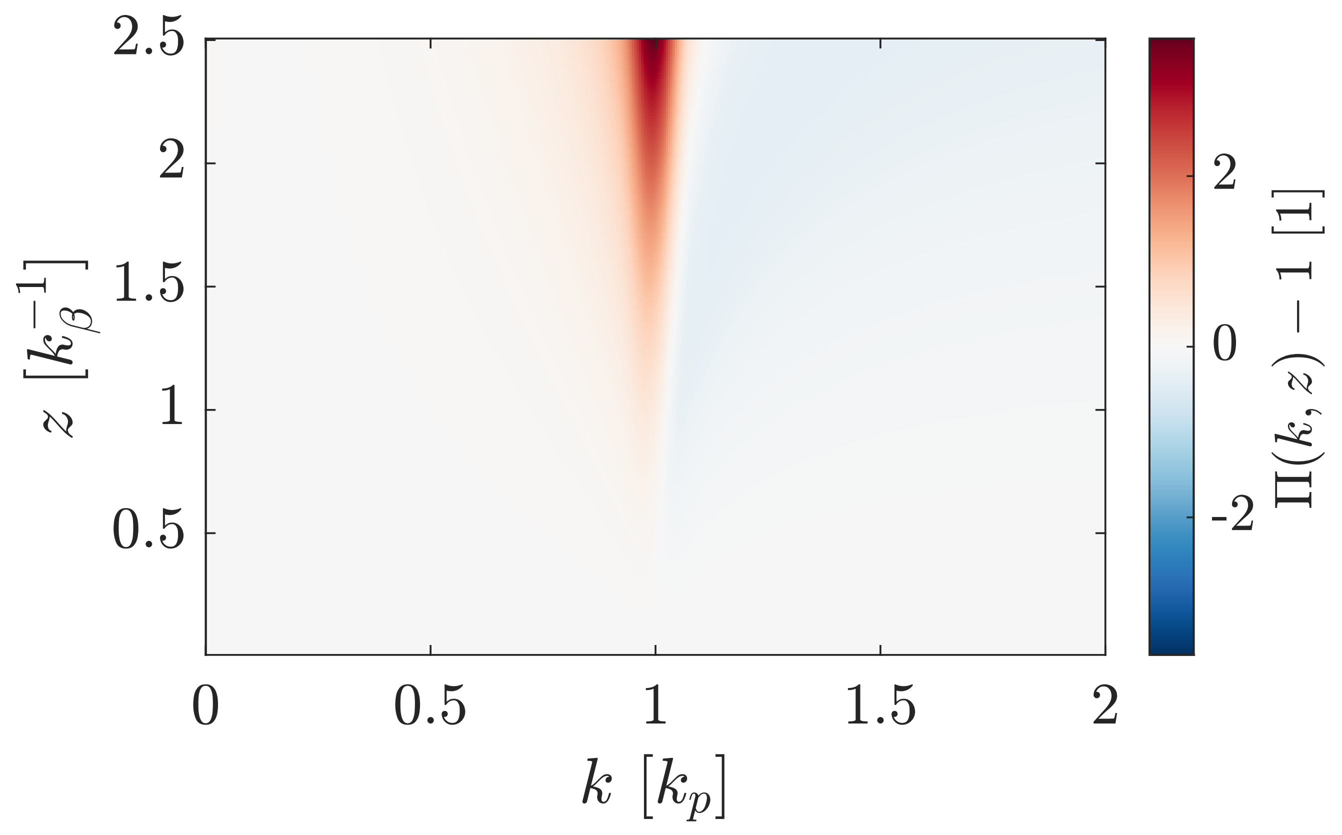
$n_{b0}/n_0 = 0.001 \quad \Rightarrow N_b = (1.9\text{-}3.8) \cdot 10^9$

- electron bunch
- bunch profile: longit. cosine and transv. Gaussian
- cold beam ($\varepsilon_N = 0$)
- head of beam, window length $L = 140\ k_p^{-1}$ ($\sim 22\ \lambda_p$)

How does the HI growth rate depend on the seed frequency?

Evolution along the propagation distance (theory)





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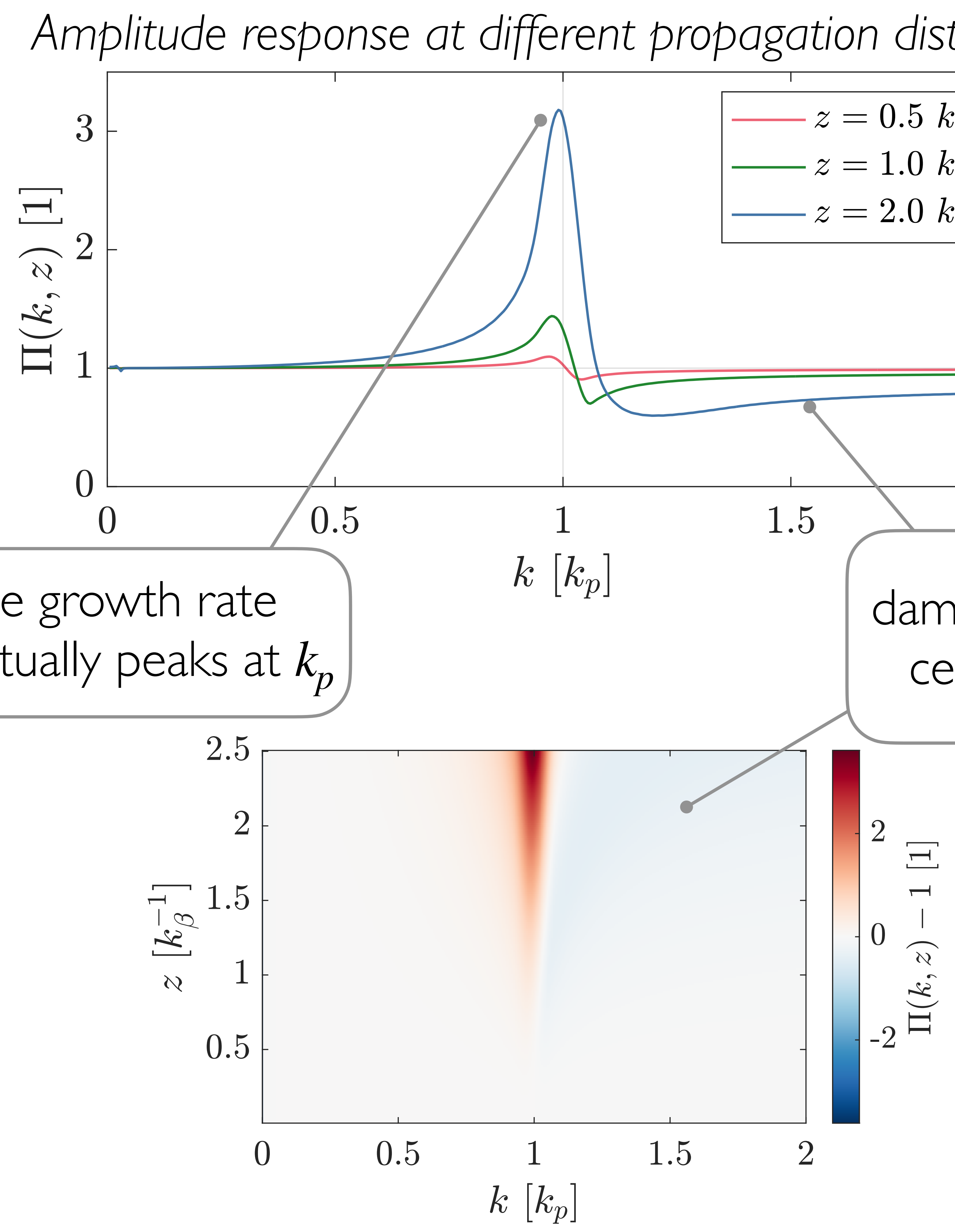

the growth rate eventually peaks at k_p

damping is possible for certain frequencies

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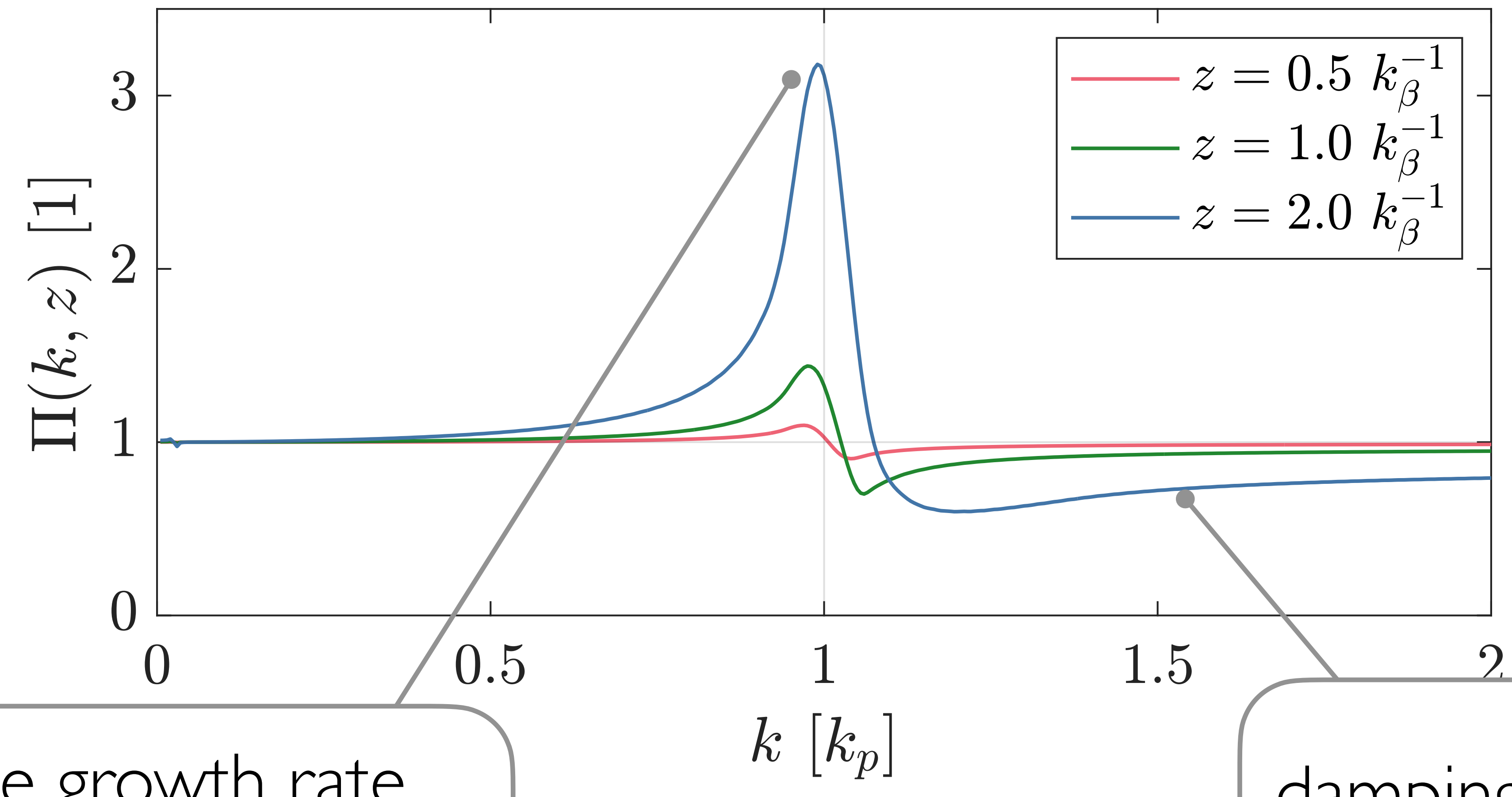
Amplitude response at different propagation distances



