A novel analytical model of space charge forces in RF-guns

M. Carillo[1,4*], M. Behtouei[2], F. Bosco[1,4], E. Chiadroni[1,2,4], L. Faillace[1,2], D. Francescone[1,4], A. Giribono[2], L . Giuliano[1,4], A. Mostacci[1,4], M. Migliorati[1,4], G.J. Silvi[1,4], B. Spataro[2], J. Rosenzweig[3] and L. Palumbo[1,4]

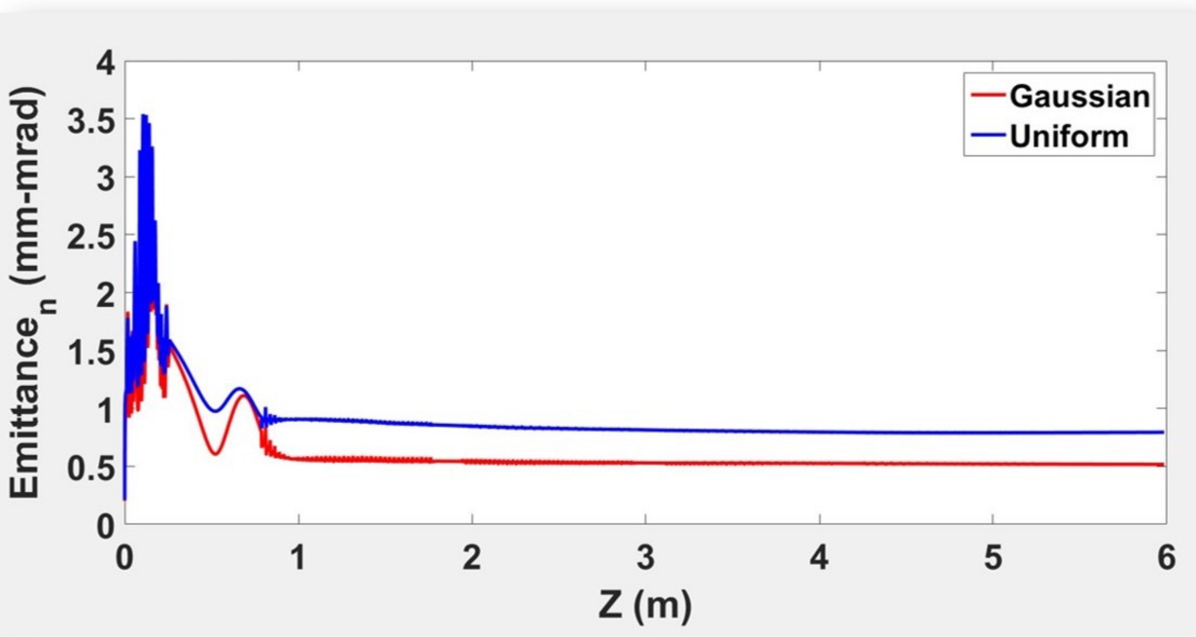
[1] La Sapienza University of Rome, 00161 Rome, Italy; [2] INFN-LNF, 00044 Frascati, Italy; [3] UCLA, Los Angeles, 90095 California, USA; [4] INFN-Sez.Roma1, 00161 Roma, Italy

*martina.carillo@uniroma1.it

Abstract

A novel model of space charge forces is proposed for a low-energy (γ~1) bunch with arbitrary charge distribution in RF-guns. By exploiting Green function method, it is possible to develop an analytical approach and derive expressions of self-induced forces for any transverse and longitudinal bunch distribution. The model is accurate also in the approximation of low energy beams, when the 3D distribution can be decoupled in the two longitudinal and transverse planes. After having reproduced the results known in literature for Gaussian and Uniform charge distribution, we derive space charge forces in parabolic charge distributions. The proposed model highlights the dependence of rms beam emittance on the non-linearities, observed in the transverse phase space due to the space charge forces. Moreover, the model shows that the strongest non-linearities of the transverse phase space are already generated at the cathode when the beam is emitted. Finally, the method allows to include a quadrupolar perturbations on the transverse charge density.

