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Eccellenza, Project-186866

Neutrino constraints on new physics in $(g-2)_\mu$

Admir Greljo

09.03.2022, La Thuile

New Physics in $(g-2)_\mu$?

4.2σ

The Muon g-2, Fermilab, 2104.03281

A word of caution:
More EXP/TH work is needed to prove NP is behind these effects.

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Muons again?

$$b \to s\mu^+\mu^-$$

3.1σ R_K LHCb, CERN, 2103.11769

$= 4.3\,\sigma$ (conservative) global significance

[Isidori, Lancierini, Owen, Serra, 2104.05631]

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The Muon g-2, Fermilab, 2104.03281

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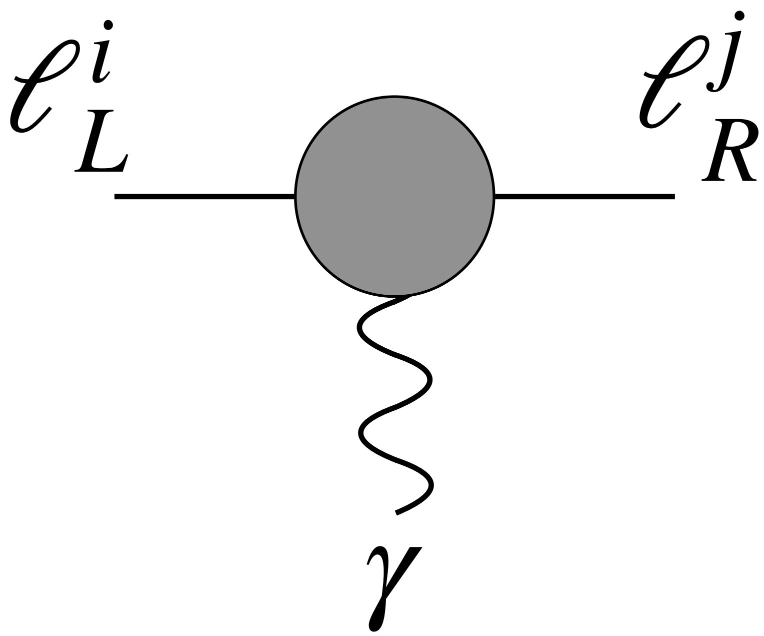
Watch out LFV!



$$\frac{Br(\mu \to e\gamma)}{3 \times 10^{-13}} \approx \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{12}}{10^{-5}}\right)^2$$

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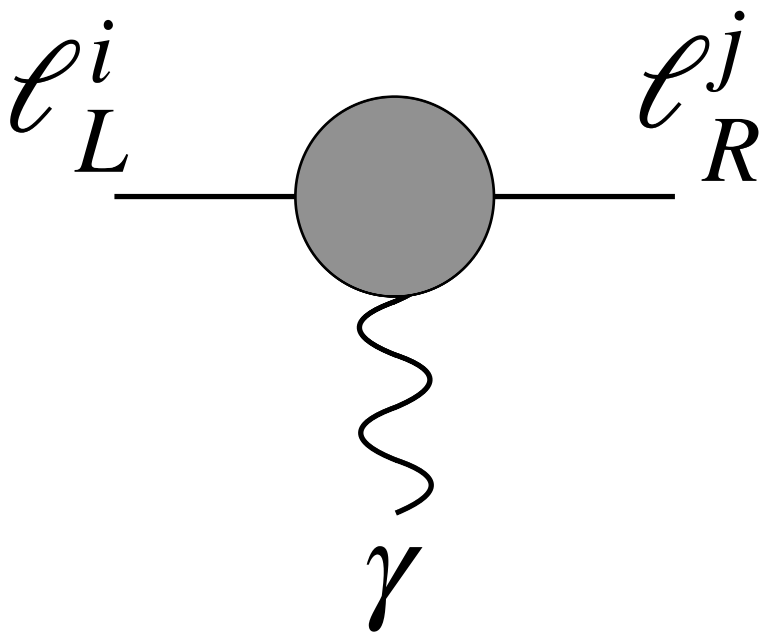

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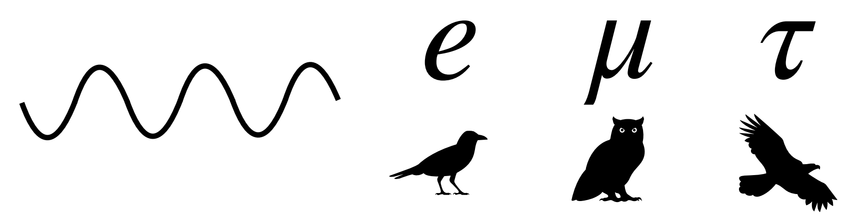
Nearly exact $\mathrm{U}(1)_\mathrm{e} \times \mathrm{U}(1)_\mu \times \mathrm{U}(1)_\tau$?

