# Early dynamics of the self-modulation instability growth rate

M. Moreira[1]*, P. Muggli[2,3], and J. Vieira[1]

[1] GoLP/Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, Lisbon, Portugal
[2] CERN, Geneva, Portugal
[3] Max-Planck Institute for Physics, Munich, Germany

*mariana.t.moreira@tecnico.ulisboa.pt

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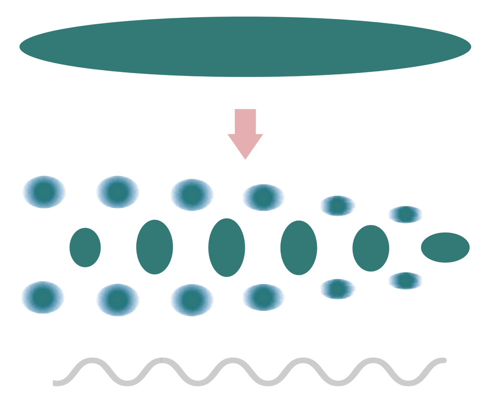
## Motivation

- Concepts for single-stage, TeV-scale **plasma wakefield acceleration (PWFA)** rely on long particle bunches as the driver
- When a long ($L \gg \lambda_p$) particle bunch propagates in plasma, it is subject to the **self-modulation instability (SMI)**
- The SMI typically modulates the bunch radius **at the plasma wavelength** $\lambda_p$
- **Self-modulation** can be seeded **(SSM)** to avoid instability and to generate high-amplitude wakefields, as in the case of the AWAKE experiment [1]



$\sigma_r(\zeta, z)$

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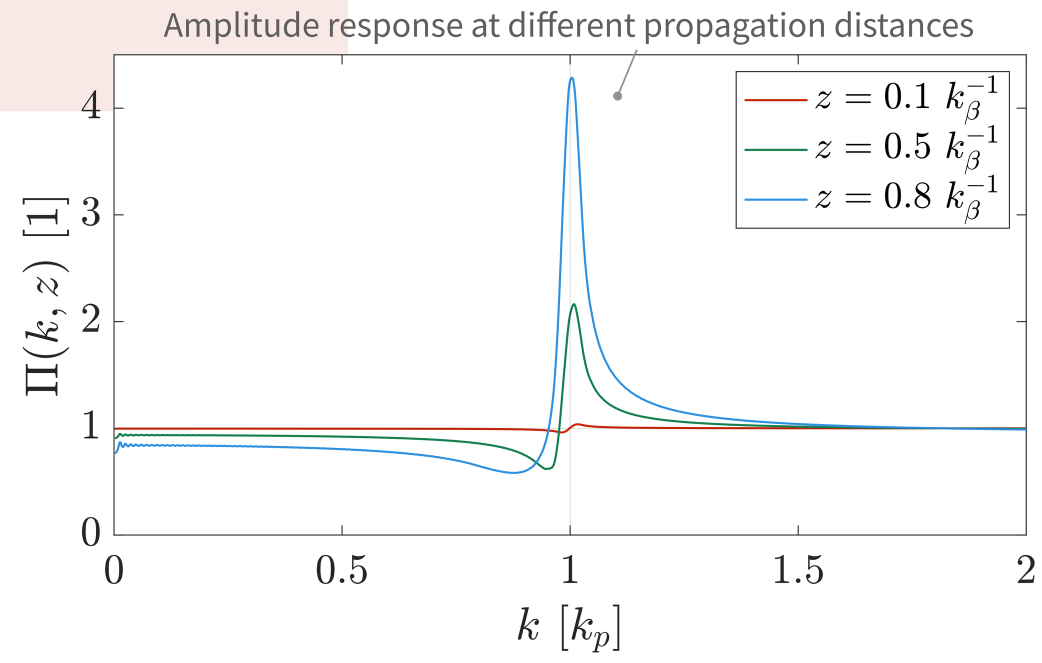
## How does the growth rate depend on the seed frequency?

