# A semi-analytical method to calculate wakefield from electron beams passing through dielectric-coated circular waveguides

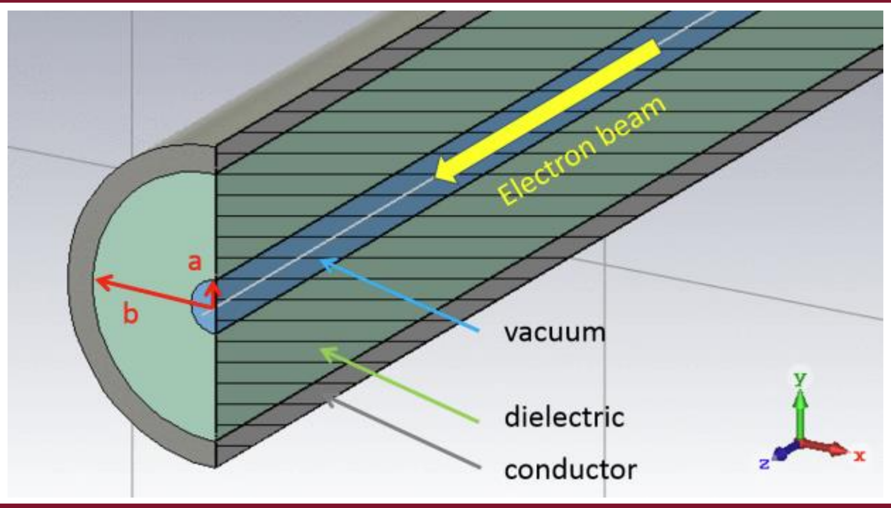
**D. Francescone**[1,2] †, F.Bosco[1,2], M. Carillo[1,2], G. J. Silvi[1,2], L. Giuliano[1,2], L. Ficcadenti[1,2] A. Mostacci[1,2], L. Palumbo[1,2], and M. Migliorati[1,2]
[1]La Sapienza University of Rome, Rome, Italy
[2]INFN-Sez. Roma1, Rome, Italy
†daniele.francescone@uniroma1.it

*This poster presentation has received support from the European Union's Horizon 2020 Research and Innovation programme under Grant Agreement No 101004730.

## Abstract

The dielectric accelerator is one of the most advanced accelerator concept, in which the ultra high accelerating field can be excited by either optical to infrared laser or ultrashort relativistic electron bunches. The beam driven dielectric wakefield accelerators (DWFA) make use of the electromagnetic Cherenkov radiation (wakefield) from the electron bunches that pass through the dielectric-lined waveguides. These high gradient fields may create strong instabilities on the beam itself causing issues in plasma acceleration experiments (PWFA), plasma lensing experiments and in recent beam diagnostic applications. We propose a semi-analytical method to calculate these high gradient fields without resorting to time consuming simulations. Ultra-relativistic bunches traveling in these dielectric capillaries can interact only with TM0n modes that travel at the speed of light. Any perturbation can be written as sum of potentially excitable harmonics, with amplitudes dictated by the shape of the power spectrum of the beam. By executing 2D simulations on a generic section of the structure it is possible to calculate the dispersion diagram of the modes and estimate the frequencies at which these modes can be excited. Finally, with these frequencies it is possible to calculate the coefficients with which to multiply the amplitudes of the corresponding harmonics

