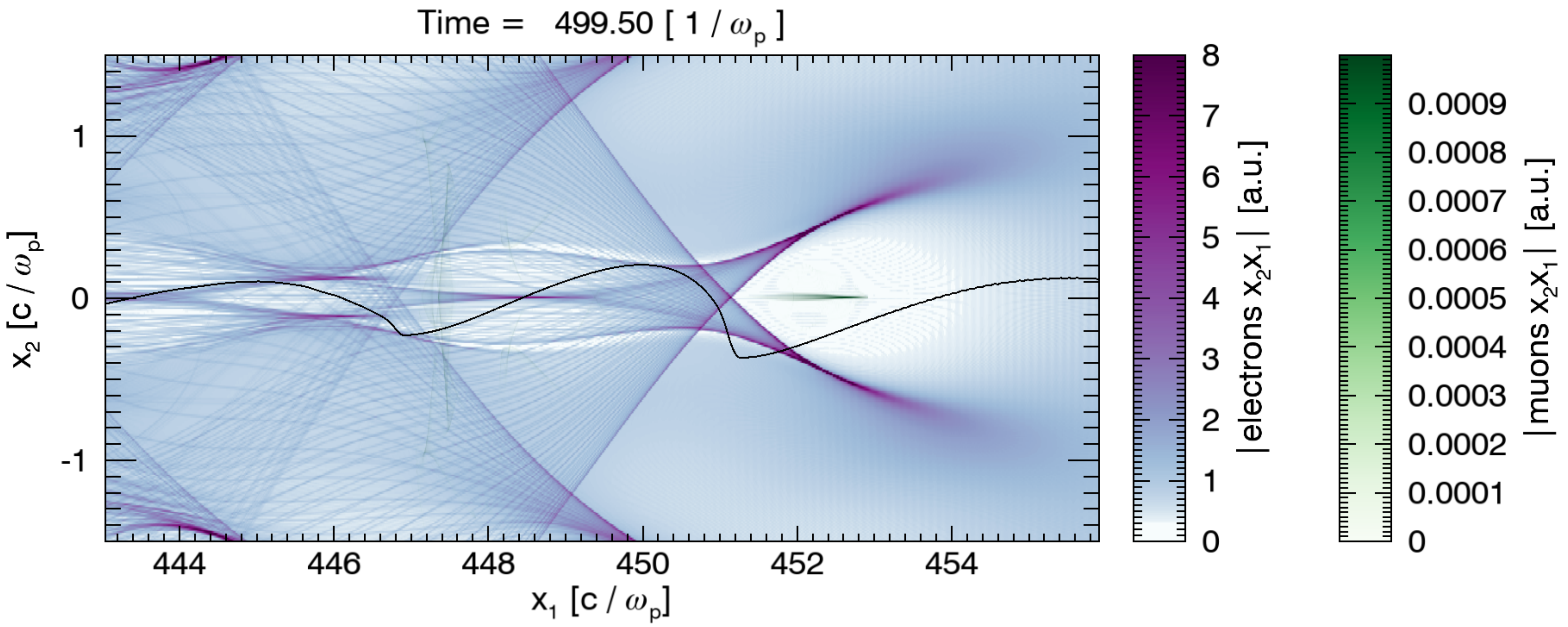
# A plasma-based acceleration method suitable for non-relativistic muons

**Chiara Badiali,**
Bernardo Malaca, Thales Silva, Ricardo Fonseca, Jorge Vieira

GoLP / Instituto de Plasmas e Fusão Nuclear Instituto Superior Técnico, Lisbon, Portugal

**epp**.tecnico.ulisboa.pt || **golp**.tecnico.ulisboa.pt

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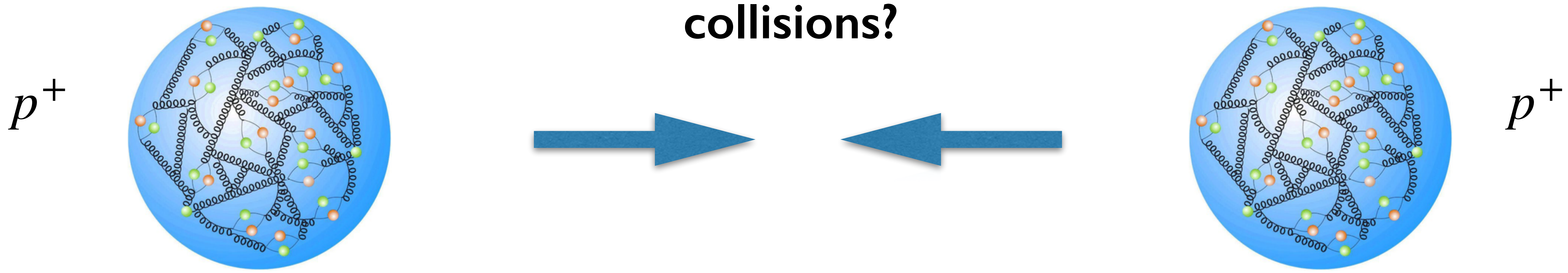
**13 TeV collisions?**



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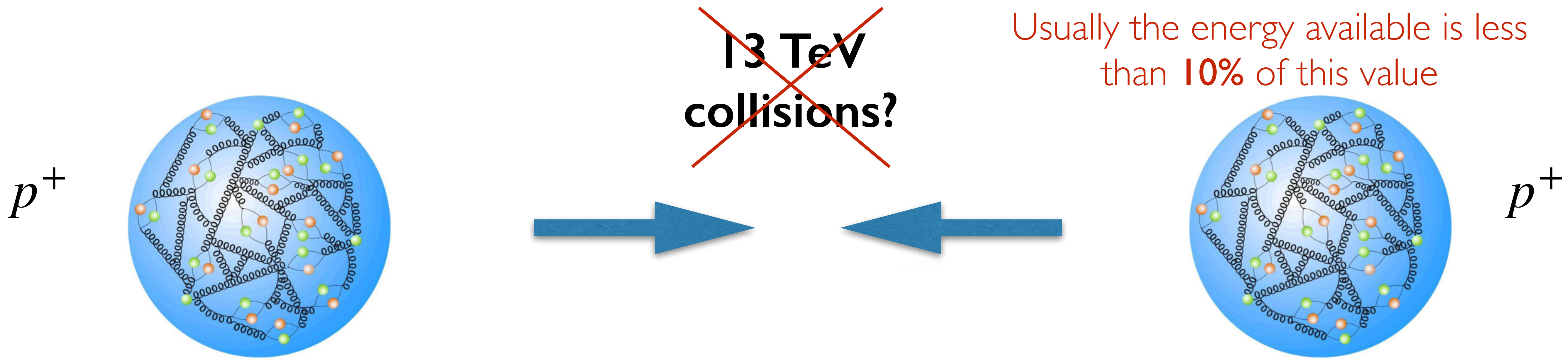

# Why muons?

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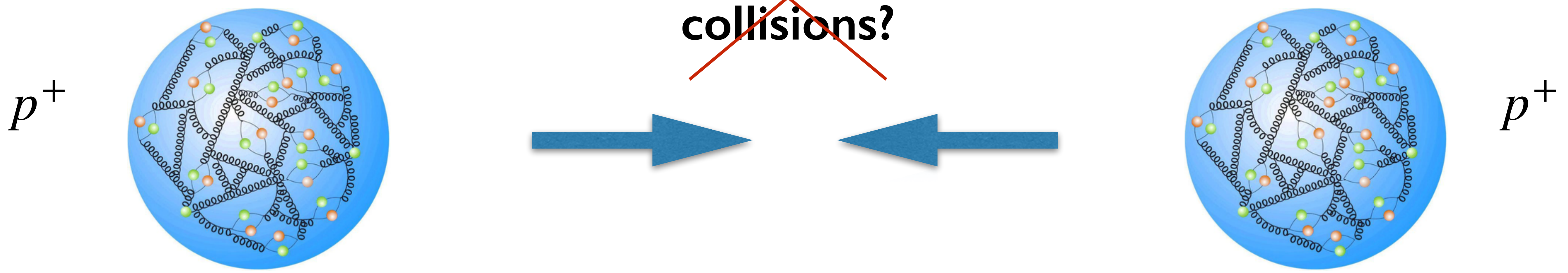
What are the best particles to collide?

~~13 TeV collisions?~~

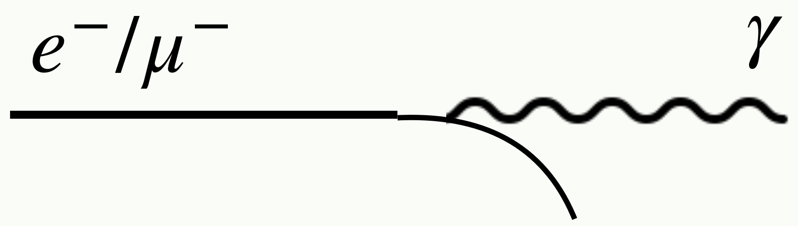
Usually the energy available is less than **10%** of this value




Muons, like electrons, are **fundamental particles**, for this reason their full energy is available in collisions

→ a **14 TeV** muon collider with sufficient enough luminosity would provide similar discovery reach as a **100 TeV** proton-proton collider.

Furthermore, $m_\mu = 200\ m_e$ so that:




$$\frac{\Delta E_\mu}{\Delta E_e} = \left(\frac{m_e}{m_\mu}\right)^4 \sim 10^{-12}$$

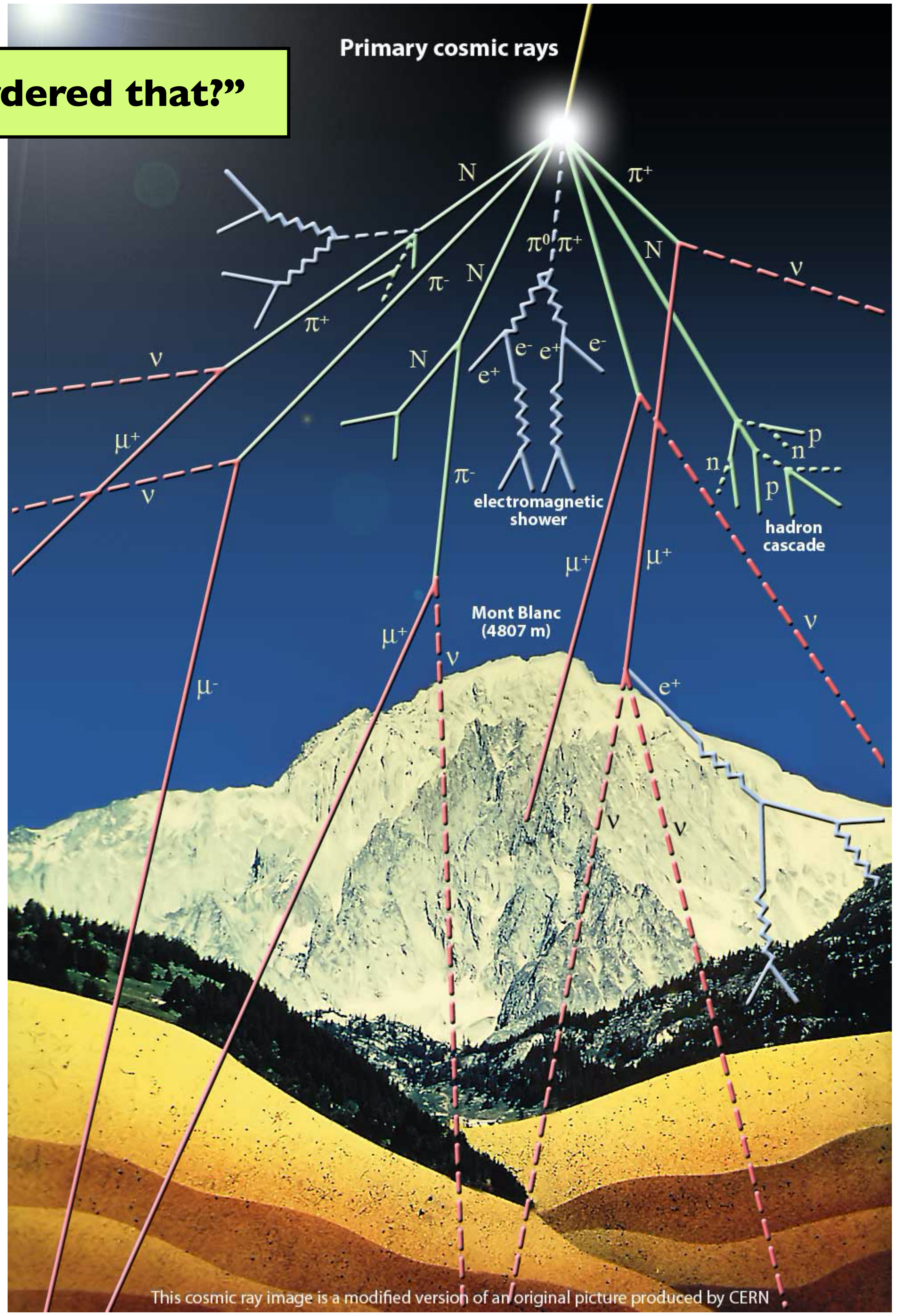
Muons have a finite life-time ($2.2\ \mu$s) → We have to accelerate them quickly to mitigate muon decay losses, exploiting the time-life dilation in the lab reference frame given by the Lorentz boost ($\tau' = \gamma\tau$)
→ possible **advantage in using plasma based accelerators.**