Flavour alignment of New Physics [Isidori, Pages, Wilsch; 2111.13724]

Gauged lepton flavor

1. Extend the SM gauge group with the **lepton flavour non-universal** $U(1)_X$.

Gauged $\mathrm{U(1)_X}$



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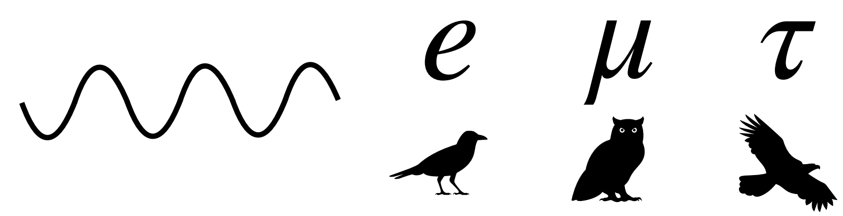

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2. Charge a **leptoquark** under $U(1)_X$.

Hambye, Heeck; 1712.04871
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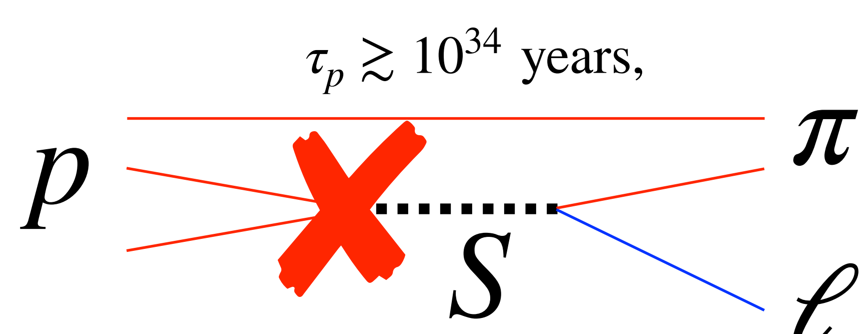
AG, Soreq, Stangl, Thomsen, Zupan; 2107.07518
Davighi, AG, Thomsen; 2202.05275

- Gauge symmetry selection rules:

✔ $q\mu S$

✘ $qeS, q\tau S, qqS^\dagger$
$qqS^\dagger H, qqS^\dagger \phi$



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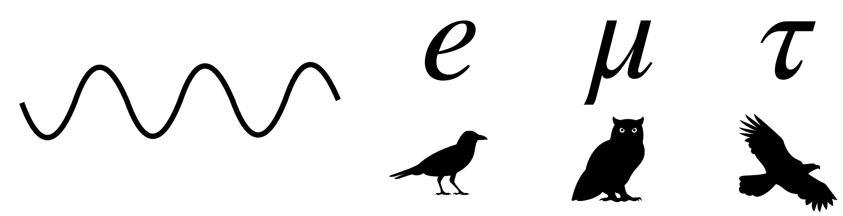
Gauged $\mathrm{U}(1)_\mathrm{X}$ 

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$\Longrightarrow$

The accidental symmetry of $\mathscr{L}^{(4)}$ is $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$ and the LQ charge is $(-1/3, 0, -1, 0)$

- Gauge symmetry selection rules: ✓ $q\mu S$ ✗ $qeS, q\tau S, qqS^\dagger$ $qqS^\dagger H, qqS^\dagger\phi$

Chiral anomaly-free ***muoquark*** models are classified in (backup):
AG, Soreq, Stangl, Thomsen, Zupan; 2107.07518

A class of $U(1)_X$ models

- Charges

$$X = 3m(B-L) - n\,(2L_\mu - L_e - L_\tau)\ , \quad \mathrm{gcd}(m,n) = 1$$

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- Field content

	Fields	$\mathrm{U}(1)_X$
Quarks	$q_i,\ u_i,\ d_i$	m
Electrons and taus	$\ell_{1,3},\ e_{1,3},\ \nu_{1,3}$	$n-3m$
Muons	$\ell_2,\ e_2,\ \nu_2$	$-2n-3m$
Higgs	H	0
Leptoquarks	$S_3,\ S_1$	$2m+2n$