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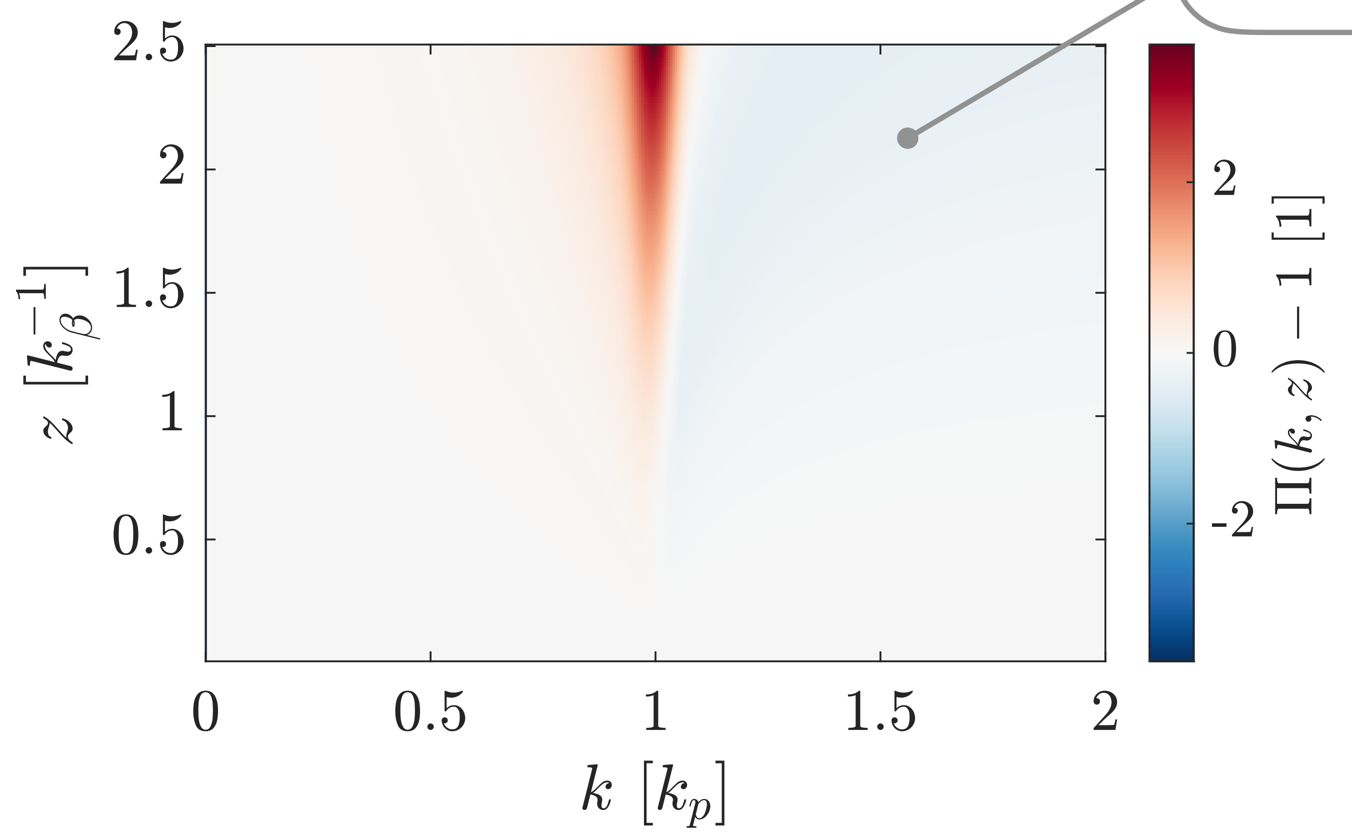
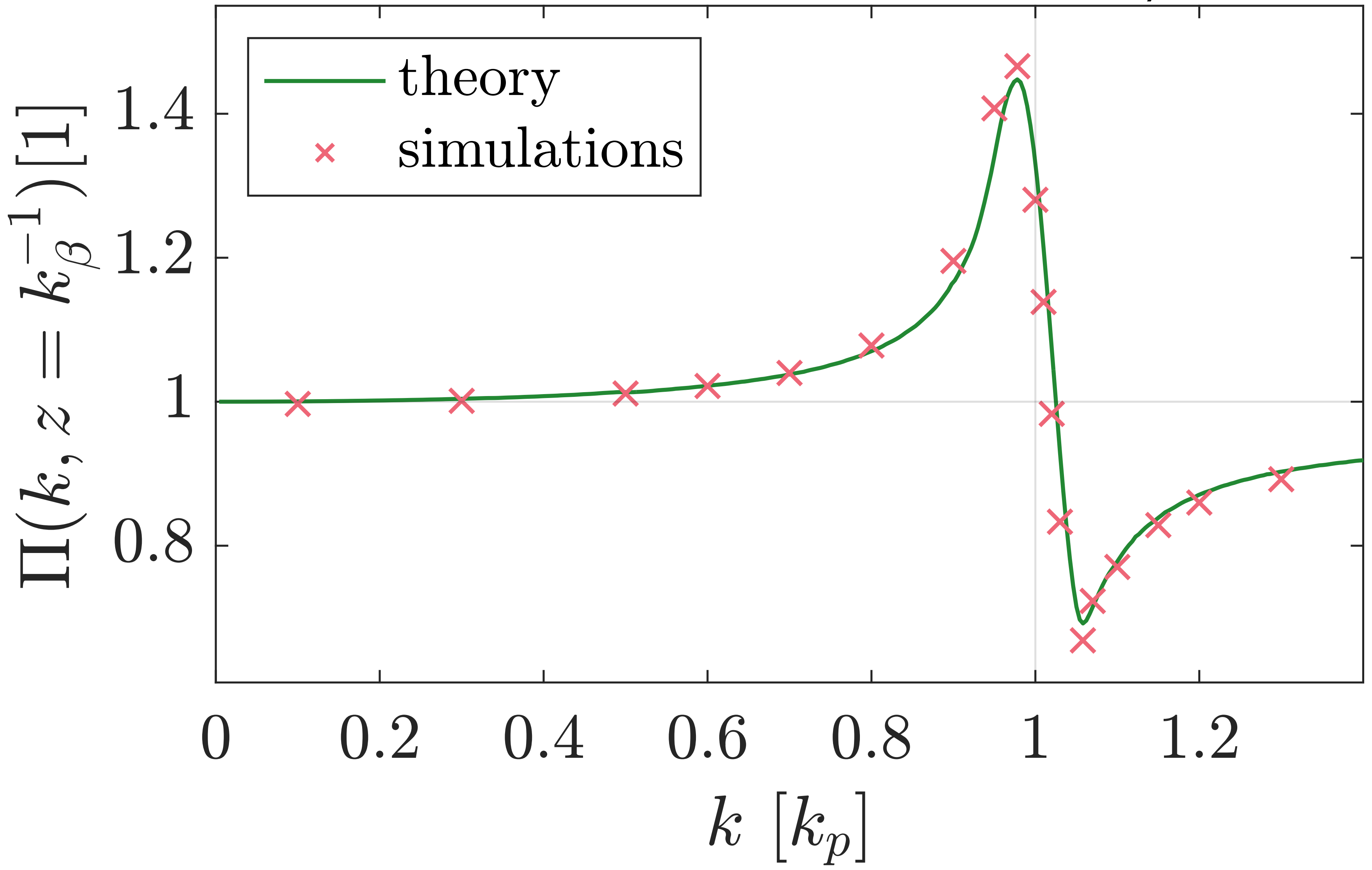


Early regime

- **excellent agreement** between theory and simulations (here: 2D Cartesian)
- early on, **significantly different growth regimes** can be accessed with a small amount of detuning

Amplitude response at $z = k_\beta^{-1}$

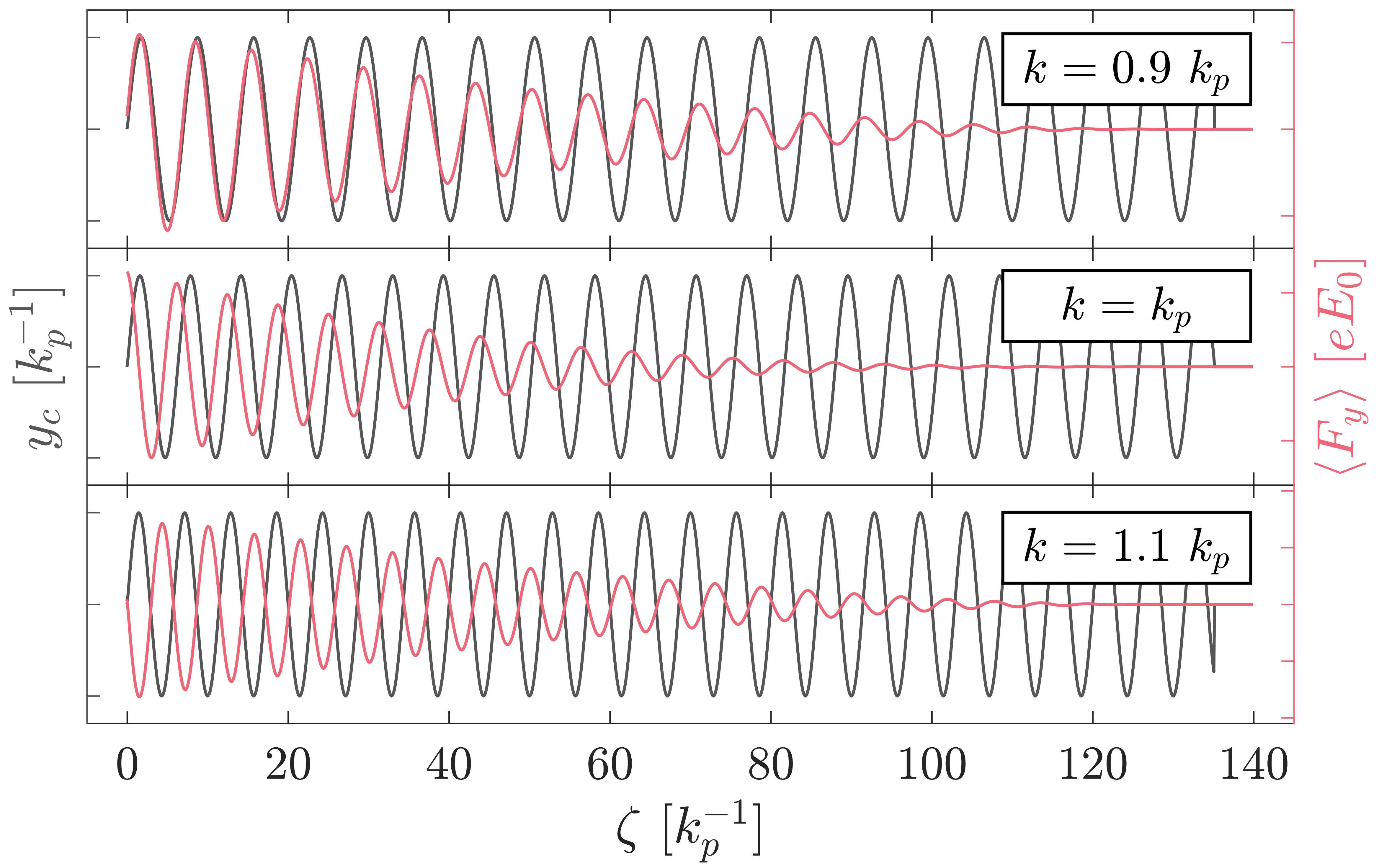


Each growth regime is associated with a phase shift

Relationship between centroid and plasma response is key

- each regime is characterised by a phase shift between the **centroid** y_c and the **plasma response** $\left\langle F_y \right\rangle$

