



DE LA RECHERCHE À L'INDUSTRIE

# Examples of PIC code limit and potential for the simulation of wakefield acceleration and accelerator applications.

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EuroNNAc4, Elba, 2022

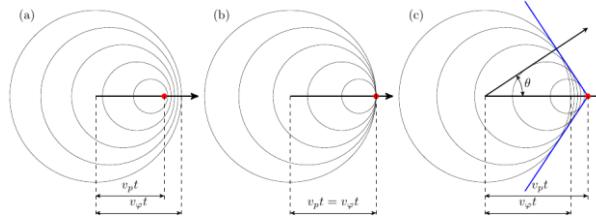
Warning: PIC simulation of wakefield are subject to errors!  
But, broader PIC code potential: toward the modeling of applications.

- Example of PIC code limits: Numerical Cerenkov Radiation / Instability (NCR/NCI)
- PIC code developments help to foster accelerator applications
  - radiation sources
  - Beam interaction with EM fields or target/plasma
  - **Needs for code developments**

## ■ (Physical) Cerenkov Radiation

It appears when a charged particle is faster than the speed of light in a medium

$$(v_p > v_\phi)$$



$$\cos(\theta) = \frac{c}{v_p n(\omega)} \quad \omega = \mathbf{v}_p \cdot \mathbf{k}$$

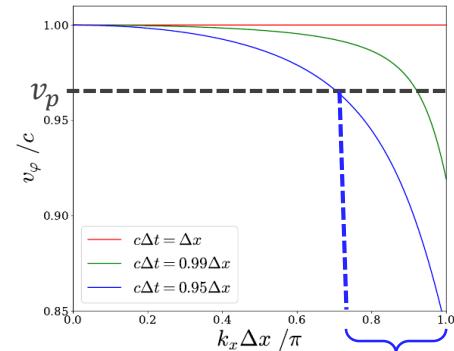
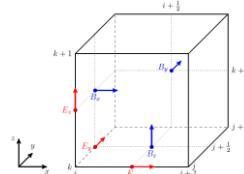
## ■ Origin of Numerical Cerenkov Radiation (NCR) in PIC code

In plasma, light phase

$$v_\phi^{num} = \frac{2}{k\Delta t} \arcsin \left( c\Delta t \sqrt{\frac{1}{\Delta x^2} \sin^2\left(\frac{k_x \Delta x}{2}\right) + \frac{1}{\Delta y^2} \sin^2\left(\frac{k_y \Delta y}{2}\right) + \frac{1}{\Delta z^2} \sin^2\left(\frac{k_z \Delta z}{2}\right)} \right)$$

velocity  $v_\phi \geq c \Rightarrow$  no CR.

In a PIC code with the standard Yee<sup>2</sup> scheme, we have  $v_\phi^{num} < c$  in vacuum  
 $\Rightarrow$  NCR...

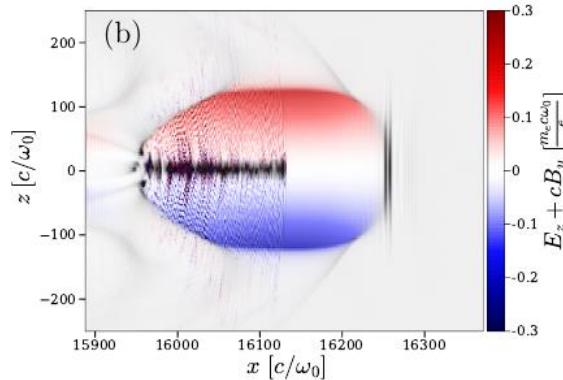
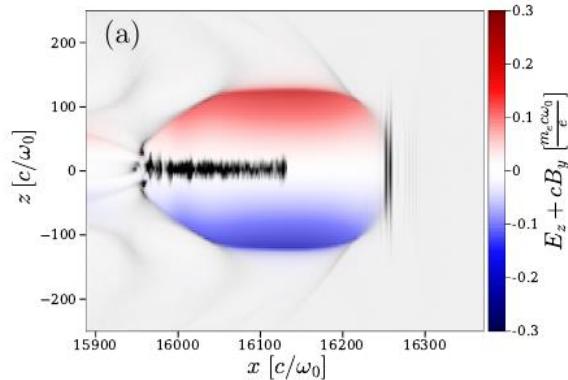


Emission of NCR in the high frequency domain

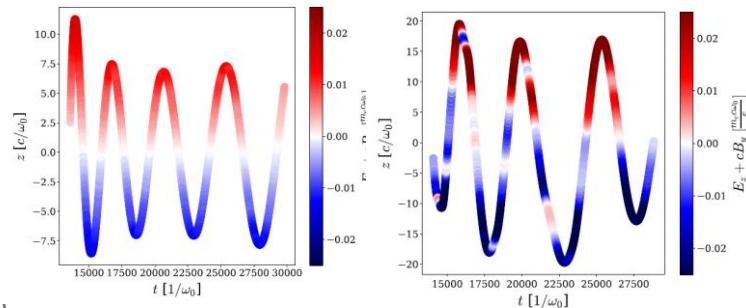
[1] B.B. Godfrey, JCP 15(4) 504–521 (1974).

