Measuring **spatio-temporal couplings** using **modal multi-spectral wavefront reconstruction**

Nils Weisse[1], Jannik Esslinger[1], Sunny Howard[12], Florian Haberstroh[1], Moritz Foerster[1], Katinka v. Grafenstein[1], Enes Travac[1], Faran Irshad[1], Jinpu Lin[1], Robin Wang[2], Peter Norreys[2], Stefan Karsch[1] & Andreas Döpp[1]

[1] Centre for Advanced Laser Applications, LMU Munich

[2] University of Oxford

Ultra-intense lasers

What happens if …

$\ll 10^{12}$ | 10^{14} | 10^{16} | 10^{18} | 10^{20} | 10^{22} | $\gg 10^{24}$ | W/cm^2

(Non-)linear optics

Plasma optics

Hydrogen ionization

Relativistic optics[1]

Electrons become relativistic

Protons become relativistic

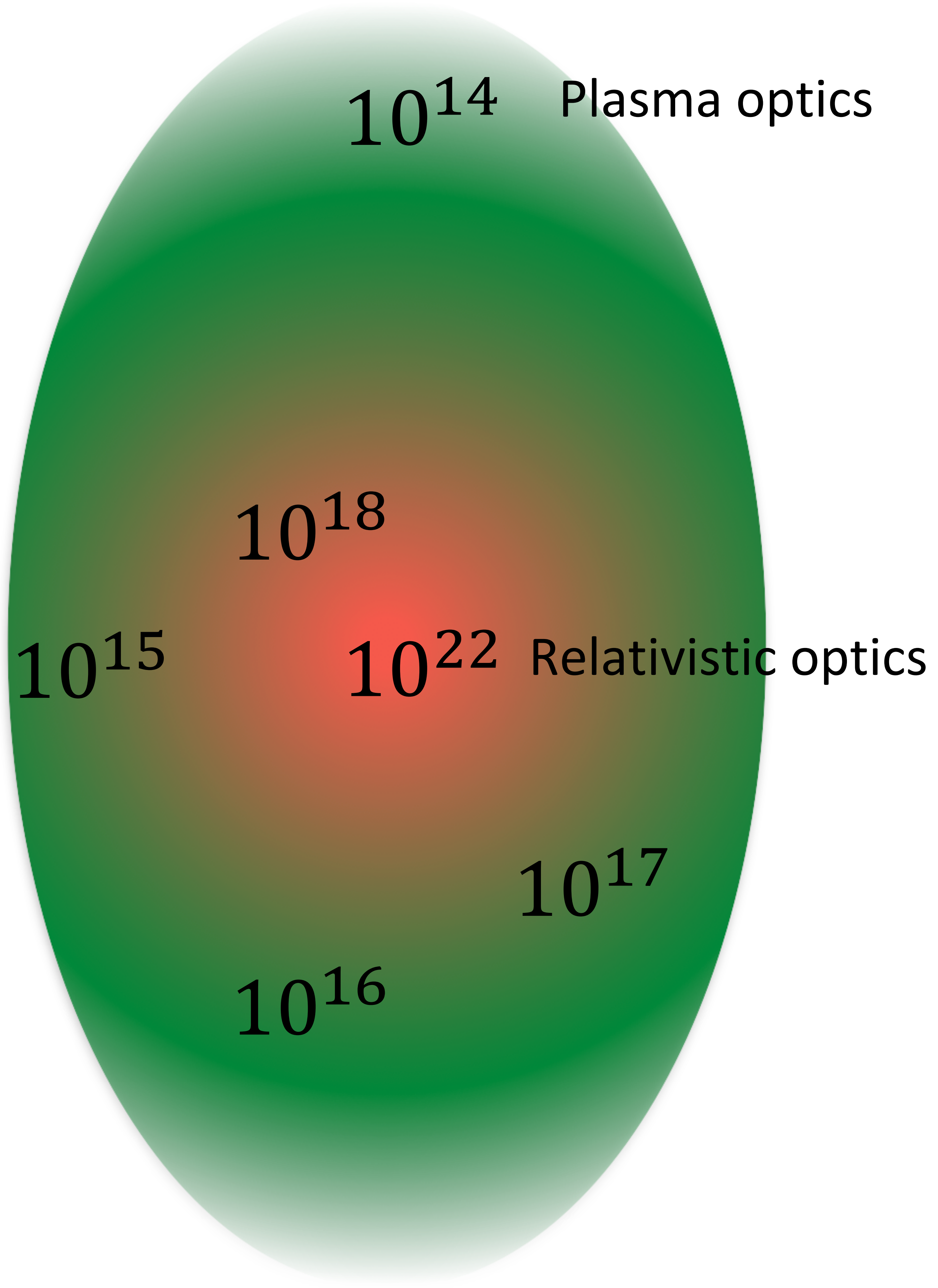
Strong-field QED optics[2]

Weird stuff happens

[1] Mourou et al. Rev. Mod. Phys. 78, 309 (2006)
[2] Di Piazza et al. Rev. Mod. Phys. 84, 1177 (2012)

Ultra-intense lasers

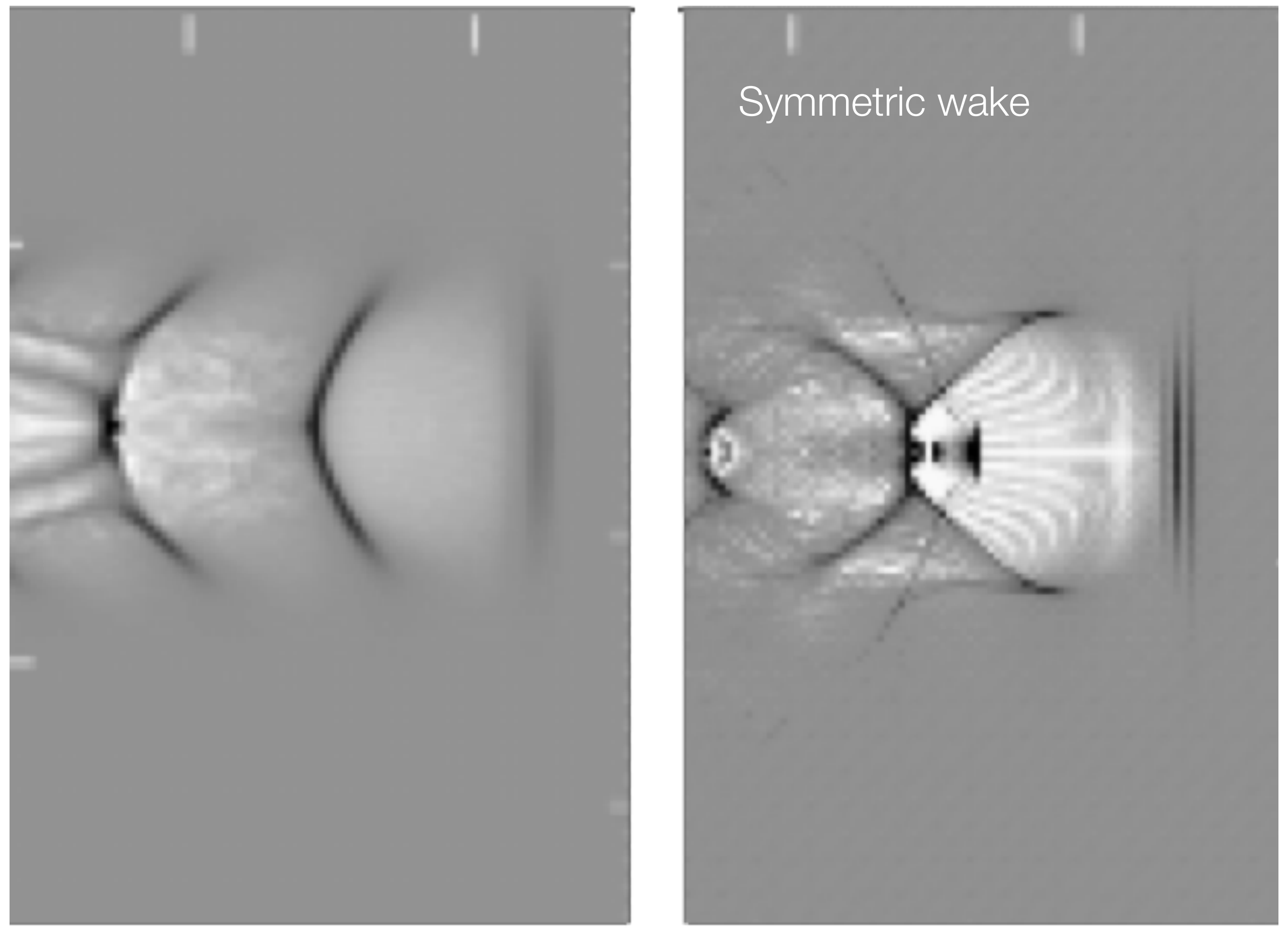
What happens *where* ...?