# Partially Dielectric Filled Circular Waveguide

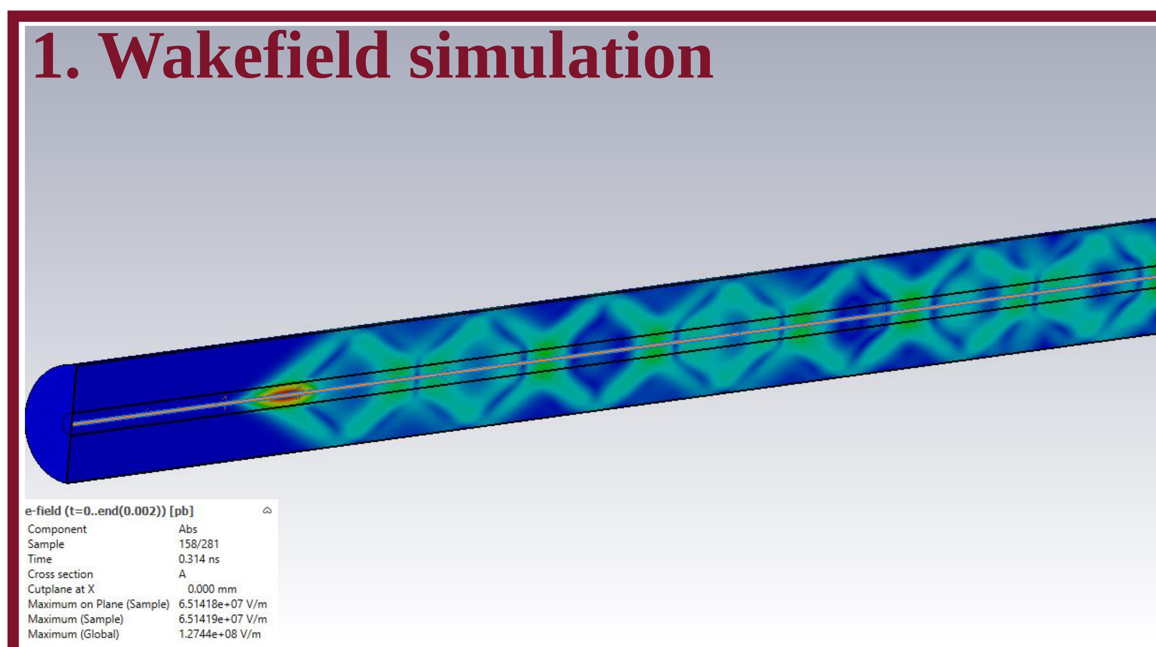



## Geometry

- Circular waveguide
- a = 0.5 mm
- b = 2.5 mm
- $\varepsilon_0 = vacum$
- $\varepsilon_r = 4.82$
- PEC cover the structure