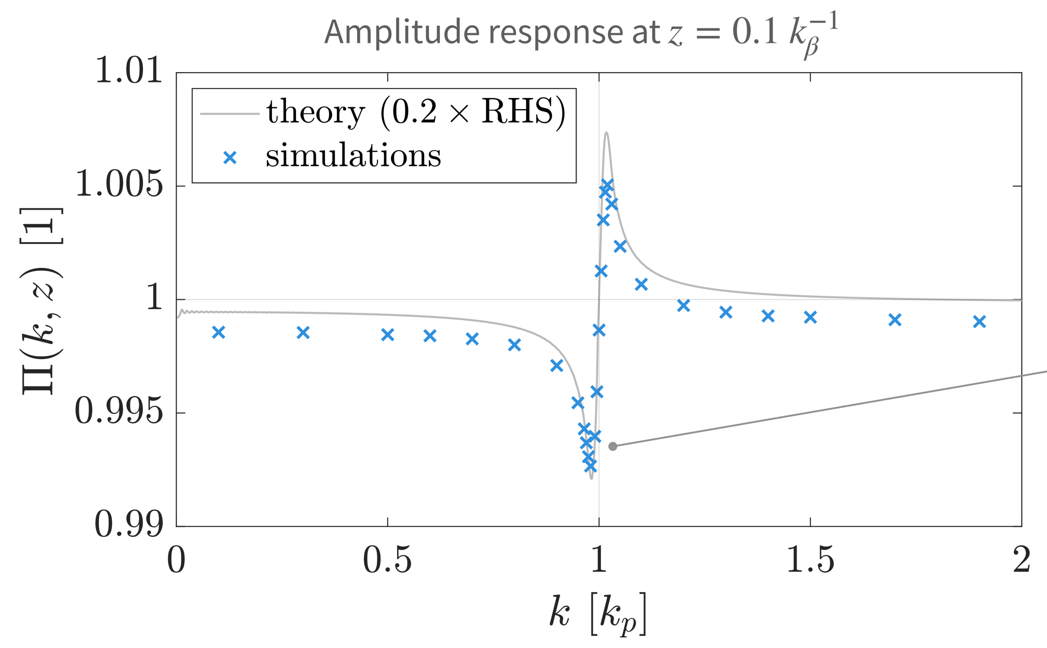
**Approach:**

- introduce an initial **sinusoidal radius perturbation** at $k$: $\Delta\sigma_{r0}(\zeta) \propto \sin(k\,\zeta)$
- obtain **evolution** of the radius perturbation $\Delta\sigma_r(\zeta, z)$
- measure an **amplitude response**: $\Pi(z) \propto \dfrac{\Delta\sigma_r(\zeta, z)}{\Delta\sigma_{r0}(\zeta)}$

the growth rate eventually peaks at $k_p$, as expected




- the growth rate evolves along the propagation distance
- **damping** [$\Pi(z) < 1$] is possible for certain frequencies




early on, significantly **different growth regimes** are accessible with a small amount of **detuning** (either via $\Delta k$ or $\Delta n_p$)

$$k_\beta^2 = \frac{1}{2\,\gamma_b}\left(\frac{\omega_b}{c}\right)^2 = \frac{1}{2\gamma_b}\,\frac{q_b^2\,n_{b0}}{\epsilon_0\,M_b}\,\frac{1}{c^2}$$

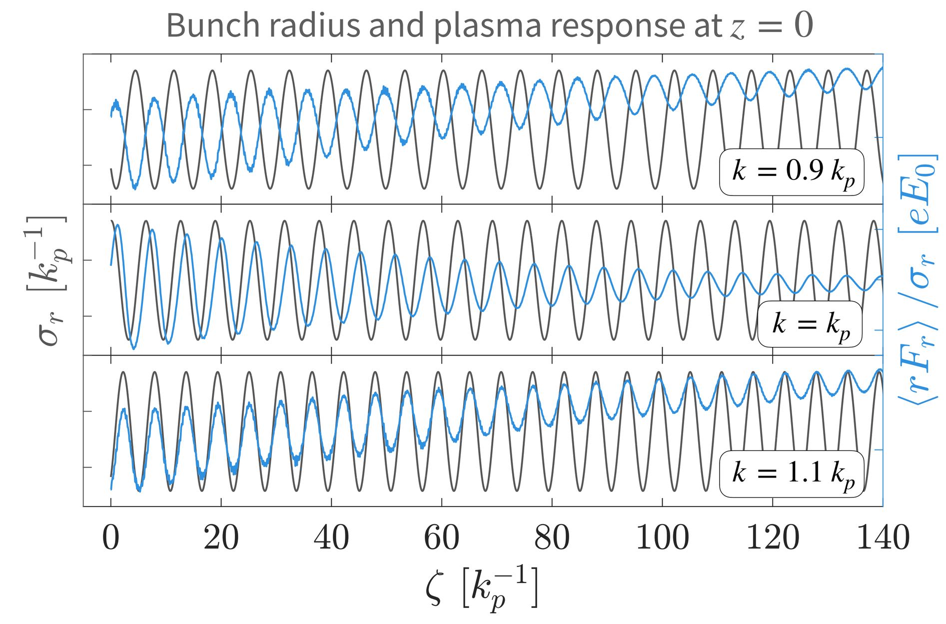
## What determines the growth regime?