$$\frac{d^2 y_c}{dz^2} = \frac{m_e}{\gamma M_b} \left\langle F_y \right\rangle$$



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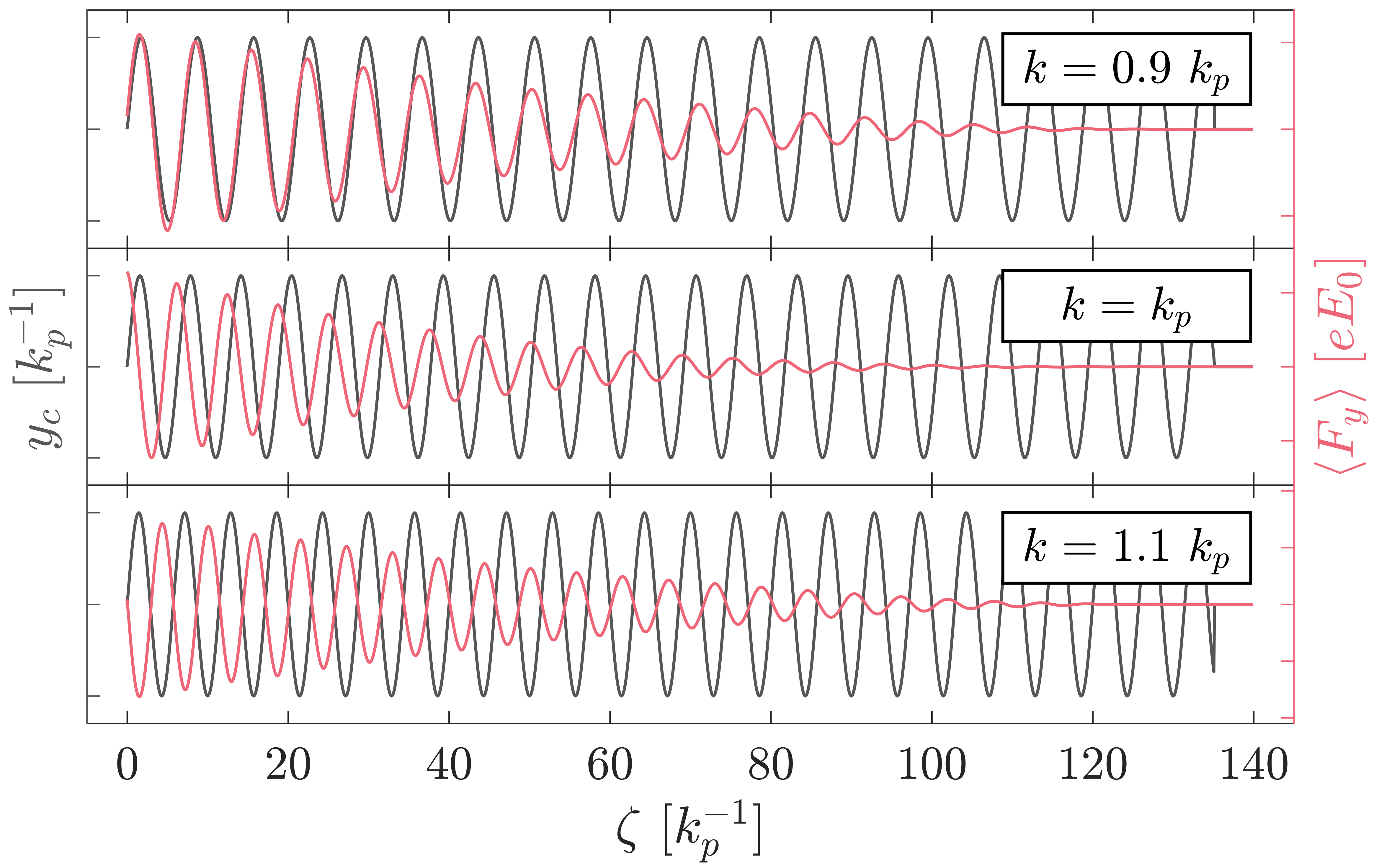
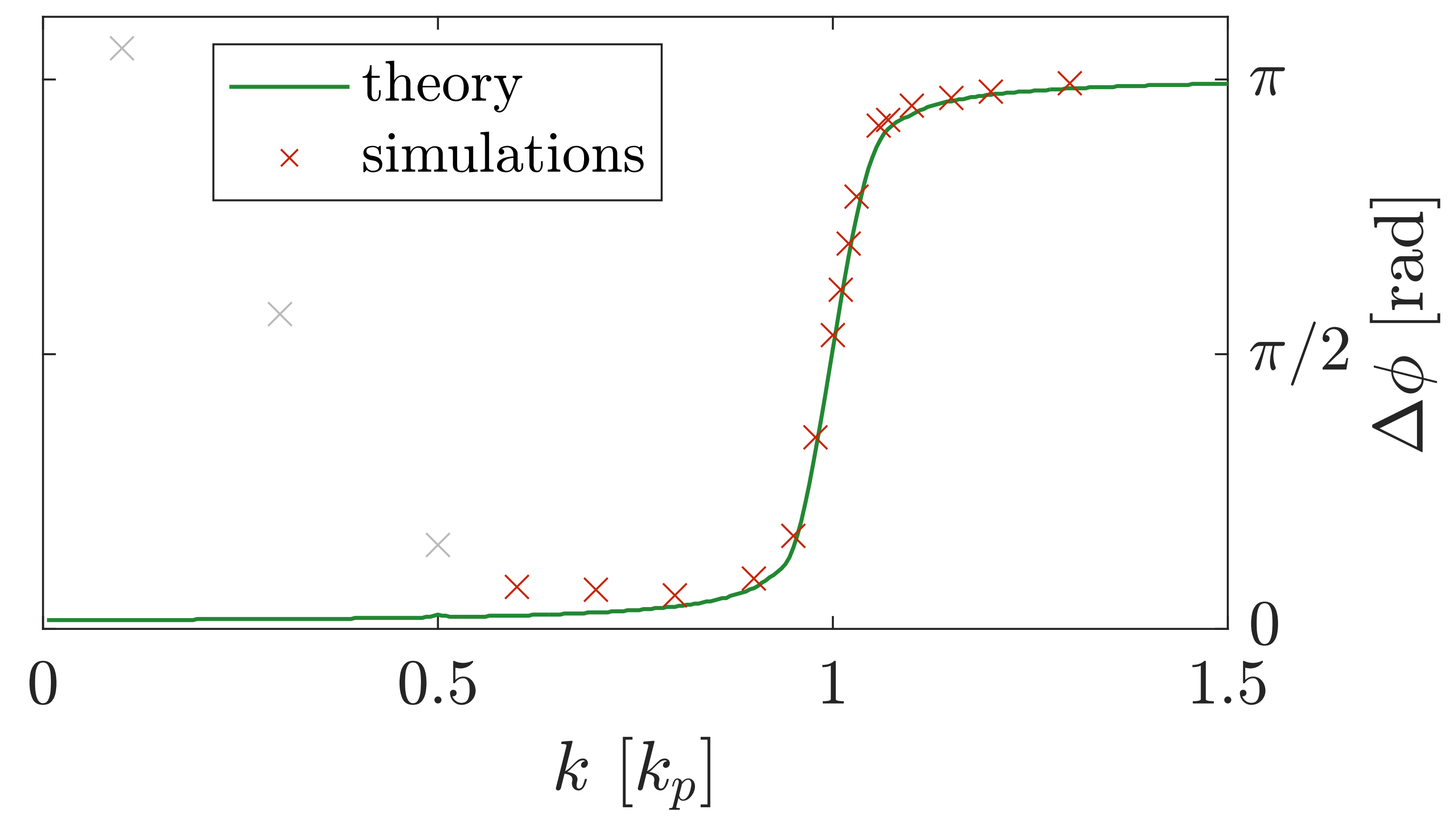


Mapping the phase shift

- the phase shift can be **measured** with a cross-correlation method*
- phase shift "spectrum" confirms three growth regimes



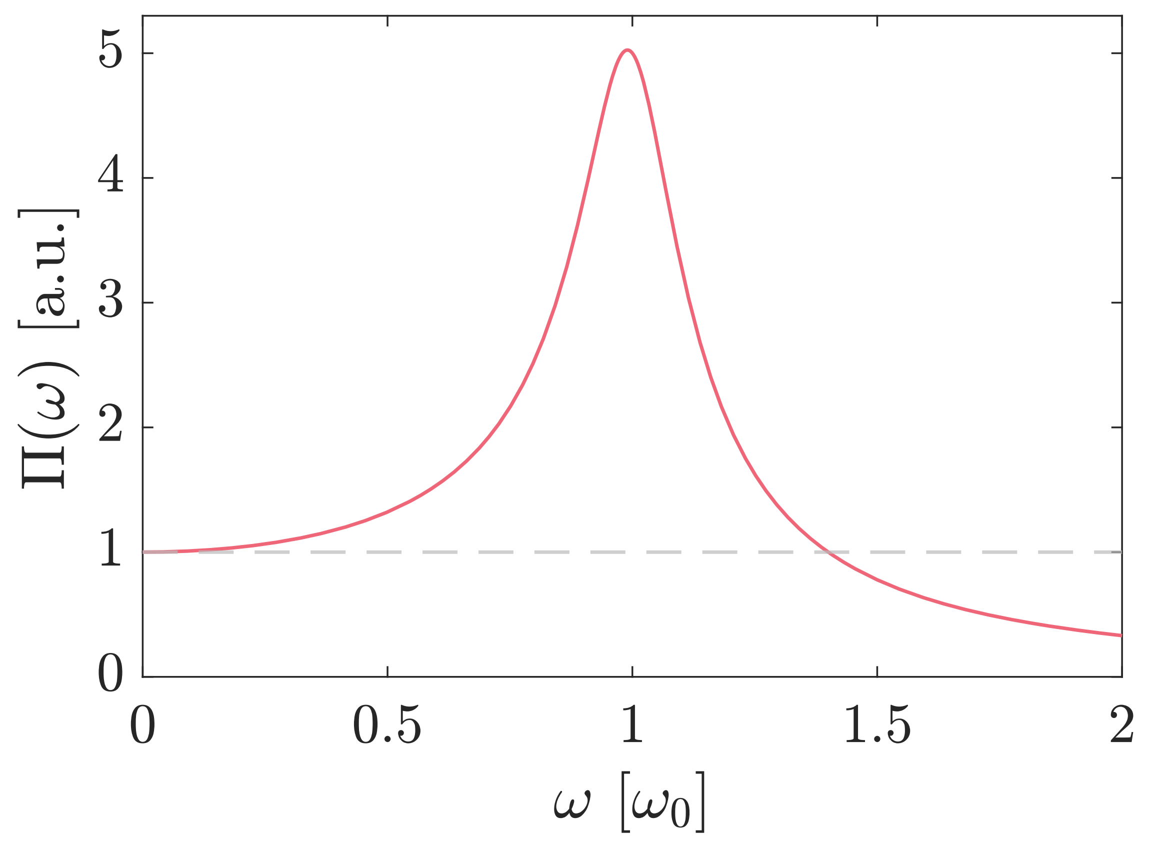
		$\Delta\phi$
$k < k_p$	slow growth	~ 0
$k = k_p$	resonant growth	$\sim \pi/2$
$k > k_p$	damping	$\sim \pi$

* For the theoretical curve, L and σ_z are scaled for each k such that the same number of wavelengths is considered in the analysis ($\sim 22\ \lambda_p$).

Behaviour is analogous with a harmonic oscillator

Sinusoidally driven damped harmonic oscillator: $\left(\partial_t^2 + 2D\,\partial_t + \omega_0^2\right) x(t) = A\ \sin(\omega t) \longrightarrow x(t) = A\,\Pi(\omega)\ \sin\Big(\omega t - \phi(\omega)\Big)$

Amplitude response vs driving frequency



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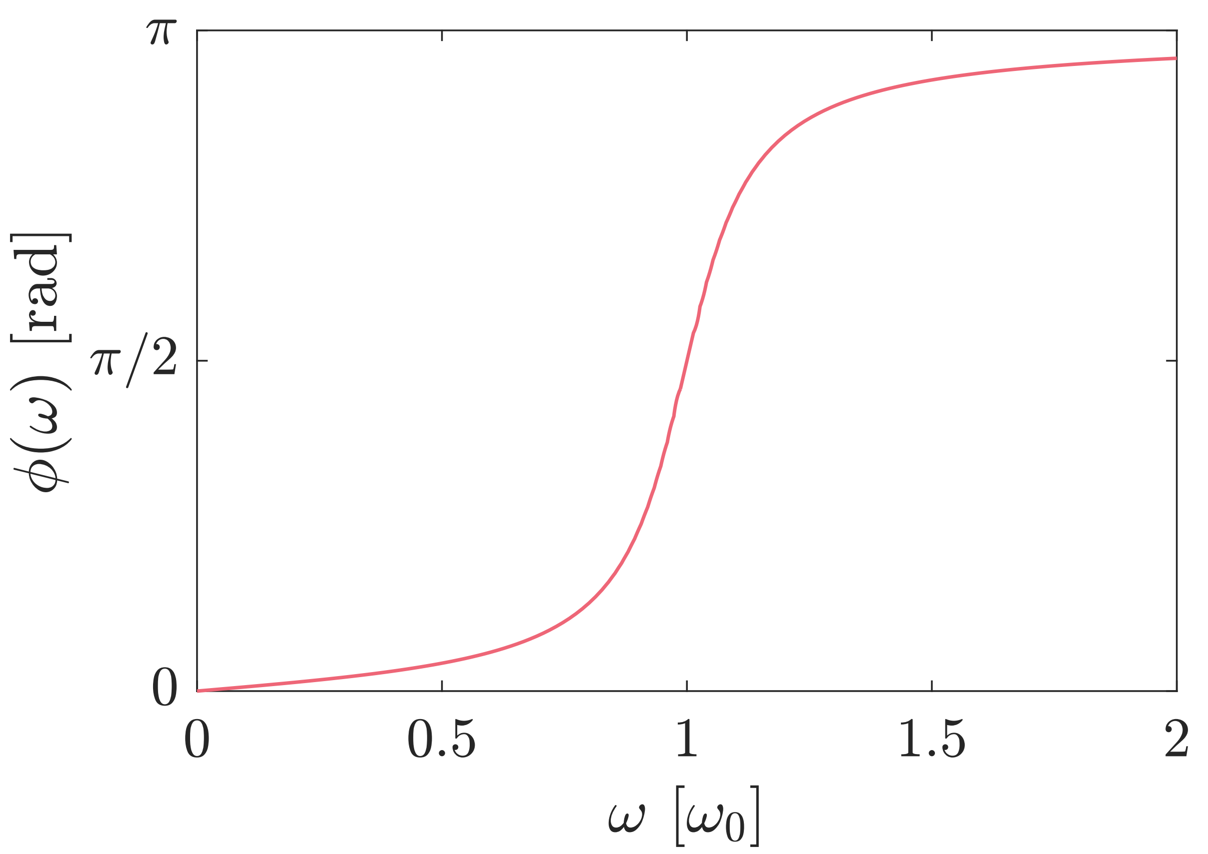