Analytical approach



Inhomogeneous wave equation:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = -\frac{\rho}{\epsilon_0}$$

Green function-based methods can be used to describe arbitrary beam distributions:

$$\phi(r,z) = \int d\tau' G(r,r',z-z')\rho(r',z')$$

Green function for a ring of a unitary charge:

$$-\epsilon_0 \,\square^2\, G(\mathbf{r},\mathbf{r}') = \frac{\delta(r-r')}{2\pi r'}\delta(z-z')$$

Solution in terms of zero-th order modified Bessel function: free-space + image charge contribution:

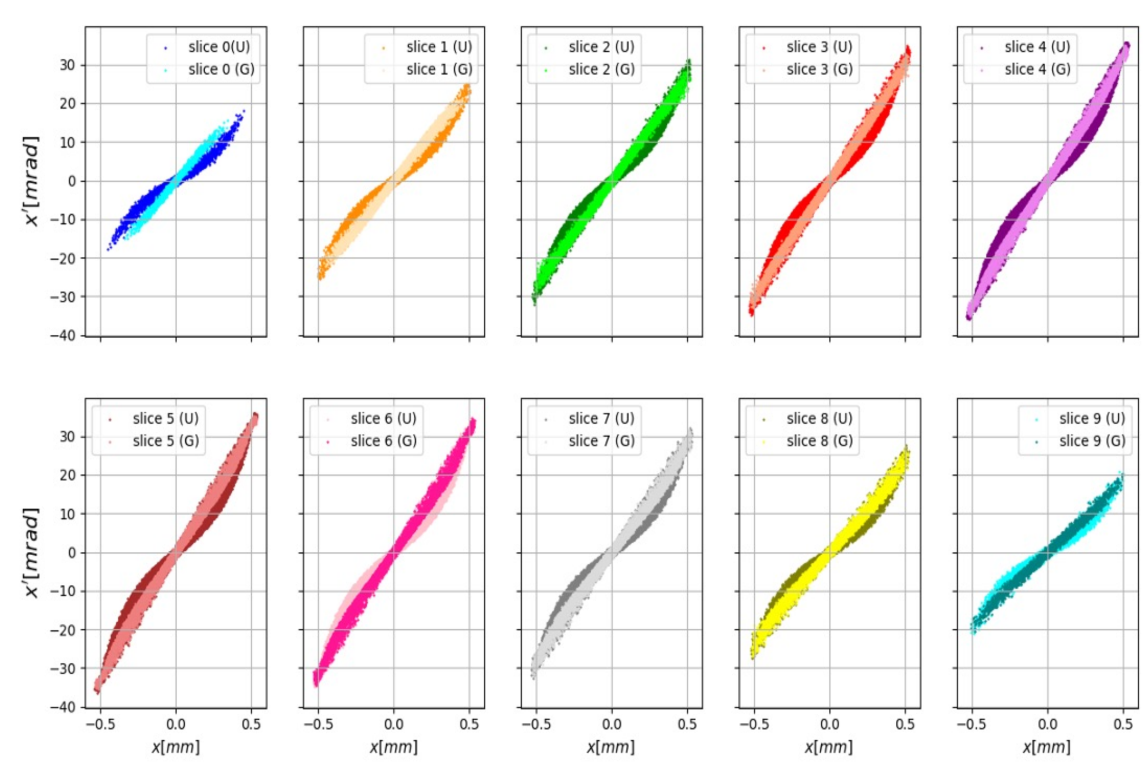
$$\tilde{G}(r,r',k) = \frac{1}{2\pi\epsilon_o}\left[I_o\left(\frac{k}{\gamma}r_<\right)K_0\left(\frac{k}{\gamma}r_>\right) - \frac{K_0\left(\frac{k}{\gamma}b\right)}{I_0\left(\frac{k}{\gamma}b\right)}I_0\left(\frac{k}{\gamma}r\right)I_0\left(\frac{k}{\gamma}r'\right)\right]$$

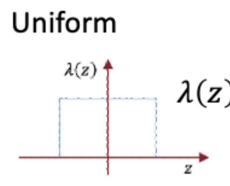
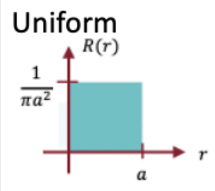
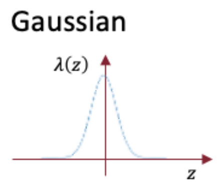
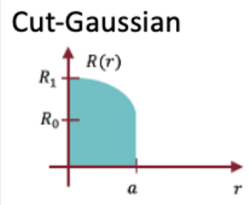
Assuming a separable form for the charge density (*low energy approximation γ~1*)

$$\rho(r,z) = QR(r)\lambda(z) \leftrightarrow \tilde{\rho}(r,k) = QR(r)\tilde{\lambda}(k)$$

