

Phase Shaping of Free-Electrons Wavefunction with fs-Laser Pulses in an  
RF-cavity-based Ultrafast Transmission Electron Microscope

Simona Borrelli

# RF-Cavity-Based Ultrafast Transmission Electron Microscope

200 keV Schottky Field Emission Gun

Projection System

Ima  
syst

203 mm

Objective  
lens

C1 lens

C1 aperture

C2 lens

C2 aperture

TM<sub>110</sub>  
cavity

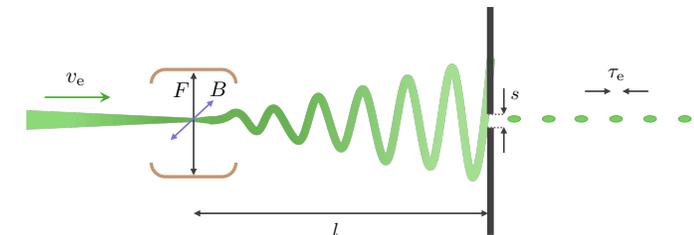
Chopping  
aperture

Specimen holder

$$Br = (7 \pm 1) \times 10^6 \text{ A/ m}^2 \text{ sr V}$$

~ 100 fs pulses  
@ 75 MHz or 3 GHz

$$\epsilon_{n,x}, \epsilon_{n,y} \sim \text{pm rad}$$



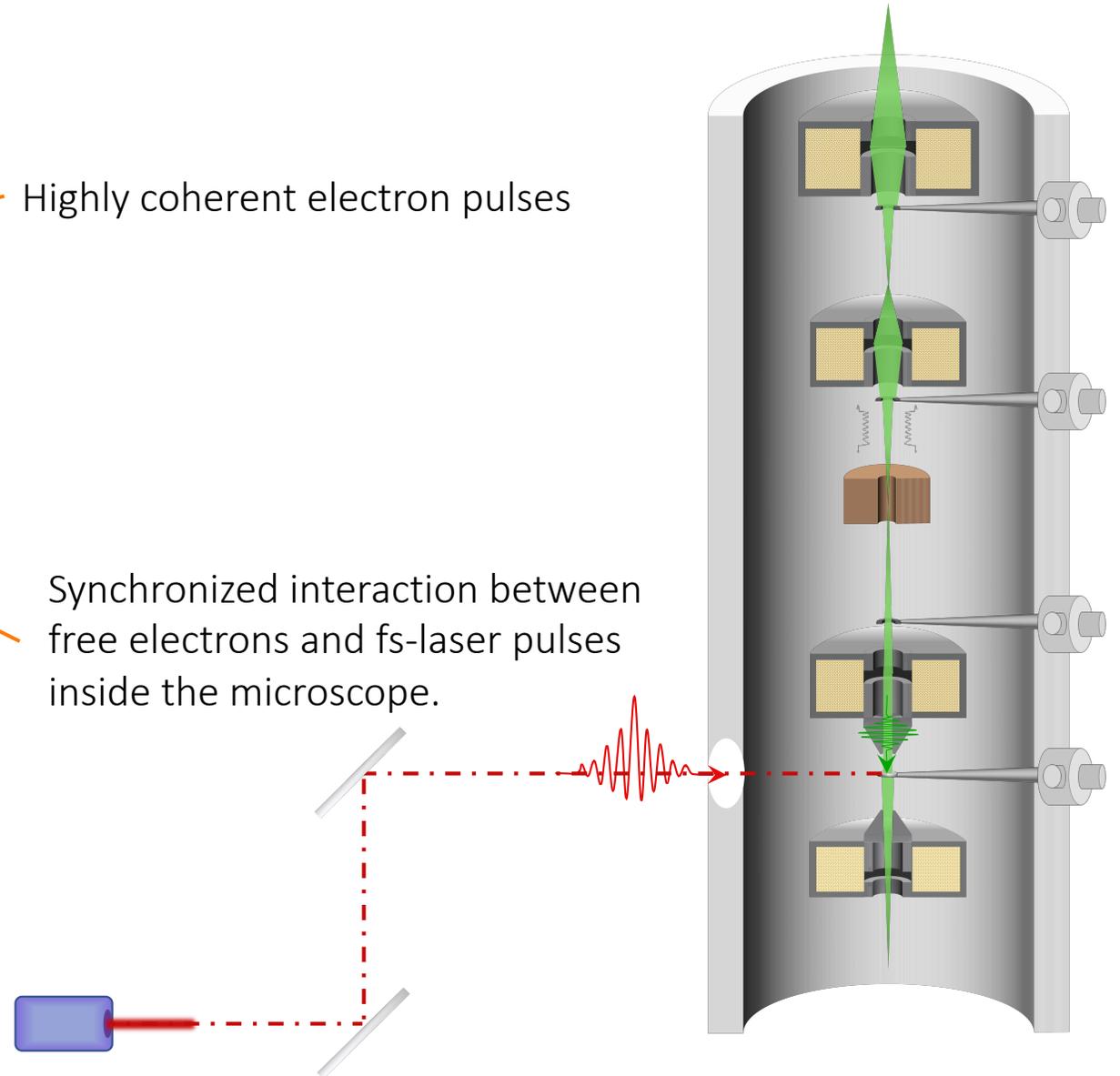
# Coherent Manipulation of Electron Wavefunction with Light

Coherent manipulation and control of the electron's wavefunction with light

Highly coherent electron pulses

Synchronized interaction between free electrons and fs-laser pulses inside the microscope.

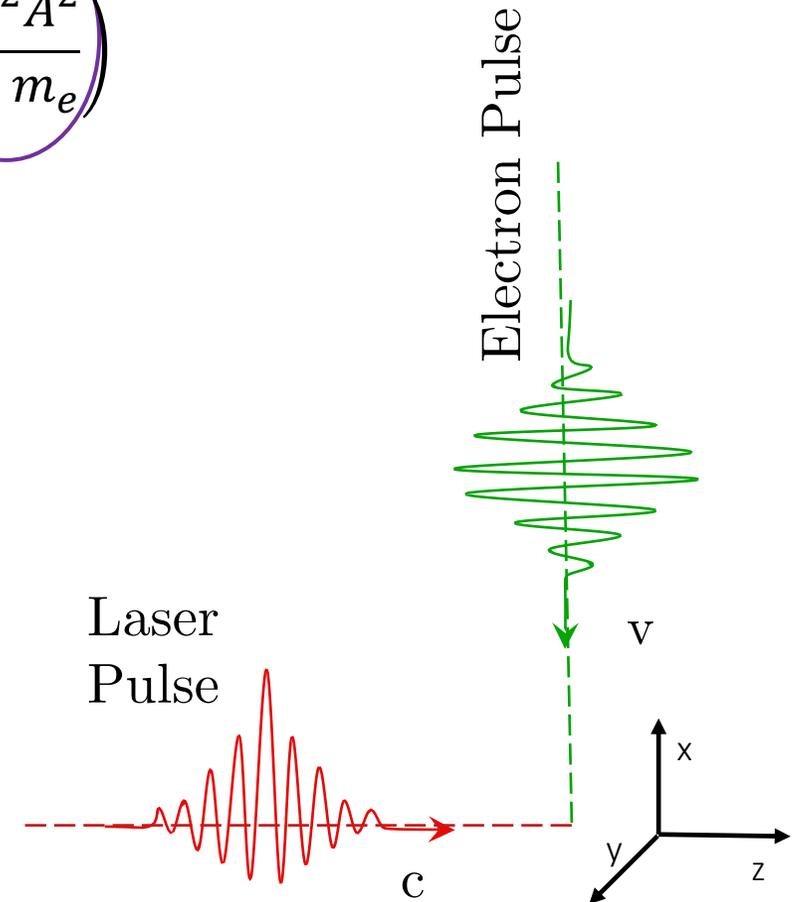
Establishing new free electron quantum optics based on the interaction of sub-ps electron pulses with the time-dependent electromagnetic fields of light.



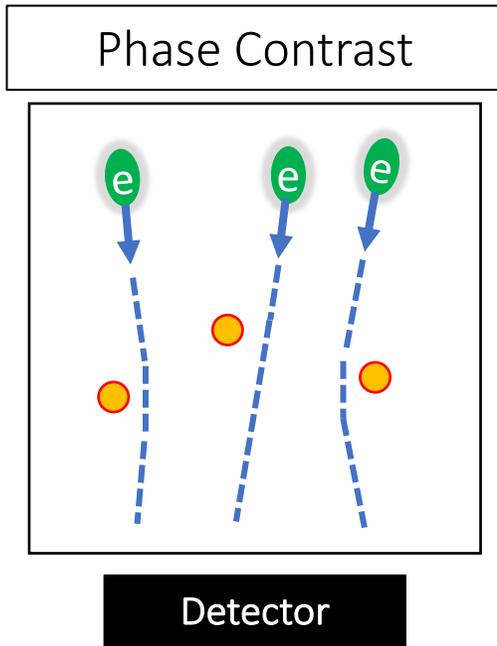
# Coherent Manipulation of Electron Wavefunction with Light

$$H_S = \left( -\frac{\hbar^2 \nabla^2}{2m_e} - \frac{ie\hbar}{m} \vec{A} \cdot \vec{\nabla} + \frac{e^2 A^2}{2m_e} \right)$$

Ponderomotive Phase  
Shaping

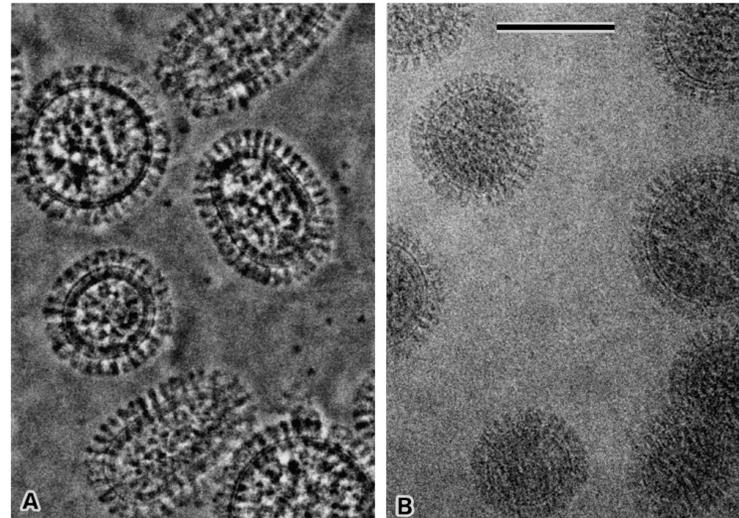


# Pulsed-Laser Zernike Phase Plate for Phase Contrast Imaging of biological samples

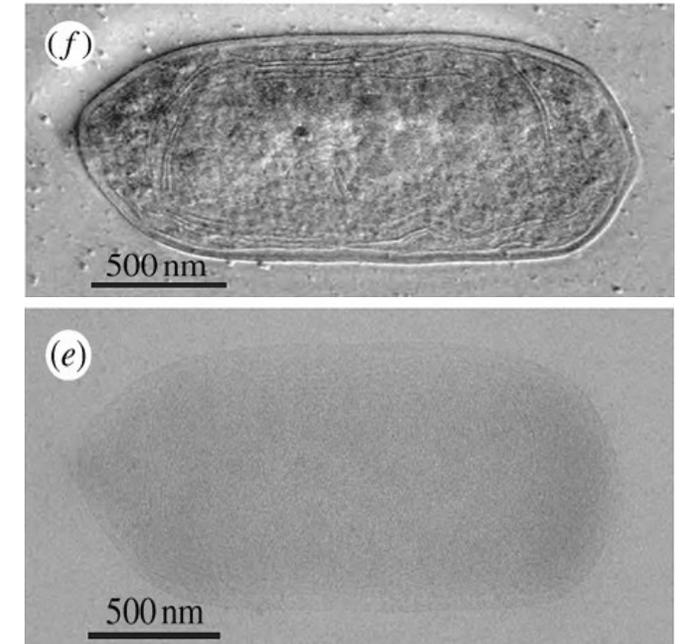


$\frac{\pi}{2}$  Phase Shift between scattered and unscattered electrons

## Weakly scattering specimens



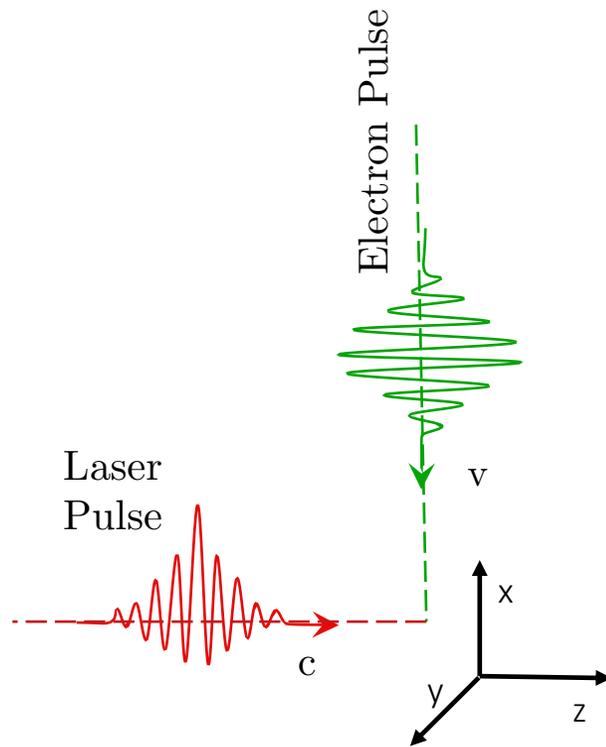
Zernike phase contrast electron micrograph (A) and a conventional electron micrograph (B) of ice-embedded influenza A viruses  
(Yamaguchi Y. et al.)



Conventional (e) and phase contrast (f) TEM image of cyanobacterial cell  
(Konyuba Y. et al.)

# Phase Shaping of Electrons Wavefunction with fs-Laser Pulses

$$H_S = \left( -\frac{\hbar^2 \nabla^2}{2 m_e} - \frac{ie\hbar}{m} \bar{A} \cdot \bar{\nabla} + \frac{e^2 A^2}{2 m_e} \right)$$



## Feasibility of a Pulsed Ponderomotive Phase Plate for Electron Beams

K.A.H. van Leeuwen<sup>1,\*</sup>, W.J. Schaap<sup>1</sup>, B. Buijsse<sup>2</sup>, S. Borelli<sup>1</sup>, S.T. Kempers<sup>1</sup>, W. Verhoeven<sup>1,3</sup> and O.J. Luiten<sup>1</sup>

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(Dated: August 31, 2022)

We propose a scheme for constructing a phase plate for use in a Zernike-type phase contrast electron microscope, based on the interaction of the electron beam with a strongly focused, high-power femtosecond laser pulse and a pulsed electron beam. Analytical expressions for the phase shift using the time-averaged ponderomotive potential and a paraxial approximation for the focused laser beam are presented, as well as more rigorous quasiclassical simulations based on the quantum phase integral along classical, relativistic electron trajectories in an accurate, non-paraxial description of the laser beam. The results are shown to agree well unless the laser beam is focused to a waist size below a wavelength. For realistic (off-the-shelf) laser parameters the optimum phase shift of  $\pi/2$  is shown to be achievable. When combined with RF-cavity based electron chopping and compression techniques to produce electron pulses, a femtosecond regime pulsed phase contrast microscope can be constructed. The feasibility and robustness of the scheme are further investigated using the simulations, leading to motivated choices for design parameters such as wavelength, focus size and polarization.