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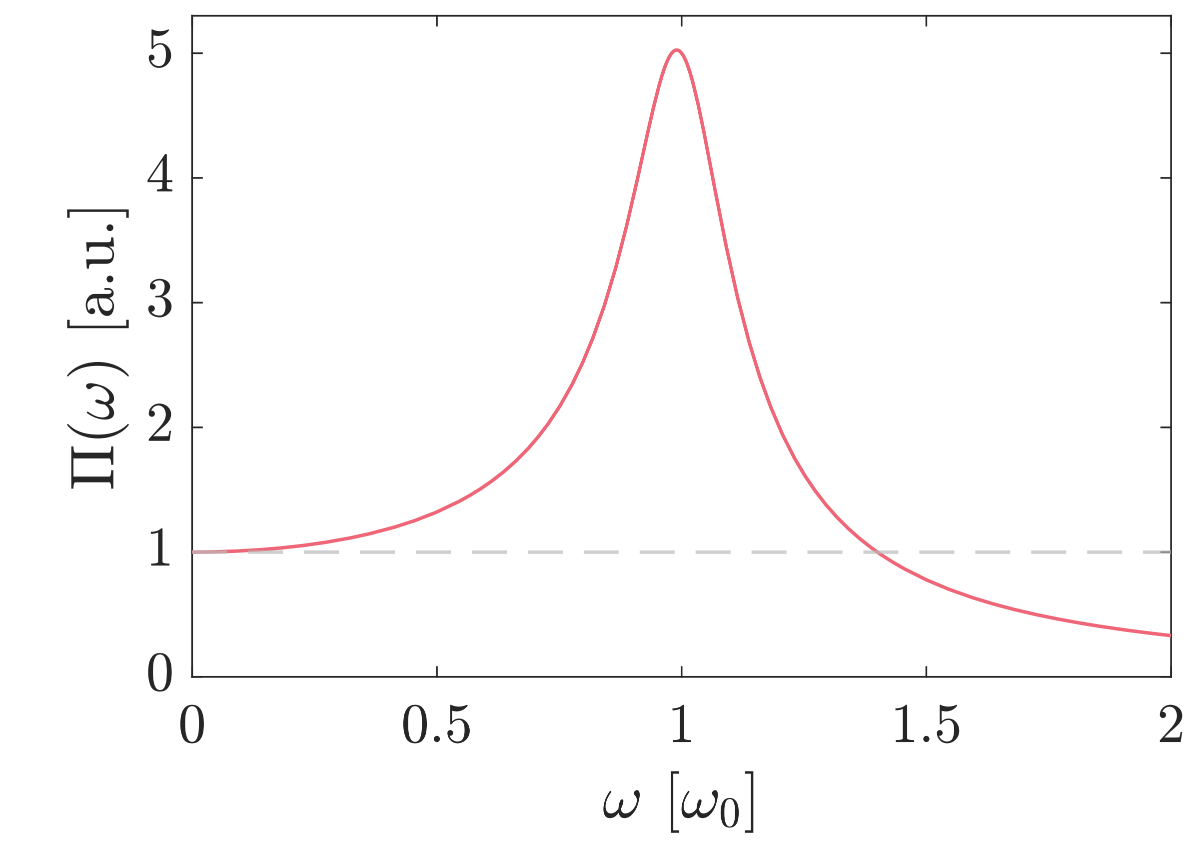

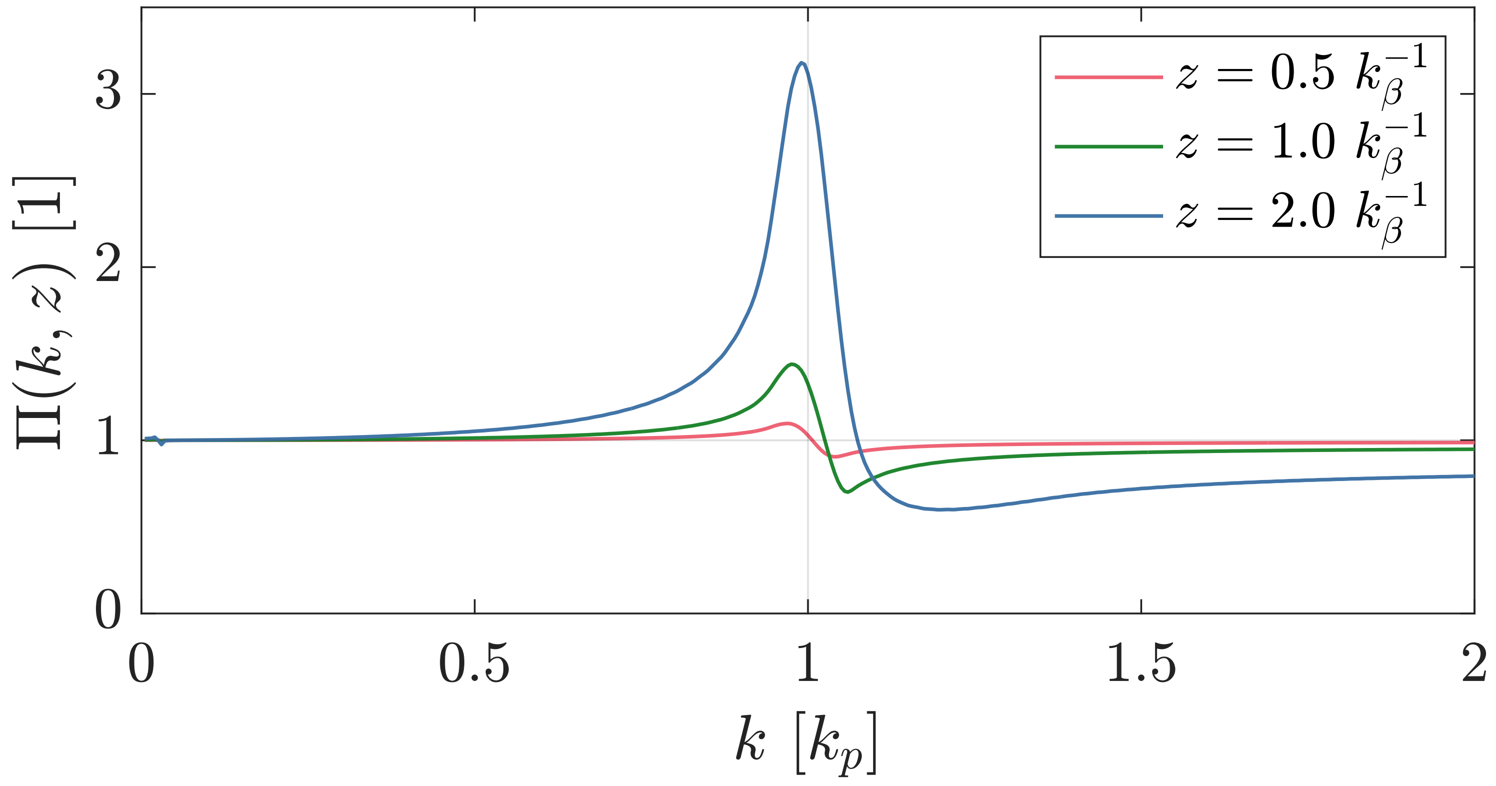

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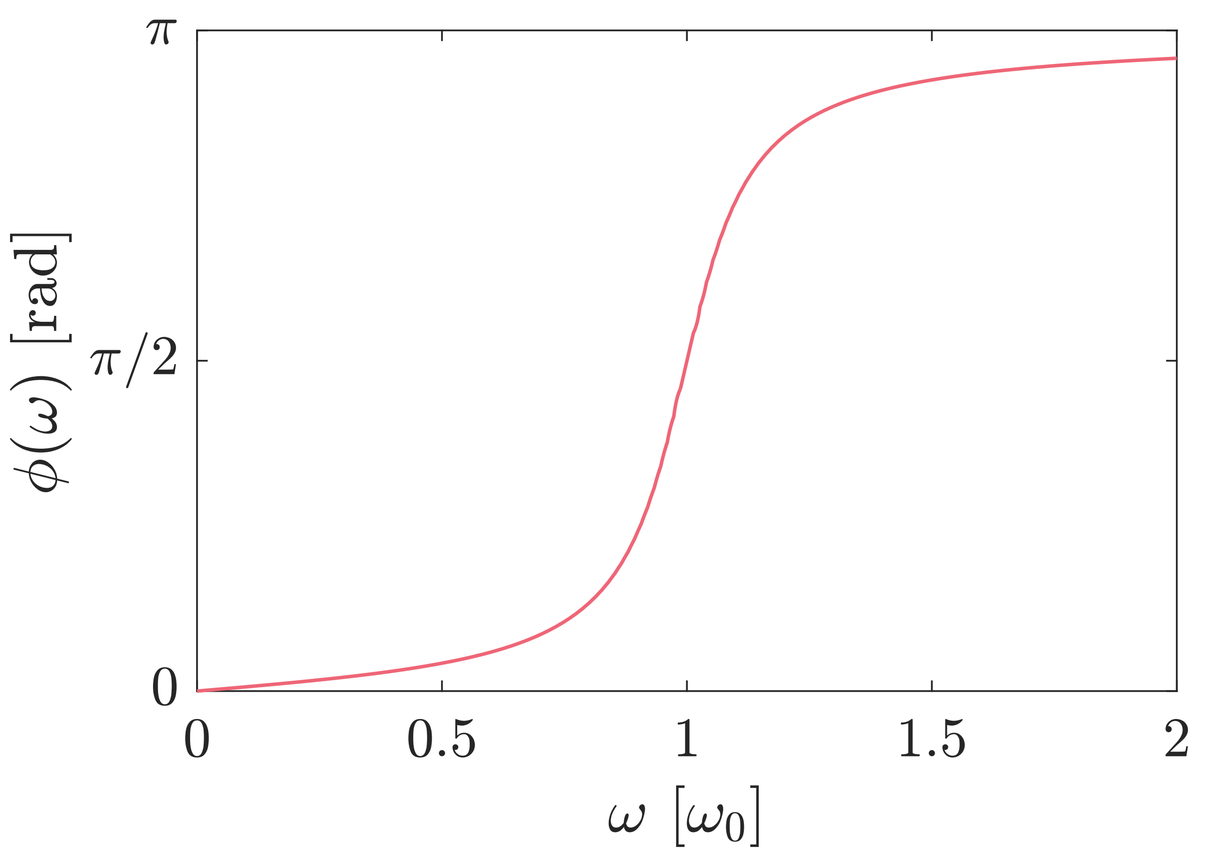

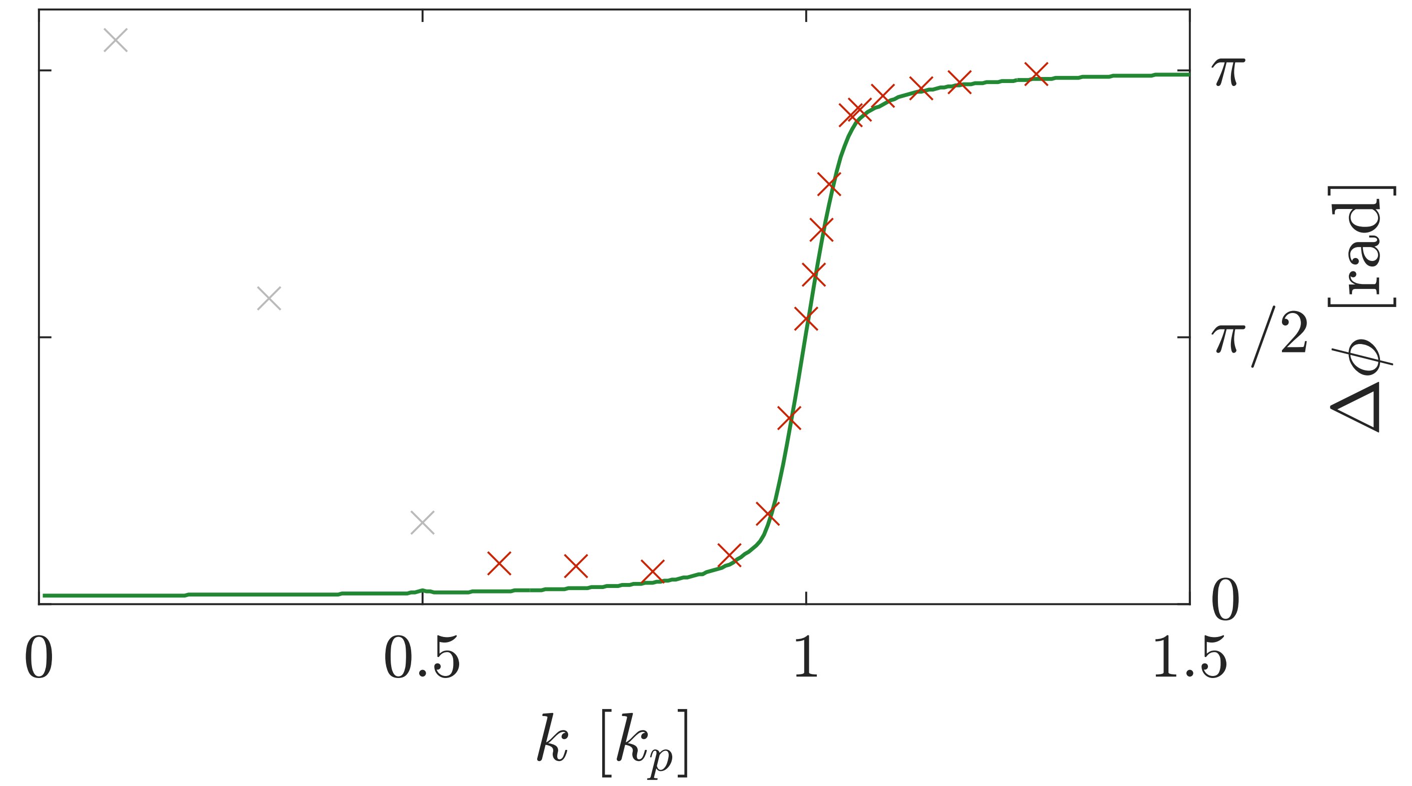

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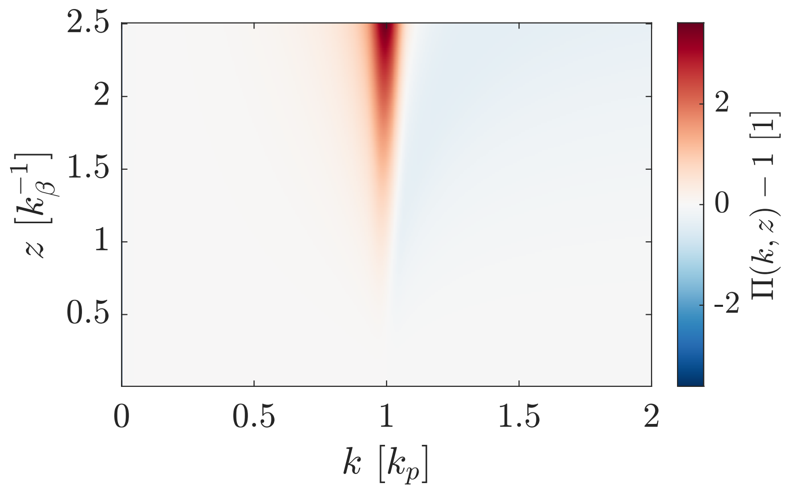
Simply staying in damping regime does not work



