Scalar leptoquarks: motivation and phenomenology

David Marzocca

INFN Trieste

INFN Trieste

La Thuile - 11/03/2022

Leptoquark is the new SUSY

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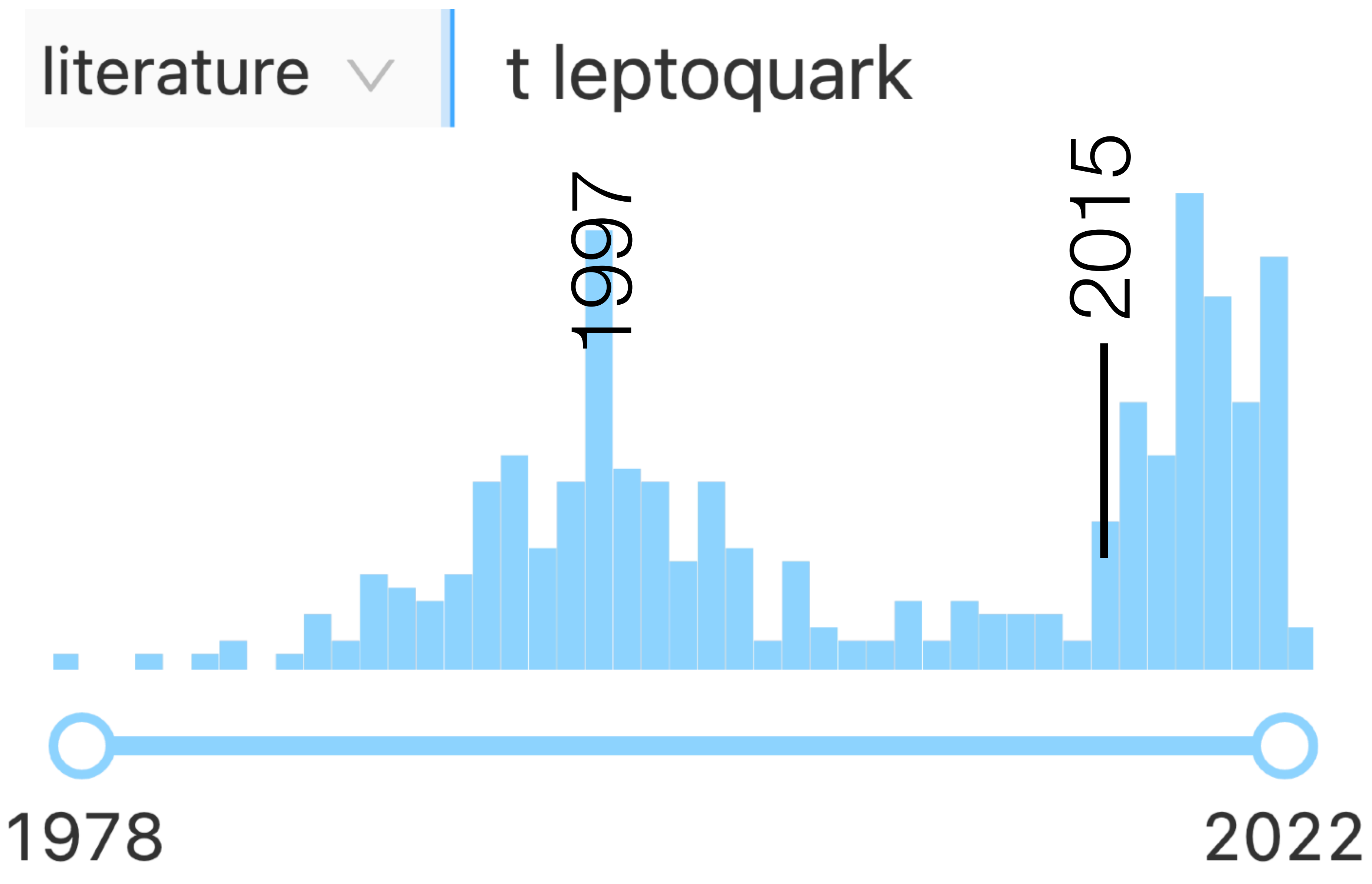

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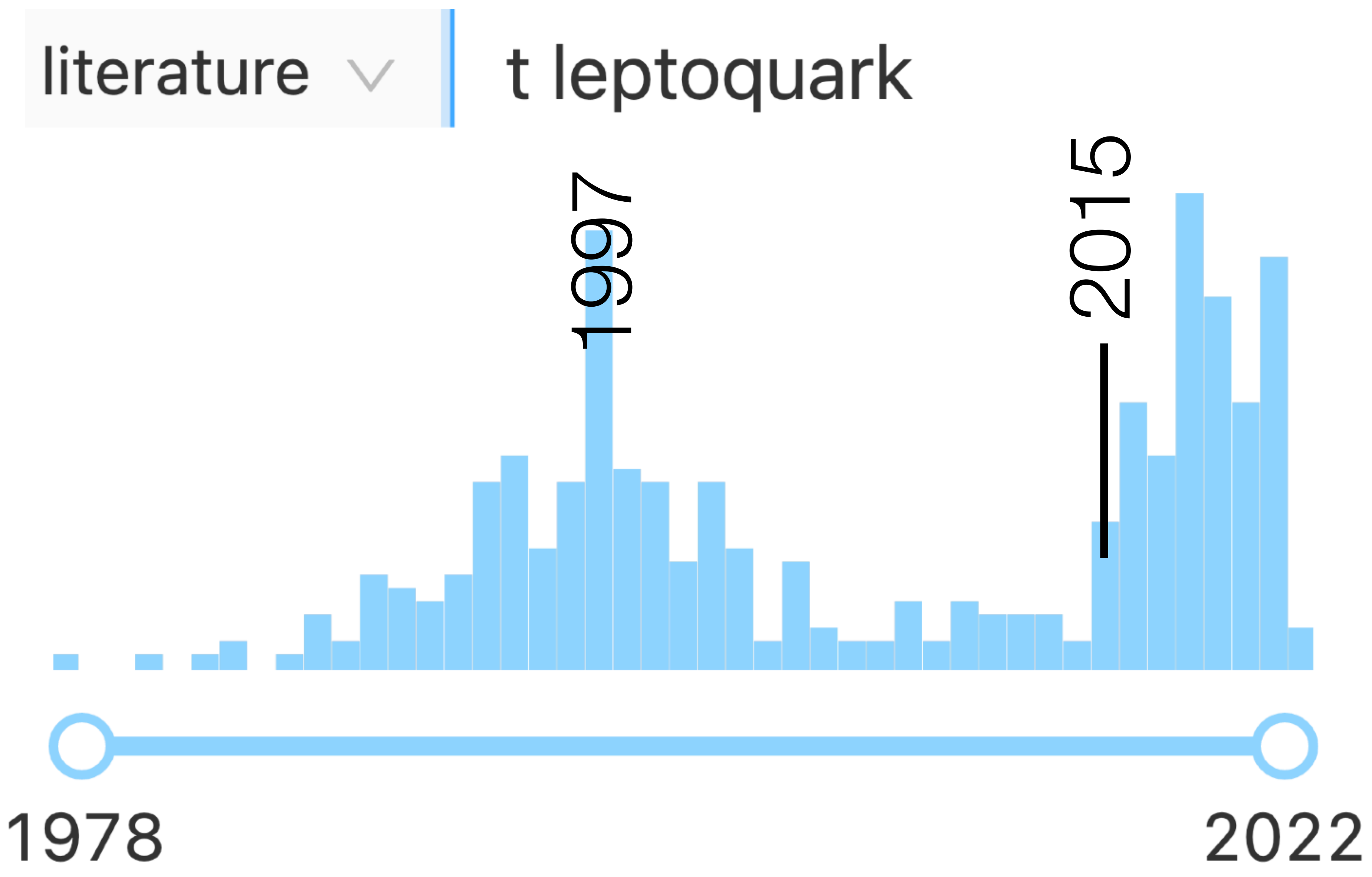

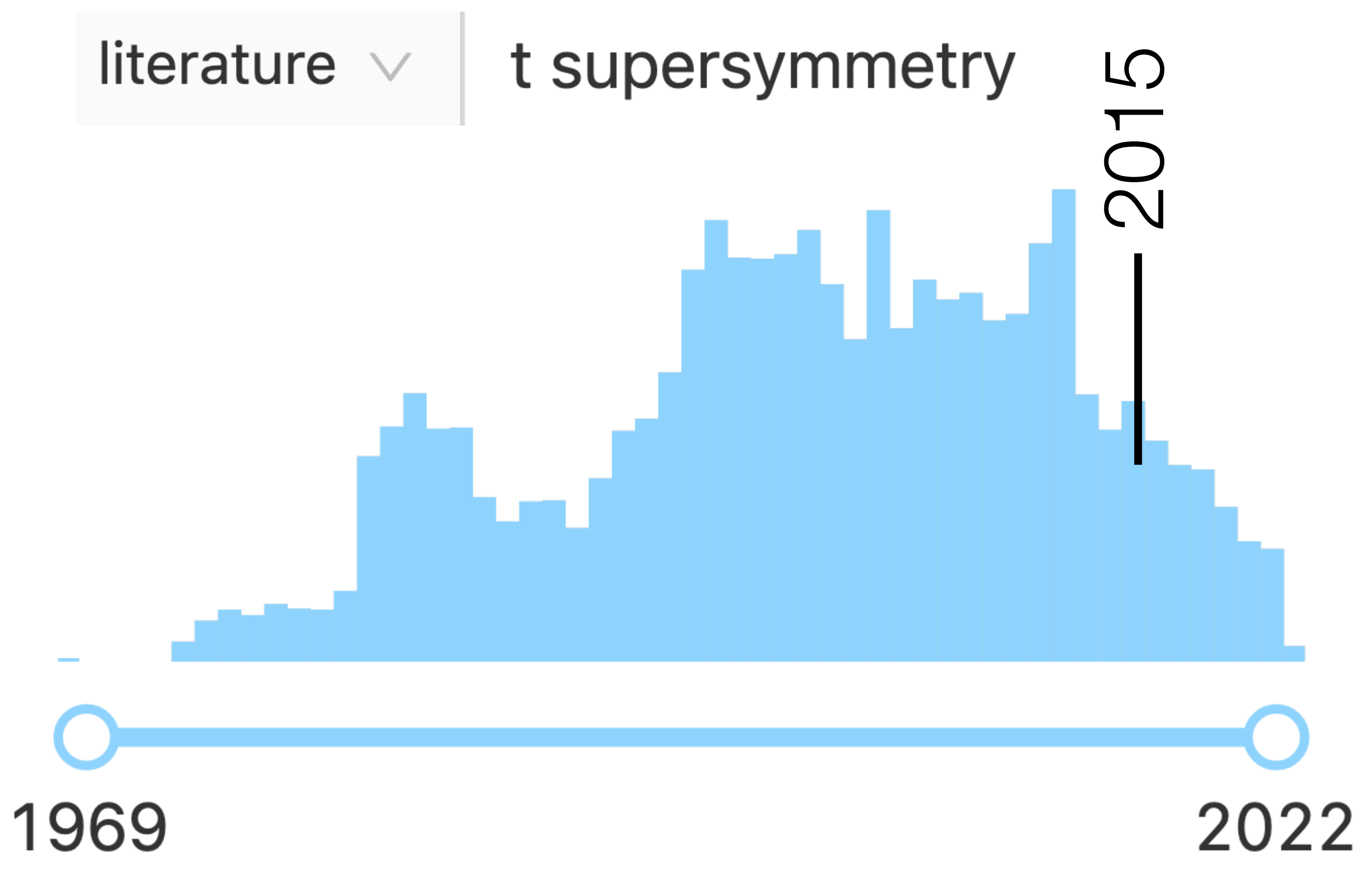

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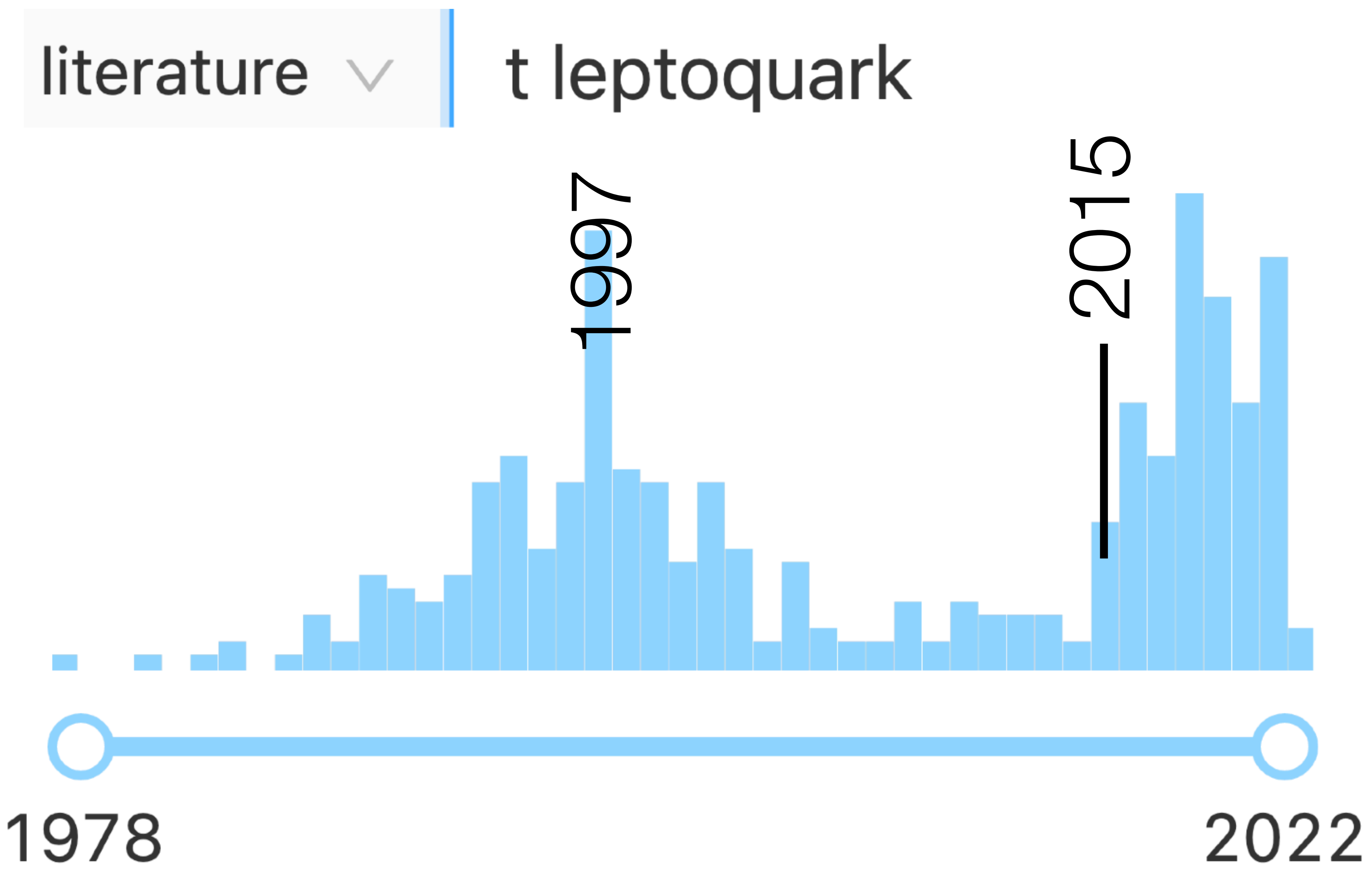
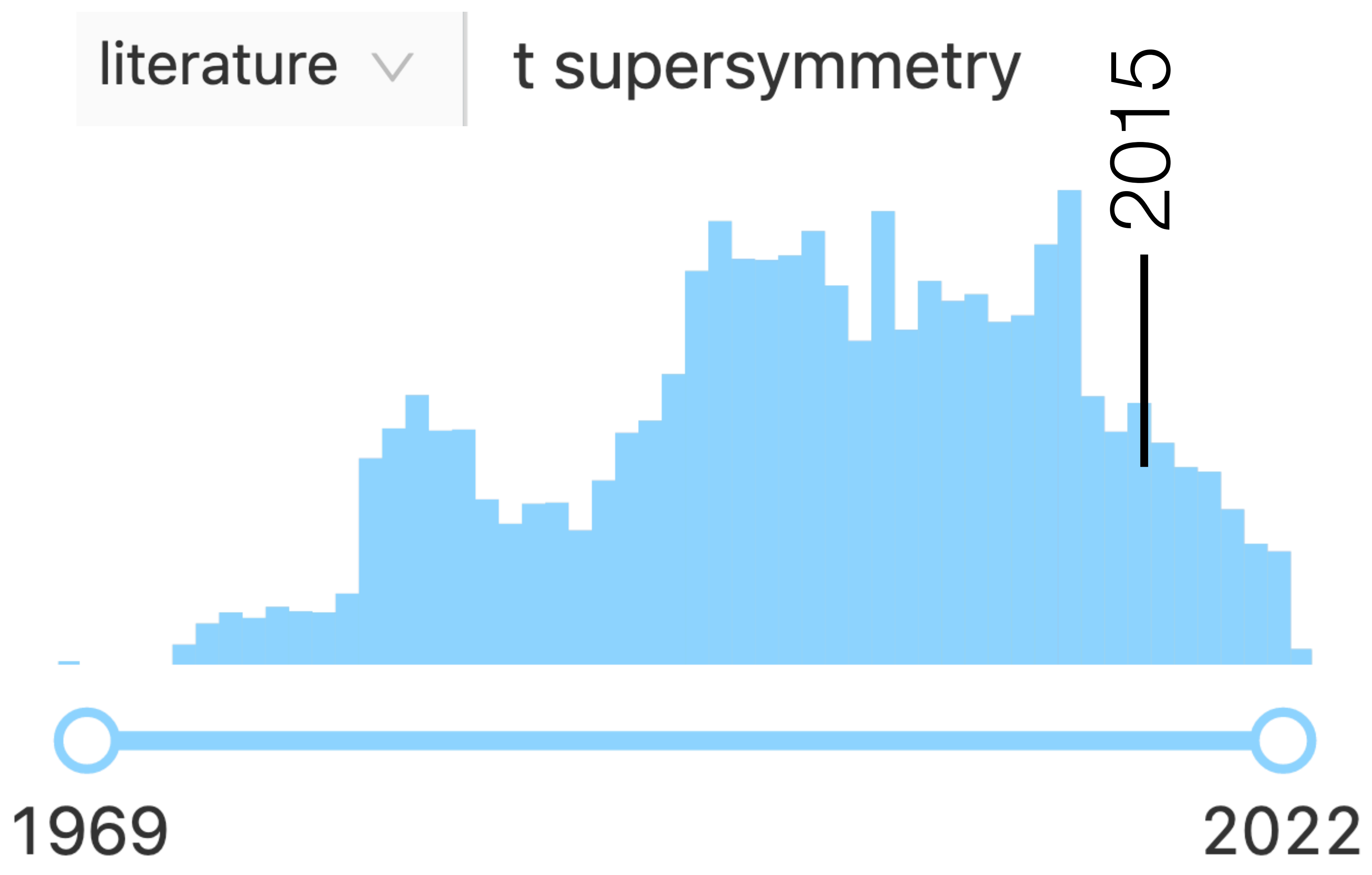
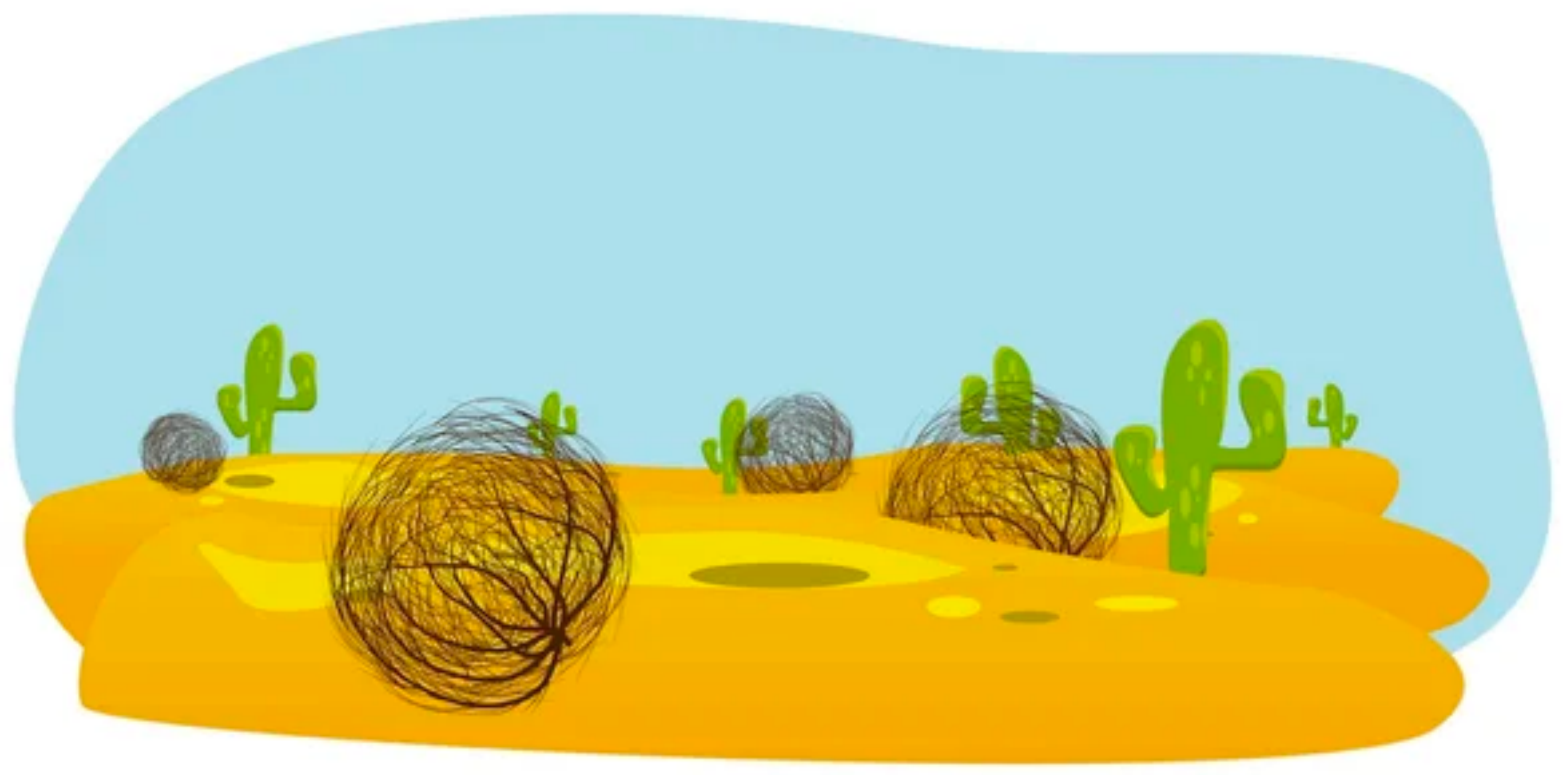
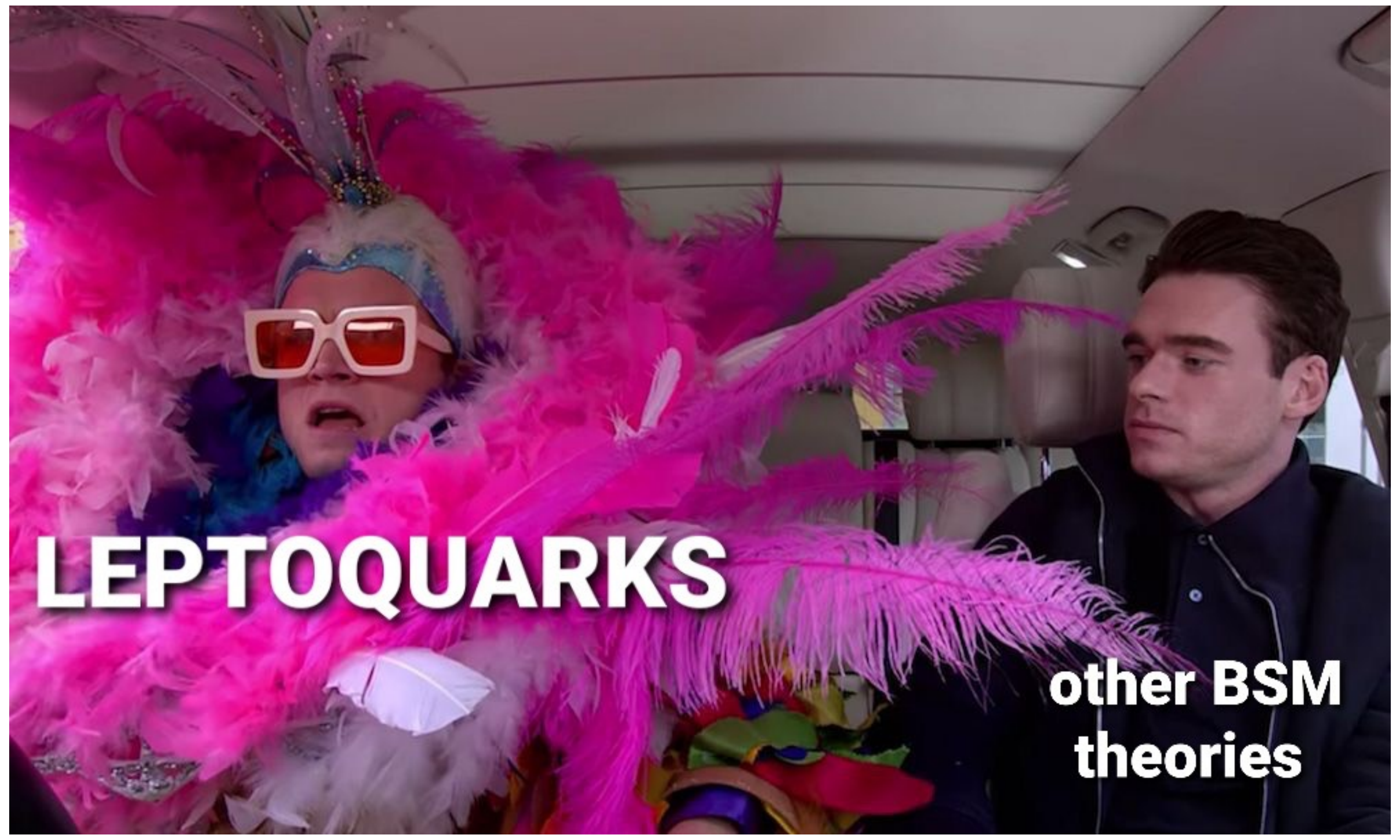