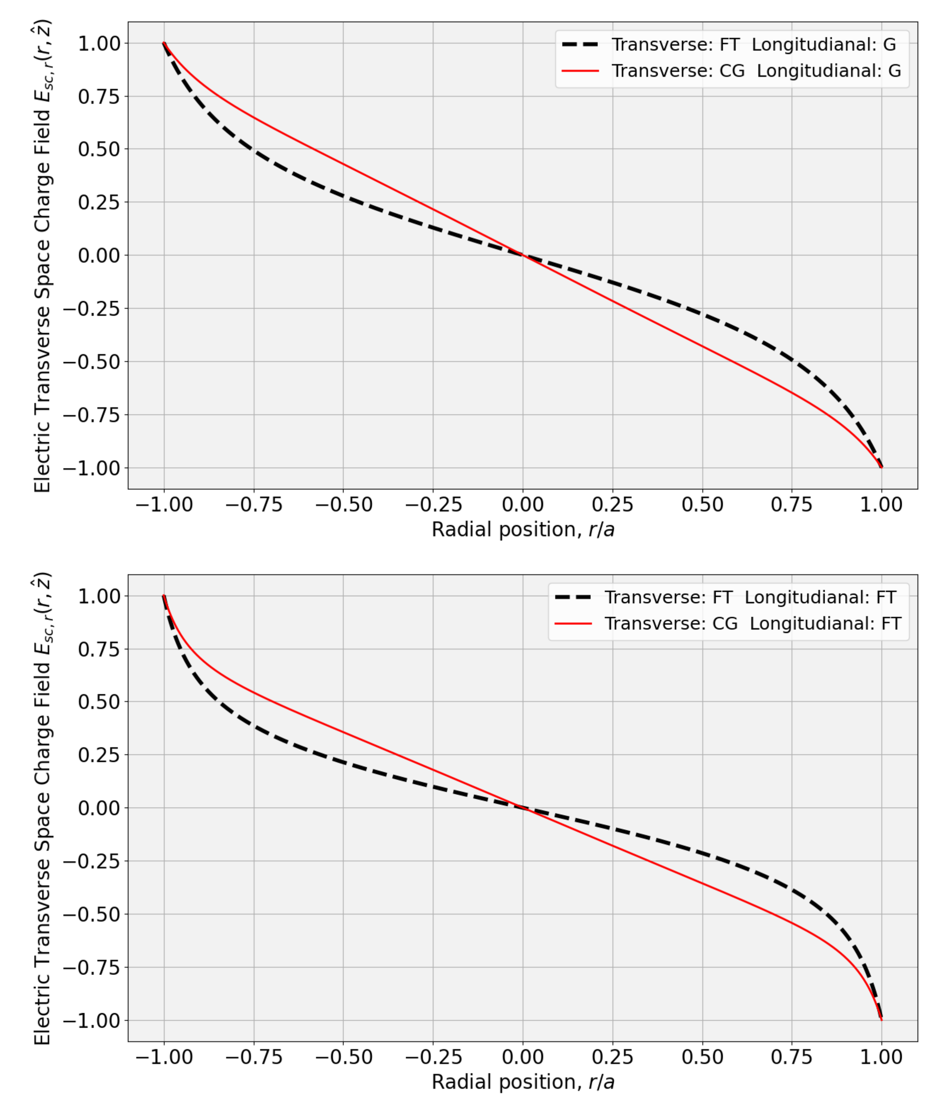
For any distribution:

$$\boxed{\phi(r,z) = Q\int\frac{dk}{2\pi}e^{ikz}\tilde{\lambda}(k)\times\int 2\pi\tilde{G}(r,r',k)R(r')r'dr'}$$