# Phase Shaping of Electrons Wavefunction with fs-Laser Pulses

Relativistic ponderomotive potential

$$U_p = \frac{e^2}{c \epsilon_0 \gamma m \omega_0^2} \frac{W}{\pi^{3/2} w(z)^2 \tau} e^{-\frac{(t-z/c)^2}{\tau^2}} e^{-2\frac{x^2+y^2}{w(z)^2}}$$

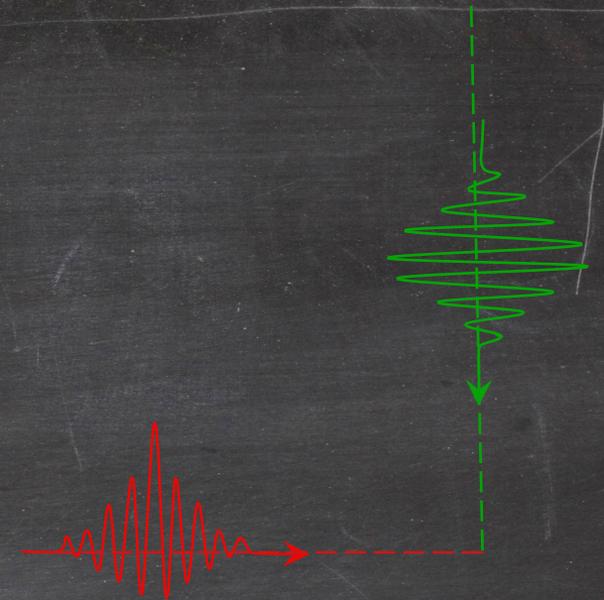
Phase Shift on an electron crossing the laser spot  $w(z)$  at  $\vec{r} = (z, y)$  at time  $t_0$

$$\Delta\varphi = -\frac{1}{\hbar} \int_{-\infty}^{+\infty} dt U_p(v(t - t_0), y, z, t)$$

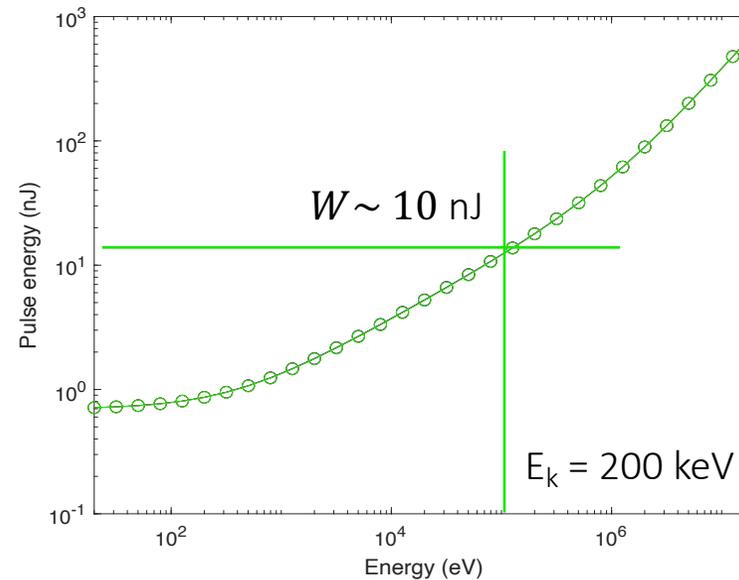
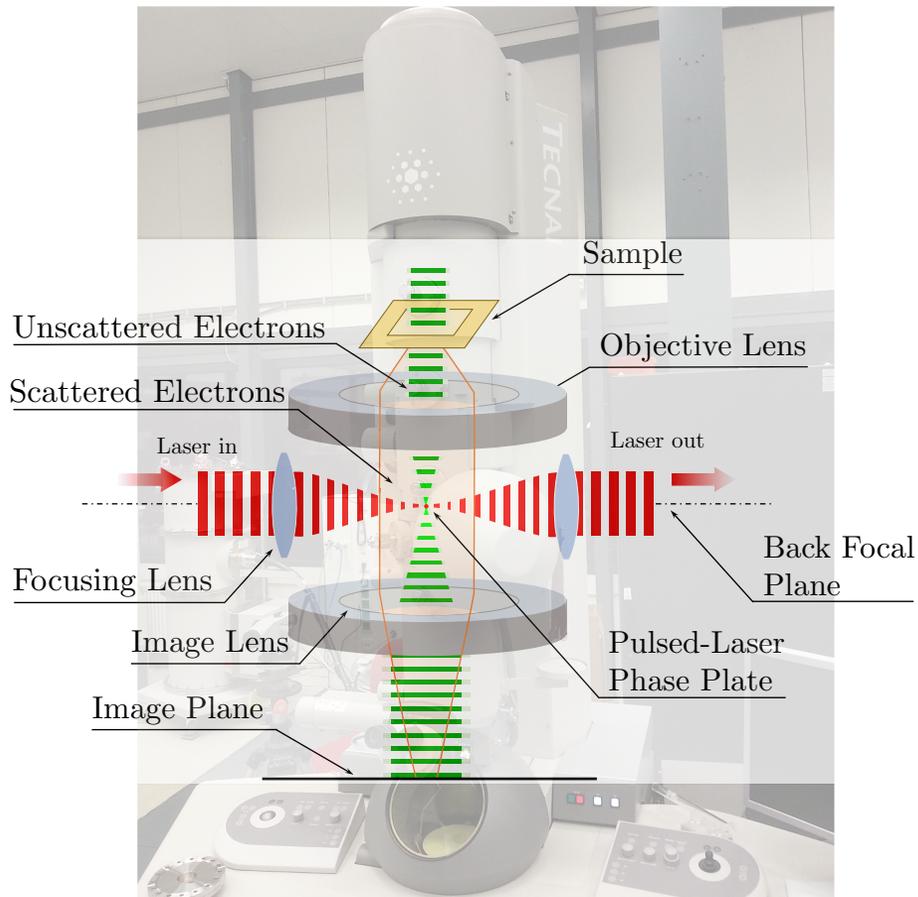
Maximum Phase Shift for perfect space-time synchronization between laser and electron pulses ( $t_0 = y = z = 0$ )

$$\Delta\varphi_{max} \sim -\frac{e^2}{\hbar c \epsilon_0 \gamma m \omega_0^2} \frac{W}{\pi w_0^2} \frac{\tau_t}{\tau}$$

Laser and electron pulses synchronized



# Pulsed-Laser Zernike Phase Plate for Phase Contrast Imaging of biological samples



$$\text{Phase Shift } \Delta\varphi_{max} = -\frac{\pi}{2}$$

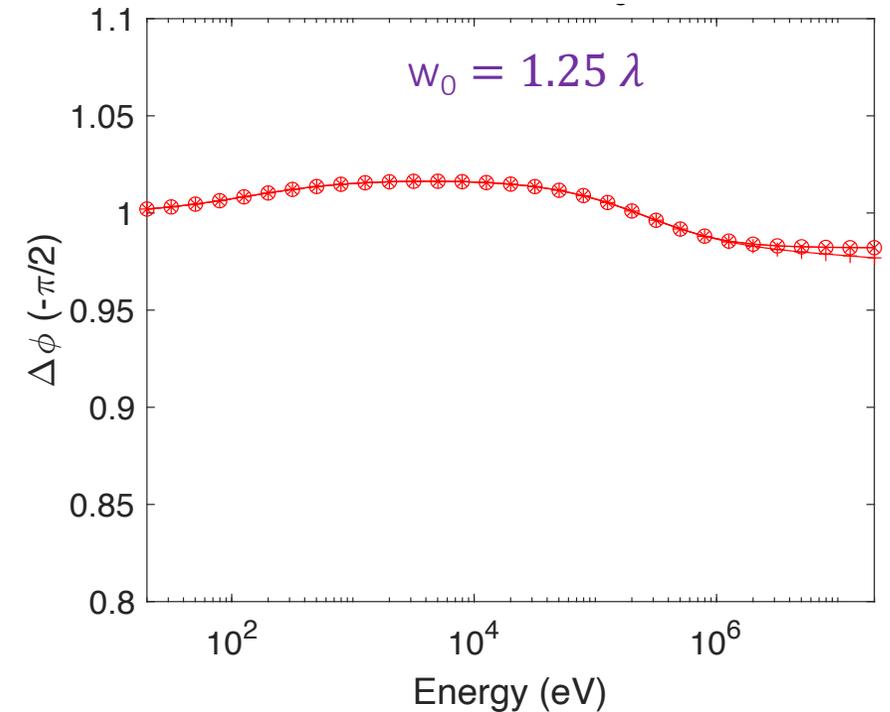
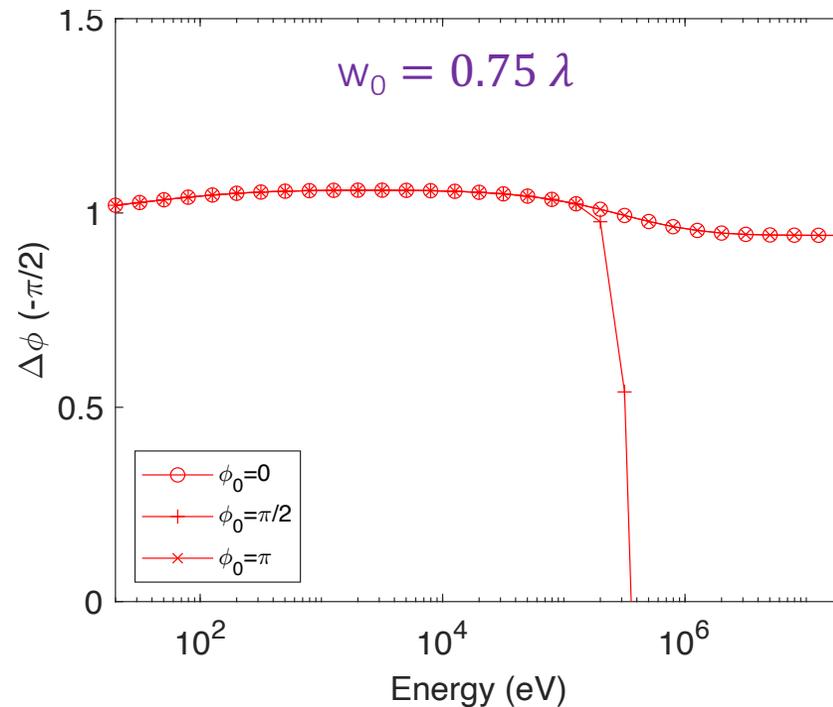
$\lambda = 800 \text{ nm}$   
 $w_0 = 2 \lambda = 1.6 \mu\text{m}$   
 $\tau = 300 \text{ fs} \quad \& \quad \tau_e = \frac{1}{3} \tau = 100 \text{ fs}$   
 $W \sim 10 \text{ nJ/pulse}$

# Phase Shaping of Electrons Wavefunction with fs-Laser Pulses

$$\Delta\varphi = \frac{1}{\hbar} \int_P dt L(x, y, z, t) = \int_P dt \left( -\frac{mc^2}{\gamma} - e \frac{\mathbf{p} \cdot \mathbf{A}}{m\gamma} - \frac{e^2 \mathbf{A} \cdot \mathbf{A}}{m\gamma} + eV \right)$$

Phase shift at high electron energies depends on:

- field phase
- field polarization
- field model chosen



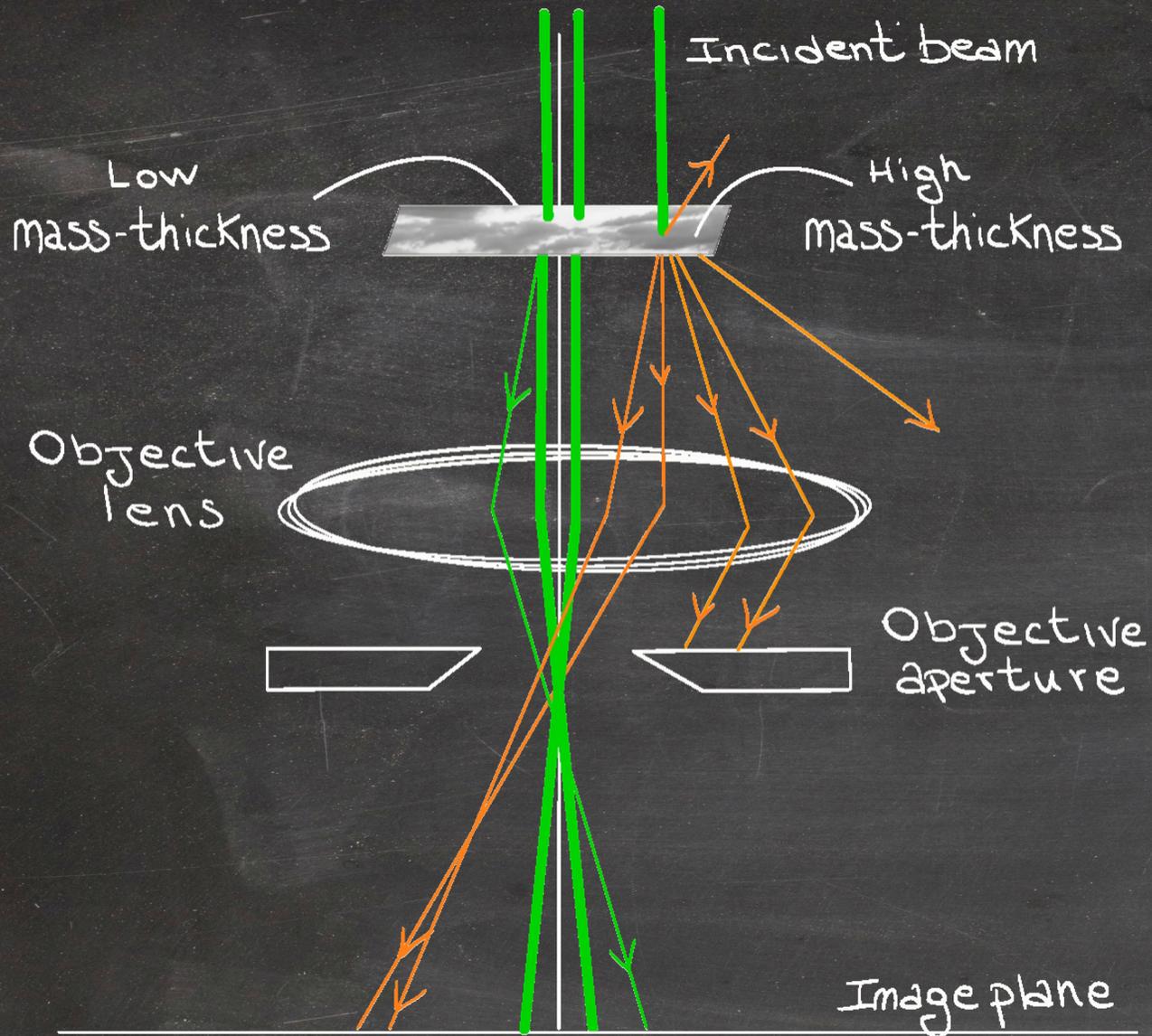
Thank you!