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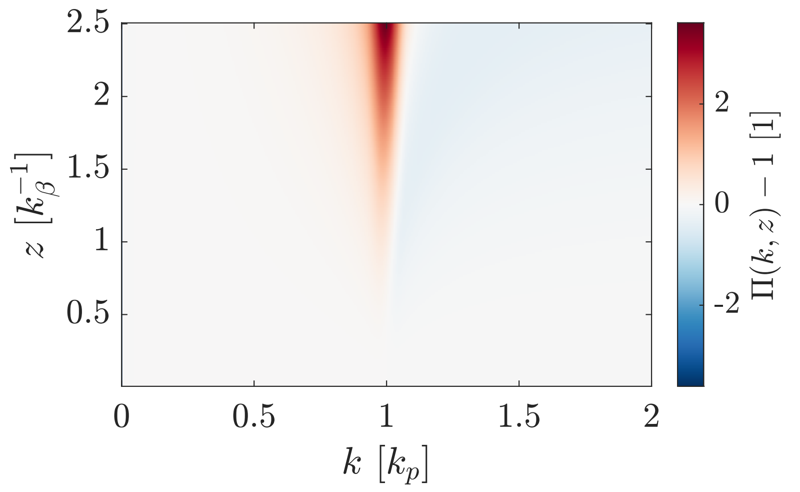

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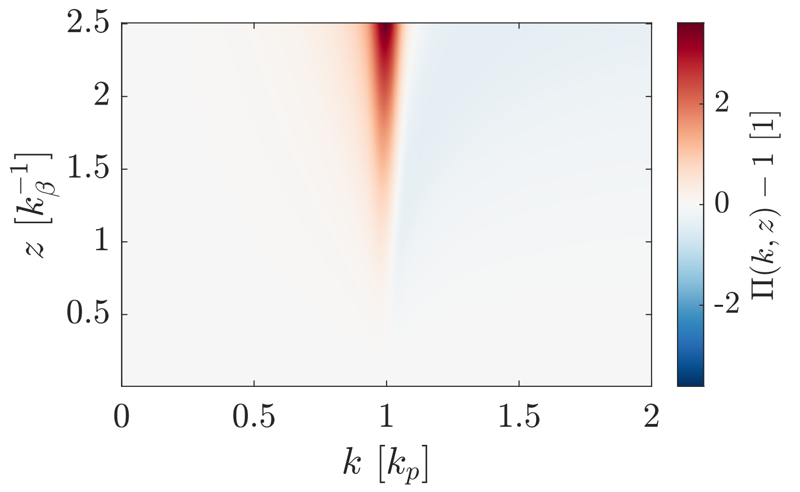

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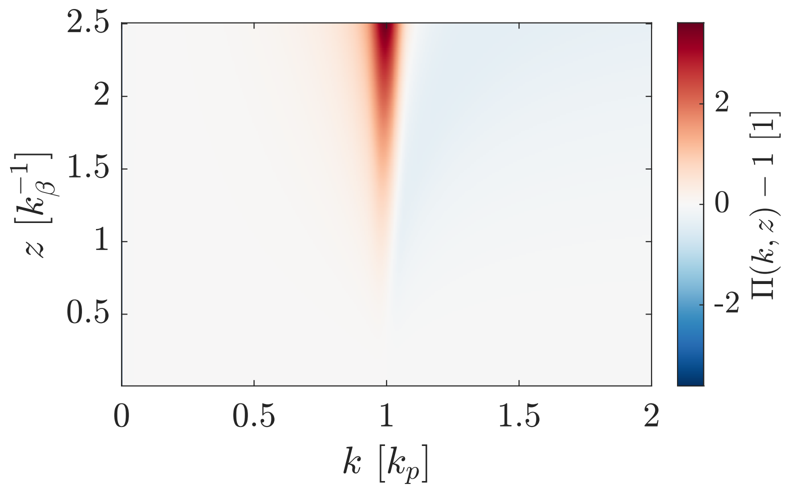

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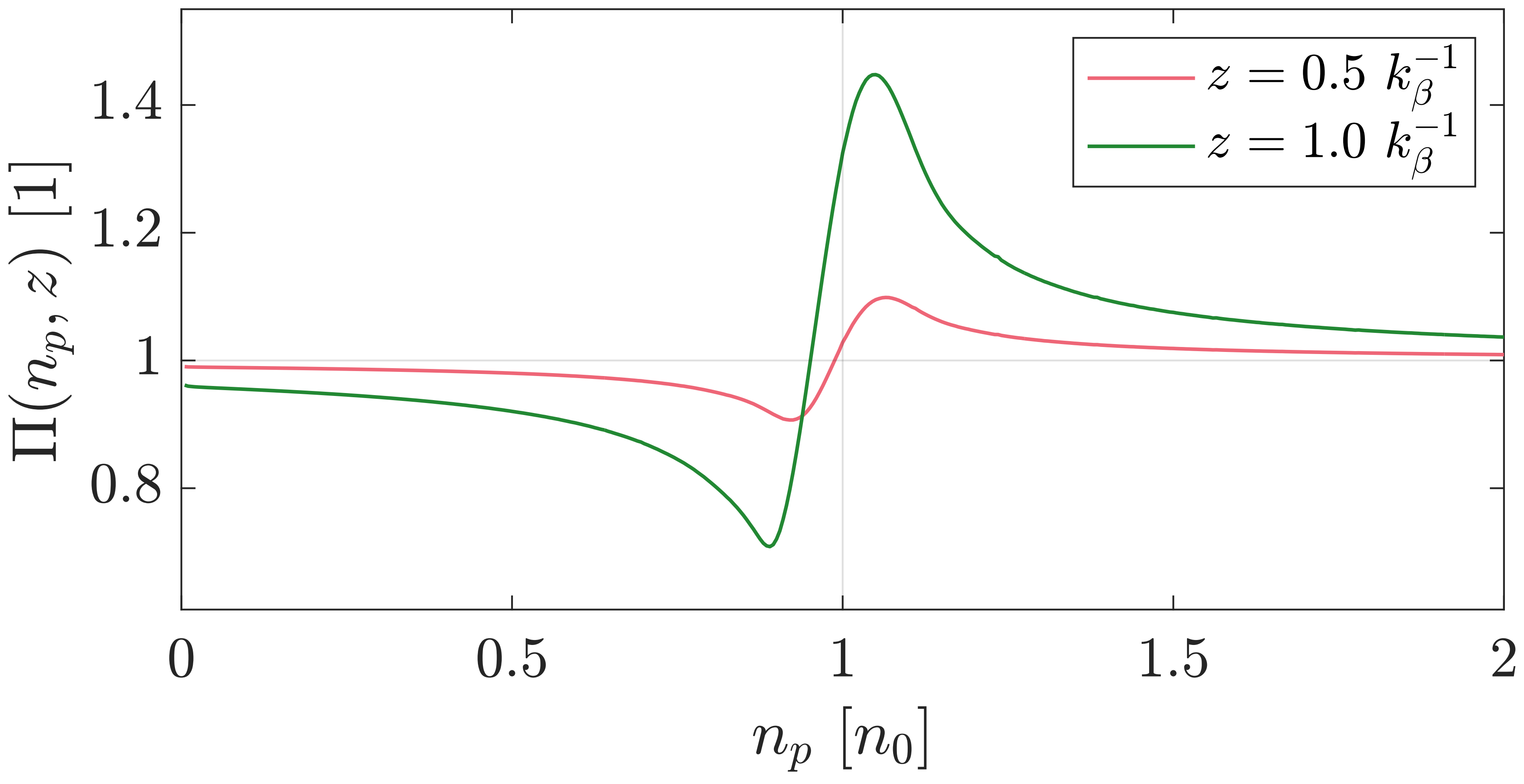
Accessing different growth regimes

control local plasma density n_p

control ratio of seed k (initial perturbation) to local k_p

Amplitude response as a function of local plasma density



Hosing can be mitigated with plasma density steps

Measuring the mitigation effectiveness

- for small centroids ($y_c \ll 1$):

$$\left(\frac{d^2}{dz^2} + k_{\mathrm{HO}}^2(\zeta, z)\right) y_c(\zeta, z) = F(\zeta, z, y_c)$$

- multiply by v_c:

$$\frac{d}{dz} \underbrace{\left(\frac{1}{2}\, v_c^2 + \frac{1}{2}\, k_{\mathrm{HO}}^2\, y_c^2 \right)}_{\mathcal{E}} = v_c\, F$$

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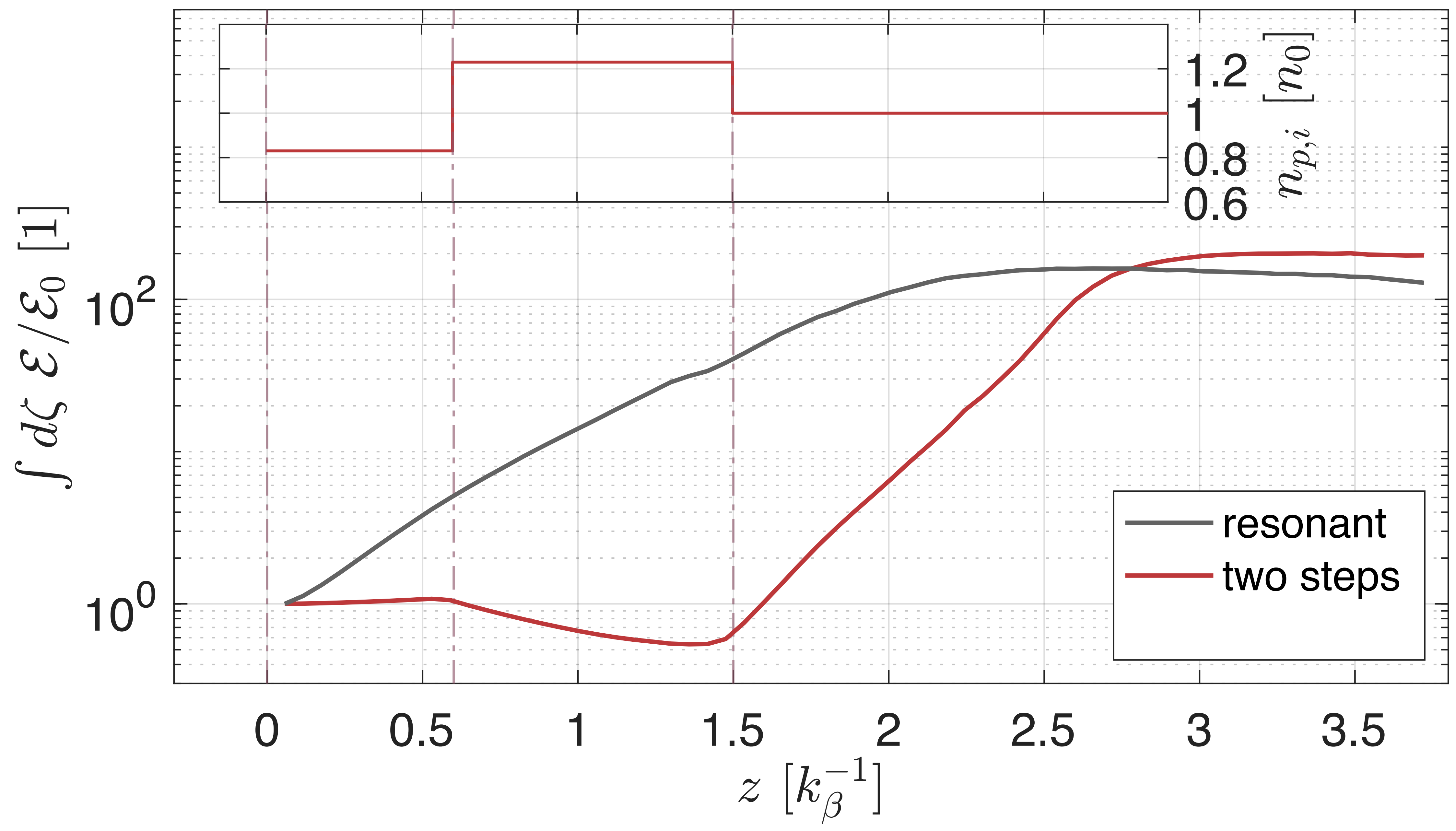
A proof-of-concept density step configuration

