La Thuile, 7-12th March 2022

The Branco Weiss Fellowship Society in Science

New ideas on the hierarchy problem

Tevong You

CERN

UNIVERSITY OF CAMBRIDGE

Outline

- **Motivation**
- **Cosmological Relaxation**
- **Cosmological Self-Organised Criticality**
- **Cosmological Censorship**
- **Conclusion**

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Introduction

- Empirical fact **(1):** descriptions of nature are **self-contained** within their respective scales

- Empirical fact **(2):** as we go to **smaller scales**, these descriptions *unify more and more* into increasingly tightly knit, **rigid frameworks**

- *This did not have to be so*, but appears to be **how nature is organised**

- The **hierarchy problem** is a *deep conflict* between (1) and (2): if the Higgs emerges from a more fundamental theory, as expected from (2), then it violates (1) if the scale of new physics is too high

Effective Field Theory

$$\mathcal{L} = \Lambda^4 + \Lambda^2\mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

- Incredibly successful
- Explains many features of our theories
- **Natural expectations** for sizes of parameters
- *Sound reasoning*, vindicated many times in the past

- However: **hierarchy problem** and **cosmological constant** *defy EFT logic*

The Hierarchy Problem

- Hierarchy problem *is still a problem*: $(m_h)^2_{tree} + (m_h)^2_{radiative} = (m_h)^2_v$

$$\delta m_\phi^2 \propto m_{\text{heavy}}^2, \quad \delta m_\psi \propto m_\psi \log\left(\frac{m_{\text{heavy}}}{\mu}\right)$$

[If Higgs mass is *calculable* in underlying UV theory]

Historical precedent

- Earliest example of an unnatural, **arbitrary** feature of a fundamental theory:

$$m_{\text{inertial}} = q_{\text{gravity}}$$

- Classical electromagnetism **fine-tuning**:

$$(m_e c^2)_{\text{obs}} = (m_e c^2)_{\text{bare}} + \Delta E_{\text{coulomb}}, \qquad \Delta E_{\text{coulomb}} = \frac{e^2}{4\pi\epsilon_0 r_e}$$

- Pions, GIM mechanism, etc.

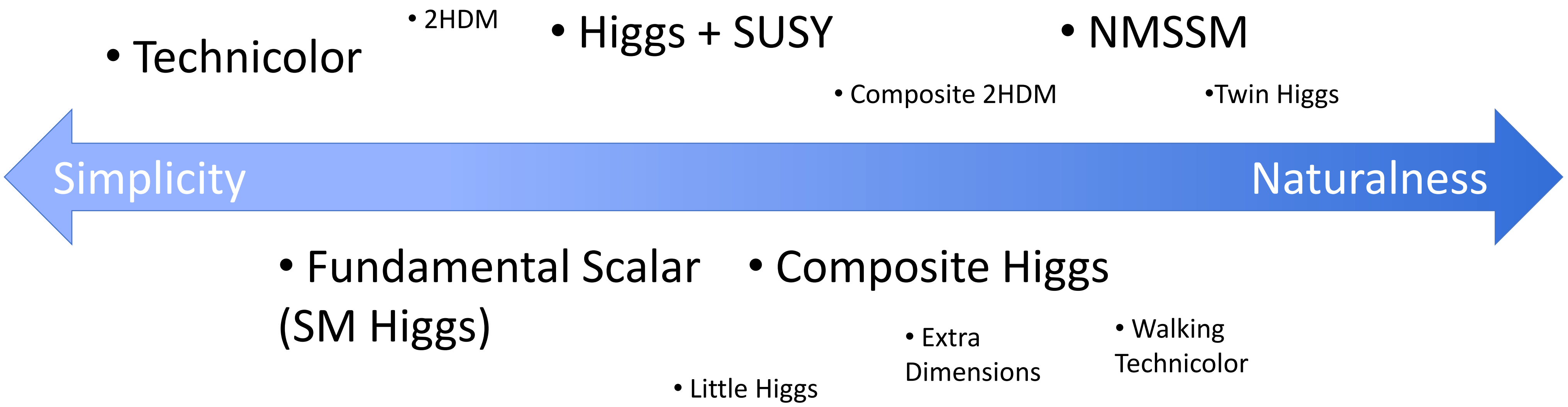
- Higgs? *Expect new physics close to weak scale*

Understanding the origin of EWSB

- The SM has many *arbitrary* features put in by hand which hint at **underlying structure**
    - *Pattern of Yukawa couplings, CKM*
    - *QCD Theta term*
    - *Neutrino mass*
    - *Higgs potential*
    - *…*
- Maybe it just is what it is ¯_(ツ)_/¯
- but we would like a **deeper understanding** i.e. an *explanation* for why things are the way they are
    - *e.g. PQ axion for Theta term, see-saw for neutrino mass, Froggat-Nielsen for Yukawas…*
- In SM, **no understanding** of Higgs sector: Higgs potential and couplings *put in by hand and unexplained*
- We feel there must be some underlying system that **explains the origin of EWSB**
- In any such theory *in which the Higgs potential is calculable*, there is a **UV sensitivity** to the Higgs mass (*that is no longer a free parameter*) which requires fine-tuned cancellations

Natural electroweak symmetry breaking?

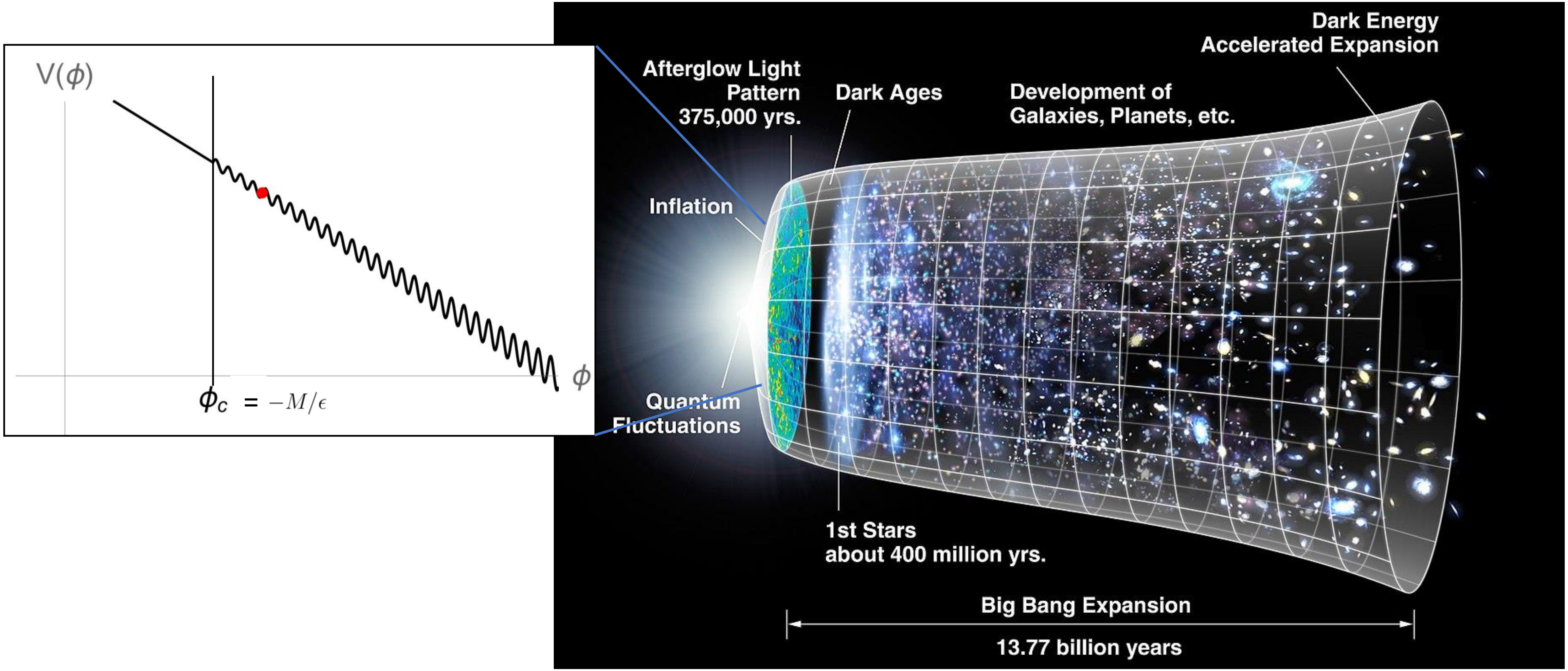
- A priori **many ways** to break electroweak symmetry



- **Tension** between *simplicity* and *naturalness*
- Driven by **lack of new physics** at weak scale
- How to reconcile this with naturalness?

Natural electroweak symmetry breaking?

- **Cosmological evolution** may play a key role!



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Cosmological relaxation

P. W. Graham, D. E. Kaplan and S. Rajendran, [arXiv:1504.07551]

L. F. Abbott, Phys. Lett. B 150 (1985) 427

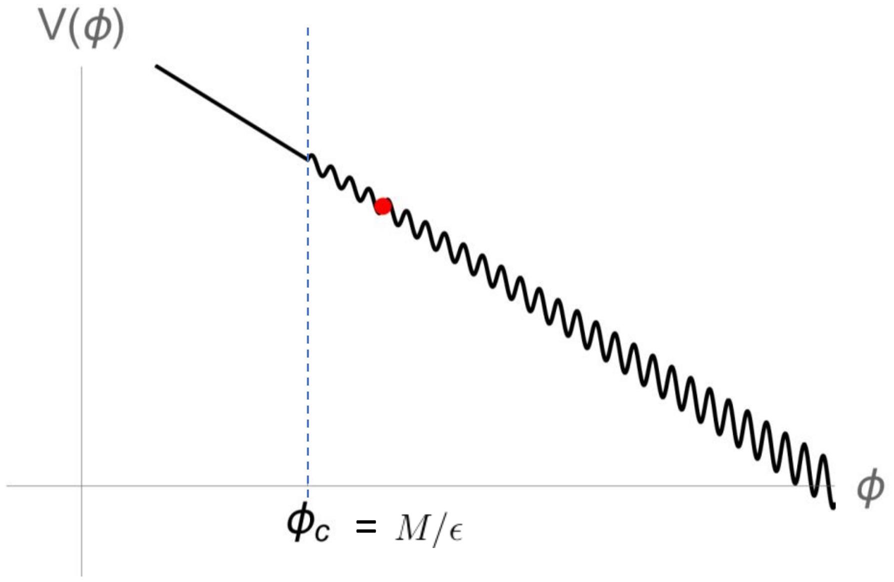
- Assume Higgs mass is naturally large at cut-off M

$$\mathcal{L} \supset (M^2 + \epsilon M\phi)|h|^2 + \epsilon M^3\phi + \ldots + \Lambda_p^{4-n} v^n \cos\left(\frac{\phi}{f_p}\right)$$

- Higgs quadratic term scanned by axion-like field ϕ during inflation

- ϕ protected by shift symmetry, explicitly broken by small parameter ε

- Backreaction when $< h > \sim v$ stops ϕ evolution at small electroweak scale v



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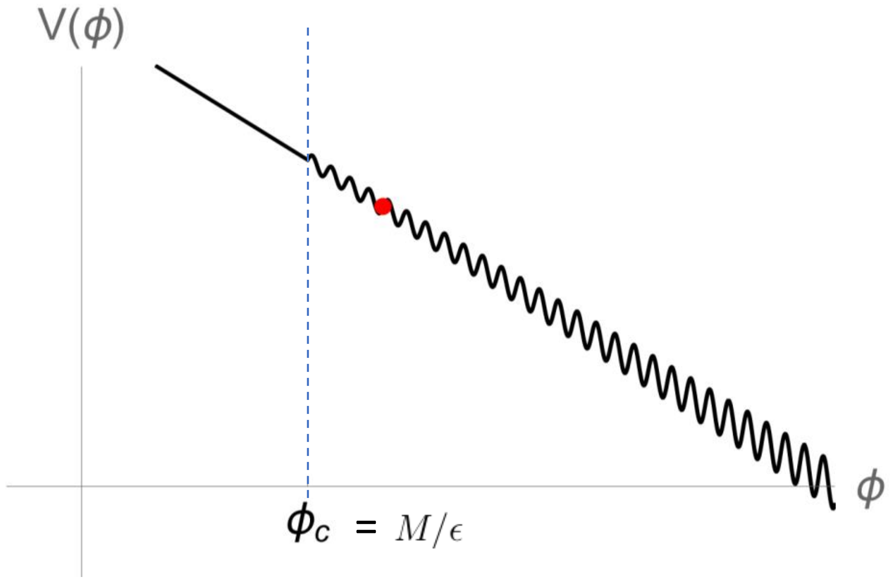

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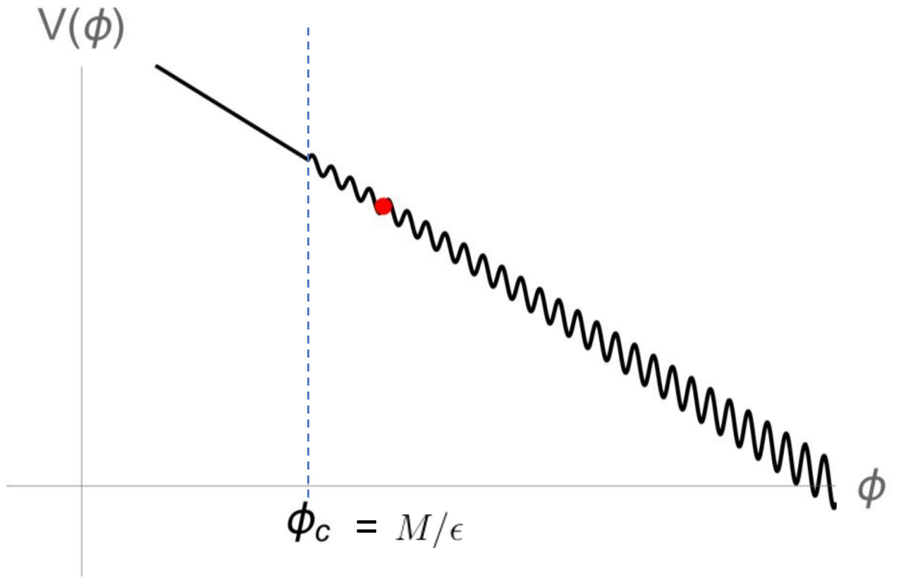

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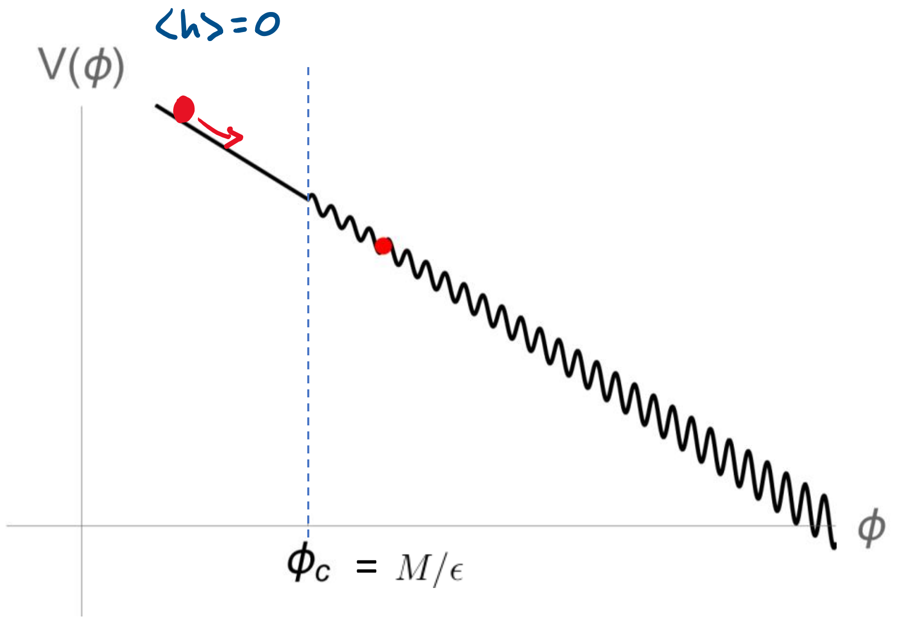

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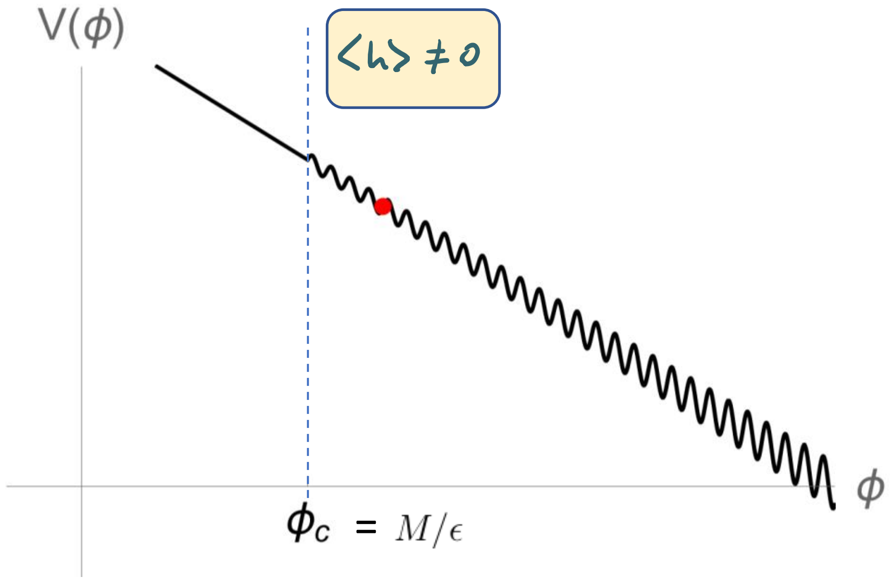

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Cosmological relaxatio

Constraints: H < v, classical rolling vs quantum, inflaton energy density dominates relaxion, etc.

