

# Simultaneous space-time focusing with radially-chirped laser pulses for ionization injection in LWFA

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## Ionization Injection

Controlled injection methods seek to **reduce shot-to-shot variations and energy spread** of electron bunches injected into laser-driven wakefield accelerators (LWFAs). **Optical ionization injection** utilises additional laser pulses to release electrons within the wakefield.

Simultaneous space-time focusing (SSTF) occurs when a transversely-chirped ultrashort laser pulse is focused using a conventional lens. SSTF ensures that the **shortest pulse duration emerges only at the focal point**, where all frequencies overlap spatially. Here we investigate the use of simultaneous space-time focusing (SSTF) to **localize ionization injection**.

## Laguerre-Gaussian

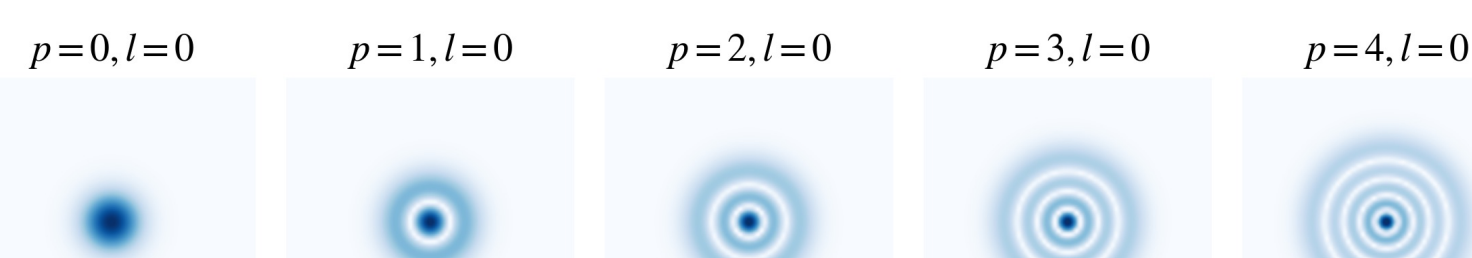
Laguerre-Gaussian beam at any longitudinal position [1]

$$LG_p^\ell(r) = \sqrt{\frac{p!}{\pi(|\ell| + p)!}} \left( -\frac{q^*(z)}{q(z)} \right)^n \frac{e^{-ikr^2/2q(z)}}{r} \left( \frac{i\sqrt{kz_R}r}{q(z)} \right)^{|\ell|+1} L_p^{|\ell|} \left( \frac{kz_R r^2}{|q(z)|^2} \right) e^{-i\ell\phi}$$

where  $q$  is the complex radius of curvature:  $q(z) = z - z_0 + iz_R$   
and  $k$  is the wavenumber, in free-space:  $k = \omega/c$