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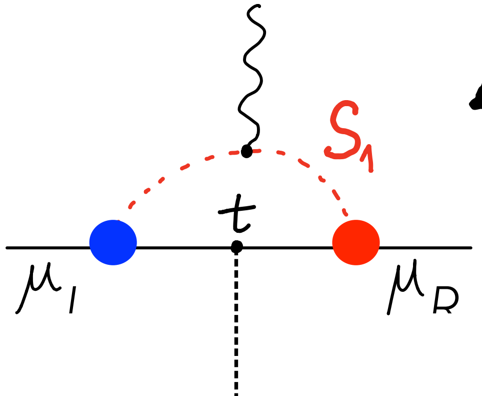
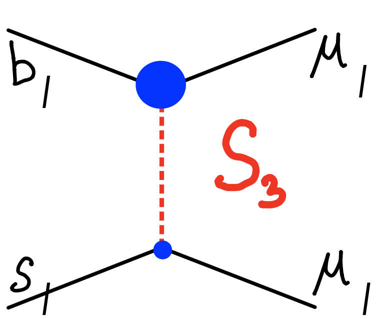
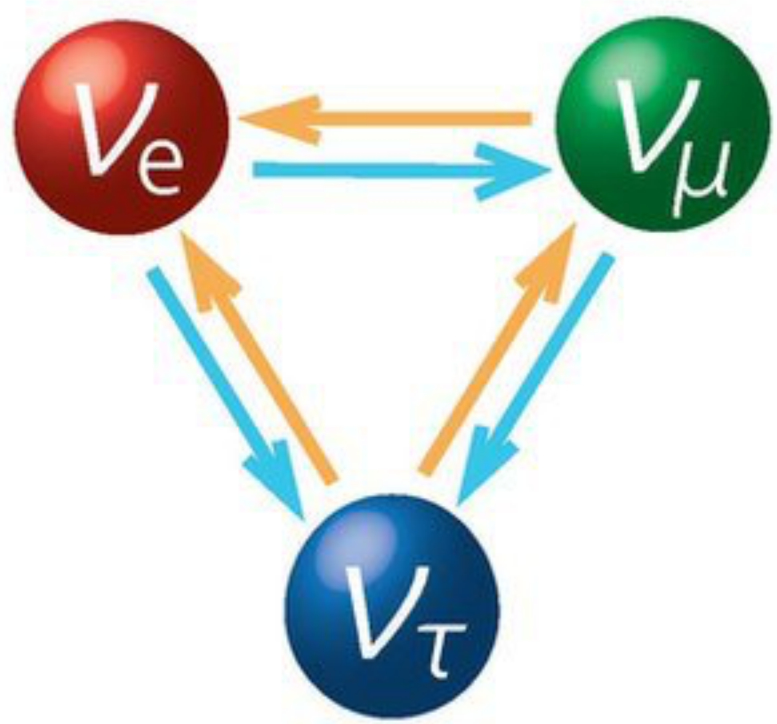
- A good fit to both $(g-2)_\mu$ and $b \to s\mu^+\mu^-$ for $m_{LQ} \sim \mathrm{TeV}$ (see backup)





Neutrino Masses

- ***LFV is observed in nature***! The PMNS is full of $\mathcal{O}(1)$ elements.
- The correct neutrino masses and mixings dictate the $U(1)_X$ breaking!



Neutrino Masses

- We introduce two $U(1)_X$-breaking scalar fields:

$\phi_{e\tau}$	$6m-2n$
ϕ_μ	$6m+n$

Type-I seesaw mechanism

$$Y_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix} \quad ; \quad \mathcal{L} \supset \bar{\nu}_{\rm R}^{i\,c} \nu_{\rm R}^{j} (\xi^{ij}_{e\tau} \phi_{e\tau} + \xi^{ij}_{\mu} \phi_\mu) \implies \frac{M_\nu}{v_X} \sim \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}$$