Phase Shaping of Free-Electrons Wavefunction with fs-Laser Pulses in an  
RF-cavity-based Ultrafast Transmission Electron Microscope

Simona Borrelli

Backup slides

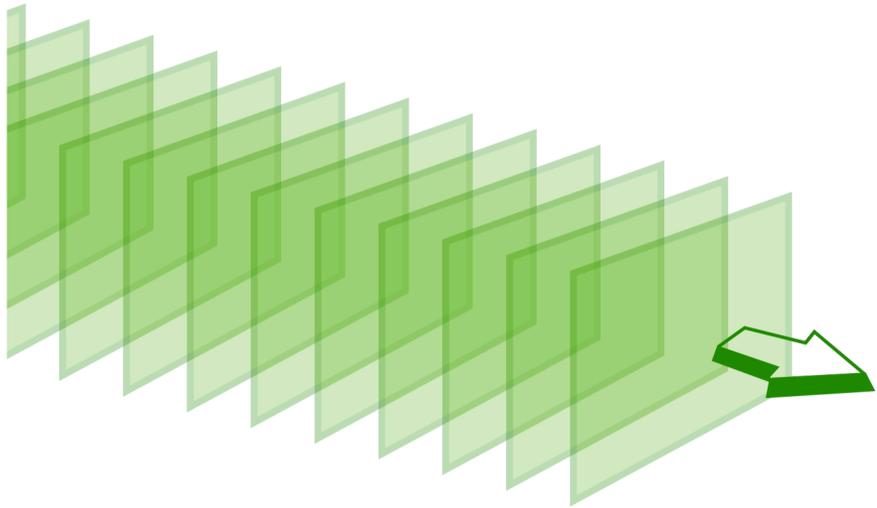
Highly scattering specimens



Amplitude Contrast  
Imaging

# Diffraction Contrast Imaging

Weakly scattering specimens



$$\psi(\vec{r}, t) = A e^{i(2\pi\vec{k}_0 \cdot \vec{r} - \omega t)}$$

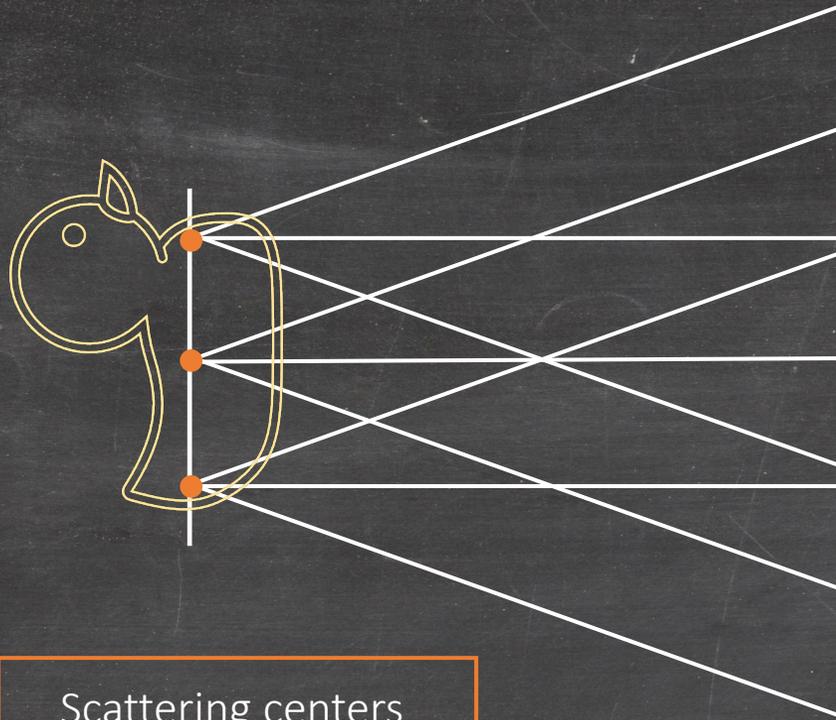
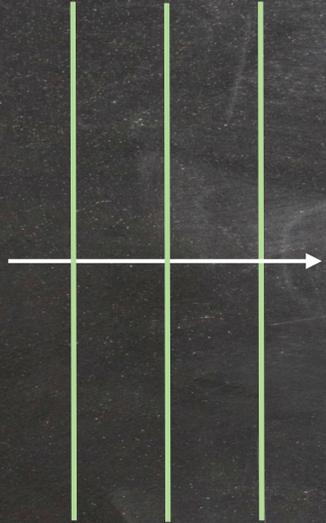


The electron planewave is modulated

Electron plane wave modulated by the object structure

Electron plane wavefunction

$$obj(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$



Scattering centers  
in the specimen

Image Formation  
in a TEM

Electron plane wave modulated by the object structure

Electron plane wavefunction

$$obj(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$

... Far field...

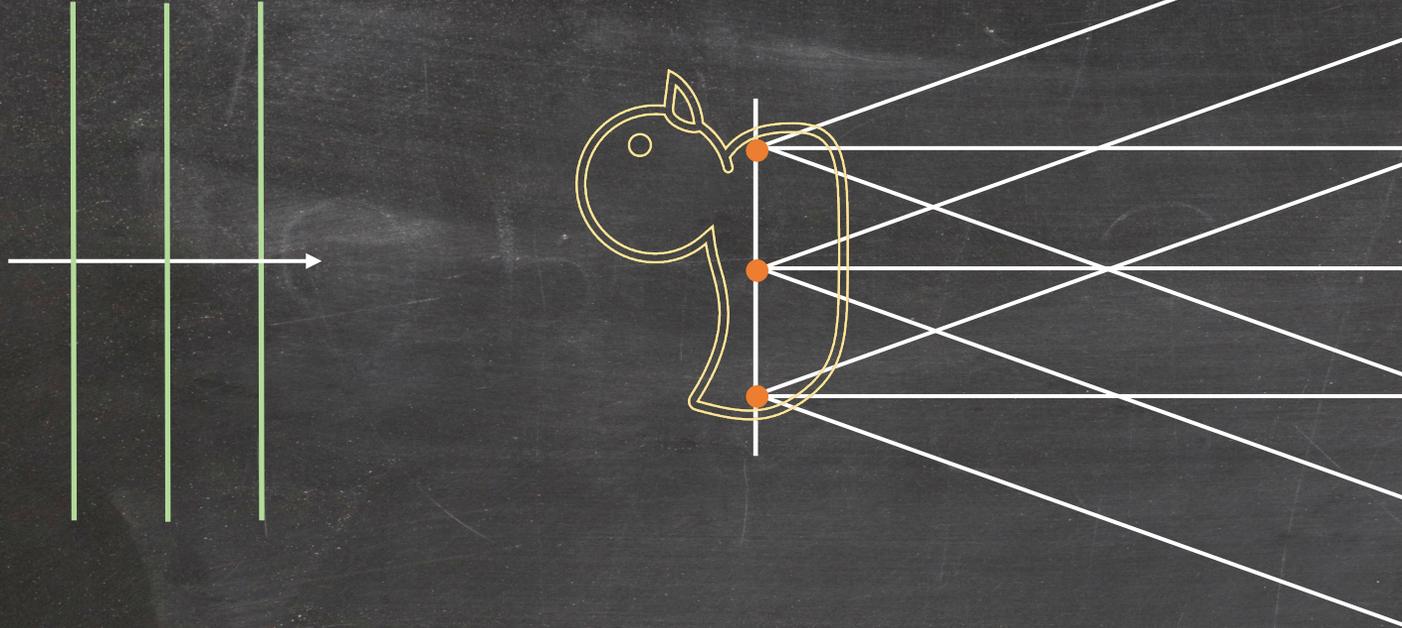
Fourier Transform of the specimen's density pattern

$$FT[obj(\vec{r}) \int_{-\infty}^{+\infty} obj(\vec{r}) e^{-i 2 \pi \vec{q} \cdot \vec{r}} d\vec{r} ]$$

Scattering Angle

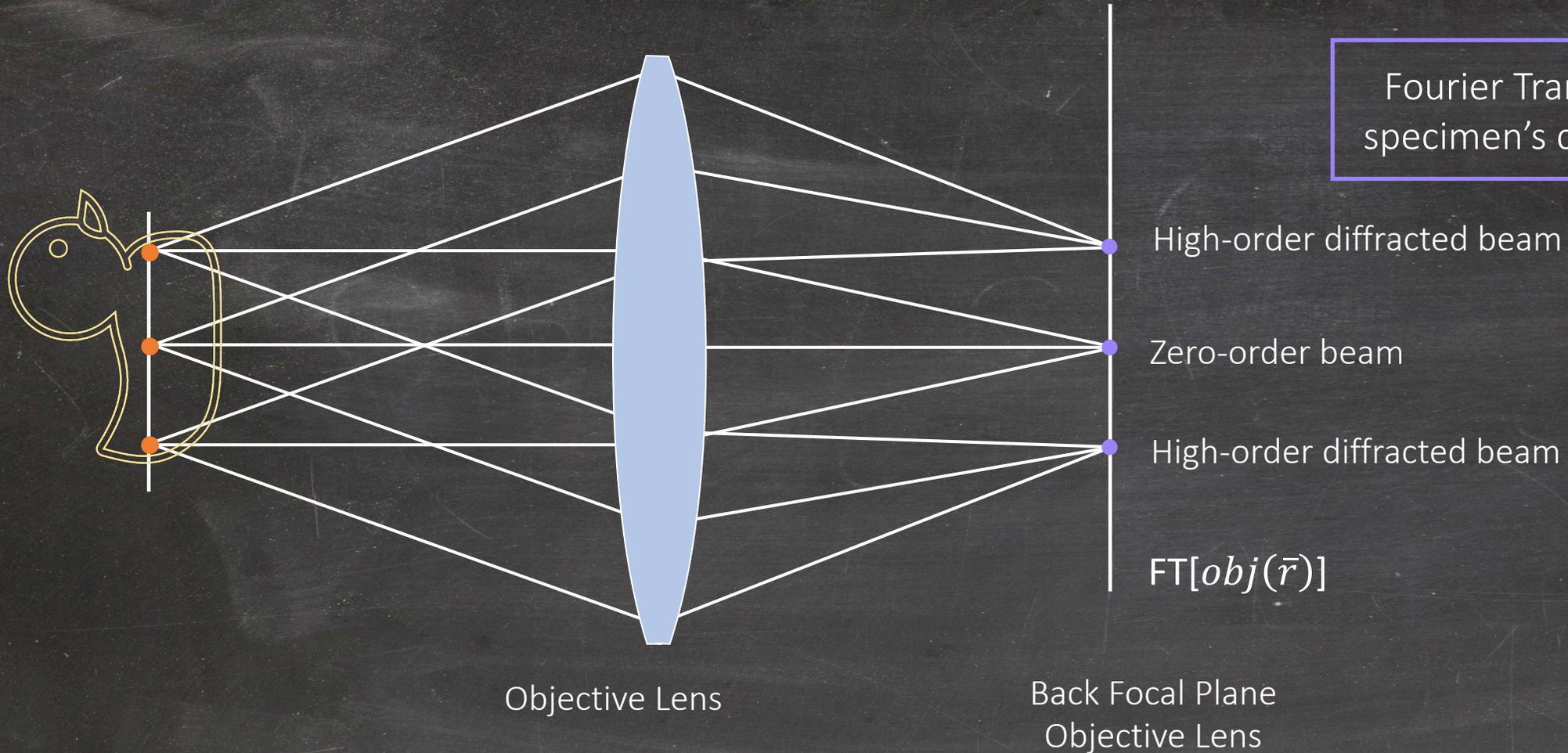
Spatial Frequency  $\vec{q} = k \vec{\vartheta}$