Very small ε and natural scanning range lead to super-planckian field excursions, exponential e-foldings…

- Assume Higgs mass is naturally l

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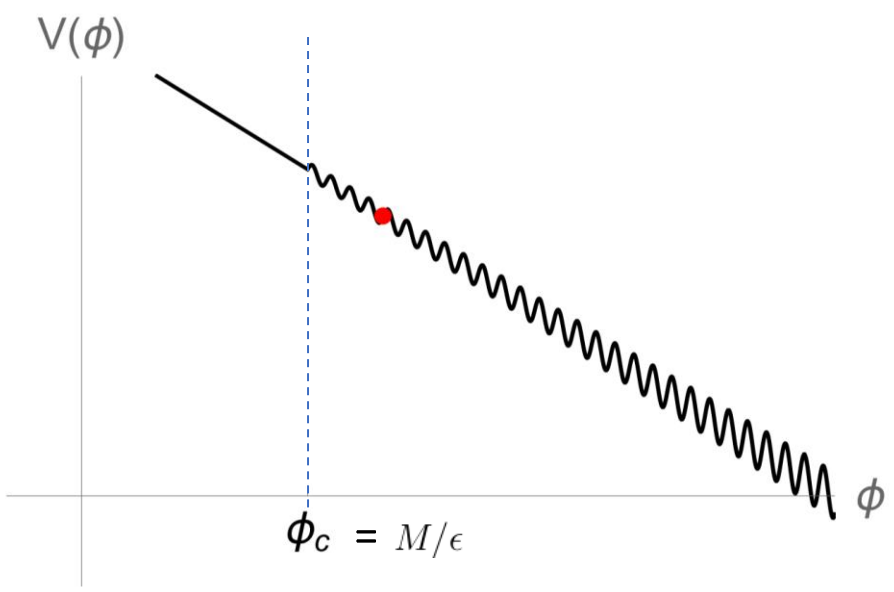

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Cosmological relaxation

$$\mathcal{L} \supset (M^2 + \epsilon M\phi)|h|^2 + \epsilon M^3\phi + ... + \boxed{\Lambda_p^{4-n} v^n} \cos\left(\frac{\phi}{f_p}\right)$$

- **n=1** models Graham et al [1504.07551]
  - Confining gauge group G=QCD: Need additional ingredients to overcome **strong-CP problem**
  - New gauge group G: new physics at weak scale + **coincidence problem**

- **n=2** models Espinosa et al [1506.09217]
  - G can be at higher scales, raises M cut-off too
  - **Requires second scalar** to relax relaxion barriers: double-scanning mechanism

- **n=0** models Hook and Marques-Tavares [1607.01786], **TY** [1701.09167]
  - **More promising**, make use of axial gauge coupling
  - Connection to **dark photons** Domcke, Schmitz, **TY** [2108.11295]

$$\mathcal{L} = \frac{1}{32\pi^2}\frac{a}{f}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}G_{\mu\nu}G_{\rho\sigma}$$

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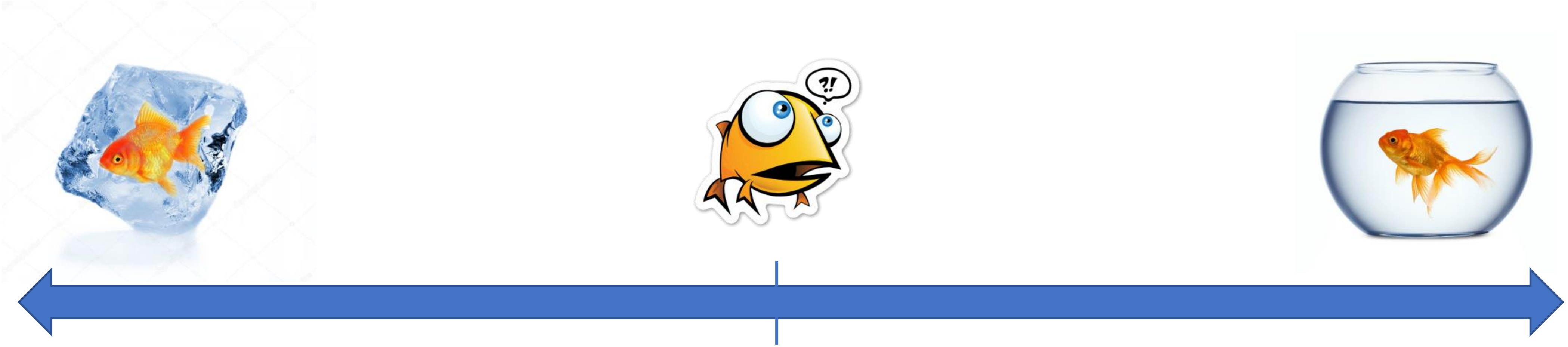
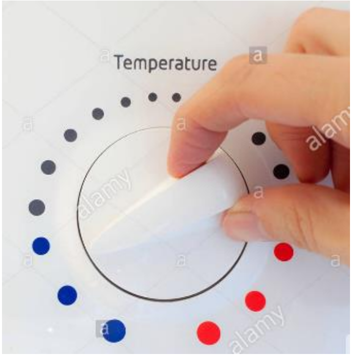
Critical points

- To be at the **critical point** of a classical phase transition **requires tuning**



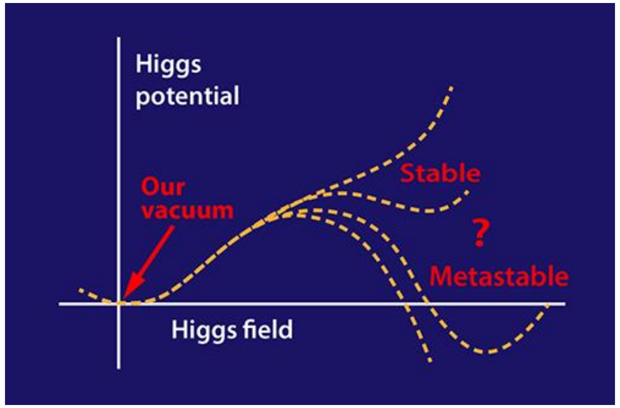


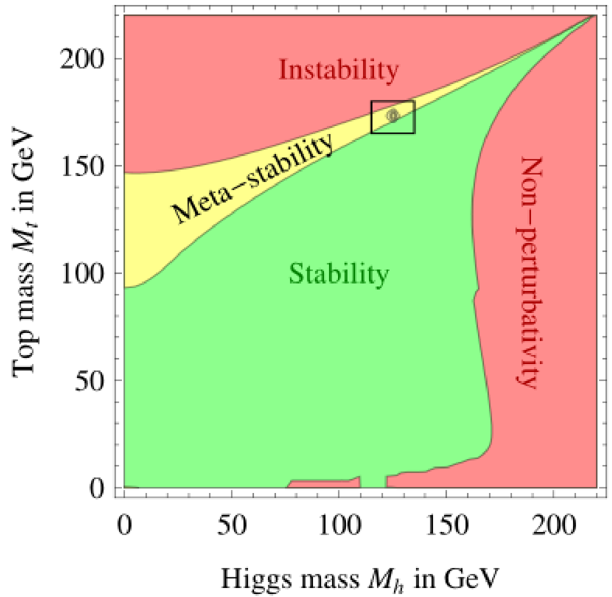
- Living **near criticality** is highly **non-generic**!

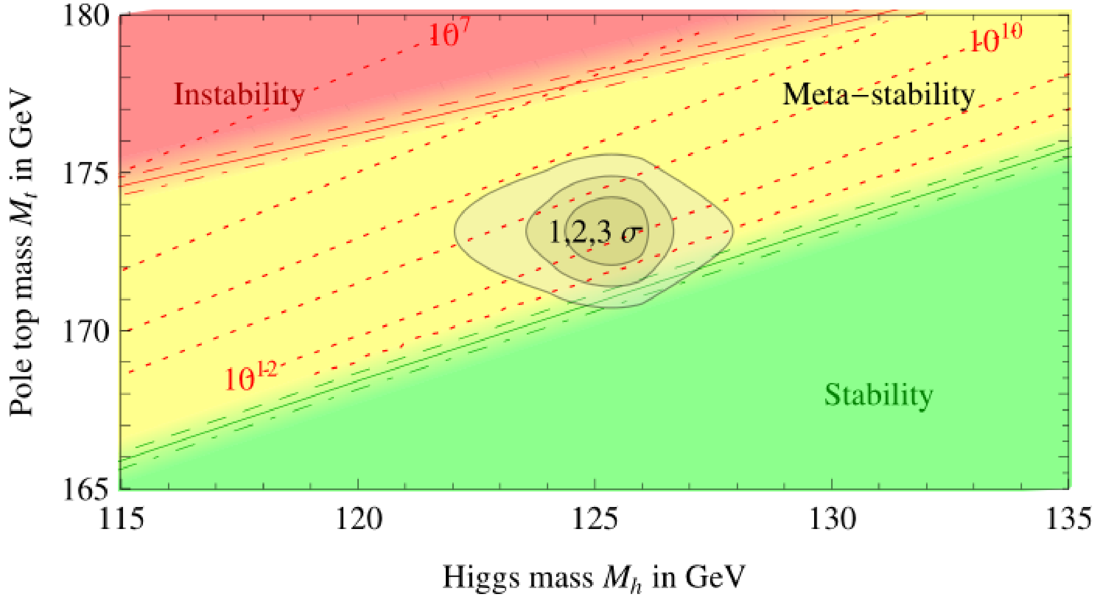
3 hints for **near-criticality** of our Universe

- 1) Higgs potential **metastability** in SM

1205.6497 Degrassi et al







- Living on critical boundary of **two phases coexisting**

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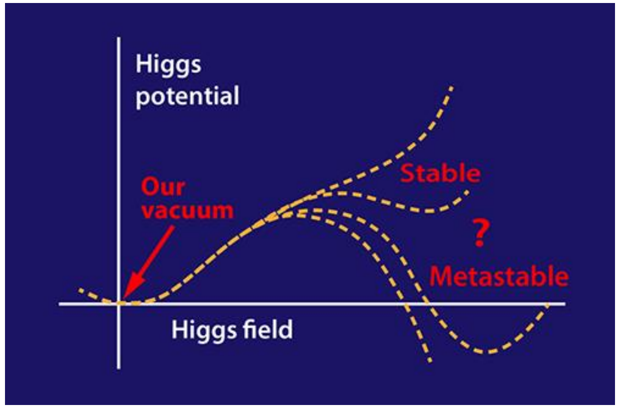

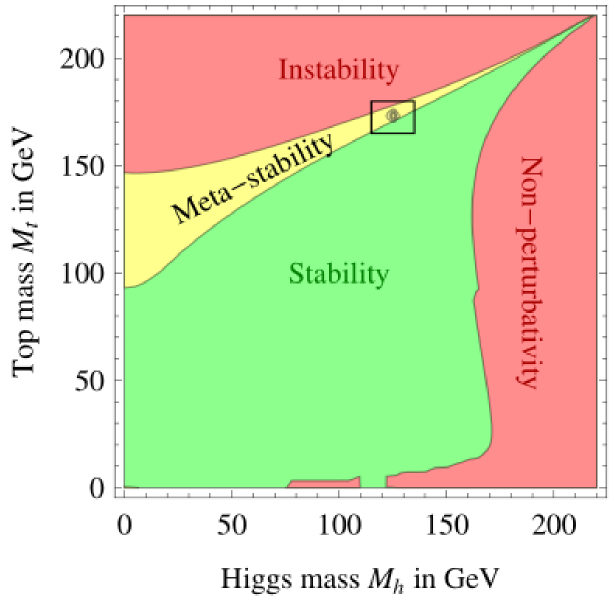

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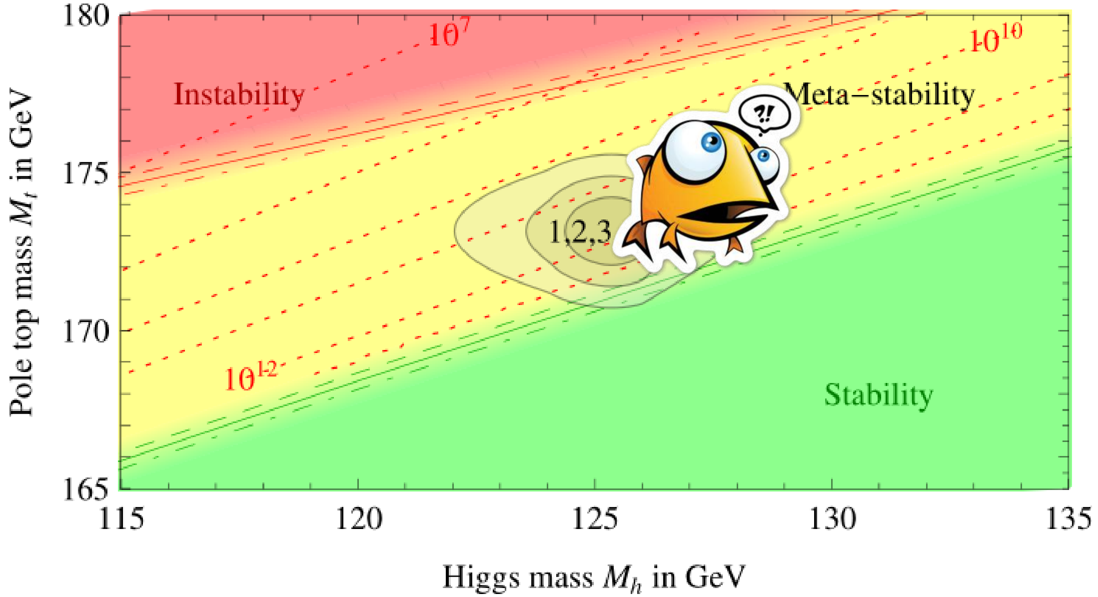