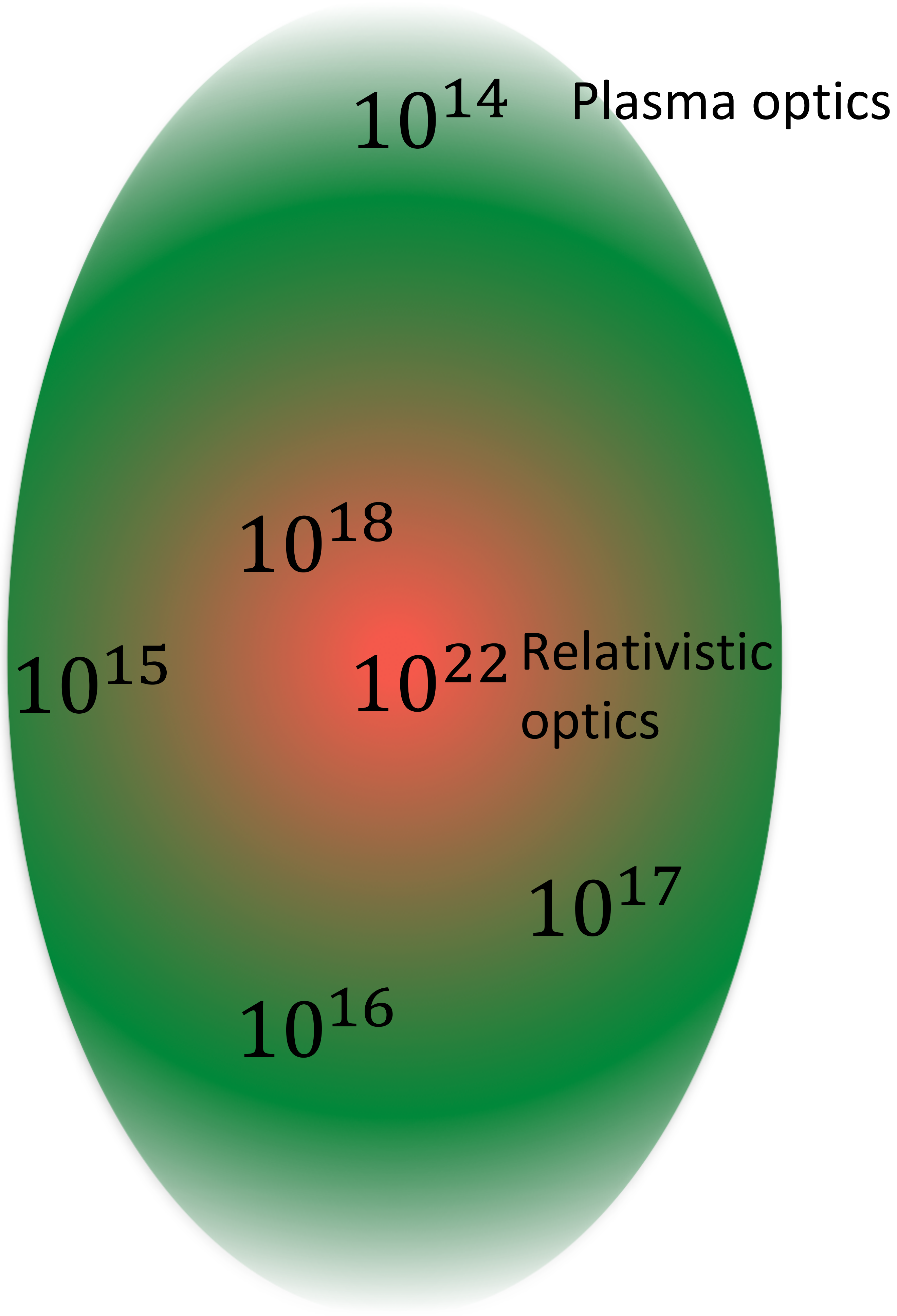


Ultra-intense lasers

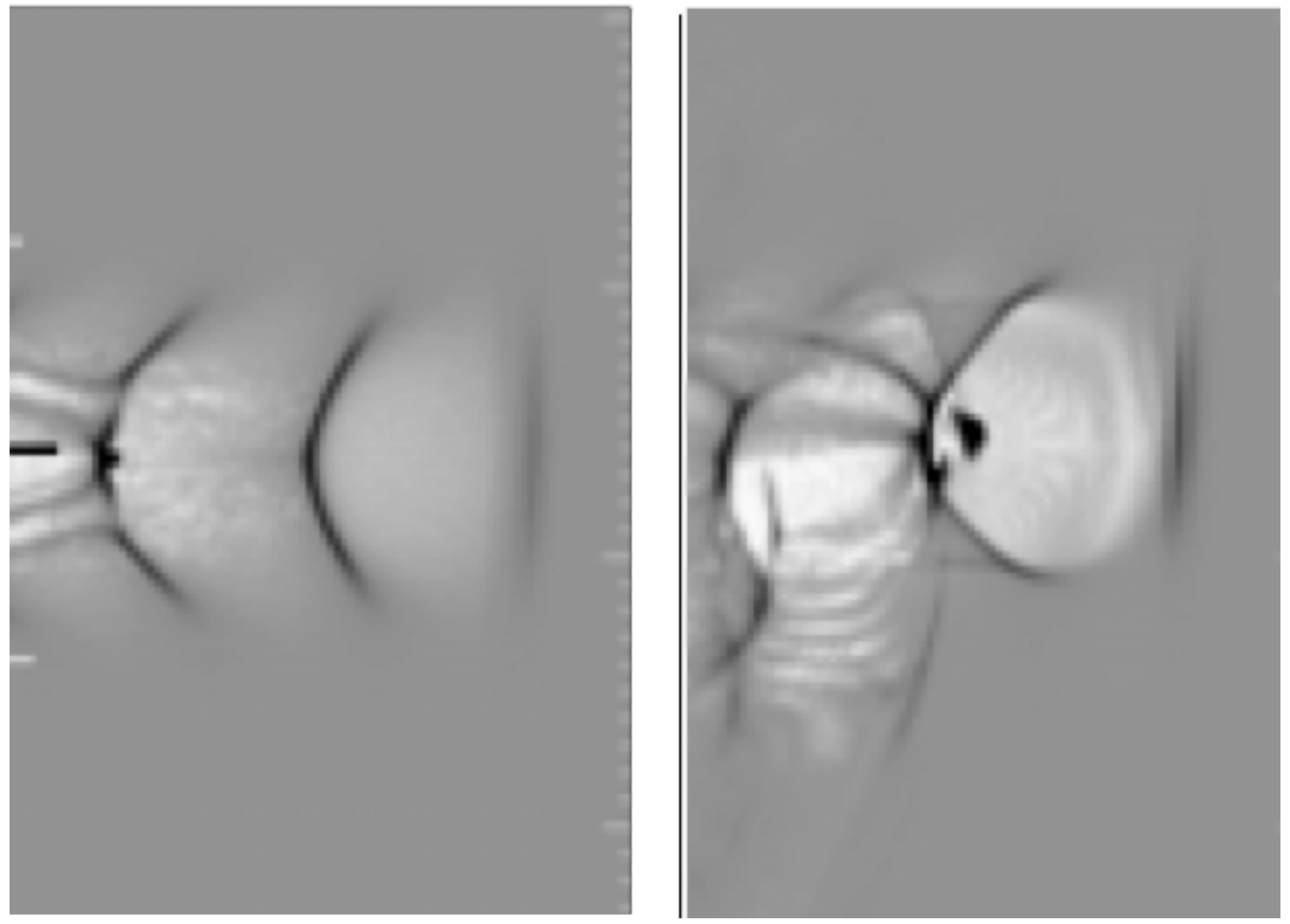
What happens *where* ...?