Wavenumber



Electron plane wave modulated by the object structure

$$obj(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$



Fourier Transform of the specimen's density pattern

High-order diffracted beam

Zero-order beam

High-order diffracted beam

$FT[obj(\vec{r})]$

Objective Lens

Back Focal Plane  
Objective Lens

Electron plane wave modulated by the object structure

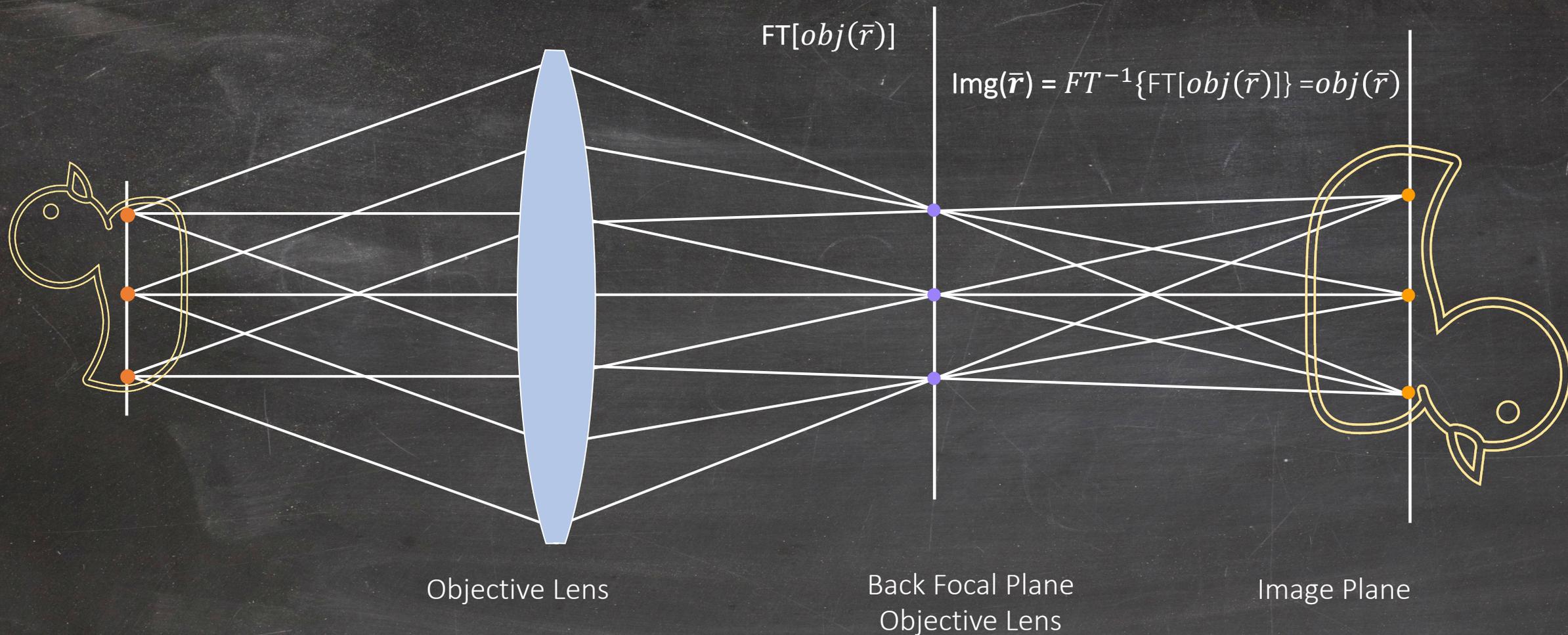
Image wave

$$obj(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$

$$Img(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$

$FT[obj(\vec{r})]$

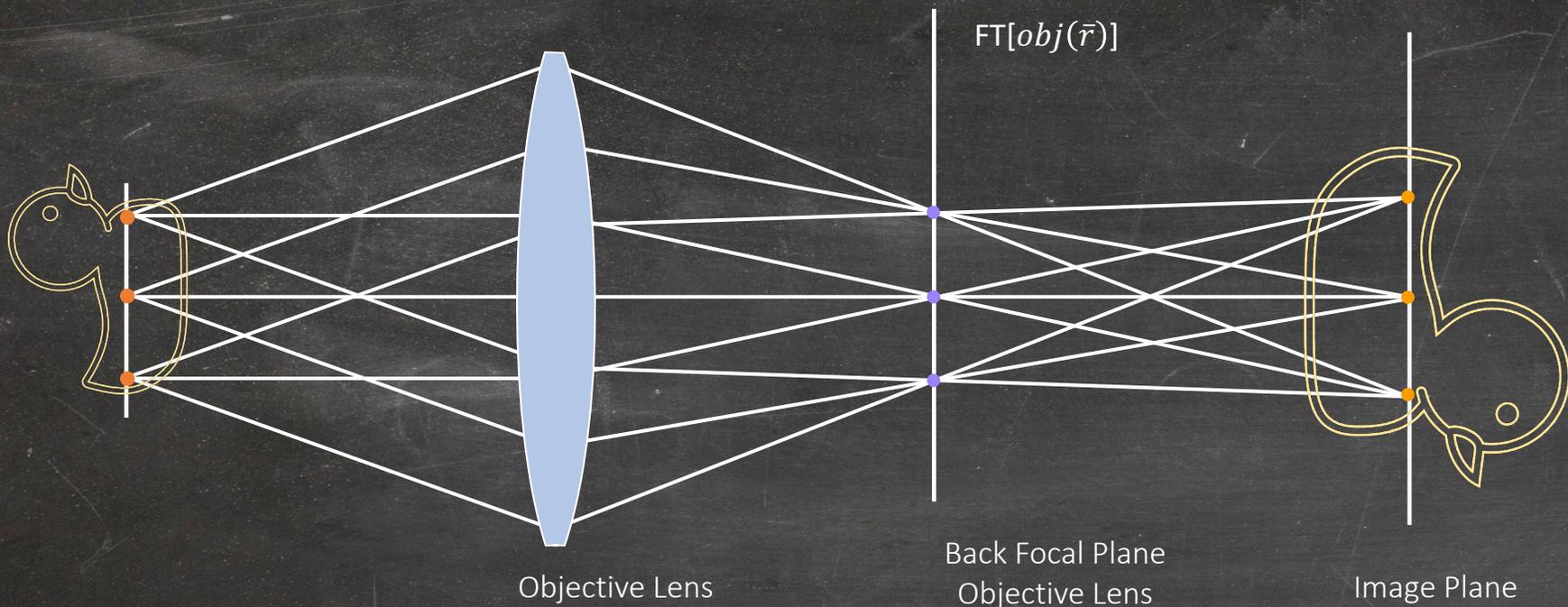
$$Img(\vec{r}) = FT^{-1}\{FT[obj(\vec{r})]\} = obj(\vec{r})$$



$$obj(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$



$$Img(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$



Detector

$$I = |Img(\vec{r})|^2 = A(\vec{r})^2$$

Amplitude  
Contrast

In the image only the amplitude information is retrieved.  
The phase information is lost.

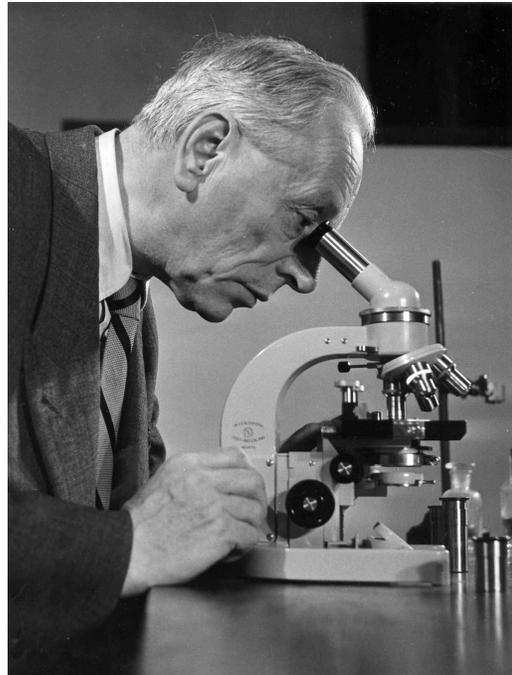


HOW RETRIEVE PHASE INFORMATION???

# Phase Contrast Imaging....

... With ...

Zernike  
Phase Plate

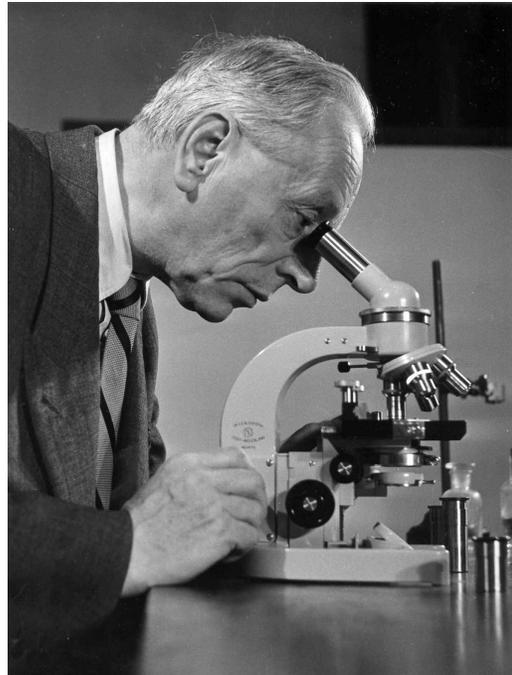


HOW RETRIEVE PHASE  
INFORMATION???

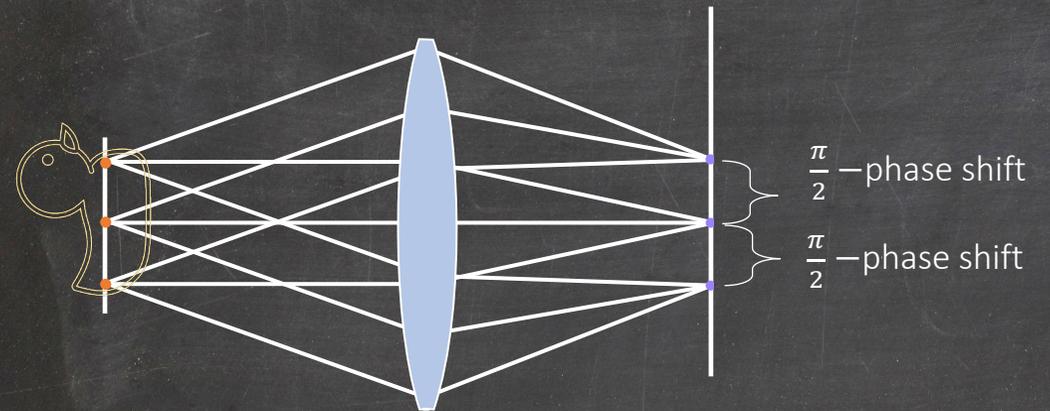
# Phase Contrast Imaging....

... With ...

## Zernike Phase Plate



Phase information can be retrieved by inserting into the Fourier space a  $\lambda/4$ -phase plate that introduces an additional  $\frac{\pi}{2}$ -phase shift between the zero-order beam ( $q = 0$ ) and the high-order diffracted beams ( $q \neq 0$ ).

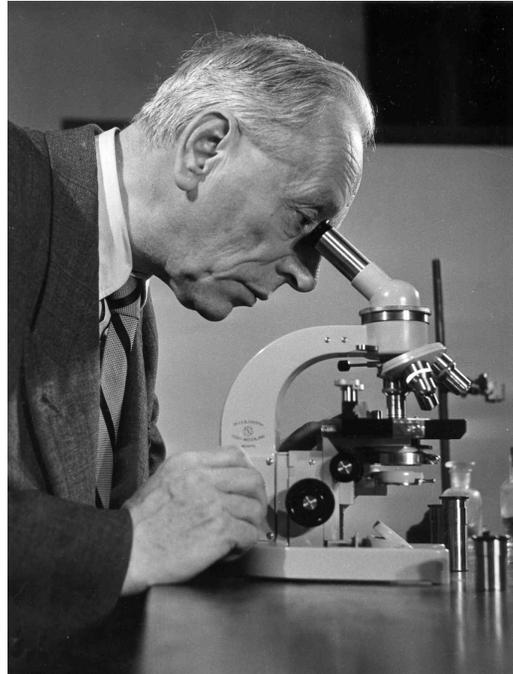




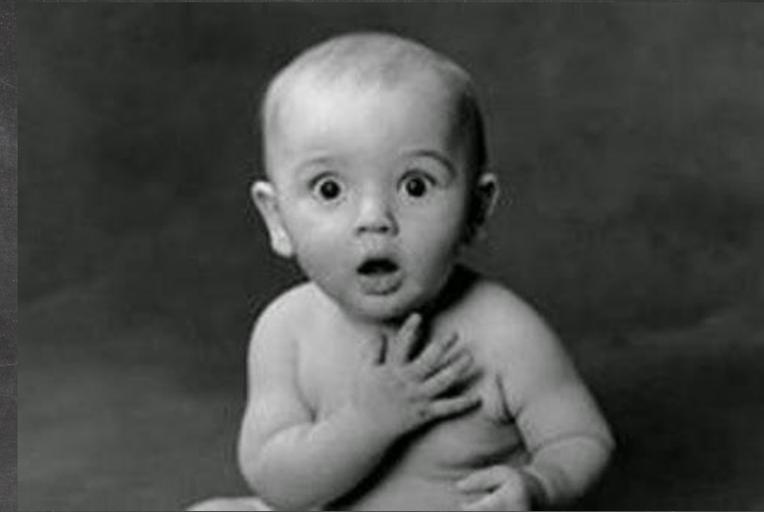
# Phase Contrast Imaging....

... With ...

## Zernike Phase Plate



$$i = e^{i\frac{\pi}{2}}$$

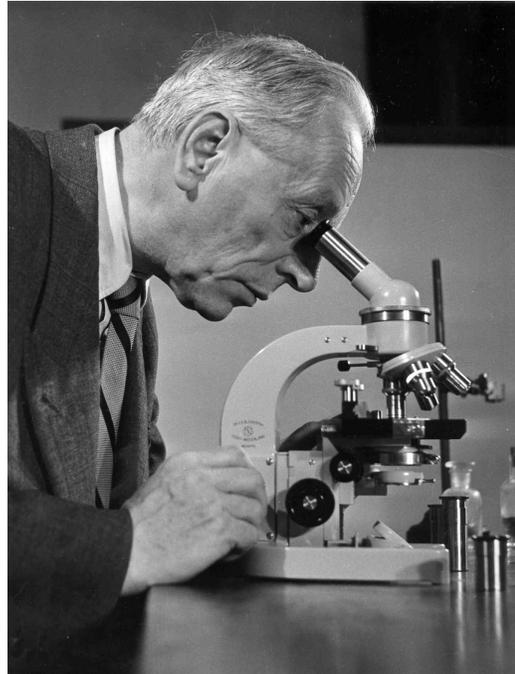


Phase information can be retrieved by inserting into the Fourier space a  $\lambda/4$ -phase plate that introduces an additional  $\frac{\pi}{2}$ -phase shift between the zero-order beam ( $q = 0$ ) and the high-order diffracted beams ( $q \neq 0$ )

# Phase Contrast Imaging....

... With ...

Zernike  
Phase Plate



Weakly scattering specimens

$$obj(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$

$$A(\vec{r}) = 1 - t(\vec{r})$$

Spatial frequency  $q_0$

$$\phi(\vec{r}) \leq 2\pi/10$$

$$t \ll 2\pi/10$$

$$obj(x) \approx 1 - (t - i\phi) \cos(2\pi q_0 \cdot x + \varepsilon) = Img(x)$$

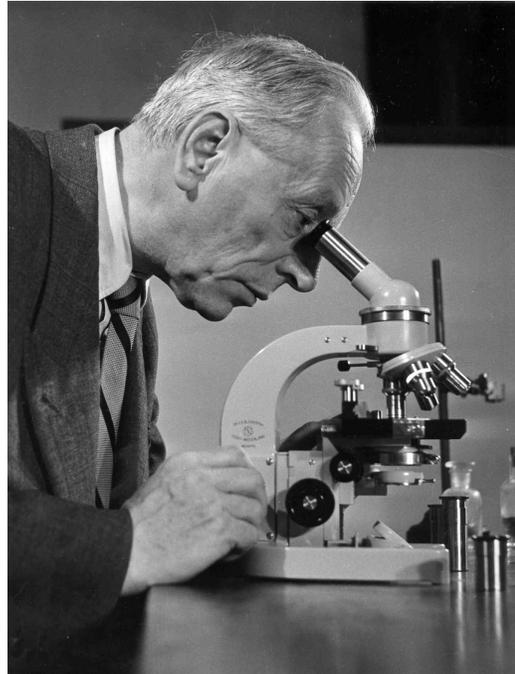
Amplitude Contrast

$$I = |Img(x)|^2 \approx 1 - 2t \cos(2\pi q_0 \cdot x + \varepsilon)$$

# Phase Contrast Imaging....

... With ...