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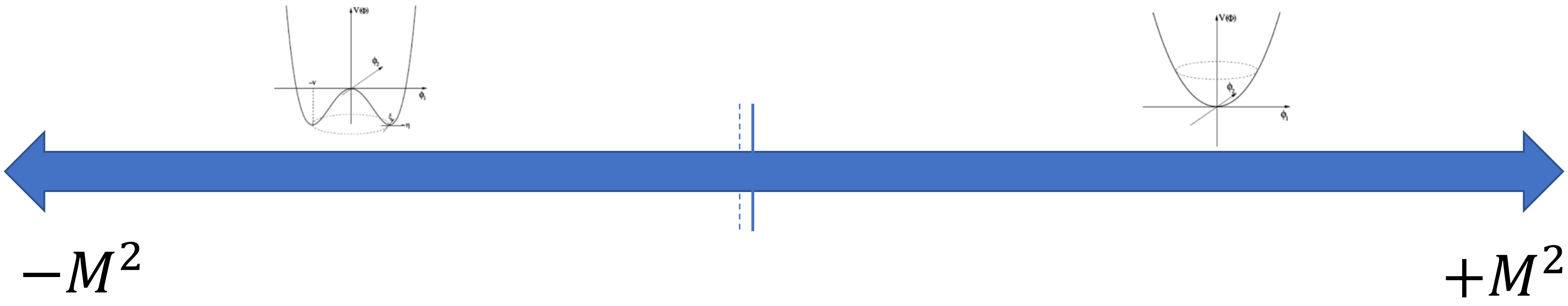
- 2) **Higgs mass**



- Tuned close to boundary between **ordered** and **disordered** phase

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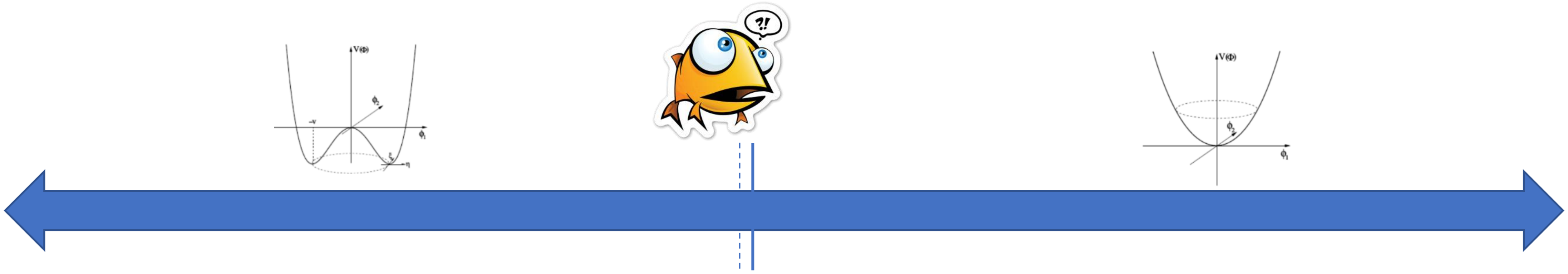
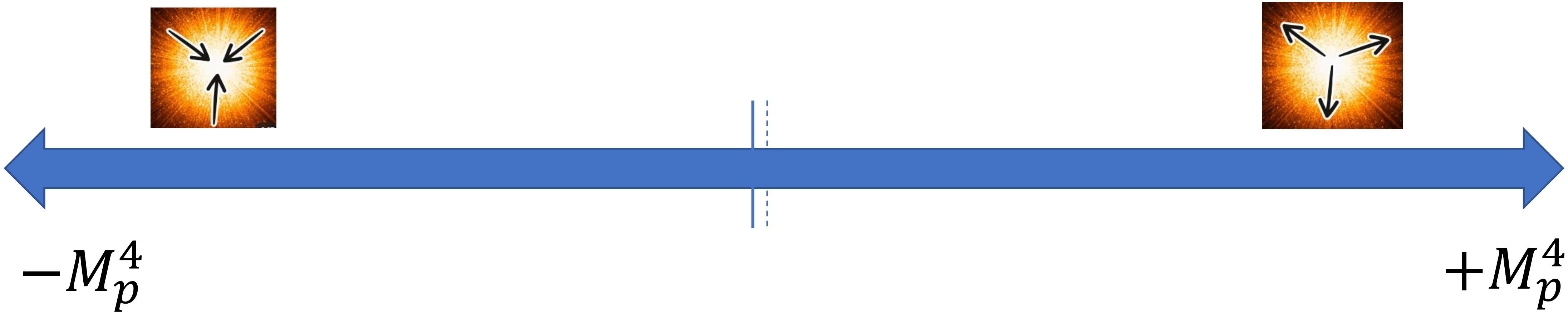


$-M^2$ $+M^2$

- Tuned close to boundary between **ordered** and **disordered** phase

3 hints for **near-criticality** of our Universe

- 3) **Cosmological constant**



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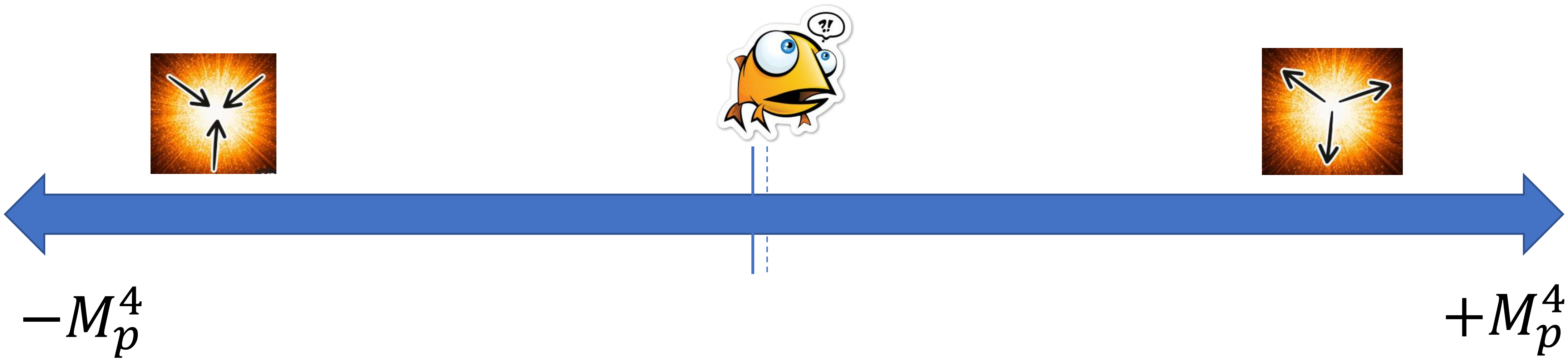
3 hints for **near-criticality** of our Universe

- 3) **Cosmological constant**



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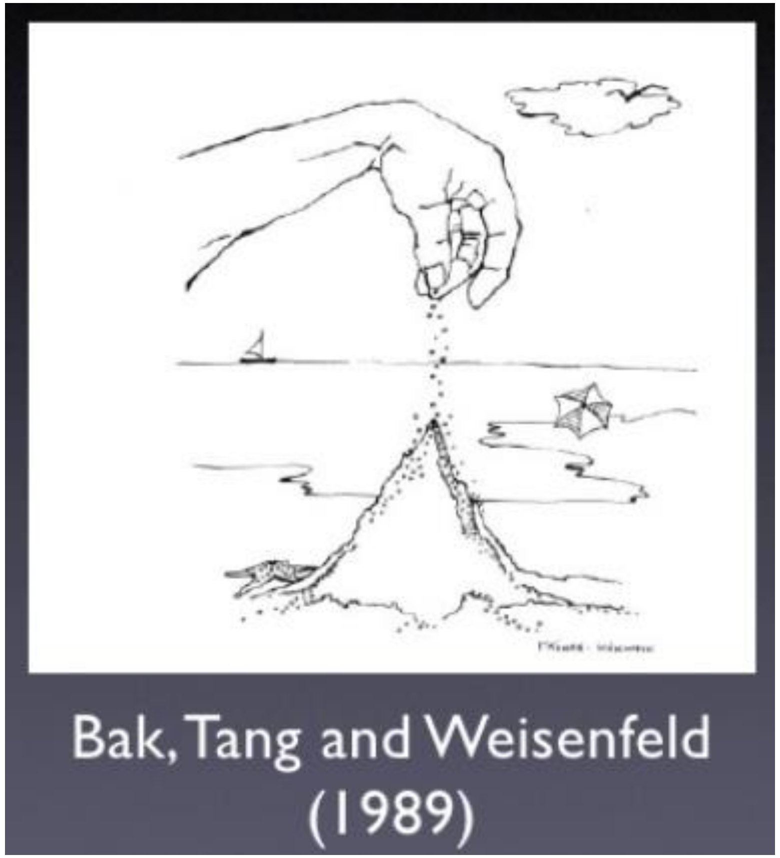
3 hints for **near-criticality** of our Universe

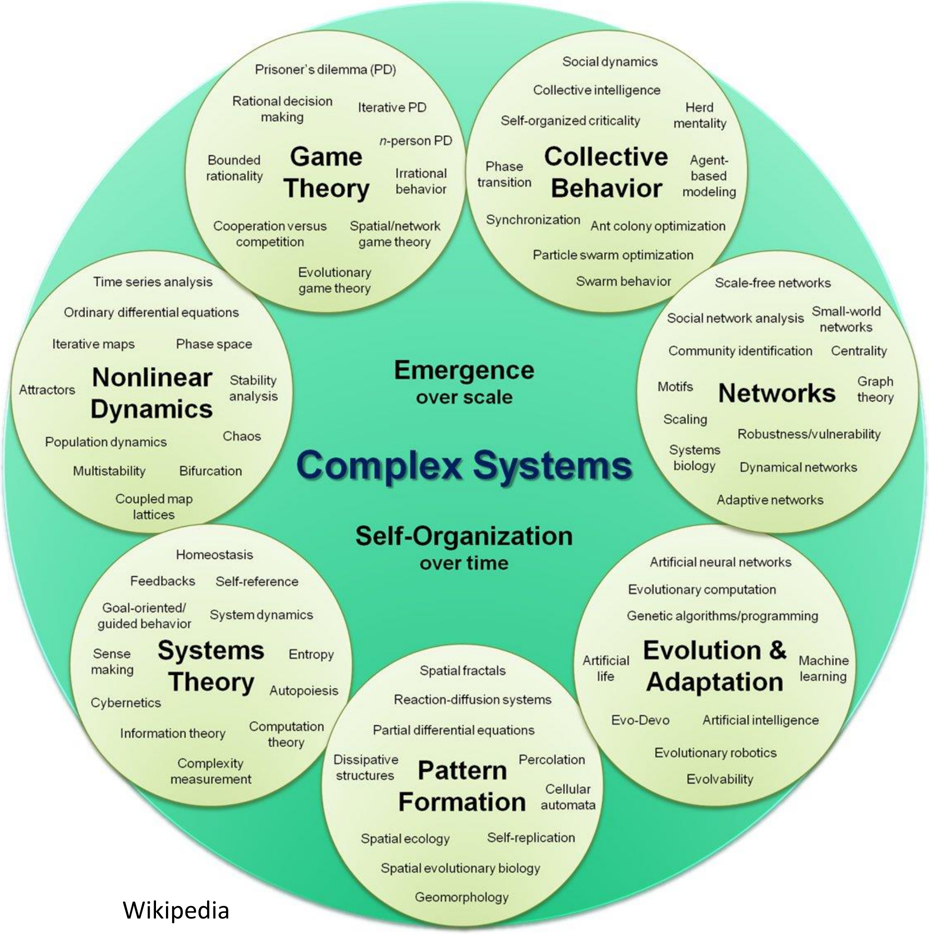
- Why do we appear to live at a **special point** close to criticality?

- Hints of a **new principle** *beyond EFT expectations* at play?

Self-Organised Criticality

- Many systems in nature **self-tuned** to live near criticality





Wikipedia

Self-Organised Criticality

- **Fundamental** self-organised criticality in our universe?

- Need a **mechanism** for self-organisation of **fundamental parameters** determined by scalar fields

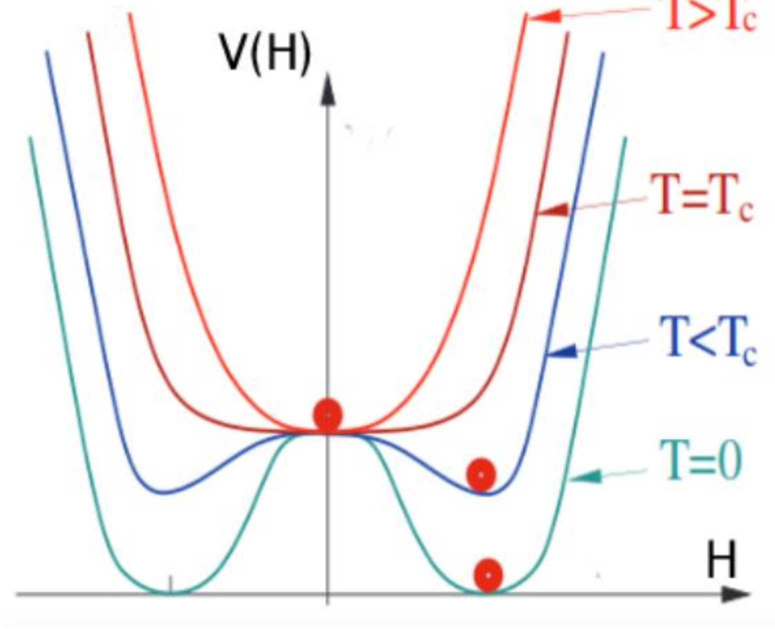
  e.g. Self-Organized Criticality in eternal inflation landscape: J. Khoury et al [1907.07693, 1912.06706, 2003.12594]

- **Self-Organised Localisation (SOL):**
  - cosmological **quantum phase transitions** localise fluctuating scalar fields during inflation at critical points

  Giudice, McCullough, TY [2105.08617]

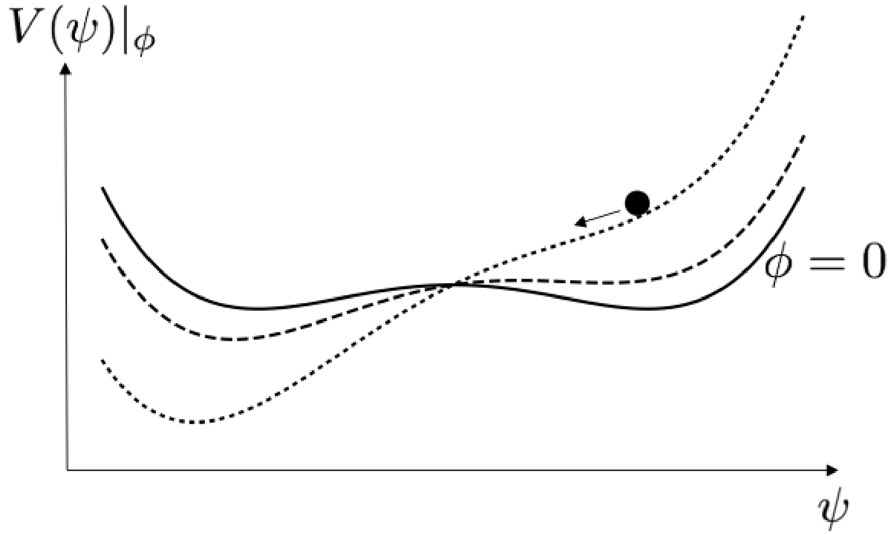
Phase Transitions (PT)

- **Classical** PT: varying background temperature

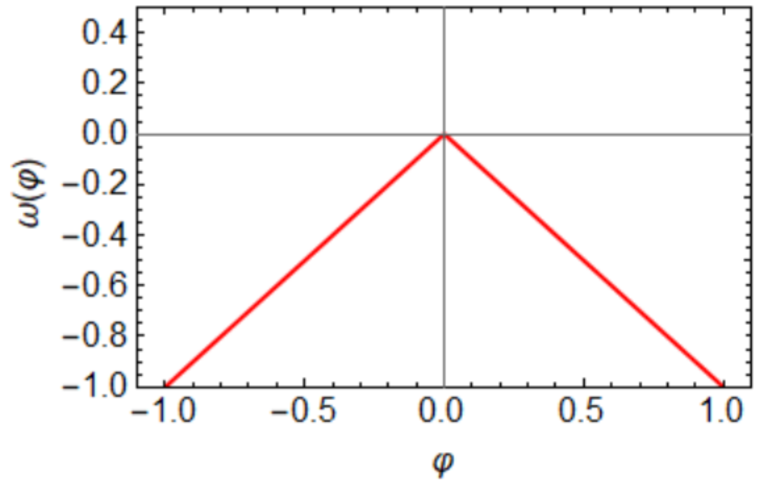


- **Quantum** PT: varying background field

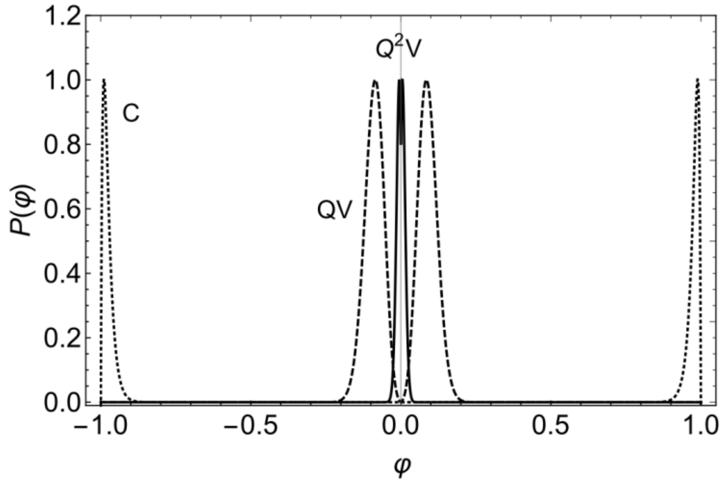
$$V = \frac{\lambda}{4}\left(\psi^2 - \rho^2\right)^2 + \kappa\phi\psi$$



Toy example



- ϕ triggers 1st order **quantum phase transition** of another field at ϕ_c
- Distribution of ϕ values **peaked at critical point**:



Of all the possible parameter values being scanned by ϕ, the value at the critical point is the attractor!