Before ~ 2015

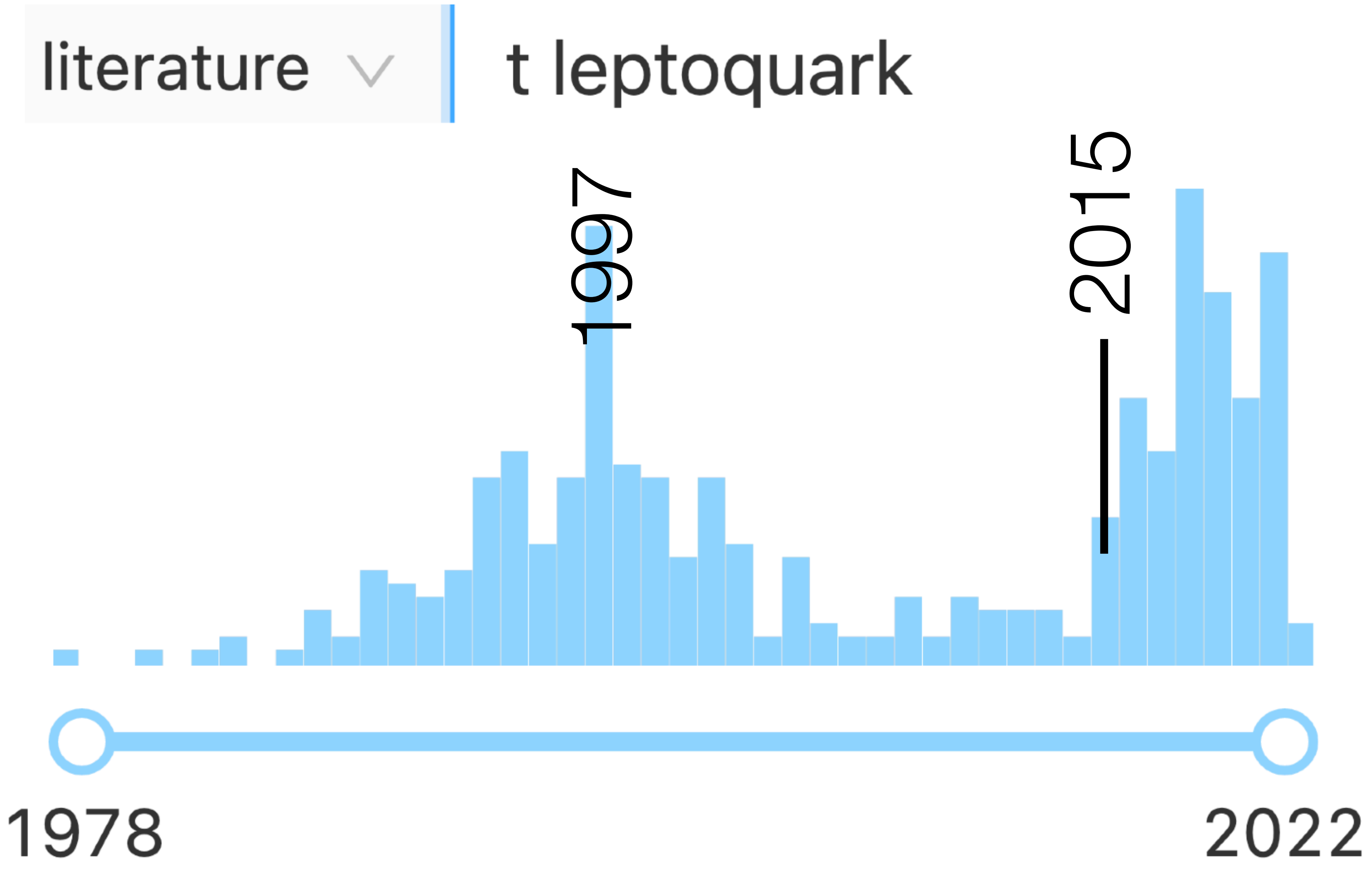
After ~ 2015

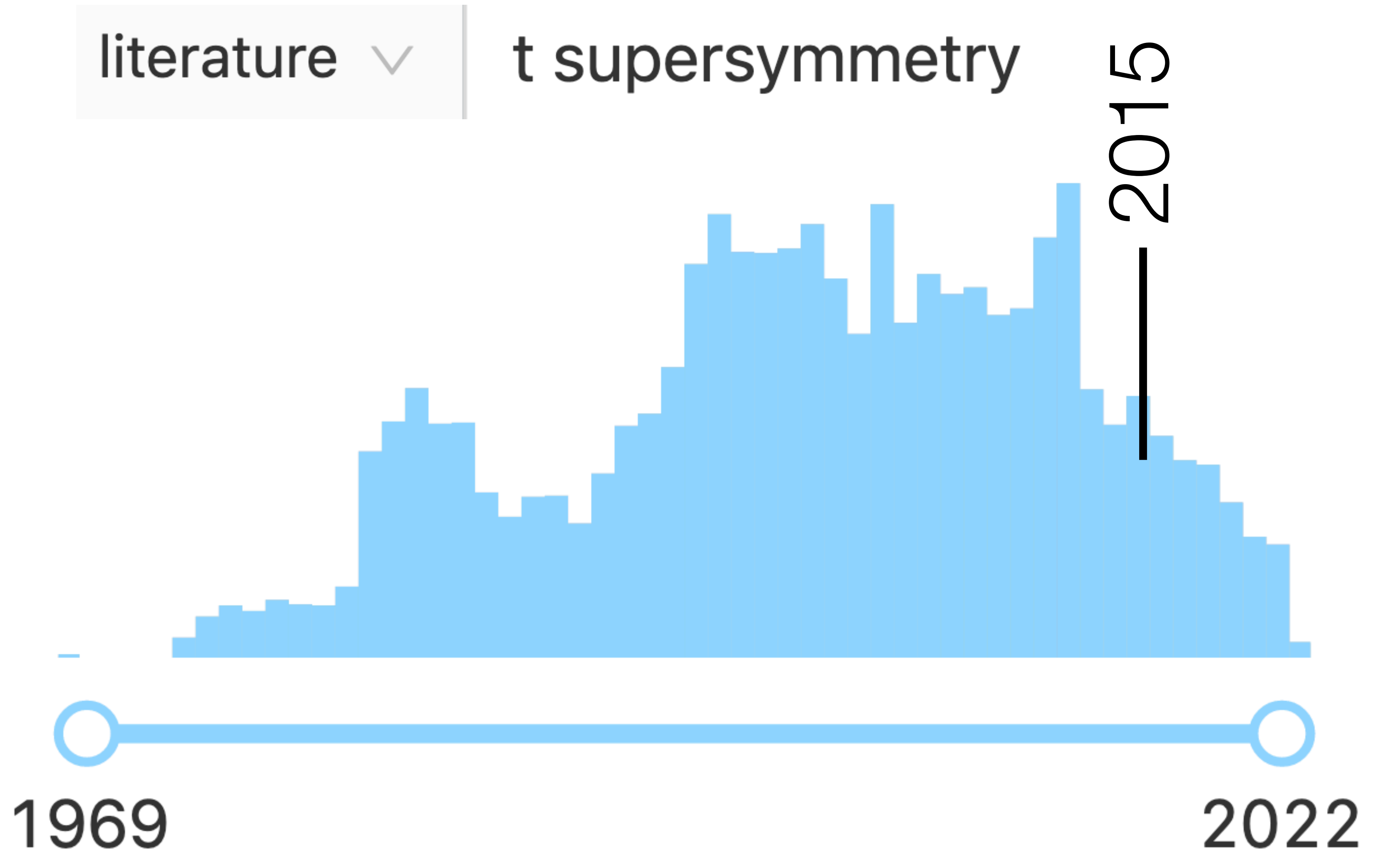




Leptoquark is the new SUSY

INSPIRE HEP





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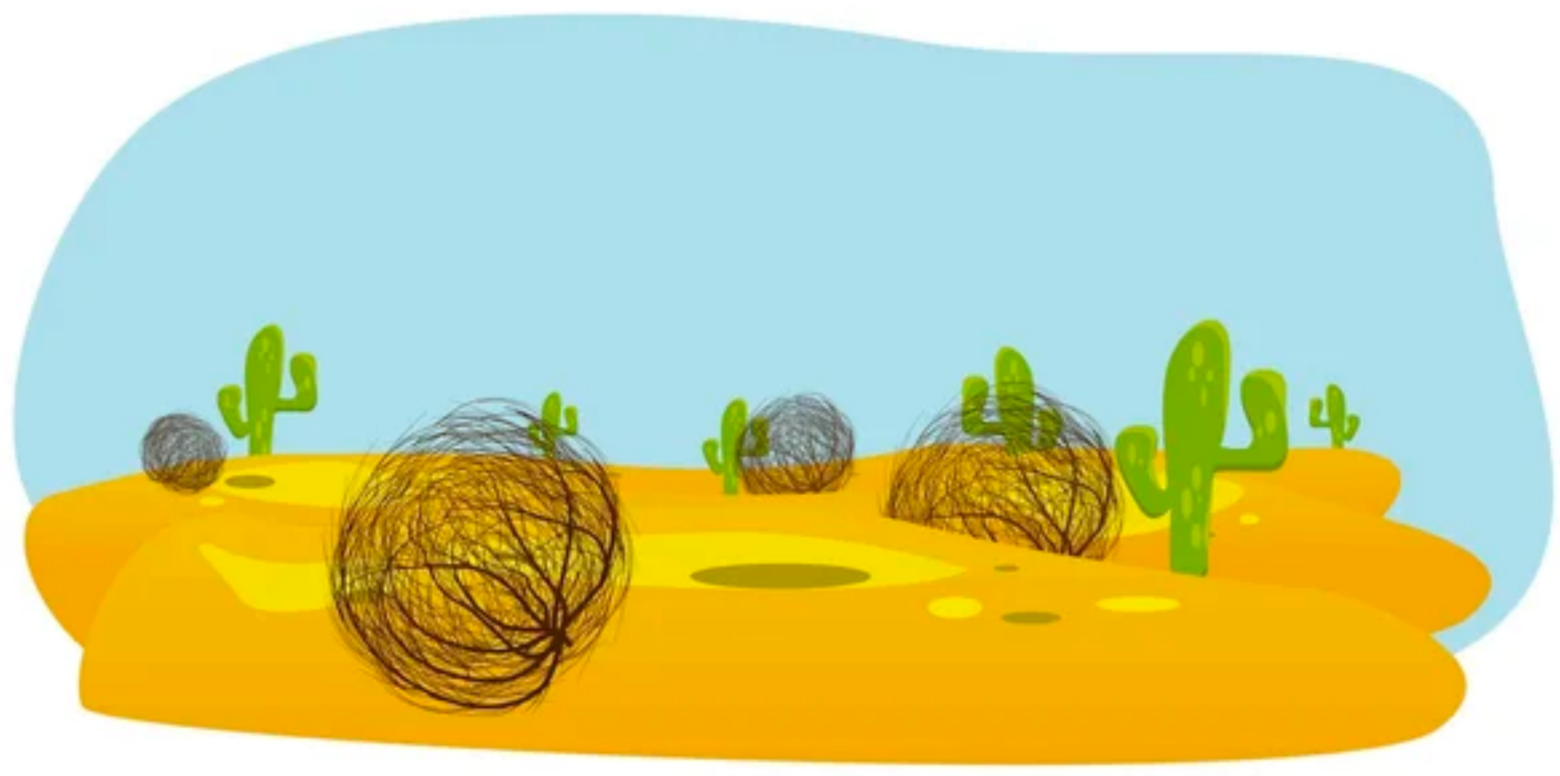

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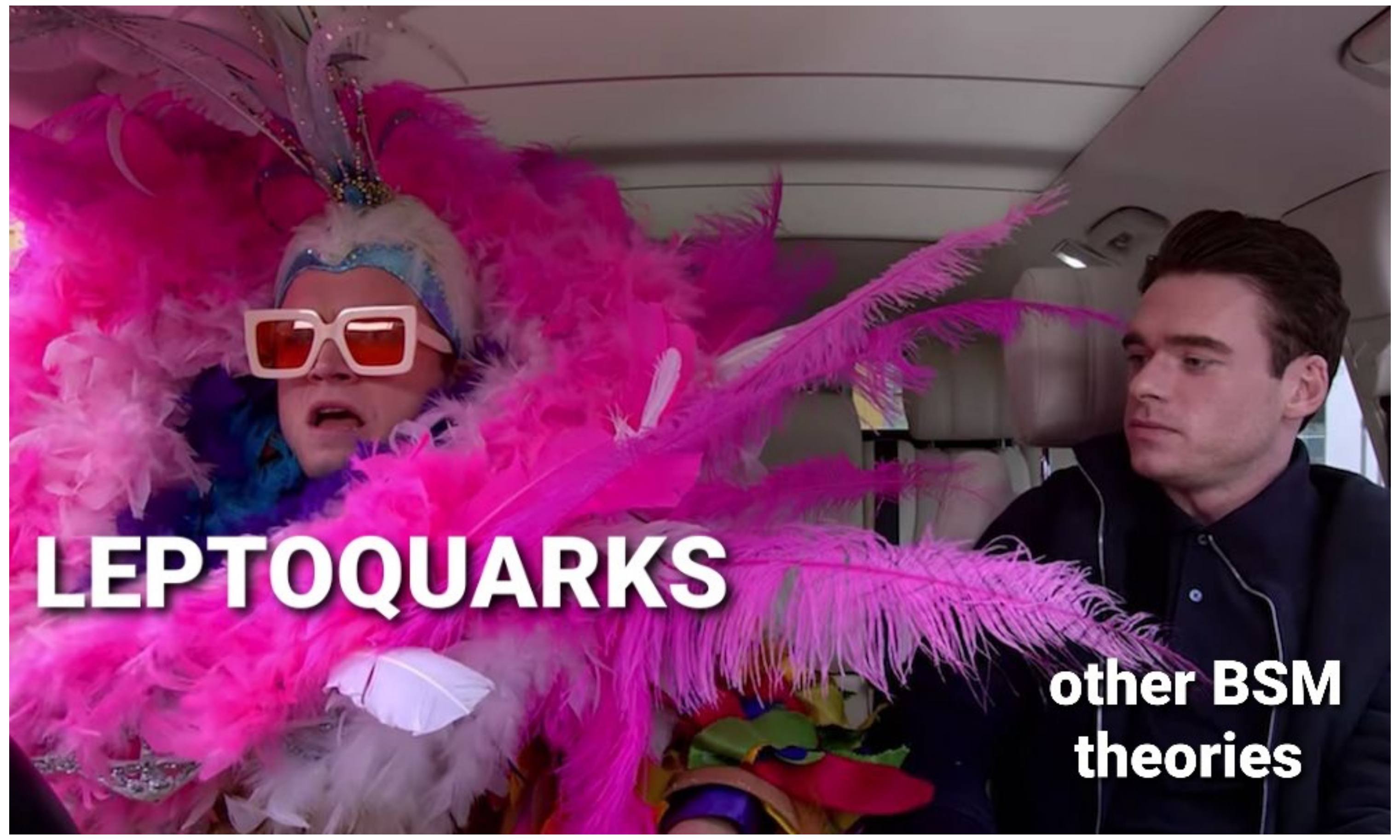

A clear trend…

Why?

Leptoquark Renaissance

This renewed interest
is mostly data-driven!

Leptoquark Renaissance

(see talks by V. Lisovskyi and L. Silvestrini)

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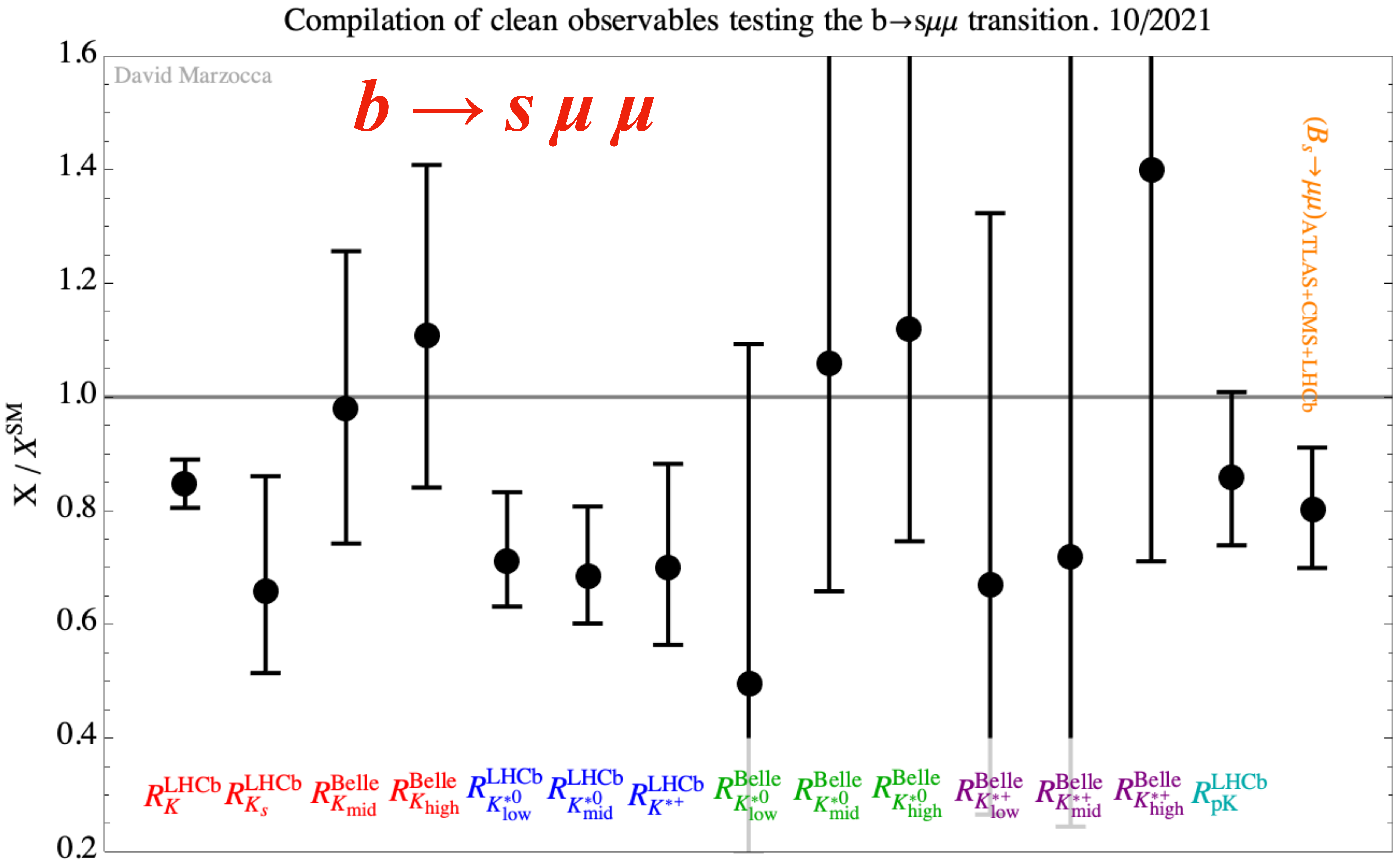