3D OSIRIS simulations



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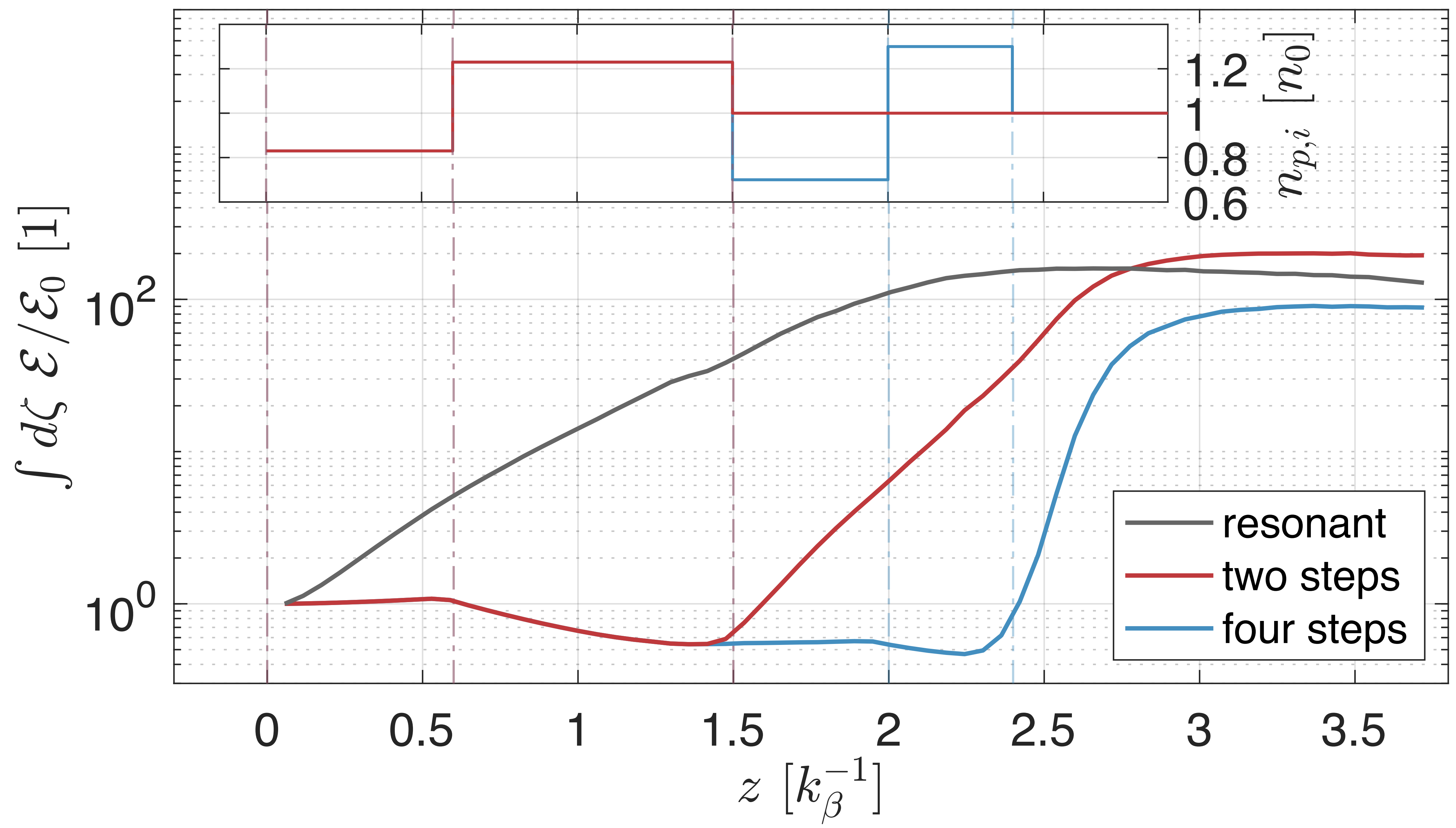

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- a **second set of steps** prolongs the suppressive effect

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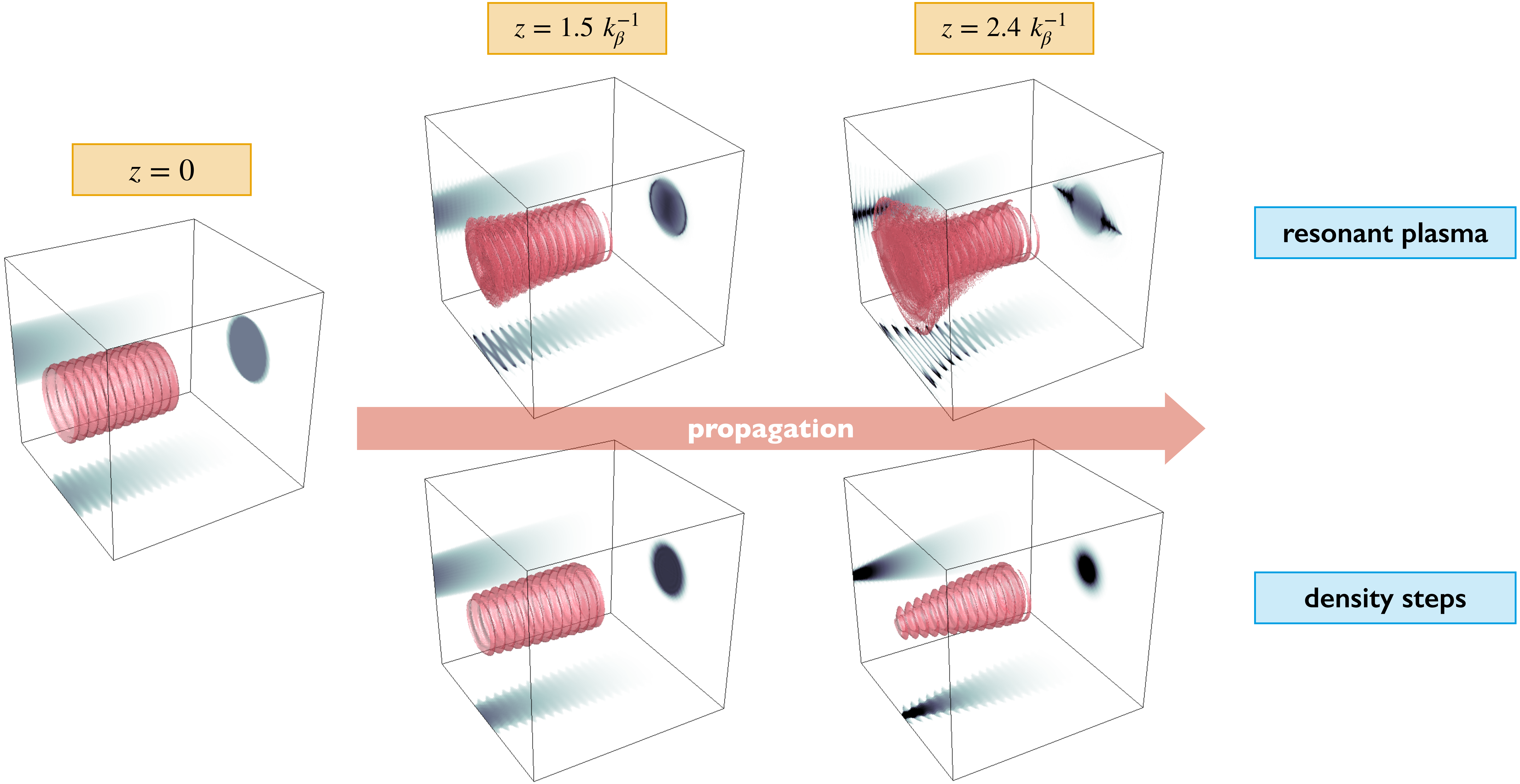

Does the mitigation set-up destroy a self-modulated bunch?

Methodology

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- submit **fully self-modulated bunch** to the two-step density profile

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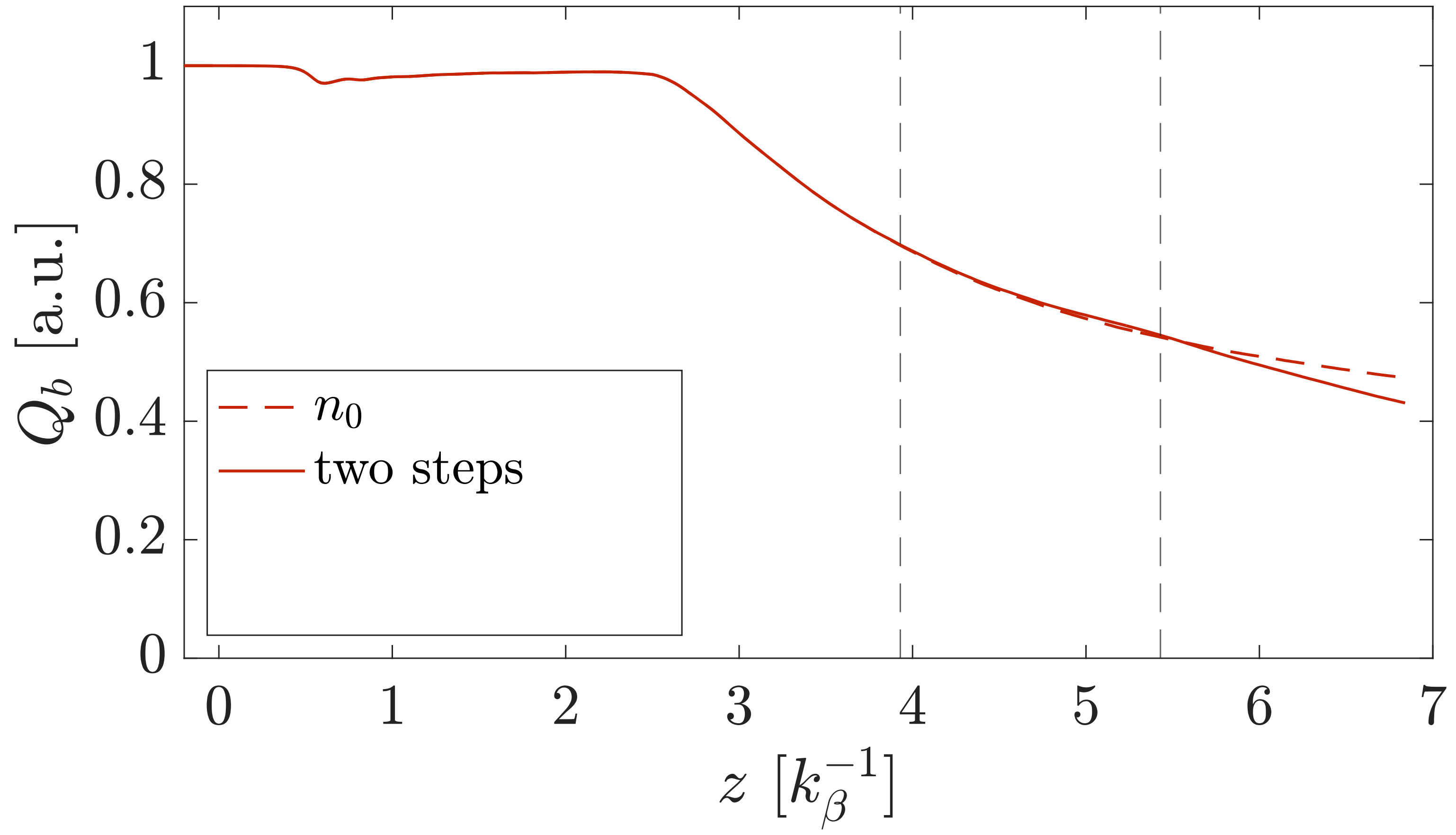
Virtually no effect on bunch charge