## Two ways to obtain electric field on axis

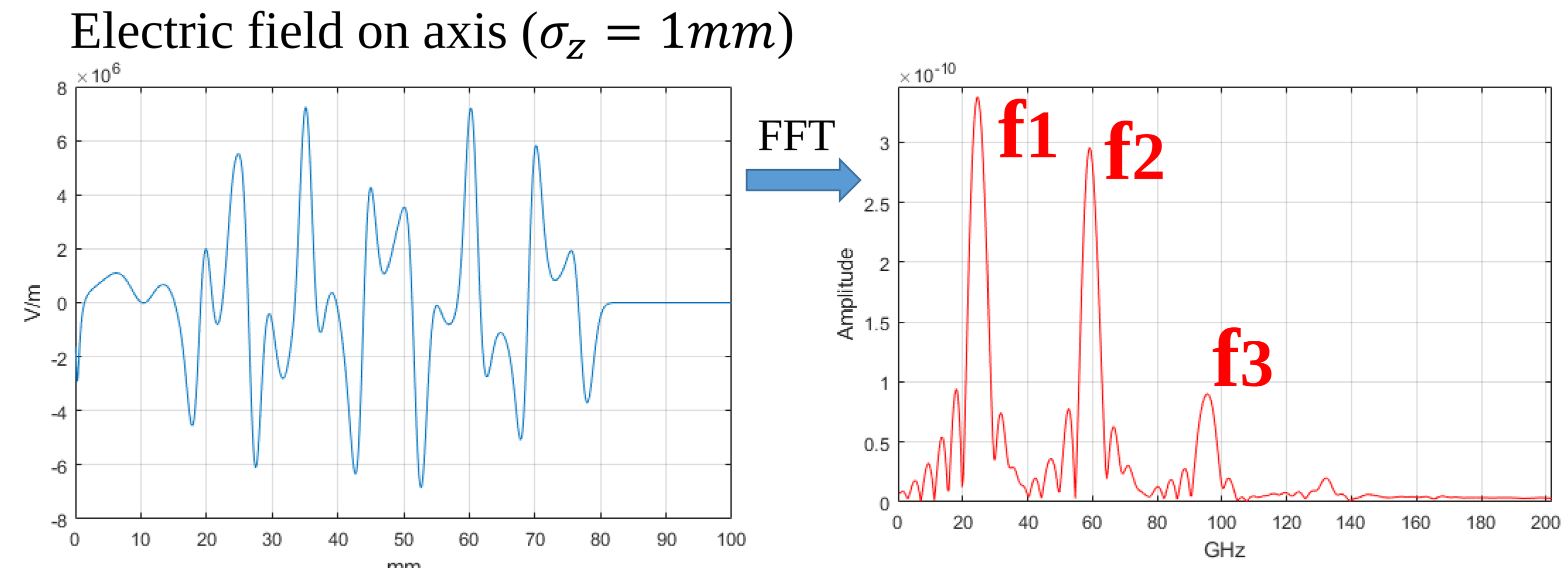
## 1. Wakefield simulation



- CST Simulation in **wakefield mode**
- The pulse excitation signal is gaussian with $\sigma_z = 1\ mm$
- $\beta$ = 1 (relativistic beta)
- L= 100 mm
- It excited an Electric field on axis **Ez** wich is composed mainly by the first three TM0n modes
- Time consumption Strongly dependent on the geometry, pulse length, mesh and time steps

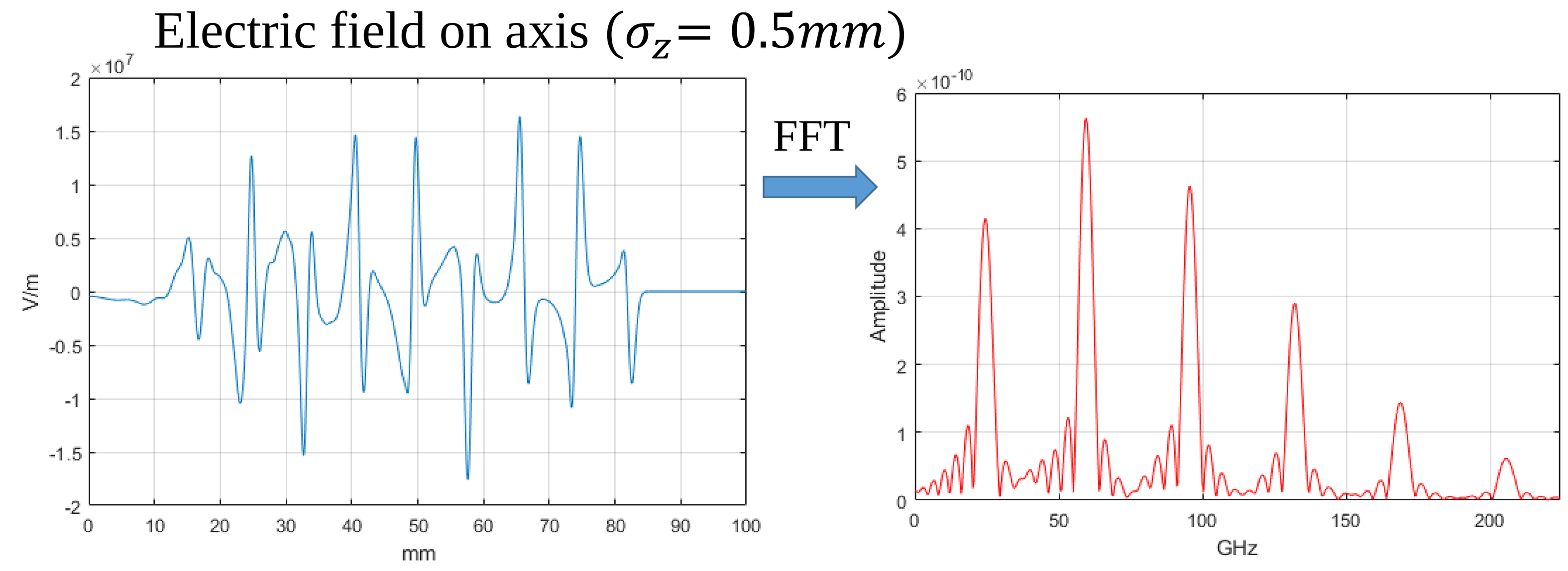
Electric field on axis ($\sigma_z = 1mm$)

- **Ez on axis**
Electric field on axis stimulated by a gaussian particle beam ($\sigma_z$= 1 $mm$)




Electric field on axis ($\sigma_z$= 0.5$mm$)