## Reconstruction of Arbitrary Beams

Arbitrary beam as a sum of Laguerre-Gaussian modes

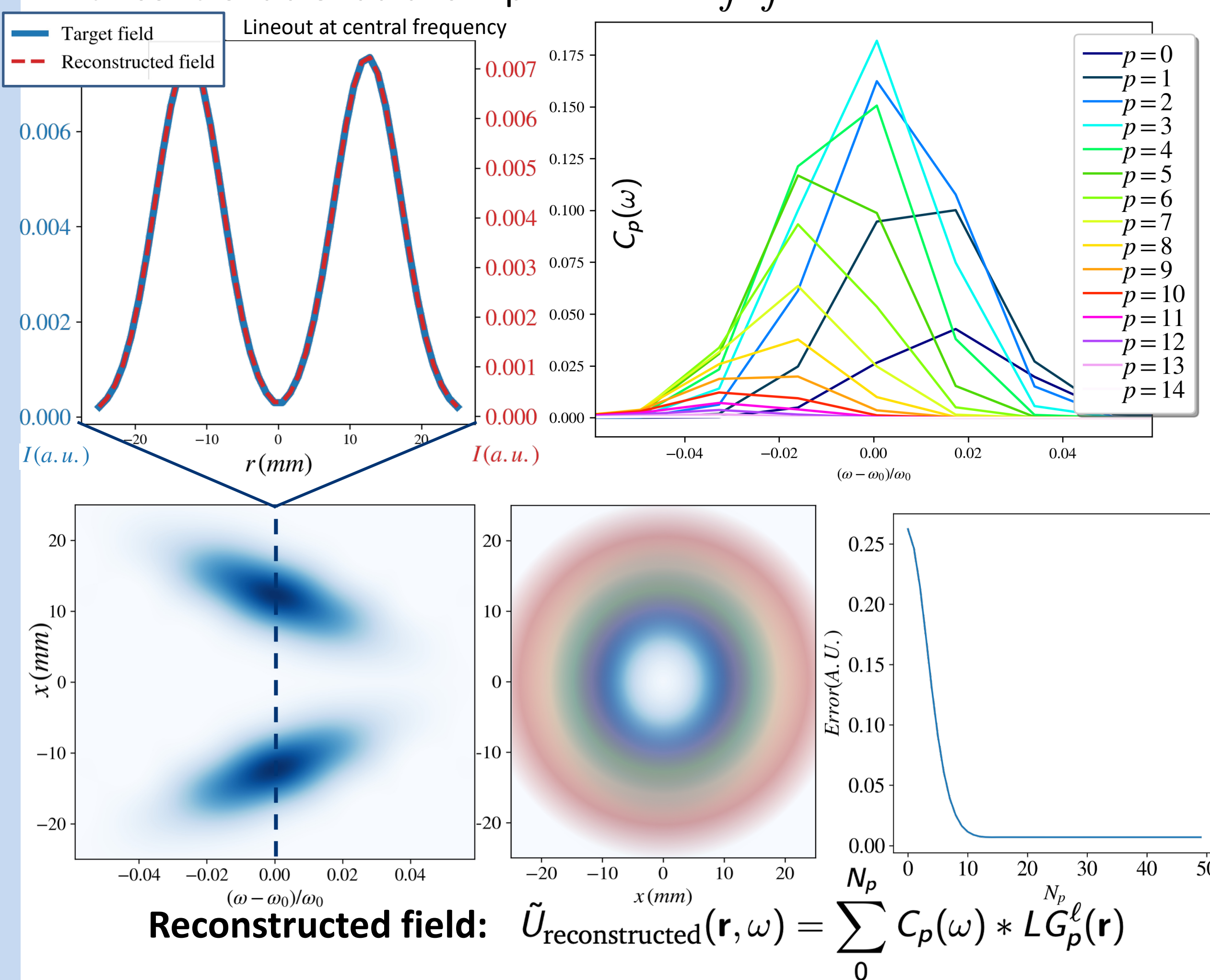


$\ell = 0$  (No OAM)

Overlap integral defines mode contributions:

Target field: Gaussian annulus with controllable radial chirp

$$C_p(\omega) = \iint \tilde{U}_{\text{target}}(\mathbf{r}, \omega) * LG_p^\ell(\mathbf{r}) dx dy$$



Parameters:  $\lambda_0 = 800$  nm,  $\tau_{p,0} = 40$  fs laser pulse with  $f/40$  imaging

## Method 1: Analytic Propagation

$q' = \frac{(A * q + B)}{(C * q + D)}$  Propagate **each** mode downstream [2], sum over  $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f(\omega) & 1 \end{pmatrix}$  weighted modes **Lens**

Additional mode-dependent Guoy phase shift at lens:

$$\dots \times e^{-i(2p+\ell+1) \left[ \psi(0, z_0, z_R) - \psi(0, z'_0, z'_R) \right]} \quad \psi(z, z_0, z_R) = \tan^{-1} \left( \frac{z - z_0}{z_R} \right)$$

## Method 2: Collins' Diffraction Integral

Propagate the superposition of **all** modes downstream directly  $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & z - z_0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f(\omega) & 1 \end{pmatrix}$  **Free-space** **Lens**