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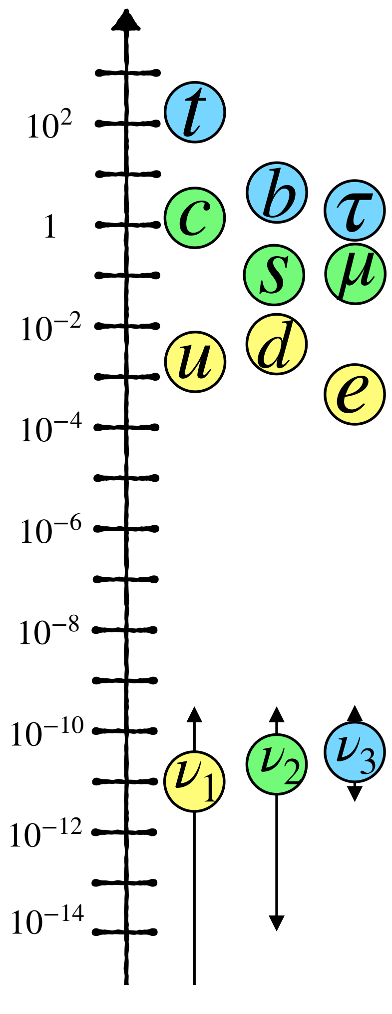
- This is enough to accommodate for:
  - *Neutrino oscillations data,*
  - *The Planck limit on the sum of neutrino masses,*
  - *The absence of neutrinoless double beta decay.*

  See e.g. [Asai 1907.04042]

Neutrino Masses



$U(1)_X$ *breaking at the high scale?*

The mass gap is explained if $\langle \phi_{e\tau} \rangle, \langle \phi_\mu \rangle \gg \langle H \rangle$ + the high-scale leptogenesis

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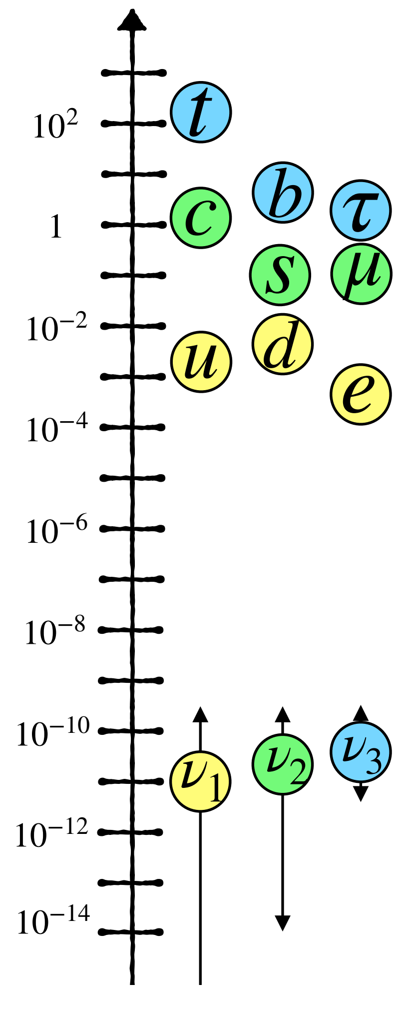

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BUT

In the $U(1)_X$ broken phase one can *naively* write renormalisable terms qqS^* and $q_i\ell_jS$ that violate $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$

- What about the TeV scale LQs?
- Is there proton decay? cLFV?

THE IR: DISCRETE GAUGE SYMMETRY

- The $U(1)_X$ gauge symmetry is broken by the VEVs of $\phi_{e\tau}$ and ϕ_μ, whose charges are uniquely fixed so as to produce *a dense* neutrino Majorana mass matrix.

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$$k = \gcd\left([\phi_{e\tau}]_X\,,\ [\phi_\mu]_X\right) \qquad \left[e^{i\frac{2\pi}{k}[\phi]_X}\phi = \phi\right] \qquad \Gamma \ \cong\ \mathbb{Z}_k$$

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Davighi, AG, Thomsen; 2202.05275

$$\boxed{\begin{aligned}&(m,\, n) = \big(3a + r,\, 9b + 3r\big), \quad \text{for} \quad r \in \{1,\, 2\},\\ &(a,\, b) \in \mathbb{Z}^2,\ \text{and}\ \gcd\big(3a + r,\, b - a\big) = 1\,.\end{aligned}}$$

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$$k = 9\,\mathrm{gcd}\big(2,\, b+r\big)$$

$$\Gamma \cong \begin{cases} \mathbb{Z}_9, & \text{for } b+r \in 2\mathbb{Z}+1 \\ \mathbb{Z}_{18}, & \text{for } b+r \in 2\mathbb{Z} \end{cases}$$

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THE IR: DISCRETE GAUGE SYMMETRY

- Charges

$b+r \pmod 2$	Γ	ℓ	q	S	$qS\ell$	qS^*q
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Tabele 1: Charges under the remnant discrete symmetry Γ

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Tabele 1: Charges under the remnant discrete symmetry Γ

- Stable proton

- The protection of baryon number by Γ goes beyond just banning the diquark operators.

$$\Delta B = 0 \pmod 3$$

Exact proton stability to all orders in the SMEFT!