[2] K. Yee, IEEE Trans. Antennas Propag. 14, 302 (1966)

- Consequences: emission of spurious and high frequency radiation, which can affect the beam/plasma dynamic



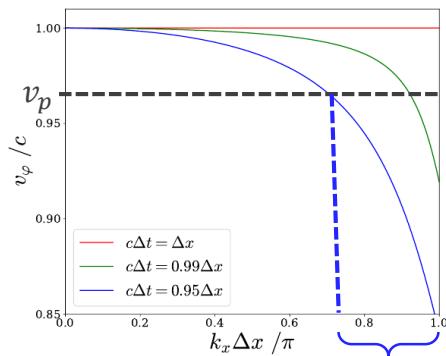
Depending on the beam charge:  
wrong beam divergence,  
transverse size, transverse  
emittance...



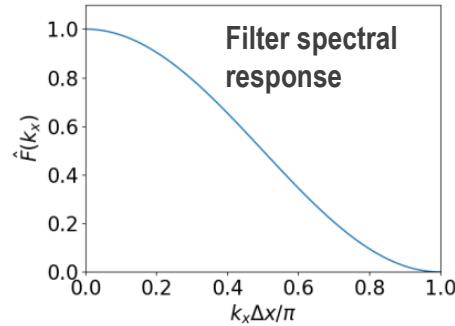
P.-L. Bourgeois, PhD, IP Paris (2020)  
P.-L. Bourgeois *et al.*, JCP **413** 109426 (2020)

- “Standard” spatial filtering.
- Higher resolution.
- Modified Maxwell solver.
- Modified field interpolation.
- **Combination of different technics.**
  - Needs to give the details in the reports/papers

- “Standard” spatial filtering<sup>1</sup>
  - Low path filters such as Binomial Filters + Compensator



$$\begin{array}{c} 1/4 \quad 1/2 \quad 1/4 \\ \hline F_{i-1} \quad F_i \quad F_{i+1} \\ \downarrow \\ \widehat{F}_i = \frac{1}{4} F_{i-1} + \frac{1}{2} F_i + \frac{1}{4} F_{i+1} \end{array}$$



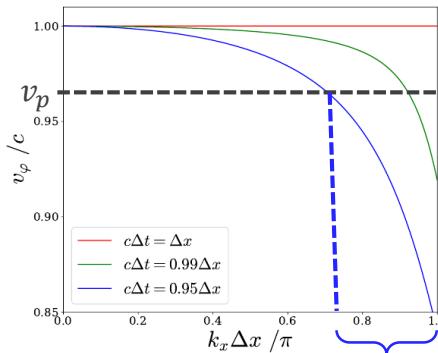
Emission of NCR in the high frequency domain

- Number of paths  $N$  per time step  $\Delta t$ ?  $\frac{N}{\Delta t} > 1$  or  $< 1$  ?
- Usually aggressive filtering of the currents can be used.
- Filtering of the fields: be careful with LWFA, the laser field should not be damped too much!

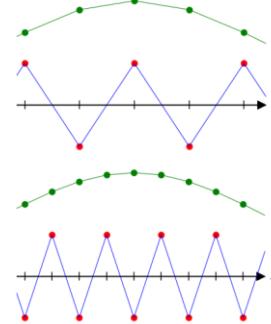
[1] J.-L. Vay *et al.*, JCP 230 5908–5929 (2011)

- Higher resolution

- Costly, not a good idea alone (NCR still exist with high resolution), but can help to use more aggressive filtering



Emission of NCR in the high frequency domain



- The use of  $\Delta z, \Delta y \gg \Delta x$  can help to get  $c\Delta t \sim \Delta x$  if Yee<sup>1</sup> scheme is used.
- Dispersion free schemes allow  $c\Delta t = \Delta x$  [2-4].

[1] K. Yee, IEEE Trans. Antennas Propag. **14**, 302 (1966)

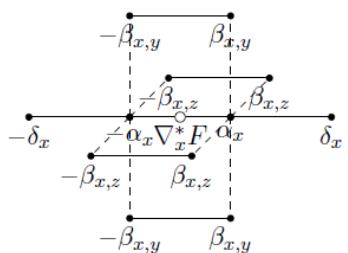
[2] A. Pukhov, JPP **61**(3) 425 (1999)

[3] M. Kärkkäinen, Proceedings of ICAP (2006)

[4] B. M. Cowan, PRSTAB **16** 041303 (2013)

- Modified Maxwell solver (FTDT)

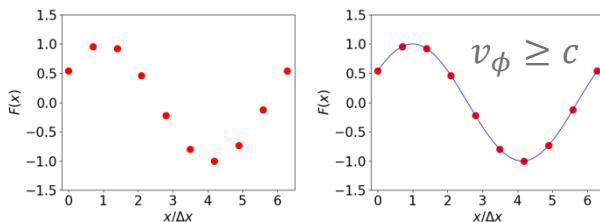
- Main idea: modification of the Maxwell solver to modify the numerical dispersion relation and avoid  $v_\phi < c$  in vacuum.
- A new stencil is often used to compute the derivative in the Maxwell equations:



- [1] A. Pukhov, JPP **61**(3) 425 (1999)
- [2] A.D. Greenwood, *et al.*, JCP. **201**(2) 665–684 (2004).
- [3] M. Kärkkäinen, Proceedings of ICAP (2006)
- [4] J.-L. Vay *et al.*, JCP **230** 5908–5929 (2011)
- [5] R. Lehe, *et al.*, PRSTAB **16** 021301 (2013).
- [6] B. M. Cowan, PRSTAB **16** 041303 (2013)
- [7] R. Nuter *et al.*, JCP **305** 664–676 (2016).
- [8] F. Li *et al.*, CPC **214** 6–17 (2017)
- [9] A. Blinne *et al.*, CPC **224** 273–281 (2018)
- [10] X. Xu *et al.*, JCP **413** 109451 (2020)
- [11] A. Pukhov, JCP **418** 109622 (2020)