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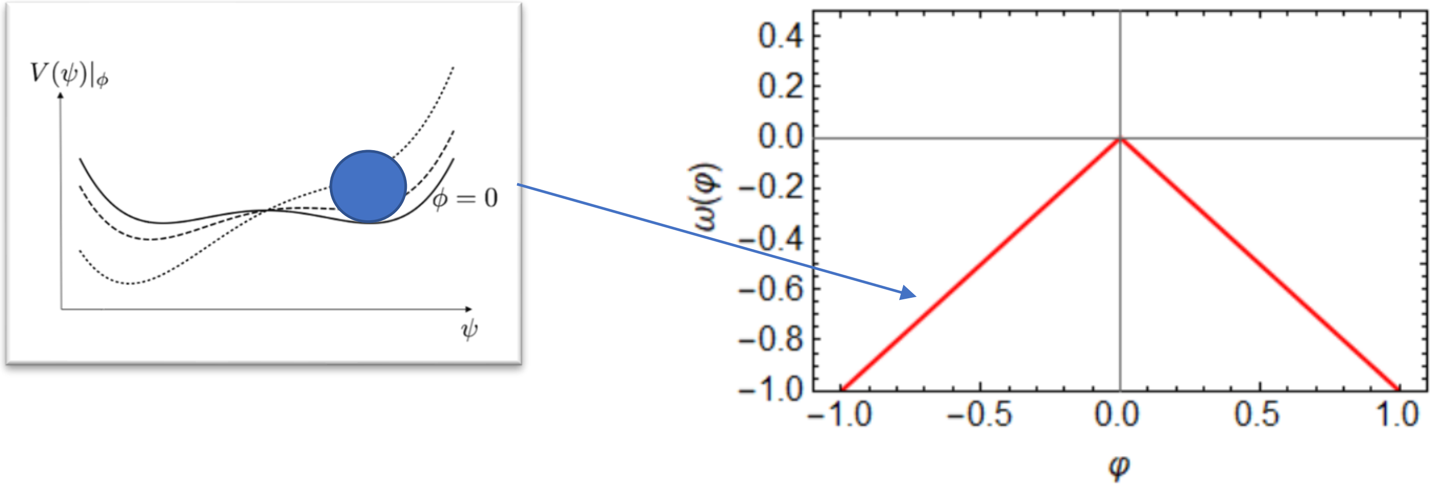

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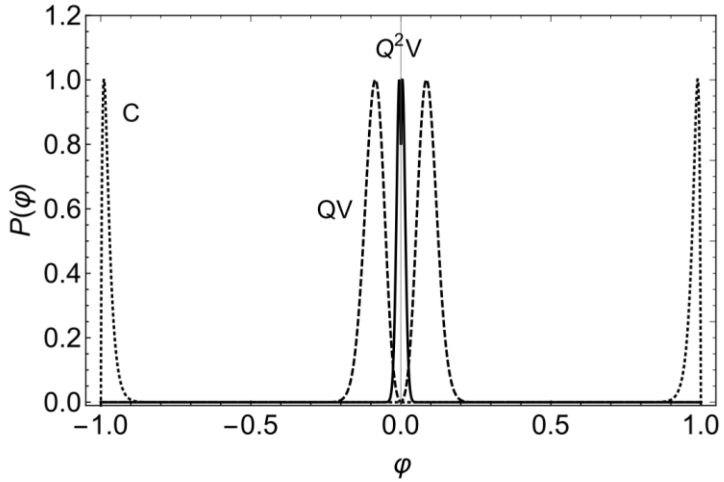

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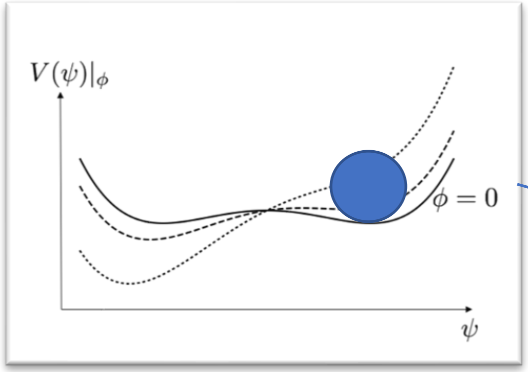

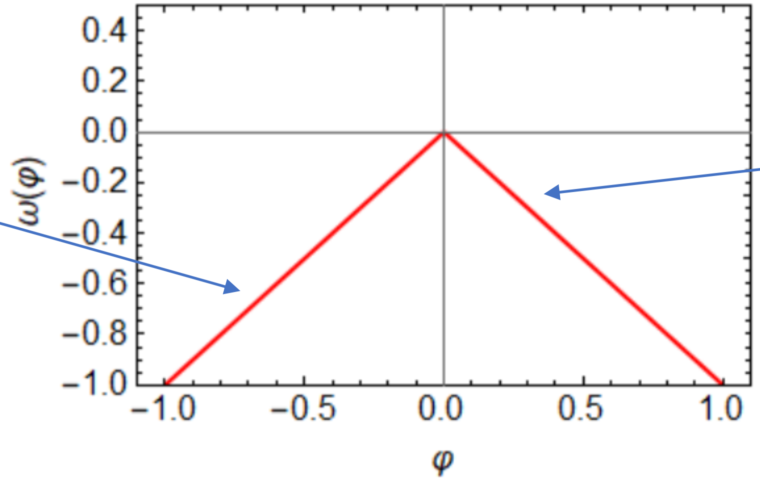

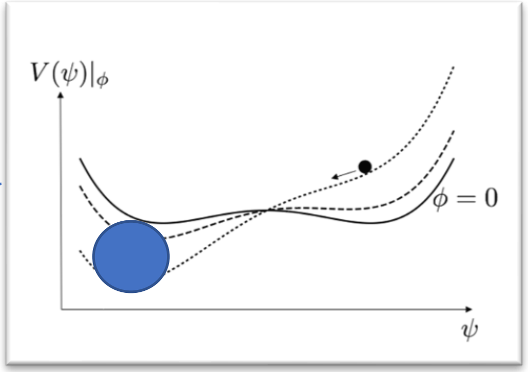

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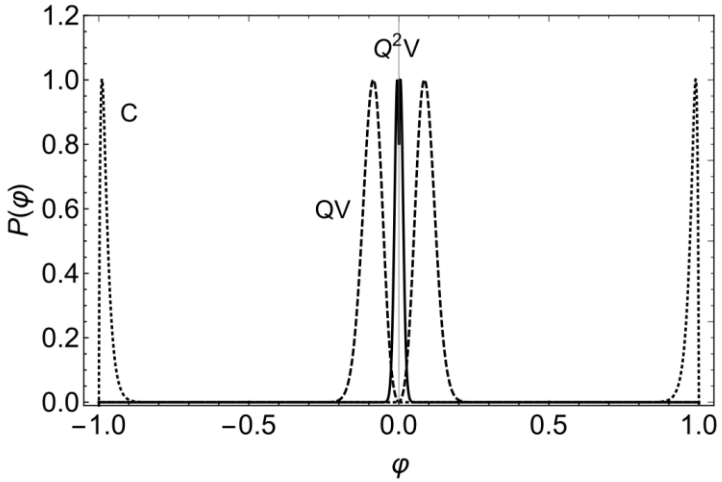

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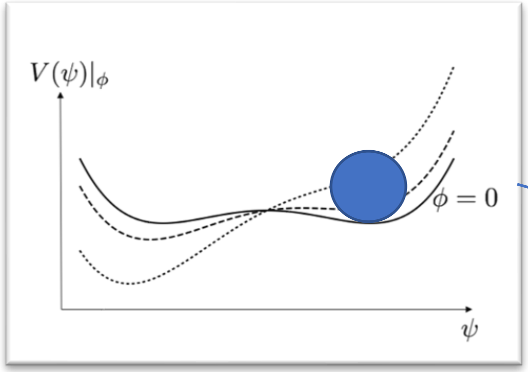

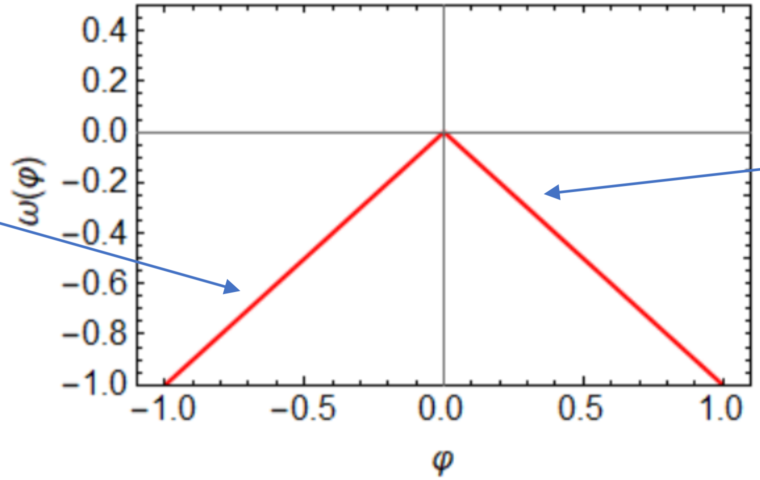

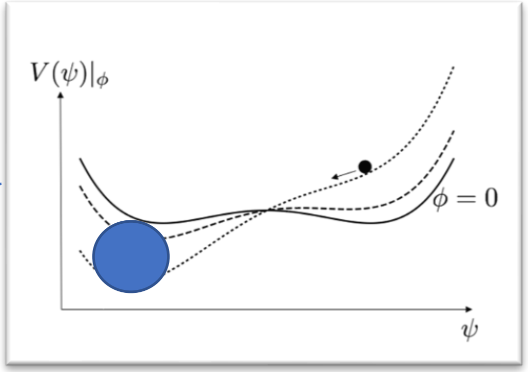

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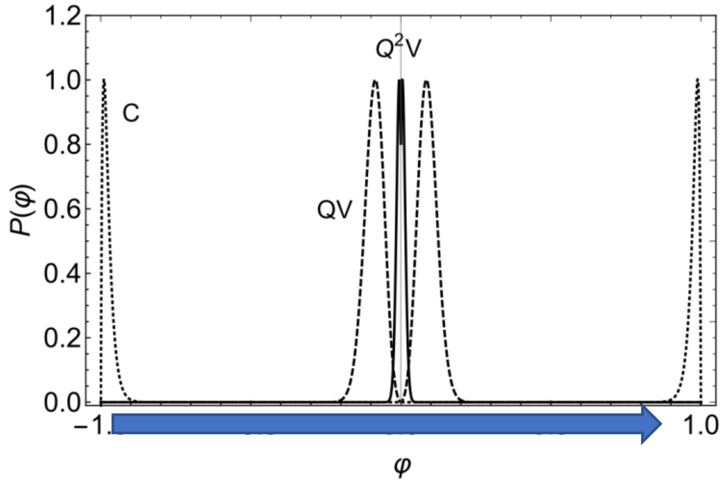

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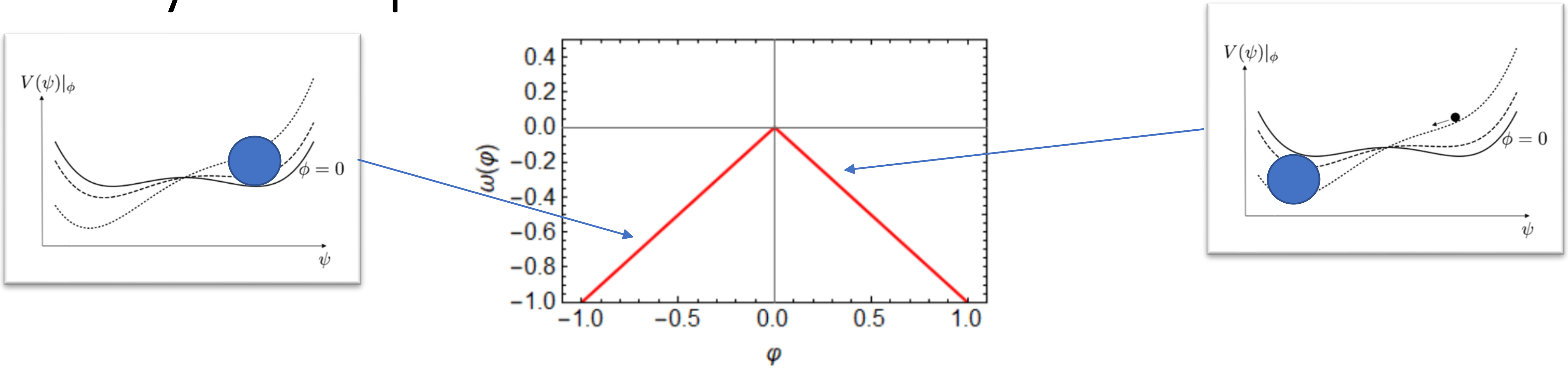

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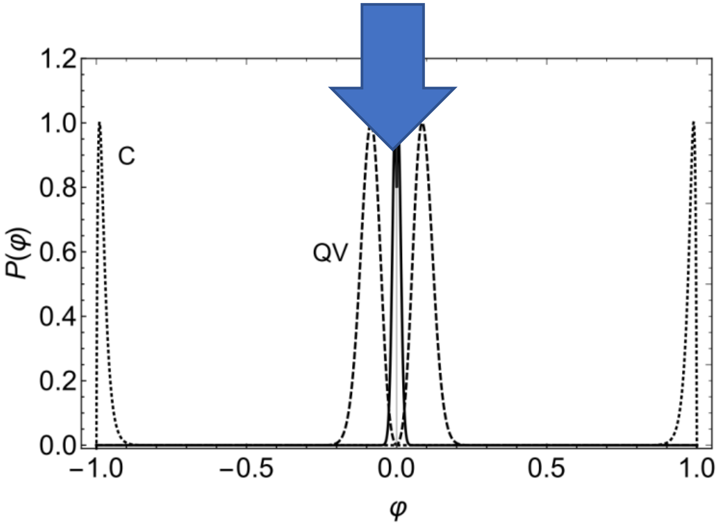

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Cosmological Censorship

- Weinberg's **anthropic argument**: censor all cosmological constant values leading to expansion rate *incompatible with life*

- **Sliding naturalness**: censor all parameter values leading to **vacuum crunch** *incompatible with life*

  D'Agnolo, Teresi [2106.04591, 2109.13249]

- **N-naturalness**: censor all Higgs mass values **too large to reheat** universe

  Arkani-Hamed et al [1607.06821]

- Many more ideas, cosmological and non-cosmological…

[Apologies for incomplete references]

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Conclusion

- Naturalness is an **aesthetic argument** but an *important piece of the puzzle*

- **Symmetry-based solutions** don't seem to be enough

- Keep an open mind for **new principles** e.g. cosmological dynamics, landscape selection rules, UV-IR mixing…

- **Exciting times**—may be analogous to early 20th century revolution

Conclusion

- **1900**: Almost all data agree spectacularly with the fundamental framework of the time, *no reason to doubt its universal applicability or completeness*.