✗ $qqq\ell$

- Other baryon number–violating processes are in principle possible through ΔB = 3 operators allowing for sphalerons.

Accidental lepton flavor conservation

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$$\frac{1}{\Lambda^2}\phi_{e\tau}\phi_\mu^*\, qS_{1/3}\ell_{1,3} \qquad \frac{1}{\Lambda^2}\phi_{e\tau}\phi_\mu^*\, uS_1 e_{1,3}$$

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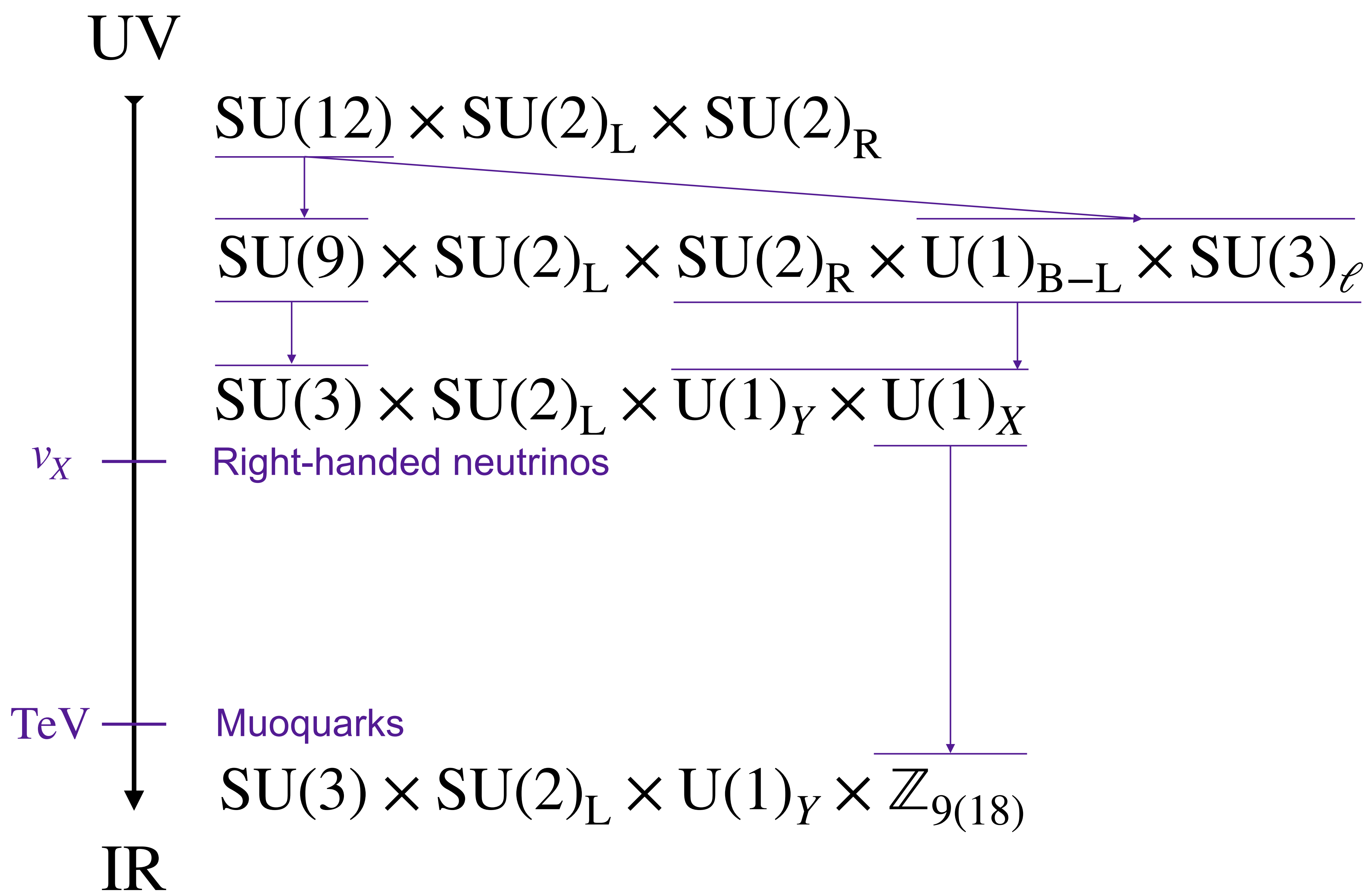
- The limits on $\mu \to e\gamma$ suggest $v_X/\Lambda \lesssim 0.01$. A modest scale separation is sufficient to suppress LFV processes to a level compatible with current bounds.