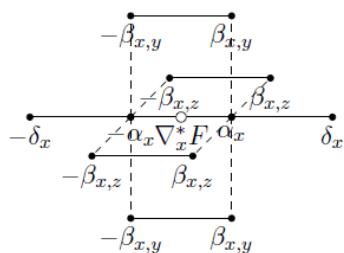
- Limitations:

- Can produce EM waves with  $v_\phi \geq c$  or other spurious results
- Difficulties to tackle aliasing modes.



- Modified Maxwell solver (FTDT)

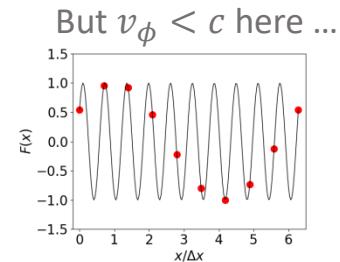
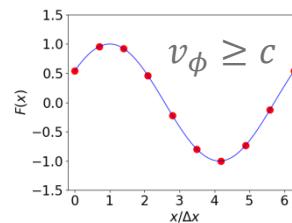
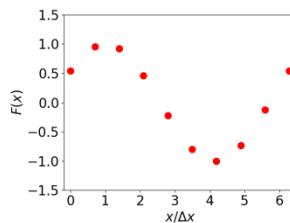
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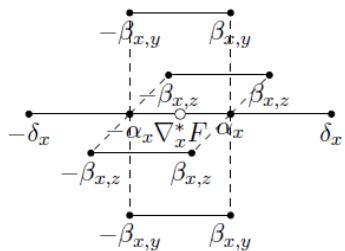
- [1] A. Pukhov, JPP **61**(3) 425 (1999)
- [2] A.D. Greenwood, *et al.*, JCP. **201**(2) 665–684 (2004).
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- [4] J.-L. Vay *et al.*, JCP **230** 5908–5929 (2011)
- [5] R. Lehe, *et al.*, PRSTAB **16** 021301 (2013).
- [6] B. M. Cowan, PRSTAB **16** 041303 (2013)
- [7] R. Nuter *et al.*, JCP **305** 664–676 (2016).
- [8] F. Li *et al.*, CPC **214** 6–17 (2017)
- [9] A. Blinne *et al.*, CPC **224** 273–281 (2018)
- [10] X. Xu *et al.*, JCP **413** 109451 (2020)
- [11] A. Pukhov, JCP **418** 109622 (2020)

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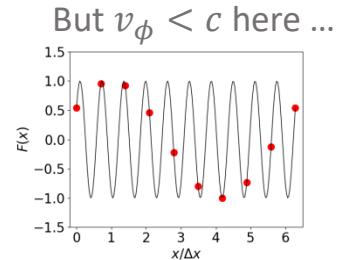
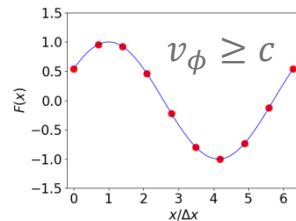
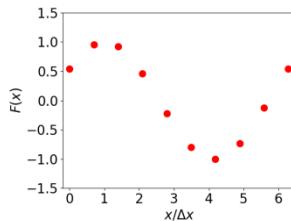


- Modified Maxwell solver (FDTD) + spectral solvers (NCI)
  - Main idea: modification of the Maxwell solver to modify the numerical dispersion relation and avoid  $v_\phi < c$  in vacuum.
  - A new stencil is often used to compute the derivative in the Maxwell equations:



- [1] A. Pukhov, JPP **61**(3) 425 (1999)
- [2] A.D. Greenwood, *et al.*, JCP. **201**(2) 665–684 (2004).
- [3] M. Kärkkäinen, Proceedings of ICAP (2006)
- [4] J.-L. Vay *et al.*, JCP **230** 5908–5929 (2011)
- [5] R. Lehe, *et al.*, PRSTAB **16** 021301 (2013).
- [6] B. M. Cowan, PRSTAB **16** 041303 (2013)
- [7] R. Nuter *et al.*, JCP **305** 664–676 (2016).
- [8] F. Li *et al.*, CPC **214** 6–17 (2017)
- [9] A. Blinne *et al.*, CPC **224** 273–281 (2018)
- [10] X. Xu *et al.*, JCP **413** 109451 (2020)
- [11] A. Pukhov, JCP **418** 109622 (2020)