K. R. Long et al., Nature Physics **17**, pages 289–292 (2021)

# Muon production: proton beam against a dense target

$\mu^-$ production in the earth's atmosphere

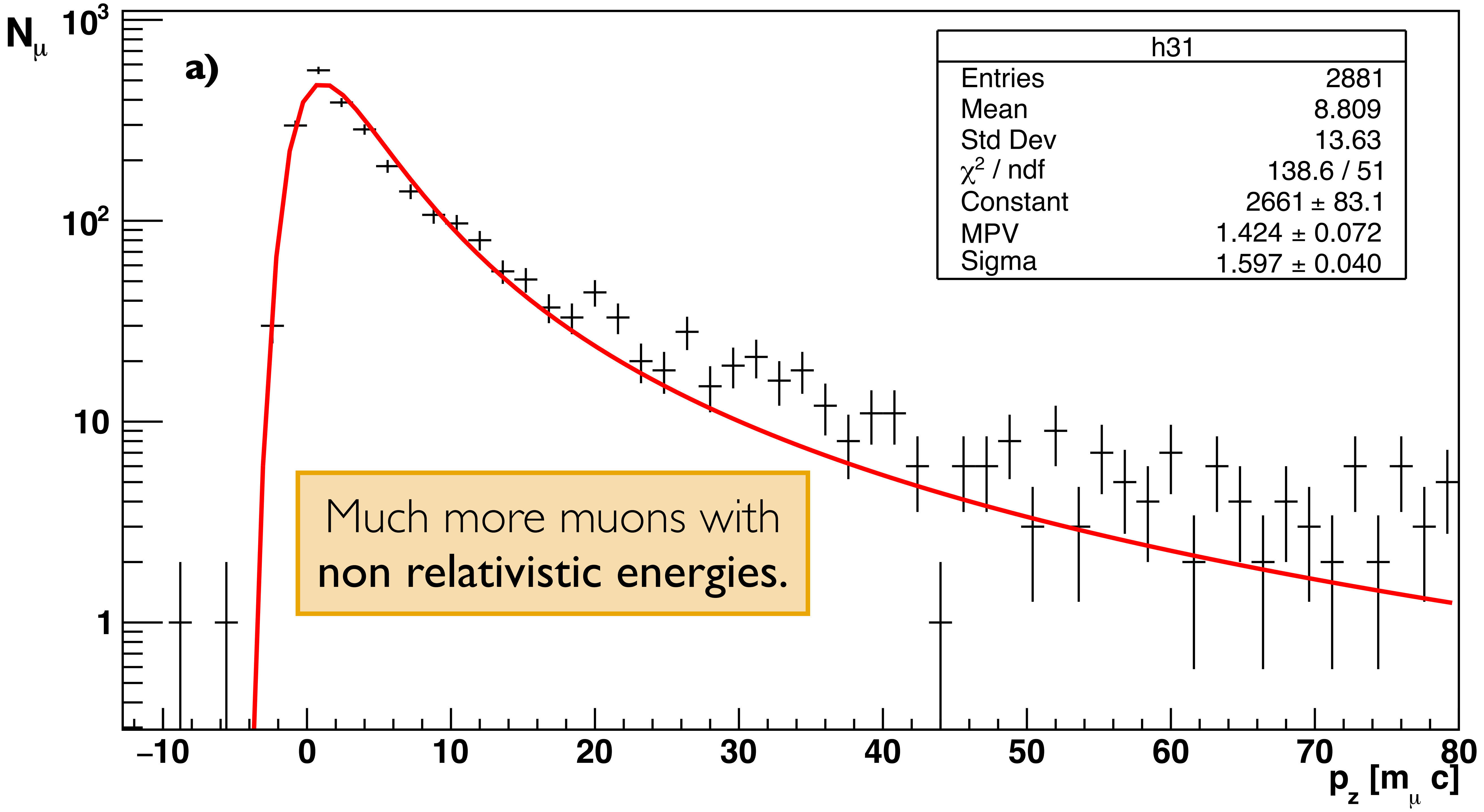
**"Who ordered that?"**




## Muons production on Earth

$p$ $Hg$

High energetic proton beam against a dense target




Much more muons with non relativistic energies.

A **GEANT4*** simulation with a proton beam of $5 \cdot 10^6$ monoenergetic protons ($E_p = 450\ GeV$) was performed (in reality $\sim 10^{12}$ protons).

* J.Allison et al., Nucl. Instrum. Meth.A 835 (2016) 186-225.

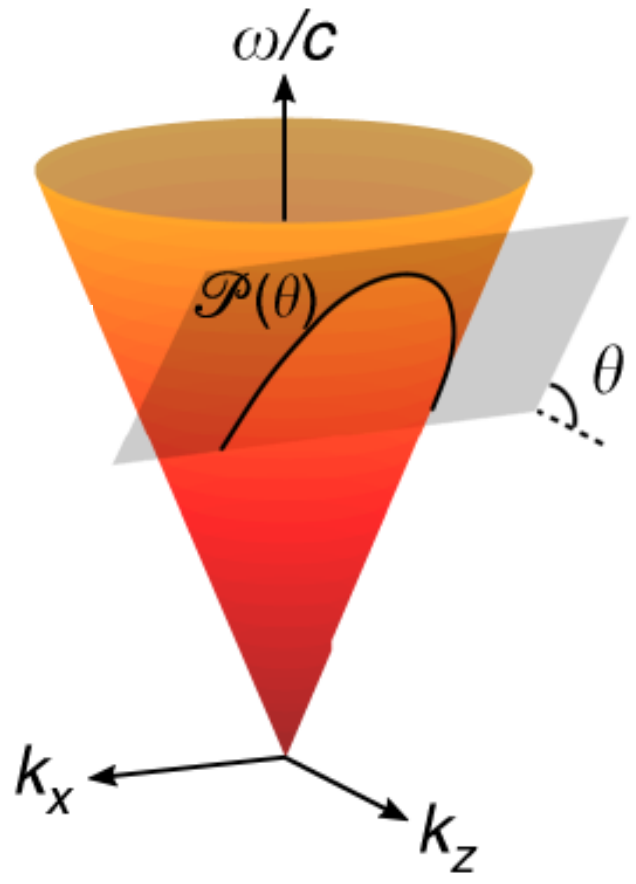
# Subluminal spatio-temporal pulses as drivers

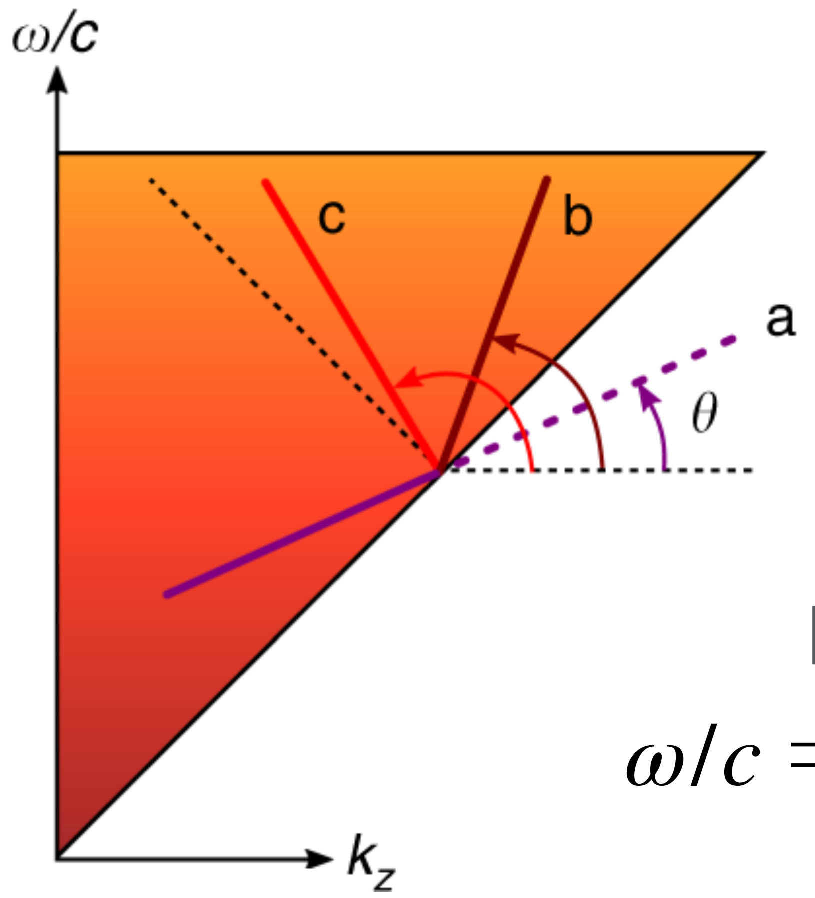
A first step toward the acceleration of non-relativistic particles is having **slower drivers ($v_g < c$)**.

In free space, we can sculpt optical pulses with a modulation of the spatio-temporal degrees of freedom *.

**Light cone**

$$k_x^2 + k_z^2 = (\omega/c)^2$$




**Hyperplane**

$$\omega/c = k_0 + (k_z - k_0)\tan\theta$$

**This allows us to have pulses propagating with a group velocity $v_g \neq c$:**

$$v_g = \frac{\partial\omega}{\partial k} = c\tan\theta$$

*H. Kondakci,, Y. F. Abouraddy,Nature Communications, 10, 929 (2019).

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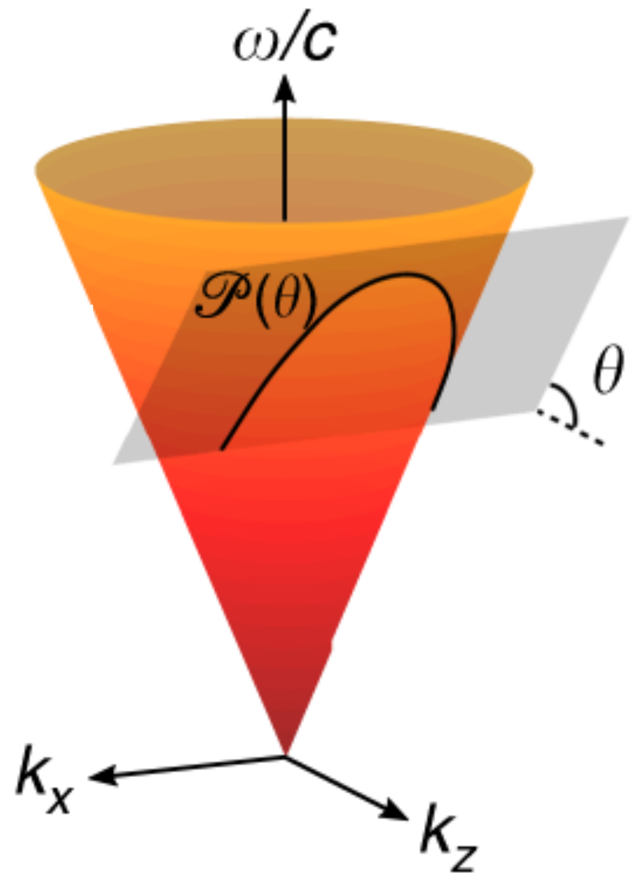