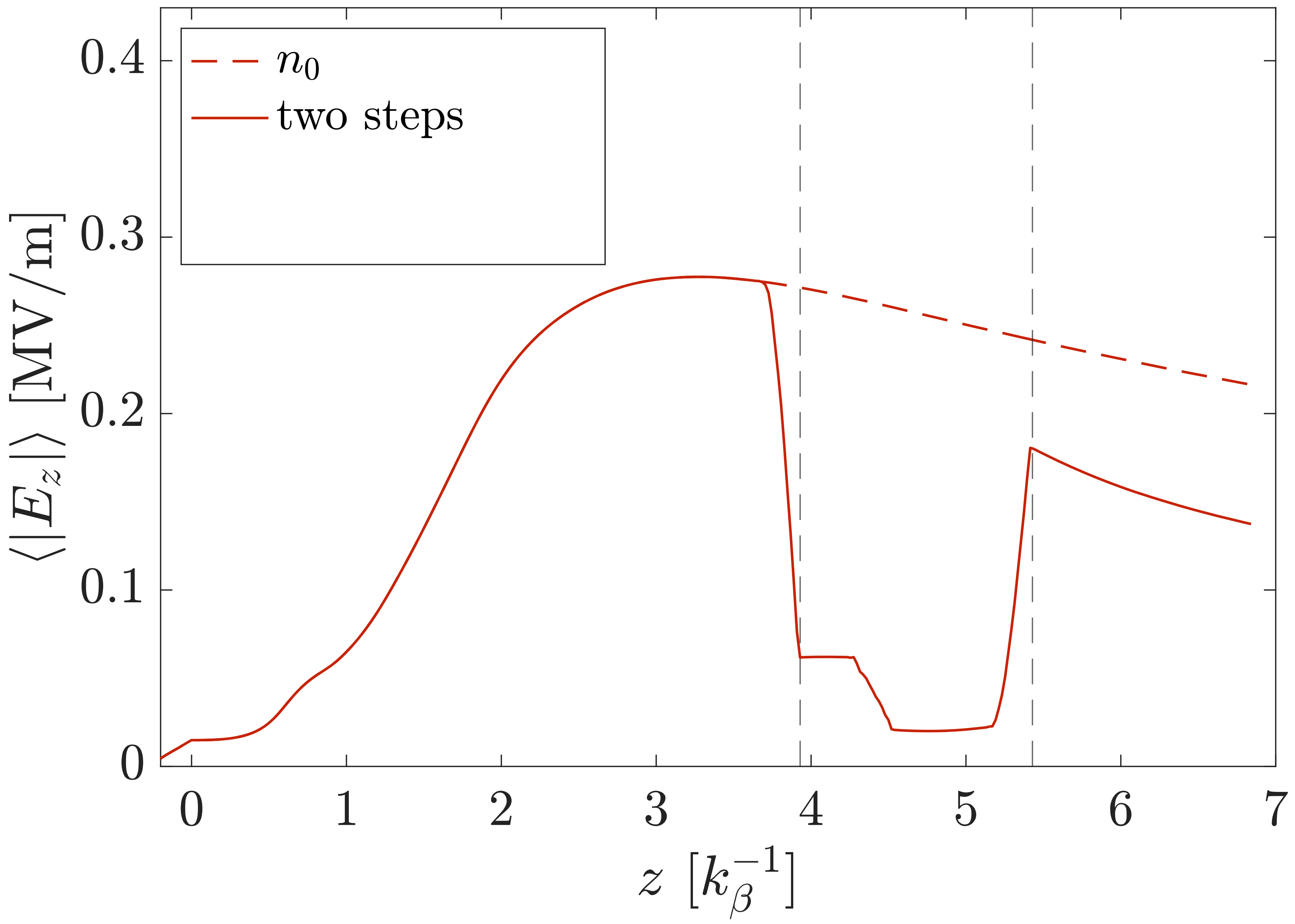
Integrated bunch charge (up to $r = k_p^{-1}$) versus propagation



There is significant impact on the accelerating field amplitude

- preliminary study indicates a **large drop** in the amplitude of E_z (~ –40%)

Average longitudinal wakefield amplitude versus propagation



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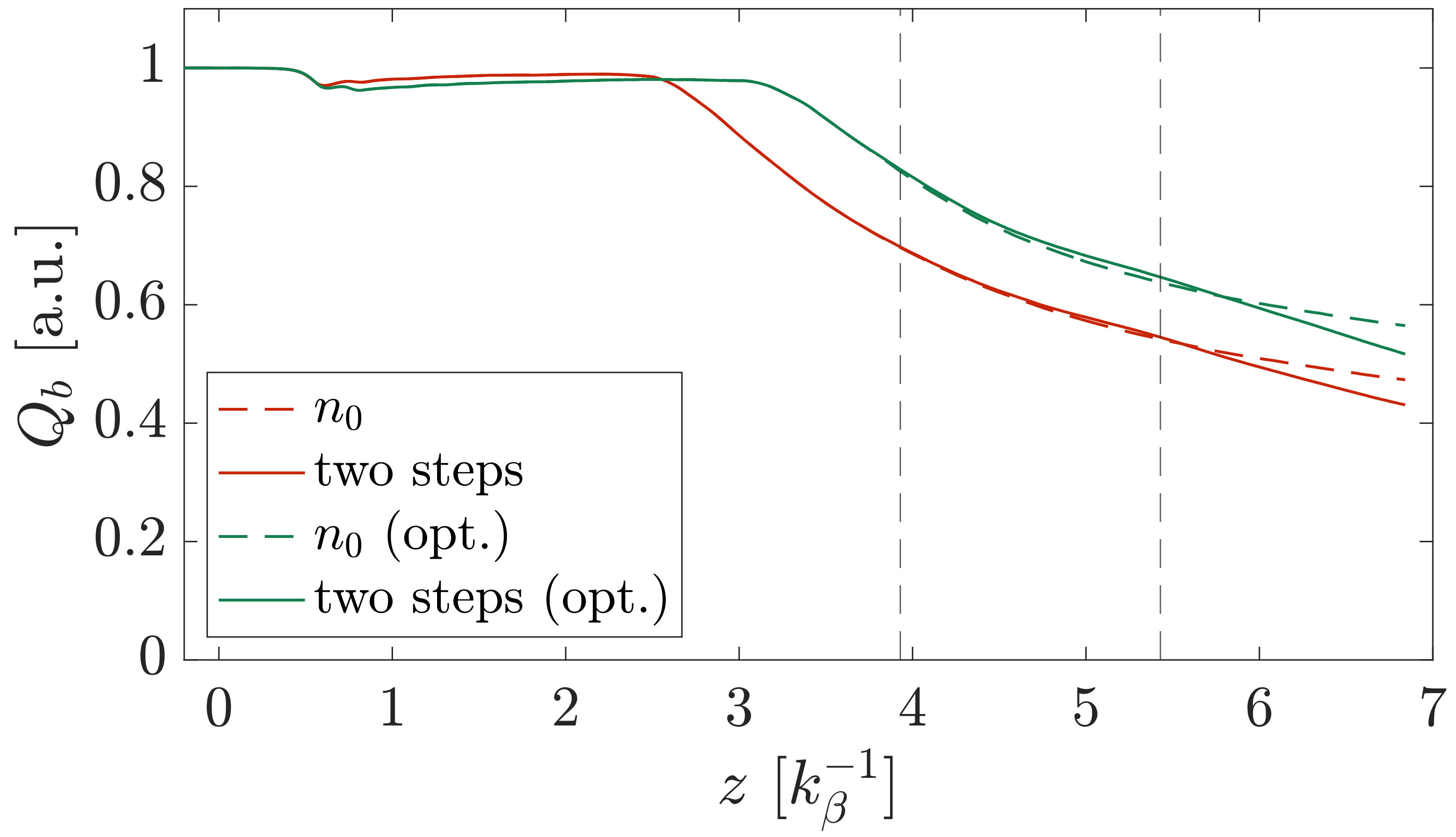

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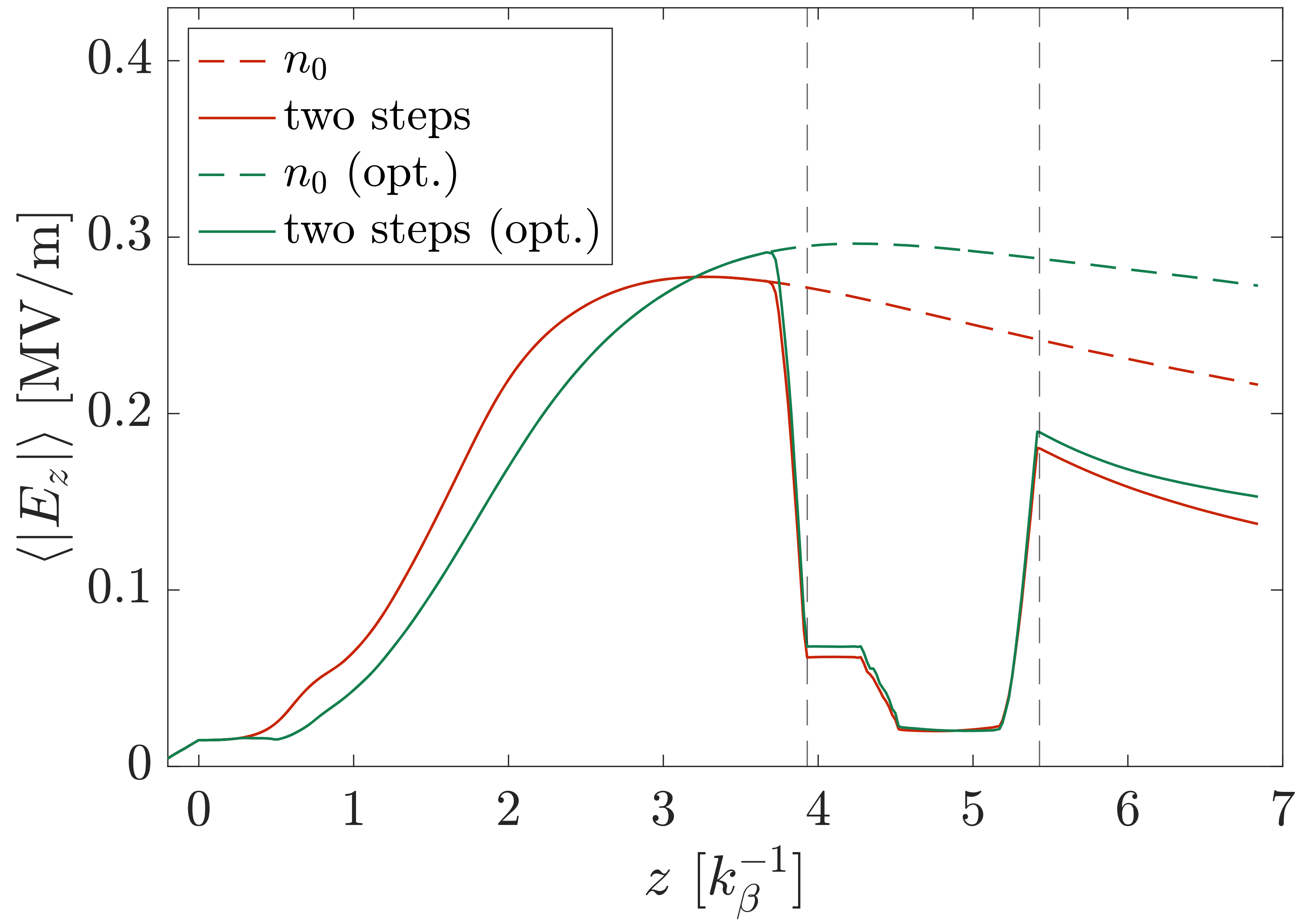


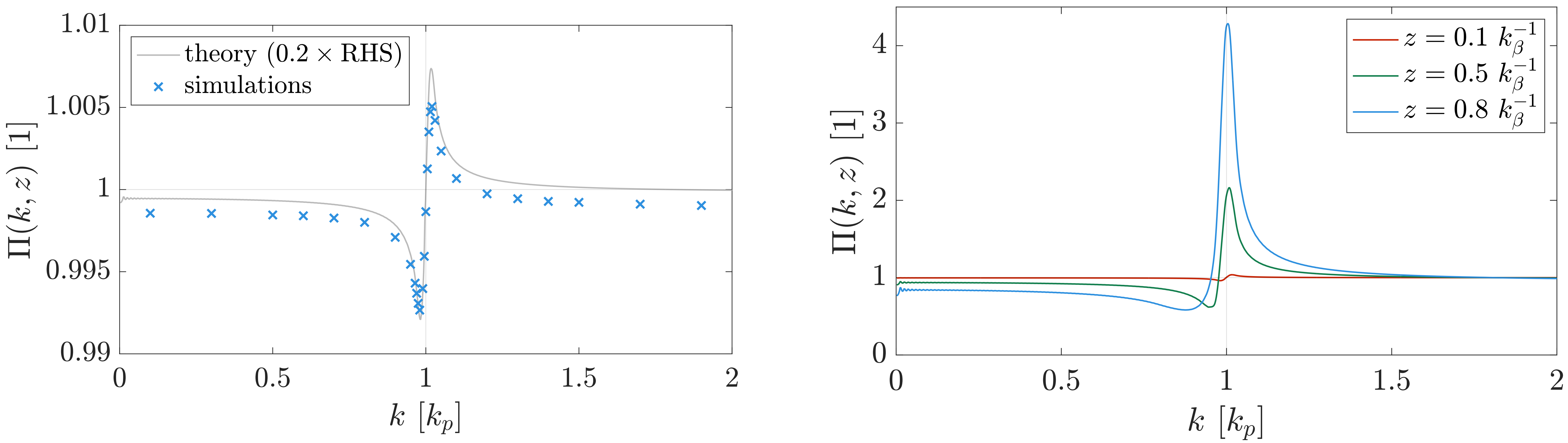
- the SMI can be **optimised** with a small, early density step*
- **similar impact** on this configuration ("opt.")