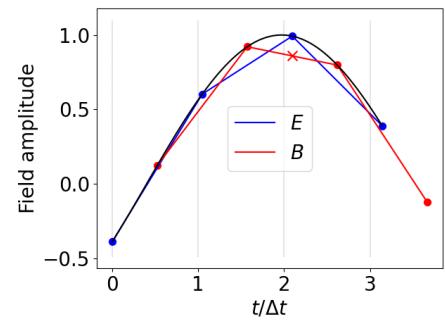
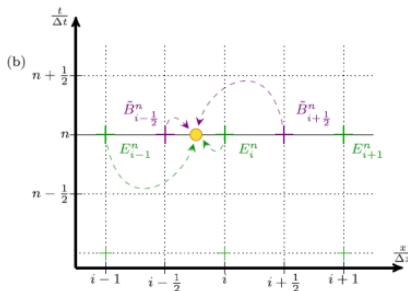
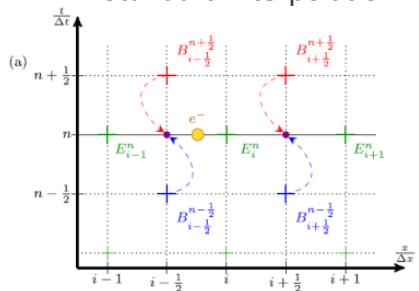
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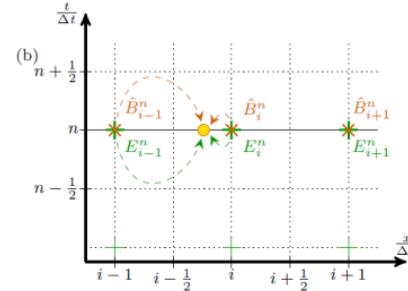
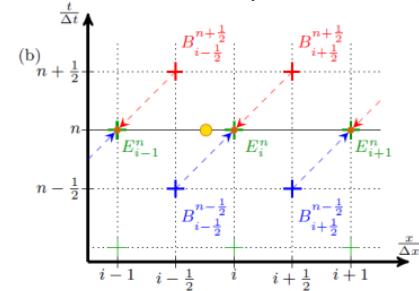
- Modified field interpolation.

- Field filters used only to push the particles (**NCI**) [1,2].
- BTIS scheme [3-5] : NCR propagates with the beam: E and B force compensate => no transverse force on the beam. Easy to implement!

Standard interpolation



BTIS3 interpolation [3-5] in CALDER [6]

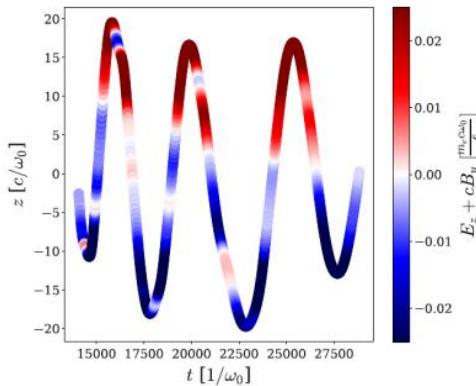
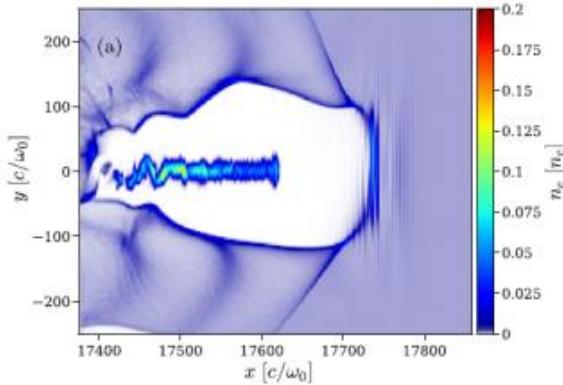


- [1] B. B. Godfrey *et al.*, JCP **267** 1–6 (2014)  
[2] Y. Lu *et al.*, JCP **413** 109388 (2020)

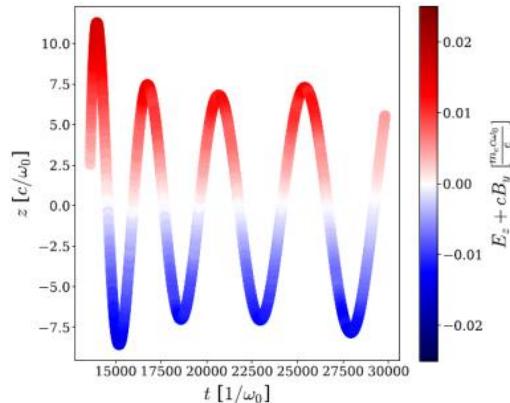
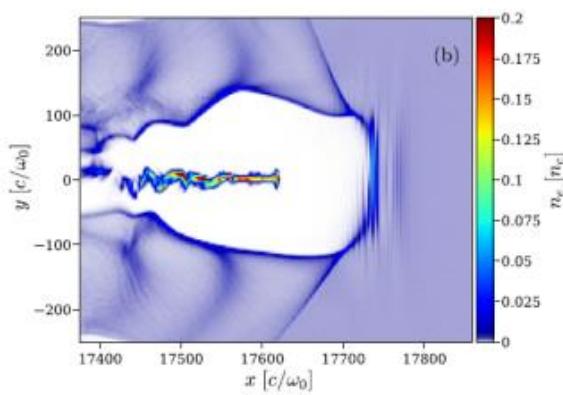
- [3] P.-L. Bourgeois, PhD, IP Paris (2020)  
[4] P.-L. Bourgeois, JCP **413** 109426 (2020)  
[5] P.-L. Bourgeois, to be published

- [6] E. Lefebvre *et al.*, Nuc. Fus. **43** 629 (2003)

## Standard interpolation



## BTIS interpolation [1-3]



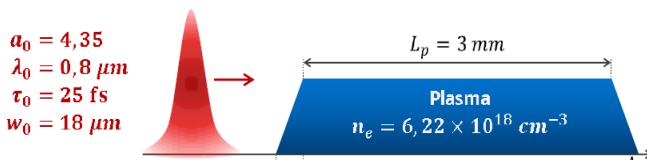
- Benchmark between Yee<sup>1</sup>, a modified Maxwell solver<sup>2</sup>, and BTIS scheme<sup>3,4</sup>