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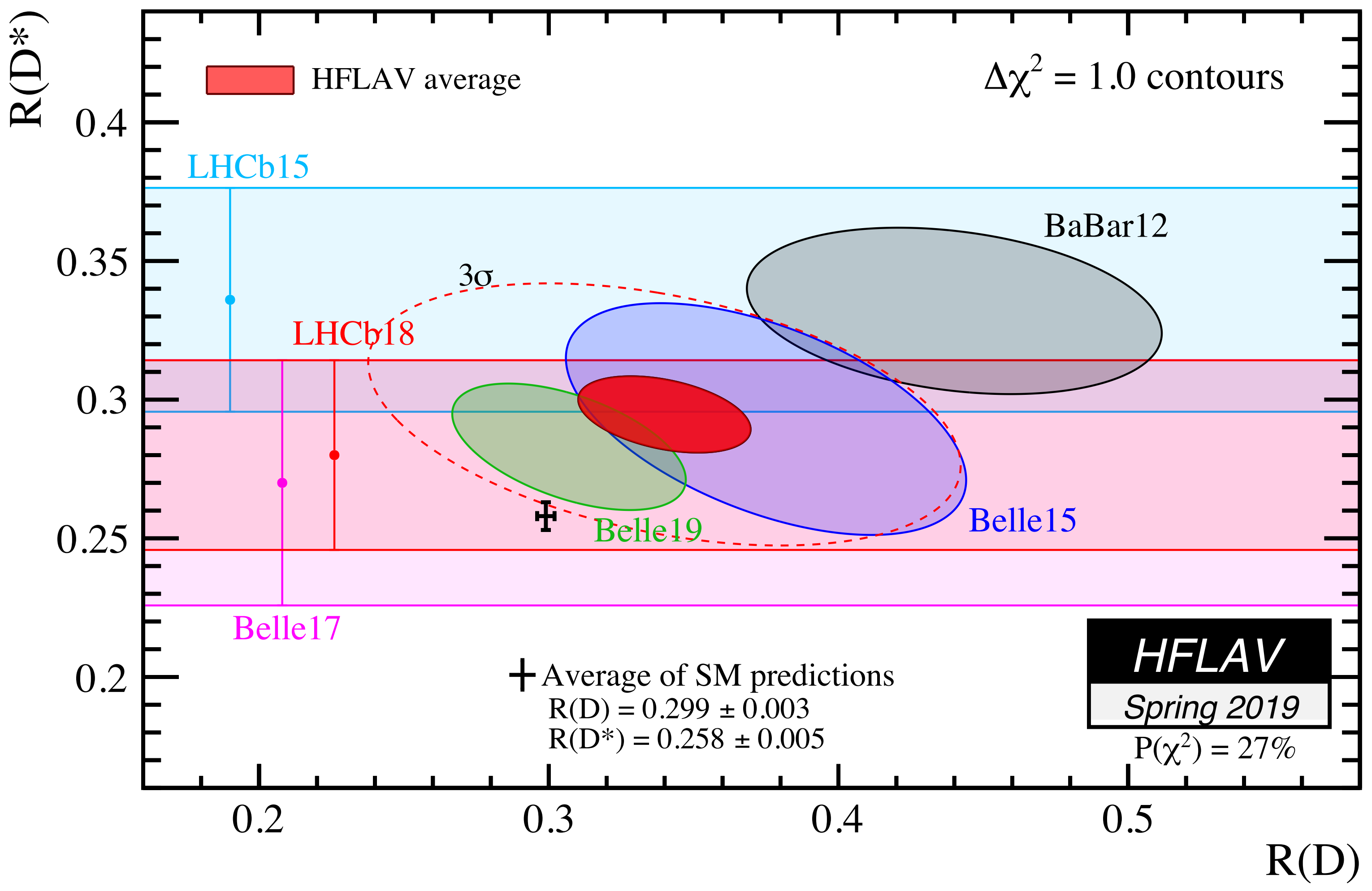
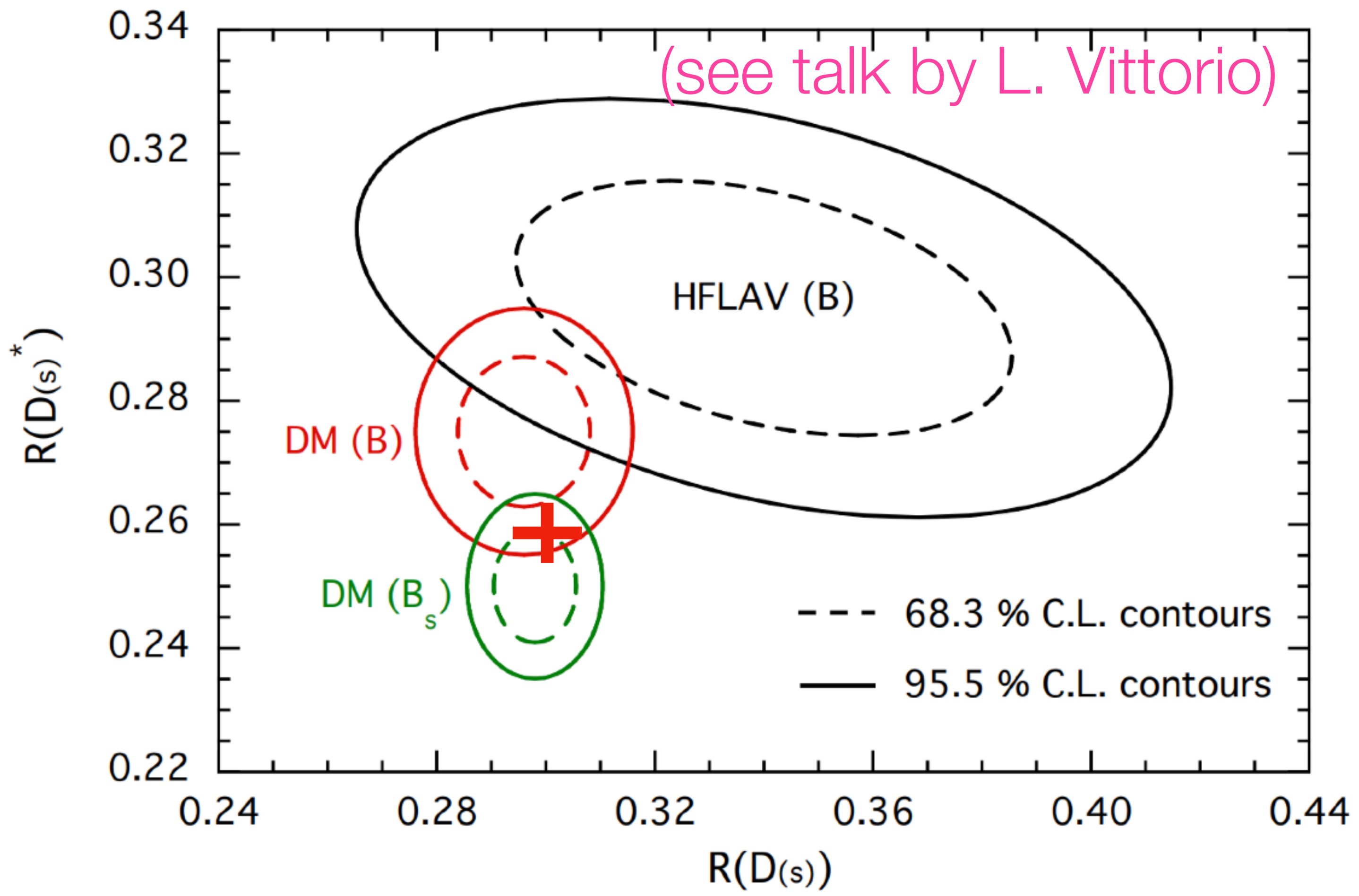
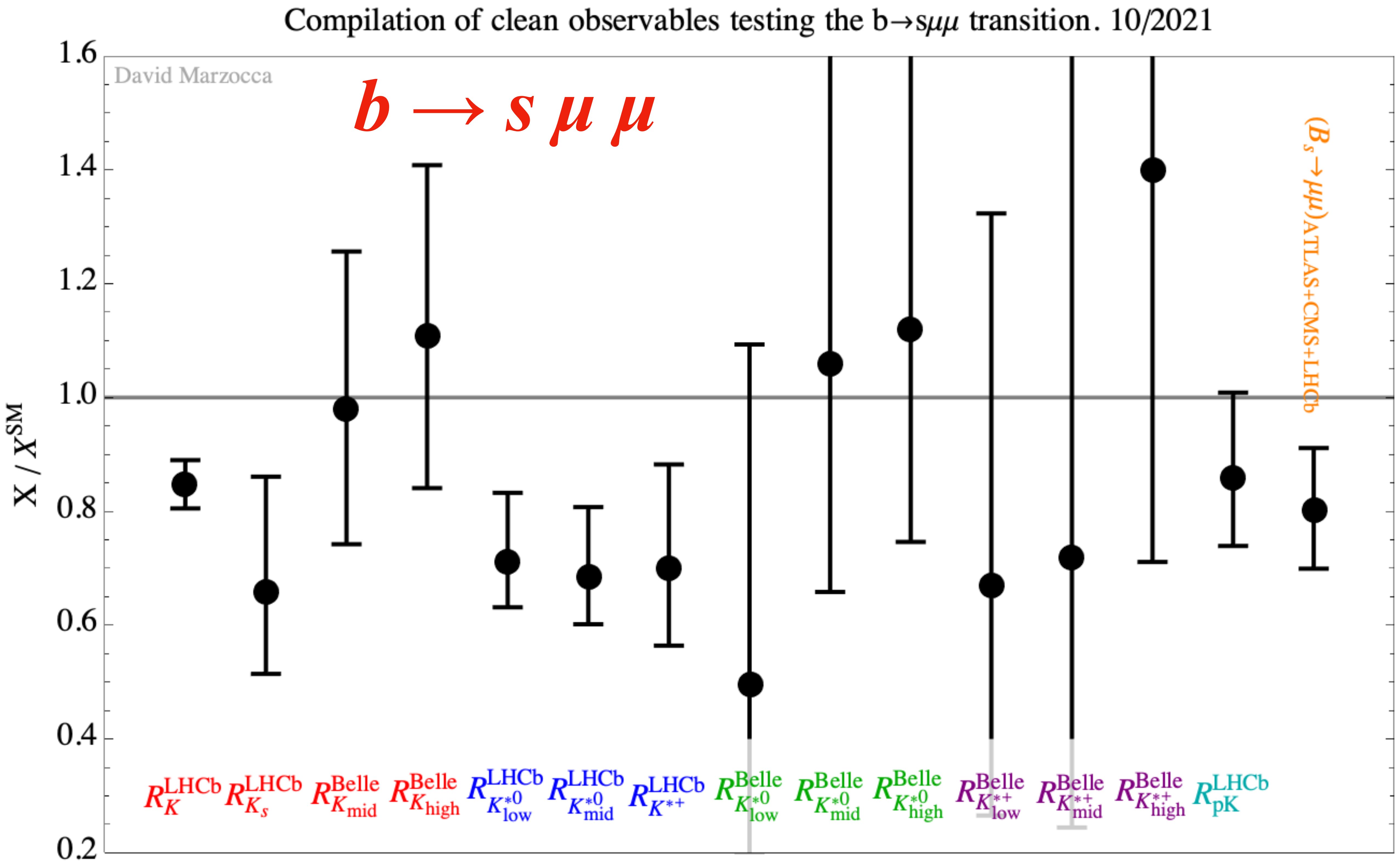


(see talk by L. Vittorio)





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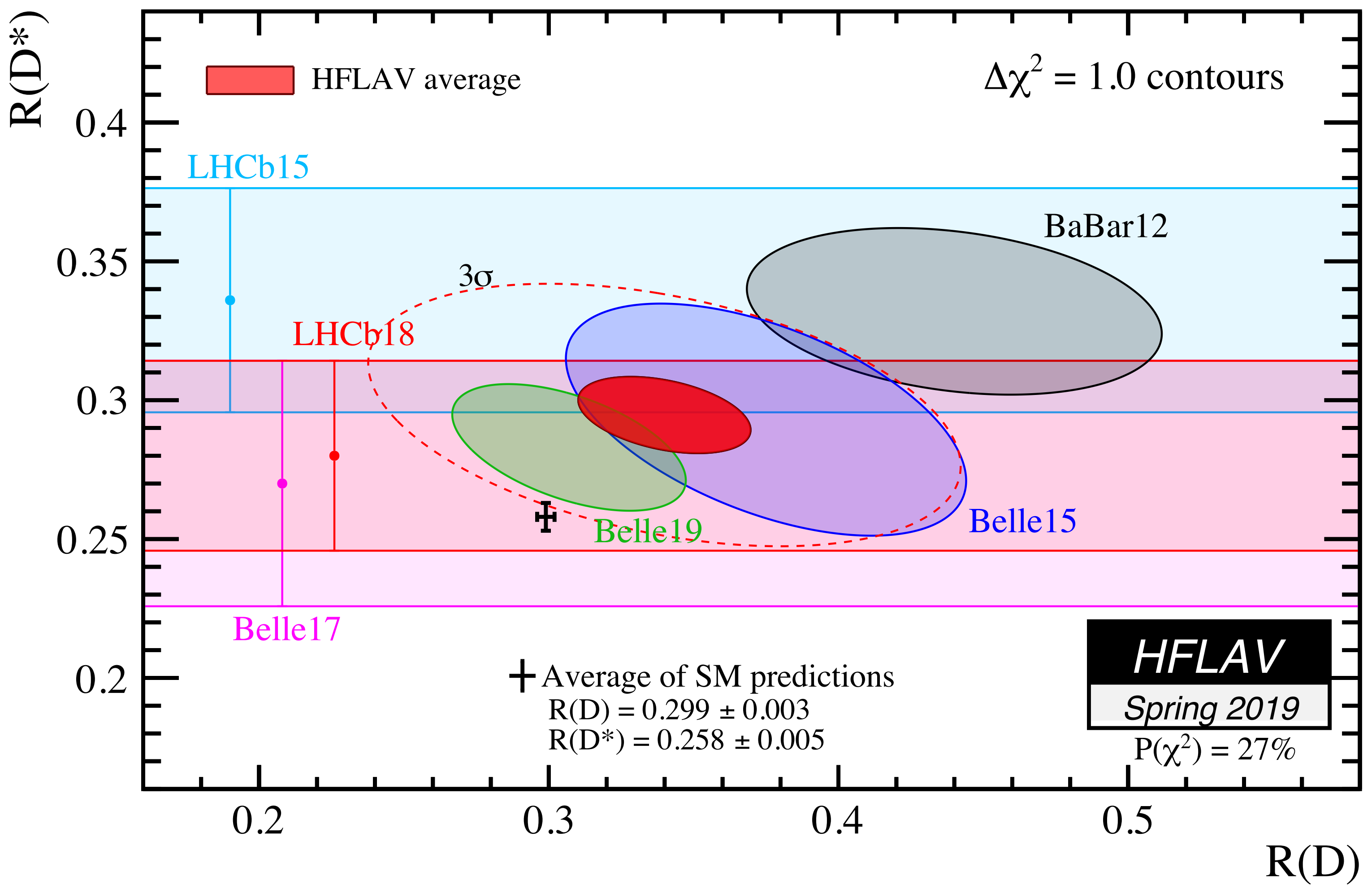
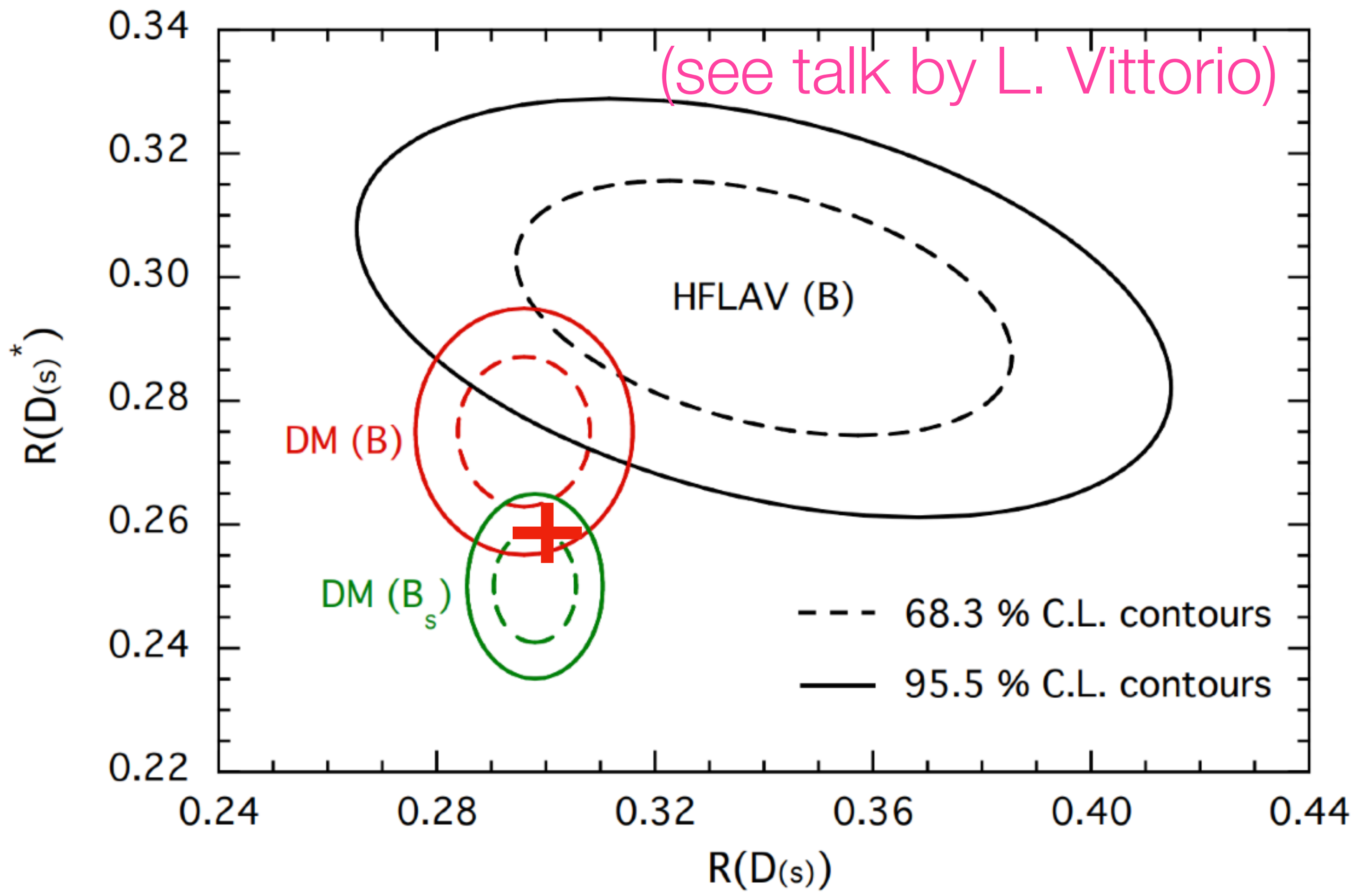
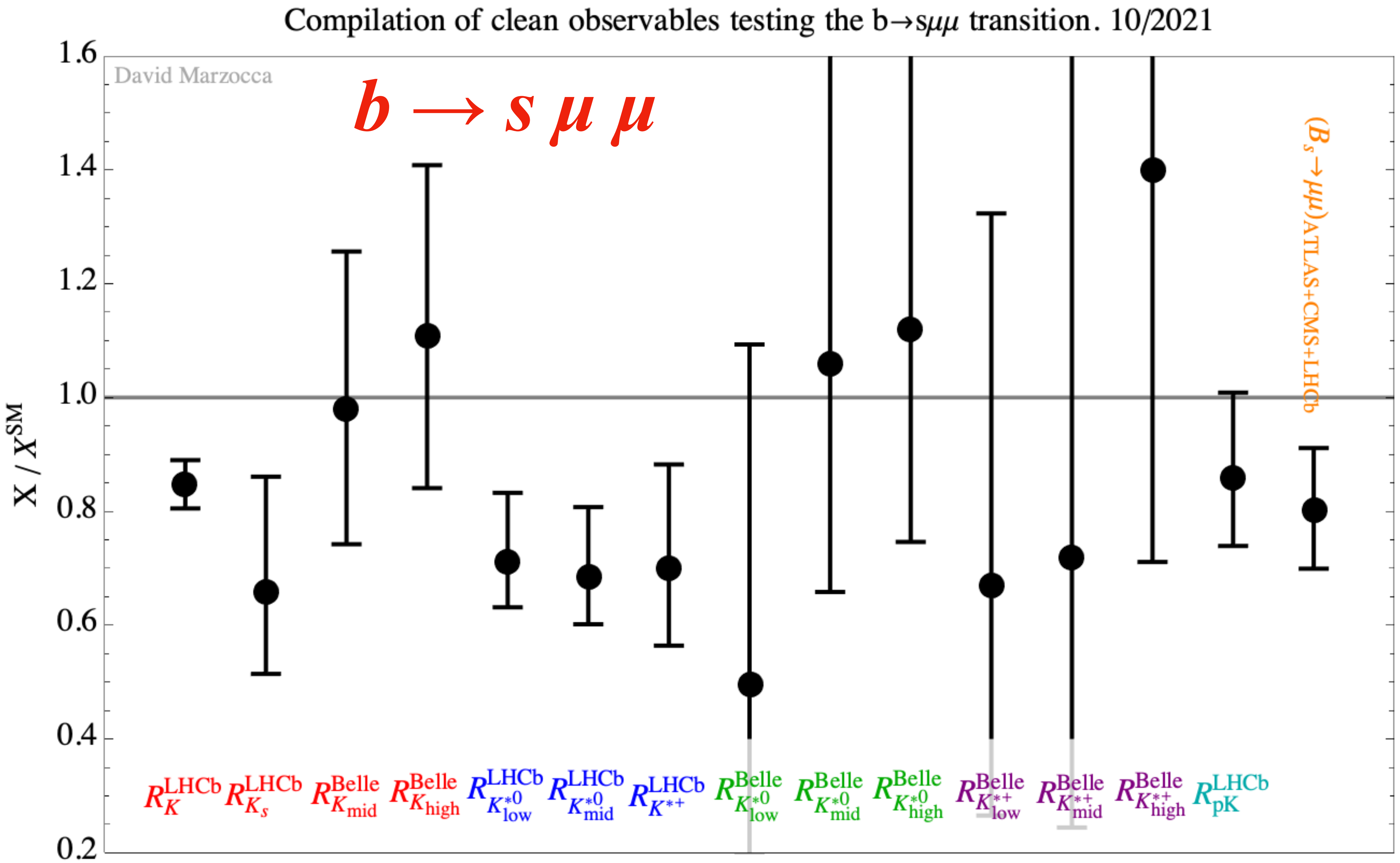
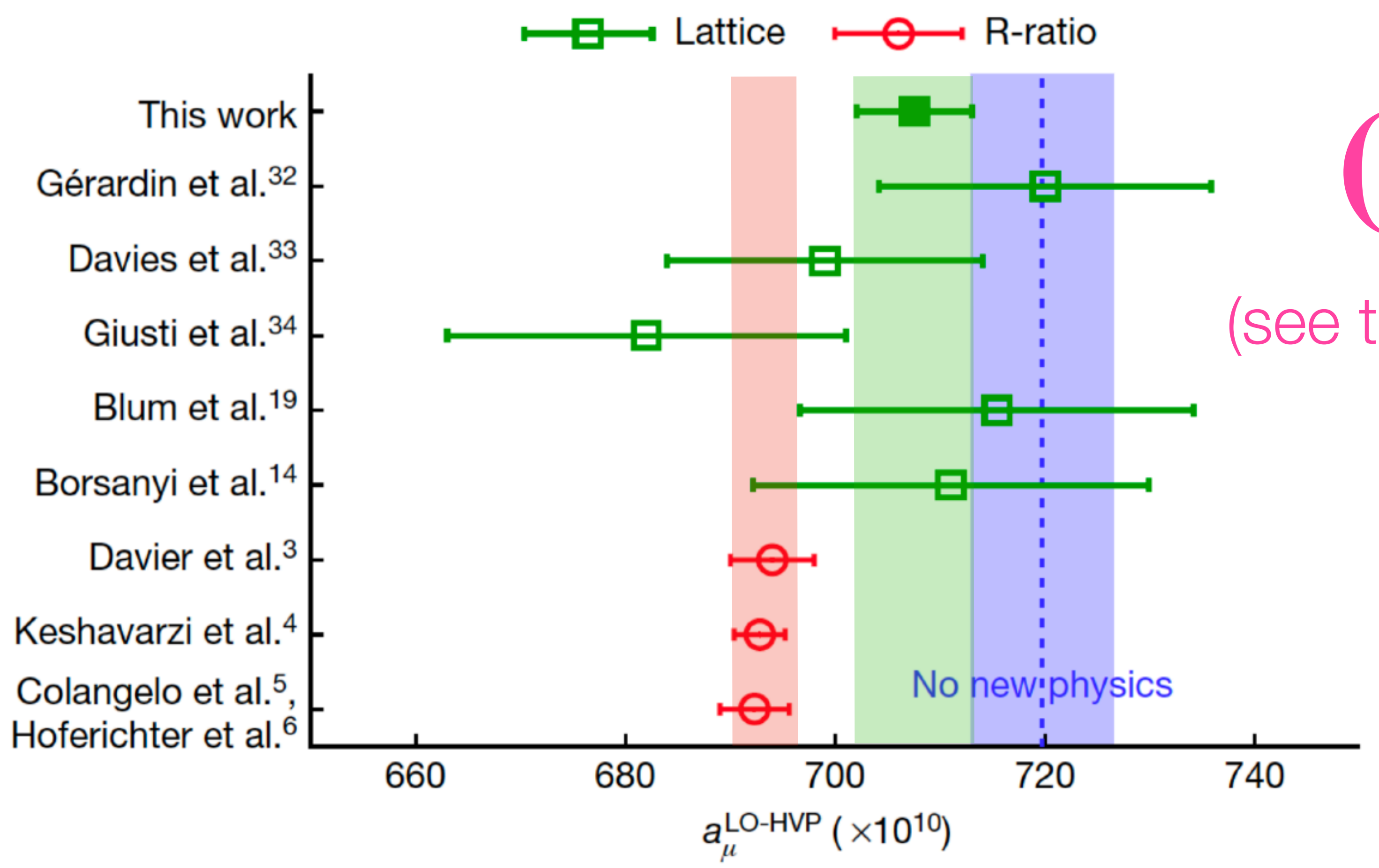






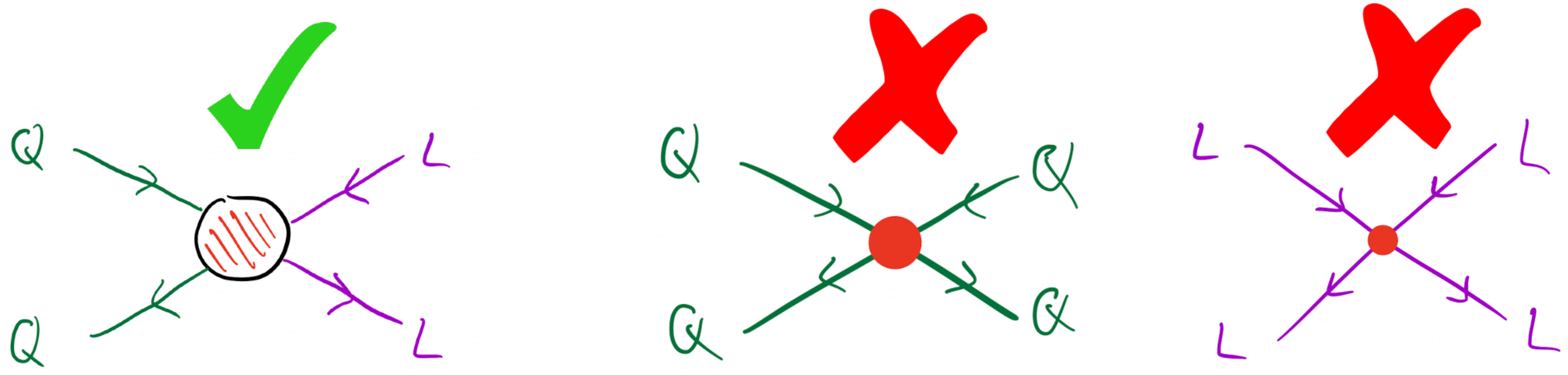


(g-2)μ

(see talks by M. Passera and A. Crivellin)

Leptoquark Renaissance

Deviations in **semileptonic** processes,
strong bounds from $\Delta F=2$ & CLFV processes.



