Lattice Boltzmann Simulations of Plasma Wakefield Acceleration

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Introduction: we explore a novel simulation route for Plasma Wakefield Acceleration (PWFA) by using the computational tool known as the **Lattice Boltzmann Method (LBM)**[1]. LBM is based on a discretization of the continuum kinetic theory while assuring the convergence towards hydrodynamics for coarse-grained fields (i.e., density, velocity, etc.). LBM is an established numerical tool in computational fluid dynamics, **able to efficiently bridge between kinetic theory and hydrodynamics**, but its application in the context of PWFA has never been investigated so far. Our work takes a step forward to fill this gap. Results of LBM simulations for PWFA are discussed and compared with those of a code (Architect[2]) implementing a Cold Fluid (CF) model for the plasma in cylindrical symmetry.

Cold Fluid Model:

- equations for the electron density n_e, momentum $\mathbf{p}_e$ and electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$;
- we consider the ions to be immobile;
- CF approximation neglects thermal effects.

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \left(n_e c \boldsymbol{\beta}_e \right) = 0 \ ;$$

$$\frac{\partial \mathbf{p}_e}{\partial t} + c\boldsymbol{\beta}_e \cdot \nabla \mathbf{p}_e = -e\left(\mathbf{E} + c\boldsymbol{\beta}_e \times \mathbf{B} \right) \ ;$$

$$\boldsymbol{\beta}_e = \frac{\mathbf{p}_e}{m_e c \sqrt{1 + |\mathbf{p}_e / m_e c|^2}} \ .$$

Lattice Boltzmann Method:

- studies the evolution of a kinetic probability distribution function $f_i(x,t)$ to find a "fluid particle" in the position x at the time t with a kinetic velocity ξ_i;
- the velocity space is discretized (i = 0, 1, …, N_{tot} -1) with a finite set of ξ_i;
- the Boltzmann equation predicts the evolution of the $f_i(x,t)$

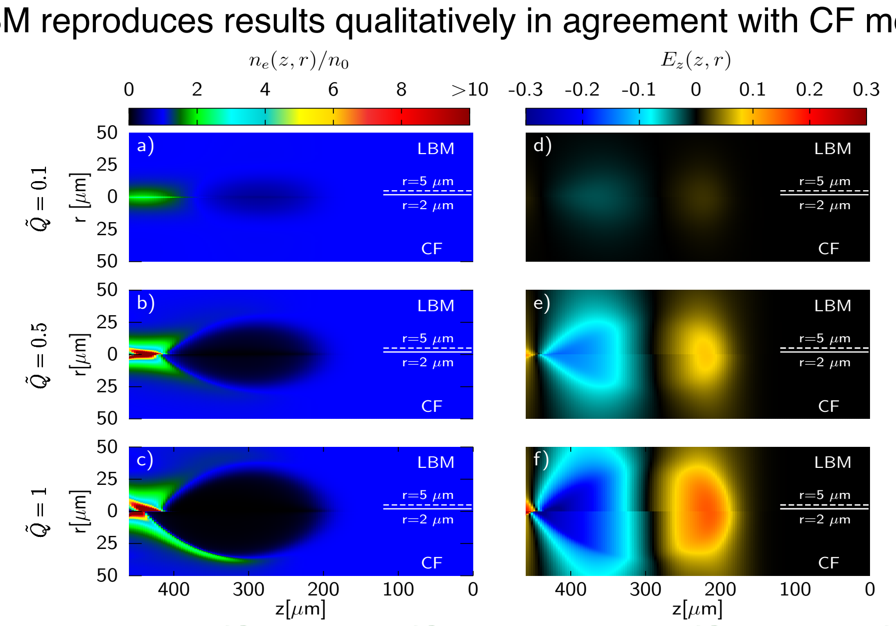
$$f_i(\mathbf{x} + \xi_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = -\frac{\Delta t}{\tau}\left[f_i(\mathbf{x}, t) - f_i^{(eq)}(\mathbf{x}, t) \right];$$

- τ represents the time that $f_i(x,t)$ takes to relax to its equilibrium value $f_i^{(eq)}(x,t)$;
- we use the LBM in its advection-diffusion framework[1];
- diffusion is a built-in property that we can tune through the parameter τ

$$D = \frac{1}{3}\left(\frac{\tau}{\Delta t} - \frac{1}{2} \right)\frac{\Delta x^2}{\Delta t}.$$

Simulations results:

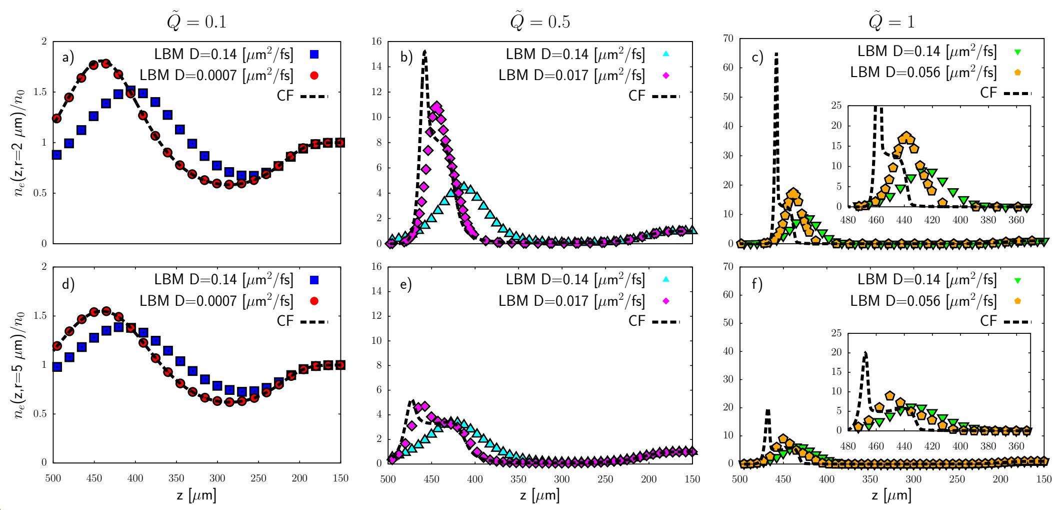
- we choose τ to result in the smallest diffusion parameter that allows for numerical stability for LBM;
- LBM reproduces results qualitatively in agreement with CF model.



Diffusion effects:

- act as a computational regularization for the density singularity predicted by the CF model in the limit of *cold wave breaking;*
- other studies already considered regularization effects[3] using:

1. non-zero temperature in the limit of a 1D model;
2. transverse fluctuations.



Conclusion: LBM introduces diffusion effects in the plasma evolution, differentiating from the CF implementation of the code Architect but still retaining a hydrodynamic character. **The results of simulations support the applicability of LBM up to the onset of the non-linear regime.**
On diffusion effects, peculiar of LBM:

- not considered before because small in early periods of the plasma waves;
- may become necessary in the high repetition rate studies when the behaviour of the plasma waves in late periods has to be understood[4].

Future perspective: modern LBM developments point to **applications that go beyond "strict hydrodynamics"** and are well justified by the fact that LBM is grounded on kinetic theory. The **Relativistic LBM**[5] could be a powerful tool to simulate:

- the hydrodynamics of 3D relativistic fluids, also in deep non-linear regimes;
- thermal effects in the plasma dynamic.

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Acknowledgement: This poster presentation has received support from the European Union's Horizon 2020 Research and Innovation programme under Grant Agreement No 101004730.