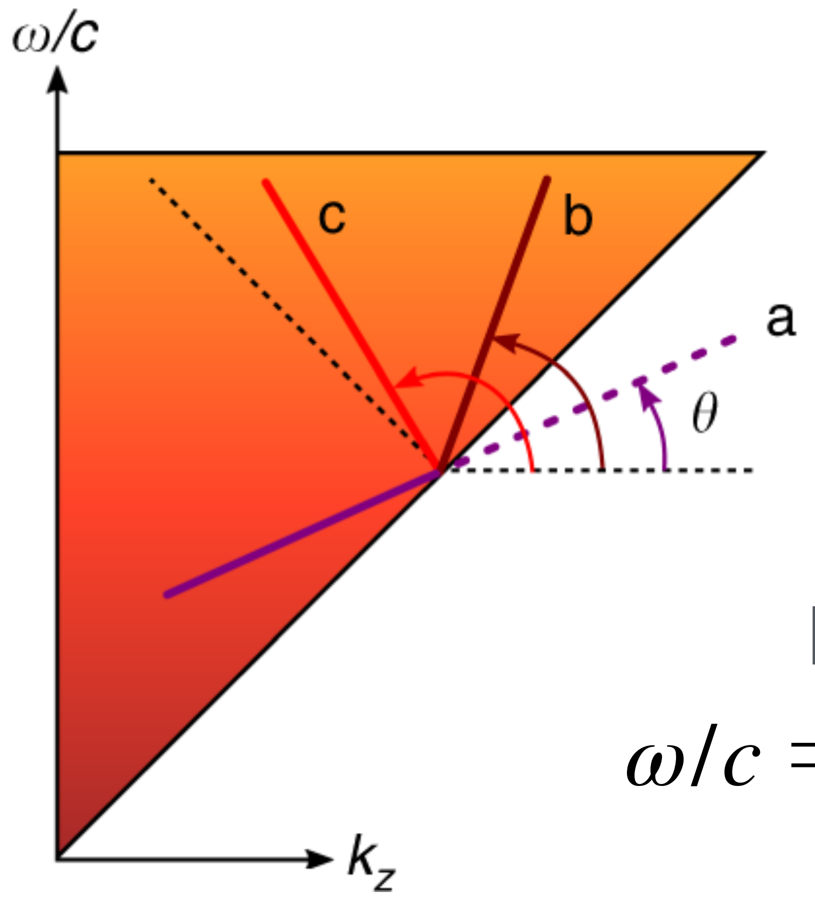


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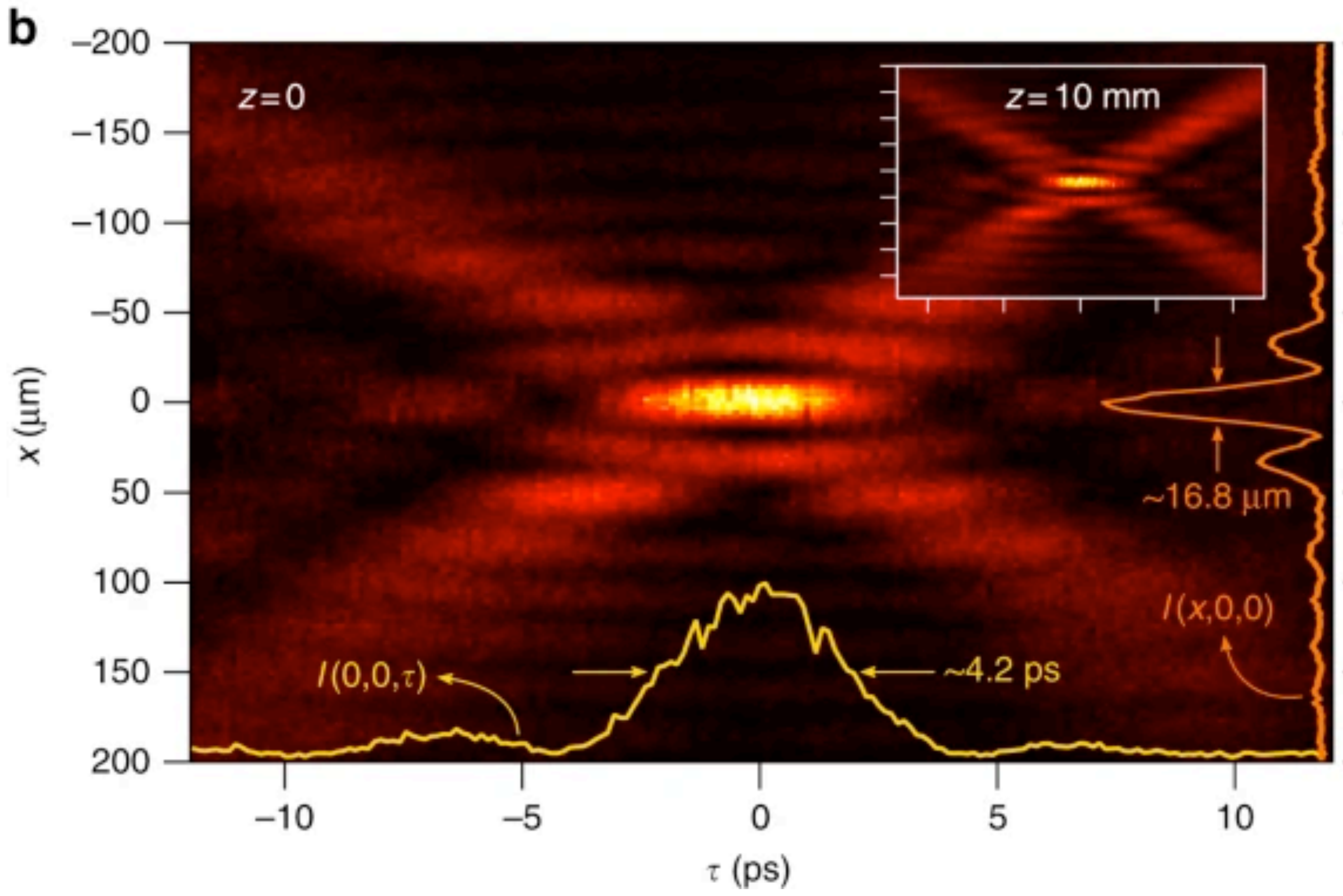


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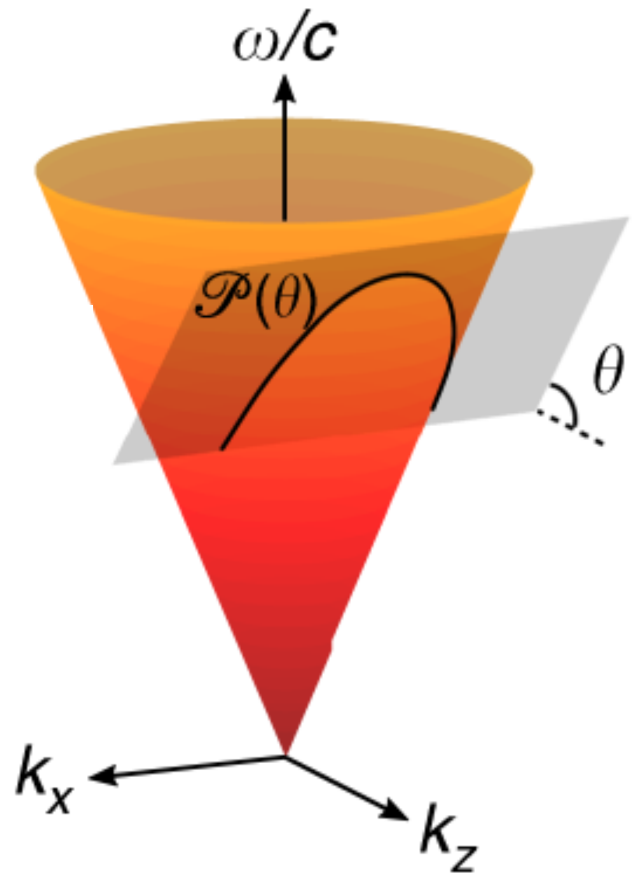


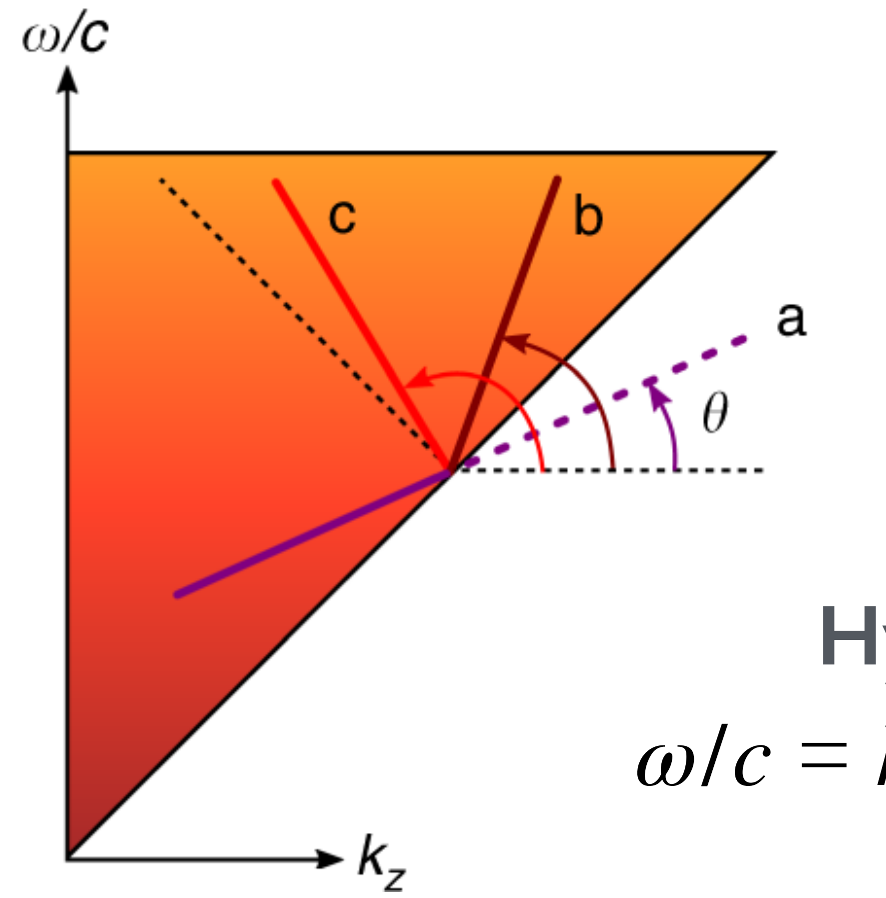


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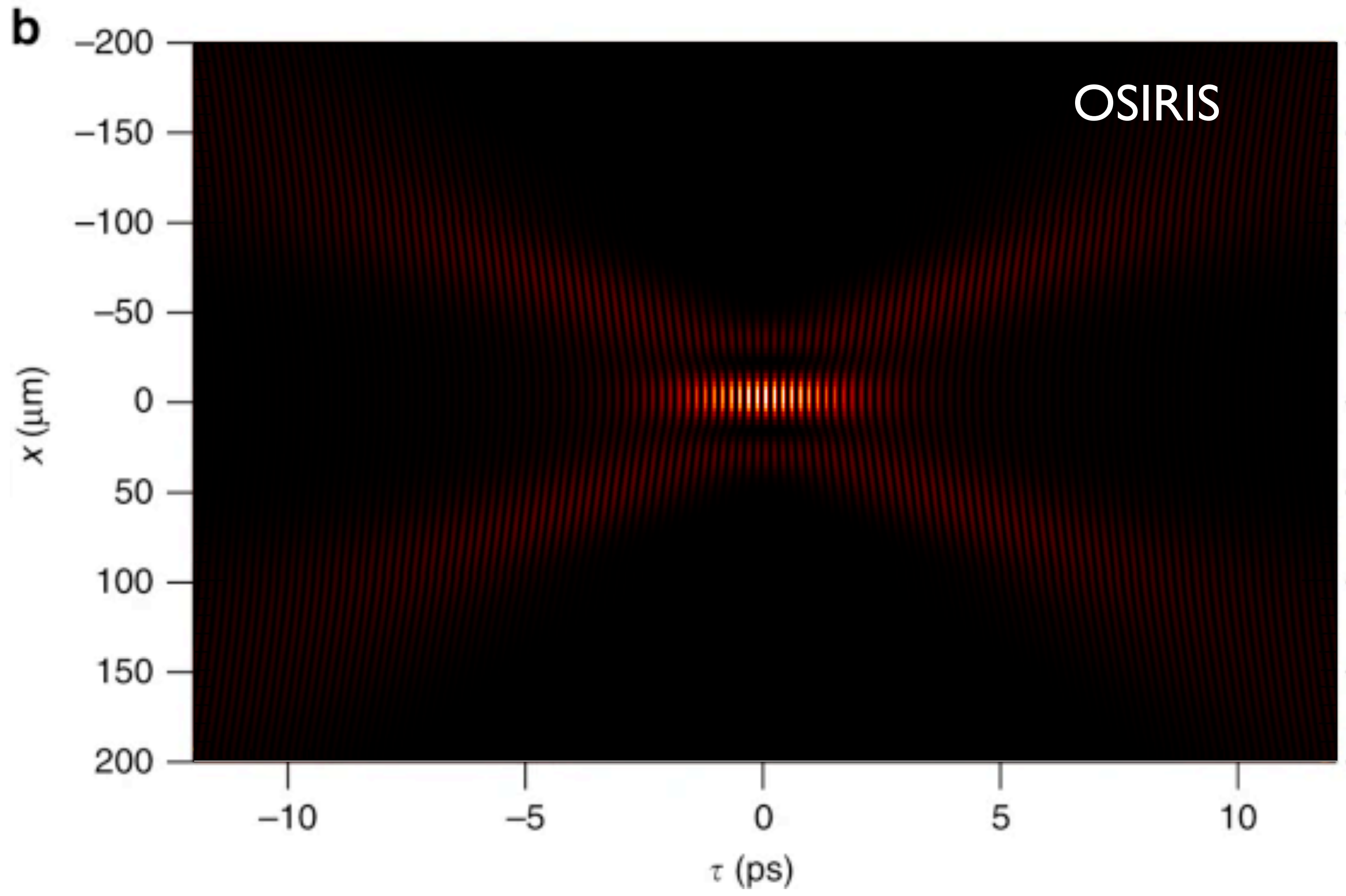