DEEPER INTO THE UV: UNIFICATION



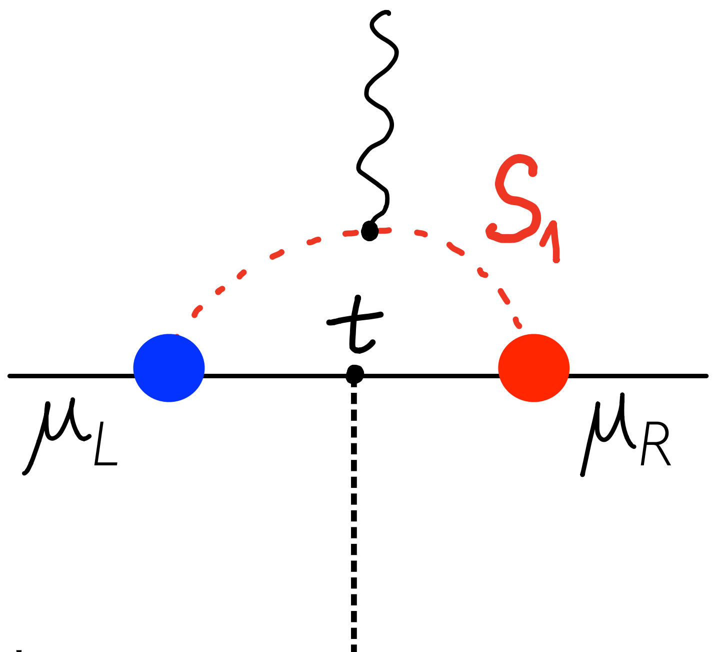
Tentative gauge–flavour unification scenario.

Conclusions

- Lepton flavour non-universal $U(1)_X$ extensions of the SM provide an elegant framework to address the flavour anomalies — explaining, in particular, why the SM accidental symmetries work so well.
- A novel mechanism to restrict TeV-scale leptoquark interactions and render the proton exactly stable to all orders in the effective field theory expansion — while explaining the neutrino masses and mixings.
- The $U(1)_X$ can emerge from a gauge–flavour unified theory at even higher energies.