## Zernike Phase Plate



Weakly scattering specimens

$$obj(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$

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Amplitude Contrast

$$|Img(x)|^2 \approx 1 - 2t \cos(2\pi q_0 \cdot x + \varepsilon)$$

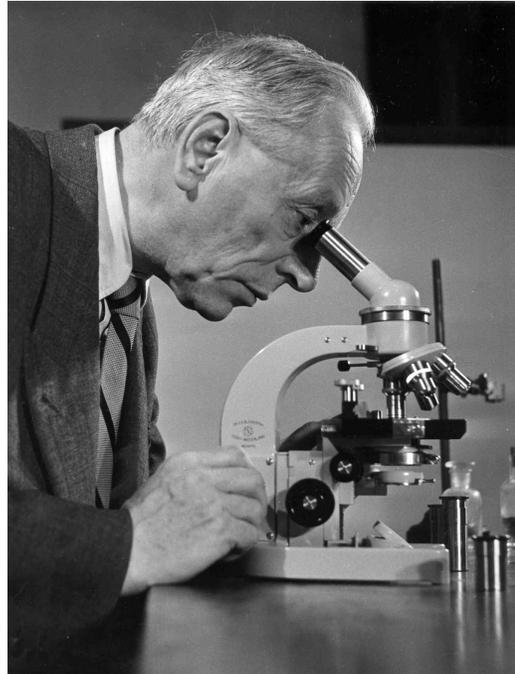
$\lambda/4$ -Phase Plate

$$e^{i\frac{\pi}{2}} \Rightarrow Img(x) \approx 1 - (it + \phi) \cos(2\pi q_0 \cdot x + \varepsilon)$$

# Phase Contrast Imaging....

... With ...

## Zernike Phase Plate



Weakly scattering specimens

$$obj(\vec{r}) = A(\vec{r}) e^{i[\phi(\vec{r})]}$$

$$A(\vec{r}) = 1 - t(\vec{r})$$

$$\phi(\vec{r}) \leq \frac{2\pi}{10}$$

At a certain spatial frequency  $q_0$

$$obj(x) \approx 1 - (t - i\phi) \cos(2\pi q_0 \cdot x + \varepsilon) = Img(x)$$

Amplitude Contrast

$$I = |Img(x)|^2 \approx 1 - 2t \cos(2\pi q_0 \cdot x + \varepsilon)$$

$\lambda/4$ -Phase Plate

$$Img(x) \approx 1 - (it + \phi) \cos(2\pi q_0 \cdot x + \varepsilon)$$

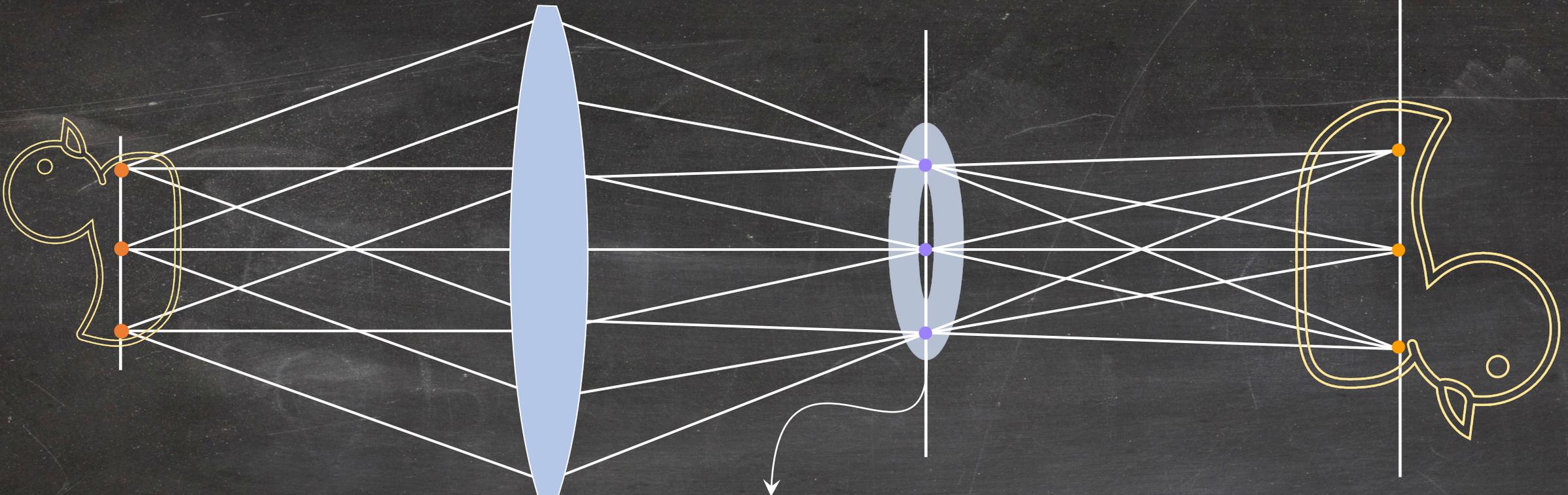
Phase Contrast

$$I = |Img(x)|^2 \approx 1 - 2\phi(\vec{r}) \cos(2\pi q_0 \cdot x + \varepsilon)$$

Objective Lens

Back Focal Plane  
(i.e., Fourier Space)

Image Plane

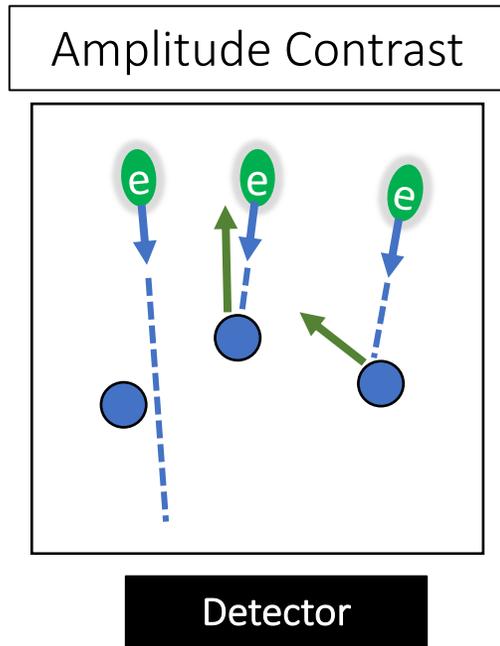


Phase Plate

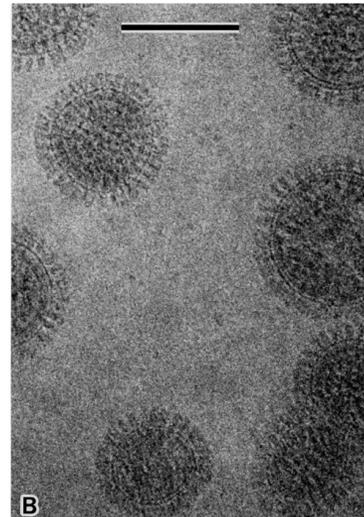
# Phase Contrast Imaging with Zernike Phase Plates

The phase modulation induced by the specimen is imaged by the detector.

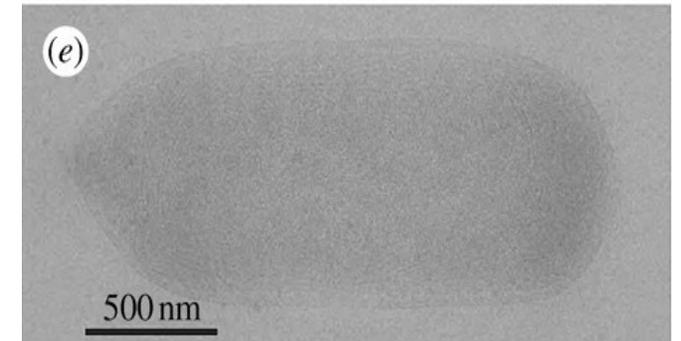
# Ponderomotive Phase Shaping



## Weakly scattering specimens

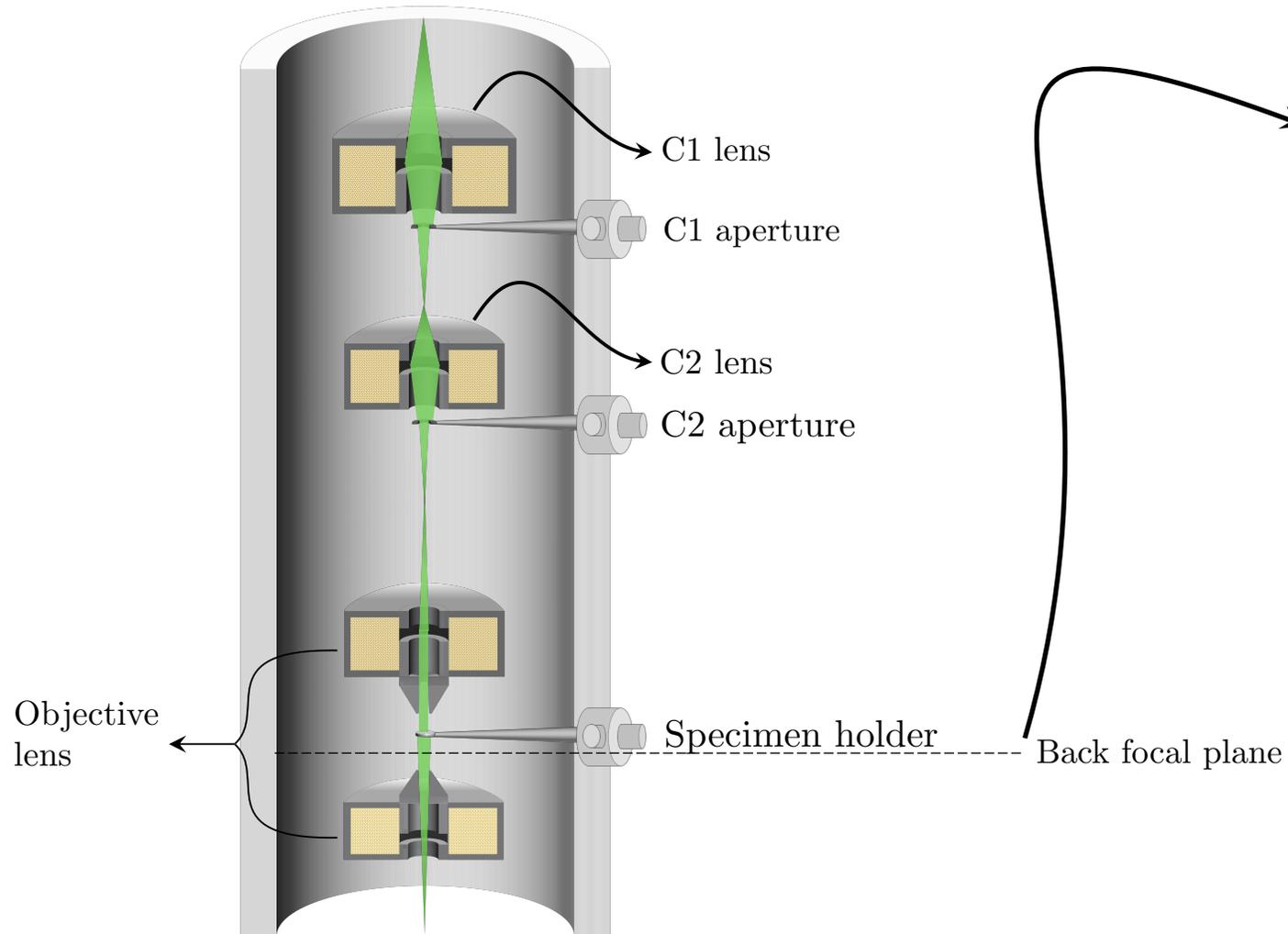


Conventional electron micrograph (B) of ice-embedded influenza A viruses  
(*Yamaguchi Y. et al.*)

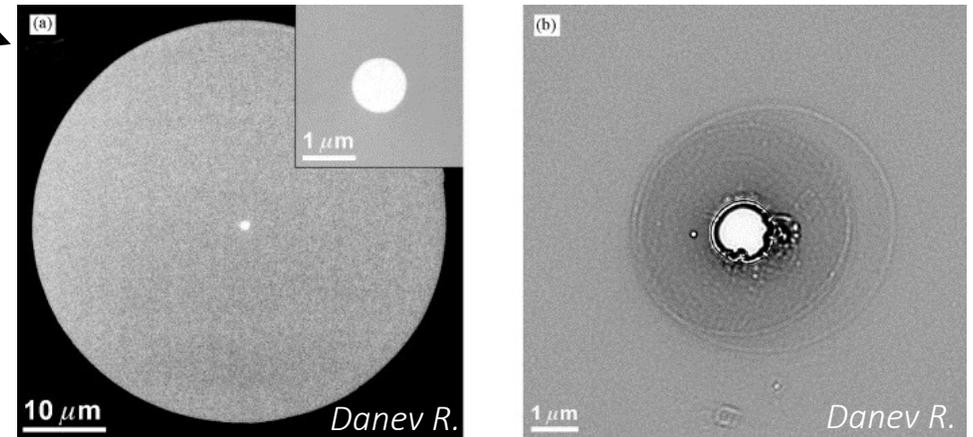


Conventional (e) TEM image of cyanobacterial cell  
(*Konyuba Y. et al.*)