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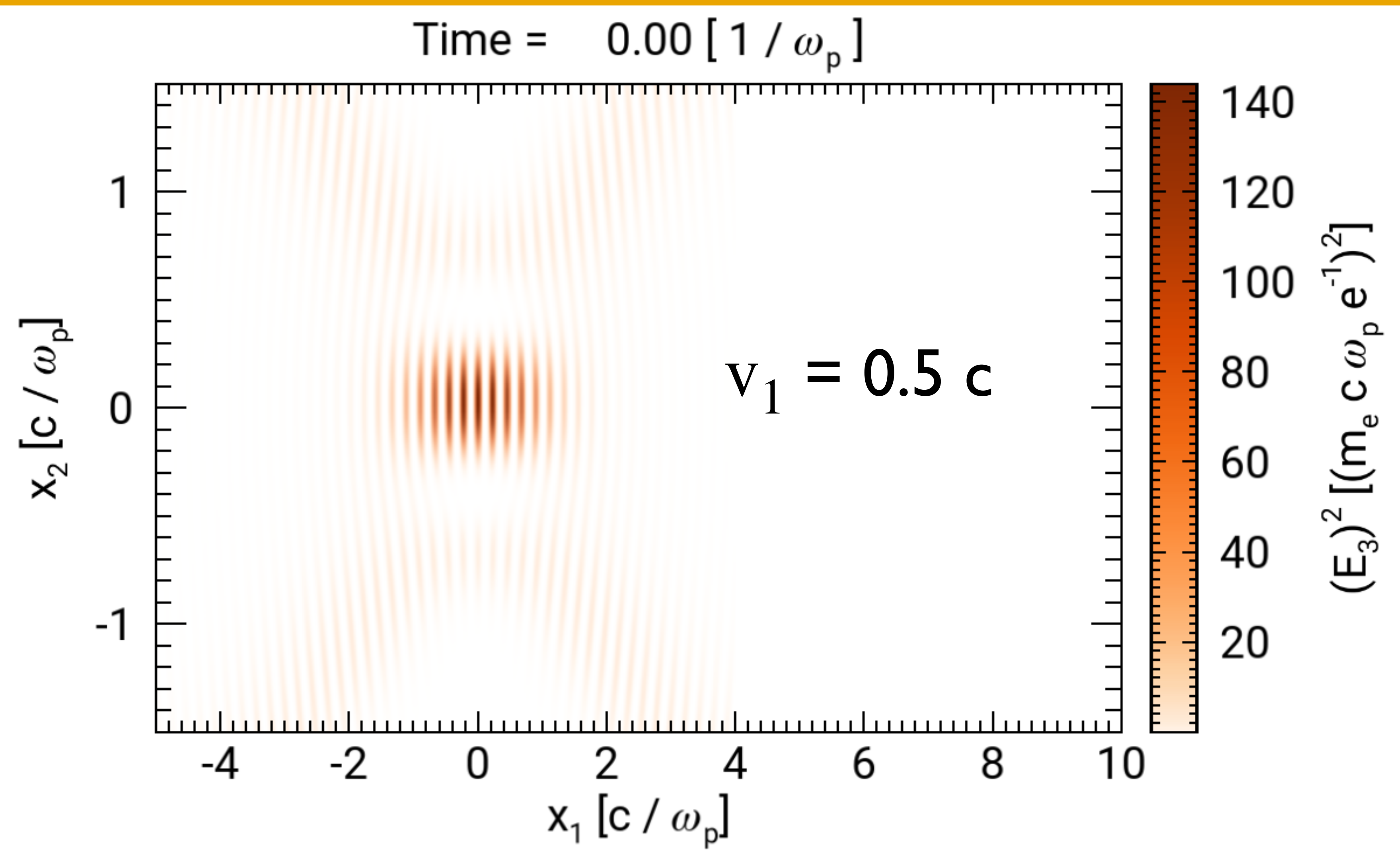
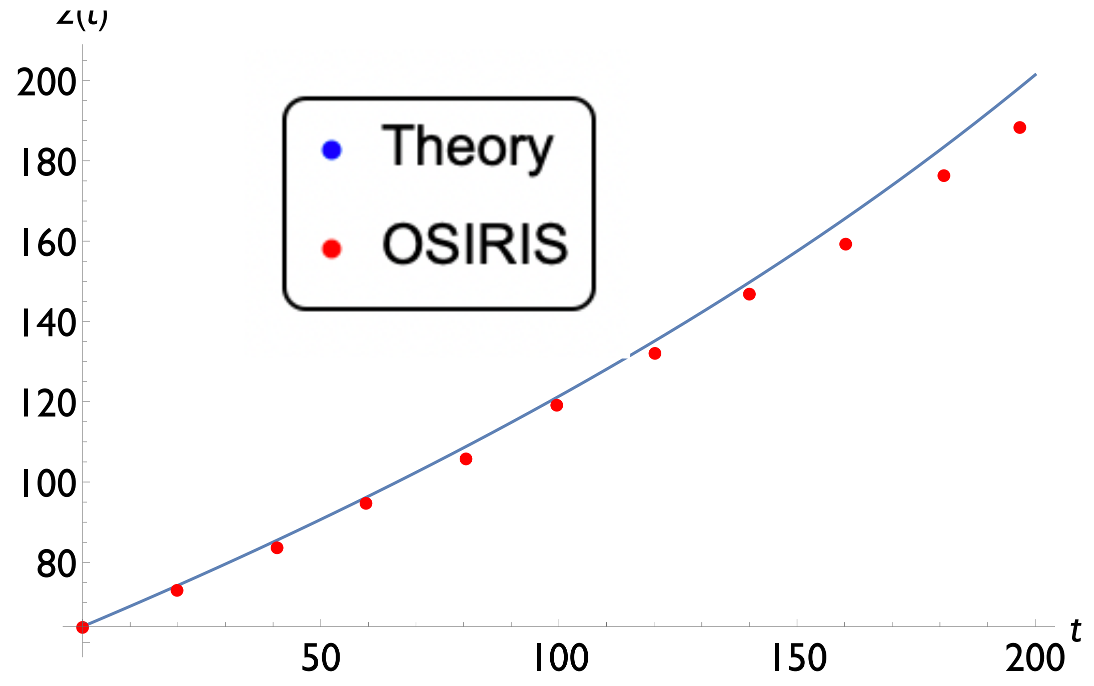
# Toward accelerating drivers

If we assign a **finite spatial spectrum** instead of a singular frequency of the stationary case, we can make these pulses accelerate with an axially encoded changing velocity*.

## Accelerating pulses




Simulation box moving at 0.5 c




## Analytical model

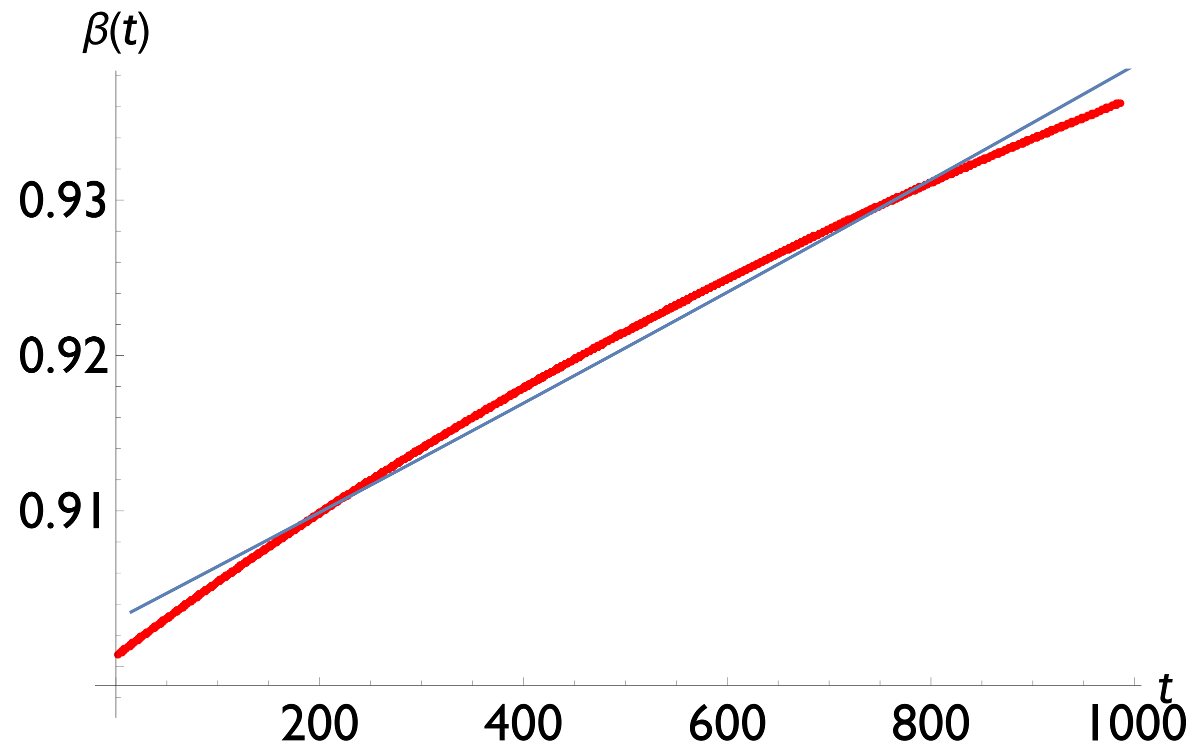
We model the energy gain of non-relativistic particles using an external field with a **time dependent phase velocity**.

Energy gain:

$$\frac{dp}{dt} = \frac{d}{dt}\left(\frac{\beta_z(t)}{\sqrt{1-\beta_z^2(t)}}\right) = eE_0 cos[k_p(z(t) - \int \beta_\phi(t)\ dt)]$$

Imposing the phase-locking condition ($\beta_z(t) = \beta_\phi(t)$), we find:

$$\beta_z^2(t) = \frac{\left(\frac{qE_0 t}{mc} + \beta_0\right)^2}{1 + \left(\frac{qE_0 t}{mc} + \beta_0\right)^2}$$

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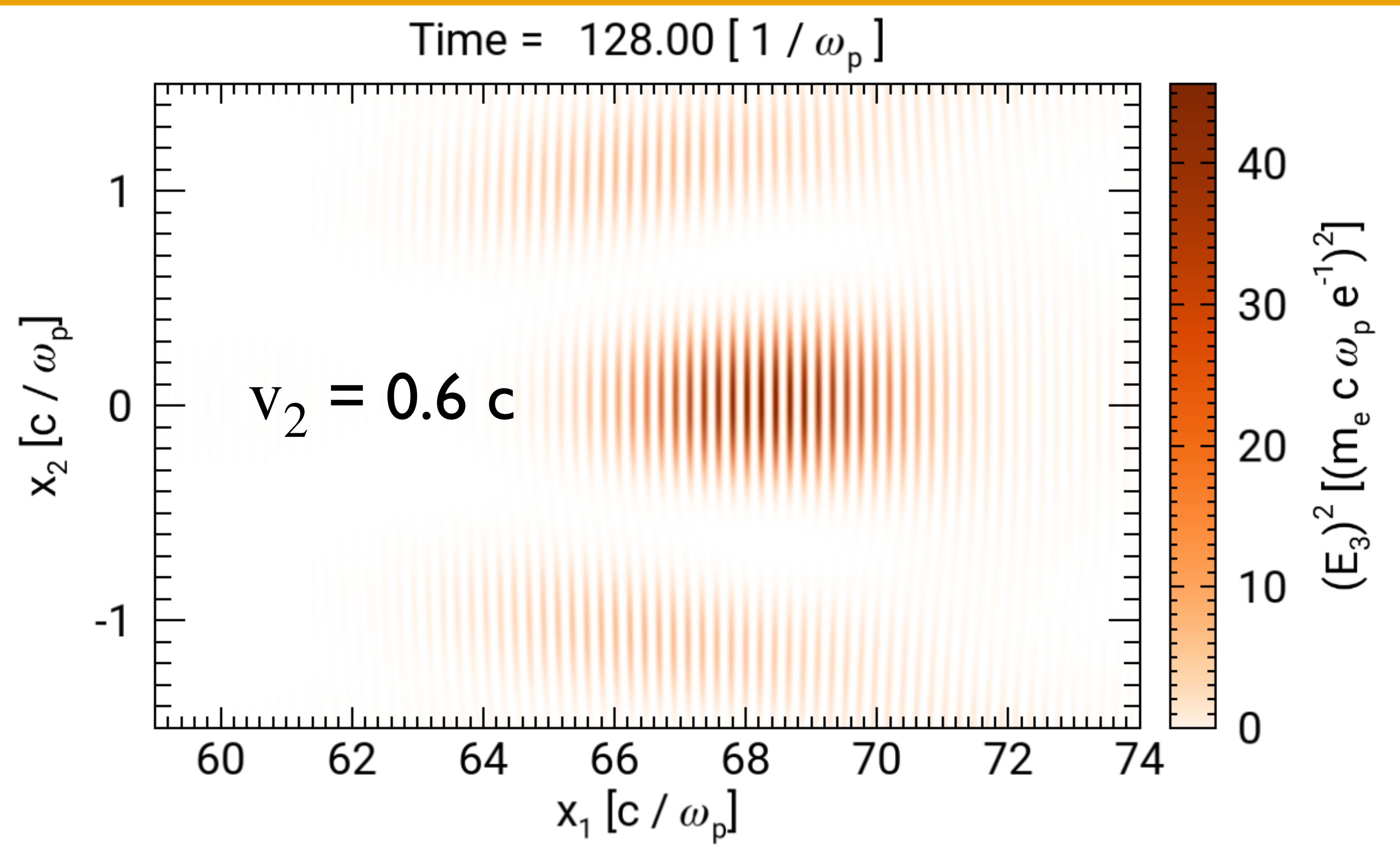