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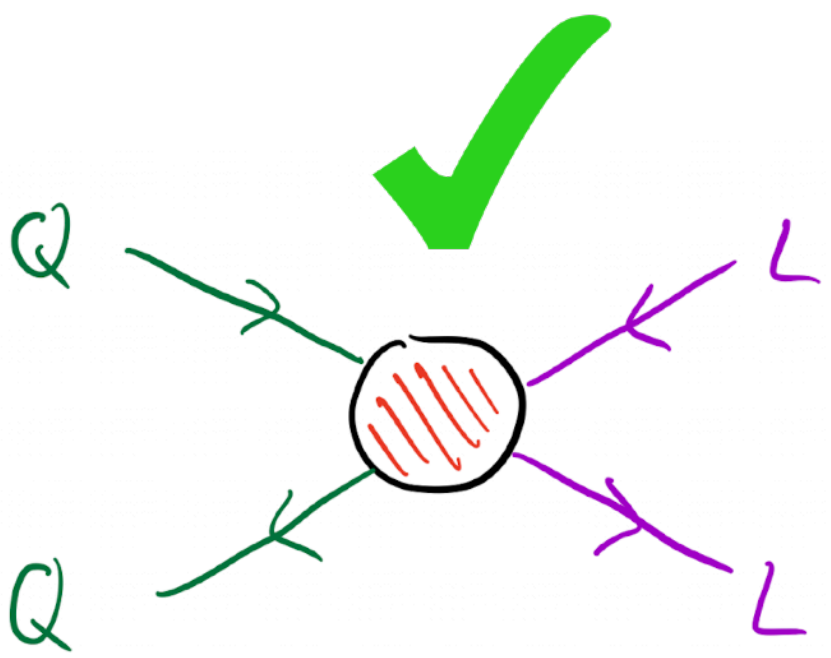

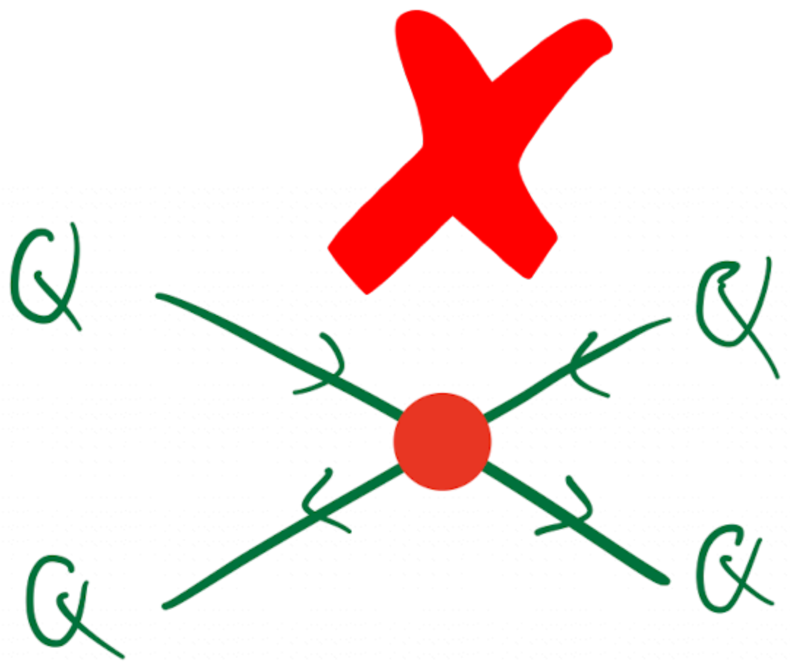

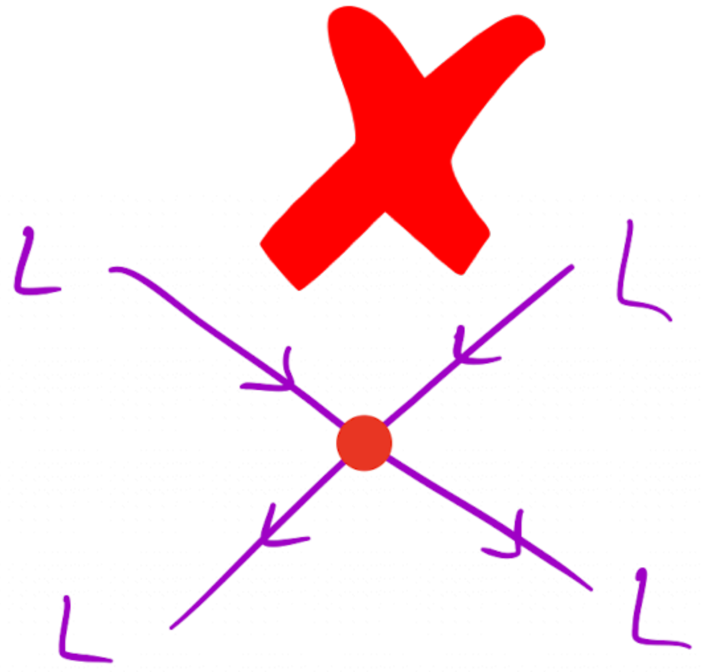

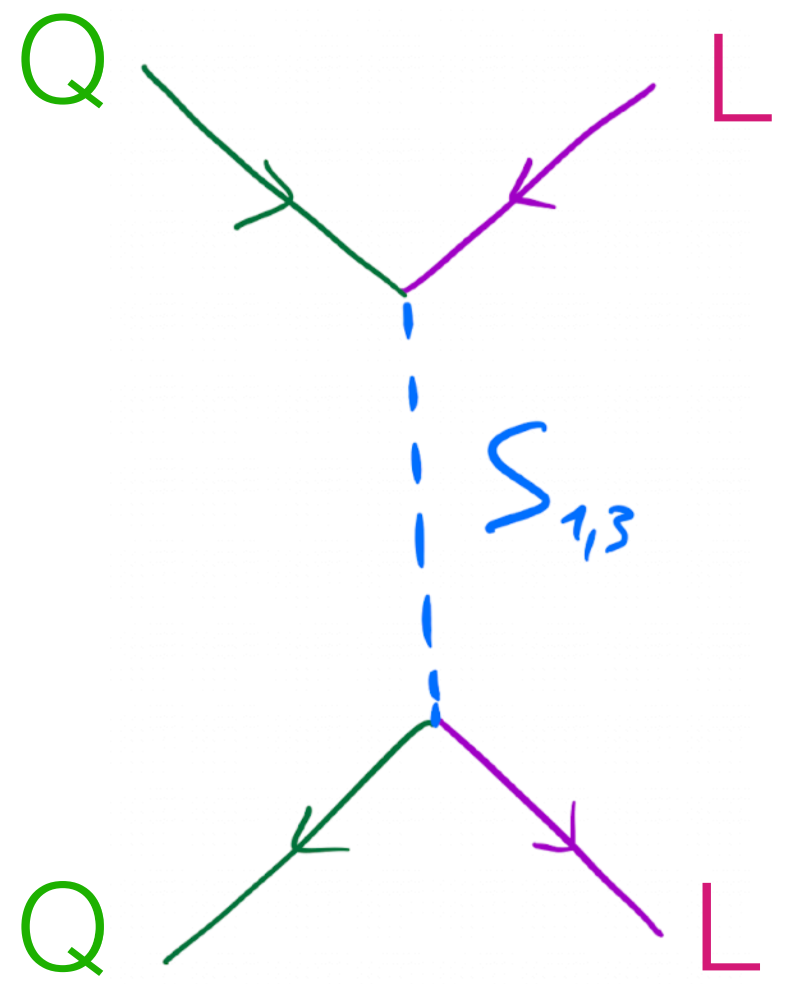

LQ induce semileptonic @ tree level,
4-quark & 4-fermion only at loop level.

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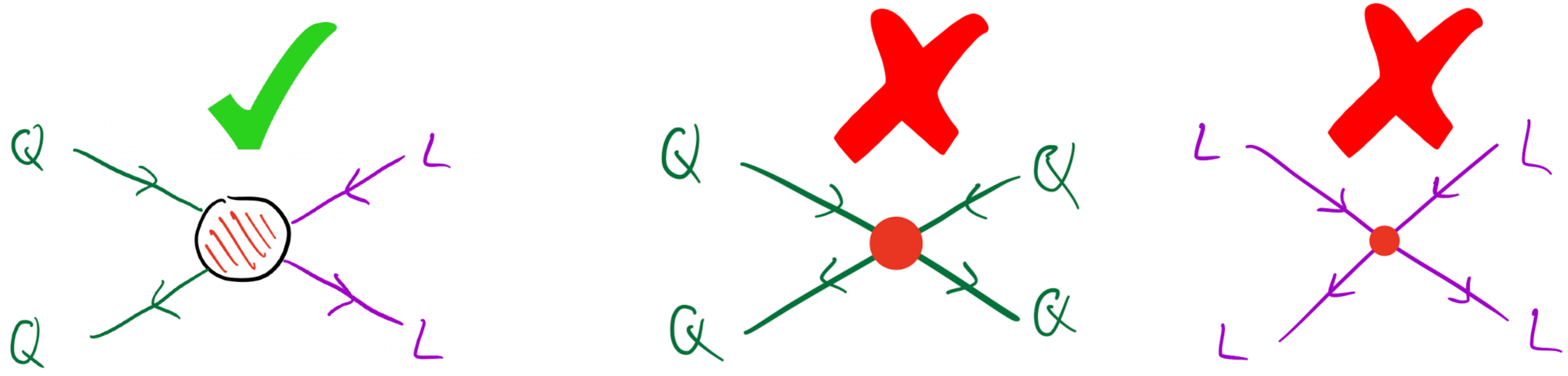

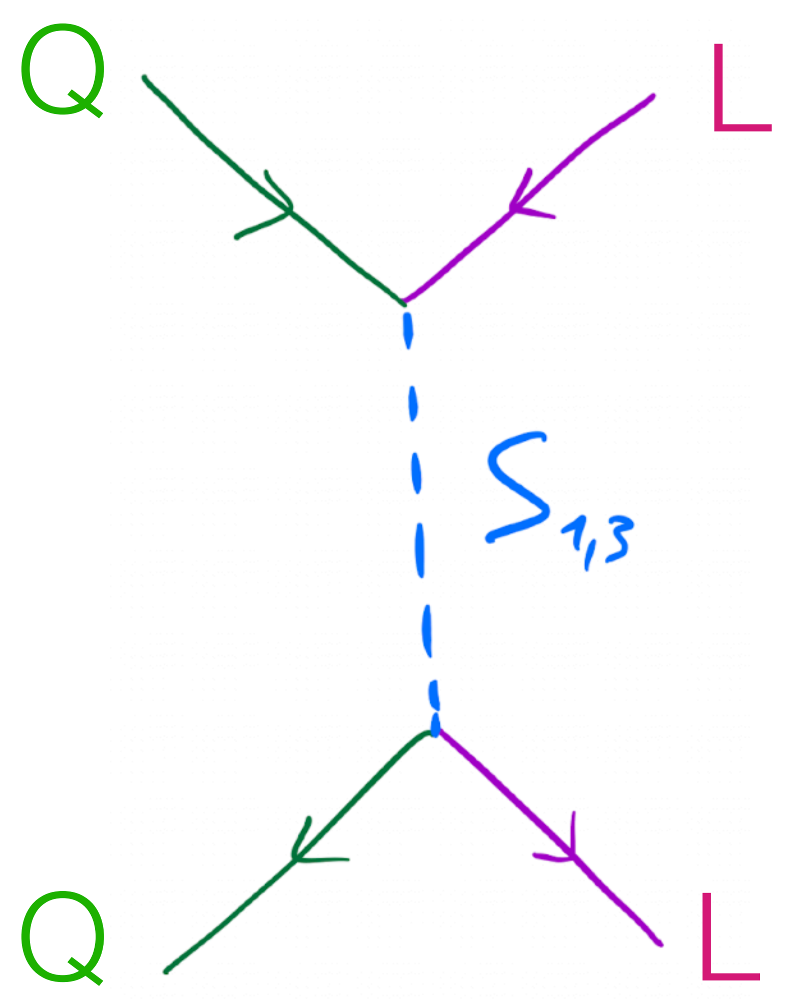

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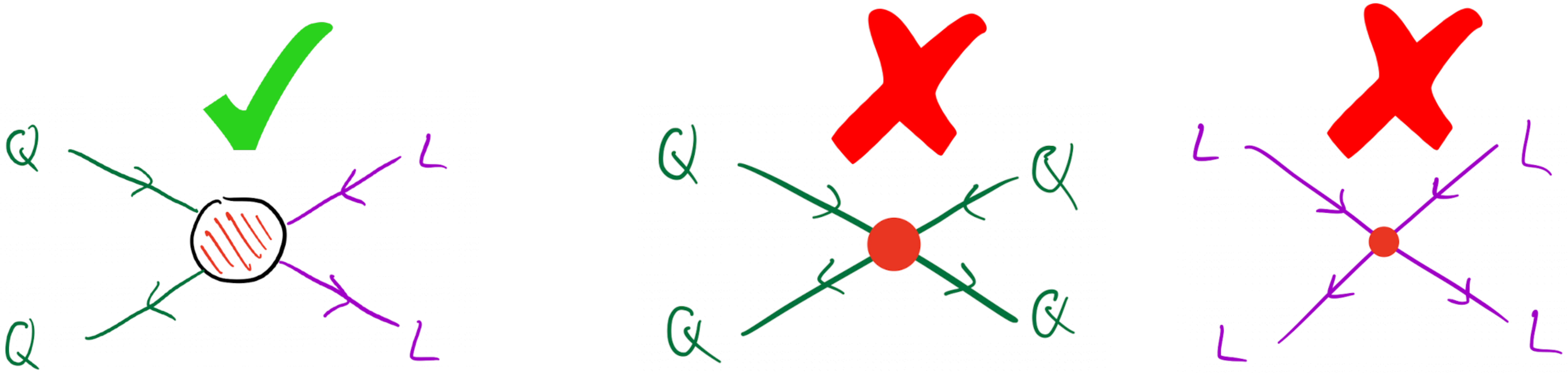

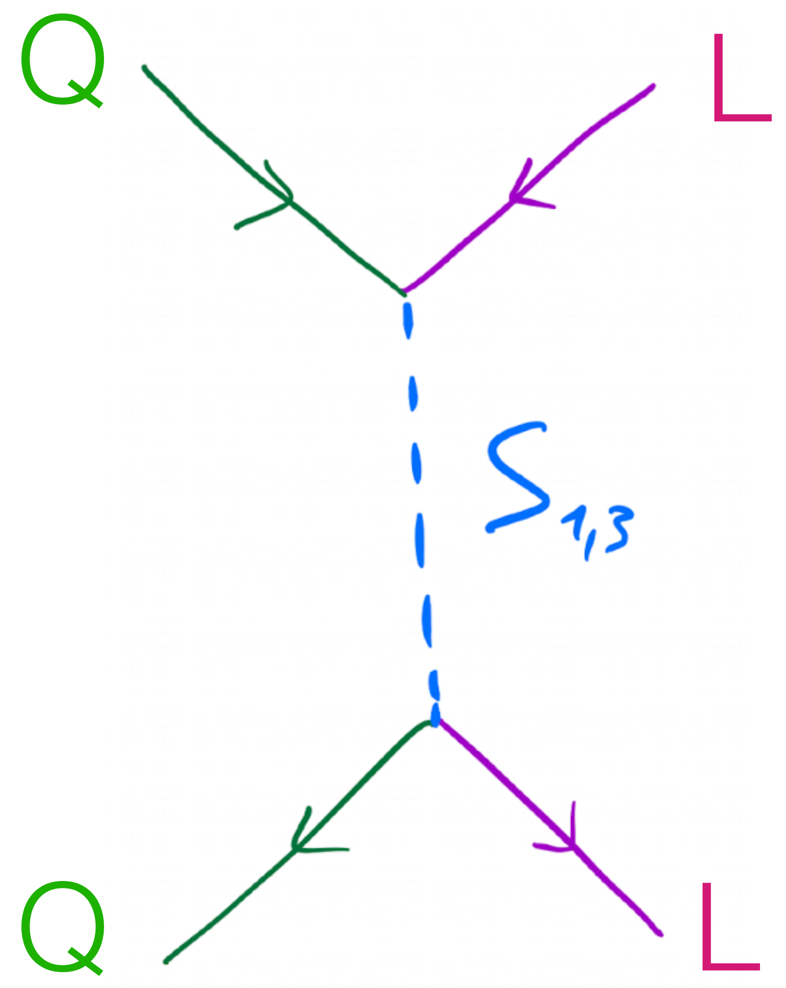

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To address both B-anomalies: (see talk by C. Cornella)

$\boxed{\textbf{TeV-scale leptoquark} \text{ coupled to } \textbf{3rd} \text{ and } \textbf{2nd} \text{ generation} \quad \mathbf{g(3rd) > g(2nd) > g(1st)}}$

From Leptoquarks to the Higgs, and back

From B-anomalies

$M_{LQ} \sim$ TeV

Hierarchical couplings to SM fermions

$$g(\mathbf{3rd}) > g(\mathbf{2nd}) > g(\mathbf{1st})$$

From Leptoquarks to the Higgs, and back

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Higgs & EW hierarchy

$M_{\text{BSM-Higgs hierarchy problem}} \sim$ TeV

Hierarchical Yukawa couplings

$y(3rd) > y(2nd) > y(1st)$

From Leptoquarks to the Higgs, and back

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$\mathbf{M_{LQ}} \sim$ **TeV**

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$\mathbf{M_{BSM\text{-}Higgs\ hierarchy\ problem}} \sim$ **TeV**

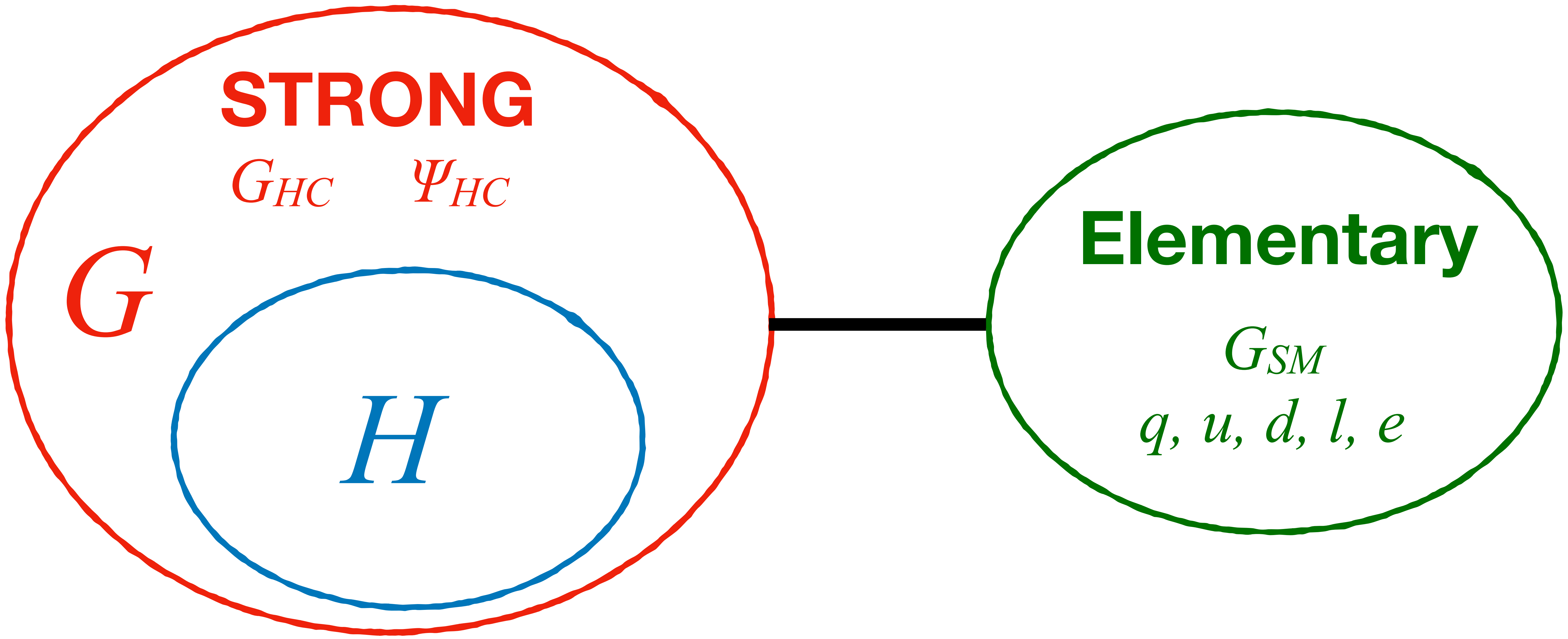
Hierarchical Yukawa couplings

$\mathbf{y(3rd) > y(2nd) > y(1st)}$

LQ from same UV responsible for the EW scale, connection between LQ couplings and Yukawa couplings.

