FAST MODELS FOR COLLECTIVE EFFECTS IN LINEAR ACCELERATORS

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Abstract

The applications envisioned for advanced linear accelerator-based facilities rely on the production of intense particle beams delivered at high repetition rates. Indeed, the demanding brightness and luminosity foreseen by electron driven radiation sources and linear colliders, respectively, imply the coexistence of high peak currents and small transverse emittances. The acceleration of such beams in high gradient machines exposes charged particles to a mutual parasitic interaction which is caused by the excitation of wakefields acting either within the same bunch or among different bunches. Moreover, electron beams produced by rf-photoinjectors enter the main linac with energies in the 4-6 MeV range which implies a non-negligible sensitivity to space-charge effects. The presence of the aforementioned self-induced fields may dilute the phase space quality and, thus, their effect has to be investigated carefully in order to ensure the design performance. Beam dynamics studies including collective effects typically require significant numerical resources and, therefore, we present here reliable methods to describe such processes by means of quasi-analytical approaches that simplify the computation. Such models are embedded in a custom tracking code that provides a fast simulation tool for the dynamics of electron linacs in presence of space charge forces and wakefield effects.

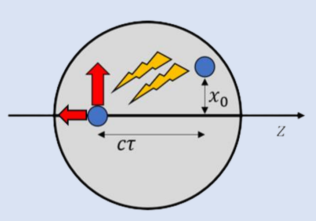
MODELING WAKEFIELD EFFECTS

Sources of wakefields in linacs

- Longitudinal monopole and transverse dipole higher order modes (**HOMs**) in the accelerating cavities are responsible for the **long-range** wakefield (LRWF) interaction ($\omega_n = \omega_0\sqrt{1-(2Q)^{-2}}$ and $\alpha = \omega_0/2Q$) [1,2]
- Longitudinal monopole and transverse dipole impedance from **diffraction** theory describe the **short-range** wakefield (SRWF) interaction in periodic accelerating structures [3-6]

$$\Delta\mathbf{p} = \int \mathbf{F}(\mathbf{r},\mathbf{r}_0,t)|_{z=vt-s}\,dt \qquad w_\parallel = -\frac{c}{qq_0}\Delta p_z \qquad \mathbf{w}_\perp = \frac{c}{qq_0}\Delta\mathbf{p}_\perp$$

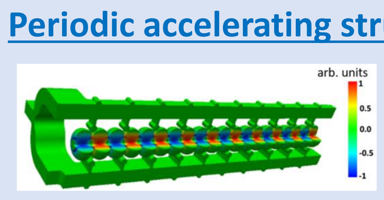
Modes in resonant cavities

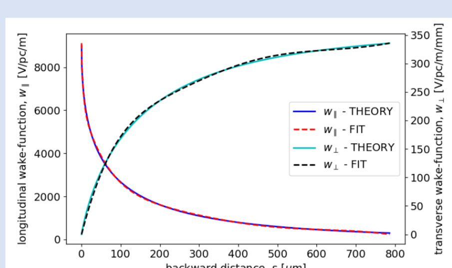


$$w_\parallel(\tau) = \frac{\omega_0 R_\parallel}{Q}e^{-\alpha\tau}\left(\cos(\omega_n\tau) - \frac{\alpha}{\omega_n}\sin(\omega_n\tau)\right)$$

$$w_\perp(\tau) = \frac{\omega_0 R_\perp}{Q}e^{-\alpha\tau}\sin(\omega_n\tau)$$

Periodic accelerating structures



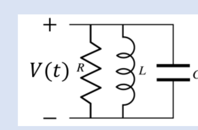


$$w_\parallel(s) = \frac{Z_0 c}{\pi a^2}\exp\left(-\sqrt{\frac{s}{s_0}}\right)$$

$$w_\perp(s) = \frac{4Z_0 c s_1}{\pi a^4}\left(1-\left(1+\sqrt{\frac{s}{s_1}}\right)\exp\left(-\sqrt{\frac{s}{s_1}}\right)\right)$$