- **1920s**: A combination of **precision measurements** (Mercury), **aesthetic arguments** (relativity) supported by **null experimental results** (Michelson-Morley), and **theoretical inconsistencies** (Rayleigh-Jeans UV catastrophe) lead to an overhaul of the fundamental picture at **smaller scales** and **higher energies** after *pushing the frontiers of technology and theory into new regimes.*

Conclusion

- **2020**: Almost all data agree spectacularly with the fundamental framework of the time, *no reason to doubt its universal applicability or completeness*.

- **2050s**: A combination of **precision measurements** (B mesons, Hubble), **aesthetic arguments** (naturalness) supported by **null experimental results** (LHC), and **theoretical inconsistencies** (black hole information paradox) lead to an overhaul of the fundamental picture at **smaller scales** and **higher energies** after *pushing the frontiers of technology and theory into new regimes*.

Backup

Outline

- Motivation
  - EFT
  - Criticality
  - Quantum phase transitions (QPT)
- **Fokker-Planck Volume (FPV) equation**
  - FPV dynamics
- FPV + QPT = SOL
  - Discontinuity
  - Flux conservation
- SOL solutions
  - Metastability
  - Higgs mass
  - Cosmological constant
- Conclusion
  - Measure problem

Fokker-Planck Volume (**FPV**) equation

- **Langevin equation**: classical slow-roll + Hubble quantum fluctuations

$$\phi(t+\Delta t) = \phi(t) - \frac{V'}{3H}\Delta t + \eta_{\Delta t}(t)$$

- Volume-averaged Langevin trajectories: **FPV for volume distribution** $P(\phi, t)$

$$\frac{\partial}{\partial \phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3 P)}{\partial \phi} + \frac{V' P}{3H}\right] + 3HP = \frac{\partial P}{\partial t} \qquad H(\phi) = \sqrt{\frac{V(\phi)}{3M_p^2}}$$

Quantum diffusion term

Classical drift term

Volume term

Fokker-Planck Volume (**FPV**) equation

- **Langevin equation**: classical slow-roll + Hubble quantum fluctuations

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- Volume-averaged Langevin trajectories: **FPV for volume distribution** $P(\phi, t)$

$$\frac{\partial}{\partial \phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^{2+\xi}P)}{\partial \phi} + \frac{V' P}{3H^{2-\xi}}\right] + 3H^{\xi}P = H_0^{\xi-1}\frac{\partial P}{\partial t_{\xi}}$$

- **Ambiguity** in choosing time "gauge" $dt_{\xi}/dt = (H/H_0)^{1-\xi}$

FPV dynamics

- ϕ is *not* the inflaton: **apeiron** field scanning parameters
- Restrict to **EFT** field range f $\quad \varphi \equiv \frac{\phi}{f} \qquad V = 3H_0^2 M_P^2 + g_\epsilon^2 f^4 \omega(\varphi)\ , \qquad \omega(\varphi) = \sum_{n=1}^{\infty} \frac{c_n}{n!}\varphi^n$
- Assume sub-dominant energy density
- Expand around constant inflationary background H_0 $\quad H(\varphi) \simeq H_0 \left(1 + \frac{\epsilon^2 f^4 \omega(\varphi)}{6M_p^2 H_0^2}\right)$

- FPV becomes

$$\frac{\alpha}{2}\frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial(\omega' P)}{\partial \varphi} + \beta\omega P = \frac{\partial P}{\partial T}$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4}\ , \quad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2}\ , \quad T \equiv \frac{t}{t_R}\ , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

Quantum diffusion

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Classical drift

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- **Maximum** number of e-folds for **non-eternal** inflation: $N_{\text{e-folds}} < S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$

FPV dynamics

- **Stationary** FPV distributions $P(\varphi, T) = \sum_{\lambda} e^{\lambda T} p(\varphi, \lambda)$

$$\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)\, p = 0$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

- Largest eigenvalue $\lambda = \lambda_{\max}$ inflates most
- **Eigenvalue** determines **peak location**

- Note: **boundary conditions** necessary input for solution

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Discriminant D>0 for **positive** solution:

$$D = \omega'^2 + 2\alpha(\lambda - \beta\omega - \omega'')$$

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e.g. D=0 at $\varphi = 1$ ➡ $\lambda_{\max} = \beta - \frac{\omega_1'^2}{2\alpha}$

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- Largest eigenvalue $\lambda = \lambda_{\max}$ inflates most
- **Eigenvalue** determines **peak location**

$$\frac{\alpha}{2} p'' + \cancel{\omega' p'} + (\omega'' + \beta\omega - \lambda)\, p = 0 \quad \Rightarrow \quad \lambda = \beta\omega(\bar\varphi) + \omega''(\bar\varphi) - \frac{\alpha}{2\sigma^2}$$

- Note: **boundary conditions** necessary input for solution

FPV dynamics

- **Stationary** FPV distributions $P(\varphi,T)=\sum_{\lambda} e^{\lambda T} p(\varphi,\lambda)$

$$\frac{\alpha}{2}p''+\omega' p'+(\omega''+\beta\omega-\lambda)\,p=0 \Rightarrow$$

Discriminant D>0 for **positive** solution:

$$D=\omega'^2+2\alpha(\lambda-\beta\omega-\omega'')$$

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e.g. D=0 at $\varphi=1$ $\Rightarrow$ $\lambda_{\max}=\beta-\frac{\omega_1'^2}{2\alpha}$

- **Eigenvalue** determines **peak location**

$$\frac{\alpha}{2}p''+\cancel{\omega' p'}+(\omega''+\beta\omega-\lambda)\,p=0 \Rightarrow \lambda=\beta\omega(\bar\varphi)+\omega''(\bar\varphi)-\frac{\alpha}{2\sigma^2} \Rightarrow \omega(\bar\varphi)=1-\frac{\omega_1'^2}{2\alpha\beta}$$

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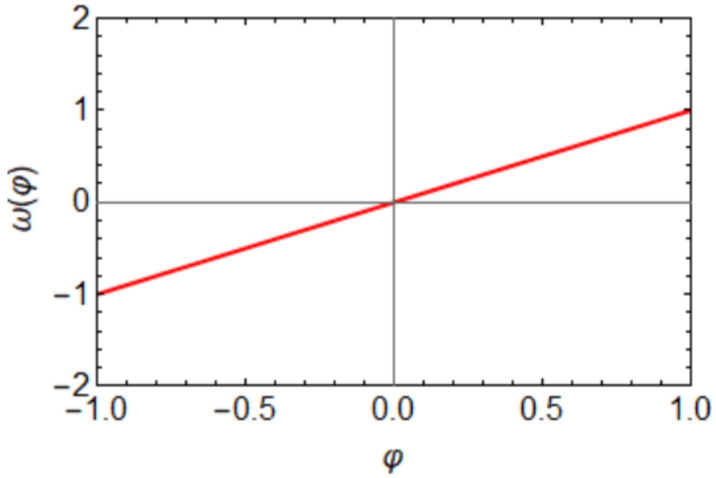

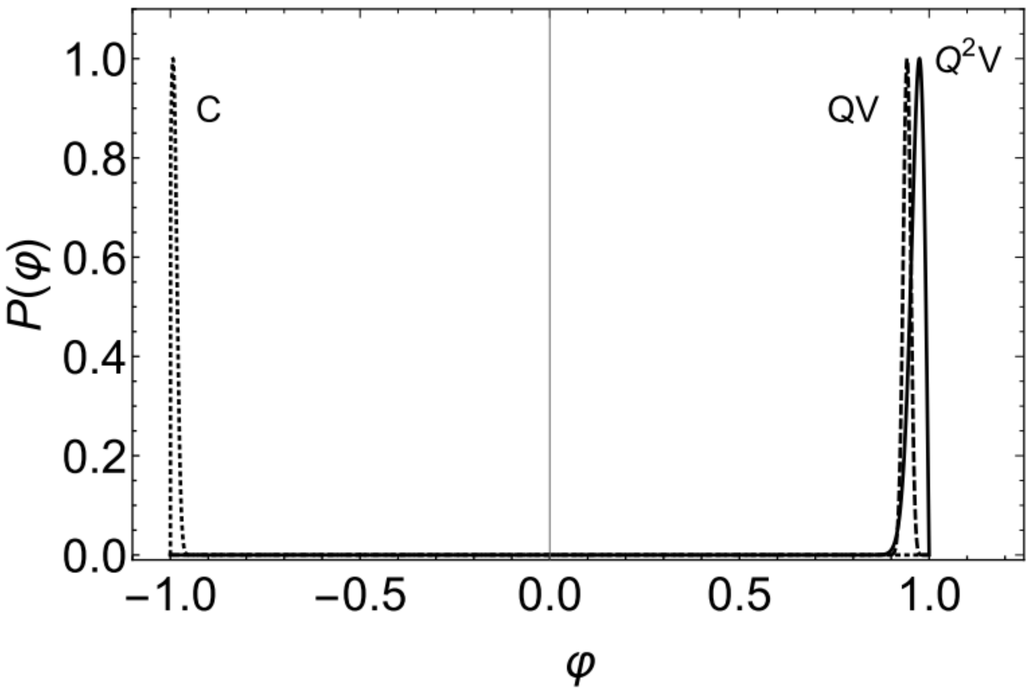

- C regime: $\alpha\beta \ll 1$. Peak is located as far down the potential as allowed by boundary condition.
- QV regime: $\alpha\beta \gg 1$, $\alpha^2\beta \ll 1$. Peak is a distance $1/(\alpha\beta)$ from the top with width $\sigma \simeq 1/\sqrt{\beta}$.
- Q^2V regime: $\alpha^2\beta \gg 1$. Peak as close to the top as possible, with a distance comparable to the width $\sigma \simeq (\alpha/\beta)^{1/3}$.

FPV dynamics

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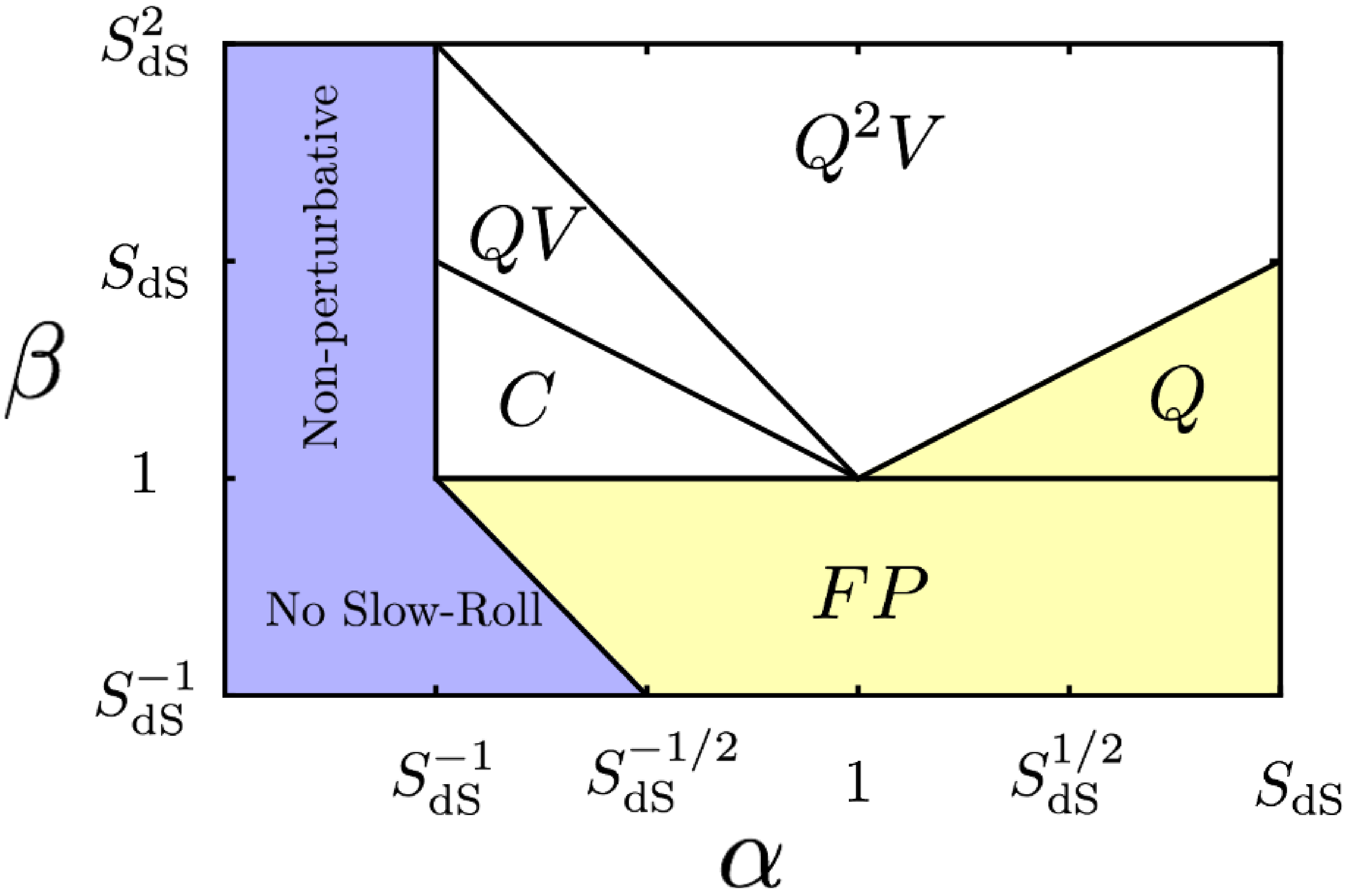

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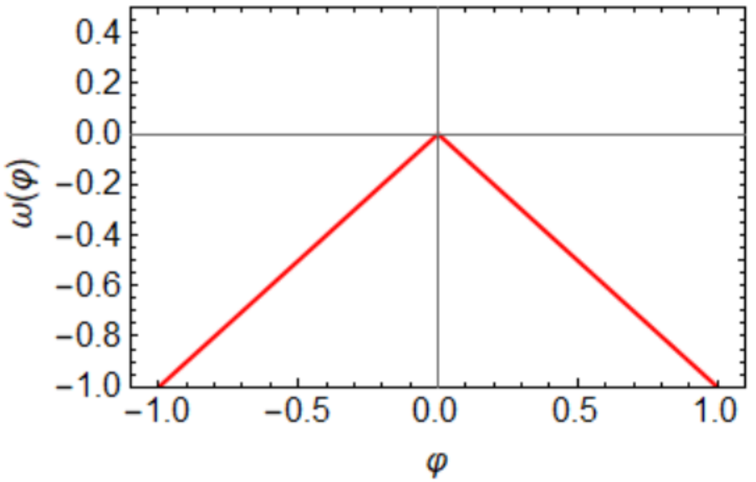
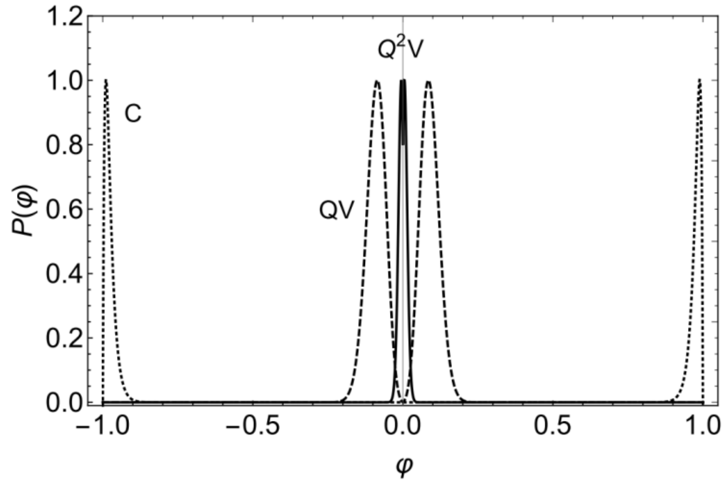
Junction conditions at phase transitions