[1] K. Yee, IEEE Trans. Antennas Propag. **14**, 302 (1966)

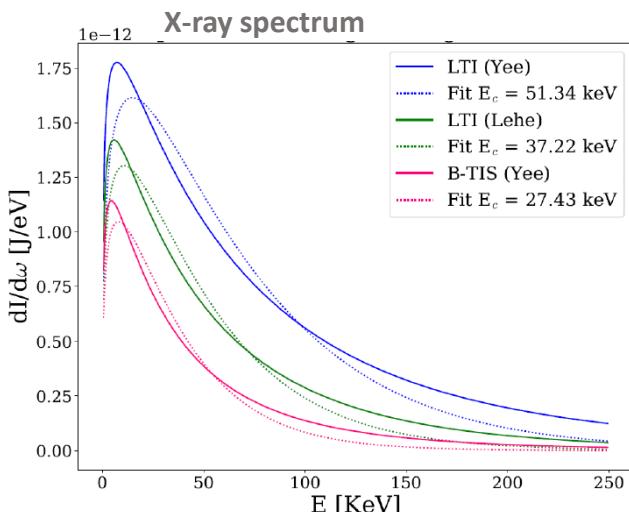
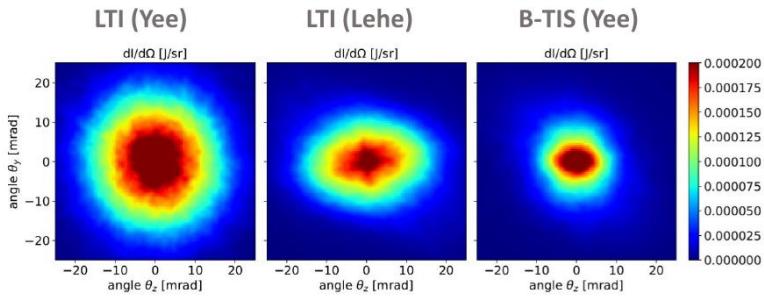
[2] R. Lehe, *et al.*, PRSTAB **16** 021301 (2013).

[3] P.-L. Bourgeois, PhD, IP Paris (2020)

[4] P.-L. Bourgeois, JCP **413** 109426 (2020)



X-ray spot



- NCR: “spontaneous” and incoherent emission
- **NCI: coupling between NCR and plasma**
  - The beam is “bunched” by the NCR and produced more coherent and amplified radiation
  - **Numerical instability: exponential growth** of the radiation and beam response
- Main goal: suppressing NCI is very difficult due to the aliasing. Instead the growth rate is often kept as low as possible to avoid significant effect of NCI during the simulation time.
- It is mainly observed when a “large” plasma/beam is drifting, like in
  - **boosted frame simulation** (used to speed up wakefield simulation)
  - **simulation of beam propagation** (applications)

- Solutions: same as before, but usually not enough. Other specific scheme have been developed.
  - Spatial filters,
  - Temporal filter: Friedman filter [1,2]. To be used carefully. Physical frequency can also be damped.
  - Adapted field filters/interpolation technics [3,4]
  - Modified Maxwell solvers (FDTD) ([5-7] and others)
  - **Spectral solvers** ( $\partial_x \rightarrow ik_x \Rightarrow$  no error in the derivative  $\Rightarrow v_\phi = c.$ )
    - Pseudo-spectral time domain (PSTD)
    - Pseudo-spectral analytic time domain (PSATD).
    - Still not enough, **specific solution to solve NCI are developed [8-16]**

[1] J.-L. Vay *et al.*, JCP **230** 5908–5929 (2011)

[2] A. Friedman, J. Comput. Phys. **90** 292312 (1990)

[3] B.B. Godfrey, J. Comput. Phys. **267** 1–6 (2014).

[4] Y. Lu *et al.*, JCP **413** 109388 (2020)

[5] F. Li *et al.*, CPC **214** 6–17 (2017)

[6] X. Xu *et al.*, JCP **413** 109451 (2020)

[7] A. Pukhov, JCP **418** 109622 (2020)

[8] X. Xu, *et al.*, CPC **184**(11) 2503–2514 (2013).

[9] B. B. Godfrey *et al.*, CPC **196** 221–225 (2015).

[10] P. Yu *et al.*, CPC **192** 32–47 (2015).

[11] R. Lehe *et al.*, PRE **94**, 053305 (2016)

[12] M. Kirchen *et al.*, POP **23**, 100704 (2016)

[13] S. Jalas *et al.*, POP **24** 033115 (2017).