Scalar LQ & Higgs: both pseudo-Goldstones?

In Composite Higgs models the Higgs arises as a pseudo-Goldstone (pNGB) of a spontaneously broken global symmetry G → H of a TeV-scale strong sector



Spontaneous global symmetry breaking at the $f \sim 1$ TeV scale

$$G \to H$$

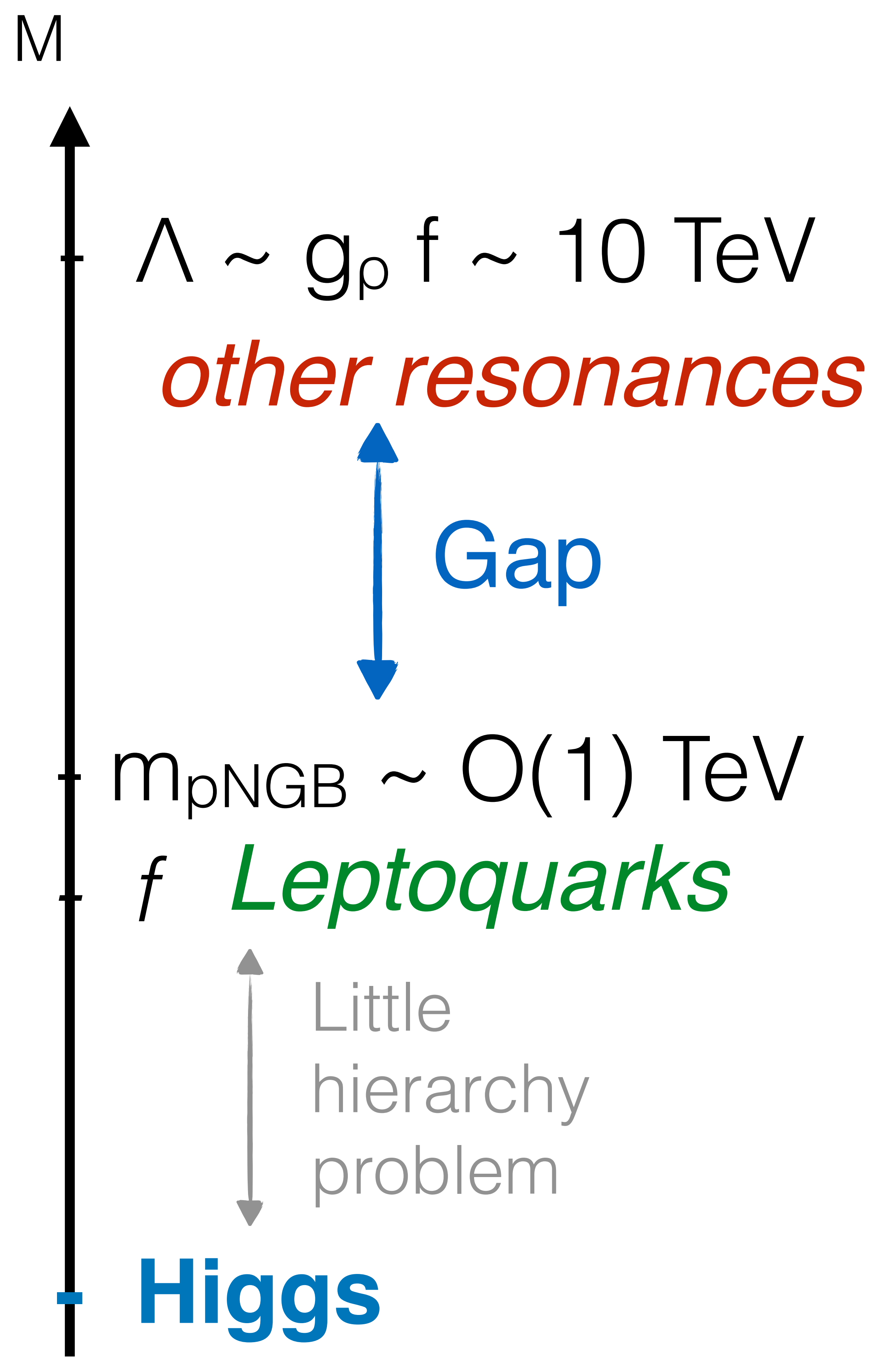
One obtains naturally

$$m_{PNGB} \ll M_{Resonances}$$

Scalar LQ & Higgs: both pseudo-Goldstones?

Scalar LQs could arise as pNGB together with the Higgs from the same G/H of the strong sector.

[Gripaios 0910.1789, Gripaios, Nardecchia, Renner 1412.1791]



Low-energy phenomenology dominated by the LQs

$$m_{SLQ} \ll \Lambda$$

Having the same origin, it is expected that LQ couplings have same structure as Higgs Yukawa couplings:

possible connection with flavour structure

A Fundamental Composite Higgs + LQ Model

D.M. 1803.10972

Gauge group: $\mathrm{SU}(N_{HC}) \times \mathrm{SU}(3)_c \times \mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$

"HyperColor"

Extra vectorlike fermions charged under SU(N_{HC}):

QCD-like!!

	$\mathrm{SU}(N_{HC})$	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_w$	$\mathrm{U}(1)_Y$
Ψ_L	$\mathbf{N_{HC}}$	$\mathbf{1}$	$\mathbf{2}$	Y_L
Ψ_N	$\mathbf{N_{HC}}$	$\mathbf{1}$	$\mathbf{1}$	$Y_L + 1/2$
Ψ_E	$\mathbf{N_{HC}}$	$\mathbf{1}$	$\mathbf{1}$	$Y_L - 1/2$
Ψ_Q	$\mathbf{N_{HC}}$	$\mathbf{3}$	$\mathbf{2}$	$Y_L - 1/3$

For similar constructions see:
Shmaltz et al 1006.1356,
Vecchi 1506.00623,
Ma, Cacciapaglia 1508.07014

SU($N_{\rm HC}$) confines at $\Lambda_{\rm HC} \sim 10$ TeV

Approximate global symmetry, spontaneously broken (as chiral symm. in QCD)

$$G = \mathrm{SU}(10)_{\rm L} \times \mathrm{SU}(10)_{\rm R} \times \mathrm{U}(1)_{\rm V} \xrightarrow[\langle \bar{\Psi}_i \Psi_j \rangle = -B_0 f^2 \delta_{ij}]{f \sim 1\mathrm{TeV}} H = \mathrm{SU}(10)_{\rm V} \times \mathrm{U}(1)_{\rm V}$$

A Fundamental Composite Higgs + LQ Model

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Like QCD pions, the pNGB are composite states of HC-fermion bilinears: $\bar{\Psi}\Psi$

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Several states are present at the **TeV scale** as pNGB, including

Two Higgs doublets: H_{SM} , $\tilde{H}_2$ $\sim \mathbf{(1,2)}_{1/2}$

Singlet and Triplet LQ: $S_1 \sim \mathbf{(3,1)}_{-1/3}$ + $S_1 \sim \mathbf{(3,3)}_{-1/3}$

H and LQ are close partners!!

$$H_1 \sim i\sigma^2(\bar{\Psi}_L \Psi_N)$$
$$H_2 \sim (\bar{\Psi}_E \Psi_L)$$
$$S_1 \sim (\bar{\Psi}_Q \Psi_L)$$
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Coupling with SM fermions from 4-fermion operators

$$\mathcal{L}_{4-\mathrm{Fermi}} \sim \frac{c_{\psi\Psi}}{\Lambda_t^2} \bar\psi_{\mathrm{SM}} \psi_{\mathrm{SM}} \bar\Psi\Psi \quad \xrightarrow{E \lesssim \Lambda_{HC}} \quad \sim y_{\psi\phi}\, \bar\psi_{\mathrm{SM}} \psi_{\mathrm{SM}}\, \phi + \ldots$$

Yukawas & LQ couplings

+ approximate $U(2)^5$ flavor symmetry to protect from unwanted flavor violation

Phenomenology of S_1 and S_3

Scalar Leptoquarks

$S_1 = (\bar{\mathbf{3}}, \mathbf{1}, 1/3)$,
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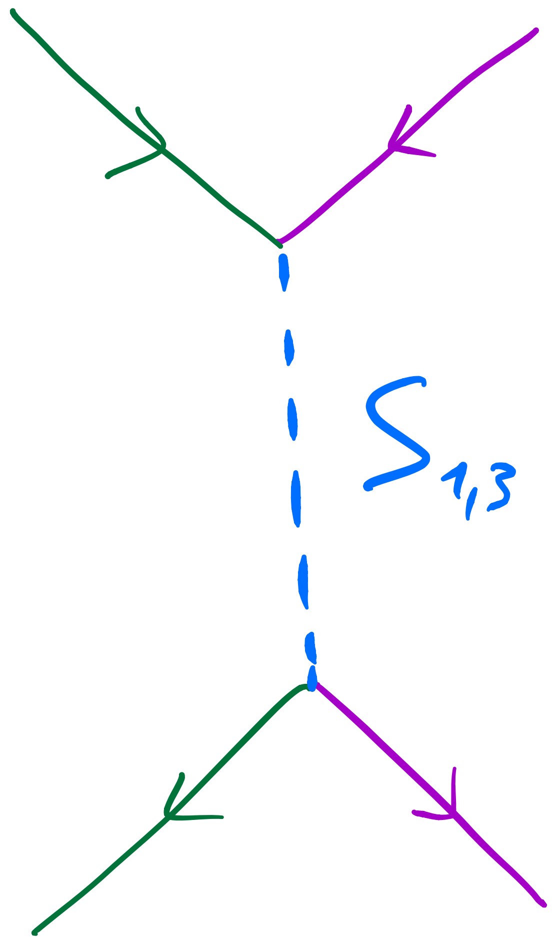

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$$\mathcal{L}_{int} \sim \left(\lambda^{1L}_{ij}\, q^i_L\, \epsilon\, \ell^j_L + \lambda^{1R}_{ij}\, u^i_R\, e^j_R\right) S_1 + \lambda^{3L}_{ij}\, q^i_L\, \epsilon\, \sigma^A\, \ell^j_L\, S_3^A + h.c.$$

1) Match **SM + S_1+S_3** to **SMEFT** @ 1-loop

(SMEFT RGE, SMEFT-LEFT 1-loop matching, LEFT RGE already done in literature)

V. Gherardi, E. Venturini, D.M. [2003.12525]

(see talk by F. Wilsch)

[Alonso, Jenkins, Manohar, Trott ’13]
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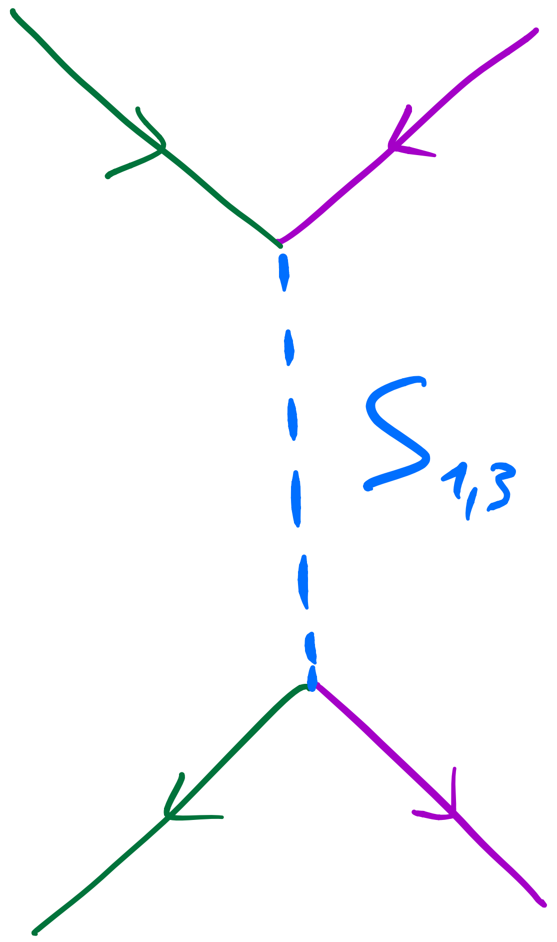

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2) Global analysis of B-anomalies + all relevant observables

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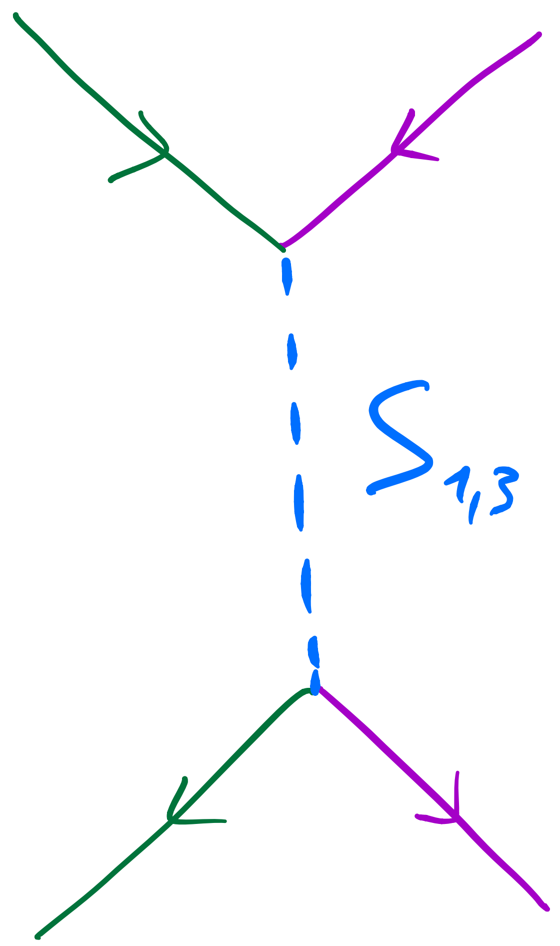

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3) Include 1st gen couplings and study **Kaon** & $\mu \to e$ observables assuming $U(2)^5$ flavor symmetry.

S. Trifinopoulos, E. Venturini, D.M. [2106.15630]

S_1 and S_3 - global analysis

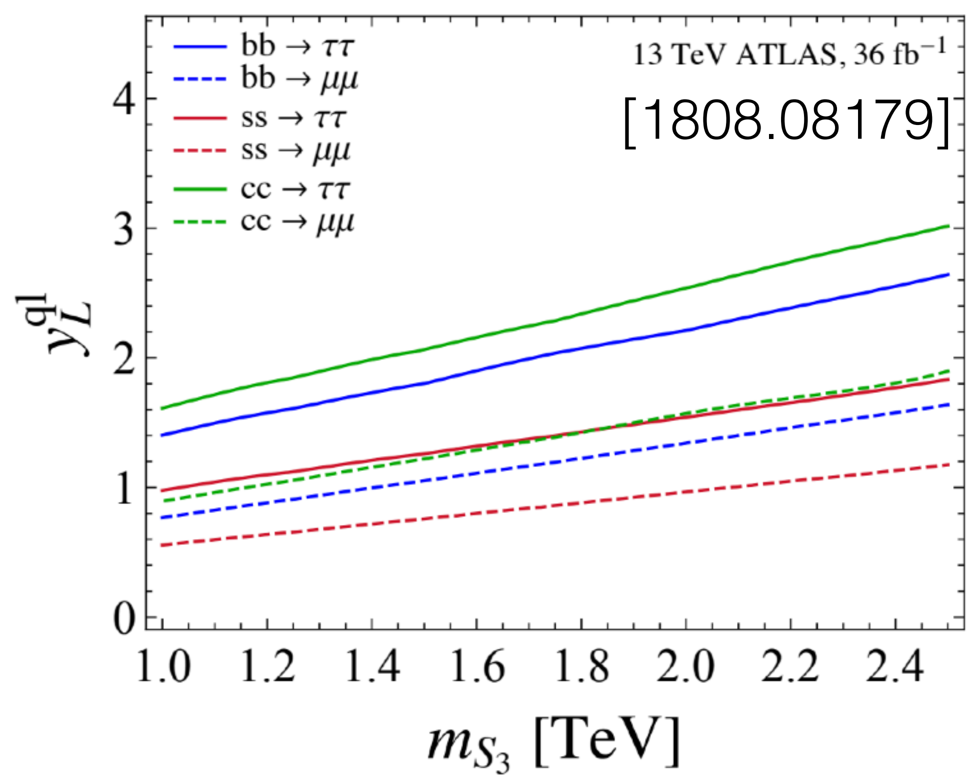
Using the complete one-loop matching to SMEFT, we include in our analysis the following observables.

All these are used to build a **global likelihood**.

$$-2\log\mathcal{L} \equiv \chi^2(\lambda_x, M_x) = \sum_i \frac{(\mathcal{O}_i(\lambda_x, M_x) - \mu_i)^2}{\sigma_i^2}$$

Observable	Experimental bounds
Z boson couplings	App. A.12
$\delta g^Z_{\mu_L}$	$(0.3 \pm 1.1)10^{-3}$ [99]
$\delta g^Z_{\mu_R}$	$(0.2 \pm 1.3)10^{-3}$ [99]
$\delta g^Z_{\tau_L}$	$(-0.11 \pm 0.61)10^{-3}$ [99]
$\delta g^Z_{\tau_R}$	$(0.66 \pm 0.65)10^{-3}$ [99]
$\delta g^Z_{b_L}$	$(2.9 \pm 1.6)10^{-3}$ [99]
$\delta g^Z_{c_R}$	$(-3.3 \pm 5.1)10^{-3}$ [99]
N_ν	2.9963 ± 0.0074 [100]

Drell-Yan



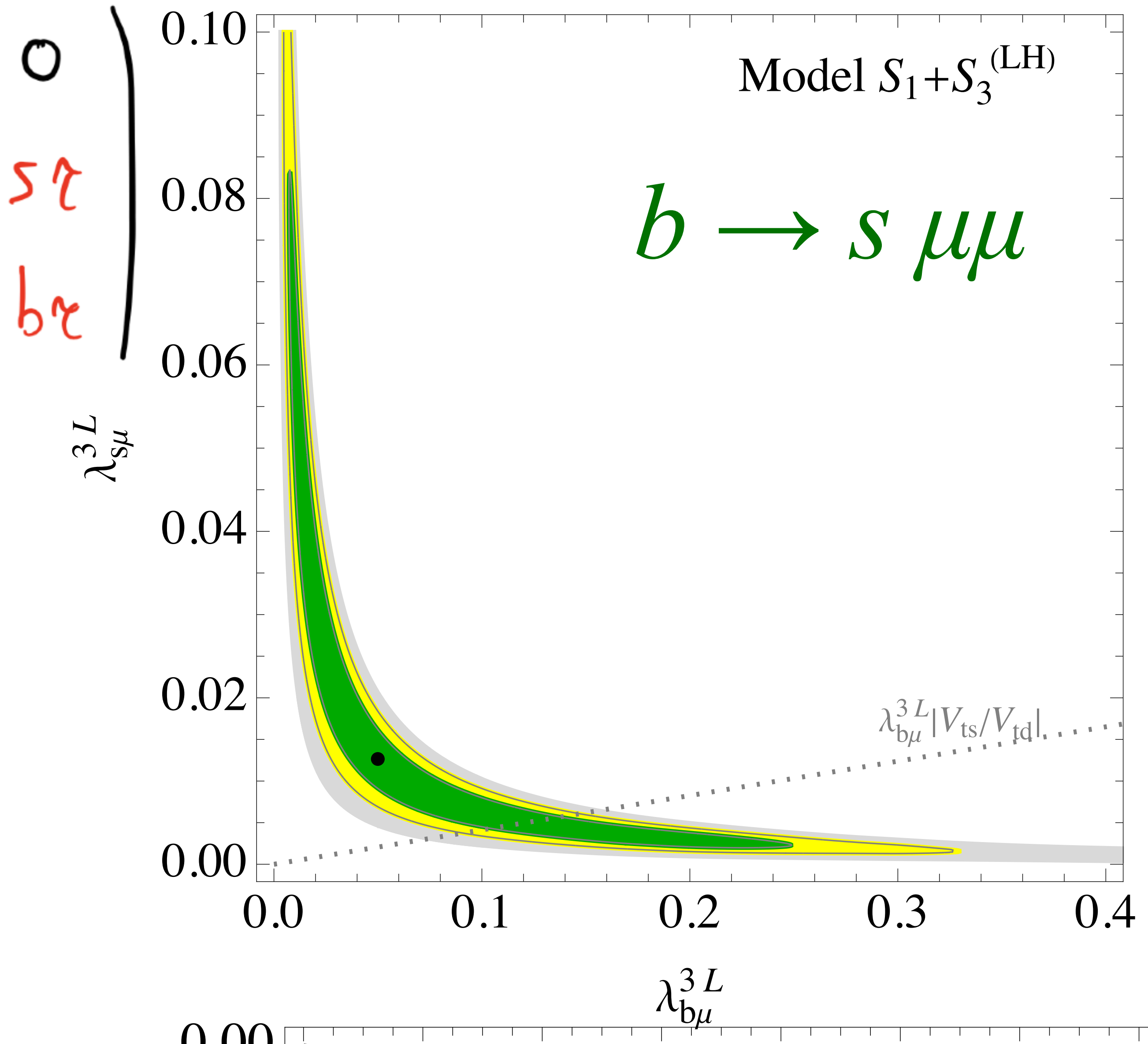
Observable	SM prediction	Experimental bounds
$b \to s\ell\ell$ observables		[37]
$\Delta\mathcal{C}_9^{sb\mu\mu}$	0	-0.43 ± 0.09 [79]
$\mathcal{C}_9^{\rm univ}$	0	-0.48 ± 0.24 [79]
$b \to c\tau(\ell)\nu$ observables		[37]
R_D	0.299 ± 0.003 [12]	$0.34 \pm 0.027 \pm 0.013$ [12]
R_D^*	0.258 ± 0.005 [12]	$0.295 \pm 0.011 \pm 0.008$ [12]
$P_\tau^{D^*}$	-0.488 ± 0.018 [80]	$-0.38 \pm 0.51 \pm 0.2 \pm 0.018$ [7]
F_L	0.470 ± 0.012 [80]	$0.60 \pm 0.08 \pm 0.038 \pm 0.012$ [81]
$\mathcal{B}(B_c^+ \to \tau^+\nu)$	2.3%	$< 10\%$ (95% CL) [82]
$R_D^{\mu/e}$	1	0.978 ± 0.035 [83, 84]
$b \to s\nu\nu$ and $s \to d\nu\nu$		[37]
R_K^ν	1 [85]	< 4.7 [86]
$R_{K^*}^\nu$	1 [85]	< 3.2 [86]
$b \to d\mu\mu$ and $b \to dee$		App. A.5
$\mathcal{B}(B^0 \to \mu\mu)$	$(1.06 \pm 0.09) \times 10^{-10}$ [87, 88]	$(1.1 \pm 1.4) \times 10^{-10}$ [89, 90]
$\mathcal{B}(B^+ \to \pi^+\mu\mu)$	$(2.04 \pm 0.21) \times 10^{-8}$ [87, 88]	$(1.83 \pm 0.24) \times 10^{-8}$ [89, 90]
$\mathcal{B}(B^0 \to ee)$	$(2.48 \pm 0.21) \times 10^{-15}$ [87, 88]	$< 8.3 \times 10^{-8}$ [51]
$\mathcal{B}(B^+ \to \pi^+ee)$	$(2.04 \pm 0.24) \times 10^{-8}$ [87, 88]	$< 8 \times 10^{-8}$ [51]
B LFV decays		[37]
$\mathcal{B}(B_d \to \tau^\pm\mu^\mp)$	0	$< 1.4 \times 10^{-5}$ [91]
$\mathcal{B}(B_s \to \tau^\pm\mu^\mp)$	0	$< 4.2 \times 10^{-5}$ [91]
$\mathcal{B}(B^+ \to K^+\tau^-\mu^+)$	0	$< 5.4 \times 10^{-5}$ [92]
$\mathcal{B}(B^+ \to K^+\tau^+\mu^-)$	0	$< 3.3 \times 10^{-5}$ [92] $< 4.5 \times 10^{-5}$ [93]