- ϕ triggers 1st order **quantum phase transition** at ϕ_c
- **Discontinuity** in V' leads to discontinuous P'
- Requiring **continuity of FPV** across the critical point gives a **junction condition** to satisfy

$$\lim_{\epsilon\to 0}\int_{\phi_c-\epsilon}^{\phi_c+\epsilon} d\phi \frac{\partial}{\partial\phi}\left[\frac{V'P}{3H}+\frac{\hbar}{8\pi^2}\frac{\partial}{\partial\phi}\left(H^3P\right)\right]=0 \quad \Longrightarrow \quad \frac{\Delta P'}{P(\varphi_c)}=-\frac{2\Delta\omega'}{\alpha}$$

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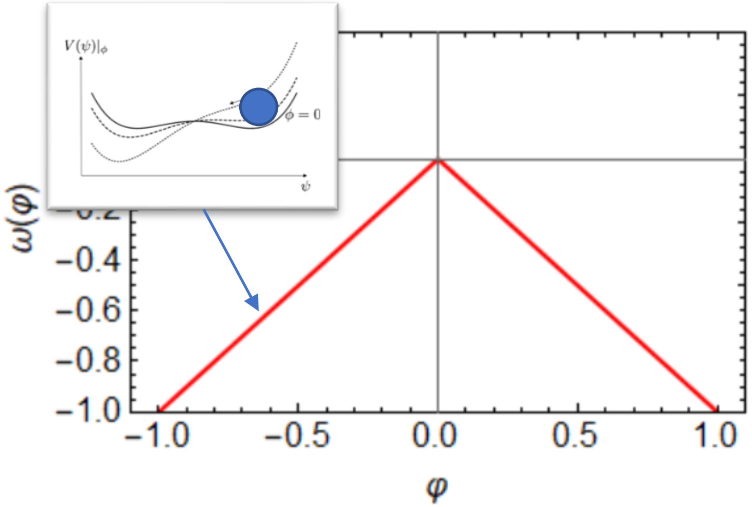

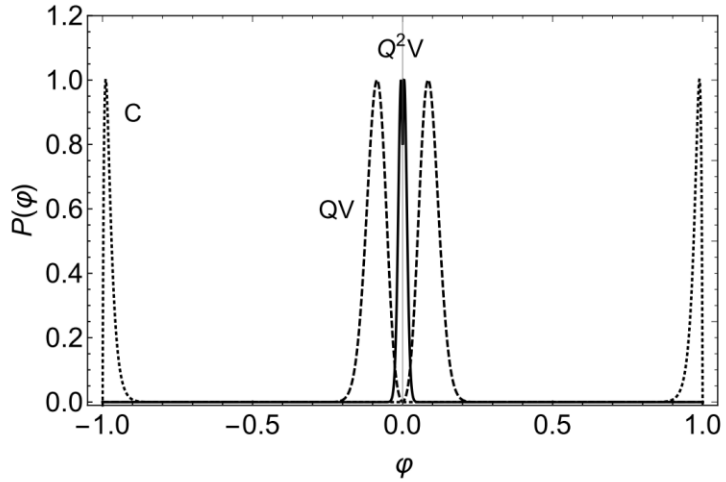

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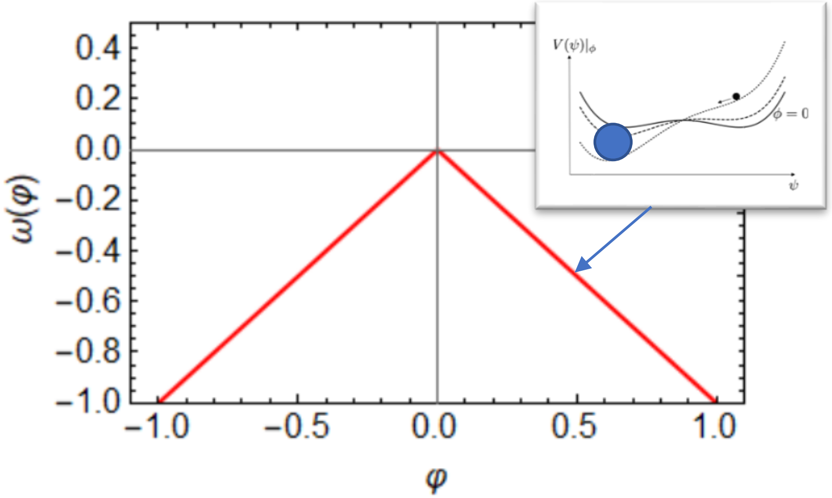

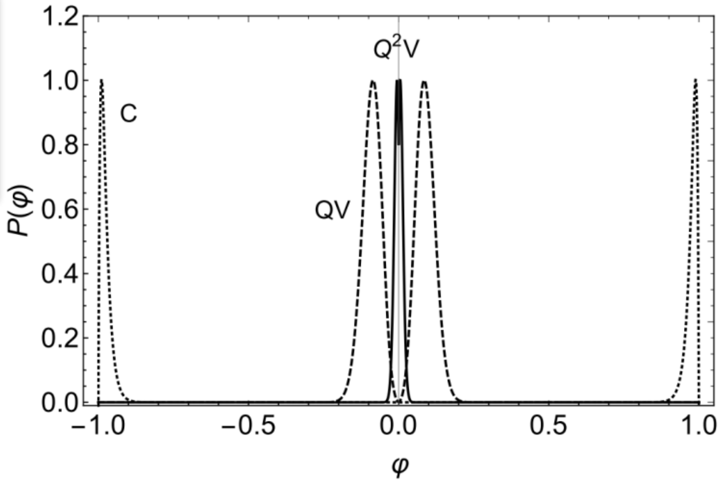

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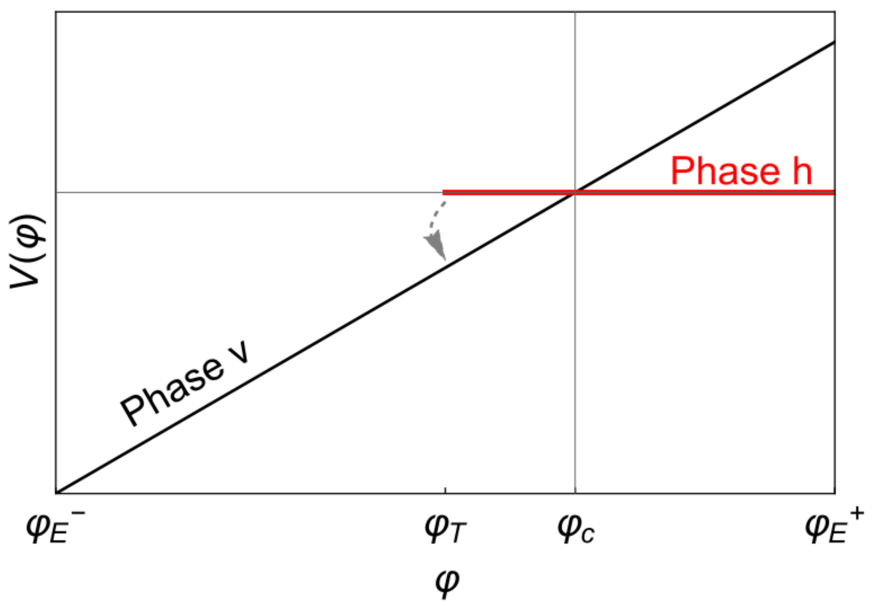
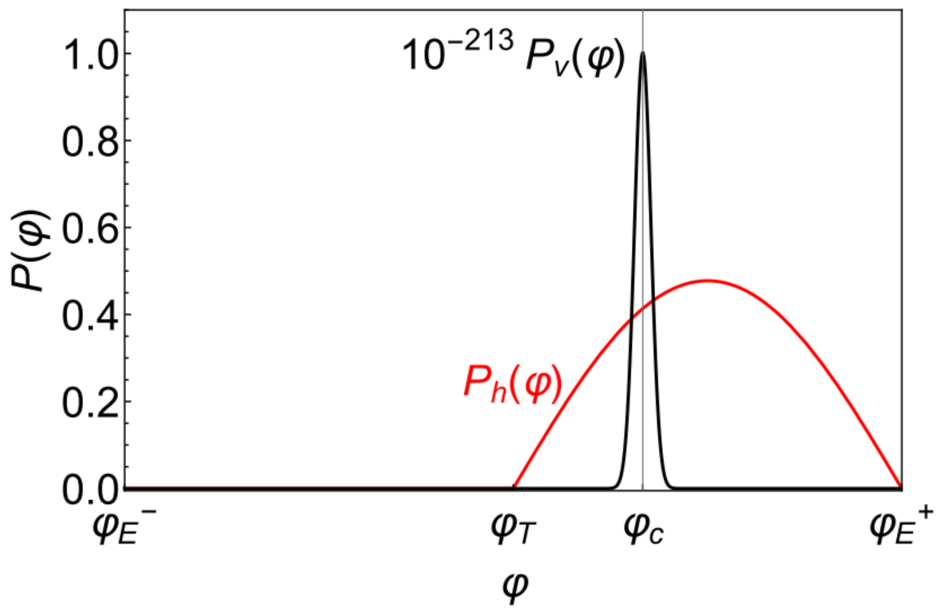
Junction conditions at phase transitions

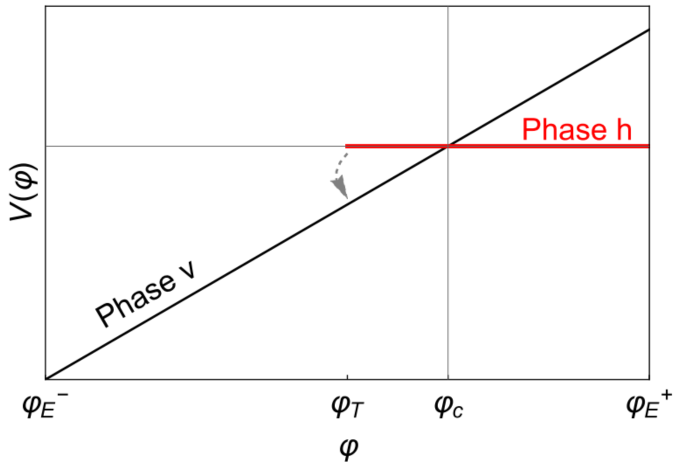


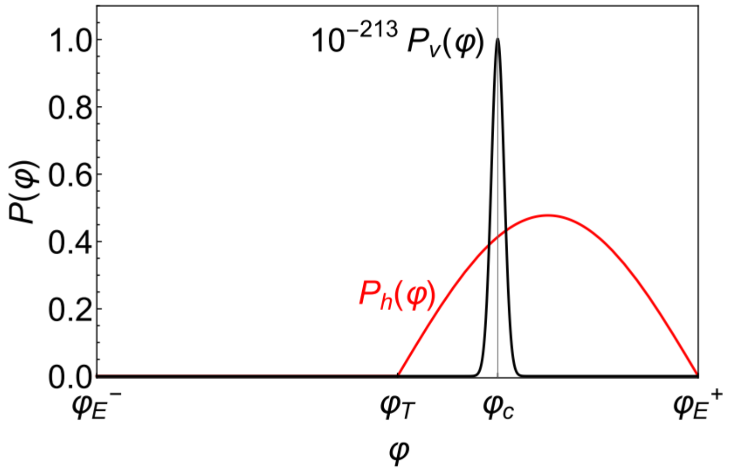


- **Coexistence** of branches of different phases, require continuity of P_V and $P_V + P_h$ in FPV at ϕ_T: **flux conservation** junction conditions

$$P_h(\phi_T) = 0 \qquad \Delta P_v' = -P_h'(\phi_T) \qquad \Delta P_v = 0$$

Junction conditions at phase transitions





- **Phase v** *must be in C regime*
- **Boundary conditions** pick out diffusionless solution over Gibbs solution
- Require **flux at boundary**

Solve FPV: $\frac{\alpha}{2}p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)p = 0\,, \qquad \omega_h(\varphi) = 0 \quad , \quad \omega_v(\varphi) = \varphi\,.$

Phase h:

$$p_h(\varphi_E^+) = 0 \quad , \quad p_h(\varphi_T) = 0\,. \qquad p_h'(\varphi_T) = \kappa_h\,.$$

$$p_h(\varphi) = \frac{(\varphi_E^+ - \varphi_T)}{\pi}\kappa_h \sin\left(\frac{\pi(\varphi - \varphi_T)}{\varphi_E^+ - \varphi_T}\right) \quad , \quad (\varphi > \varphi_T)\,.$$

$$\lambda = -\frac{\alpha}{2}\frac{\pi^2}{(\varphi_E^+ - \varphi_T)^2}\,.$$

Phase v:

$$1)\ P_v^-(-1) = 0 \quad , \quad 2)\ P_v^{+\prime}(1) = -k_v\,,$$

$$3)\ P_v^+(\varphi_T) = P_v^-(\varphi_T) \quad , \quad 4)\ P_v^{+\prime}(\varphi_T) = P_v^{-\prime}(\varphi_T) - k_h\,,$$

$$P_v^\pm(\varphi, \lambda) = e^{-\frac{\varphi}{\alpha}}\left[g_a^\pm(\lambda)Ai(x) + g_b^\pm(\lambda)Bi(x)\right]\,,$$

$$x = \frac{1 + 2\alpha\lambda - 2\alpha\beta\varphi}{(2\alpha^2\beta)^{2/3}}\,.$$