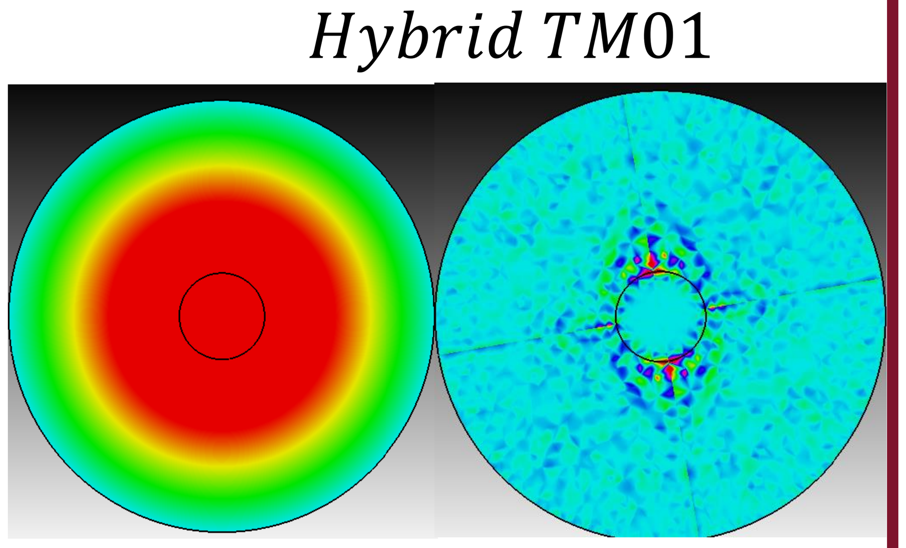
- **Ez on axis**
Electric field on axis stimulated by a gaussian particle beam ($\sigma_z$= 0.5 $mm$)




## 2. Electromagnetic simulation

- **Classical electromagnetic 2D simulations**:
Simulate the modes only on the waveguide port in frequency domain
- **We are only interested in TM0n modes**
Beam on axis it can couple only with TM0n modes
- **Excitation of hybrid modes**
Unlike empty waveguides, in this case, due to the dielectric, hybrid modes are excited.