- three growth regimes can be identified
- each regime is associated with a phase shift between $\sigma_r$ and the plasma response $\langle r\,F_r\rangle/\sigma_r$

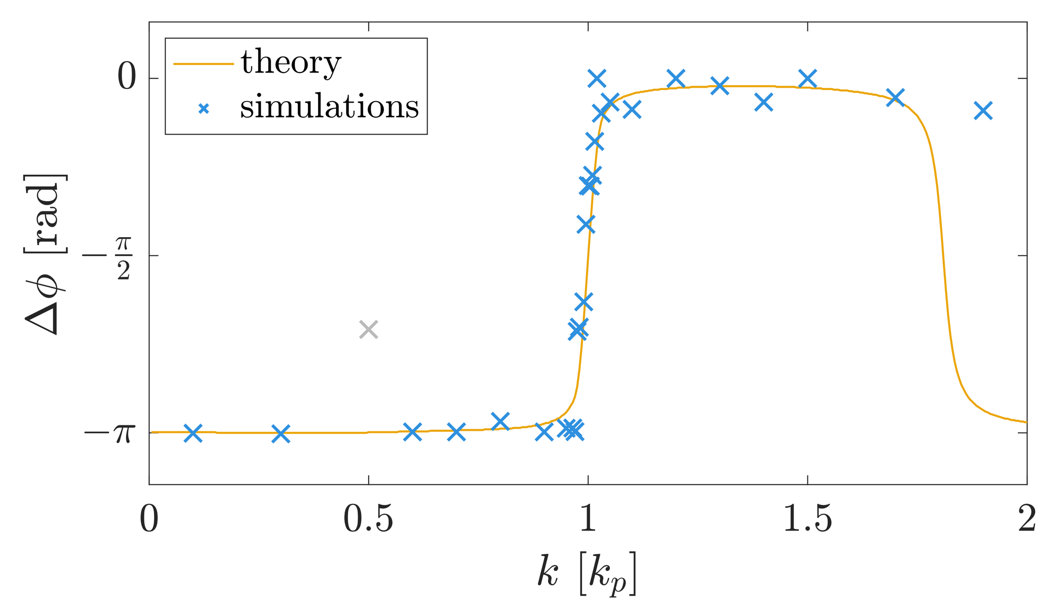
Beam envelope equation (SMI):

$$\frac{d^2\sigma_r}{dz^2} - \frac{1}{4}\frac{\epsilon^2}{\sigma_r^3} = 2\,\frac{m_e}{\gamma_b M_b}\,\frac{\langle r\,F_r\rangle}{\sigma_r}$$

| | | $\Delta\phi$ |
|---|---|---|
| $k < k_p$ | damping | $-\pi$ |
| $k = k_p$ | resonant | $-\pi/2$ |
| $k > k_p$ | slow growth | 0 |




- the phase shift can be **measured** with a cross-correlation method*




* For the theoretical curve, $L$ and $\sigma_z$ are scaled for each $k$ such that the same number of wavelengths is considered in the analysis ($\sim 44\ \lambda_p$).

## Methods

**Theory**

Evolution of radius perturbation:

$$\frac{d^2 r_1}{dz^2} = \mathrm{RHS}(r_1)$$

assuming:

- flat-top transverse profile with radius $r_b$
- small perturbation: $r_b = r_0 + r_1, \quad r_1 \ll r_0$

First-order evolution of $r_1$ (valid for $z \lesssim k_\beta^{-1}$):

$$r_1(\zeta, z) = r_{10} + \mathrm{RHS}(r_{10})\ \frac{1}{2}\ z^2$$

$$\Pi(z) = \frac{\int d\zeta\ |r_1(\zeta, z)|}{\int d\zeta\ |r_{10}(\zeta)|}$$

**Parameters**

$n_0 = 2 \cdot 10^{14}\ \mathrm{cm}^{-3}$
$\gamma_b = 427$
$\sigma_r = 200\ \mu\mathrm{m} \quad \approx 0.53\ k_p^{-1}$
$\sigma_z = 12\ \mathrm{cm} \quad \approx 320\ k_p^{-1}$
$M_b = 50\ m_e \quad \Rightarrow k_\beta^{-1}/k_p^{-1} \approx 1500$