# Zernike Phase Plates for Phase Contrast Imaging



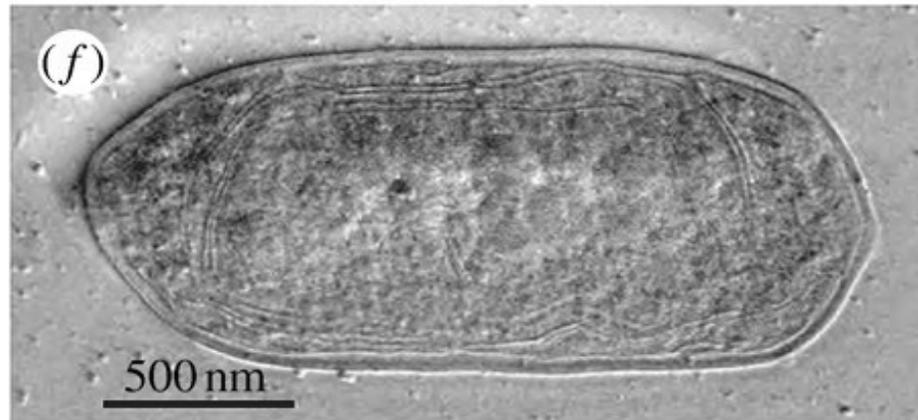
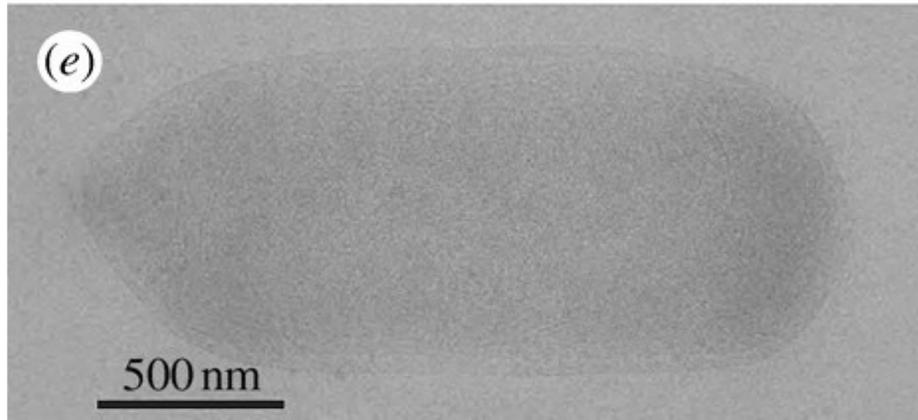
SEM image Zernike phase plates



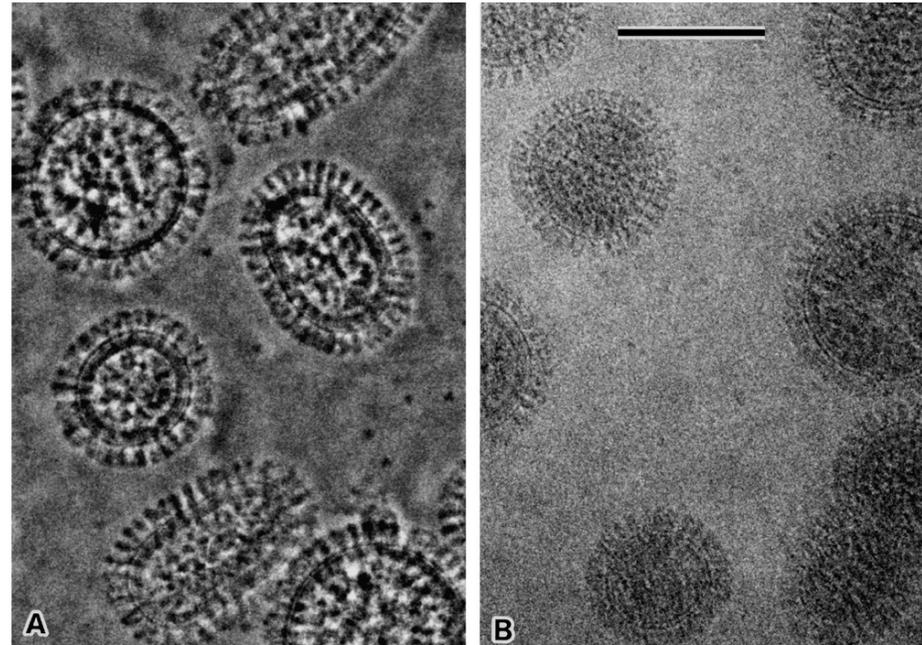
Limitations inherently related to the presence of matter:

- Image distortion due to charging
- Loss of electrons and decoherence
- Limited Spectroscopic capabilities
- Contamination from the electron beam
- Non-tunable and time-varying phase shift

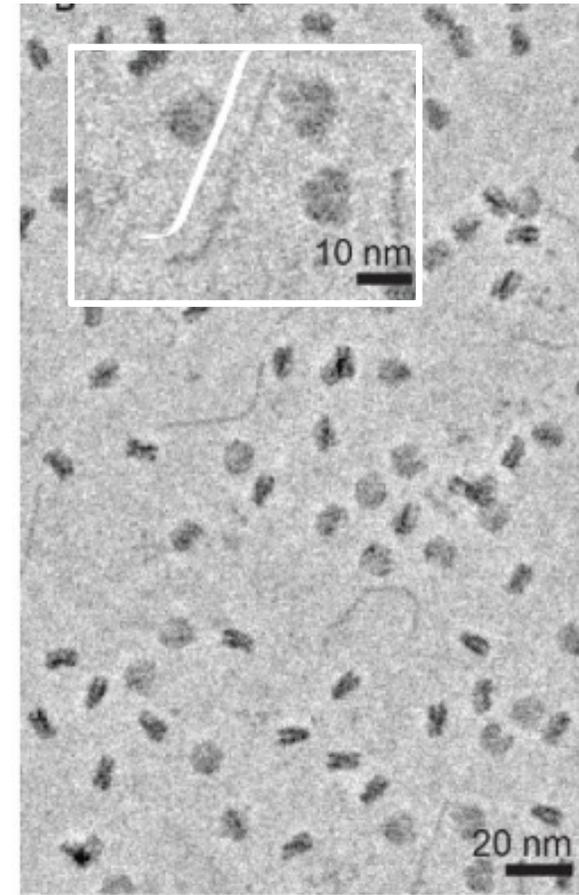
# Phase Contrast Imaging with Zernike Phase Plates



Conventional (e) and phase-Contrast (f) TEM image of cyanobacterial cell (*Konyuba Y. et al.*)

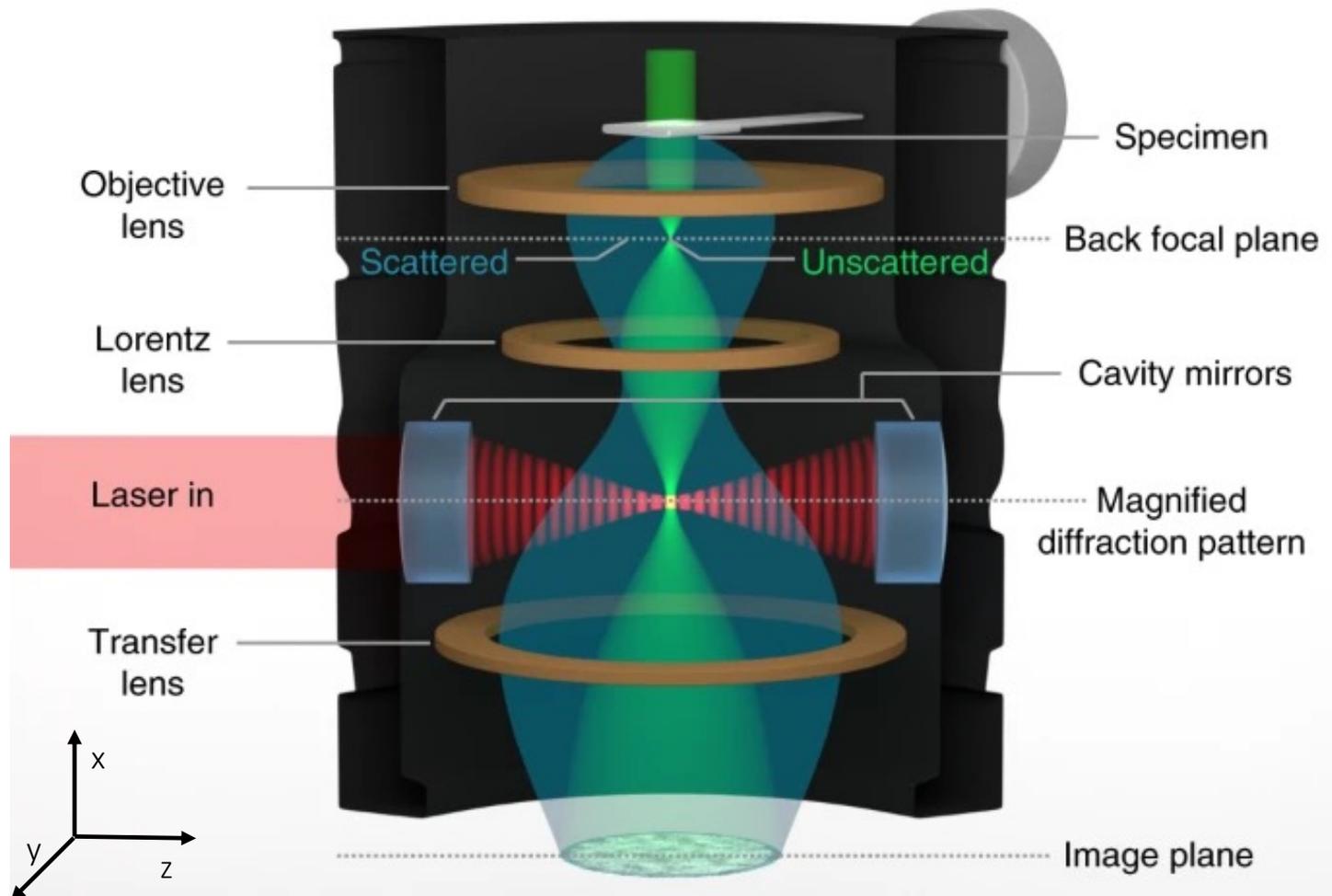


Zernike phase contrast electron micrograph (A) and a conventional electron micrograph (B) of ice-embedded influenza A viruses (*Yamaguchi Y. et al.*)



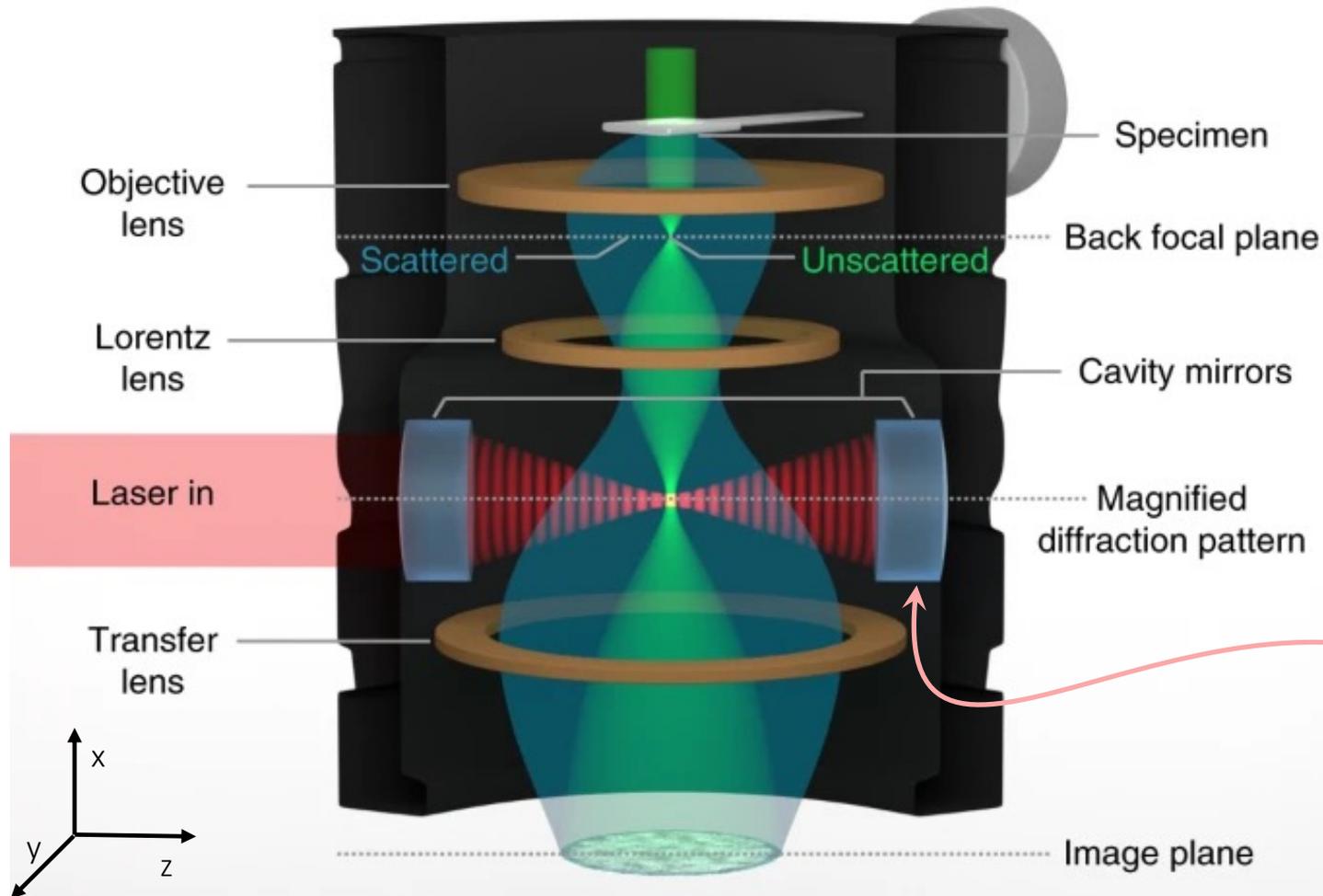
Phase plate cryo-EM image of nucleosome core particles and DNA (*E.Y.D. Chua et al.*)

# Laser-Based Zernike Phase Plate



Phase Plate for TEM based on the ponderomotive interaction between a tightly focused CW standing laser wave and the continuous zero-order electron beam transmitted by the sample.