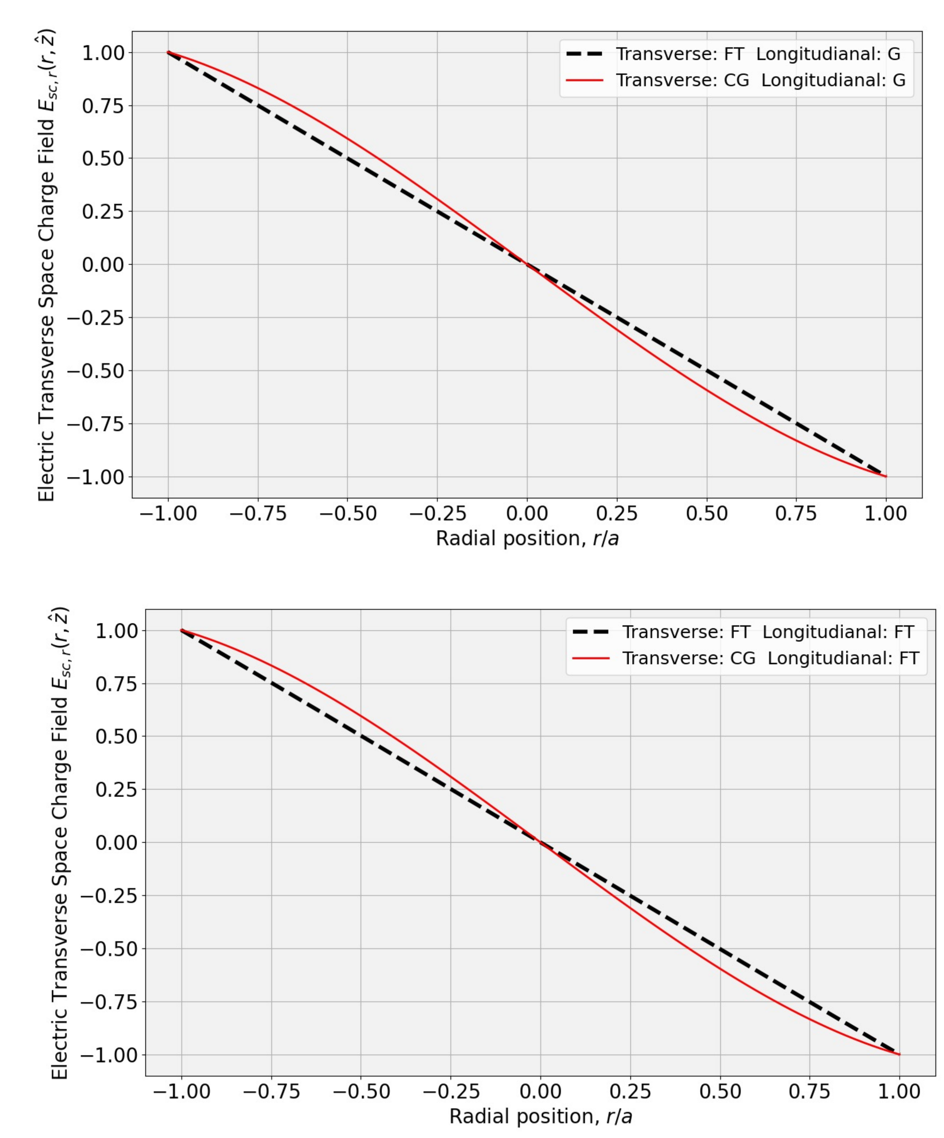


Longitudinal distribution		Trasverse distribution	
Uniform 	$\lambda(z) = \frac{1}{L}\begin{cases}1, & \lvert z\rvert \le \frac{L}{2} \\ 0, & otherwise\end{cases}$	Uniform 	$S(r,k) = \frac{1}{\epsilon_0\pi a^2k^2}[1 - kaK_1(ka)I_0(kr)]$
Gaussian 	$\lambda(z) = \frac{e^{-\frac{z^2}{2\sigma_z^2}}}{\sqrt{2\pi}\sigma_z}$	Cut-Gaussian 	$S(r,k) = \frac{1}{\epsilon_0 a^2k^4}\begin{bmatrix}(4+k^2r^2)R_0 + \lvert -4 + k^2(a-r)(a+r)\rvert R_1 + \\ akI_0(kr)\begin{vmatrix}2ak(-R_0+R_1)K_0(ak) - \\ (4R_0 + a^2k^2R_0 - 4R_1)K_1(ak)\end{vmatrix}\end{bmatrix}$