Backup

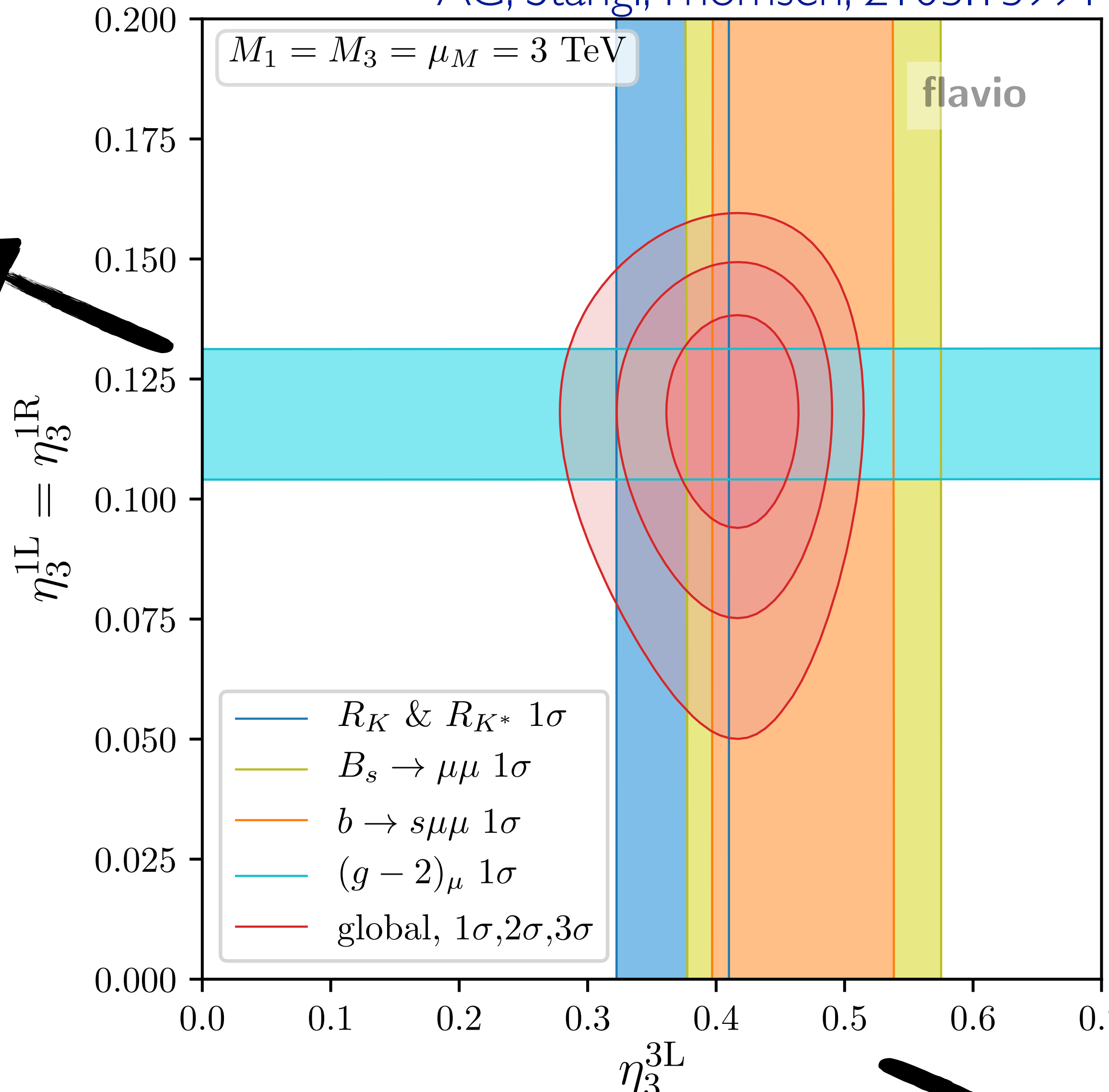
$U(1)_{B-3L_\mu}$ ***example***



* m_t/m_μ enhancement

Queiroz, Shepherd; 1403.2309,
Dorsner, Fajfer, AG, Kamenik, Kosnik; 1603.04993,
Coluccio Leskow, Crivellin, D'Ambrosio, Müller; 1612.06858
Dorsner, Fajfer, Sumensari; 1910.03877
Gherardi, Marzocca, Venturini; 2008.09548
+ many more

AG, Stangl, Thomsen, 2103.13991



$$\eta_i^{3L} = (V_{td},\, V_{ts},\, 1)\,\eta_3^{3L}$$

* V-A structure

Hiller, Schmaltz, 1408.1627,
Dorsner, Fajfer, AG, Kamenik, Kosnik; 1603.04993,
Buttazzo, AG, Isidori, Marzocca; 1706.07808,
Gherardi, Marzocca, Venturini; 2008.09548
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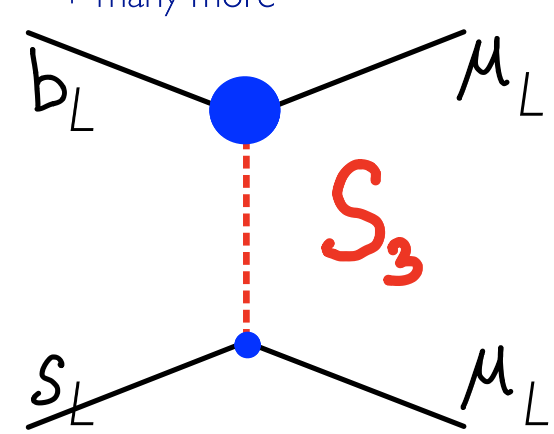

- One-loop matching to SMEFT from 2003.12525
- 399 observables in **smelli** 1810.07698
- EW and flavor opservables, LFV, LFU, magnetic moments, neutral meson mixing, semileptonic and rare B, D, K decays, etc.

The $U(1)_X$ atlas

- $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_X$ gauge group

- Chiral fermions:

$$Q_i \sim (\mathbf{3}, \mathbf{2}, \tfrac{1}{6}, X_{Q_i}), \qquad U_i \sim (\mathbf{3}, \mathbf{1}, \tfrac{2}{3}, X_{U_i}), \qquad D_i \sim (\mathbf{3}, \mathbf{1}, -\tfrac{1}{3}, X_{D_i}),$$
$$L_i \sim (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}, X_{L_i}), \qquad E_i \sim (\mathbf{1}, \mathbf{1}, -1, X_{E_i}), \qquad N_i \sim (\mathbf{1}, \mathbf{1}, 0, X_{N_i})$$

Left-handed $\longleftrightarrow$ Right-handed

- The symmetry breaking scalar fields:

$$H = (\mathbf{1}, \mathbf{2}, \frac{1}{2}, X_H) \ , \qquad \phi = (\mathbf{1}, \mathbf{1}, 0, X_\phi)$$

- Without loss of generality $X_H = 0$

* By field redefinitions, shifting $X_f \to X_f - aY_f$ for all fields, gives an equivalent theory.