Observable	SM prediction	Experimental bounds
D leptonic decay		[37] and App. A.4
$\mathcal{B}(D_s \to \tau\nu)$	$(5.169 \pm 0.004) \times 10^{-2}$ [94]	$(5.48 \pm 0.23) \times 10^{-2}$ [51]
$\mathcal{B}(D^0 \to \mu\mu)$	$\approx 10^{-11}$ [95]	$< 7.6 \times 10^{-9}$ [96]
$\mathcal{B}(D^+ \to \pi^+\mu\mu)$	$\mathcal{O}(10^{-12})$ [97]	$< 7.4 \times 10^{-8}$ [98]
Rare Kaon decays ($\nu\nu$)		App. A.1
$\mathcal{B}(K^+ \to \pi^+\nu\nu)$	8.64×10^{-11} [99]	$(11.0 \pm 4.0) \times 10^{-11}$ [100]
$\mathcal{B}(K_L \to \pi^0\nu\nu)$	3.4×10^{-11} [99]	$< 3.6 \times 10^{-9}$ [101]
Rare Kaon decays ($\ell\ell$)		App. A.3 and A.2
$\mathcal{B}(K_L \to \mu\mu)_{SD}$	8.4×10^{-10} [102]	$< 2.5 \times 10^{-9}$ [76]
$\mathcal{B}(K_S \to \mu\mu)$	$(5.18 \pm 1.5) \times 10^{-12}$ [76, 103, 104]	$< 2.5 \times 10^{-10}$ [105]
$\mathcal{B}(K_L \to \pi^0\mu\mu)$	$(1.5 \pm 0.3) \times 10^{-11}$ [106]	$< 4.5 \times 10^{-10}$ [107]
$\mathcal{B}(K_L \to \pi^0 ee)$	$(3.2^{+1.2}_{-0.8}) \times 10^{-11}$ [108]	$< 2.8 \times 10^{-10}$ [109]
LFV in Kaon decays		App. A.3 and A.2
$\mathcal{B}(K_L \to \mu e)$	0	$< 4.7 \times 10^{-12}$ [110]
$\mathcal{B}(K^+ \to \pi^+\mu^-e^+)$	0	$< 7.9 \times 10^{-11}$ [111]
$\mathcal{B}(K^+ \to \pi^+e^-\mu^+)$	0	$< 1.5 \times 10^{-11}$ [112]
CP-violation		App. A.8
ϵ'_K/ϵ_K	$(15 \pm 7) \times 10^{-4}$ [113]	$(16.6 \pm 2.3) \times 10^{-4}$ [51]

Observable	SM prediction	Experimental bounds
$\Delta F = 2$ processes		[37]
$B^0 - \overline{B}^0$: $\lvert C^1_{B_d}\rvert$	0	$< 9.1 \times 10^{-7}$ TeV^{-2} [114, 115]
$B^0_s - \overline{B}^0_s$: $\lvert C^1_{B_s}\rvert$	0	$< 2.0 \times 10^{-5}$ TeV^{-2} [114, 115]
$K^0 - \overline{K}^0$: Re$[C^1_K]$	0	$< 8.0 \times 10^{-7}$ TeV^{-2} [114, 115]
$K^0 - \overline{K}^0$: Im$[C^1_K]$	0	$< 3.0 \times 10^{-9}$ TeV^{-2} [114, 115]
$D^0 - \overline{D}^0$: Re$[C^1_D]$	0	$< 3.6 \times 10^{-7}$ TeV^{-2} [114, 115]
$D^0 - \overline{D}^0$: Im$[C^1_D]$	0	$< 2.2 \times 10^{-8}$ TeV^{-2} [114, 115]
$D^0 - \overline{D}^0$: Re$[C^4_D]$	0	$< 3.2 \times 10^{-8}$ TeV^{-2} [114, 115]
$D^0 - \overline{D}^0$: Im$[C^4_D]$	0	$< 1.2 \times 10^{-9}$ TeV^{-2} [114, 115]
$D^0 - \overline{D}^0$: Re$[C^5_D]$	0	$< 2.7 \times 10^{-7}$ TeV^{-2} [114, 115]
$D^0 - \overline{D}^0$: Im$[C^5_D]$	0	$< 1.1 \times 10^{-8}$ TeV^{-2} [114, 115]
LFU in τ decays		[37]
$\lvert g_\mu/g_e\rvert^2$	1	1.0036 ± 0.0028 [116]
$\lvert g_\tau/g_\mu\rvert^2$	1	1.0022 ± 0.0030 [116]
$\lvert g_\tau/g_e\rvert^2$	1	1.0058 ± 0.0030 [116]
LFV observables		[37]
$\mathcal{B}(\tau \to \mu\phi)$	0	$< 1.00 \times 10^{-7}$ [117]
$\mathcal{B}(\tau \to 3\mu)$	0	$< 2.5 \times 10^{-8}$ [118]
$\mathcal{B}(\tau \to \mu\gamma)$	0	$< 5.2 \times 10^{-8}$ [119]
$\mathcal{B}(\tau \to e\gamma)$	0	$< 3.9 \times 10^{-8}$ [119]
$\mathcal{B}(\mu \to e\gamma)$	0	$< 5.0 \times 10^{-13}$ [120]
$\mathcal{B}(\mu \to 3e)$	0	$< 1.2 \times 10^{-12}$ [121]
$\mathcal{B}^{(\rm Ti)}_{\mu e}$	0	$< 5.1 \times 10^{-12}$ [122]
$\mathcal{B}^{(\rm Au)}_{\mu e}$	0	$< 8.3 \times 10^{-13}$ [123]
EDMs		[37]
$\lvert d_e\rvert$	$< 10^{-44}$ e · cm [124, 125]	$< 1.3 \times 10^{-29}$ e · cm [126]
$\lvert d_\mu\rvert$	$< 10^{-42}$ e · cm [125]	$< 1.9 \times 10^{-19}$ e · cm [127]
d_τ	$< 10^{-41}$ e · cm [125]	$(1.15 \pm 1.70) \times 10^{-17}$ e · cm [37]
d_n	$< 10^{-33}$ e · cm [128]	$< 2.1 \times 10^{-26}$ e · cm [129]
Anomalous Magnetic Moments		[37]
$a_e - a_e^{SM}$	$\pm 2.3 \times 10^{-13}$ [130, 131]	$(-8.9 \pm 3.6) \times 10^{-13}$ [132]
$a_\mu - a_\mu^{SM}$	$\pm 43 \times 10^{-11}$ [42]	$(279 \pm 76) \times 10^{-11}$ [40, 42]
$a_\tau - a_\tau^{SM}$	$\pm 3.9 \times 10^{-8}$ [130]	$(-2.1 \pm 1.7) \times 10^{-7}$ [133]

S_1 and S_3 — only LH couplings

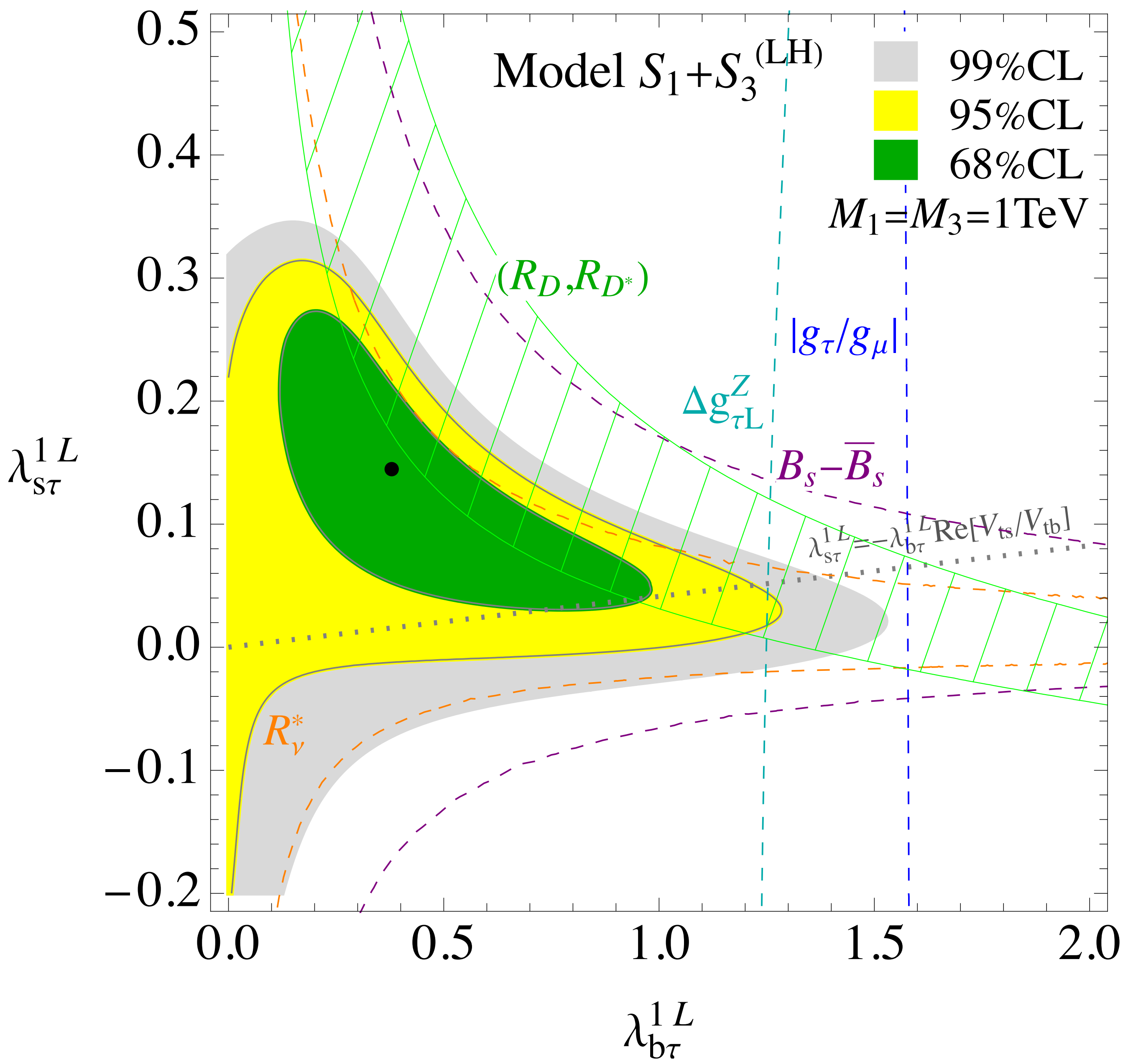
$$\lambda^{1L} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & s\tau \\ 0 & 0 & b\tau \end{pmatrix} \quad \lambda^{3L} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s\mu & s\tau \\ 0 & b\mu & b\tau \end{pmatrix}$$

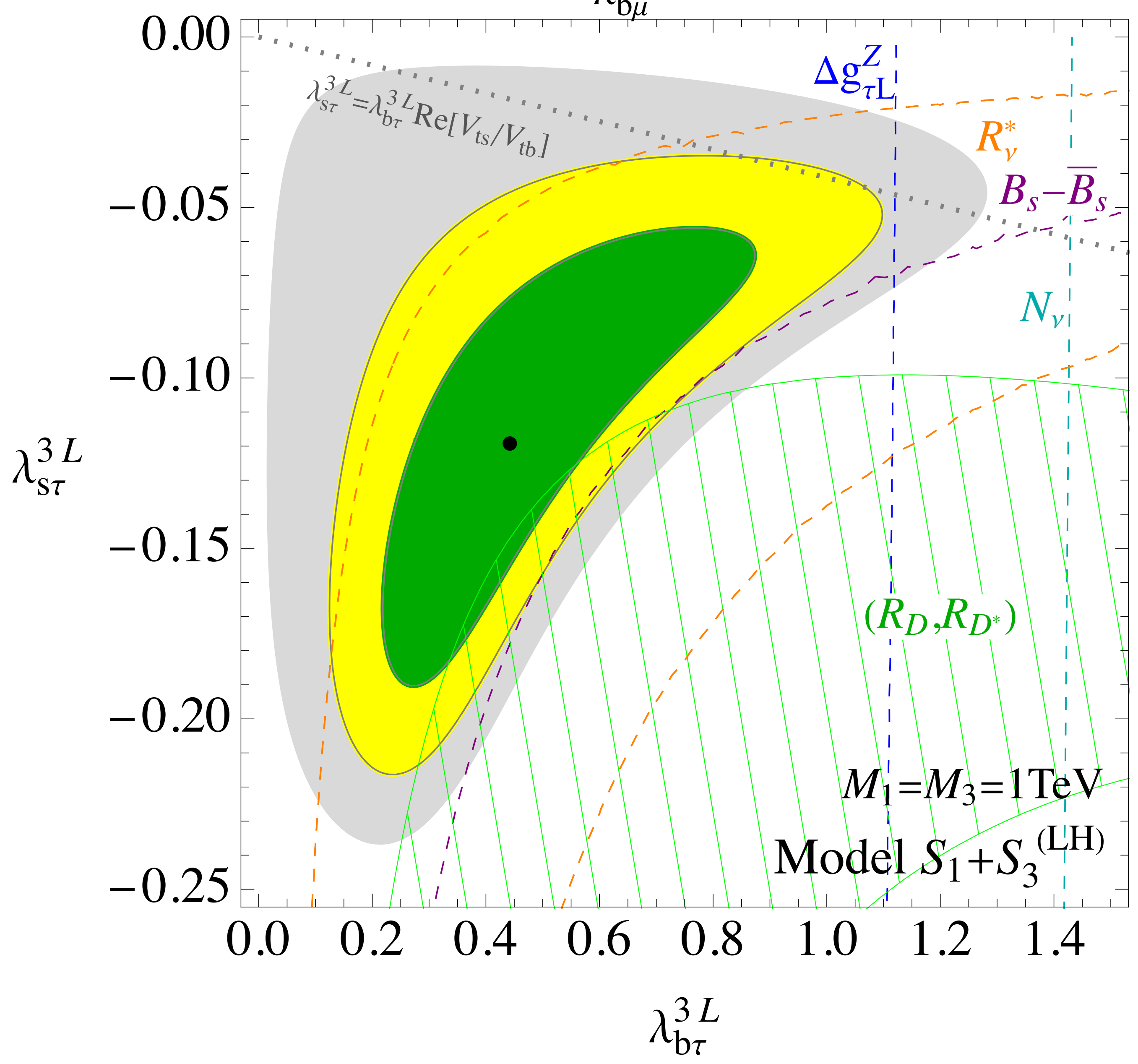


$\lambda^{1R} = 0$ → Cannot fit $(g-2)_\mu$

(see backup slides for a S_1+S_3 scenario that addresses also the muon magnetic moment)

R(D(*))





Plots updated w.r.t. [v3:2008.09548]

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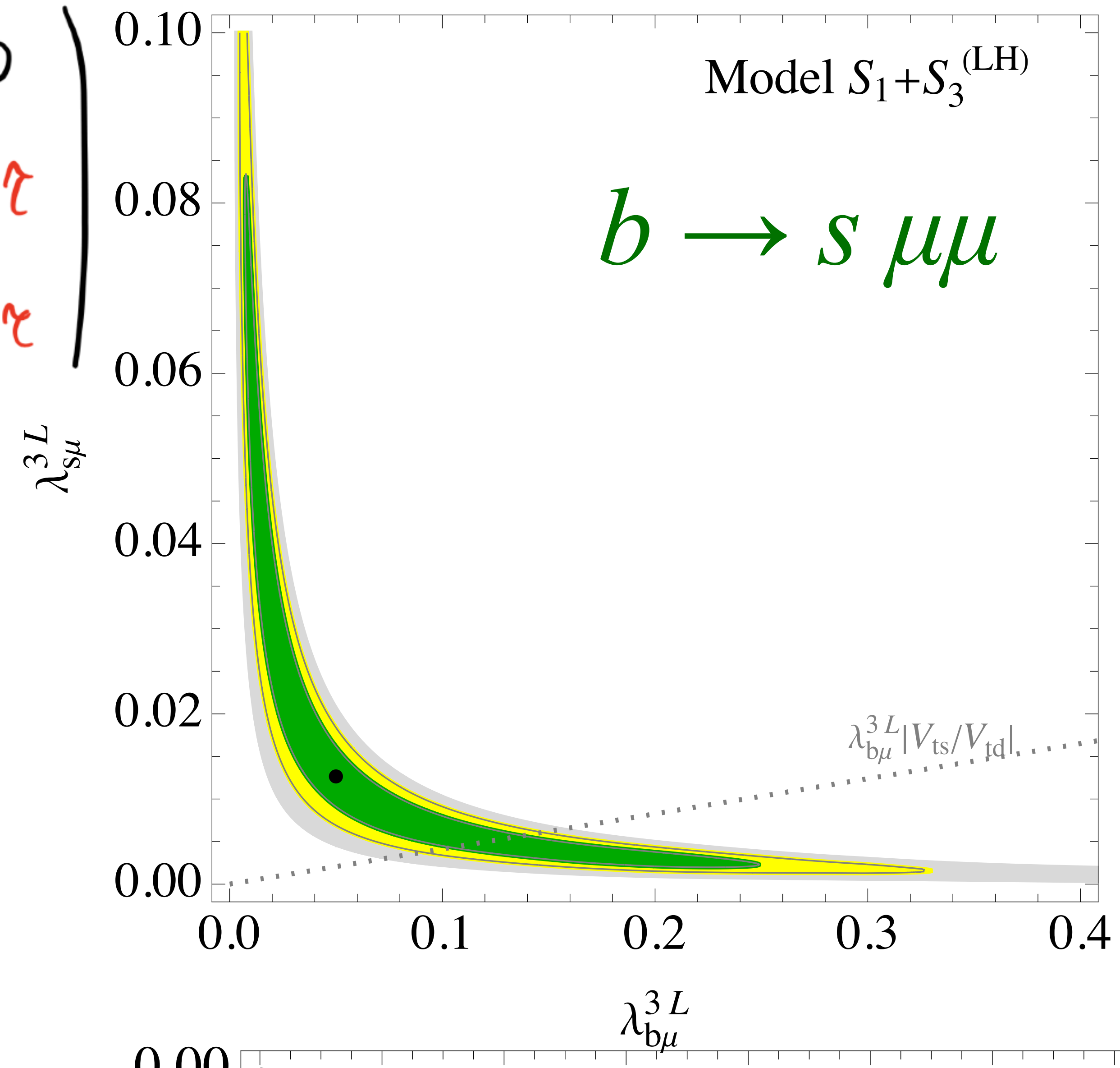
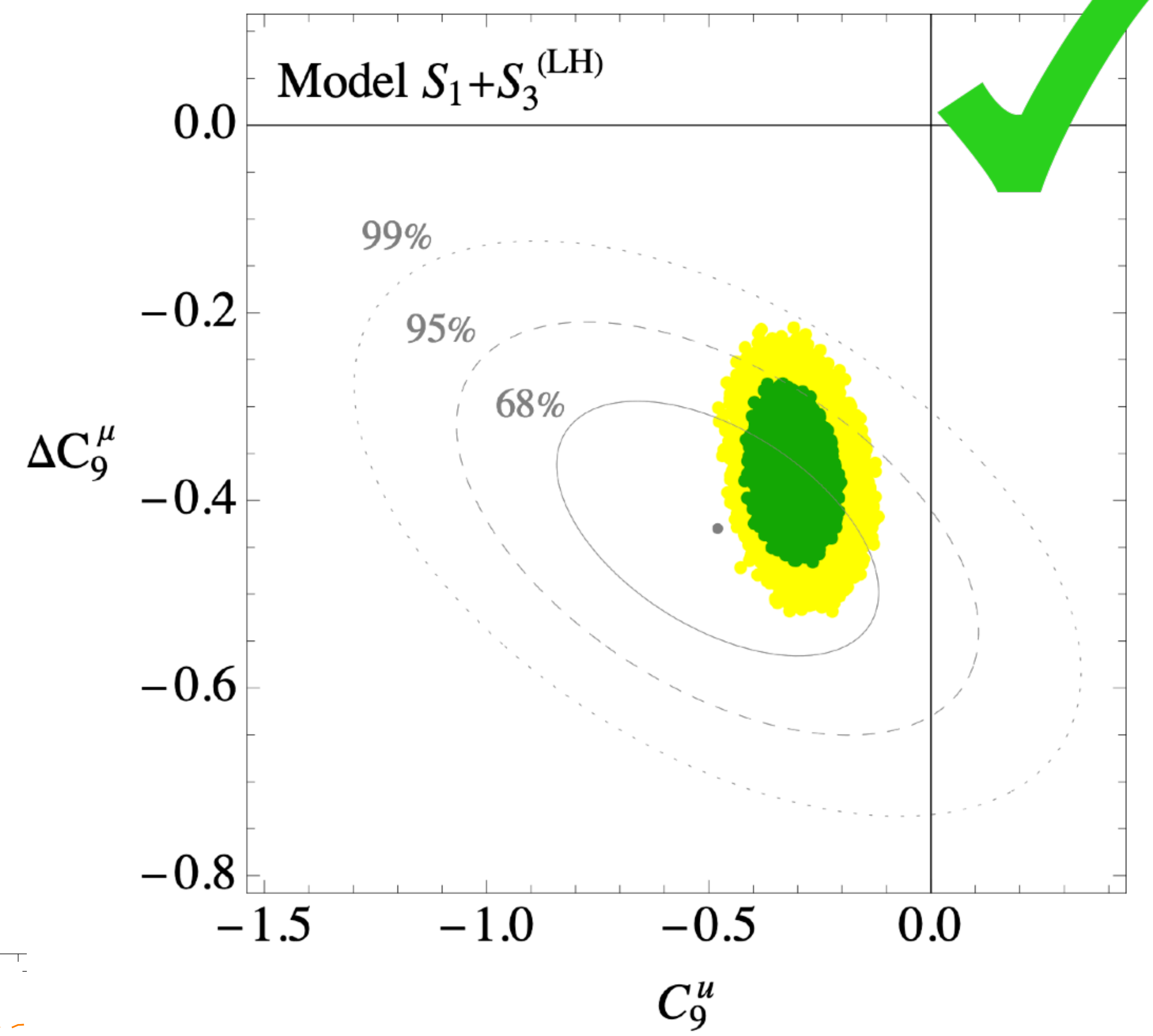
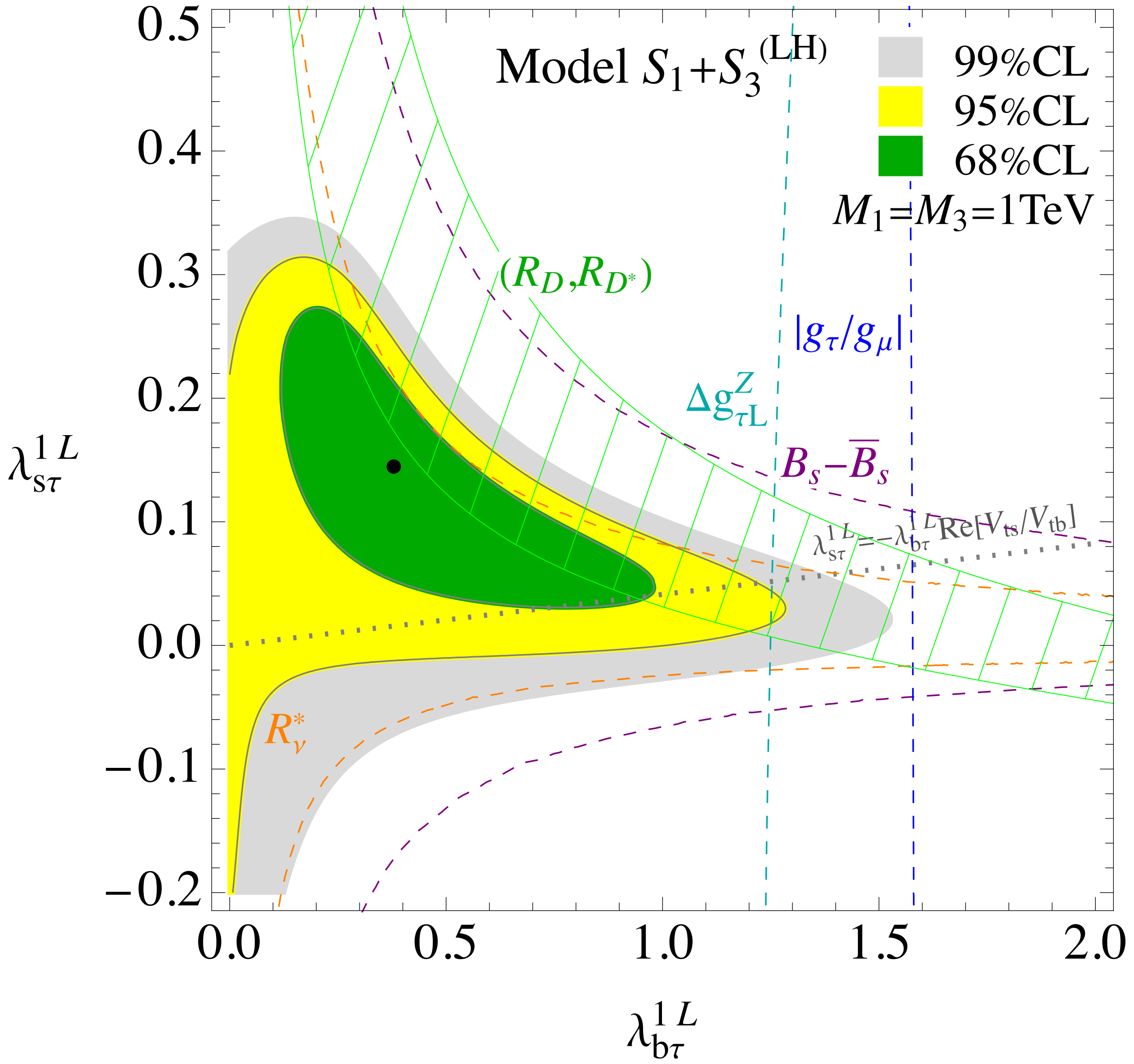
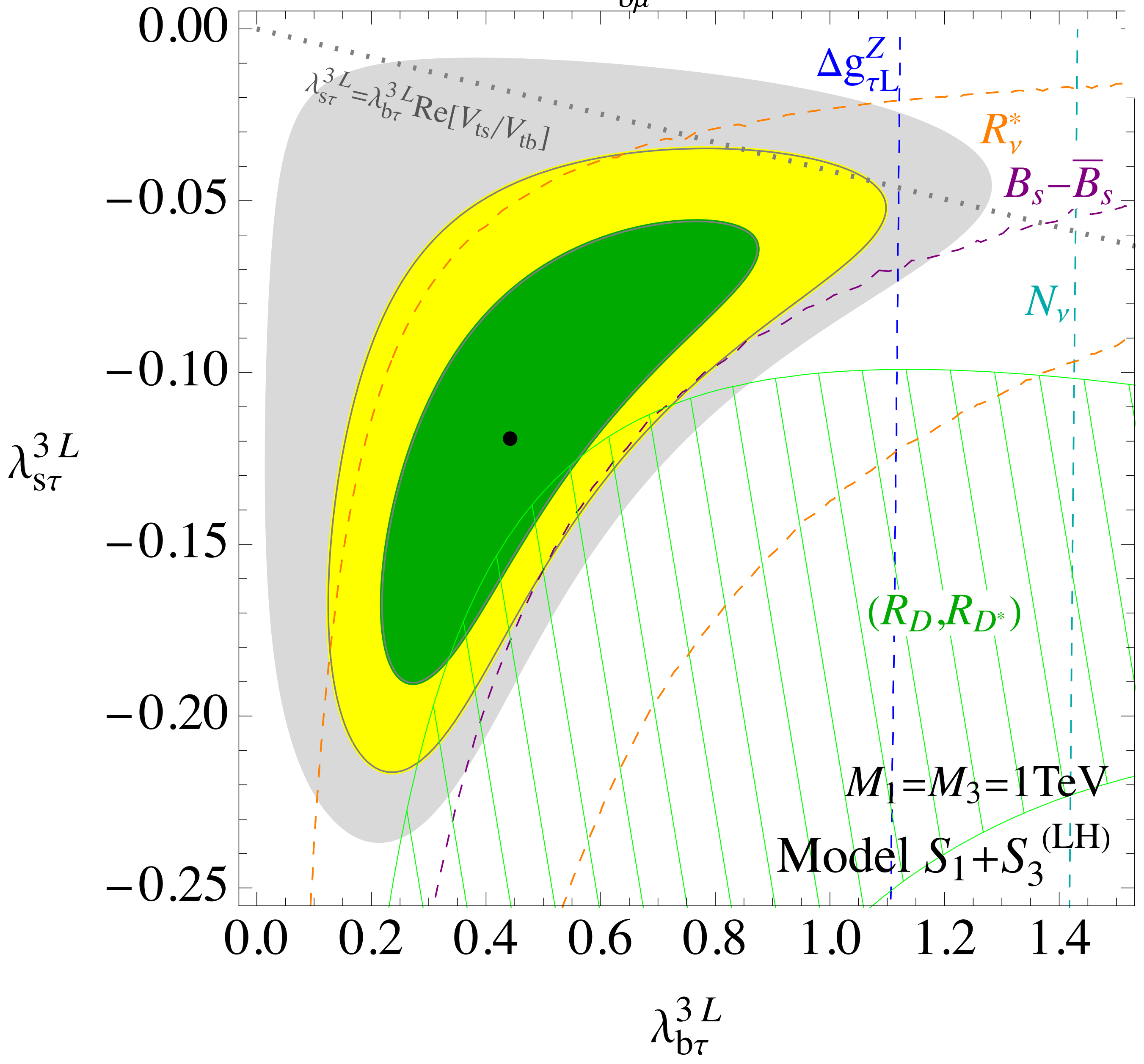