Ideal laser pulse





With **pulse front tilt**

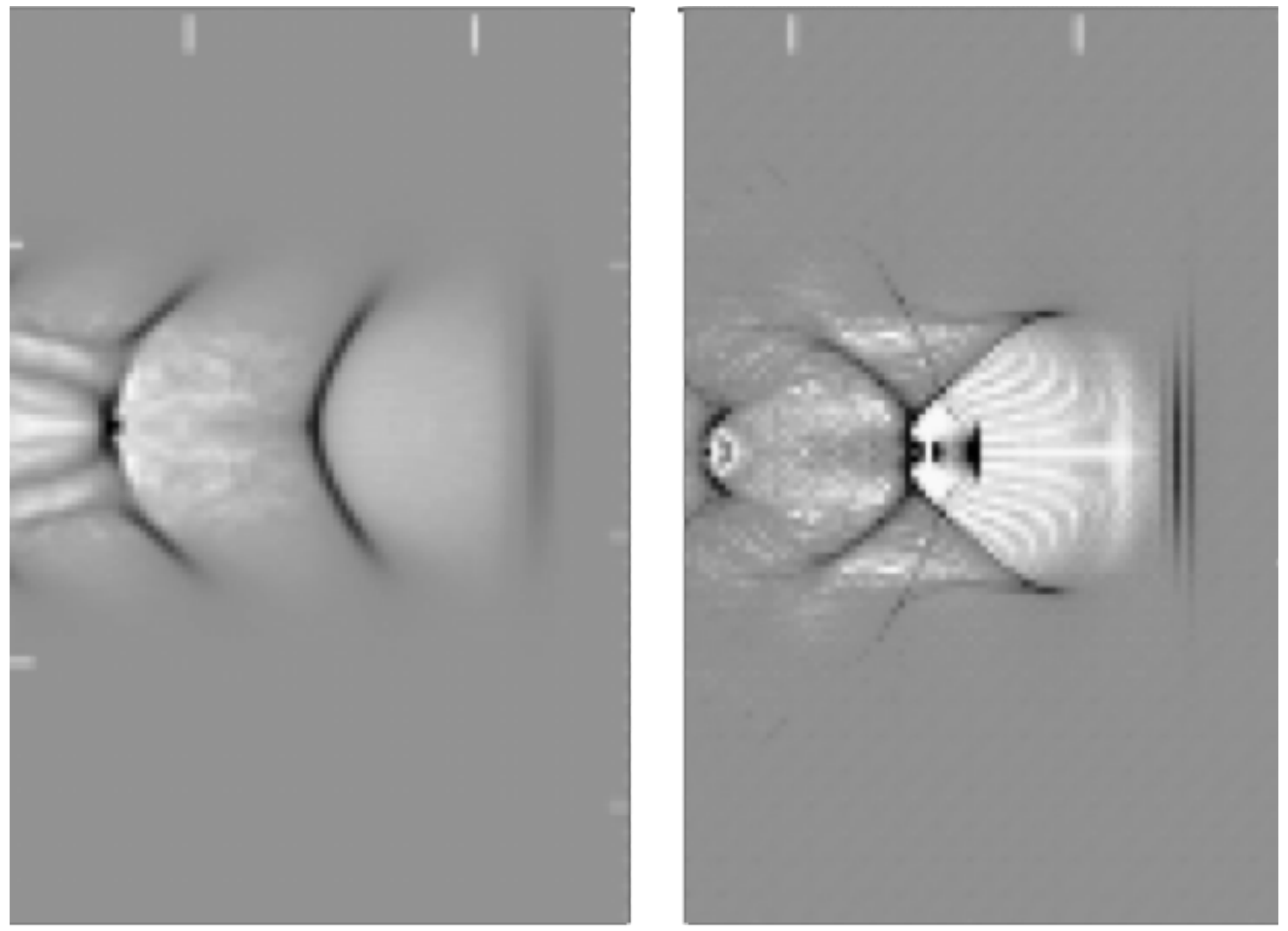


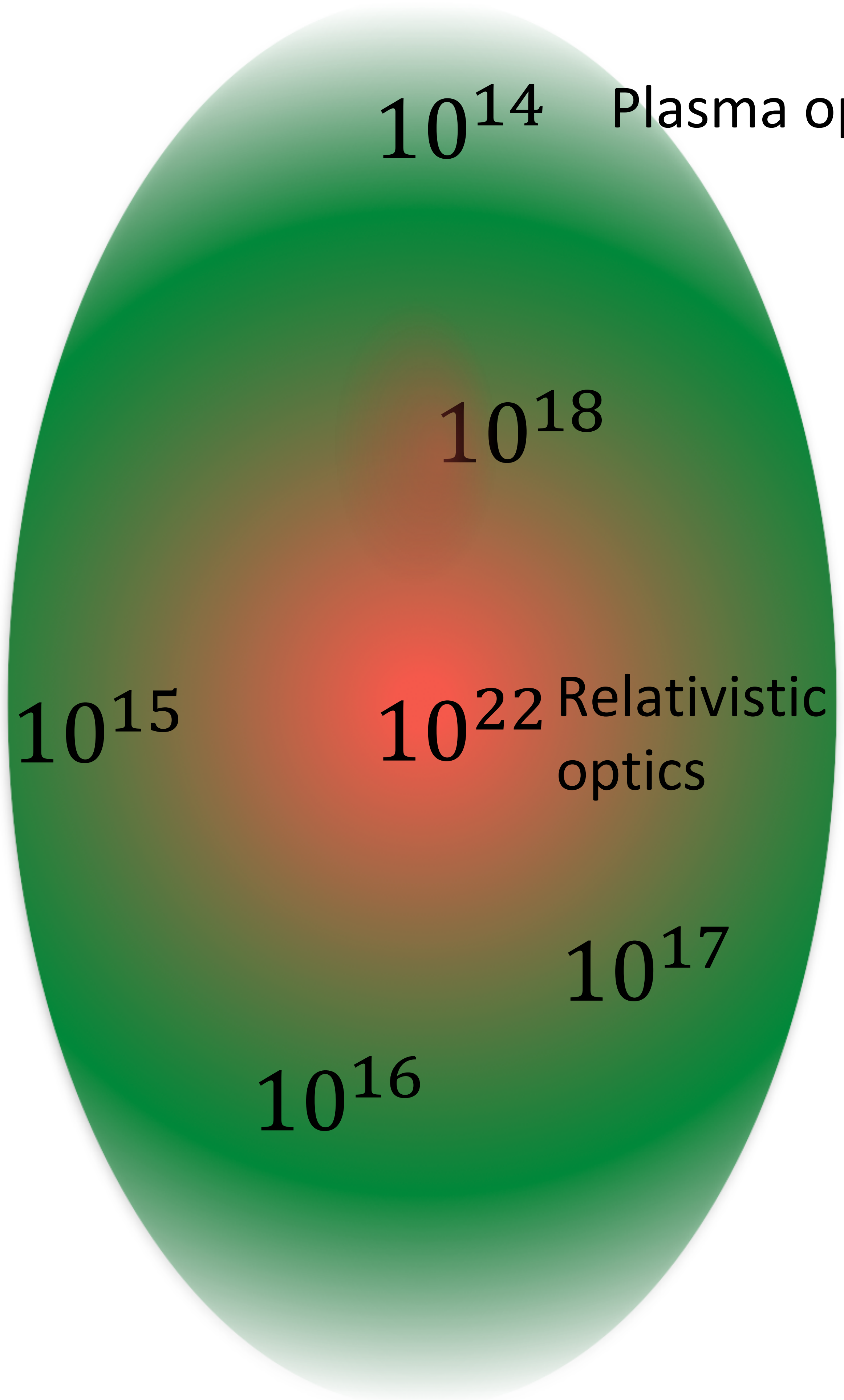
- A. Popp, PhD Tesis (2011)

Ultra-intense lasers

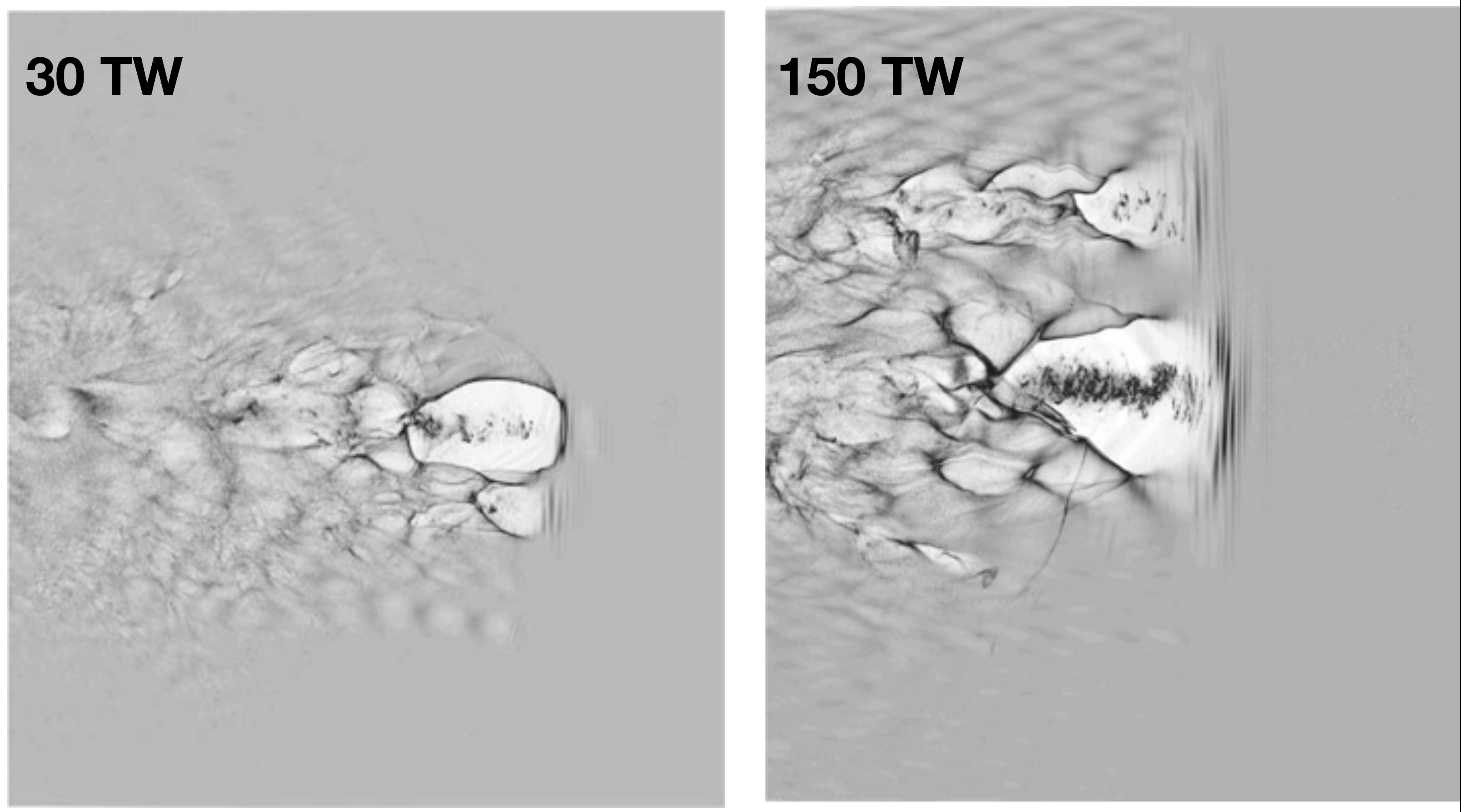
What happens *where* …?