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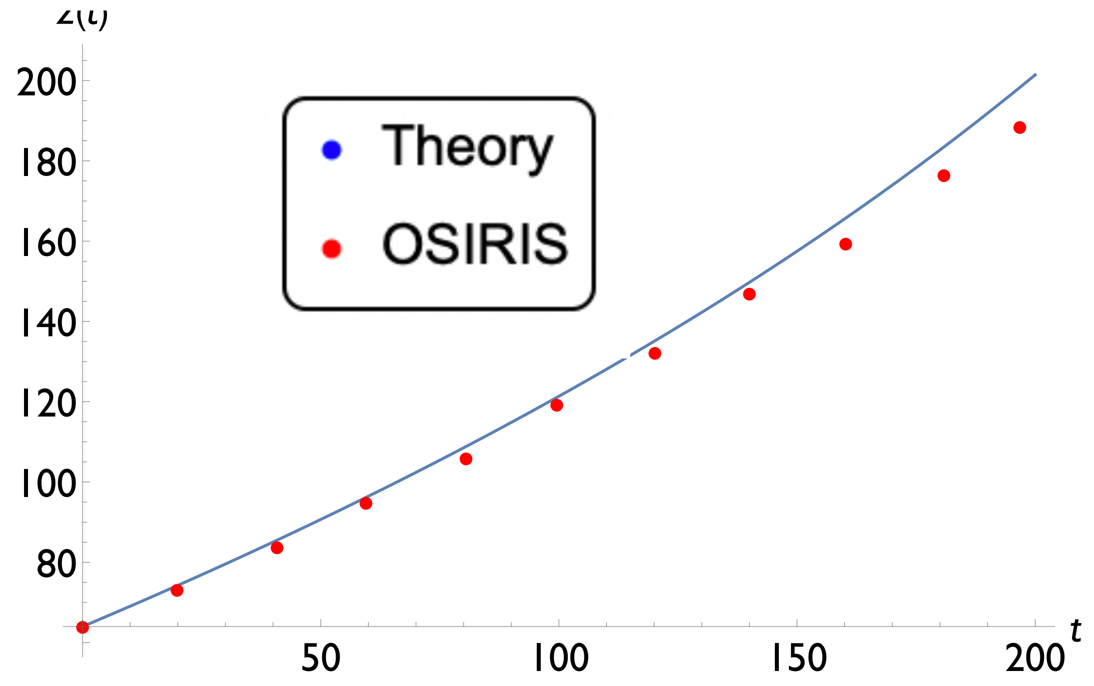


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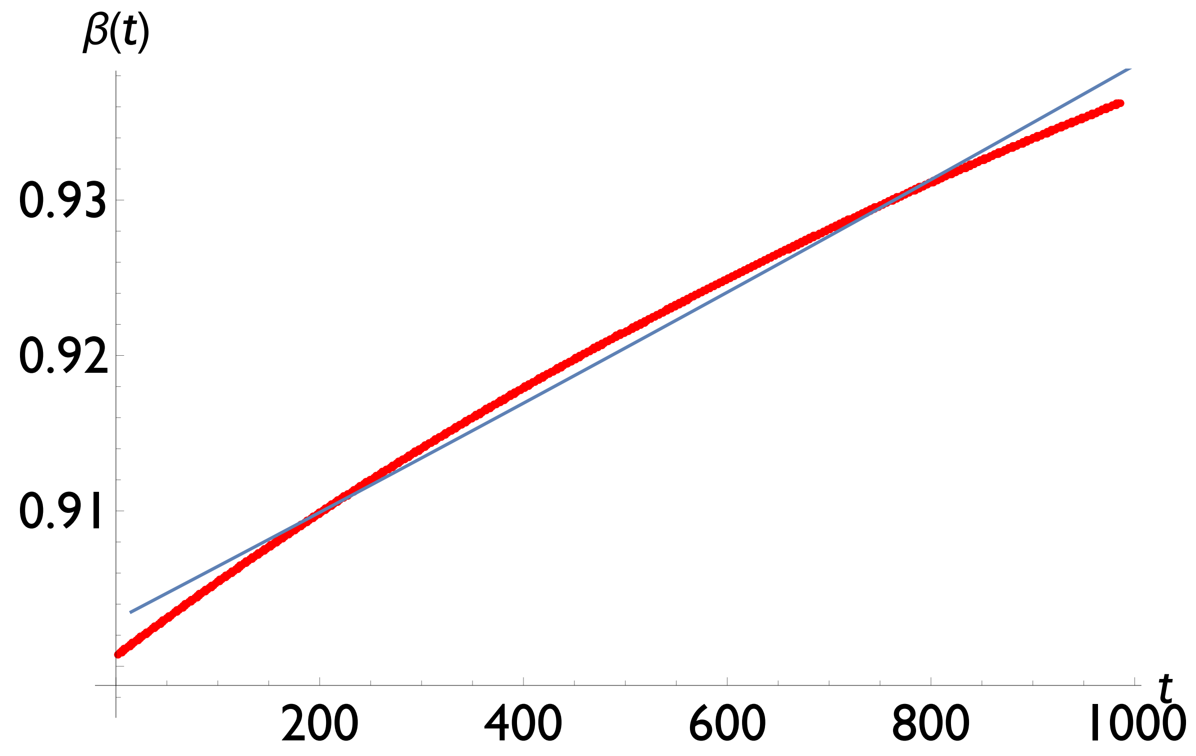


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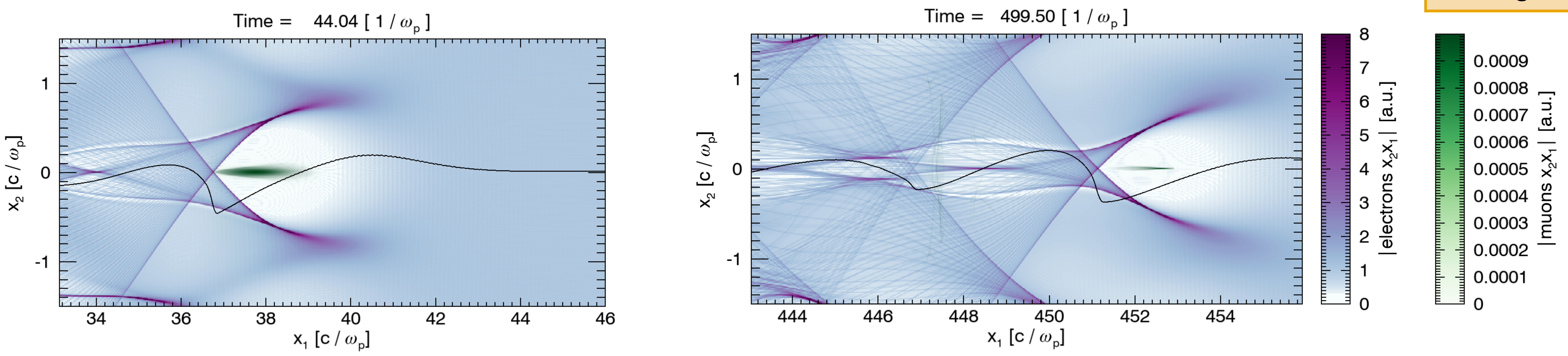
# 2D tests in OSIRIS of the use of these accelerating pulses as drivers

2D simulations have been performed using OSIRIS, testing the acceleration in the quasi-linear regime, with $a_0 = 0.8$.

Simulation box moving at 0.9 c



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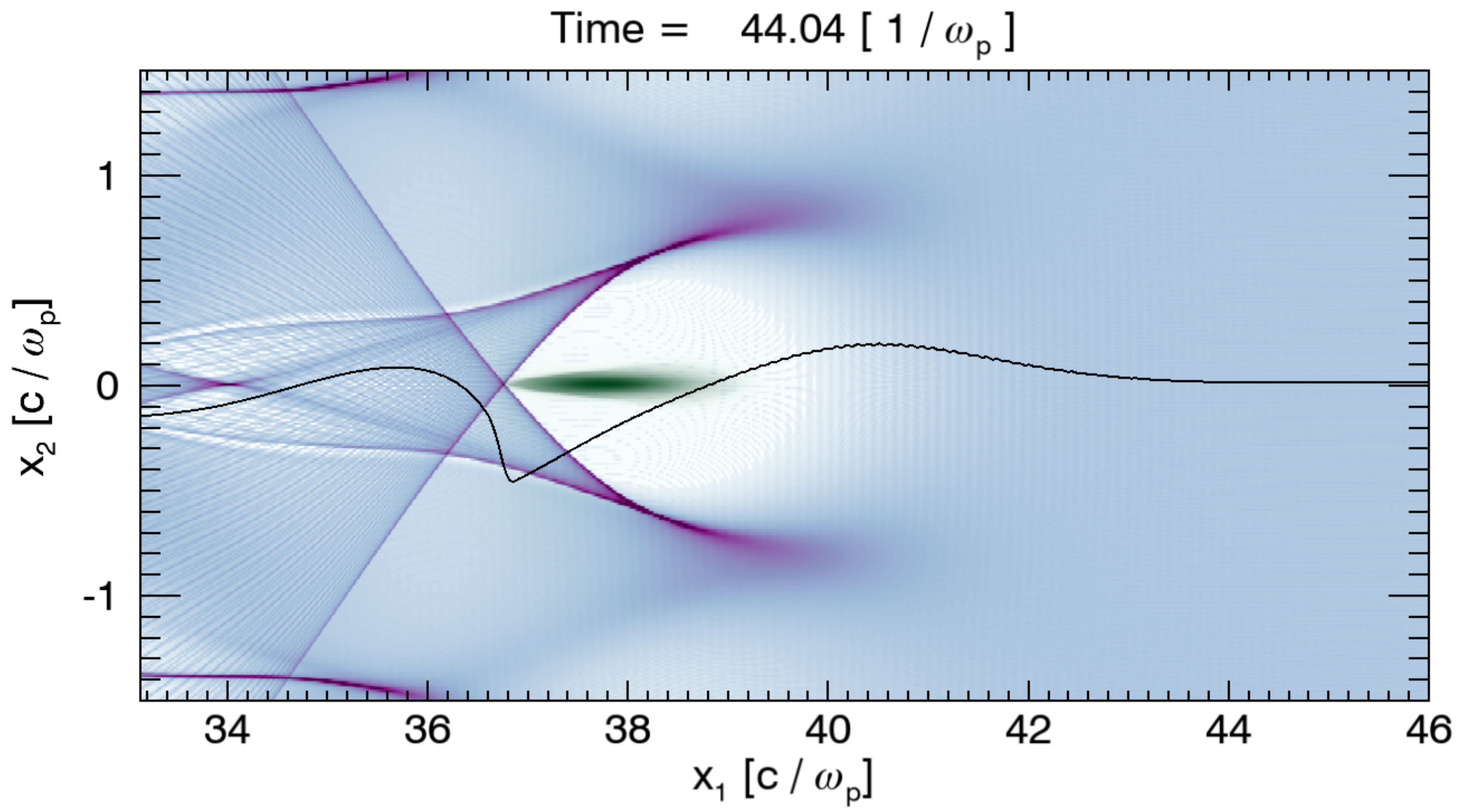