Wakefield Matrix Formalism

- Interaction of a charged particle with a cavity resonant **mode**
- The change in momentum (energy/deflection) is described in terms of the equivalent shunt RLC-circuit **voltage**



$$c\Delta p_z = c\int F_z\,dt = -qq_0 w_\parallel(\tau) \doteq -qV_\parallel(\tau)$$

$$c\Delta\mathbf{p}_\perp = c\int \mathbf{F}_\perp\,dt = qq_0\boldsymbol{\rho}_0 w_\perp(\tau) \doteq q\mathbf{V}_\perp(\tau)$$

Evolution of the voltage state

$$\begin{pmatrix}V(t)\\ \dot V(t)\end{pmatrix} \Rightarrow \mathcal{T}(\cdot) = \mathbf{p} + M(\cdot)$$

Perturbation:
Kick induced by the passing charges

$$\mathbf{p}_\mathbf{i}^{(\parallel)} = q\frac{\omega_0 R_\parallel}{Q}\begin{pmatrix}1\\ -\omega_0/Q\end{pmatrix} \qquad \mathbf{p}_\mathbf{i}^{(\perp)} = q\omega_n\frac{\omega_0 R_\perp}{Q}x_i\begin{pmatrix}0\\ 1\end{pmatrix}$$

Free-evolution:
Homogenous 2nd order differential equation

$$\ddot V(t) + 2\alpha\dot V(t) + \omega_0^2 V(t) = 0$$

$$\begin{pmatrix}V(t)\\ \dot V(t)\end{pmatrix} \mapsto \begin{pmatrix}V(t+\tau)\\ \dot V(t+\tau)\end{pmatrix} = M(\tau)\begin{pmatrix}V(t)\\ \dot V(t)\end{pmatrix}$$

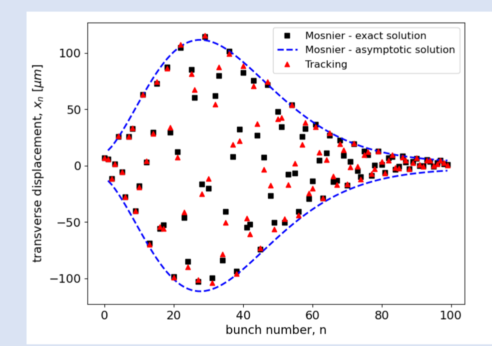
$$M(\tau) = e^{-\alpha\tau}\begin{pmatrix}\cos\omega_n\tau + \frac{\alpha}{\omega_n}\sin\omega_n\tau & \frac{1}{\omega_n}\sin\omega_n\tau \\ -\frac{\omega_0^2}{\omega_n}\sin\omega_n\tau & \cos\omega_n\tau - \frac{\alpha}{\omega_n}\sin\omega_n\tau\end{pmatrix}$$

Recursive formula

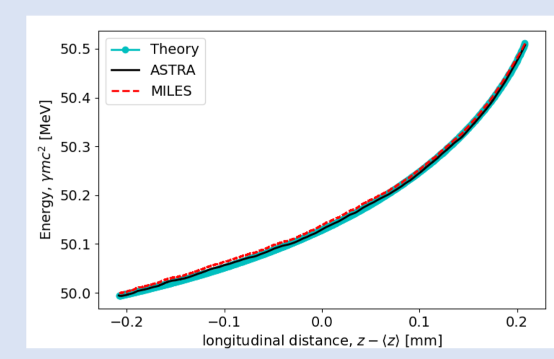
- Voltage **state** after the passage of n charges re-exciting the mode [7-9]
- N particles $\Rightarrow N-1$ operations instead of $N(N-1)/2$ (convolution with the wake-function)

$$\mathcal{T}_n\mathcal{T}_{n-1}\cdots\mathcal{T}_1(\cdot) = (\mathbf{p_n}+M_n)(\mathbf{p_{n-1}}+M_{n-1})\cdots(\mathbf{p_1}+M_1)(\cdot) \Rightarrow \begin{pmatrix}V_n\\ \dot V_n\end{pmatrix} = \mathbf{p_n} + M(\tau_n-\tau_{n-1})\begin{pmatrix}V_{n-1}\\ \dot V_{n-1}\end{pmatrix}$$