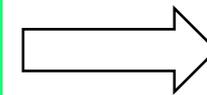
# Laser-Based Zernike Phase Plate



Short electron-light interaction time in a micron-scale laser focus

Phase Shift

$$\Delta\varphi = \frac{\pi}{2}$$



$$P_{\text{laser}} \sim 10 \text{ GW/cm}^2$$

Fabry-Pérot Cavity into TEM to magnify optical power

$$P_{\text{laser}} = 43 \text{ GW/cm}^2$$

(Schwartz O. et al., 2019)

# Pulsed Laser Phase Plate



*To develop a Pulsed Laser Phase Plate*



Laser beam tightly focused at the interaction point inside the TEM



Dedicated fs-laser with ad-hoc operational parameters



Electron pulse compression



Space-Time synchronization between laser and electron pulses



Laser and electron beam diagnostics

# Pulsed Laser Phase Plate



*To develop a Pulsed Laser Phase Plate*



Laser beam tightly focused at the interaction point inside the TEM



Dedicated fs-laser with ad-hoc operational parameters



Electron pulse compression



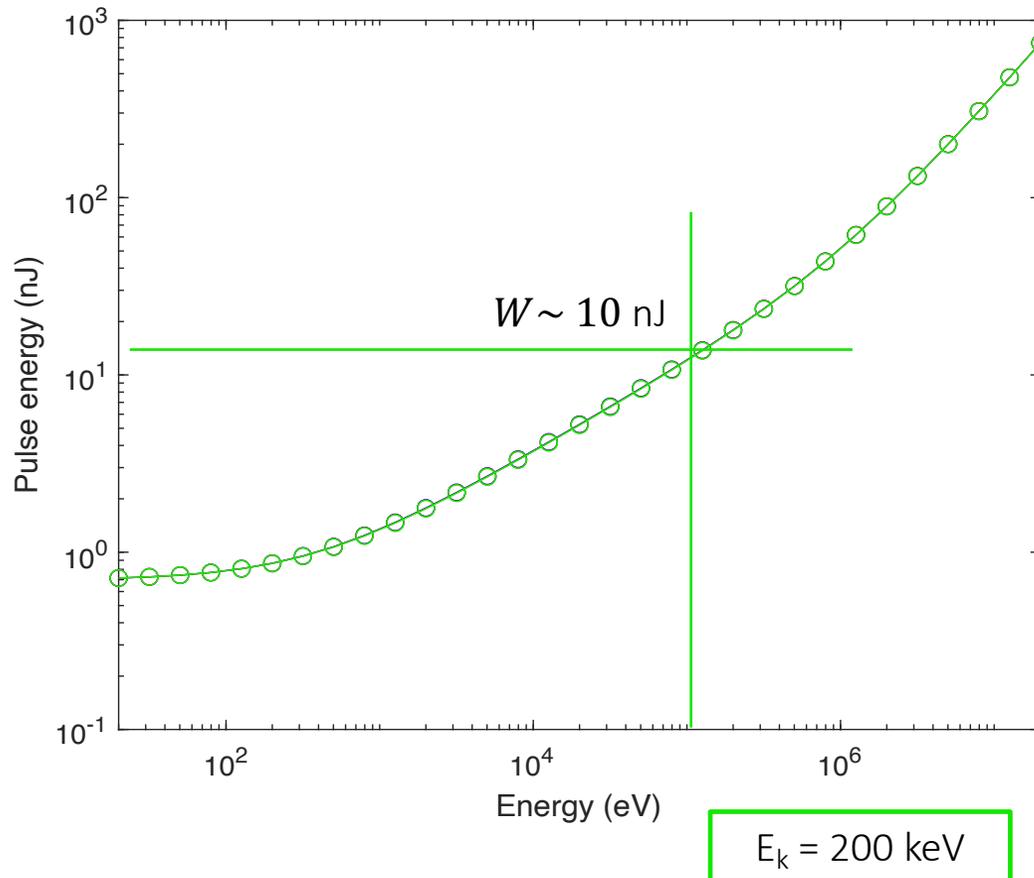
Space-Time synchronization between laser and electron pulses



Laser and electron beam diagnostics

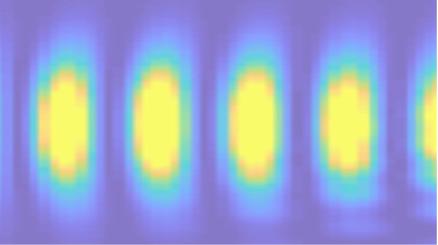
# Pulsed-Laser-Based Zernike Phase Plate

$$\Delta\varphi_{max} \sim \frac{e^2}{\hbar c \epsilon_0 \gamma m \omega_0^2} \frac{W}{\pi w_0^2} \frac{\tau_t}{\tau} = \frac{\pi}{2}$$

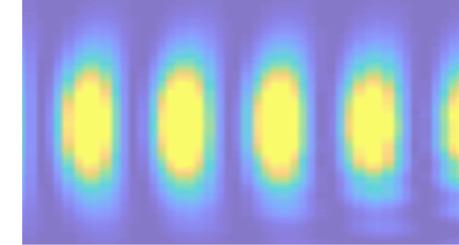


## Experimental Parameters At first glance

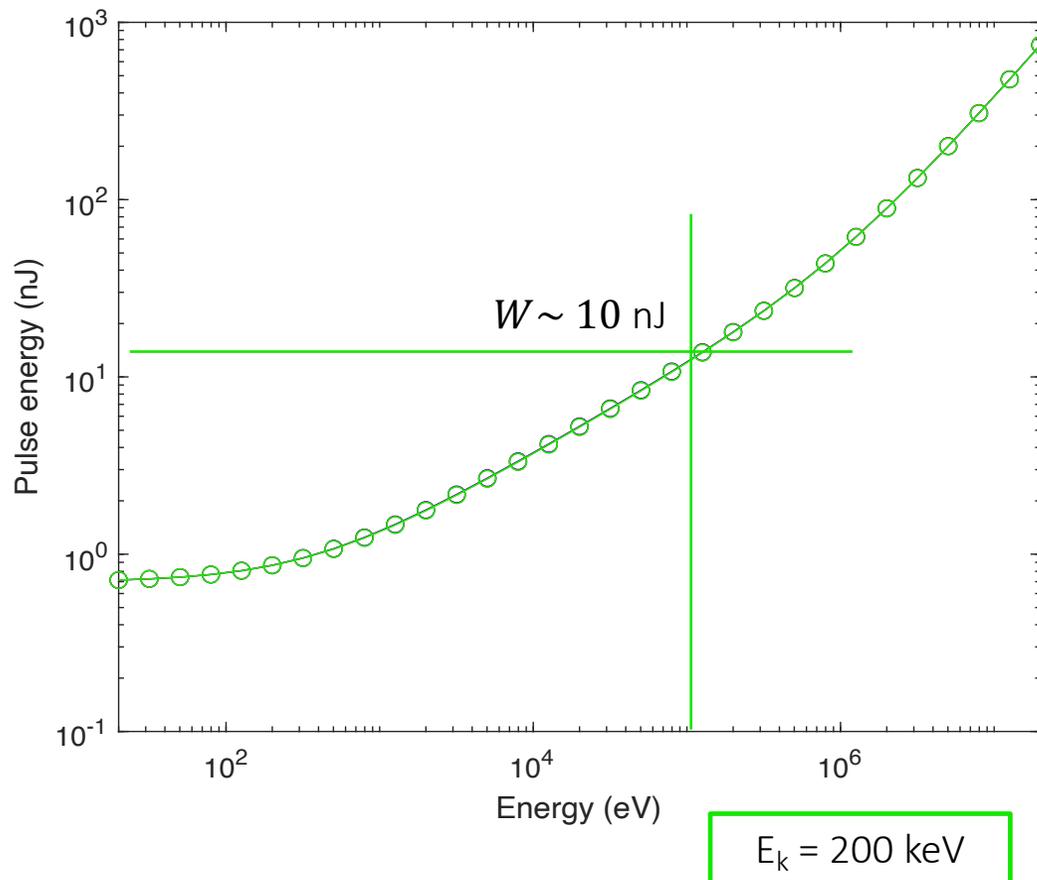
- $\lambda = 800$  nm
- $w_0 = 2 \lambda$
- $\tau = 100$  fs  $\Rightarrow \tau_e = 0.3 \tau \sim 33$  fs
- $W \sim 10$  nJ/pulse



# Pulsed-Laser-Based Zernike Phase Plate



$$\Delta\varphi_{max} \sim \frac{e^2}{\hbar c \epsilon_0 \gamma m \omega_0^2} \frac{W}{\pi w_0^2} \frac{\tau_t}{\tau} = \frac{\pi}{2}$$

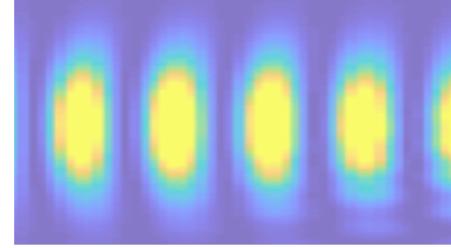
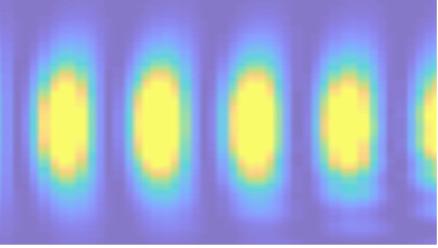


## Experimental Parameters At first glance

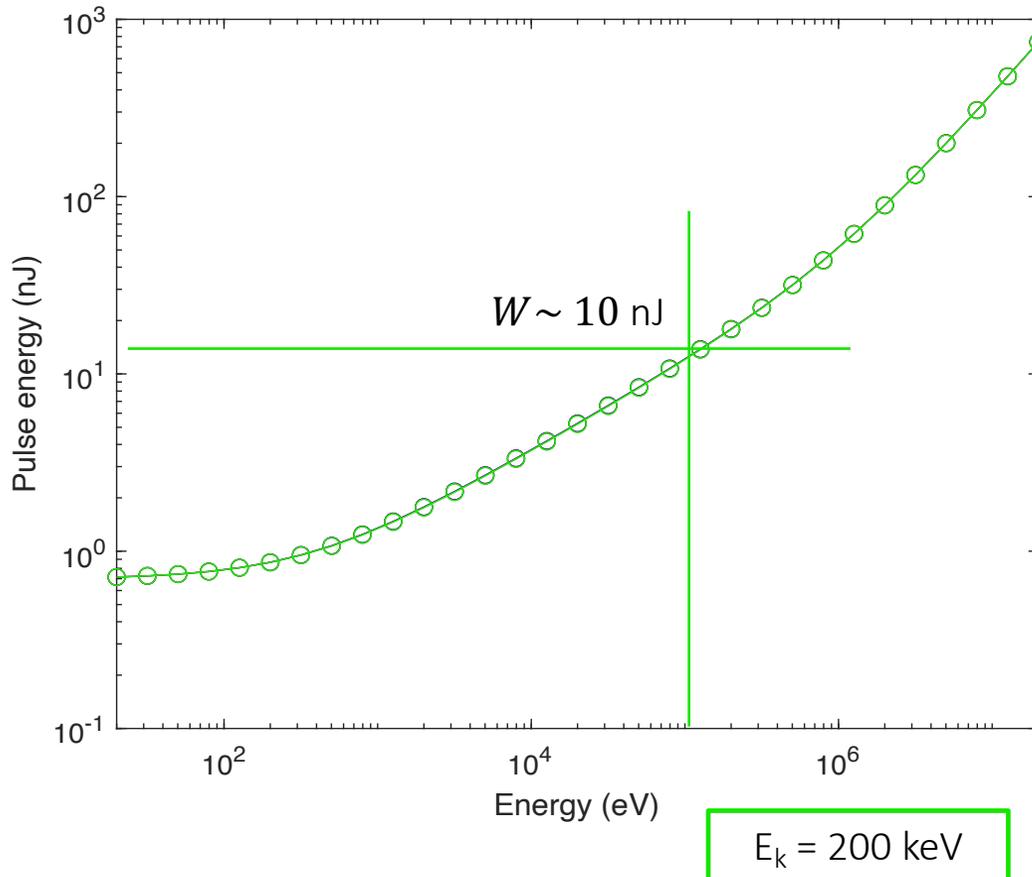
- $\lambda = 800$  nm
- $w_0 = 2 \lambda$
- $\tau = 100$  fs  $\Rightarrow \tau_e = 0.3 \tau \sim 33$  fs
- $W \sim 10$  nJ/pulse

Compression &  
Synchronization < 10 fs

# Pulsed-Laser-Based Zernike Phase Plate



$$\Delta\varphi_{max} \sim \frac{e^2}{\hbar c \epsilon_0 \gamma m \omega_0^2} \frac{W}{\pi w_0^2} \frac{\tau_t}{\tau} = \frac{\pi}{2}$$



## Experimental Parameters At first glance

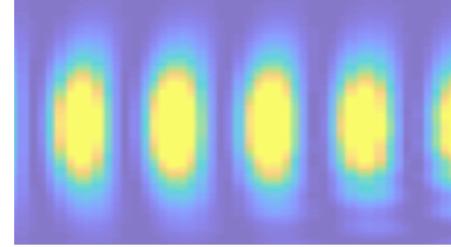
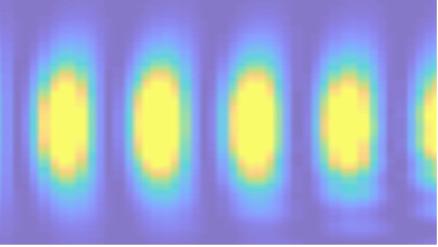
- $\lambda = 800$  nm
- $w_0 = 2 \lambda$
- $\tau = 100$  fs  $\Rightarrow \tau_e = 0.3 \tau \sim 33$  fs
- $W \sim 10$  nJ/pulse

Average Power

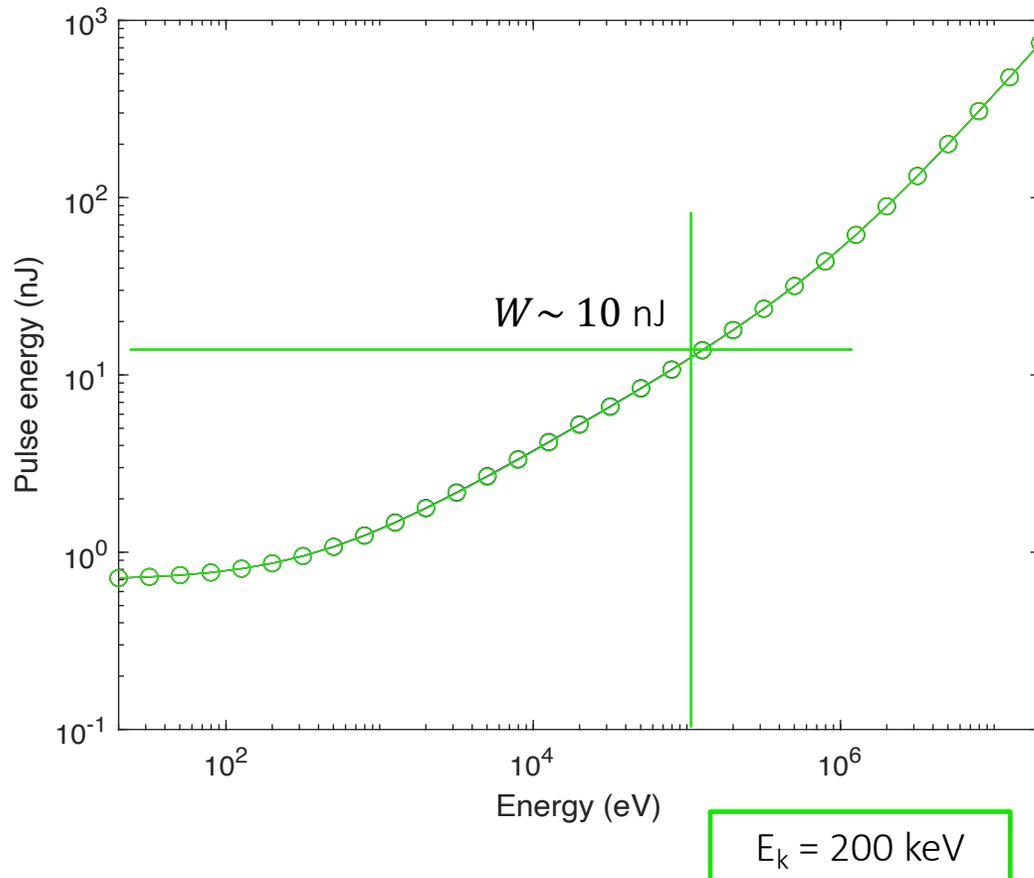
$$P_{avg} = \frac{W(\lambda) \text{ rep rate}}{\sqrt{\pi}}$$