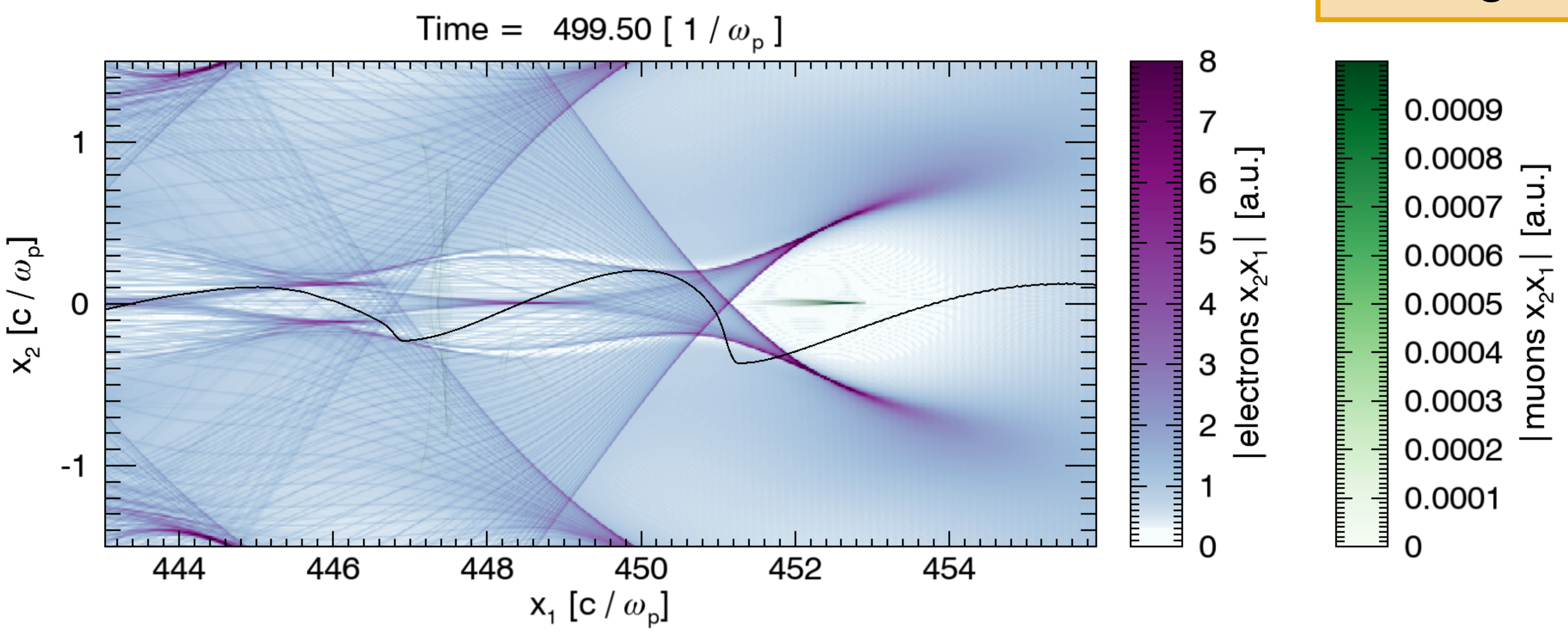


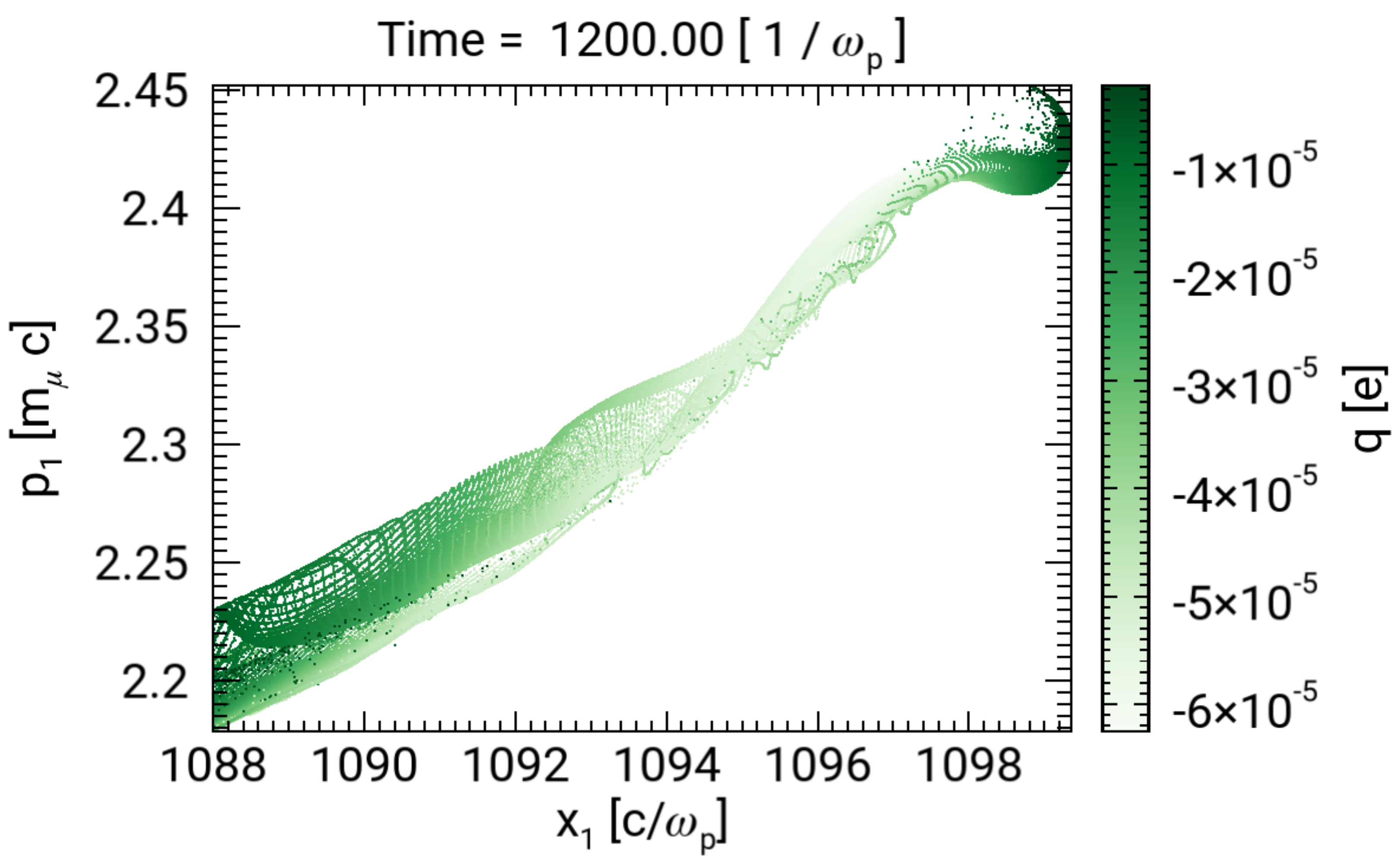


With a plasma density of $n_0 = 10^{18}\mathrm{cm}^{-3}$:

energy gain of $50$ MeV in $5$ mm
(acceleration gradient of ~10 GeV/m)

# Conclusions & Future Work

**Plasma accelerators so far are only applicable to relativistic particles.**

➡ To fill this gap, we propose the possibility of accelerating non-relativistic particles using optical wave packets with a group velocity smaller than the speed of light.

**Accelerating space time wave packets** have been implemented into OSIRIS, and then tested as drivers for the acceleration of non-relativistic muons.

In the future, we will investigate the **non-linear** regime to see if we can improve the energy gain.

Thank you for your attention! :)

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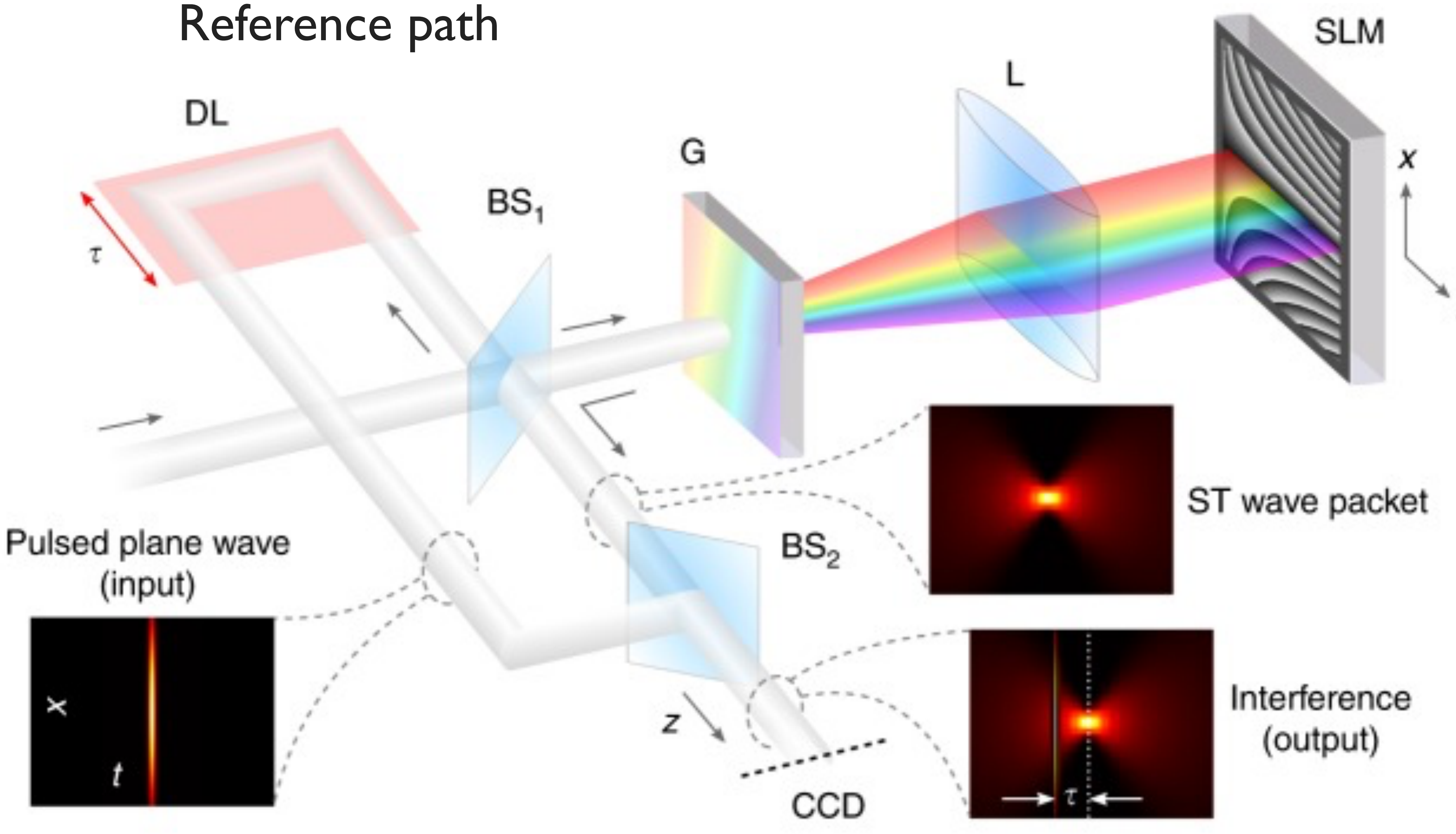
# BACK UP SLIDES

A pulsed plane wave is split into two paths:

Reference path

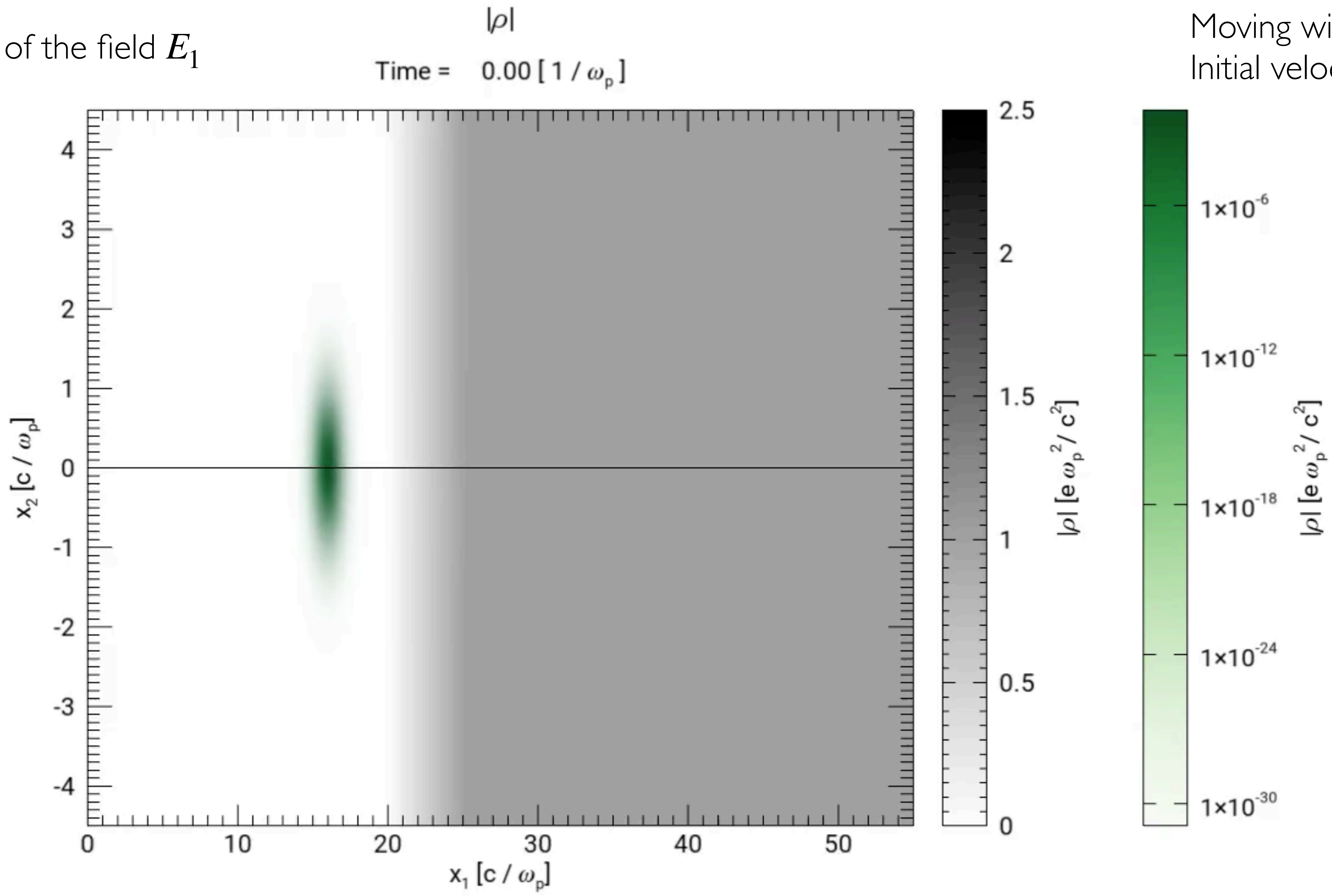



the ST wave packet is synthesized using a two-dimensional pulse shaper formed of a diffraction grating (G), cylindrical lens (L), and spatial light modulator (SLM)