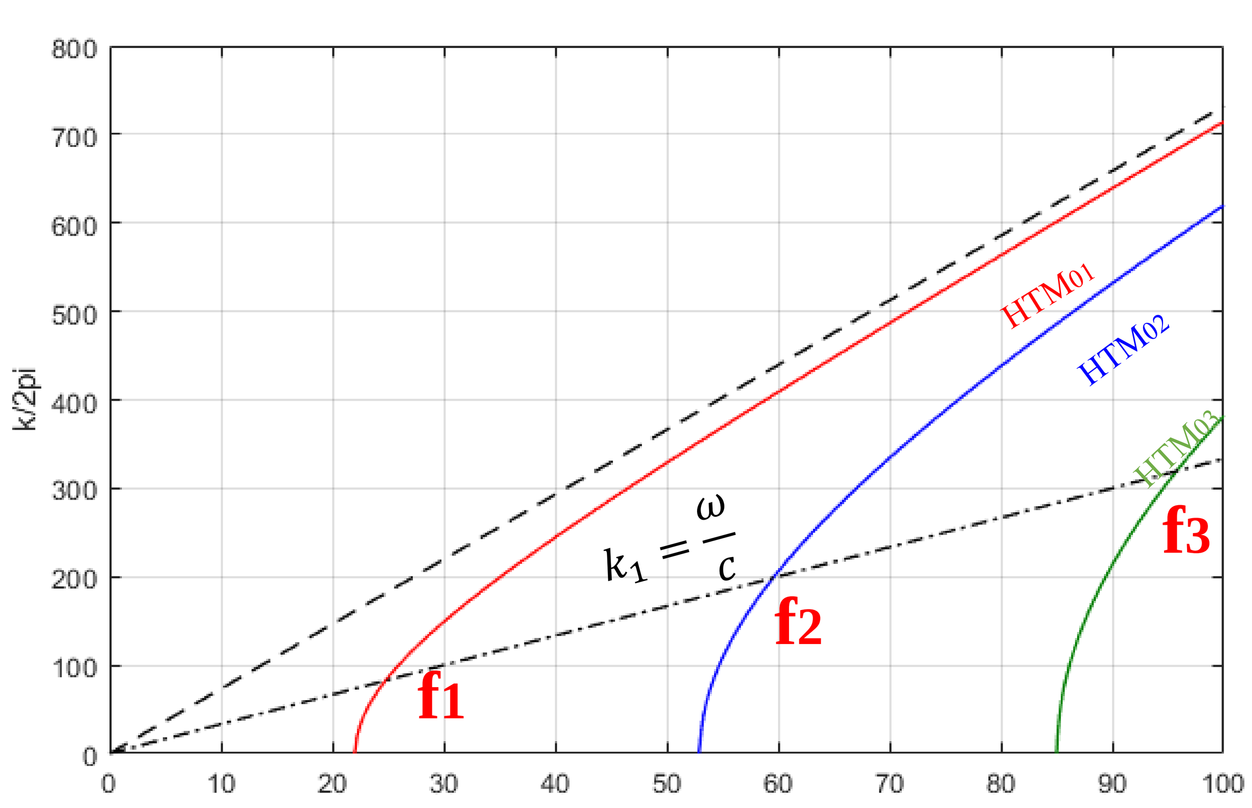
$Hybrid\ TM01$



$E_z \neq 0$ $H_z \neq 0$

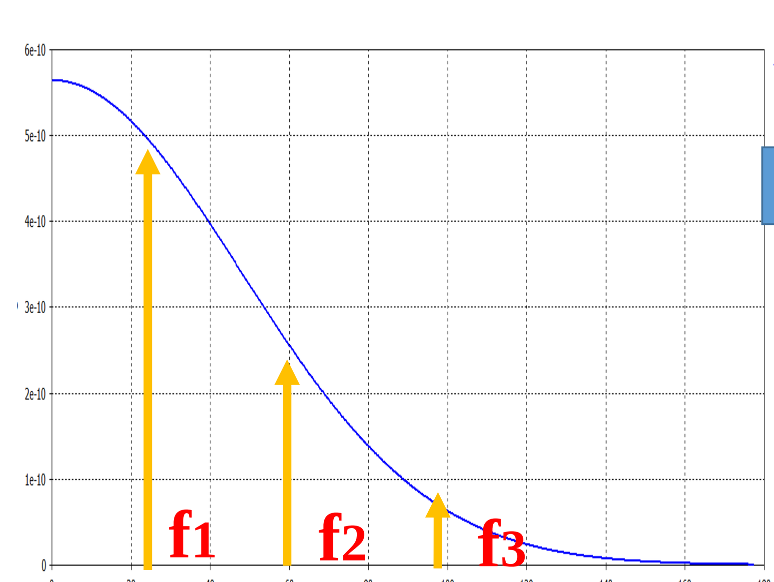
A generic hybrid TM mode could have a longitudinal component of the magnetic field

Dispersion diagram




- **Case of study**
If we consider a simulation with a frequency range from 0 to 100 GHz, we set the correct planes of symmetry, we obtain a list of excited modes of which we only have to take the frequencies of the TM modes (f1,f2,f3), with phase velocity c
- **Weighing of modes**
Once the frequencies f1, f2, f3 have been obtained, they must be weighted with the spectrum of the excitation signal