AG, Soreq, Stangl, Thomsen, Zupan; 2107.07518

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$$L_i \sim (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}, X_{L_i}), \quad E_i \sim (\mathbf{1}, \mathbf{1}, -1, X_{E_i}), \quad N_i \sim (\mathbf{1}, \mathbf{1}, 0, X_{N_i})$$

Anomaly cancelation conditions:

$$\begin{aligned}
\mathrm{SU}(3)_C^2 \times \mathrm{U}(1)_X :&\quad \sum_{i=1}^{3}(2X_{Q_i} - X_{U_i} - X_{D_i}) = 0 \ , \\
\mathrm{SU}(2)_L^2 \times \mathrm{U}(1)_X :&\quad \sum_{i=1}^{3}(3X_{Q_i} + X_{L_i}) = 0 \ , \\
\mathrm{U}(1)_Y^2 \times \mathrm{U}(1)_X :&\quad \sum_{i=1}^{3}(X_{Q_i} + 3X_{L_i} - 8X_{U_i} - 2X_{D_i} - 6X_{E_i}) = 0 \ , \\
\mathrm{Gravity}^2 \times \mathrm{U}(1)_X :&\quad \sum_{i=1}^{3}(6X_{Q_i} + 2X_{L_i} - 3X_{U_i} - 3X_{D_i} - X_{E_i} - X_{N_i}) = 0 \ , \\
\mathrm{U}(1)_Y \times \mathrm{U}(1)_X^2 :&\quad \sum_{i=1}^{3}(X_{Q_i}^2 - X_{L_i}^2 - 2X_{U_i}^2 + X_{D_i}^2 + X_{E_i}^2) = 0 \ , \\
\mathrm{U}(1)_X^3 :&\quad \sum_{i=1}^{3}(6X_{Q_i}^3 + 2X_{L_i}^3 - 3X_{U_i}^3 - 3X_{D_i}^3 - X_{E_i}^3 - X_{N_i}^3) = 0 \ .
\end{aligned}$$

- Unification => Rational charges. Rescale g_X => Integer charges.

$-10 \leq X_{F_i} \leq 10$ => 21'546'920 inequivalent solutions (i.e. up to flavor permutation, etc)

*to be explored

Allanach, Davighi, Melville; 1812.04602

The $U(1)_X$ atlas

Quark flavor universal

$-10 \leq X_{F_i} \leq 10$

- $Y^{u,d}$ are allowed => $X_{Q_i} = X_{U_j} = X_{D_k}$ $(X_H = 0)$ [276 inequivalent solutions]

- Muoquark requirement

  eg. S_3 LQ: $X_{L_2} \neq \{X_{L_{1,3}}, -3X_q\}$ [273 inequivalent solutions]

  Y^e allowed => $\textbf{vector category}: \quad X_{L_i} = X_{E_i}$ [252 inequivalent solutions]

  $\textbf{chiral category}: \quad \text{the rest.}$ [21 inequivalent solutions]

AG, Soreq, Stangl, Thomsen, Zupan; 2107.07518

The $U(1)_X$ atlas

Third-family-quark

- The "2+1" charge assignment

$$X_{Q_i} = X_{U_j} = X_{D_k} \equiv X_{q12} \quad \text{for all} \quad i, j, k = 1, 2, \quad \text{and}$$
$$X_{Q_3} = X_{U_3} = X_{D_3} \equiv X_{q3} \ . \quad (X_H = 0)$$

- The CKM elements (V_{td}, V_{ts}) at dim-5:

$$\mathcal{L} \supset \frac{x_i^u}{\Lambda}\overline{Q}_i \tilde{H}\phi U_3 + \frac{x_i^d}{\Lambda}\overline{Q}_i H\phi D_3 \ + \ \text{H.c.}$$

- The ACC conditions are satisfied provided

$$2X_{q12} + X_{q3} = 3X_q$$

*The quark flavor-universal solutions can immediately be extended to the 2 + 1 case.

- The muoquark conditions slightly change: $X_{q12} = 0$

eg. S_3 LQ: $X_{L_2} \neq \{X_{L_{1,3}}, X_{L_{1,3}} - X_{q3}, -X_{q3}, -2X_{q3}, -3X_{q3}\}$ [171 inequivalent sol.]

$-10 \leq X_{F_i} \leq 10$

AG, Soreq, Stangl, Thomsen, Zupan; 2107.07518