$$
\begin{aligned}
E(x, z; t) &= e^{i(k_o z-\omega_o t)} \int z \mathrm{d}k_x \tilde{\psi}(k_x) e^{i(k_x x+[k_z-k_o][z-ct \tan \theta])} \\
&= e^{i(k_o z-\omega_o t)} \psi(x, z-v_g t).
\end{aligned}
$$

# Testing of the analytical model in 2D using OSIRIS

In black, the line out of the field $E_1$




Moving window at **0.95c**
Initial velocity of the muons of **0.9c**

# Optical space-time wave packets with arbitrary group velocities

The group velocity of an optical pulse can usually be modified in the propagation in a material.

In free space, we can sculpt optical pulses with a modulation of the spatial and temporal degrees of freedom.

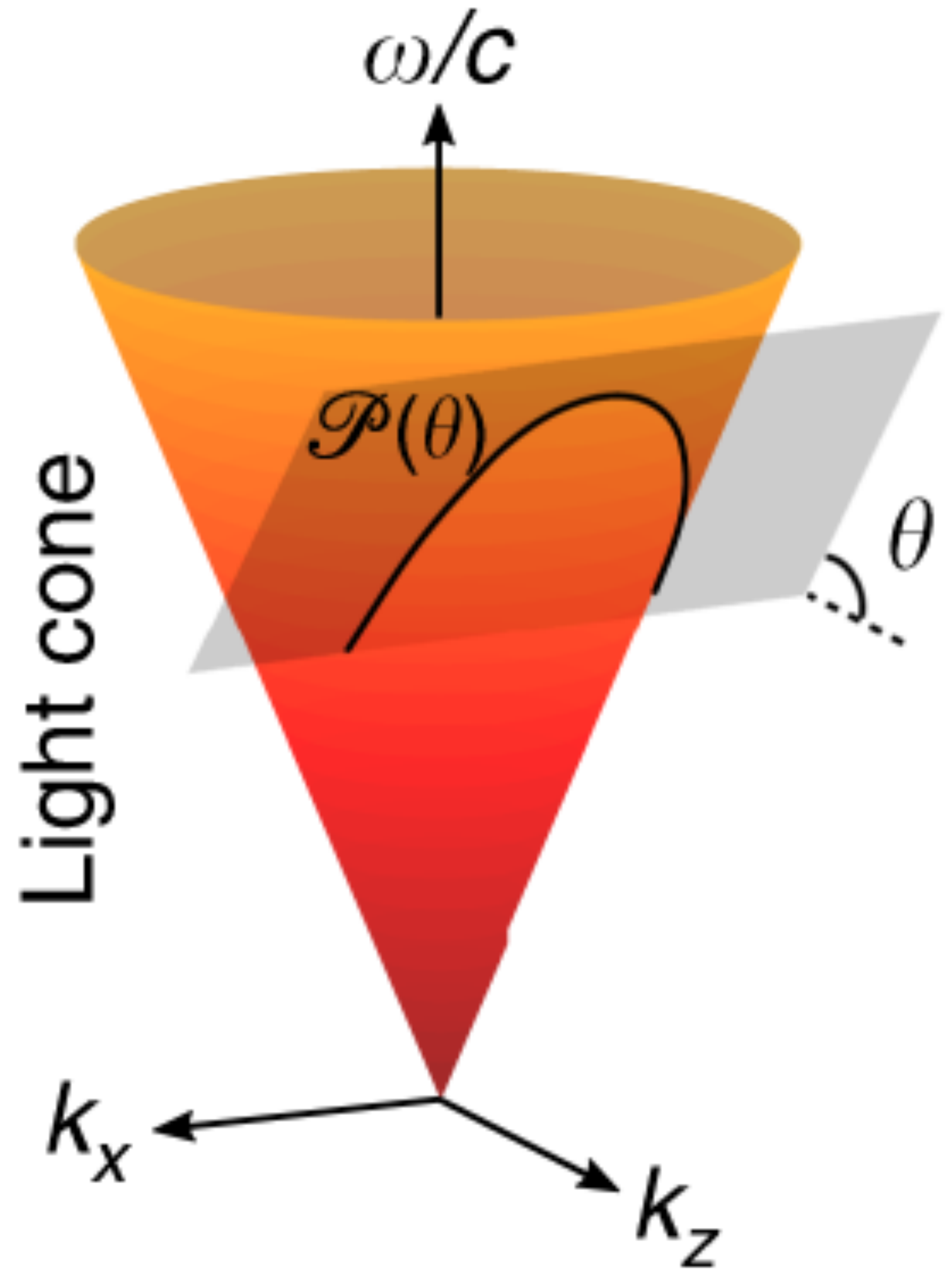
**Spatio-temporal wave packets**: each spatial frequency is uniquely associated with a specific temporal frequency (or wavelength)

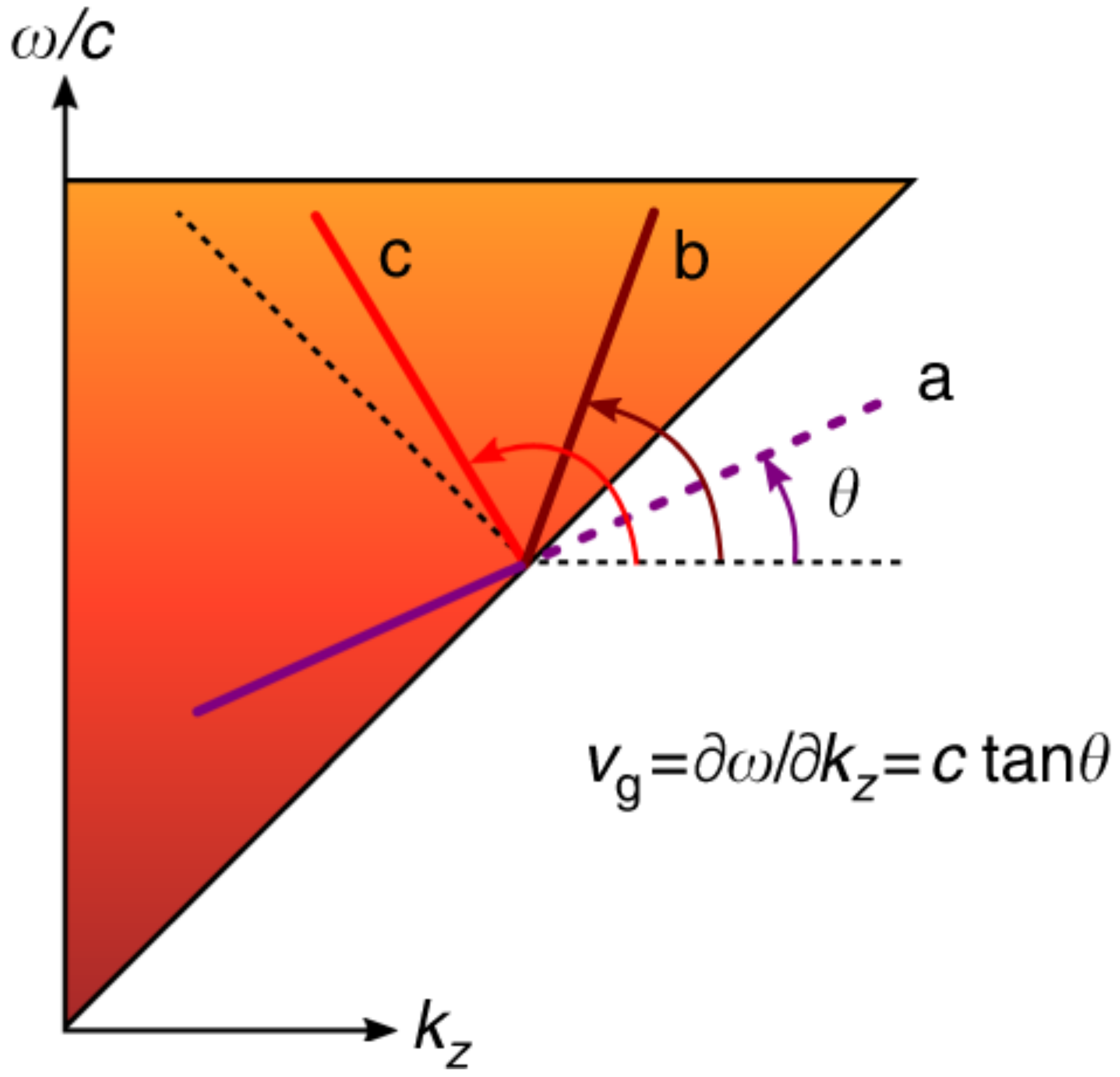
Light cone

$$k_x^2 + k_z^2 = (\omega/c)^2$$

Hyperplane

$$\omega/c = k_0 + (k_z - k_0)\tan\theta$$

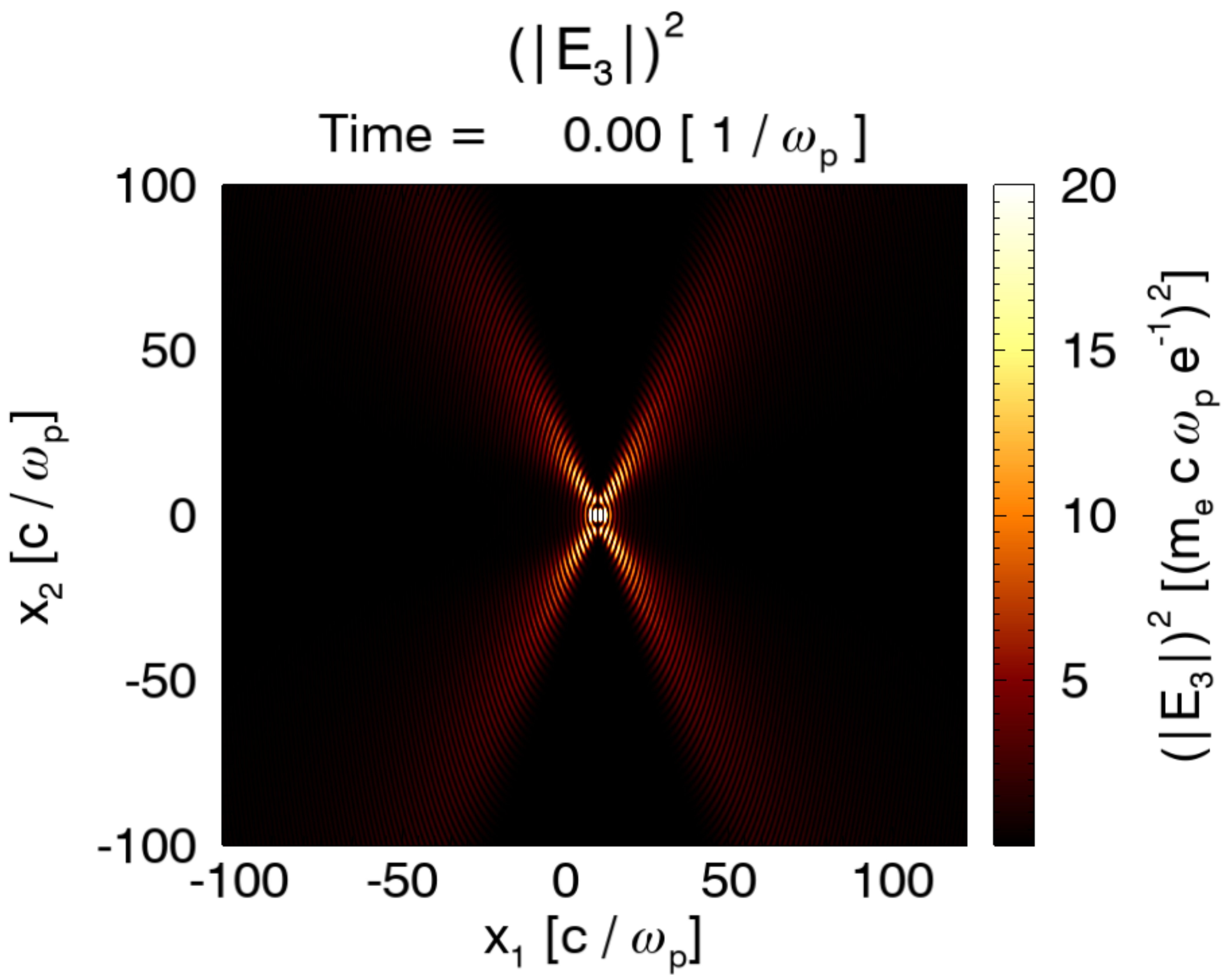



Kondakci, H. Esat; Abouraddy, Ayman F., Nature Communications, **10**, Article number: 929 (2019) https://doi.org/10.1038/s41467-019-08735-8