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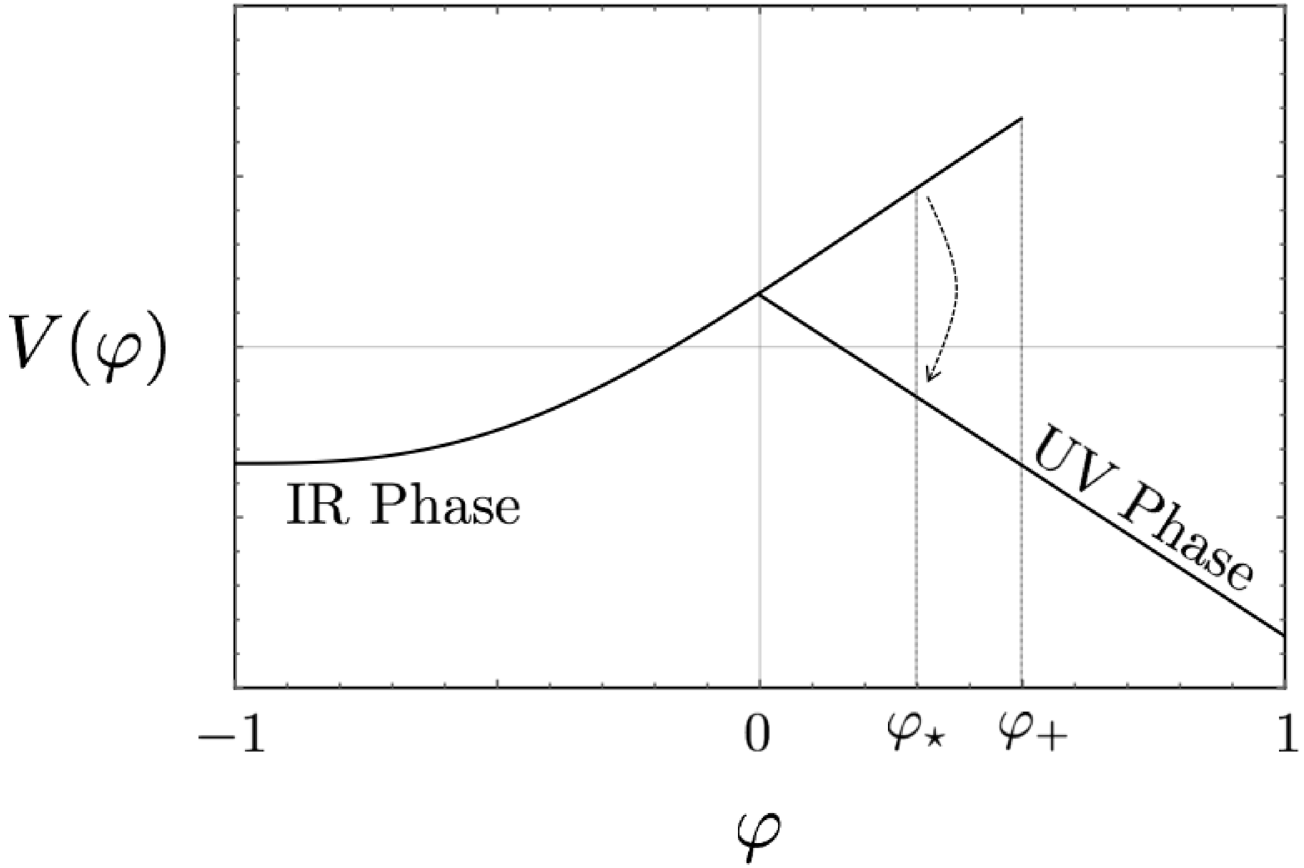
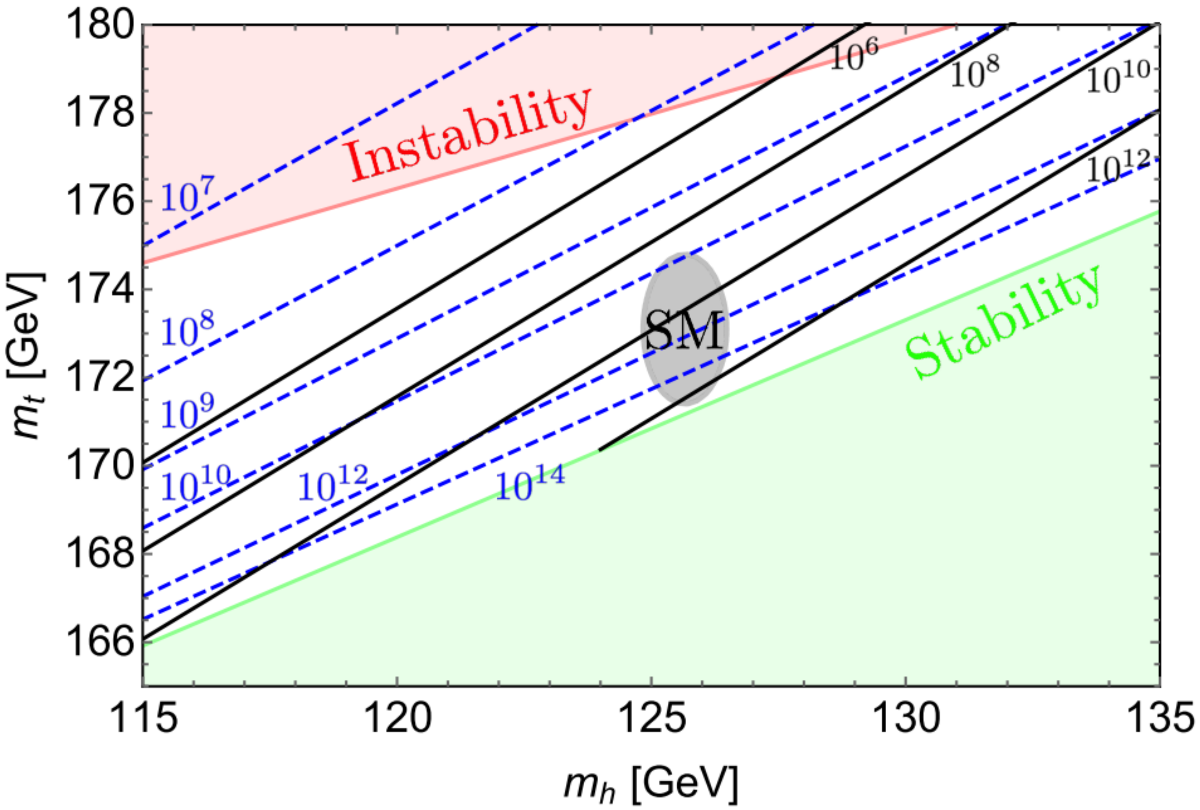
Higgs metastability

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) + \frac{\lambda(\varphi, h)}{4}\left(h^2 - v^2\right)^2$$

$$\lambda(\varphi, M/g_*) = -g_*^2\,\varphi$$





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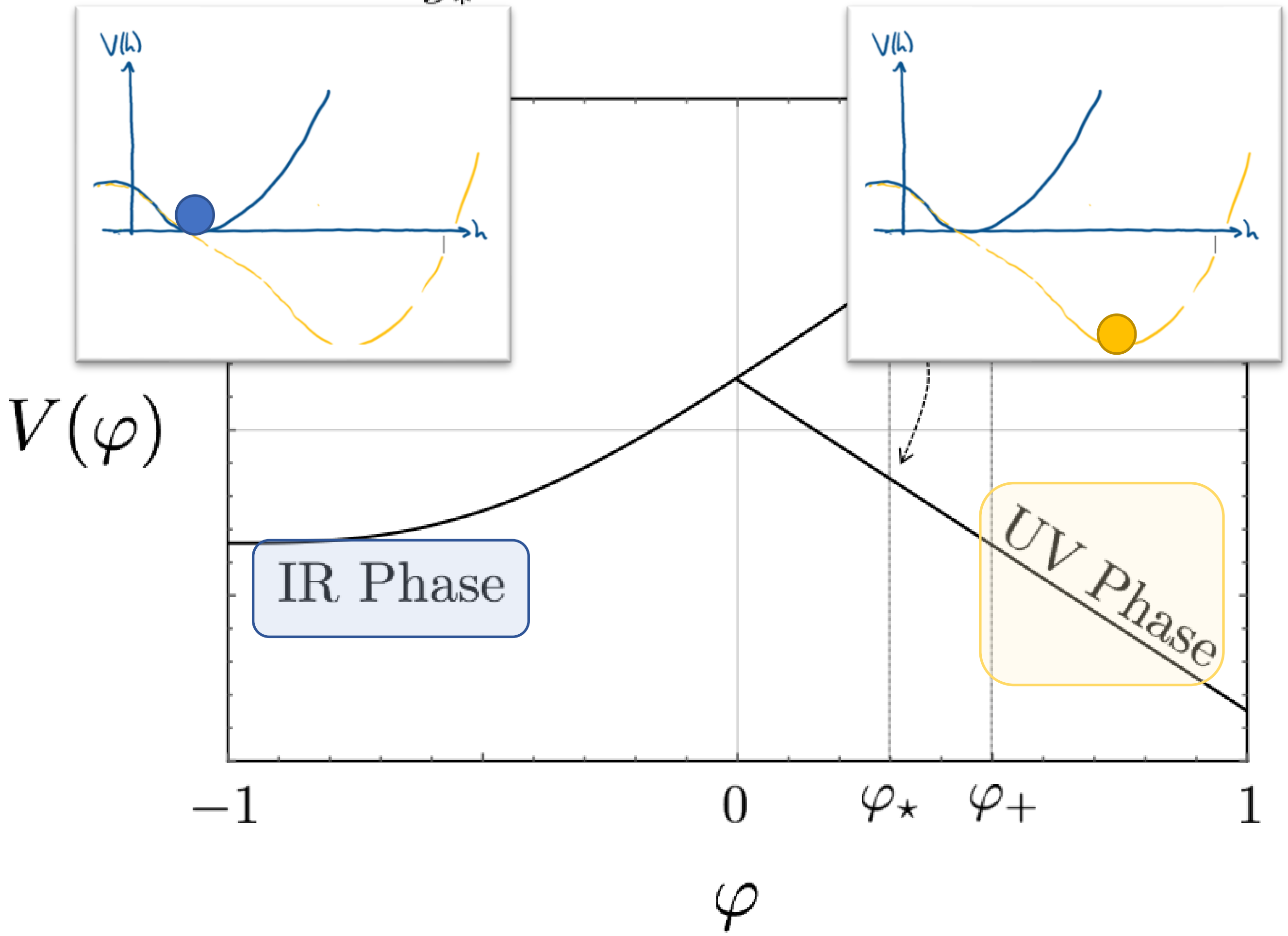

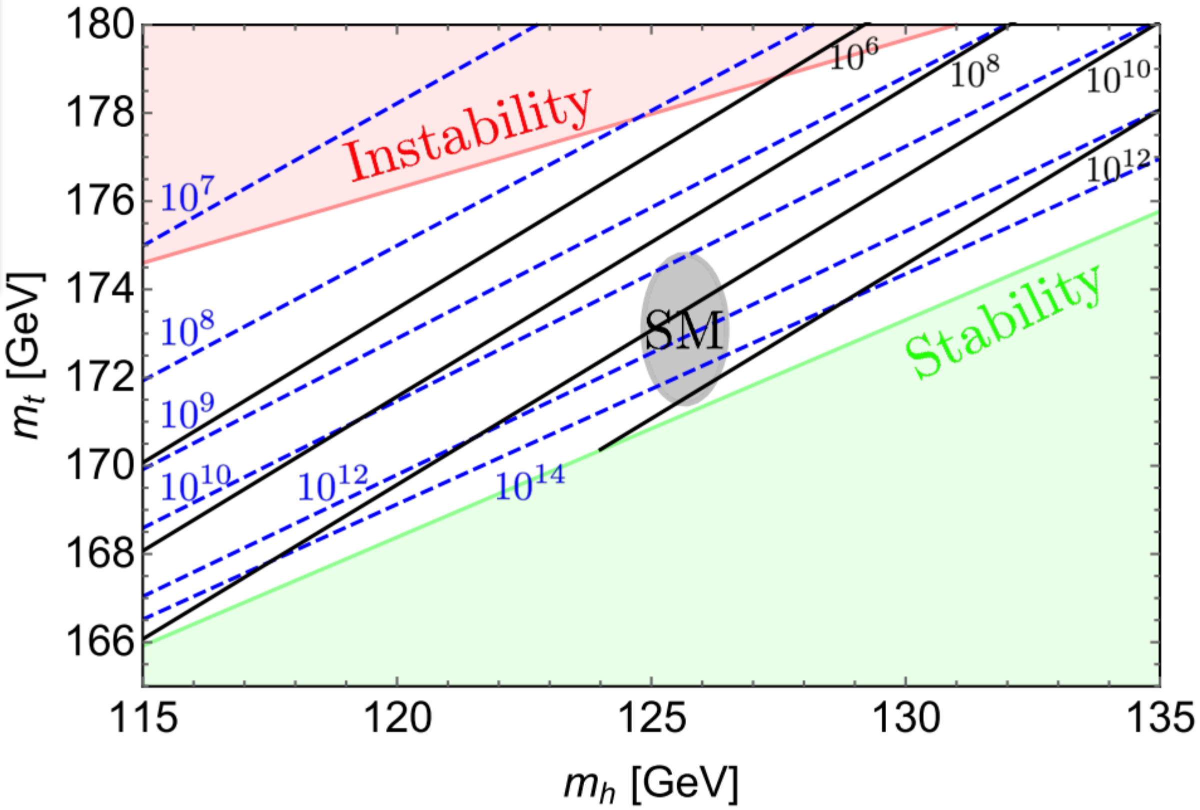

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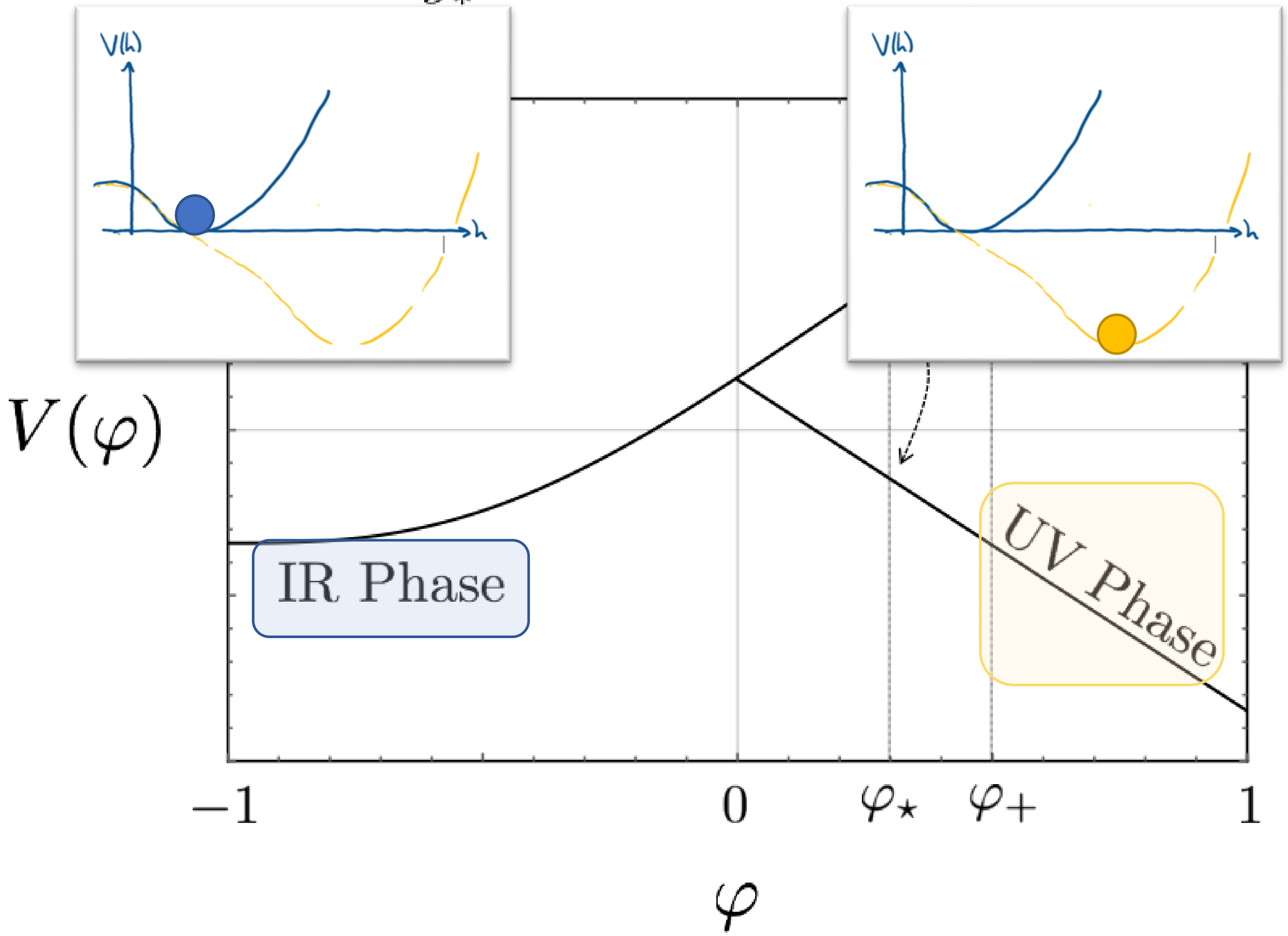
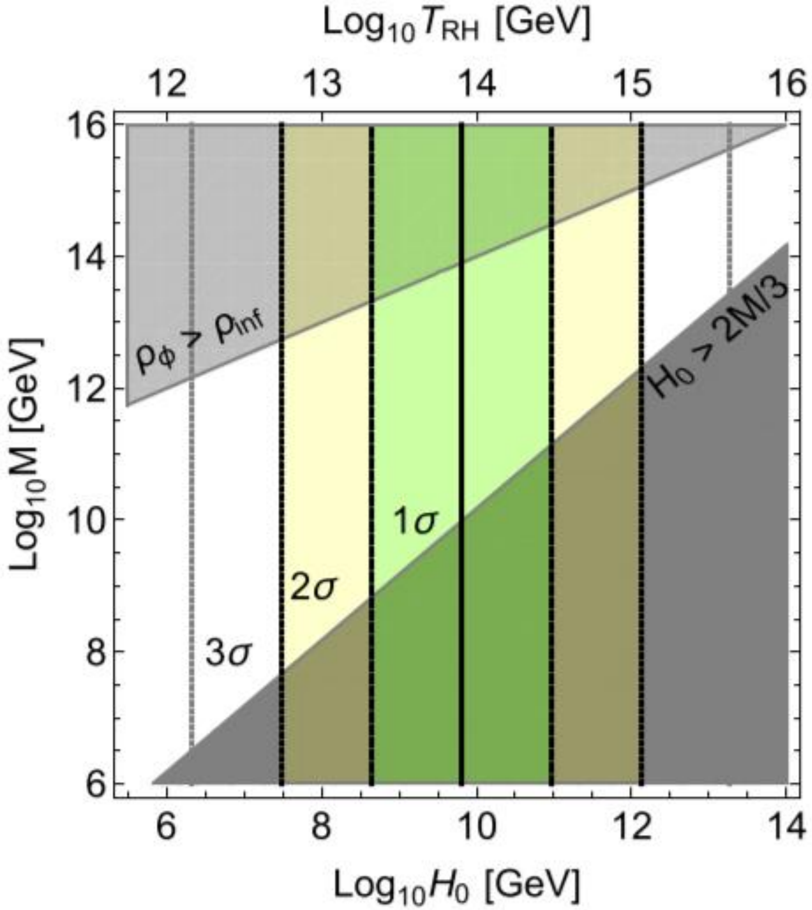