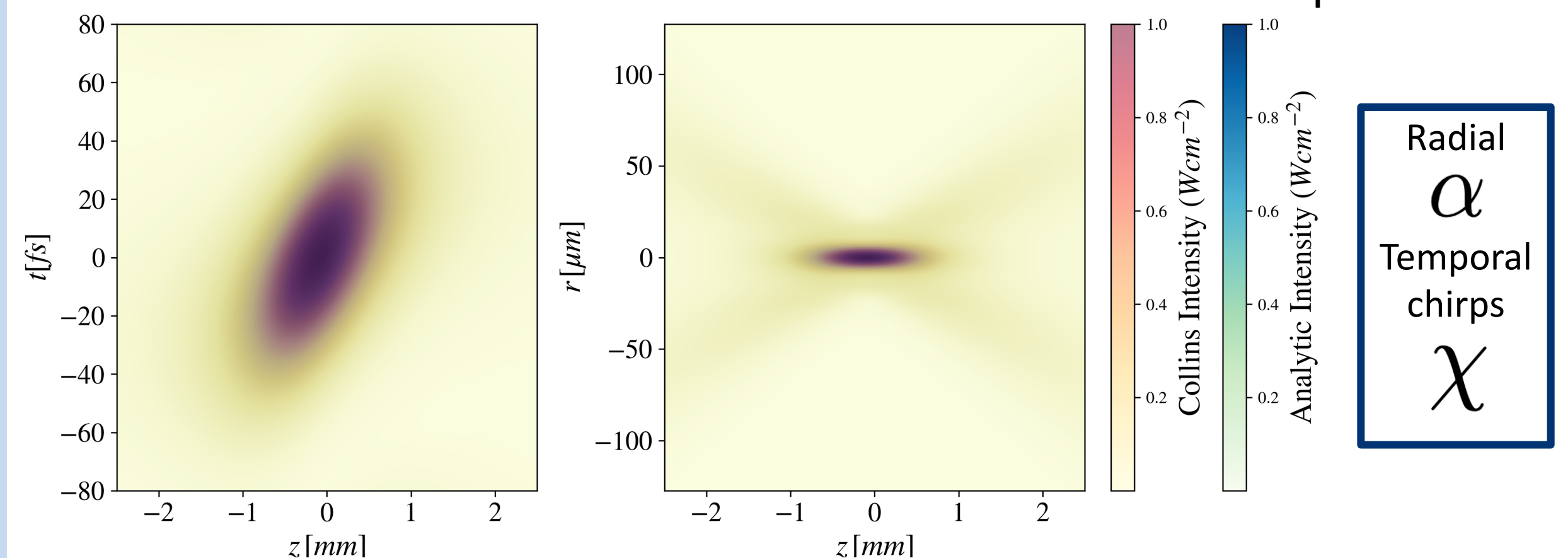
$$\tilde{U}_{\text{out}}(\mathbf{r}, \omega) = \frac{1}{i\lambda B} e^{ik(z-z_0)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{U}_{\text{in}}(\mathbf{r}_0, \omega) e^{ikS} dx_0 dy_0$$

Also based on the ray-matrix [3], diffraction integral calculated numerically via FFT [4]