[14] M. Kirchen *et al.*, PRE **102**, 013202 (2020)

[15] O. Shapoval *et al.*, PRE **104**, 055311 (2021)

[16] E. Zoni *et al.*, CPC **279** 108457 (2022)

- Example of PIC code limits: Numerical Cerenkov Radiation / Instability (NCR/NCI)
  - Solutions are diverse, they should be known, used, mention in publications
- PIC code developments help to foster accelerator applications
  - radiation sources
  - Beam interaction with EM fields or target/plasma
  - Needs for code developments

- X/ $\gamma$ -ray sources

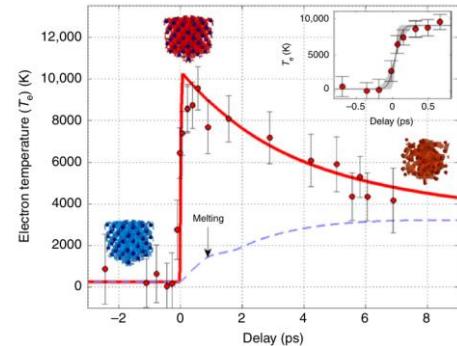
- Betatron sources (X-ray) particles tracking and use of Liénard-Wiechert potential (or not [1,2]) + NCR should be limited!

- See the talk of S. Mangles on Thursday

Warm dense matter probing:

- [1] B. Mahieu *et al.*, Nat. Comm. **9** 3276 (2018)
- [2] A. Grolleau *et al.*, PRL **127** 275901 (2021)

For this application, LWFA can already be considered as a beamline used by external users!



- Bremsstrahlung ( $\gamma$ -ray)
- Compton sources (“peaked”  $\gamma$ -ray) Synchrotron radiation
- Others... (e.g. “Boosted” betatron producing  $\gamma$ -ray [3])

- THz sources [4,5]

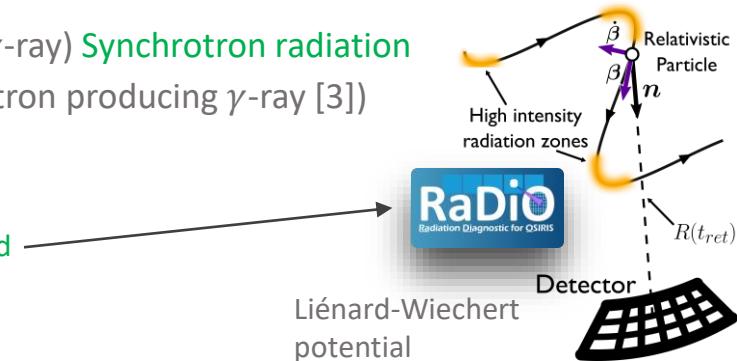
[1] R. Pausch *et al.*, NIMA **740** 250–25 (2014)

[2] M. Pardal, J. Vieira, RaDiO, to be published

[3] J. Ferri *et al.*, PRL **120** 254802 (2018)

[4] J. Dechard *et al.*, PRL **120** 144801 (2018)

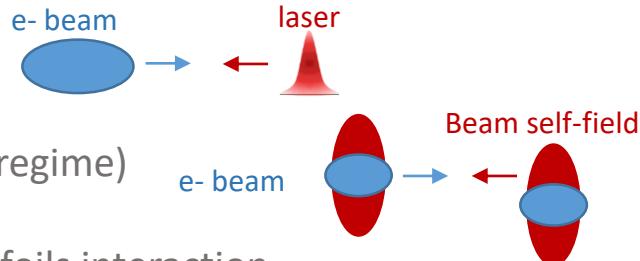
[5] J. Dechard *et al.*, PRL **123** 264801 (2019)



- Example of PIC code limits: Numerical Cerenkov Radiation / Instability (NCR/NCI)
  - Solutions are diverse, they should be known, used, mention in publications
- PIC code developments help to foster accelerator applications
  - radiation sources
  - **Beam interaction with EM fields or target/plasma. (beams from conventional accelerators, PWFA, LWFA)**
  - **Needs for code developments**

- Laser-Beam interaction

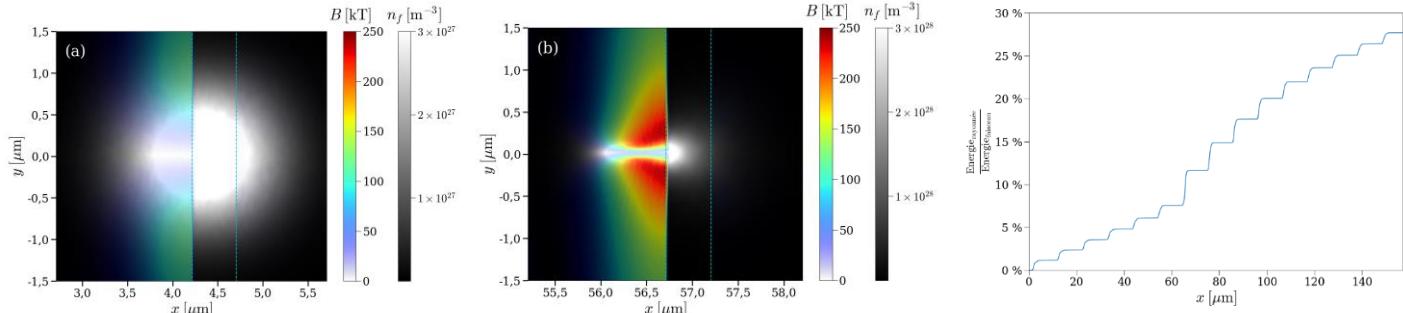
M. Lobet et al., PRAB 20 043401 (2017), ...