* K.V. Lotov, Phys. Plasmas 18, 024501 (2011); K.V. Lotov, Phys. Plasmas 22, 103110 (2015)

The self-modulation instability obeys similar physics

The symmetric mode can also be controlled via detuning

- the growth rate of the **bunch radius perturbation** displays similar behaviour
- **opposite growth regimes** to hosing



⇒ Poster session tonight!

#49 - "Early dynamics of the self-modulation instability growth rate"

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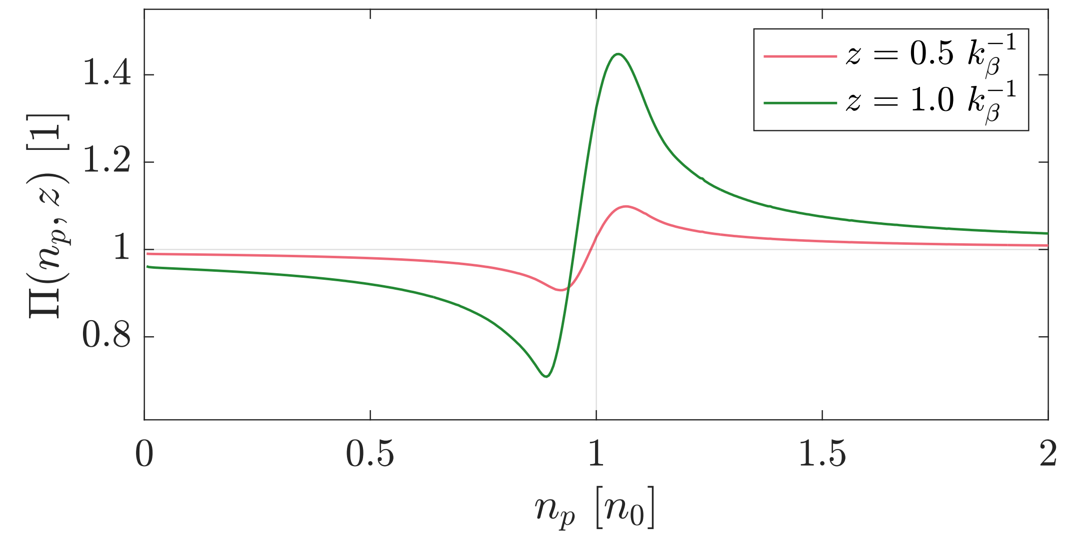
Conclusions

The hosing growth rate depends on the perturbation wavelength

- the amplitude response evolves along the propagation
- the amplitude "spectrum" can be probed via plasma density detuning (such as a density step)



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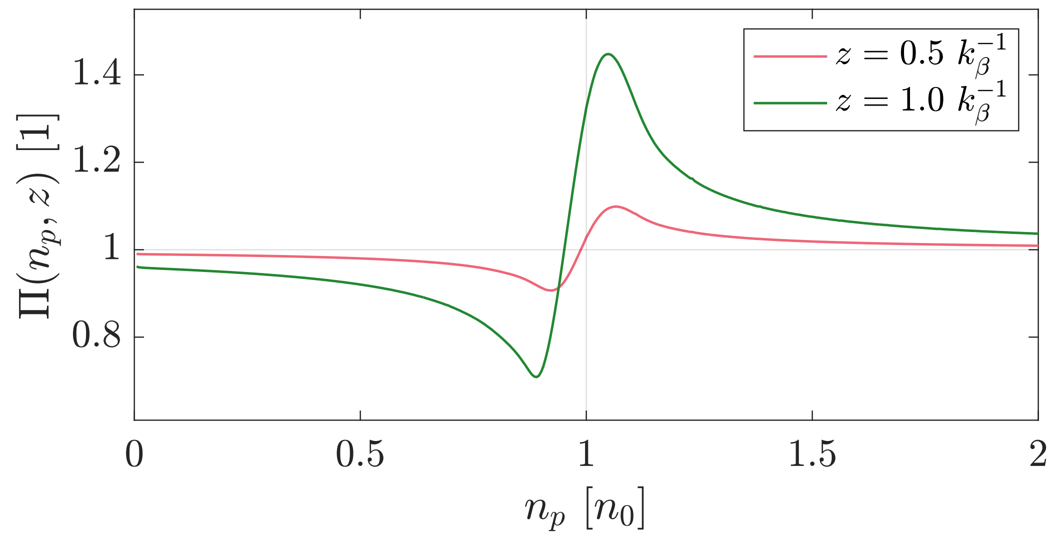

There is a particular amplitude response early in the development of hosing

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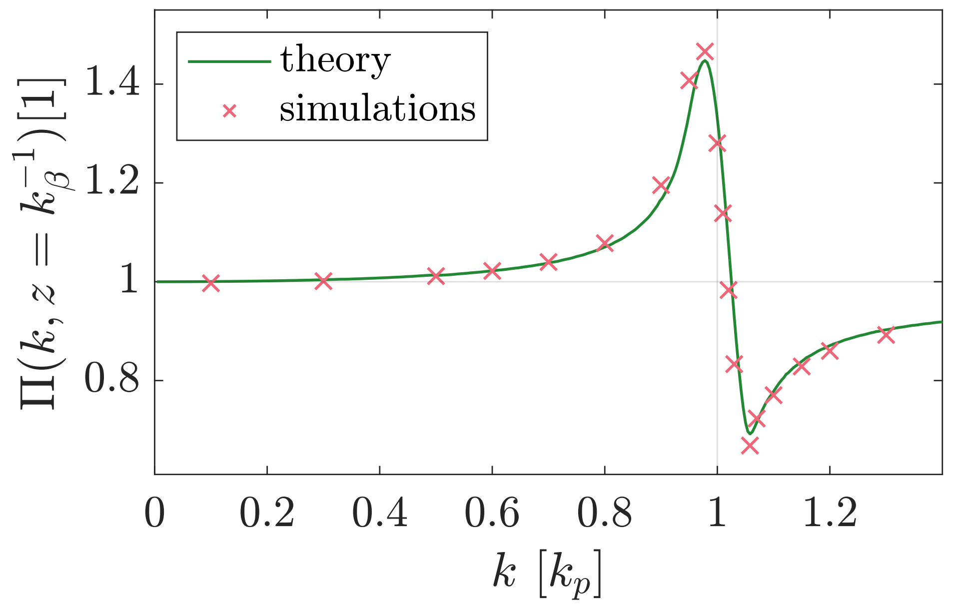
- these growth regimes are associated with a characteristic phase shift between the radius and the plasma response



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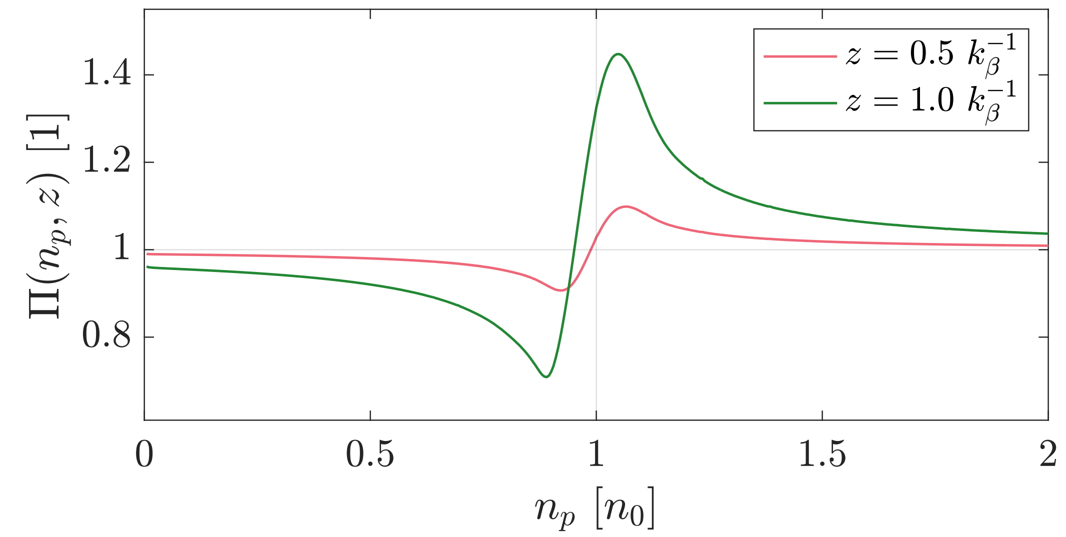

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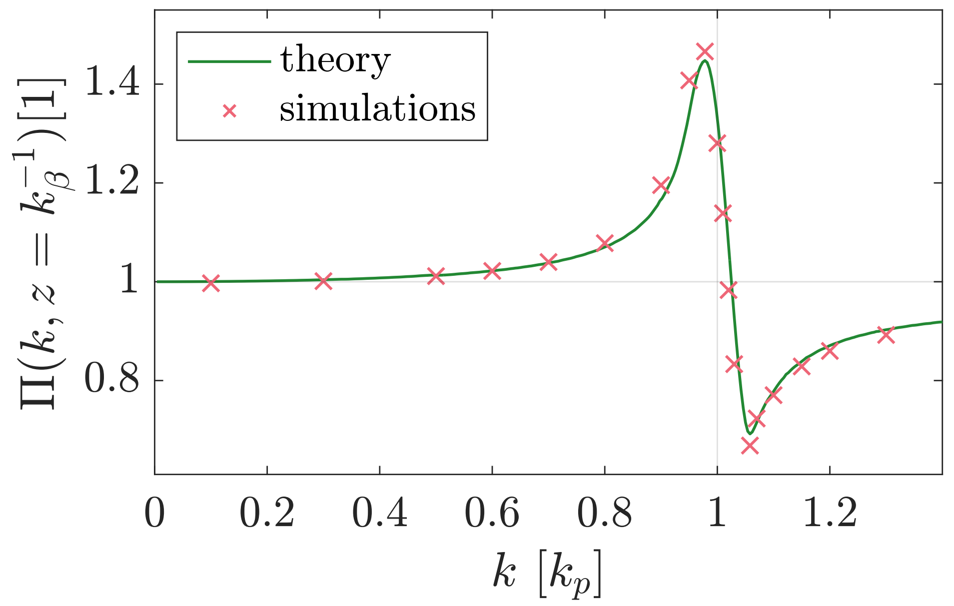
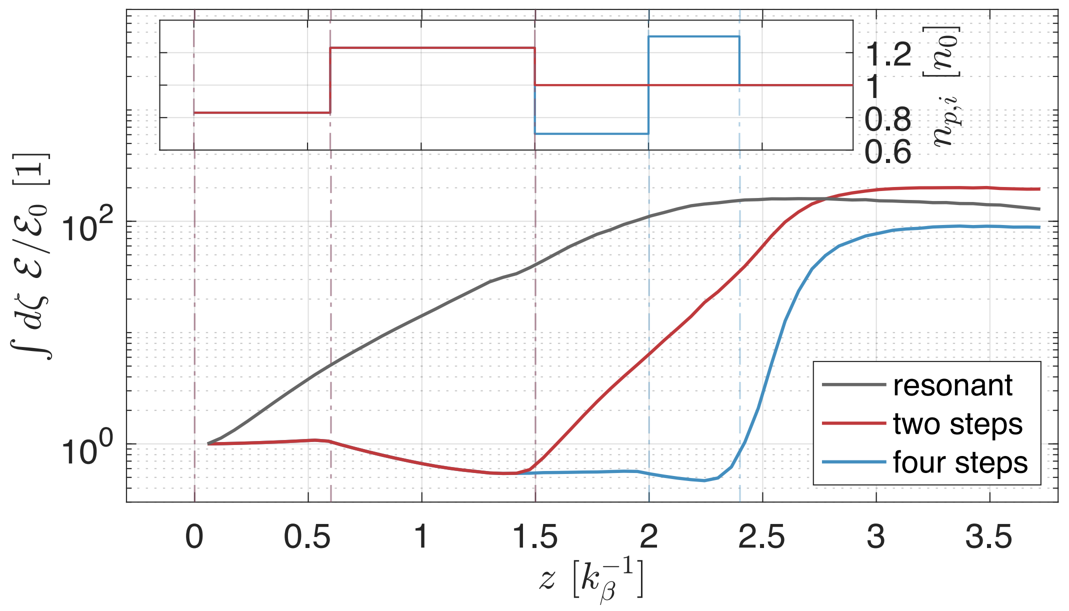


A hosing seed can be suppressed through a series of plasma density steps

- however, set-up may significantly impact the wakefield amplitude driven by a self-modulated bunch
- implications for the control of the growth of transverse beam-plasma instabilities in general



⇒ **For more information:** https://doi.org/10.48550/arXiv.2207.14763

https://www.physik.uni-konstanz.de/vs/themen/mechanik/schwingung-resonanz/resonanz/phasensprung-bei-resonanz/