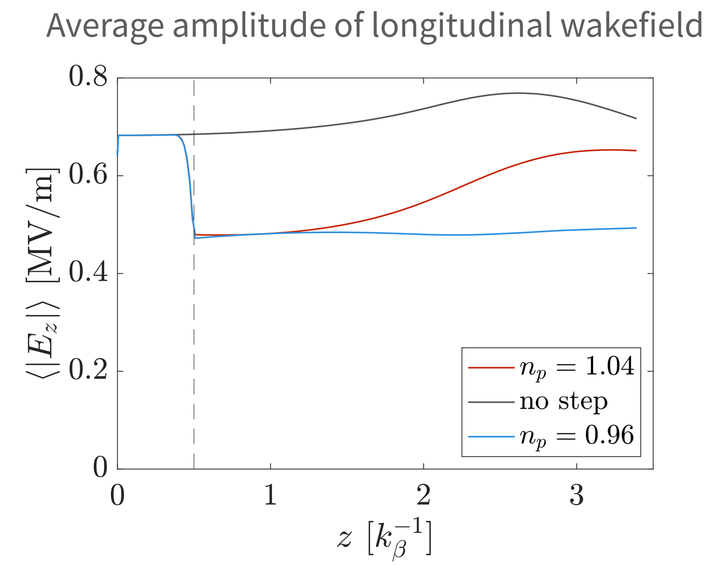
**Simulation**

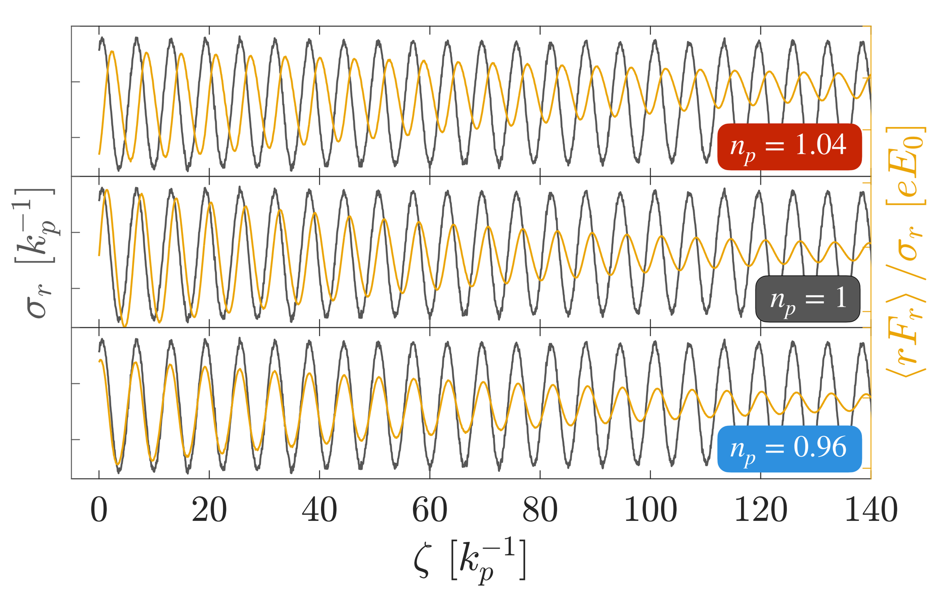
- **2D cylindrical** simulations seeded at different $k$'s $\Rightarrow \sigma_{r,k}(\zeta, z)$
- one simulation without perturbation $\Rightarrow \sigma_{r,\mathrm{adiab}}(\zeta, z)$

$$\Pi(z) = \frac{\int d\zeta\ |\sigma_{r,k}(\zeta, z) - \sigma_{r,\mathrm{adiab}}(\zeta, z)|}{\int d\zeta\ |\sigma_{r,k}(\zeta, 0) - \sigma_{r,\mathrm{adiab}}(\zeta, 0)|}$$

## Application: wakefield amplitude optimization

- a plasma **density step** has been proposed to solve the problem of a falling wakefield amplitude after saturation of the SMI in AWAKE [2]




- **effect** of the density step on the **phase shift** is consistent with understanding of the growth rate presented here

## Conclusions

**The growth rate of the SMI is a function of the seed frequency**

- at an early phase, very different growth regimes are accessible with a small amount of detuning
- these growth regimes are associated with a characteristic phase shift between $\sigma_r$ and the plasma response

**These results help us understand why it is possible to control the growth of the SMI with plasma density steps**

- a single density step early in the SMI development shifts the wakefields w.r.t. the bunch radius oscillation

## References

[1] M. Turner *et al.* (The AWAKE Collaboration), Phys. Rev. Lett. **122**, 054801 (2019)

[2] K. V. Lotov, Phys. Plasmas **18**, 024501 (2011); K. V. Lotov, Phys. Plasmas **22**, 103110 (2015)