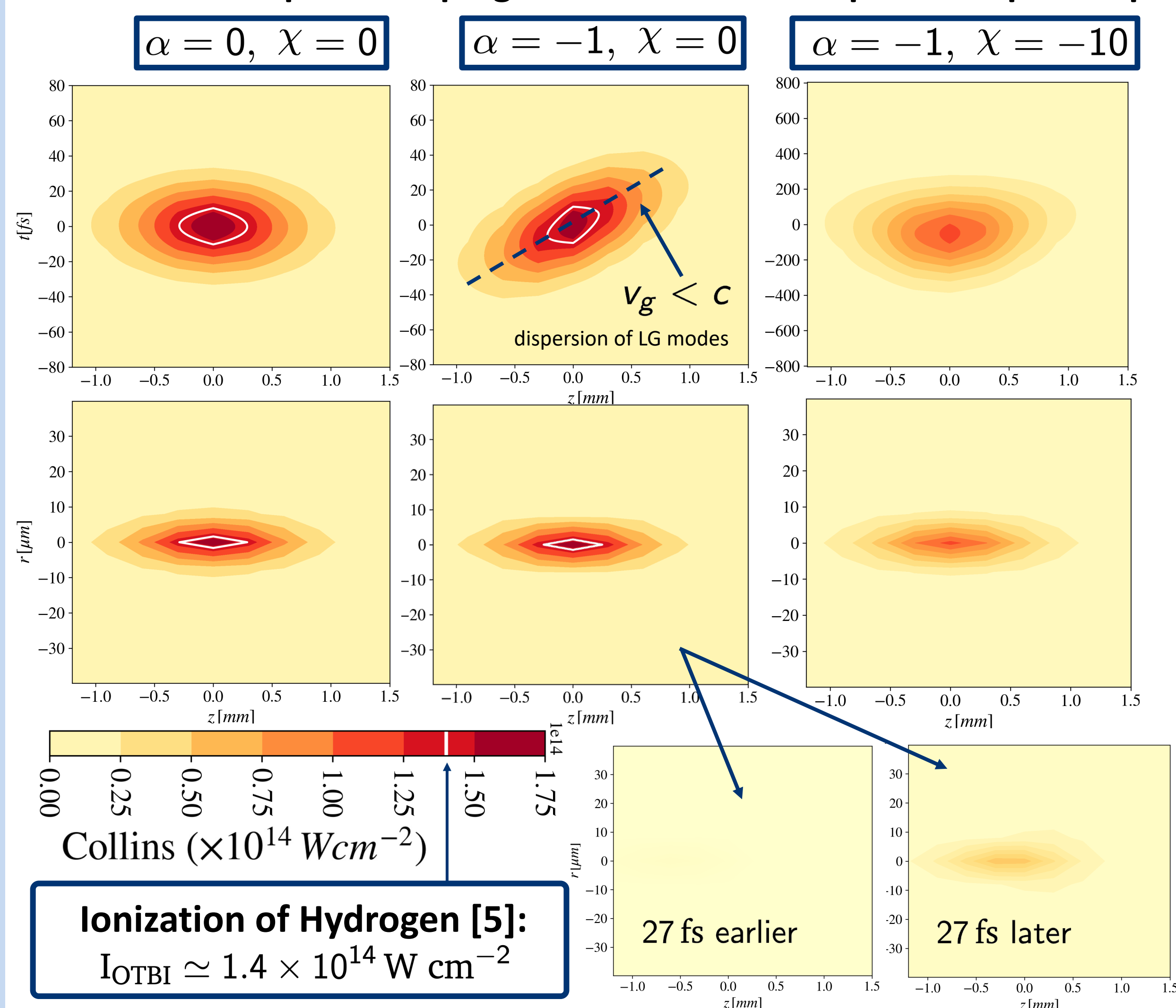
$$S = \left[ \frac{1}{2B} (A x_0^2 + y_0^2) + D(x^2 + y^2) - 2(x x_0 + y y_0) \right]$$

## Results

Normalised on-axis field for both methods:  $< 0.5\%$  least squares error



Collins' results: pulse shaping with radial and temporal chirps of input



## Conclusions and Future Work

Found good agreement between two different methods used to propagate radially-chirped beams. Potentially observed dispersion of structured beams in free-space [6]

Work planned to analyse focal volumes on finer grids and to model ionisation injection by implementing these beams in PIC codes

- [1] Giuseppe Vallone, "Role of beam waist in Laguerre-Gauss expansion of vortex beams," Opt. Lett. 42, 1097-1100 (2017)
- [2] J. P. Taché, "Derivation of ABCD law for Laguerre-Gaussian beams," Appl. Opt. 26, 2698-2700 (1987)
- [3] Stuart A. Collins, "Lens-System Diffraction Integral Written in Terms of Matrix Optics," J. Opt. Soc. Am. 60, 1168-1177 (1970)
- [4] J. W. Goodman, Introduction to Fourier Optics, (2005)
- [5] P. Gibbon, Short Pulse Laser Interactions with Matter (2005)
- [6] Bareza, N., Hermosa, N. Subluminal group velocity and dispersion of Laguerre Gauss beams in free space. Sci Rep 6, (2016)



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