Rep Rate (MHz)	$P_{avg}$ (W)
75	0.2
250	0.8
$3 * 10^3$	9.2

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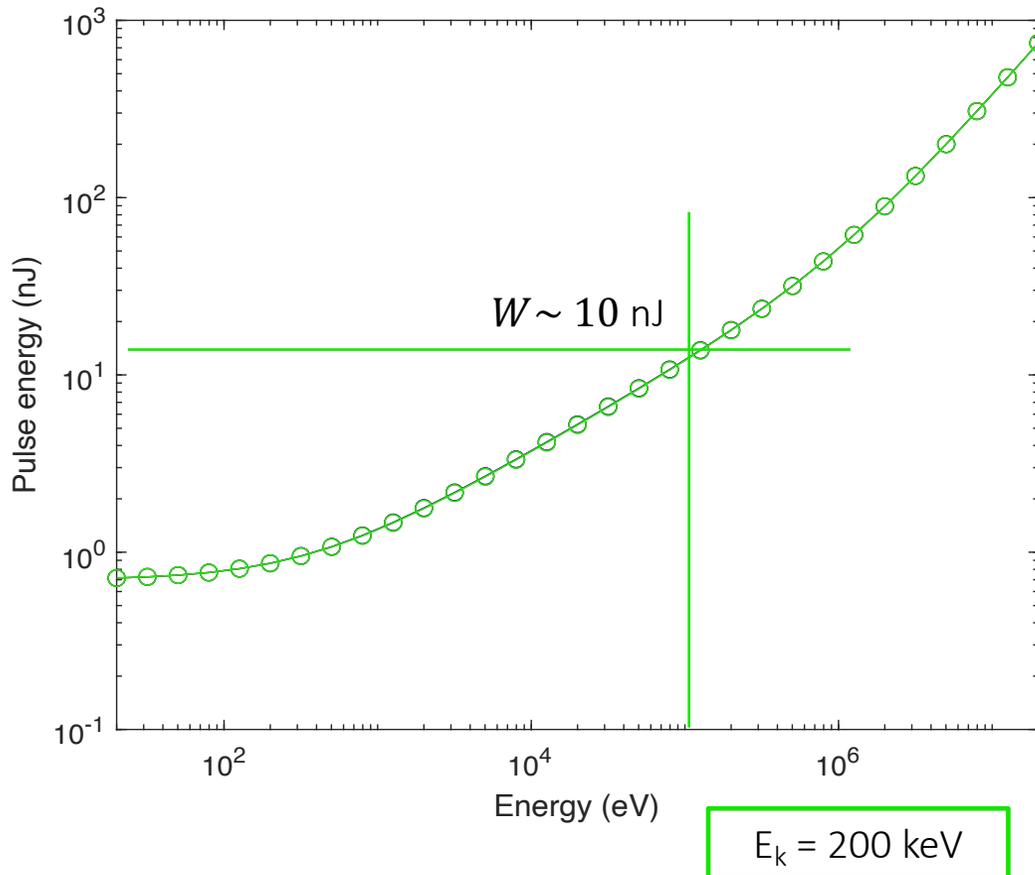
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Rep Rate (MHz)	$P_{avg}$ (W)
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250	0.8
$3 * 10^3$	9.2

Jom says: "Let's think big!"

# Pulsed-Laser-Based Zernike Phase Plate

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## Experimental Parameters At first glance

- $\lambda = 800$  nm
- $w_0 = 2 \lambda$
- $\tau = 100$  fs  $\Rightarrow \tau_e = 33$  fs
- $W \sim 10$  nJ nJ/pulse

- $\tau = 30$  fs  $\Rightarrow \tau_e \sim 10$  fs
- $W \sim 5$  nJ/pulse

# Pulsed Laser Phase Plate



*To develop a Pulsed Laser Phase Plate*



Laser beam tightly focused at the interaction point inside the TEM



Dedicated fs-laser with ad hoc operational parameters



Electron pulse compression

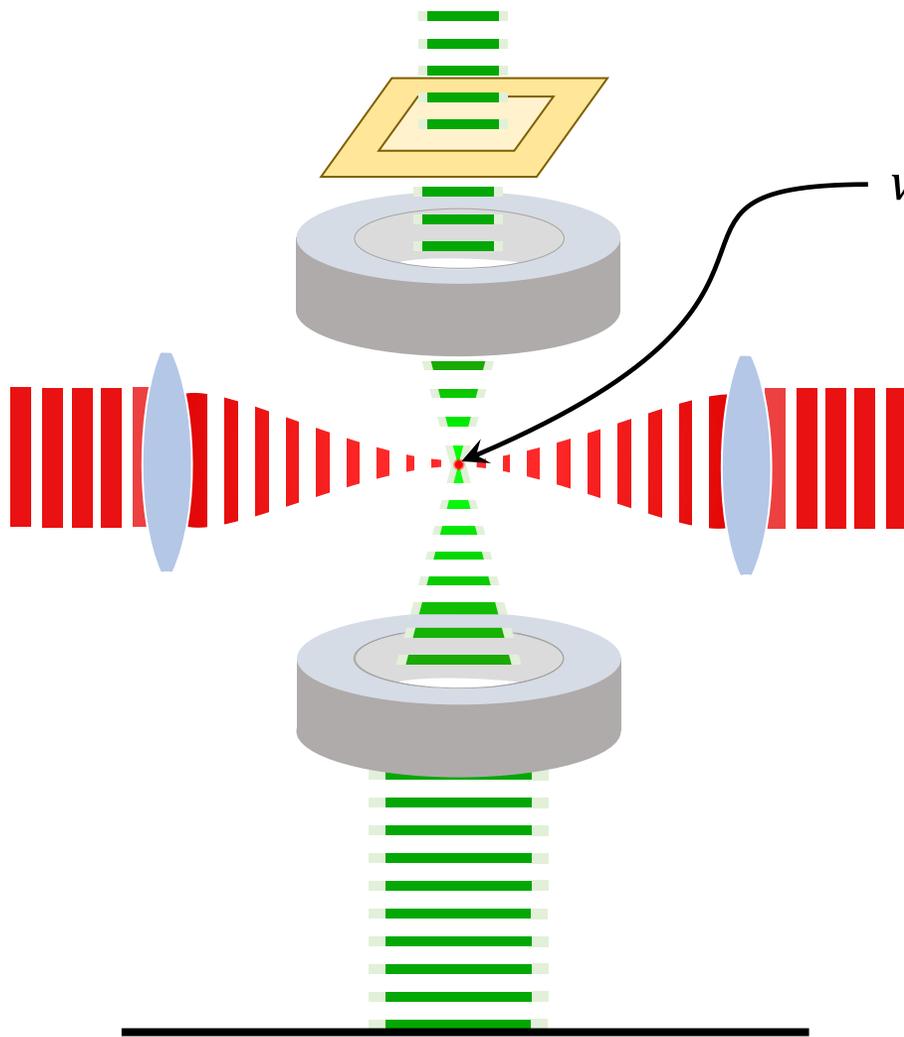


Space-Time synchronization between laser and electron pulses

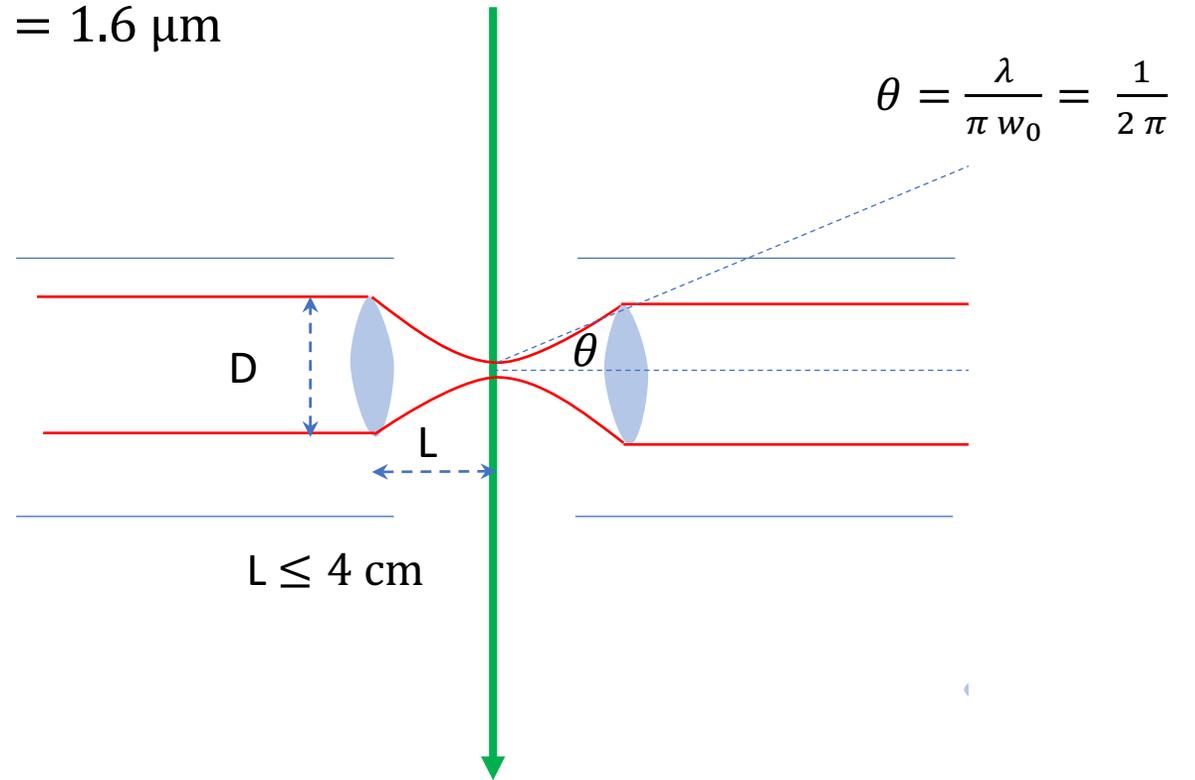


Laser and electron beam diagnostics

# Pulsed-Laser-Based Zernike Phase Plate



$$w_0 = 2\lambda = 1.6 \mu\text{m}$$



# Pulsed Laser Phase Plate



*To develop a Pulsed Laser Phase Plate*



Laser beam tightly focused at the interaction point inside the TEM



Dedicated fs-laser with ad hoc operational parameters



Electron pulse compression



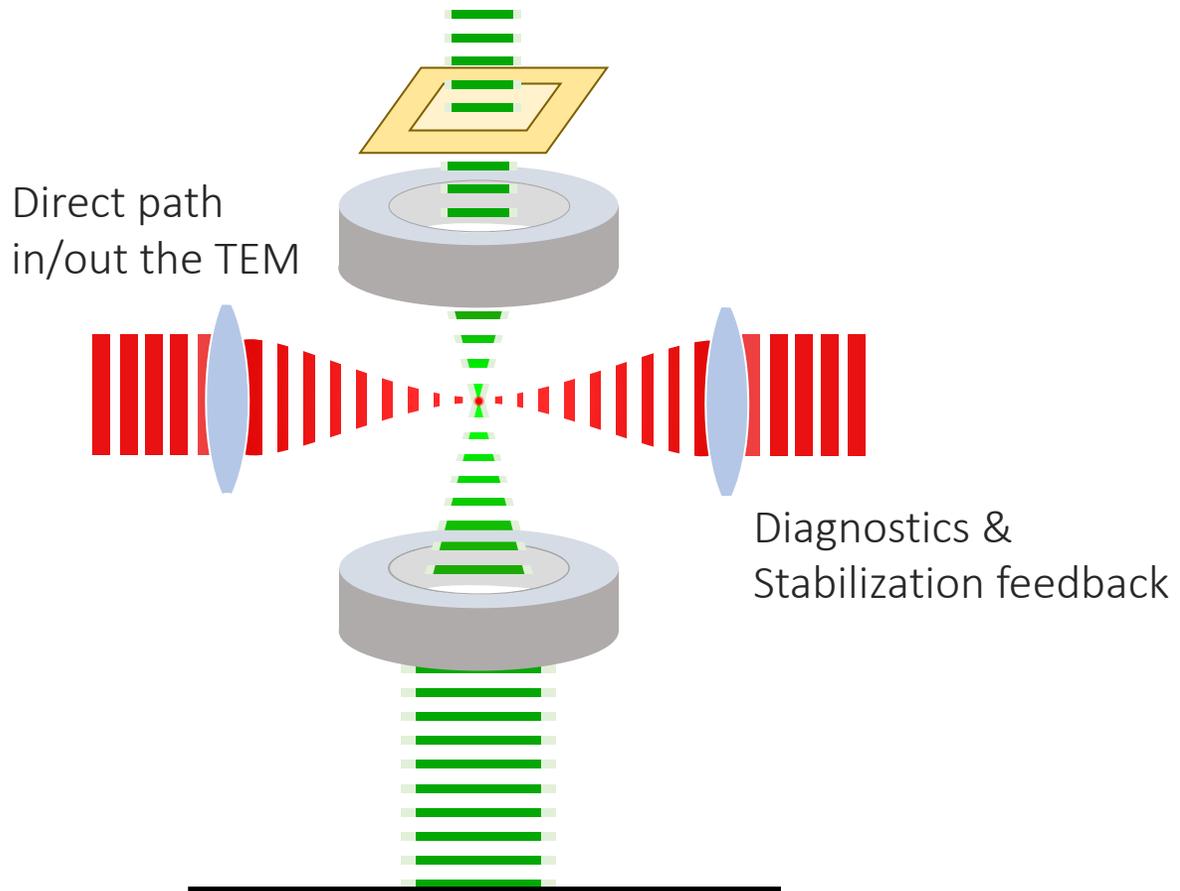
Space-Time synchronization between laser and electron pulses



Laser and electron beam diagnostics

# Pulsed-Laser-Based Zernike Phase Plate

## Spatial Alignment and Diagnostics



## Temporal Alignment