Ideal laser pulse





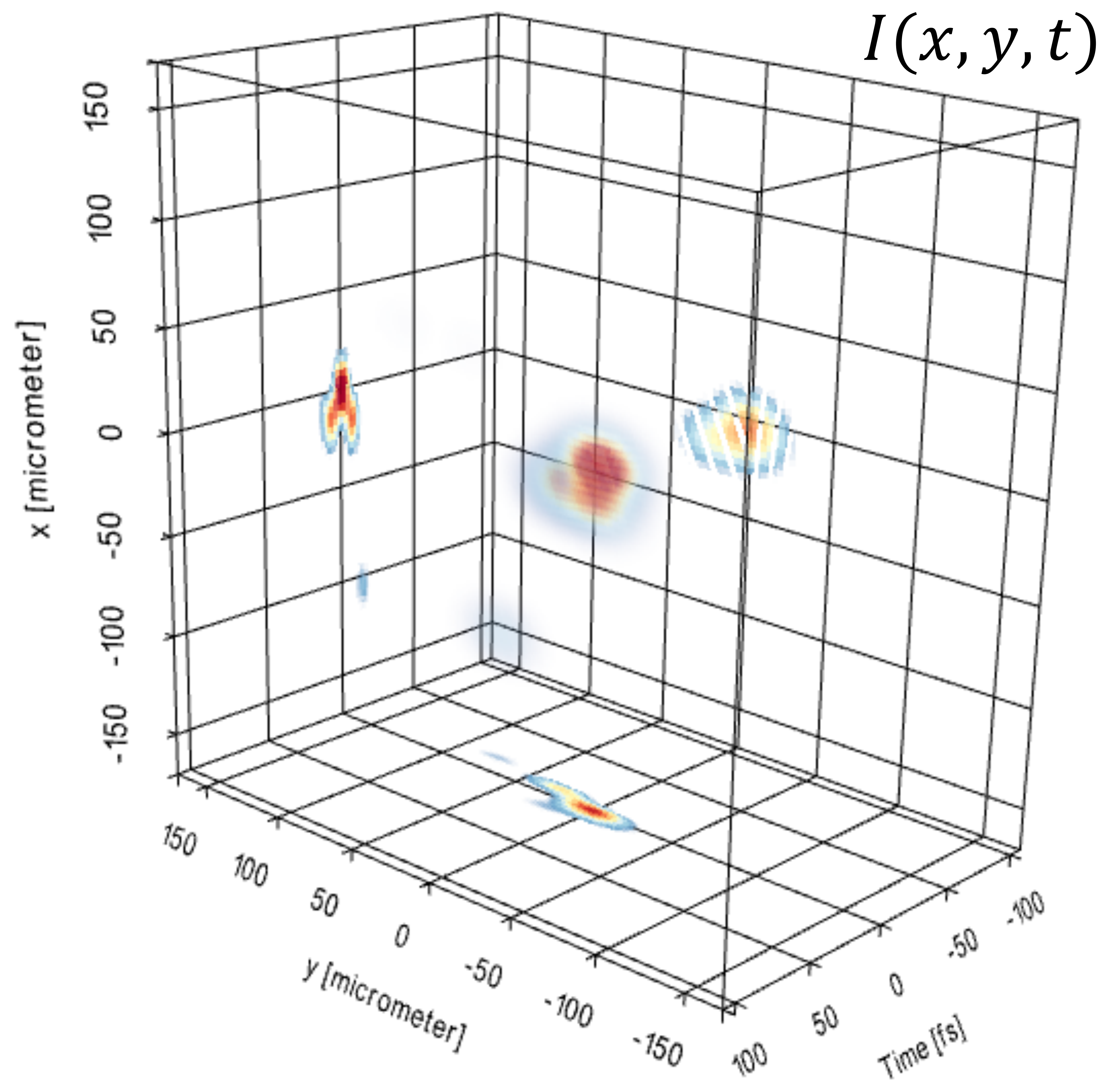
More aberrations | And **more power**



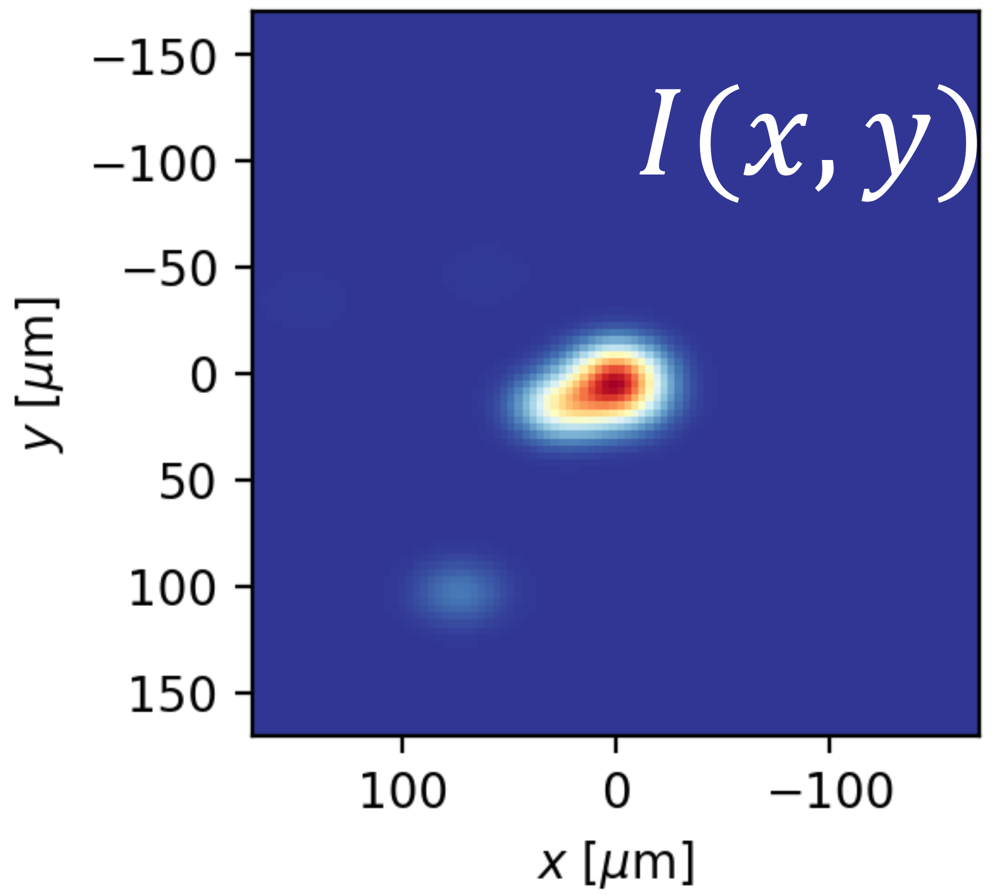
- N. Pathak et al., **Physics of Plasmas** 23, 113106 (2016)