- Beam-Beam collision (beamstrahlung regime)

V. Yakimenko et al., PRL 122, 190404 (2019)

- Beam-foil interaction and beam-multi foils interaction



10 GeV, 2 nC, 0.55  $\mu\text{m}$   
0.5  $\mu\text{m}$  Al foil

A. Sampath et al., PRL 126, 064801 (2021)  
Collaborators of the SLAC E332 experiment

PHYSICAL REVIEW LETTERS 126, 064801 (2021)

## Extremely Dense Gamma-Ray Pulses in Electron Beam-Multifoil Collisions

Archana Sampath,<sup>1</sup> Xavier Davoine,<sup>2,3</sup> Sébastien Corde,<sup>4</sup> Laurent Gremillet,<sup>2,3</sup> Max Gilljohann,<sup>4</sup> Maitreyi Sangal,<sup>1</sup> Christopher H. Keitel,<sup>1</sup> Robert Arinello,<sup>5</sup> John Cary,<sup>6</sup> Henrik Ekerfelt,<sup>6</sup> Claudio Emma,<sup>6</sup> Frederico Fiuzza,<sup>6</sup> Hiroki Fujii,<sup>7</sup> Mark Hogan,<sup>7</sup> Chan Jishi,<sup>7</sup> Alexander Knetsch,<sup>8</sup> Olena Kononenko,<sup>4</sup> Valentine Lee,<sup>5</sup> Mike Litos,<sup>5</sup> Kenneth Marsh,<sup>7</sup> Zan Nic,<sup>7</sup> Brendan O'Shea,<sup>6</sup> Doug Storey,<sup>6</sup> Yipeng Wu,<sup>7</sup> Xinlu Xu,<sup>7</sup> Chaojie Zhang,<sup>7</sup> and Michael Tamuzhi<sup>1,7</sup>

<sup>1</sup>Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany

<sup>2</sup>CEA, DAM, DIF, 91297 Arpajon, France

<sup>3</sup>Université Paris-Saclay, CEA, LALCE, 91198 Gif-sur-Yvette Cedex, France

<sup>4</sup>LOA, ENSTA Paris, CNRS, Ecole Polytechnique, Institut Polytechnique de Paris, 91760 Palaiseau, France

<sup>5</sup>Boulder, University of Colorado Boulder, Department of Physics, Center for Integrated Plasma Studies,

Boulder, Colorado 80309, USA

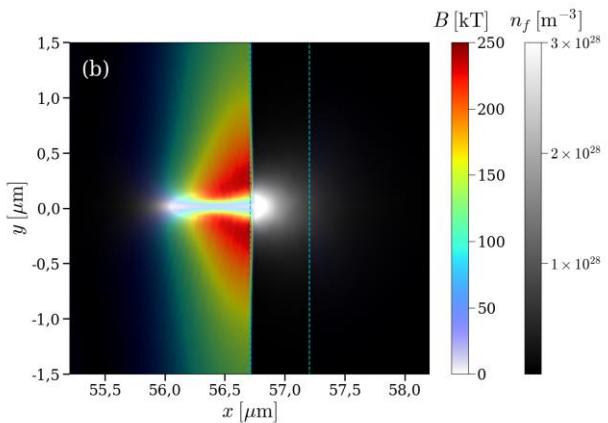
<sup>6</sup>SLAC National Accelerator Laboratory, Menlo Park, California 94025, USA

<sup>7</sup>University of California Los Angeles, Los Angeles, California 90095, USA

<sup>8</sup>Stanford University, Physics Department, Stanford, California 94305, USA

# Study of the Strong-Field QED regime (SF-QED)

## Production of intense $\gamma$ ray sources, e+e- pairs



Implemented in CALDER [1-4]:

Field ionization

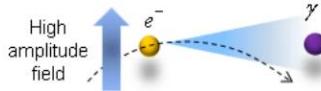
Collisions (elastic + ionization)

SF-QED processes

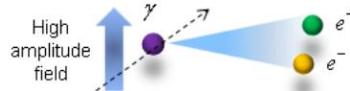
Coulomb processes

- [1] M. Lobet *et al.*, J. Phys.: Conf. Ser. **688**, 012058 (2016).
- [2] B. Martinez *et al.*, POP **26**, 103109 (2019).
- [3] M. Lobet, PhD, Université de Bordeaux (2015)
- [4] B. Martinez, PhD, Université de Bordeaux (2018)

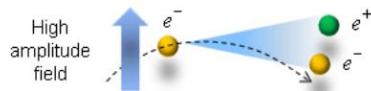
Nonlinear inverse  
Compton



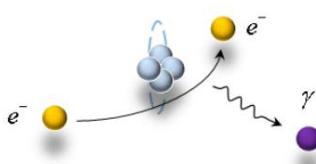
Nonlinear  
Breit-Wheeler



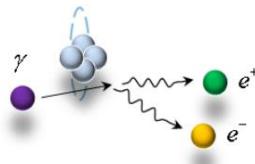
Electromagnetic Trident



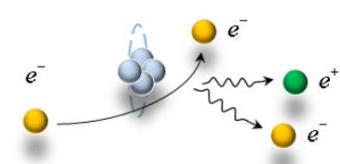
Bremsstrahlung



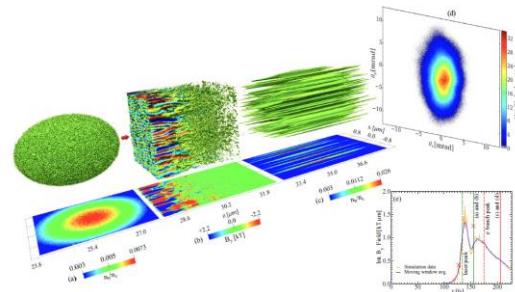
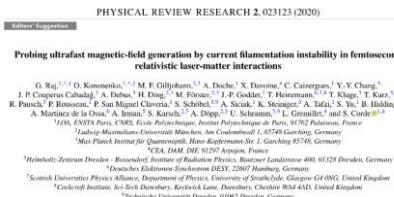
Bethe-Heitler