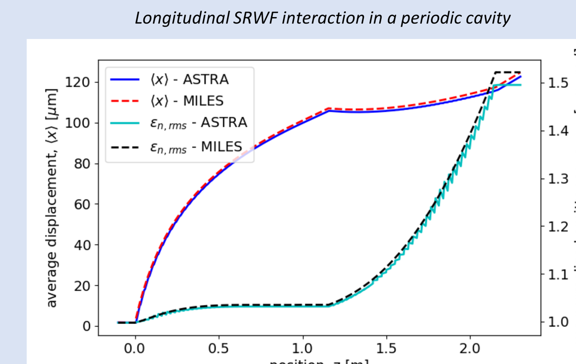
Benchmark Tests



Dipole LRWF in multi-bunch operation. Comparison with Mosnier's theory [10]



Longitudinal SRWF interaction in a periodic cavity



Dipole SRWF interaction in a misaligned two section linac with $\Delta x_1 = 100\ \mu m$ *and* $\Delta x_2 = 0$

MODELING SPACE CHARGE EFFECTS

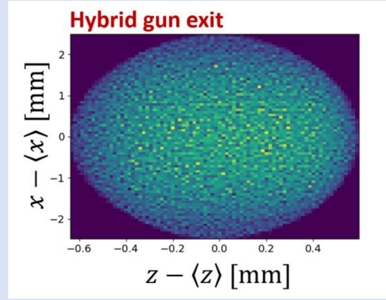
Ellipsoidal beam distributions

- The **electrostatic field** (beam-frame K') produced by an uniform ellipsoid of charge is known analytically: for a point inside the ellipsoid [11-13]

$$\mathcal{E}' : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z'^2}{c'^2} = 1 \qquad \begin{cases} a &= \sigma_x\sqrt{5} \\ b &= \sigma_y\sqrt{5} \\ c' &= \gamma\sigma_z\sqrt{5}\end{cases}$$

$$\Phi'(x,y,z') = \frac{3Q}{16\pi\epsilon_0}\int_0^{+\infty}\left(1 - \frac{x^2}{a^2+t} - \frac{y^2}{b^2+t} - \frac{z'^2}{c'^2+t}\right)\frac{dt}{\sqrt{\varphi(t)}} \equiv D_0 - A_0x^2 - B_0y^2 - C_0z'^2$$