Ultra-intense laser characterization

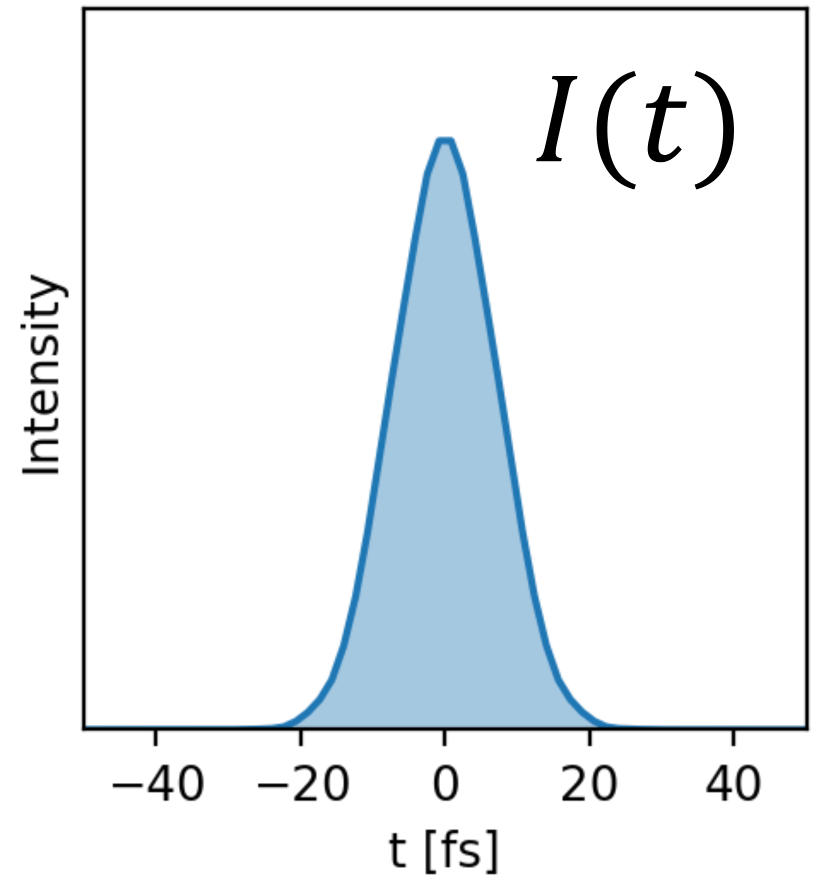
Forward projection



Focus camera

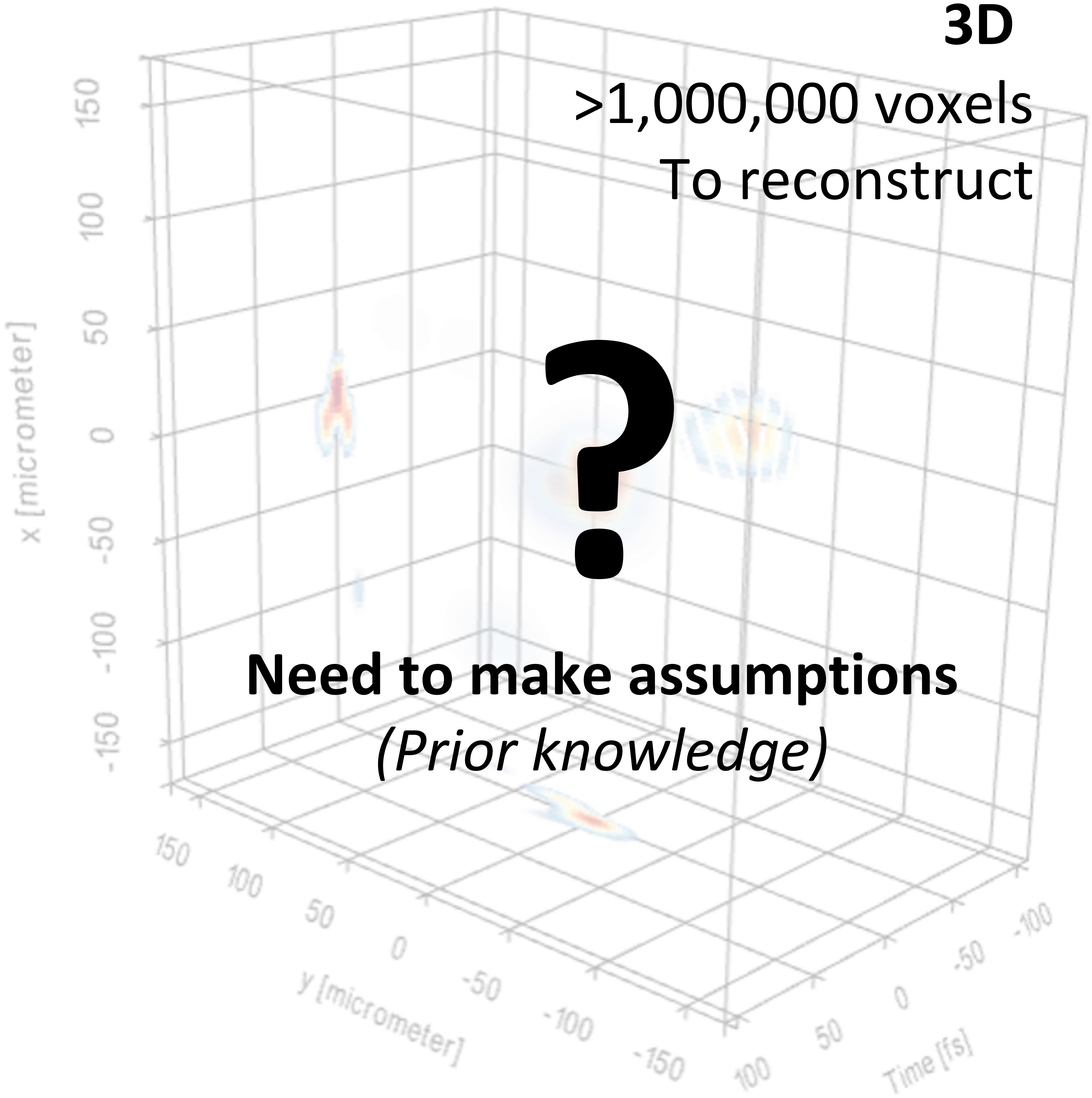


FROG, SPIDER,
WIZZLER, etc.



Ultra-intense laser characterization

Back projection: An inversion problem



3D

>1,000,000 voxels

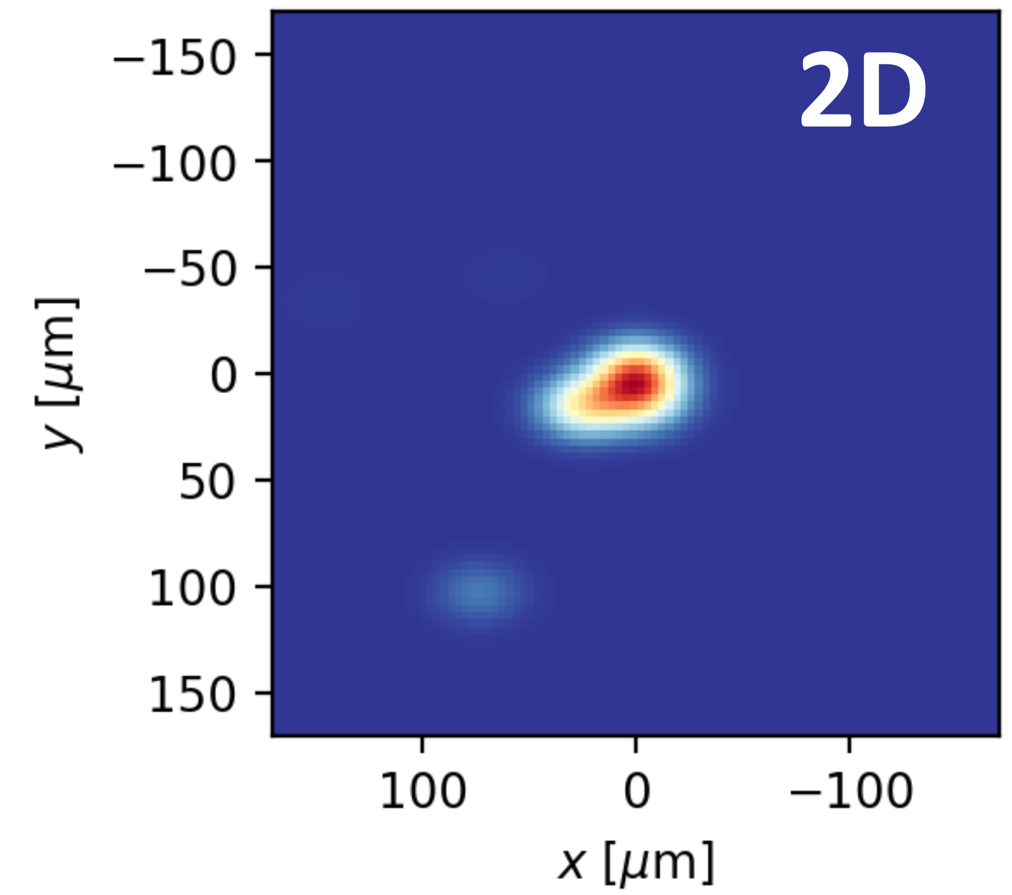
To reconstruct

?

Need to make assumptions

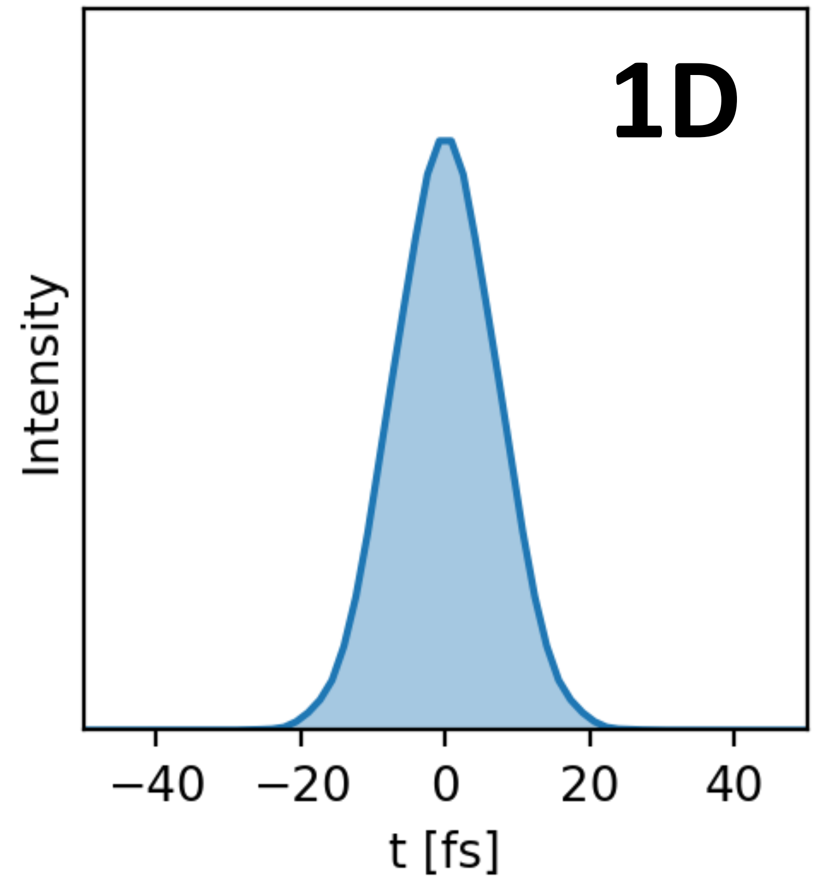
(Prior knowledge)