Pancake-like distribution



Cigar-like distribution

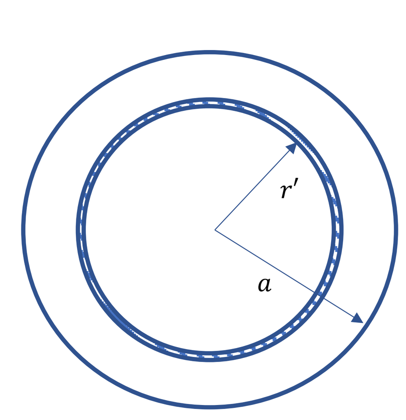


Perturbation: superficial quadrupole term

Assuming the general form of the Green function, without stopping at zero-th order, it is possible to model the perturbations on the beam:

$$G(r,r',z-z') = \frac{1}{2\pi}\sum_{m=0}^{\infty}\cos(m\phi)\int_{-\infty}^{\infty}dk\, e^{ik(z-z')}C_m\begin{cases}K_m(kr)I_m(kr') & r \ge r' \\ K_m(kr')I_m(kr), & r' \ge r\end{cases}$$

No Perturbation:

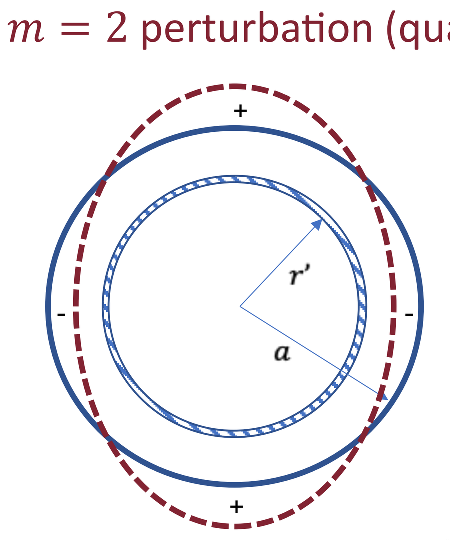


$$S(r,k) = \frac{1}{\epsilon_0\pi a^2k^2}[1 - kaK_1(ka)I_0(kr)]$$

Without perturbation: No θ dependence

$$E_r^{sc}(r,z) = -Q\int\frac{dk}{2\pi}e^{ikz}\tilde{\lambda}(k)\left(\frac{\partial}{\partial r}S(k,r)\right)$$

$m = 2$ perturbation (quadrupolar term):



Only on the surface: $r' = a$

$$\widetilde{G_2}(r,a,k,\theta,0) = \frac{1}{\pi\varepsilon_0}\cos(2\theta)\,K_2(ka)I_2(kr)$$

$$\boxed{\delta S(r,k,\theta) = A_2\frac{1}{\pi\varepsilon_0}\cos(2\theta)\,K_2(ka)I_2(kr}$$

$$\phi(r,z) + \delta\phi(r,z,\theta) = Q\int\frac{dk}{2\pi}e^{ikz}\tilde{\lambda}(k)(S(k,r) + \delta S(k,r,\theta))$$

With perturbation: θ dependence

$$E_r^{sc}(r,z) = -Q\int\frac{dk}{2\pi}e^{ikz}\tilde{\lambda}(k)\frac{\partial}{\partial r}(S(k,r,\theta) + \delta S(k,r,\theta))$$

$$E_\theta^{sc}(r,z) = -Q\int\frac{dk}{2\pi}e^{ikz}\tilde{\lambda}(k)\frac{\partial}{\partial\theta}(S(k,r,\theta) + \delta S(k,r,\theta))$$

References

[1] Feng Zhou et al., "Impact of the spatial laser distribution on photocathode gun operation", SLAC National Accelerator LABORATORY

[2] Han Chen et al., "Analysis of slice transverse emittance evolution in a very-High-frequency gun photoinjector", Accelerator laboratory, Department of Engineering Physics, Tsinghua University, Beijing, China

[3] M. Ferrario, M. Migliorati, and L. Palumbo, *"Space charge effects"*, arXiv preprint arXiv:1601.05214 (2016).

[4] John David Jackson, *"Classical Electrodynamics",* John Wileyand Sons, INC. 1998

[5] M. Carillo, et al., *"Three-Dimensional Space Charge Oscillations in a Hybrid Photoinjector"*, in 12th Int. Particle Accelerator Conf.(IPAC'21), Campinas, Brazil (2021).

[6] M. Carillo et al., "Space Charge Analysis for Low Energy Photoinjector", presented at the 13th Int.Particle Accelerator Conf. (IPAC'21), Bangkok, Thailand, June2022

Acknowledgement

This poster presentation has received support from the European Union's Horizon 2020 Research and Innovation programme under Grant Agreement No 101004730

EUROPEAN NETWORK FOR NOVEL ACCELERATORS
EuroNNAc4
SPACE supported by I.FAST
IFAST