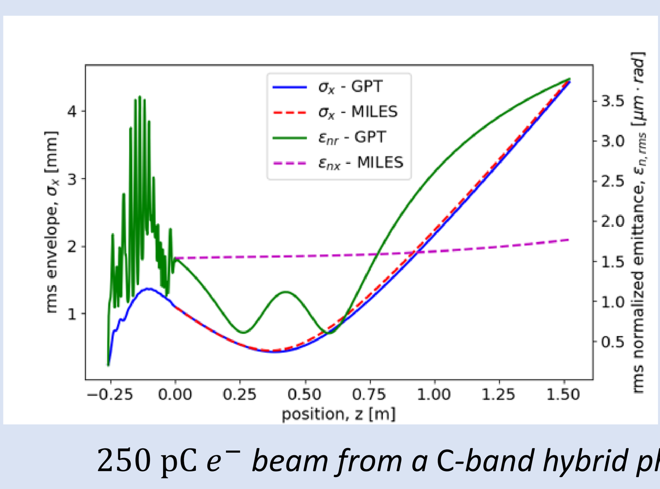
Hybrid gun exit

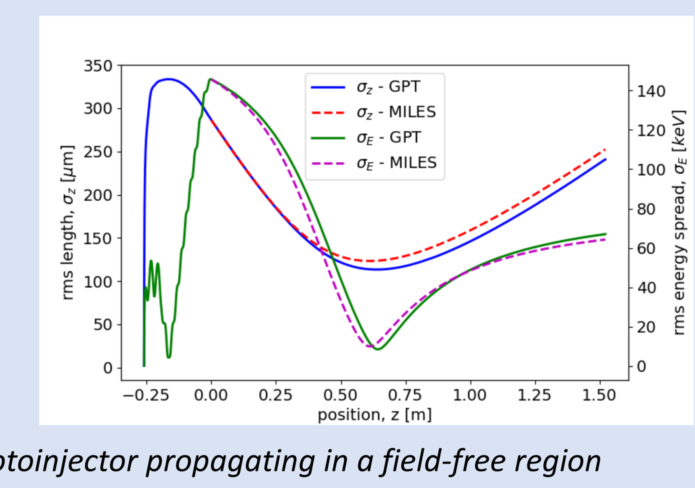


Courtesy of M. Carillo

- A Lorentz transformation provides the **forces** in the laboratory-frame K and thus the corresponding change in **momentum** (it only requires evaluation of the field **coefficients** A_0-C_0)
- Linearity preserves the uniform ellipsoidal distribution: **self-consistent** model

Benchmark test





250 pC e^- *beam from a C-band hybrid photoinjector propagating in a field-free region*

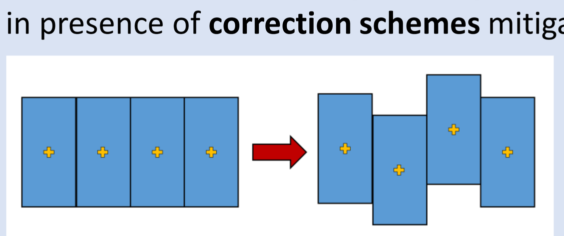
Limitations of the model

Absence of correlation

- The field coefficients A_0, B_0, C_0 are not **slice dependent**
- **Emittance dynamics** induced by slices with different equilibrium-like solutions or plasma frequencies cannot be described thoroughly (*e.g.* double *minimum* [14], *compensation* process [15])

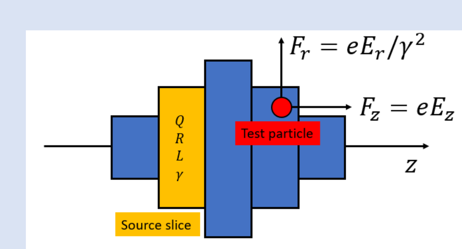
Loss of axial symmetry

- Transverse **wakefields** displace the slice centroids breaking the axial symmetry implied by the ellipsoid
- Not valid in case of **strong-BBU** regime but still useful in presence of **correction schemes** mitigating the BBU



Set of cylindrical beam slices

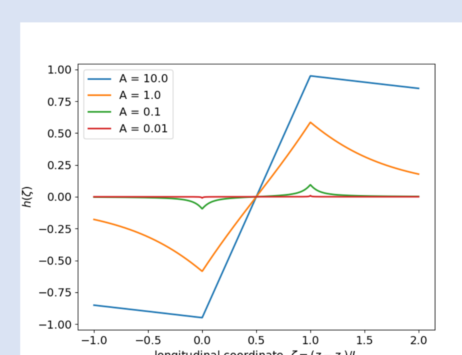
- Divide the beam in **cylindrical slices** with individual size (R, L), energy (γmc^2) and aspect ratio $(A = R/\gamma L)$
- Each slice produces a **field** in the surrounding space (assumptions: axisymmetric slice, $\partial/\partial\theta = 0$, and near-axis motion, $\partial E_z/\partial r = 0$) [16-18]
- Each particle experiences a force given by the **superposition** of the fields produced by all the slices

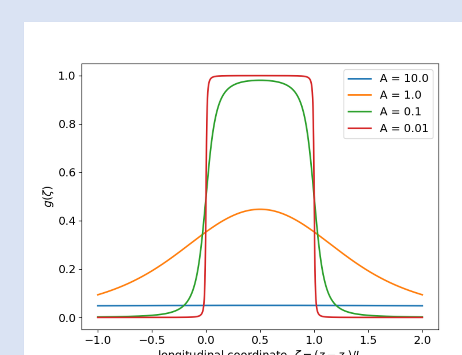


$$E_z(0,\zeta) = \frac{Q}{2\pi\epsilon_0 R^2}h(\zeta, A) \qquad E_r(r,\zeta) = \frac{Qr}{2\pi\epsilon_0 R^2 L}g(\zeta, A)$$

$$h(\zeta, A) = |\zeta| - |1-\zeta| + \sqrt{A^2+(1-\zeta)^2} - \sqrt{A^2+\zeta^2}$$