No **temporal** information



>10,000 pixels

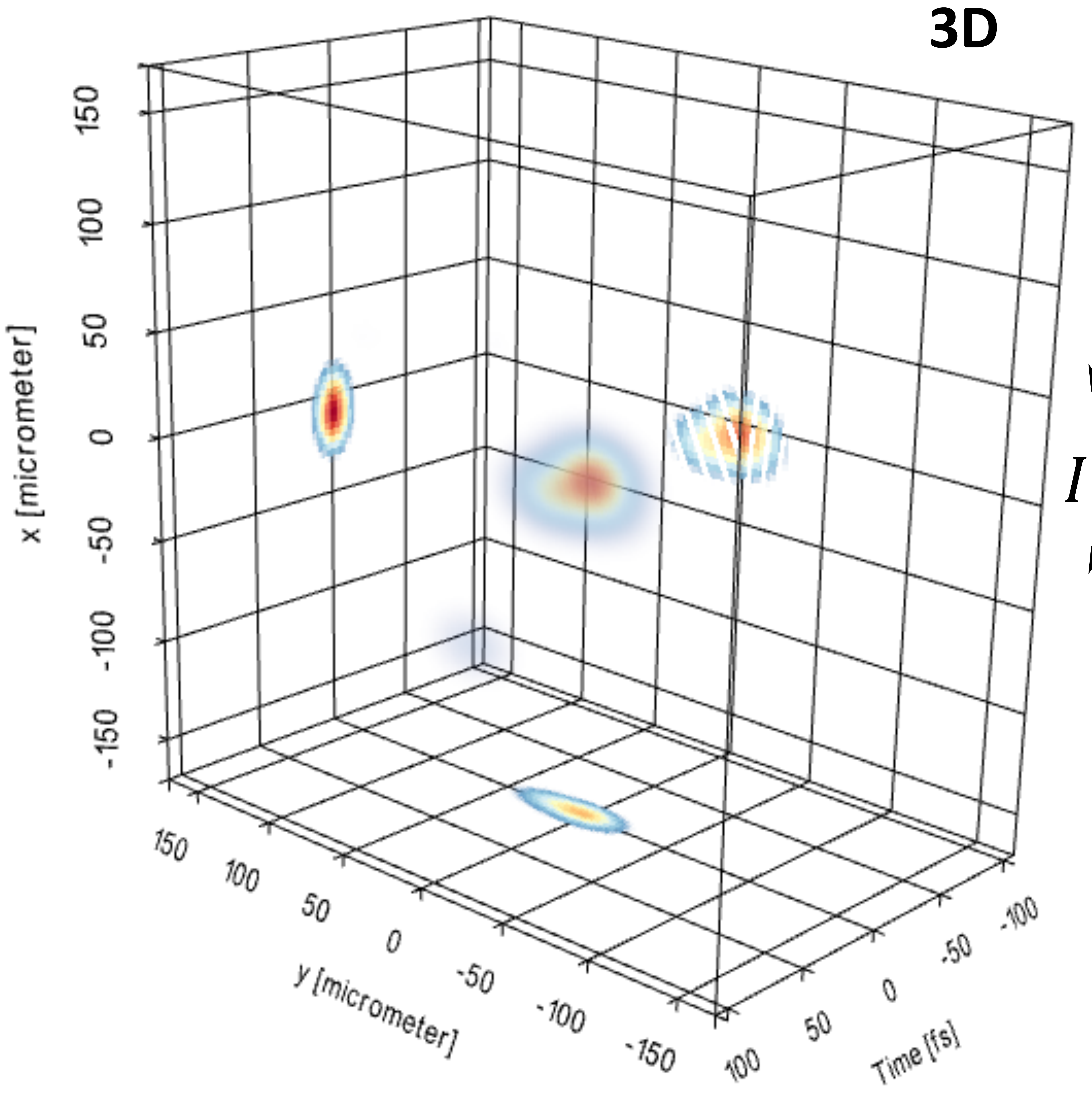
No **spatial** information



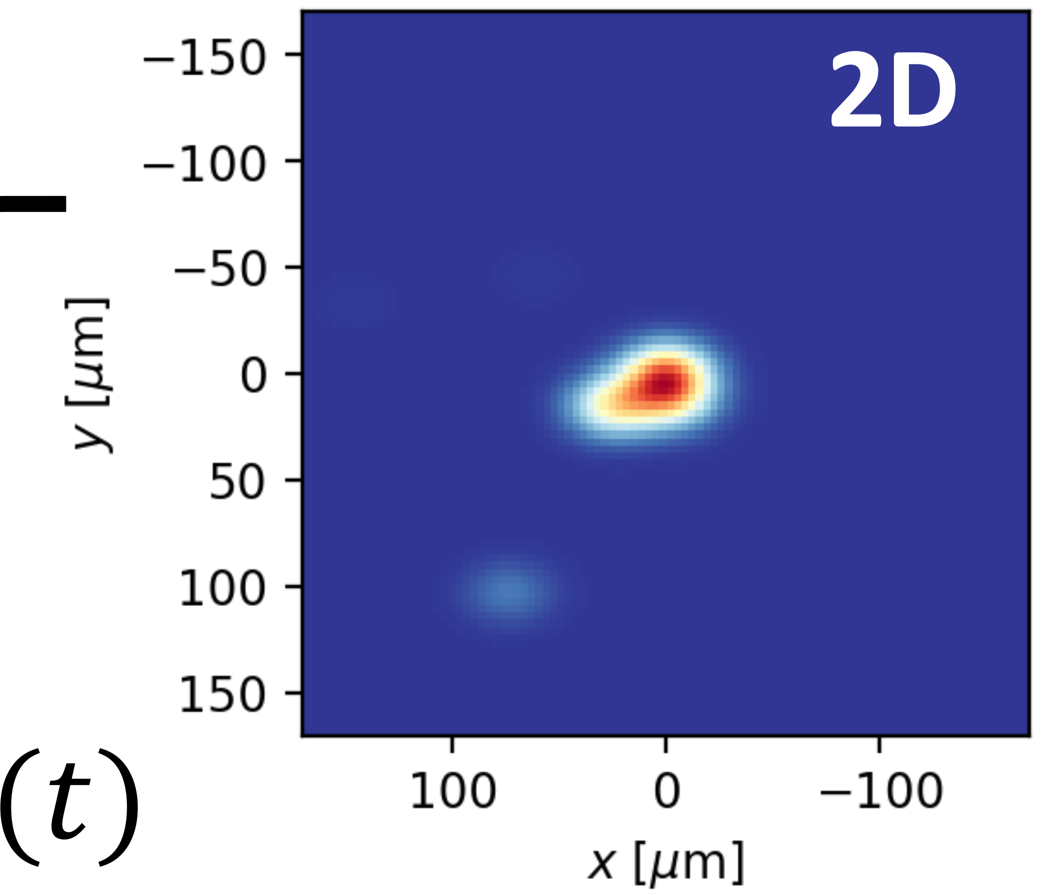
>100 data points

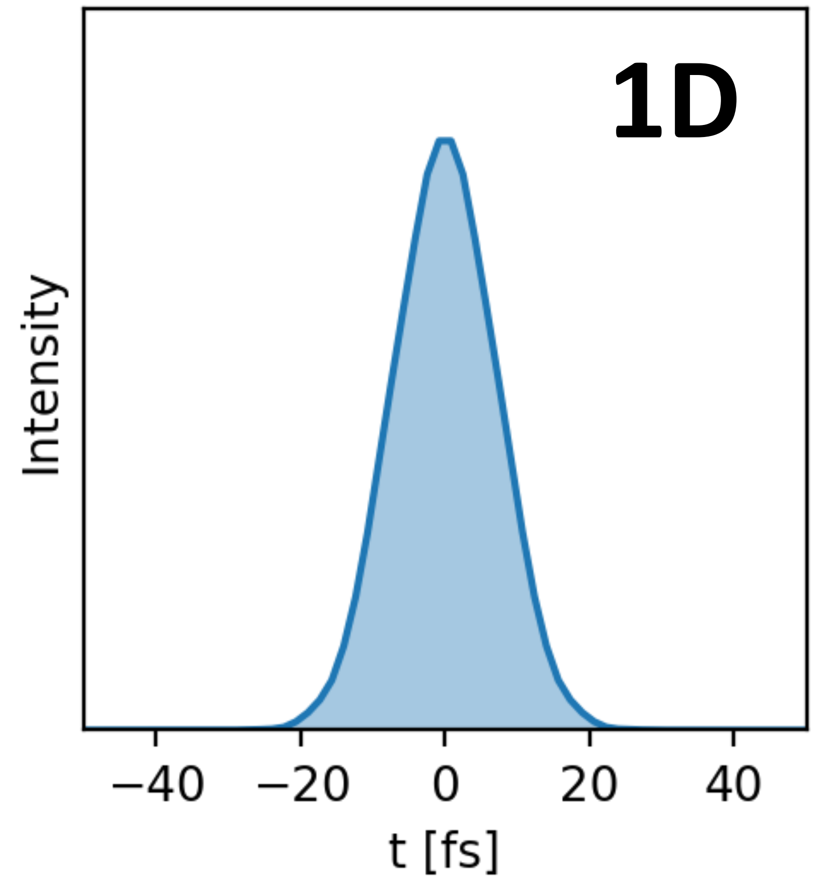
Ultra-intense laser characterization

An inversion problem



$$I(x,y,t) \approx I(x,y) \times I(t)$$





Common assumption:

No spatio-temporal couplings?

Ultra-intense laser characterization

An inversion problem

$$I(x,y,t) = ||\mathcal{F}\left[\sqrt{I(x,y,\omega)} \cdot \exp(i\Phi(x,y,\omega))\right]||^2$$

↑

$$\Phi(x,y,\omega) = \sum_{m,n,i} a^i_{m,n}(\omega-\omega_0)^i Z^m_n(x,y)$$

Can describe the **hyperspectral wavefront** using **Zernike-modes and Taylor-expansion in frequency**

Instead of > 1,000,000 voxels we only need to reconstruct dominant mode coefficients

Allows us to retrieve spatio-temporal couplings (STCs) with only a few measurements (5-10 shots)

- N. Weiße, J. Esslinger et al. Measuring spatial-temporal couplings using modal multi-spectral wavefront reconstruction (prep.)

Ultra-intense laser characterization

An inversion problem

$$I(x,y,t) = ||\mathcal{F}\left[\sqrt{I(x,y,\omega)} \cdot \exp(i\Phi(x,y,\omega))\right]||^2$$

$$\Phi(x,y,\omega) = \sum_{m,n,i} a^i_{m,n}(\omega - \omega_0)^i Z^m_n(x,y)$$

Examples Pulse Front Tilt is defined as

$$\frac{d^2\phi(x,y,\omega)}{d\omega dx} = a^1_{1,1}\frac{dZ^1_1(x,y)}{d\omega}$$

Note that "classical" spectral phase is position independent and entirely given by

$$\frac{d^n\phi(x,y,\omega)}{d\omega^n} = a^n_{0,0}\frac{d^nZ^0_0(x,y)}{d\omega^n}$$

Need no Wizzler, FROG, etc. to measure STCs!