Spectrum of exctation signal



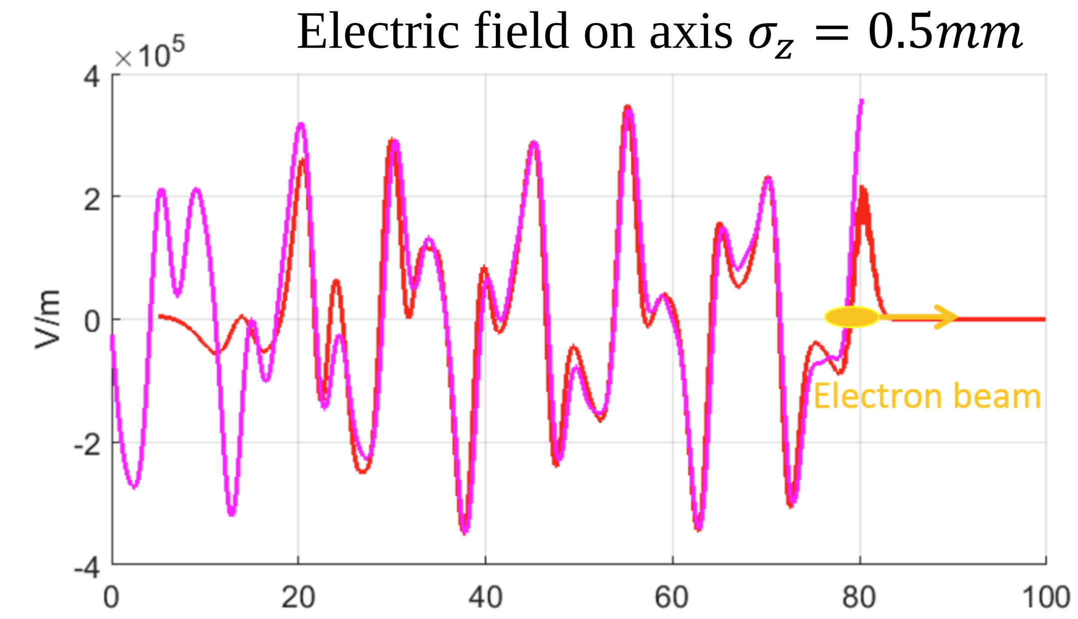

## Electric field on axis reconstruction

- **Sum of modes**
To reconstruct the electric field, a sum of the modes at the various frequencies (f1, f2, f3) is carried out, weighed by the amplitudes of the modes themselves and by the spectrum of the excitation signal

f1 f2 f3

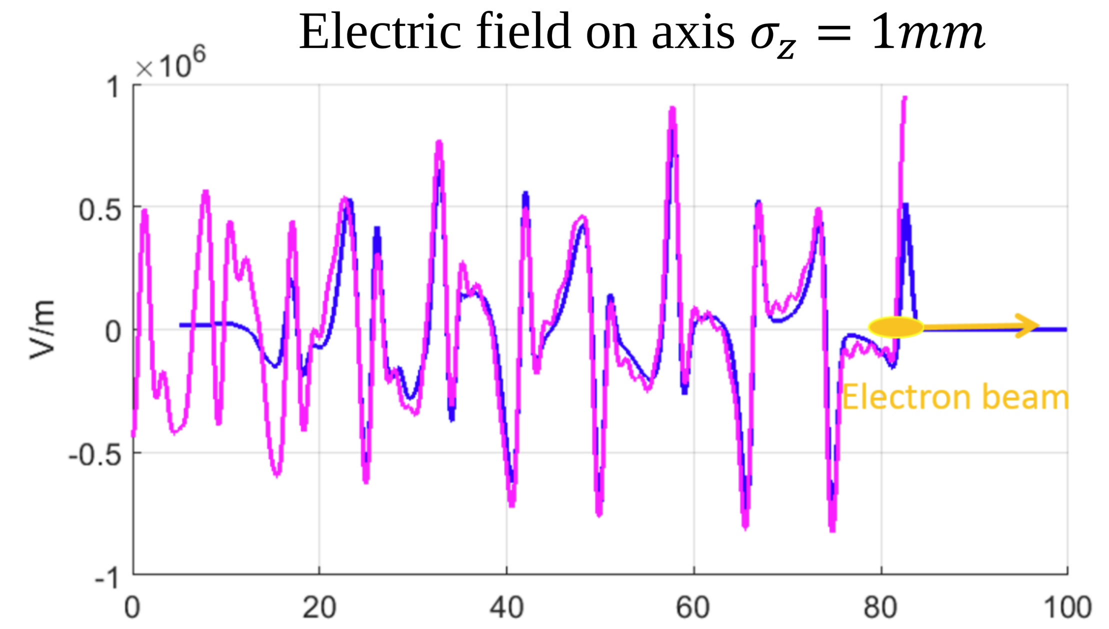
$$E = A\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{mn}(a, b, r_0, \sigma_z, \varepsilon_r, \omega_r)\cos\left(\frac{\omega_{mn}}{c} z_0\right)$$

Electric field on axis $\sigma_z = 0.5mm$




Ez behind the electron beam with a σz = 1 mm. PIC results (red), sum of the first three TM0n modes (magenta)

Electric field on axis $\sigma_z = 1mm$




Ez behind the electron beam with a σz = 0.5 mm. PIC results (blue), sum of the first five TM0n mode (magenta).

[1] L. Ficcadenti et al, '' Longitudinal and transverse wakefield simulations and studies in dielectric-coated circular waveguides'', IPAC2018, Vancouver, BC, Canada
[2] A. Biagioni et al., "Wake field effect in dielectric capillary", Nuclear Inst. and Methods in Physics Research, A, Article In Press, Jan. 2018