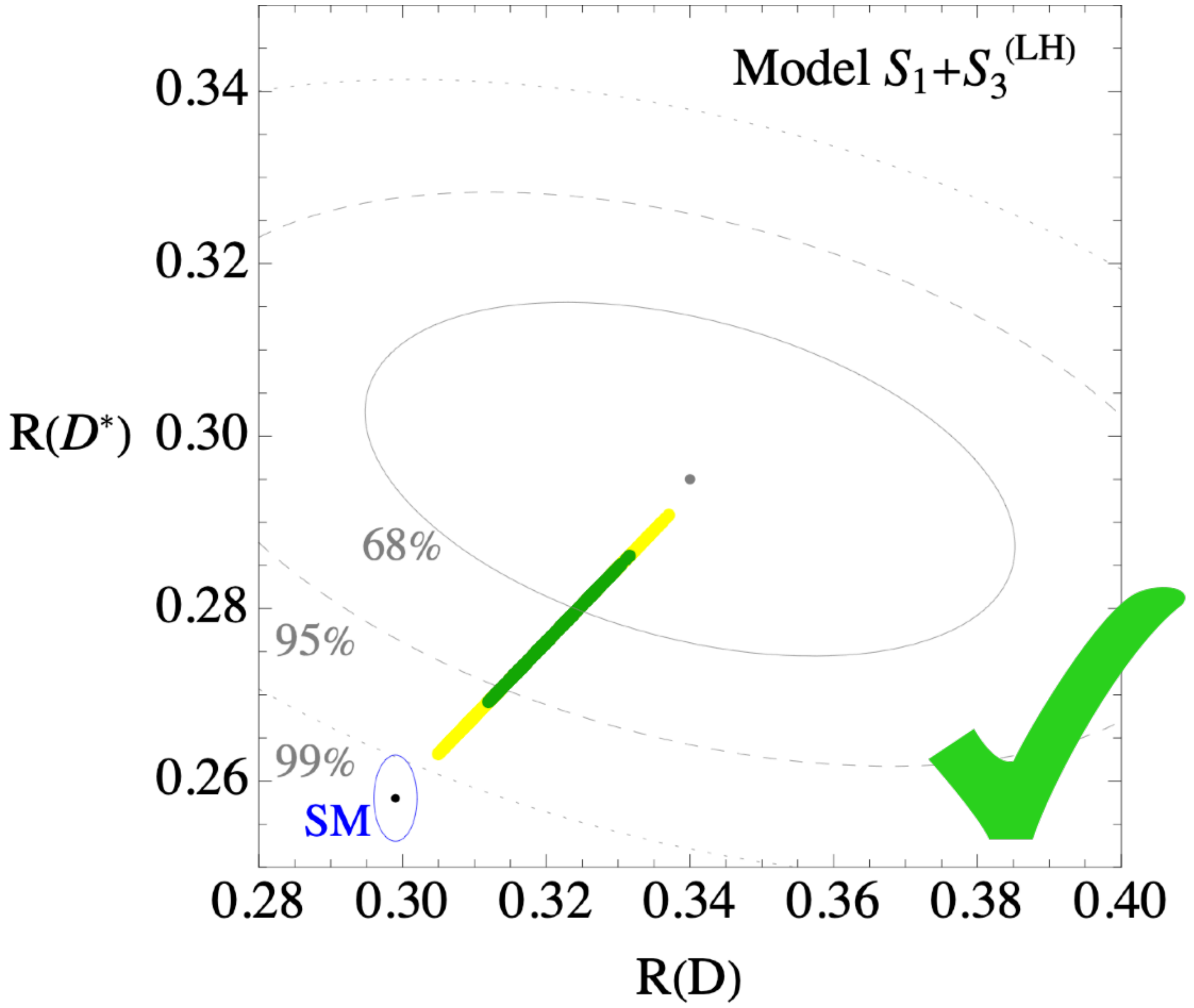


very good fit of B-anomalies

R(D(*))







Plots updated w.r.t. [v3:2008.09548]

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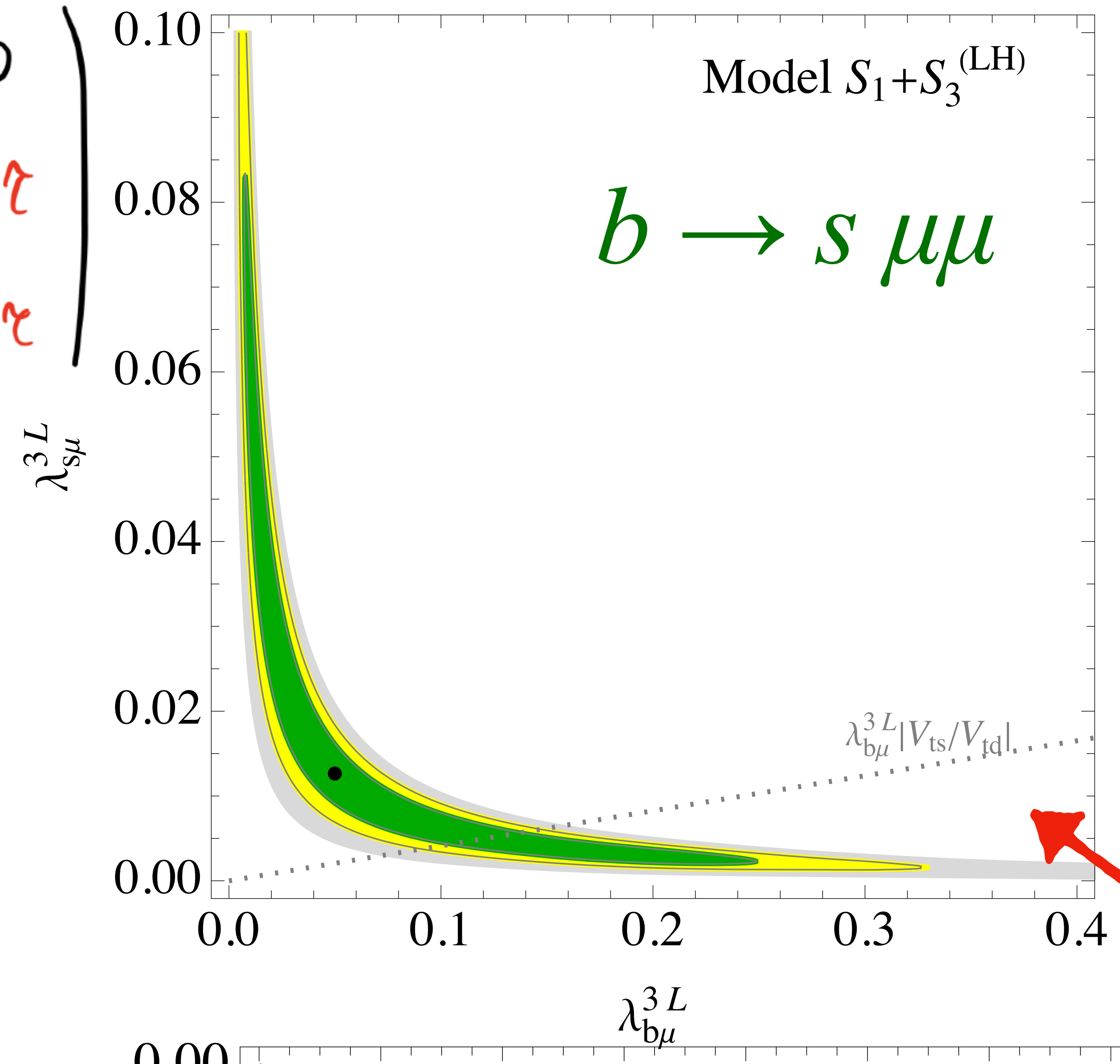
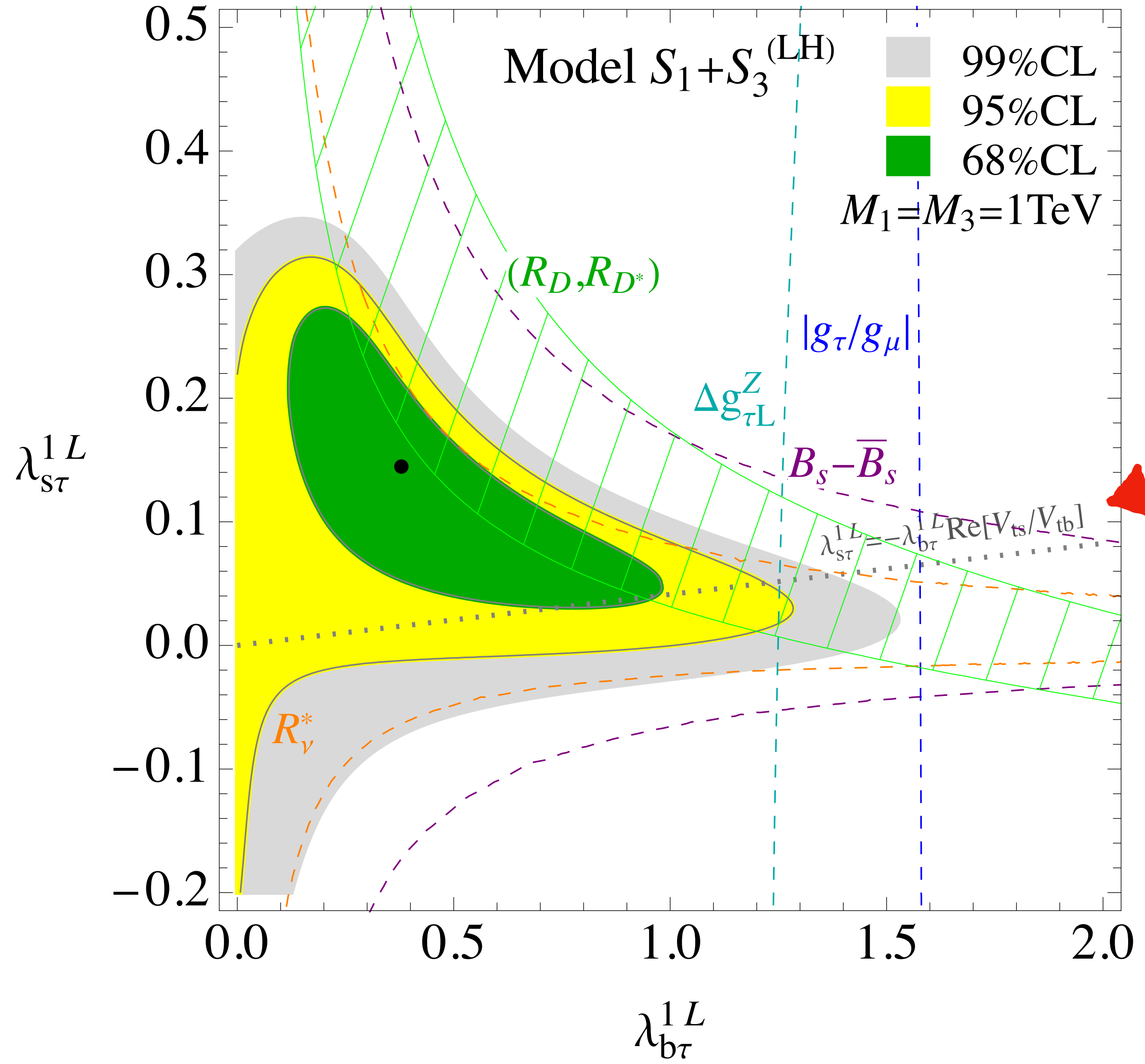
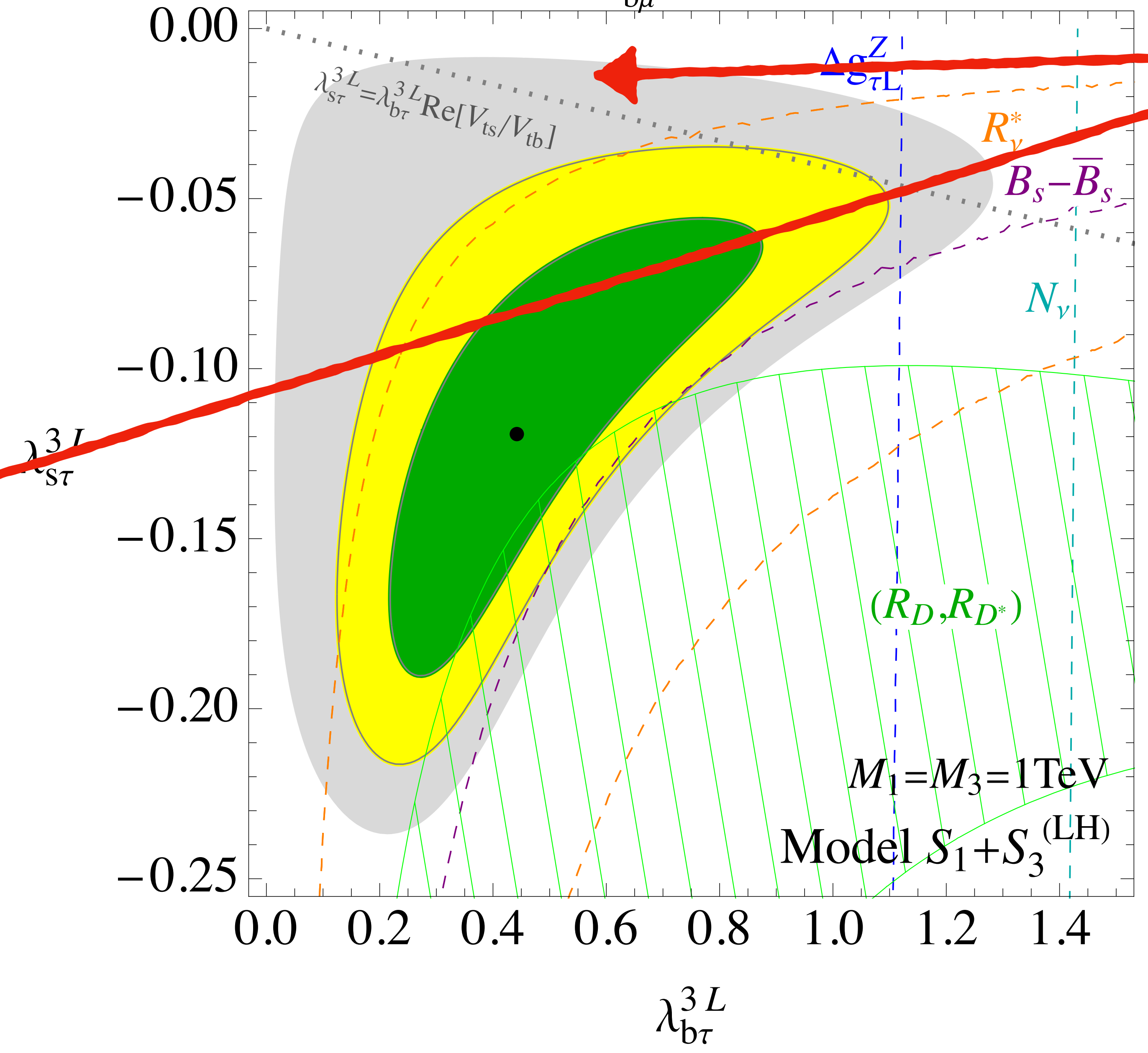


R(D$^{(*)}$)





The relation between couplings to s-quark and b-quark is compatible with a U(2)^5 flavour symmetry, that would predict:

$$\lambda_{s\alpha} = c_{U(2)} V_{ts} \lambda_{b\alpha}$$

$c_{U(2)} = 1$

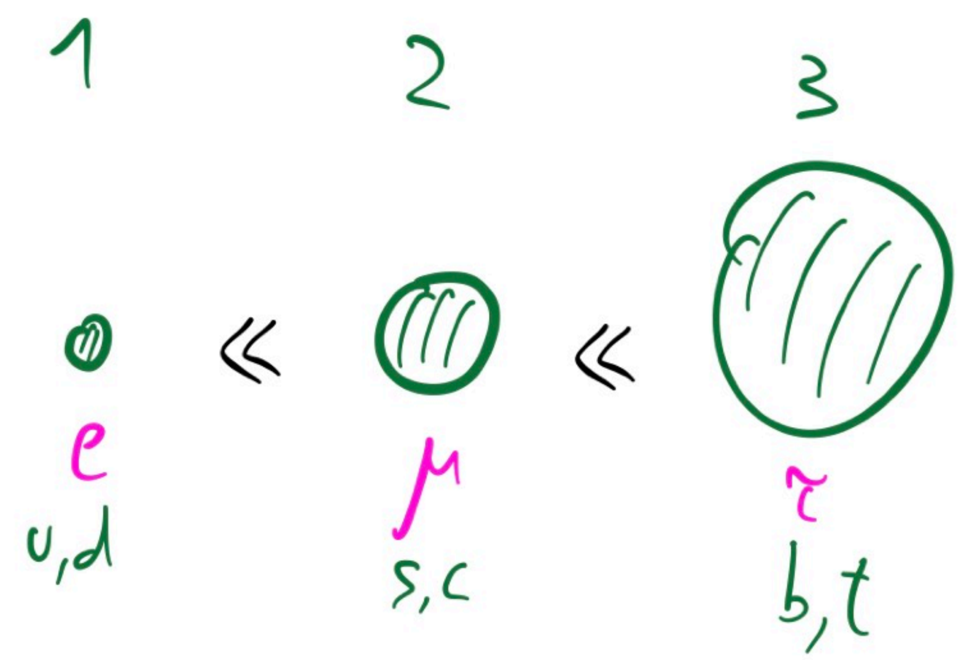
$c_{U(2)} \sim \mathcal{O}(1)$ e.g. 3 - 5

See also Buttazzo, Greljo, Isidori, D.M. 1706.07808

Plots updated w.r.t. [v3:2008.09548]

A hint for a flavor struture: U(2)^5

In first approximation only the 3rd generation couples to the Higgs.