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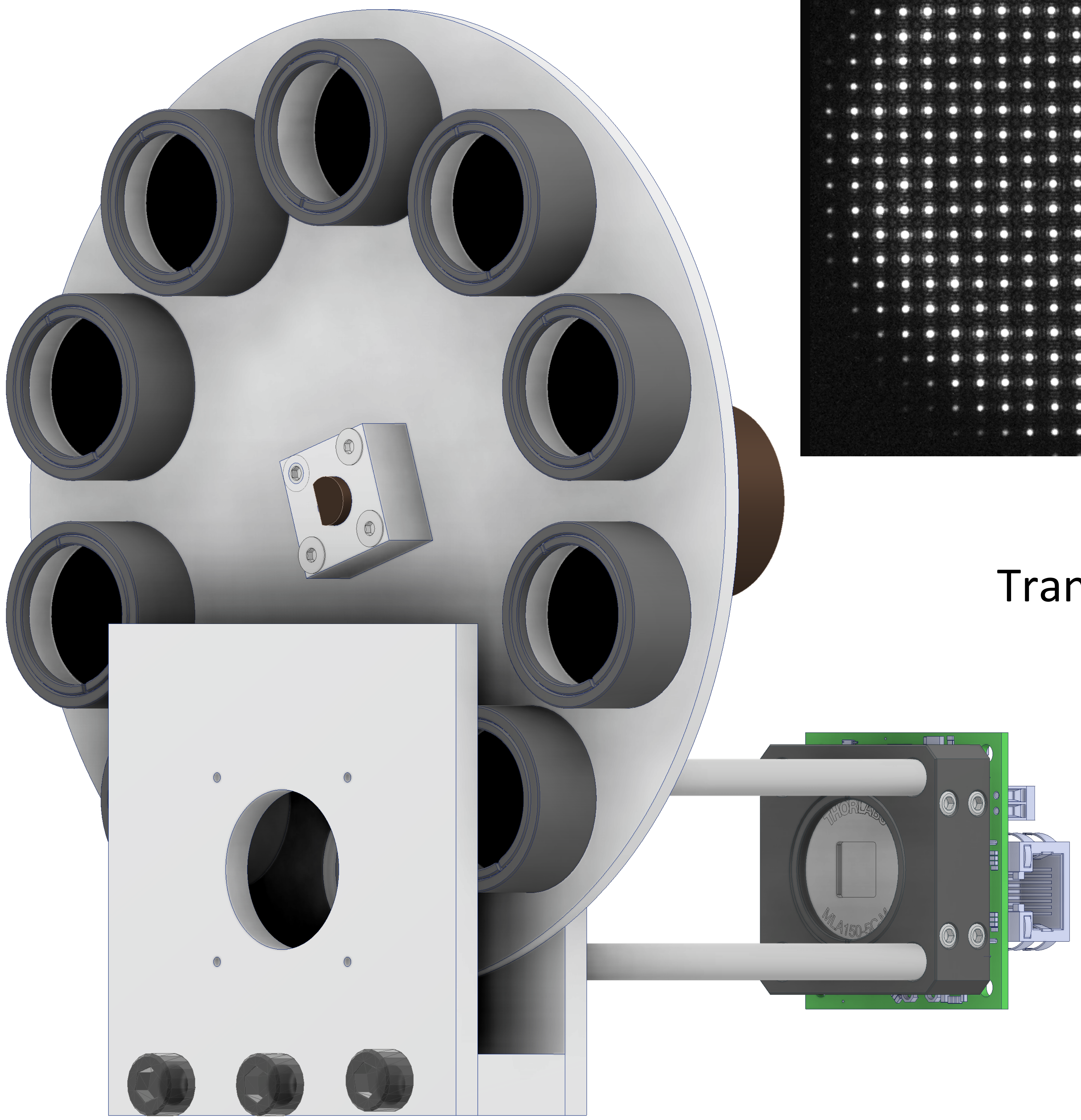
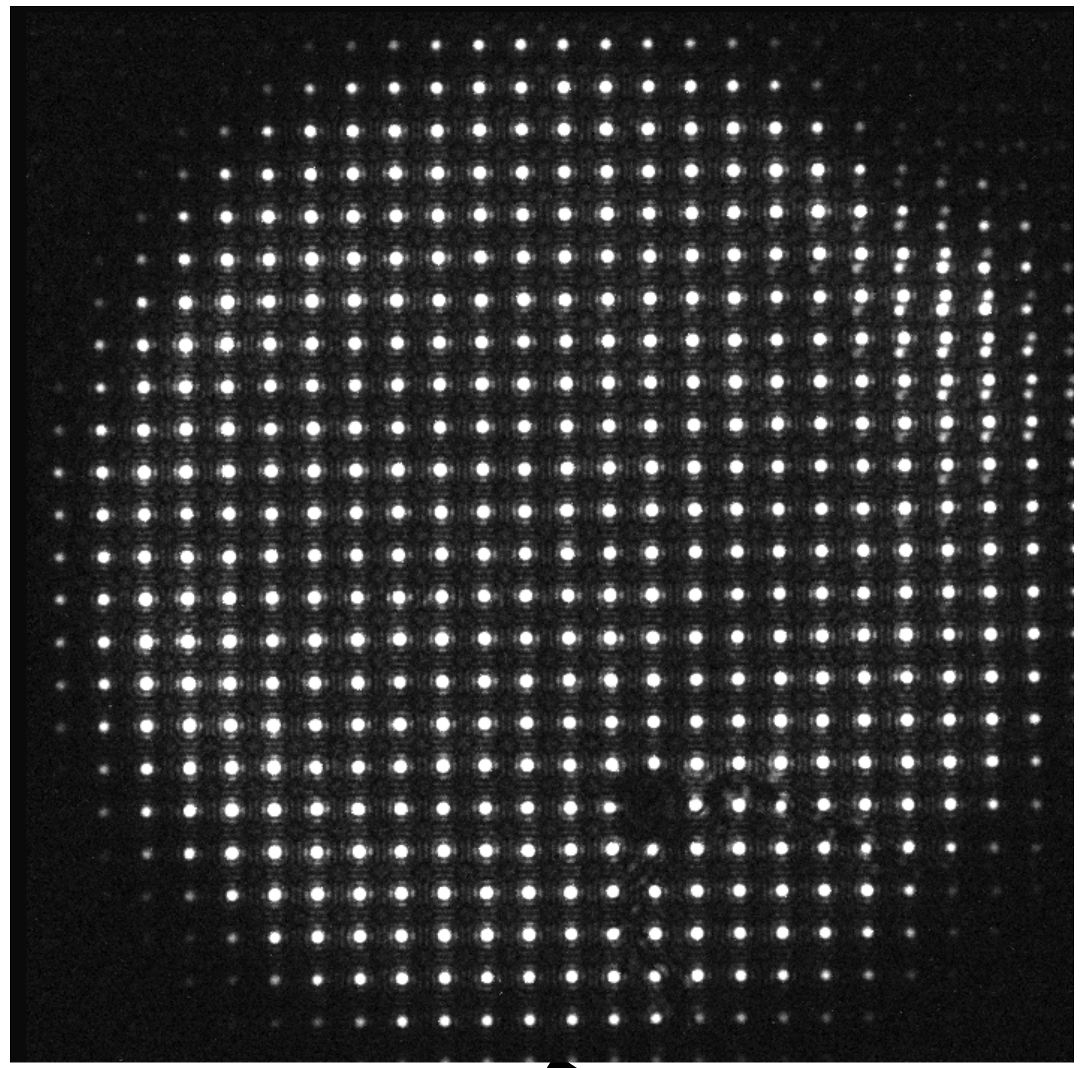
Ultra-intense laser characterization