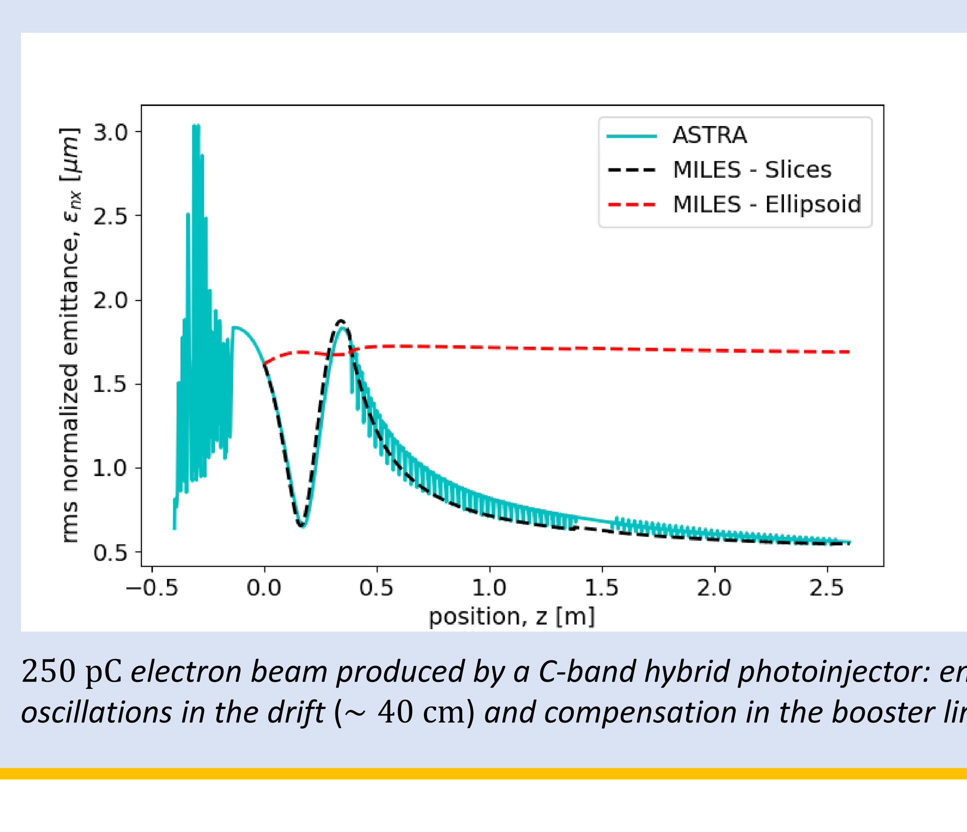
$$g(\zeta, A) = \frac{1}{2}\left[\frac{1-\zeta}{\sqrt{A^2+(1-\zeta)^2}} + \frac{\zeta}{\sqrt{A^2+\zeta^2}}\right]$$





Benchmark test

- Emittance **compensation** process: a beam is produced by a hybrid gun and matched to its invariant envelope in a booster linac
- The model successfully describes the evolution of the rms **correlated** emittance



250 pC *electron beam produced by a C-band hybrid photoinjector: emittance oscillations in the drift* (~ 40 cm) *and compensation in the booster linac*

Comments

- The model **overcomes** both limitations and allows for more **arbitrary** beam shapes
- Simulation **times** increase compared to ellipsoid method (still faster than PIC)
- Emittance dynamics are expected at **low energy** (highly space charge-dominated beams)
- Possibility of **hybrid** simulations: cylindrical slices for low energy regime (up to ~100-150 MeV) and ellipsoidal model for higher energies

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