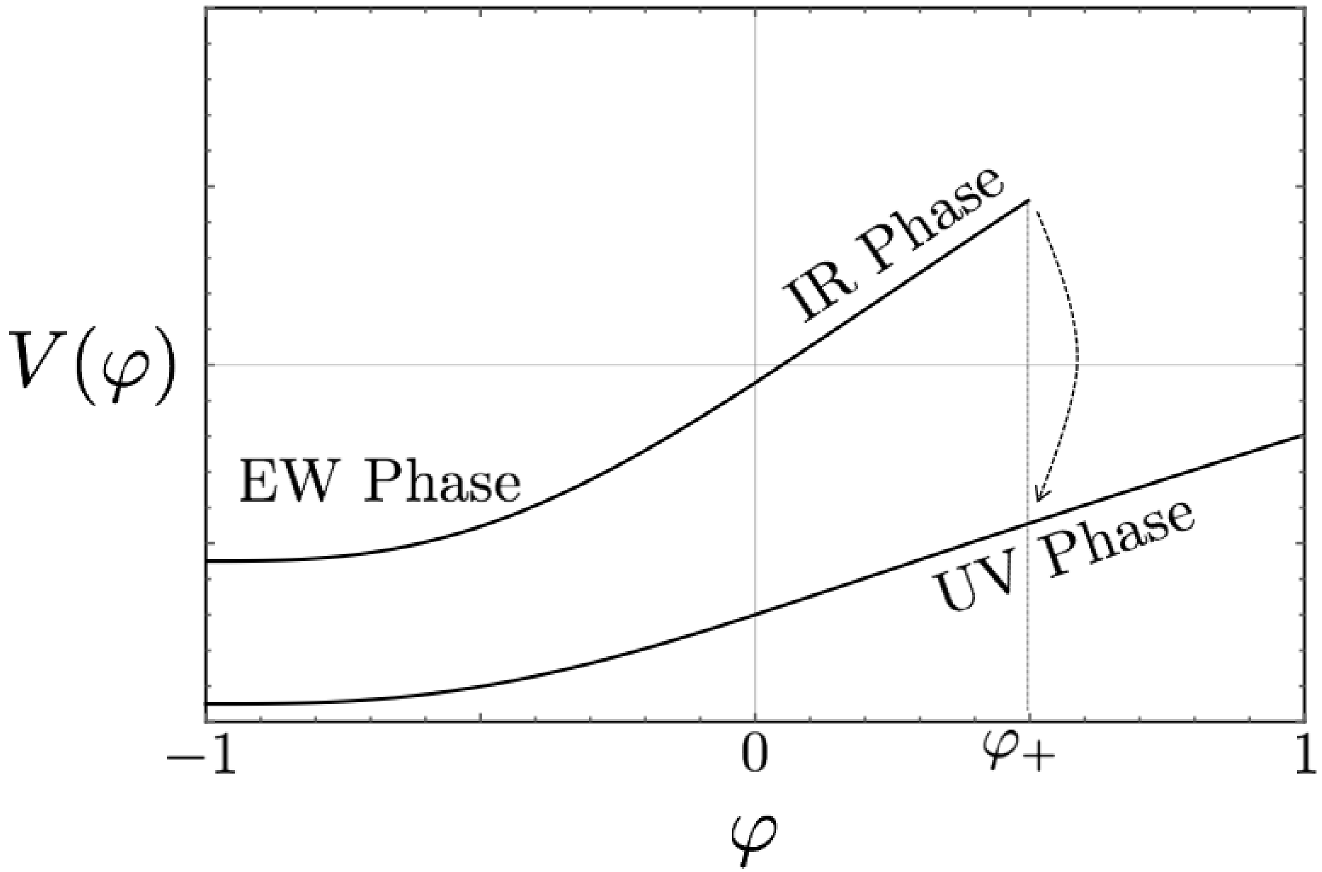


Higgs mass naturalness

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) - \frac{\varphi M^2 h^2}{2} + \frac{\lambda(h)\,h^4}{4}$$

$$\frac{V(\varphi,\langle h\rangle)}{M^4} = \begin{cases} \kappa_{\rm EW}\varphi + \kappa_2\varphi^2 + \dots & \text{for } \varphi<0 \quad \text{(unbroken EW: } \langle h\rangle = 0) \\ \kappa_{\rm EW}\varphi + \kappa_{\rm IR}\varphi^2 + \dots & \text{for } 0<\varphi<\varphi_+ \quad \text{(IR phase: } \langle h\rangle = v) \\ -\kappa_0 + \kappa_{\rm UV}\varphi + \kappa_2\varphi^2 + \dots & \text{for any } \varphi \quad \text{(UV phase: } \langle h\rangle = c_{\rm UV}M) \end{cases}$$

$$\kappa_{\rm EW} = \frac{\omega'(0)}{g_*^2}\,, \quad \kappa_2 = \frac{\omega''(0)}{2g_*^2}\,, \quad \kappa_{\rm IR} = \kappa_2 - \Delta\kappa\,, \quad \kappa_0 = \frac{-\lambda_{\rm UV}c_{\rm UV}^4}{4}\,, \quad \kappa_{\rm UV} = \kappa_{\rm EW} - \frac{c_{\rm UV}^2}{2}$$



- Unbroken to broken transition **not sufficient**

- Use **broken IR to broken UV** phase transition

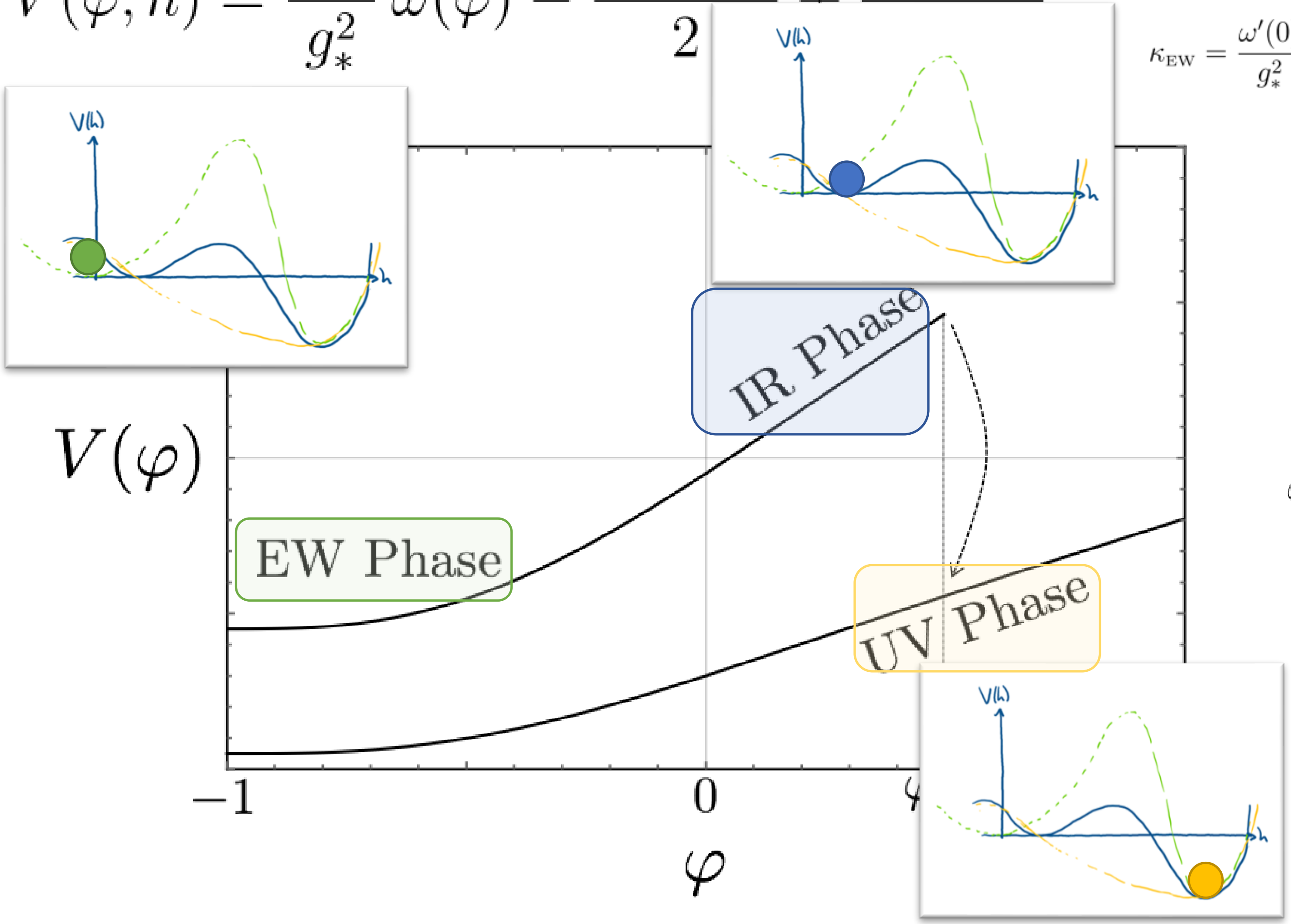
$$\varphi_+ = \frac{-\beta_I\, e^{-\frac{3}{2}}\Lambda_I^2}{M^2} \quad \Rightarrow \quad v = e^{-\frac{3}{4}}\,\Lambda_I$$

- Need **lower instability scale $\boldsymbol{\Lambda_I}$**: ~TeV through VL fermions

- (Naturalness motivation: scalars and vectors heavy, **only VL fermions at TeV scale**)

Higgs mass naturalness

$$V(\varphi,h)=\frac{M^4}{g_*^2}\,\omega(\varphi)-\frac{\varphi M^2 h^2}{2}+\lambda(h)\,h^4$$



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$$\kappa_{\rm EW}=\frac{\omega'(0)}{g_*^2}\,,\quad \kappa_2=\frac{\omega''(0)}{2g_*^2}\,,\quad \kappa_{\rm IR}=\kappa_2-\Delta\kappa\,,\quad \kappa_0=\frac{-\lambda_{\rm UV}c_{\rm UV}^4}{4}\,,\quad \kappa_{\rm UV}=\kappa_{\rm EW}-\frac{c_{\rm UV}^2}{2}$$

- Unbroken to broken transition **not sufficient**

- Use **broken IR to broken UV** phase transition

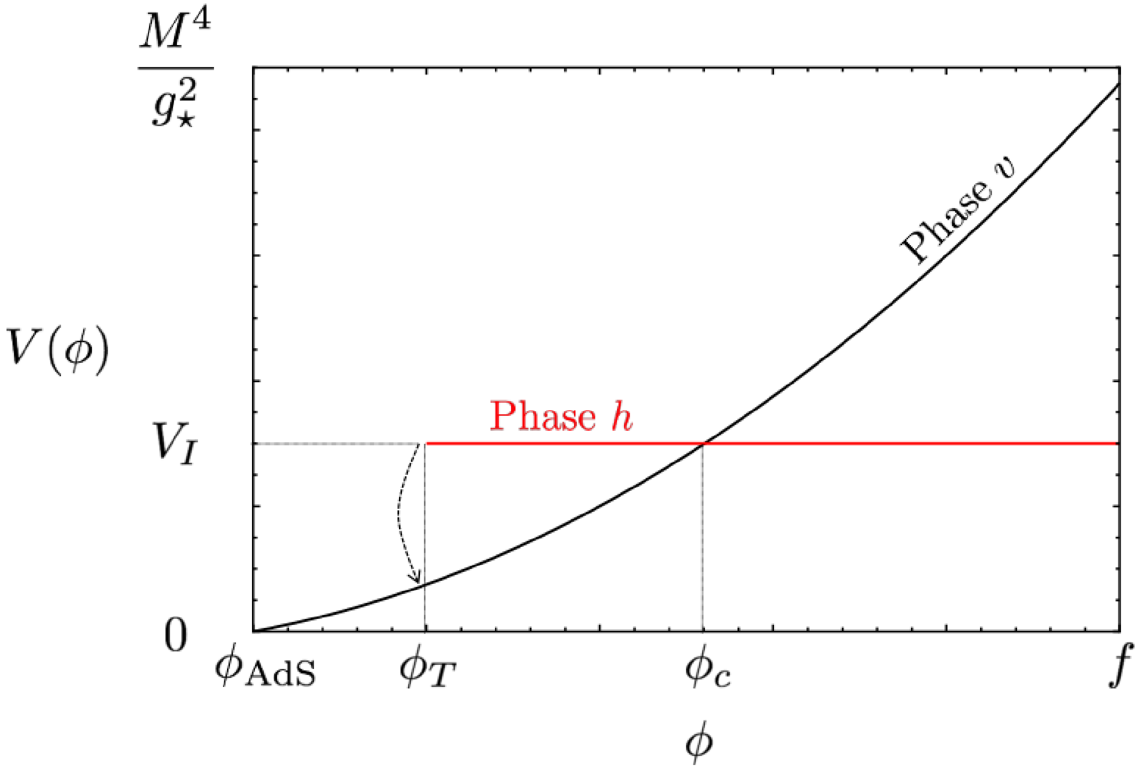
$$\varphi_+=\frac{-\beta_I\,e^{-\frac{3}{2}}\Lambda_I^2}{M^2}\quad\Rightarrow\quad v=e^{-\frac{3}{4}}\,\Lambda_I$$

- Need **lower instability scale Λ_I**: ~TeV through VL fermions

- (Naturalness motivation: scalars and vectors heavy, **only VL fermions at TeV scale**)

Cosmological constant

- **Hidden phase**: vanishing cosmological constant by R-symmetry
- **Visible phase**: SOL localises at vacuum degeneracy point



$$p_h(\phi) = \sin\left[\sqrt{\frac{6(1-\lambda_H)}{\hbar}}\,\frac{2\pi(\phi-\phi_T)}{H_I}\right]$$

$$\lambda_H = 1 - \frac{\hbar\, H_I^2}{24(f-\phi_T)^2}$$

$$V_v(\bar\phi) = V_I\,\lambda_H^{2/\xi}\ , \qquad \sigma = \sqrt{\frac{2}{3\xi}}\,M_P$$

$$\Rightarrow \quad V_v(\bar\phi) = V_I\left(1 - \frac{\hbar\, H_I^2}{12\xi f^2}\right)$$

- Solution must be in **C regime** with appropriate **boundary conditions**

Outline

- Motivation
  - EFT
  - Criticality
  - Quantum phase transitions (QPT)
- Fokker-Planck Volume (FPV) equation
  - FPV dynamics
- FPV + QPT = SOL
  - Discontinuity
  - Flux conservation
- SOL solutions
  - Metastability
  - Higgs mass
  - Cosmological constant
- **Conclusion**
  - Measure problem

Take-home message

- Scalar fields undergoing quantum fluctuations during inflation can be **localised at the critical points** of quantum phase transitions: **SOL**

- SOL suggests **our Universe lives at the critical boundary** of coexistence of phases

- **Measure problem**: ambiguous choice of time parametrisation (recall $\beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2}$)
- Related to regularisation of **infinite reheating surface**
- We have **not specified** the inflaton sector: decoupled from our scalar
- SOL prediction is quantitative but dependent on chosen solution of measure problem: **exponential localisation can remain a feature**

Introduction