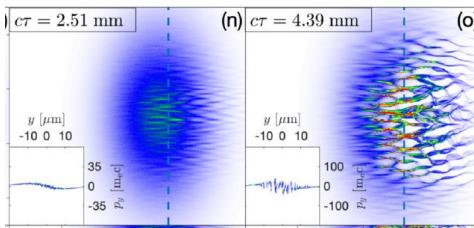
Coulomb Trident



- Probing Current Filament Instability (CFI) on short time-scale
  - CFI in laser-solid target interaction
  - Probing CFI with LWFA e- beam



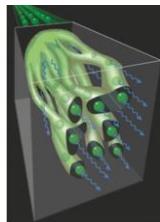
- Competition between CFI and Oblique Two Stream Instability (OTSI).
- PHYSICAL REVIEW RESEARCH 4, 023085 (2022)
- E305 SLAC experiment**



- Fundamental studies relevant to beam propagation, laser-plasma interaction, astrophysics,  $\gamma$  ray emission.

**Validity of the collisional algorithm in beam-solid interaction?**

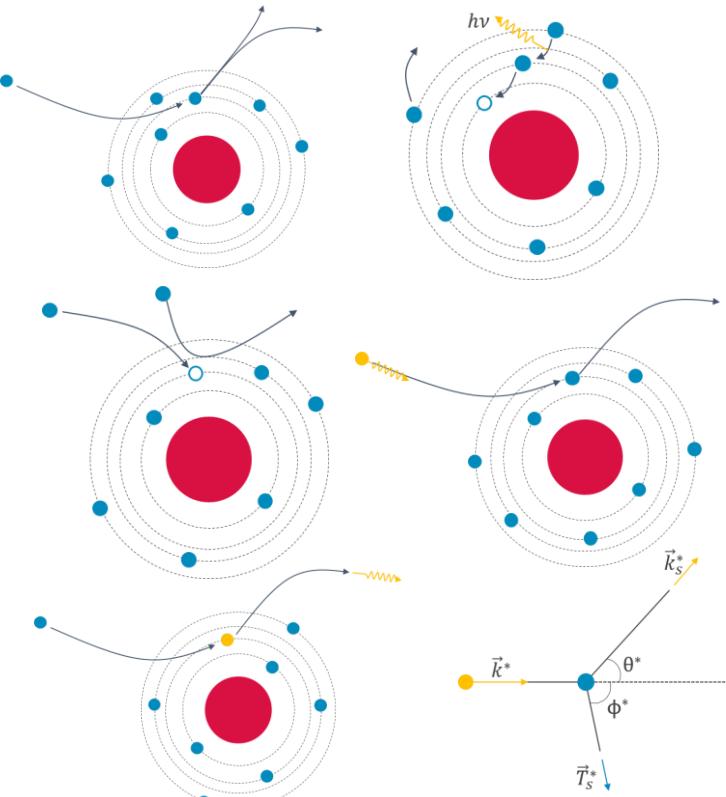
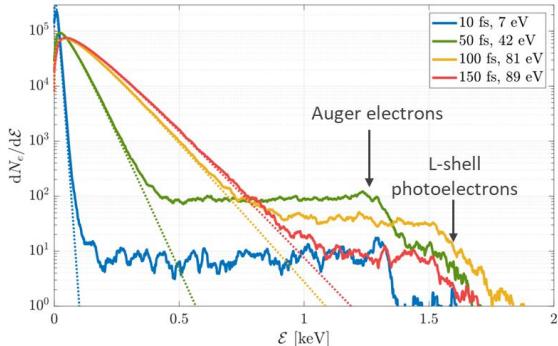
A. Benedetti et al., Nat. Phot. **12** 319–323 (2018)



# XFEL pulse interaction with matter implementation of “simple” atomic model

- Implemented in CALDER:
  - Collisional ionization
  - Atomic deexcitation
  - Three-body recombination
  - Photoionization (from X-ray)
  - Radiative recombination
  - Compton scattering
  - Ionization potential depression

Self-consistent simulation of XFEL pulse interacting with Al target



D. Tordeux, PhD, Université Paris-Saclay (2022)  
R. Royle *et al.*, Phys. Rev. E **95**, 063203 (2017).

- Numerical errors limiting PIC code prediction should be understood and reduced, e.g.:
  - NCR can impact the beam divergence, emittance, betatron source in standard wakefield simulation.
  - Reducing NCI is crucial in propagating beam or boosted frame simulation.
  - Solutions now exist to reduce NCR/NCI impact, but these solutions should be known and used by the community when needed.
- PIC code is a good tool to study some accelerator applications
  - Can foster the collaboration between the accelerator, plasma and laser communities
  - Dedicated developments are needed (physical modules, numerical schemes, ...)

- 1) Future developments needed and planned as seen from the speakers and their groups
- 2) Do the planned activities address the requirements from funded projects (AWAKE, EuPRAXIA, ...) and from various roadmaps for plasma accelerators? Are there urgent holes?
- 3) Does simulations and theory require its own roadmap or is work adequately driven/supported through funded projects and through overall plasma accelerator roadmaps?