An inversion problem



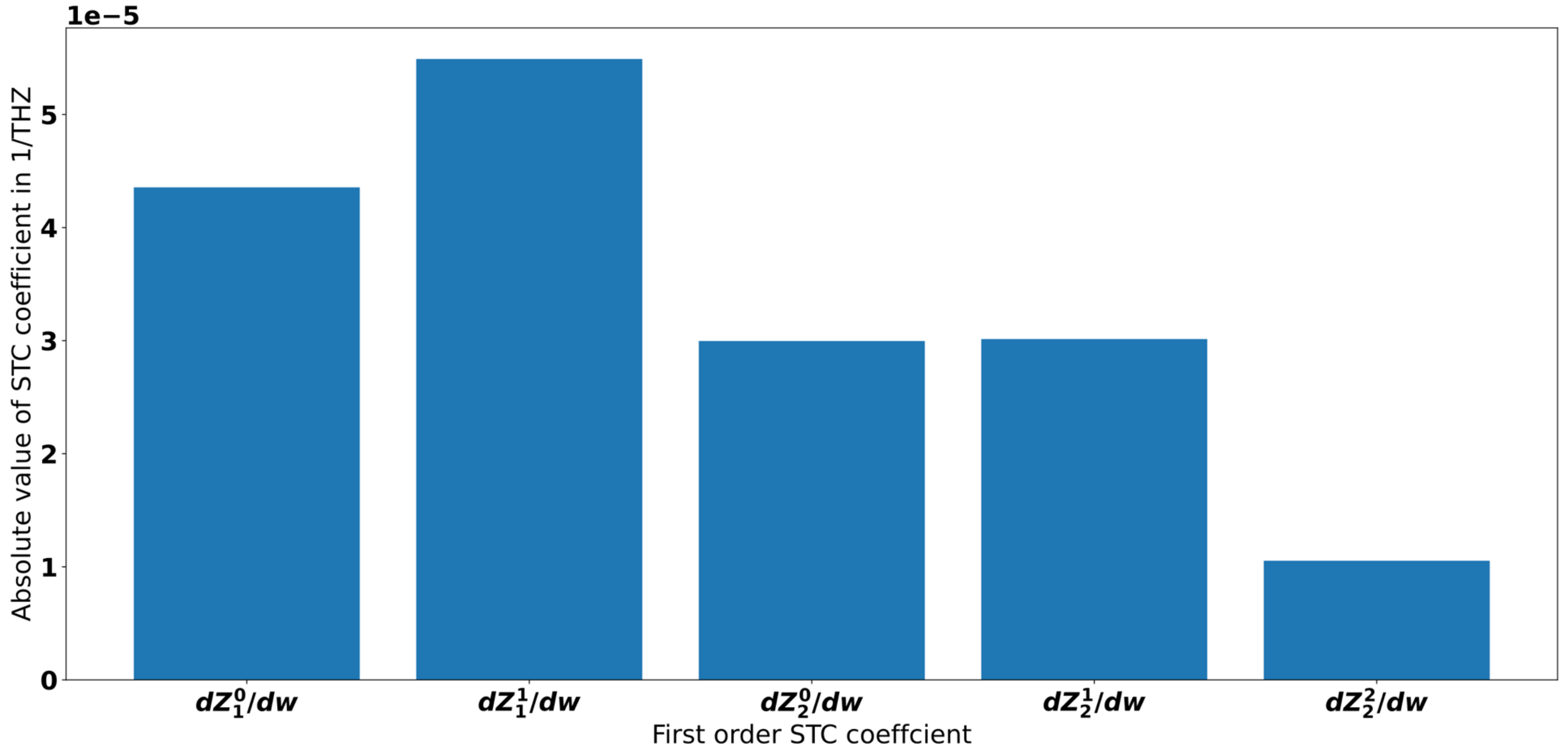


5-10 shots

$$\begin{pmatrix} s_{x,\omega_1}(0,0) \\ s_{x,\omega_1}(0,1) \\ s_{x,\omega_1}(1,0) \\ s_{x,\omega_1}(1,1) \\ s_{y,\omega_1}(0,0) \\ s_{y,\omega_1}(0,1) \\ s_{y,\omega_1}(1,0) \\ s_{y,\omega_1}(1,1) \\ s_{x,\omega_2}(0,0) \\ s_{x,\omega_2}(0,1) \\ s_{x,\omega_2}(1,0) \\ s_{x,\omega_2}(1,1) \\ s_{y,\omega_2}(0,0) \\ s_{y,\omega_2}(0,1) \\ s_{y,\omega_2}(1,0) \\ s_{y,\omega_2}(1,1) \end{pmatrix} = \begin{pmatrix}
\left.\frac{dZ_1^{-1}}{dx}\right|_{0,0} & \left.\frac{dZ_1^{1}}{dx}\right|_{0,0} & \left.\frac{dZ_0^{0}}{dx}\right|_{0,0}\tilde{\omega}_1^1 & \left.\frac{dZ_1^{-1}}{dx}\right|_{0,0}\tilde{\omega}_1^1 & \left.\frac{dZ_1^{1}}{dx}\right|_{0,0}\tilde{\omega}_1^1 \\
\left.\frac{dZ_1^{-1}}{dx}\right|_{0,1} & \left.\frac{dZ_1^{1}}{dx}\right|_{0,1} & \left.\frac{dZ_0^{0}}{dx}\right|_{0,1}\tilde{\omega}_1^1 & \left.\frac{dZ_1^{-1}}{dx}\right|_{0,1}\tilde{\omega}_1^1 & \left.\frac{dZ_1^{1}}{dx}\right|_{0,1}\tilde{\omega}_1 \\
\left.\frac{dZ_1^{-1}}{dx}\right|_{1,0} & \left.\frac{dZ_1^{1}}{dx}\right|_{1,0} & \left.\frac{dZ_0^{0}}{dx}\right|_{1,0}\tilde{\omega}_1^1 & \left.\frac{dZ_1^{-1}}{dx}\right|_{1,0}\tilde{\omega}_1^1 & \left.\frac{dZ_1^{1}}{dx}\right|_{1,0}\tilde{\omega}_1^1 \\
\left.\frac{dZ_1^{-1}}{dx}\right|_{1,1} & \left.\frac{dZ_1^{1}}{dx}\right|_{1,1} & \left.\frac{dZ_0^{0}}{dx}\right|_{1,1}\tilde{\omega}_1^1 & \left.\frac{dZ_1^{-1}}{dx}\right|_{1,1}\tilde{\omega}_1^1 & \left.\frac{dZ_1^{1}}{dx}\right|_{1,1}\tilde{\omega}_1^1 \\
\left.\frac{dZ_1^{-1}}{dy}\right|_{0,0} & \left.\frac{dZ_1^{1}}{dy}\right|_{0,0} & \left.\frac{dZ_2^{-2}}{dy}\right|_{0,0}\tilde{\omega}_1^1 & \left.\frac{dZ_2^{0}}{dy}\right|_{0,0}\tilde{\omega}_1^1 & \left.\frac{dZ_2^{2}}{dy}\right|_{0,0}\tilde{\omega}_1^1 \\
\left.\frac{dZ_1^{-1}}{dy}\right|_{0,1} & \left.\frac{dZ_1^{1}}{dy}\right|_{0,1} & \left.\frac{dZ_2^{-2}}{dy}\right|_{0,1}\tilde{\omega}_1^1 & \left.\frac{dZ_2^{0}}{dy}\right|_{0,1}\tilde{\omega}_1^1 & \left.\frac{dZ_2^{2}}{dy}\right|_{0,1}\tilde{\omega}_1^1 \\
\left.\frac{dZ_1^{-1}}{dy}\right|_{1,0} & \left.\frac{dZ_1^{1}}{dy}\right|_{1,0} & \left.\frac{dZ_2^{-2}}{dy}\right|_{1,0}\tilde{\omega}_1^1 & \left.\frac{dZ_2^{0}}{dy}\right|_{1,0}\tilde{\omega}_1^1 & \left.\frac{dZ_2^{2}}{dy}\right|_{1,0}\tilde{\omega}_1^1 \\
\left.\frac{dZ_1^{-1}}{dy}\right|_{1,1} & \left.\frac{dZ_1^{1}}{dy}\right|_{1,1} & \left.\frac{dZ_2^{-2}}{dy}\right|_{1,1}\tilde{\omega}_1^1 & \left.\frac{dZ_2^{0}}{dy}\right|_{1,1}\tilde{\omega}_1^1 & \left.\frac{dZ_2^{2}}{dy}\right|_{1,1}\tilde{\omega}_1^1 \\
\left.\frac{dZ_1^{-1}}{dx}\right|_{0,0} & \left.\frac{dZ_1^{1}}{dx}\right|_{0,0} & \left.\frac{dZ_0^{0}}{dx}\right|_{0,0}\tilde{\omega}_2^1 & \left.\frac{dZ_1^{-1}}{dx}\right|_{0,0}\tilde{\omega}_2^1 & \left.\frac{dZ_1^{1}}{dx}\right|_{0,0}\tilde{\omega}_2^1 \\
\left.\frac{dZ_1^{-1}}{dx}\right|_{0,1} & \left.\frac{dZ_1^{1}}{dx}\right|_{0,1} & \left.\frac{dZ_0^{0}}{dx}\right|_{0,1}\tilde{\omega}_2^1 & \left.\frac{dZ_1^{-1}}{dx}\right|_{0,1}\tilde{\omega}_2^1 & \left.\frac{dZ_1^{1}}{dx}\right|_{0,1}\tilde{\omega}_2^1 \\
\left.\frac{dZ_1^{-1}}{dx}\right|_{1,0} & \left.\frac{dZ_1^{1}}{dx}\right|_{1,0} & \left.\frac{dZ_0^{0}}{dx}\right|_{1,0}\tilde{\omega}_2^1 & \left.\frac{dZ_1^{-1}}{dx}\right|_{1,0}\tilde{\omega}_2^1 & \left.\frac{dZ_1^{1}}{dx}\right|_{1,0}\tilde{\omega}_2^1 \\
\left.\frac{dZ_1^{-1}}{dx}\right|_{1,1} & \left.\frac{dZ_1^{1}}{dx}\right|_{1,1} & \left.\frac{dZ_0^{0}}{dx}\right|_{1,1}\tilde{\omega}_2^1 & \left.\frac{dZ_1^{-1}}{dx}\right|_{1,1}\tilde{\omega}_2^1 & \left.\frac{dZ_1^{1}}{dx}\right|_{1,1}\tilde{\omega}_2^1 \\
\left.\frac{dZ_1^{-1}}{dy}\right|_{0,0} & \left.\frac{dZ_1^{1}}{dy}\right|_{0,0} & \left.\frac{dZ_2^{-2}}{dy}\right|_{0,0}\tilde{\omega}_2^1 & \left.\frac{dZ_2^{0}}{dy}\right|_{0,0}\tilde{\omega}_2^1 & \left.\frac{dZ_2^{2}}{dy}\right|_{0,0}\tilde{\omega}_2^1 \\
\left.\frac{dZ_1^{-1}}{dy}\right|_{0,1} & \left.\frac{dZ_1^{1}}{dy}\right|_{0,1} & \left.\frac{dZ_2^{-2}}{dy}\right|_{0,1}\tilde{\omega}_2^1 & \left.\frac{dZ_2^{0}}{dy}\right|_{0,1}\tilde{\omega}_2^1 & \left.\frac{dZ_2^{2}}{dy}\right|_{0,1}\tilde{\omega}_2^1 \\
\left.\frac{dZ_1^{-1}}{dy}\right|_{1,0} & \left.\frac{dZ_1^{1}}{dy}\right|_{1,0} & \left.\frac{dZ_2^{-2}}{dy}\right|_{1,0}\tilde{\omega}_2^1 & \left.\frac{dZ_2^{0}}{dy}\right|_{1,0}\tilde{\omega}_2^1 & \left.\frac{dZ_2^{2}}{dy}\right|_{1,0}\tilde{\omega}_2^1 \\
\left.\frac{dZ_1^{-1}}{dy}\right|_{1,1} & \left.\frac{dZ_1^{1}}{dy}\right|_{1,1} & \left.\frac{dZ_2^{-2}}{dy}\right|_{1,1}\tilde{\omega}_2^1 & \left.\frac{dZ_2^{0}}{dy}\right|_{1,1}\tilde{\omega}_2^1 & \left.\frac{dZ_2^{2}}{dy}\right|_{1,1}\tilde{\omega}_2^1
\end{pmatrix} \cdot \begin{pmatrix} a_{0,0}^0 \\ a_{1,-1}^0 \\ a_{1,1}^0 \\ a_{0,0}^1 \\ a_{1,-1}^1 \\ a_{1,1}^1 \end{pmatrix}$$

Translate into an inverse problem

Penrose pseudo-inverse calculation



- N. Weiße, J. Esslinger et al. Measuring spatial-temporal couplings using modal multi-spectral wavefront reconstruction (prep.)

Summary

- Measurement of spatio-temporal couplings does not require a full reconstruction of the electric field of the laser pulse
- 5-10 shots are sufficient to reconstruct the spatio-spectral phase enough to detect low-order spatio-temporal couplings

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Thank you for your attention!

Nils Weiße
nils.weisse@physik.uni-muenchen.de

Andreas Döpp
a.doepp@lmu.de

Am Coulombwall 1 · 85748 Garching · Germany · www.pulse.physik.lmu.de