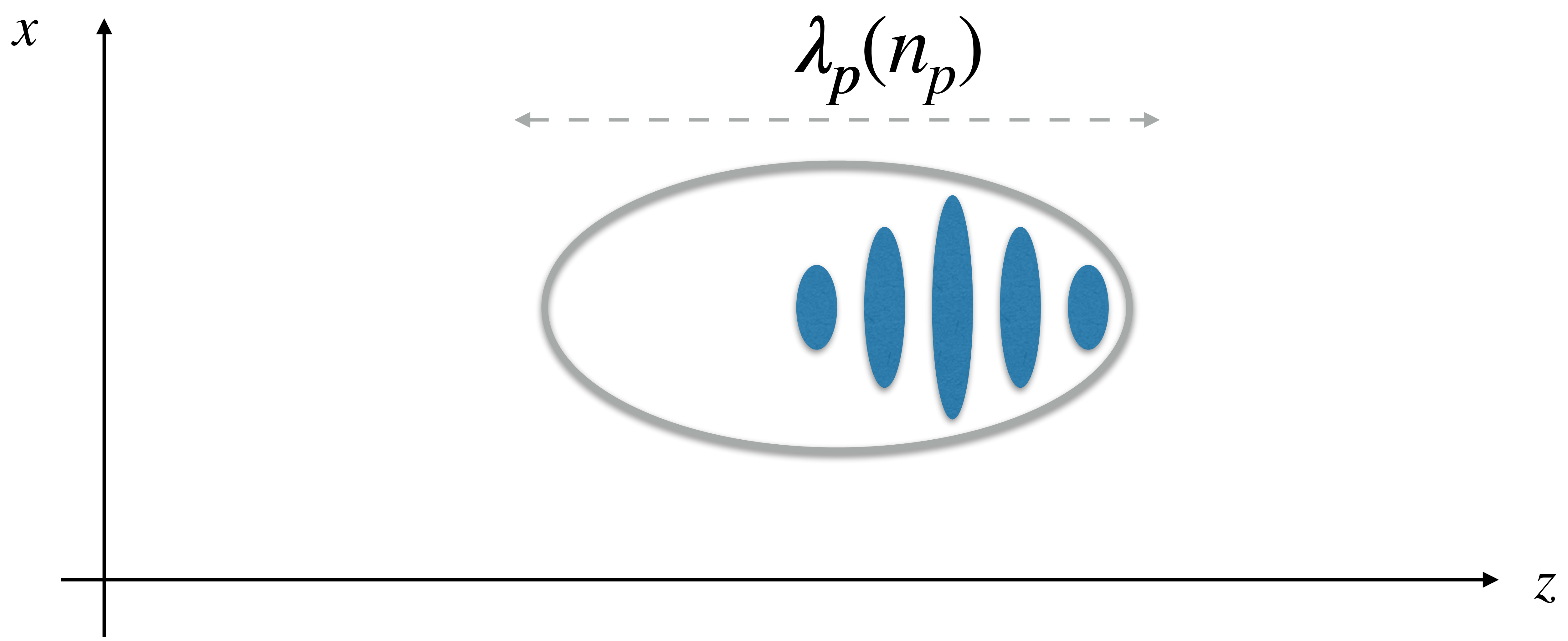
# Optical space-time wave packets with arbitrary group velocities

OSIRIS 2D simulations of subluminal space-time wake packets*




*B. Malaca et al., in preparation

# Plasma ramp for the “acceleration” of the plasma wake




S. Bulanov et al, Phys. Rev. E **58**, R5257(R) – Published 1 November (1998), https://doi.org/10.1103/PhysRevE.58.R5257

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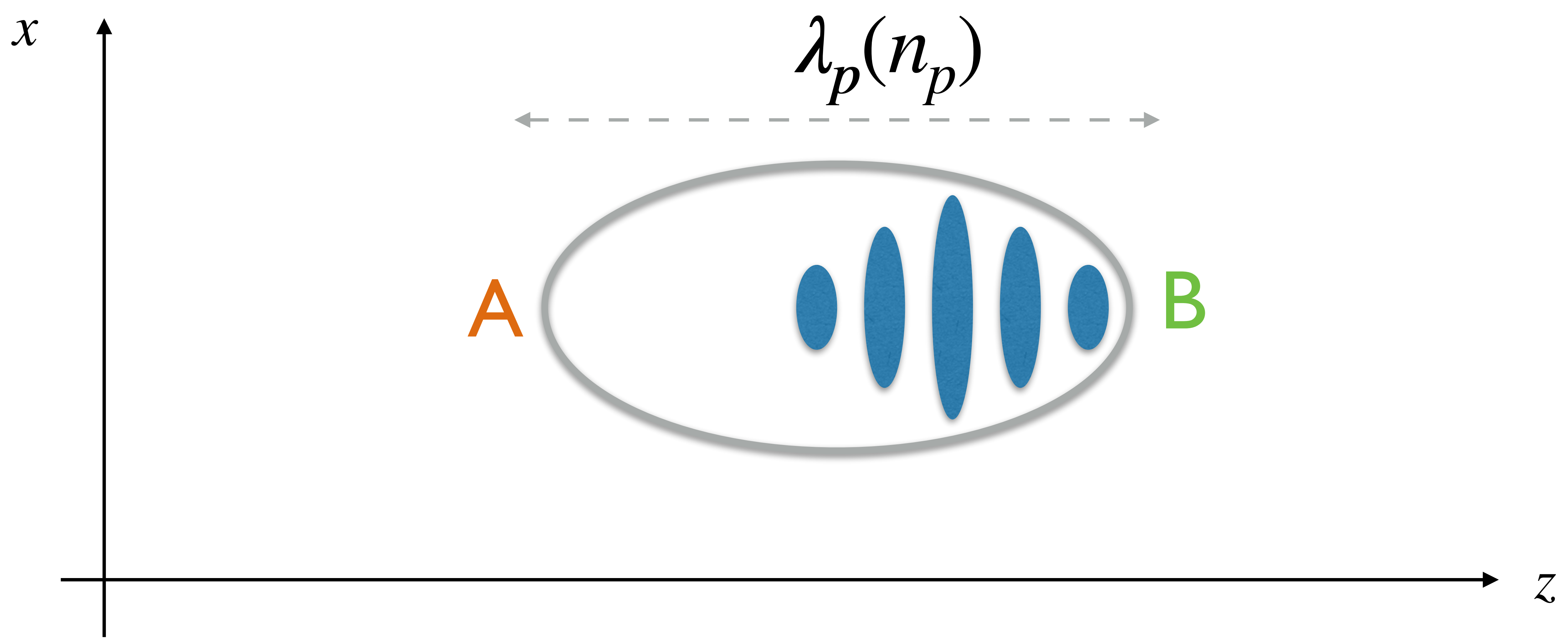


$$z_B = v_g t$$

$$v_B = v_g$$

$$z_A = v_g t - \lambda_p(n_p)$$

$$v_A = \frac{dz_A}{dt} = v_g - \frac{d\lambda_p(n_p)}{dt} = v_g - \frac{\partial \lambda_p}{\partial n_p}\boxed{\frac{\partial n_p}{\partial t}}$$

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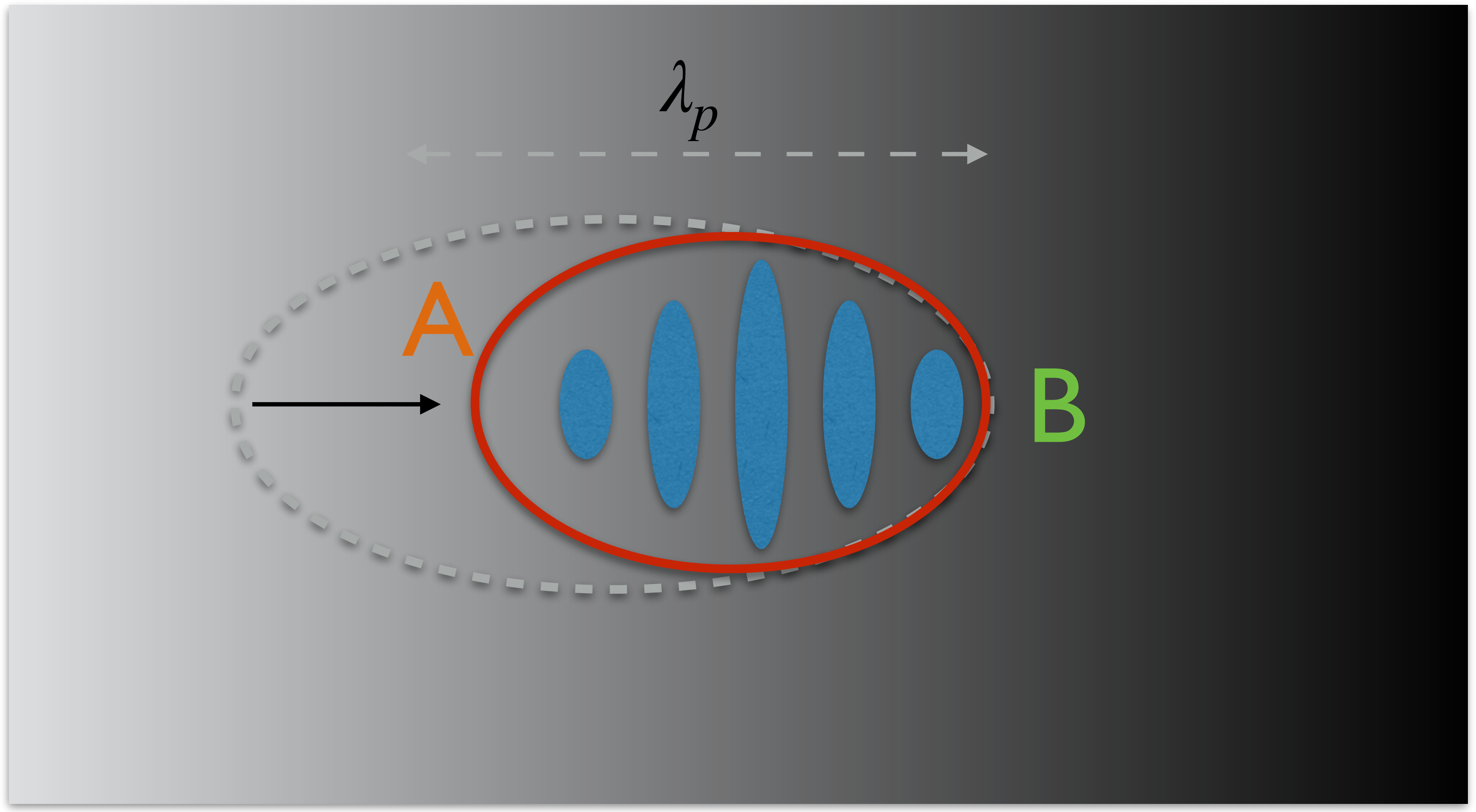


$$z_B = v_g t \qquad v_B = v_g$$

$$z_A = v_g t - \lambda_p(n_p) \qquad v_A = \frac{dz_A}{dt} = v_g - \frac{d\lambda_p(n_p)}{dt} = v_g - \frac{\partial \lambda_p}{\partial n_p}\frac{\partial n_p}{\partial t}$$

This **accordion** effect results in an acceleration of the back of the plasma wake → it could help us to **extend the acceleration distance.**

S. Bulanov et al, Phys. Rev. E **58**, R5257(R) – Published 1 November (1998), https://doi.org/10.1103/PhysRevE.58.R5257

# Acknowledgments

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**Thank you for your attention**

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