In this case the theory enjoys a U(2)^5 global symmetry

$$G_F = U(2)_q \times U(2)_\ell \times U(2)_u \times U(2)_d \times U(2)_e$$

Barbieri et al. [1105.2296, 1203.4218, 1211.5085]

The **minimal breaking** of this symmetry to reproduce the SM Yukawas is described by a set of **spurions**:

$$Y_{u,d} \sim \begin{pmatrix} \boxed{\Delta_{u,d}} & \boxed{V_q} \\ 0 \quad 0 & 1 \end{pmatrix} \qquad Y_e \sim \begin{pmatrix} \boxed{\Delta_e} & \boxed{V_\ell} \\ 0 \quad 0 & 1 \end{pmatrix}$$

Diagonalizing quark masses, the **V_q doublet spurion is fixed** to be $\mathbf{V}_q = \kappa_q (V_{td}^*, V_{ts}^*)^T$

See also Fuentes-Martin, Isidori, Pagès, Yamamoto [1909.02519]

$\kappa_q \sim O(1)$

U(2)^5 flavour symmetry and leptoquarks

Applying the same symmetry assumptions to the leptoquark couplings to SM fermions we get a structure:

$$\lambda^{1(3)L} \sim \begin{pmatrix} \boxed{V_q^{\,*}} \times \boxed{V_\ell} & \boxed{V_q^{\,*}} \\ \boxed{V_\ell} & 1 \end{pmatrix}$$

i.e.

$$\lambda^{1(3)L} = \lambda^{1(3)} \begin{array}{c} \begin{array}{ccc} e_L & \mu_L & \tau_L \end{array} \\ \begin{pmatrix} X_{q\ell}^{1(3)} s_e V_\ell V_{td} & X_{q\ell}^{1(3)} V_\ell V_{td} & X_q^{1(3)} V_{td} \\ X_{q\ell}^{1(3)} s_e V_\ell V_{ts} & X_{q\ell}^{1(3)} V_\ell V_{ts} & X_q^{1(3)} V_{ts} \\ X_\ell^{1(3)} s_e V_\ell & X_\ell^{1(3)} V_\ell & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ d_L \\ s_L \\ b_L \end{array}$$

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V_ℓ : leptonic doublet spurion

$s_e = \sin\vartheta_e$: rotation diagonalizing electrons and muon masses

$\Bigg\}$ ***Arbitrary parameters***

$x^{1(3)}$: ***O(1)*** arbitrary complex parameters.

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Arbitrary parameters

$x^{1(3)}$: ***O(1)*** arbitrary complex parameters.

The leptoquark **couplings to first generations** are now **fixed** in terms of couplings to the second generation:

$$\lambda^{1(3)L}_{d\alpha} = \lambda^{1(3)L}_{s\alpha} \frac{V_{td}}{V_{ts}}$$

$$\lambda^{1(3)L}_{ie} = \lambda^{1(3)L}_{i\mu} \sin\theta_e$$

Exact relations (selection rules)

We can now **correlate Kaon physics** observables **to B-anomalies**!

Global analysis with U(2)^5

S. Trifinopoulos, E. Venturini, D.M. [2106.15630]

We perform a global fit in the U(2)^5 flavour structure.

The parameters are consistent with the symmetry: **all x's are O(1)**, $V_\ell \sim 0.1$, $|s_e| \lesssim 0.02$

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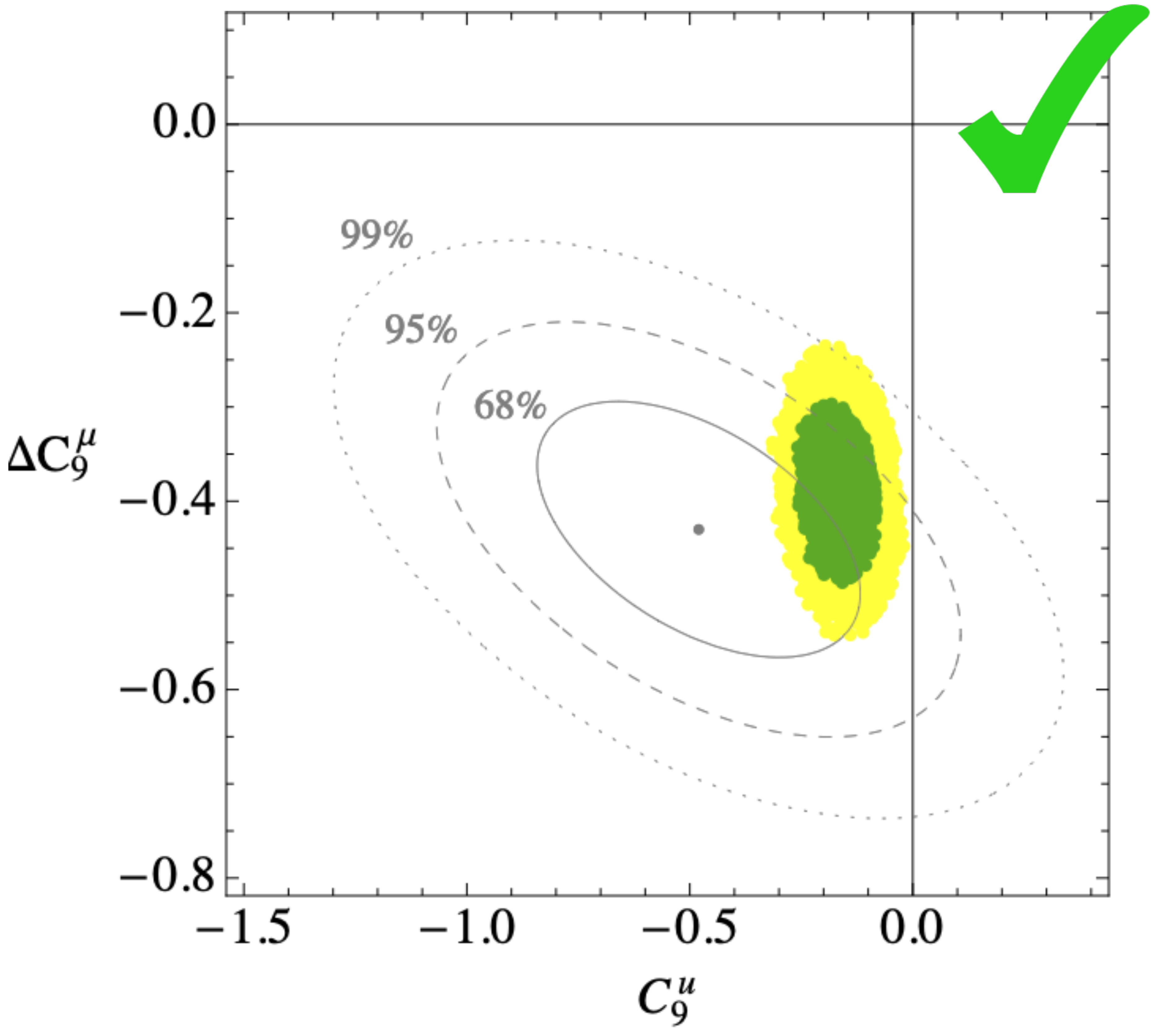

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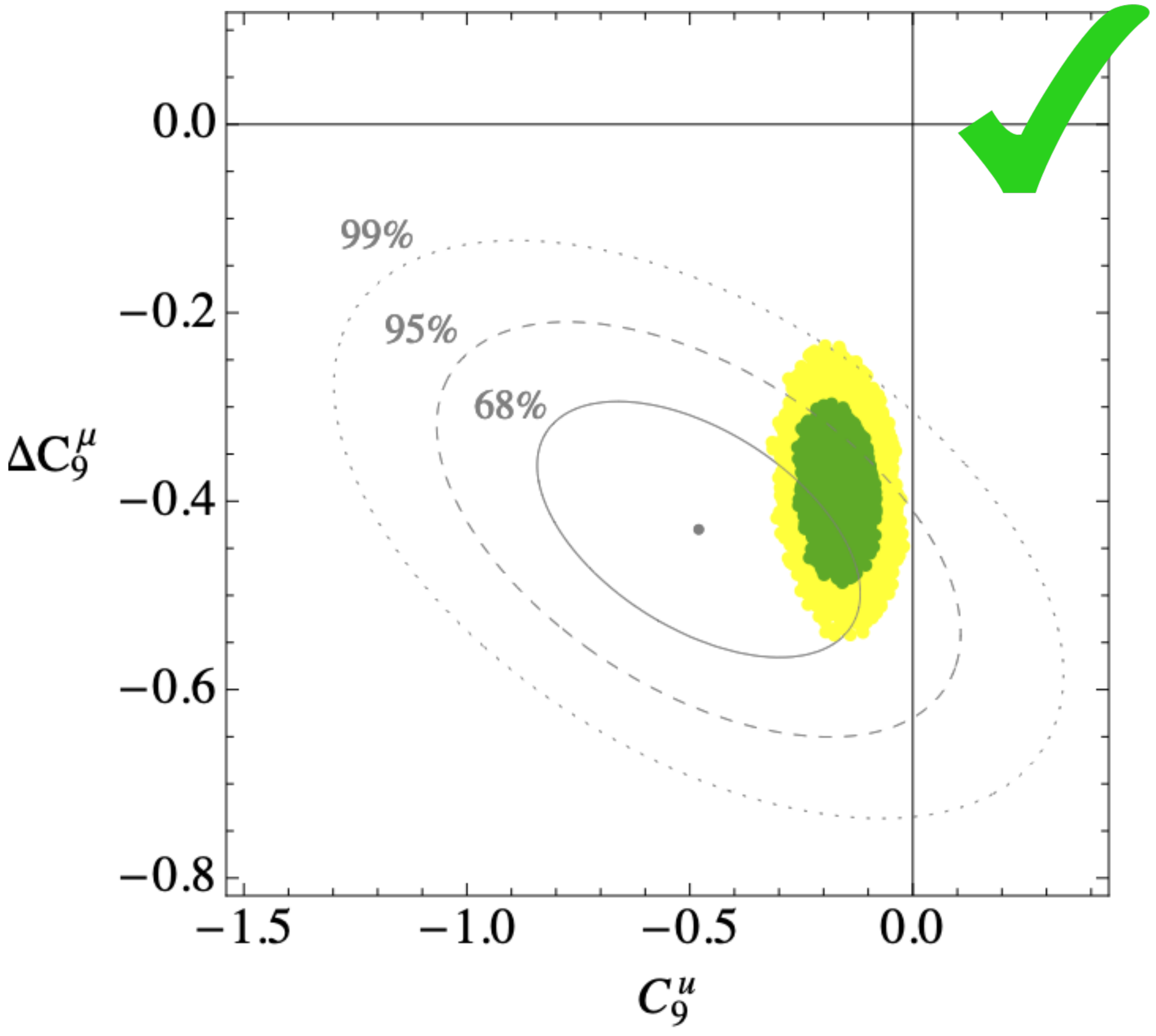
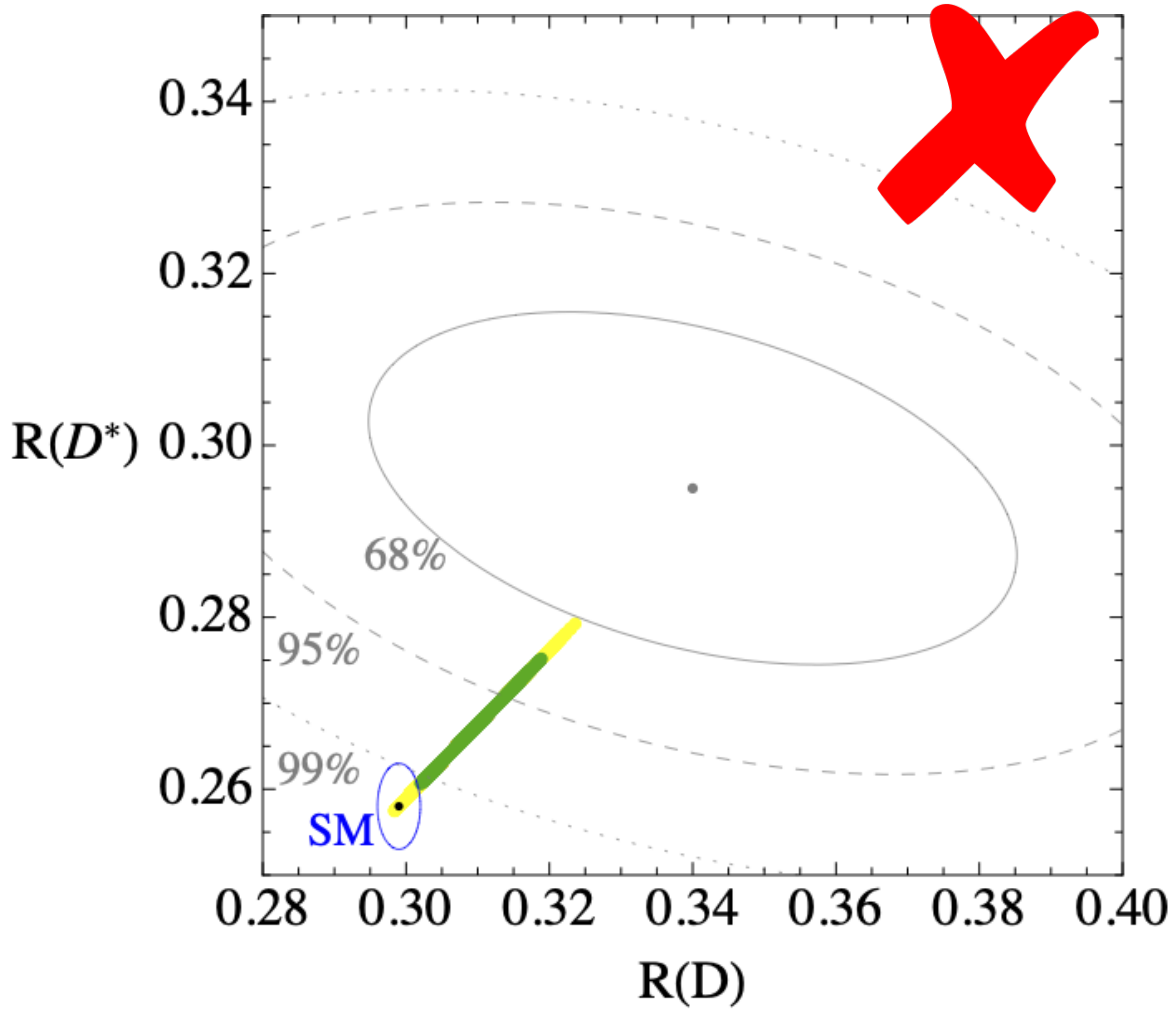
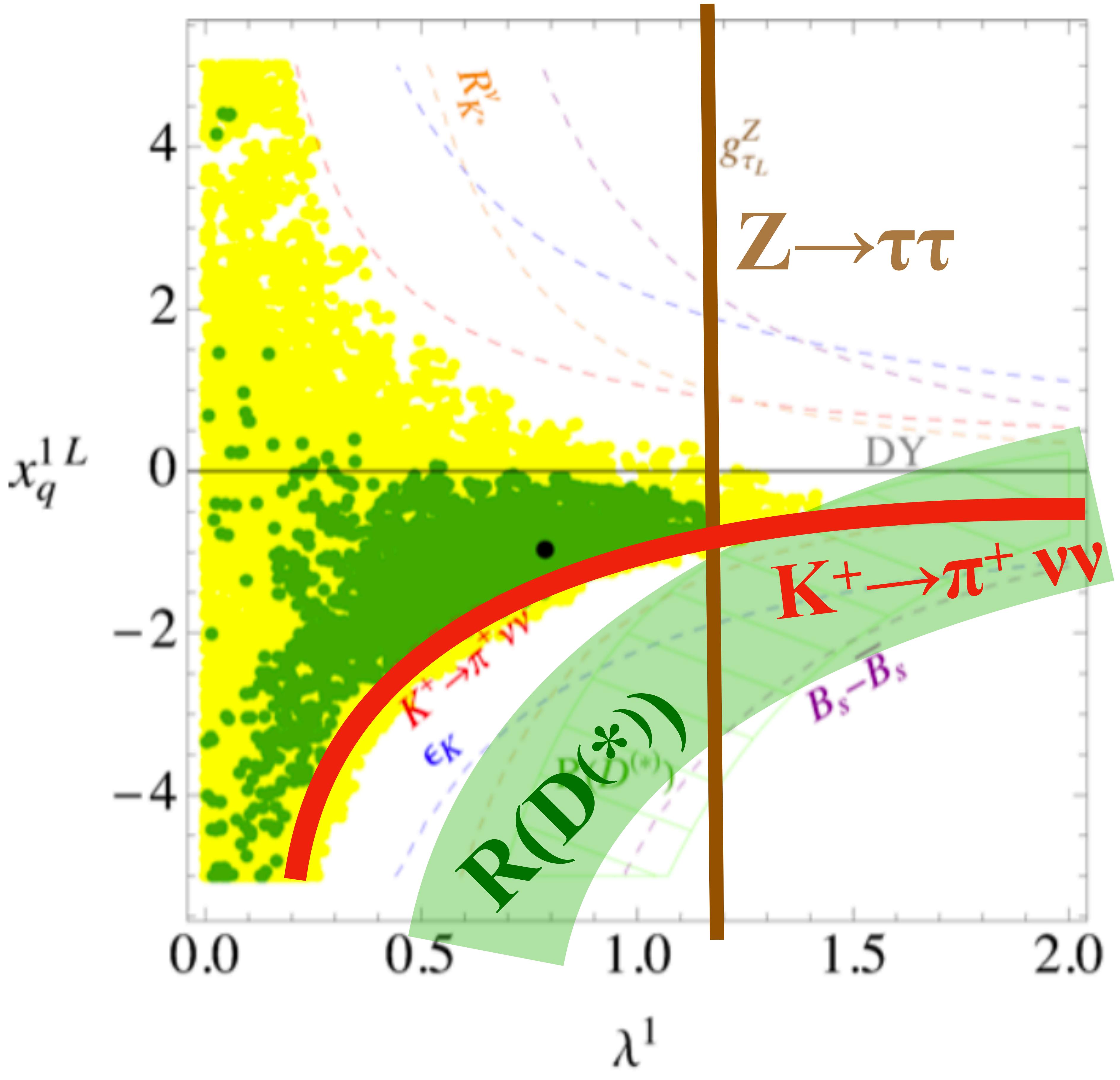


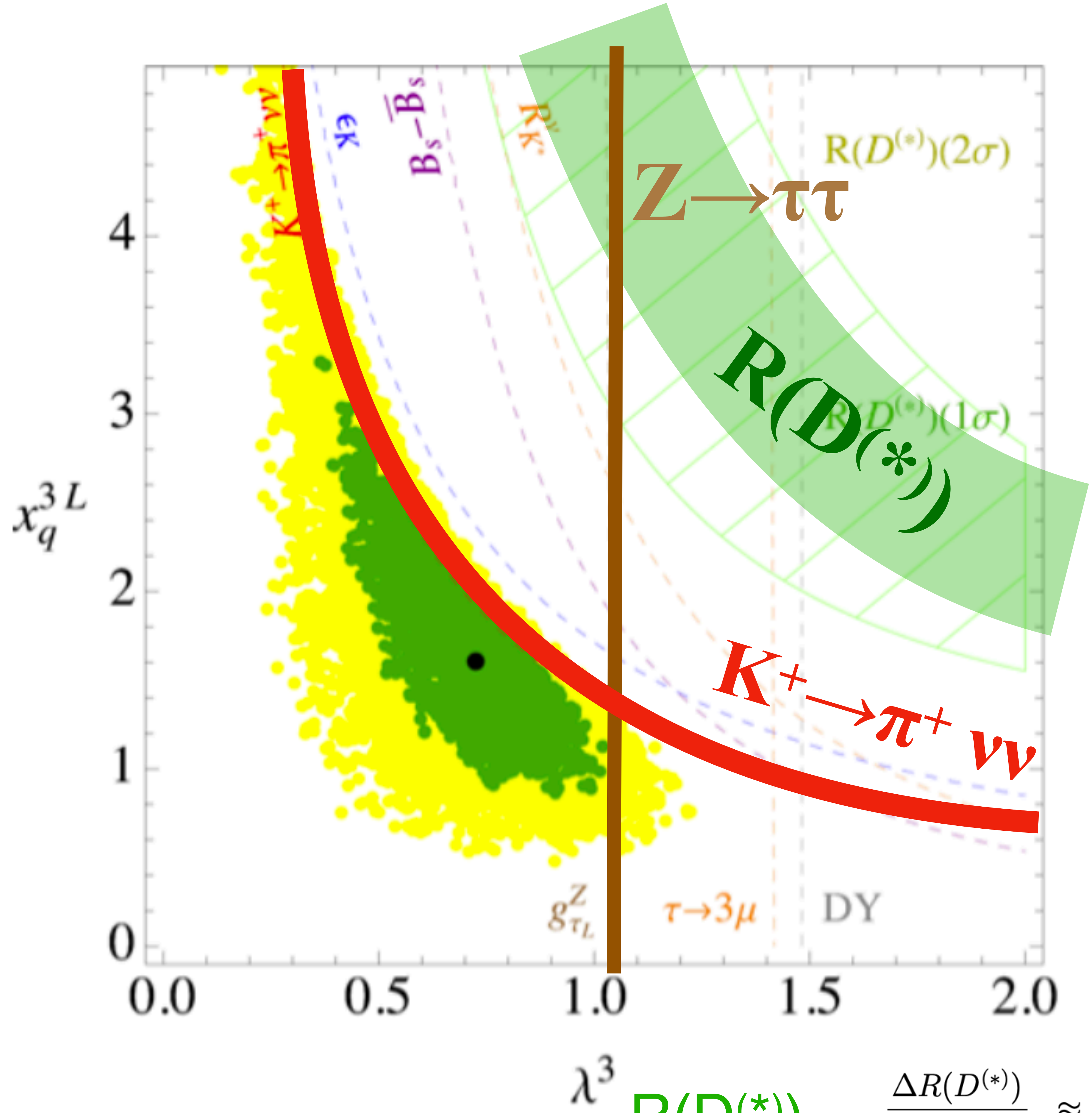
R(D$^{(*)}$) instead can only be addressed at 2σ:



Global analysis with U(2)^5

This is due to the **combination** of the **constraints from Z→ττ and K⁺→π⁺ νν**



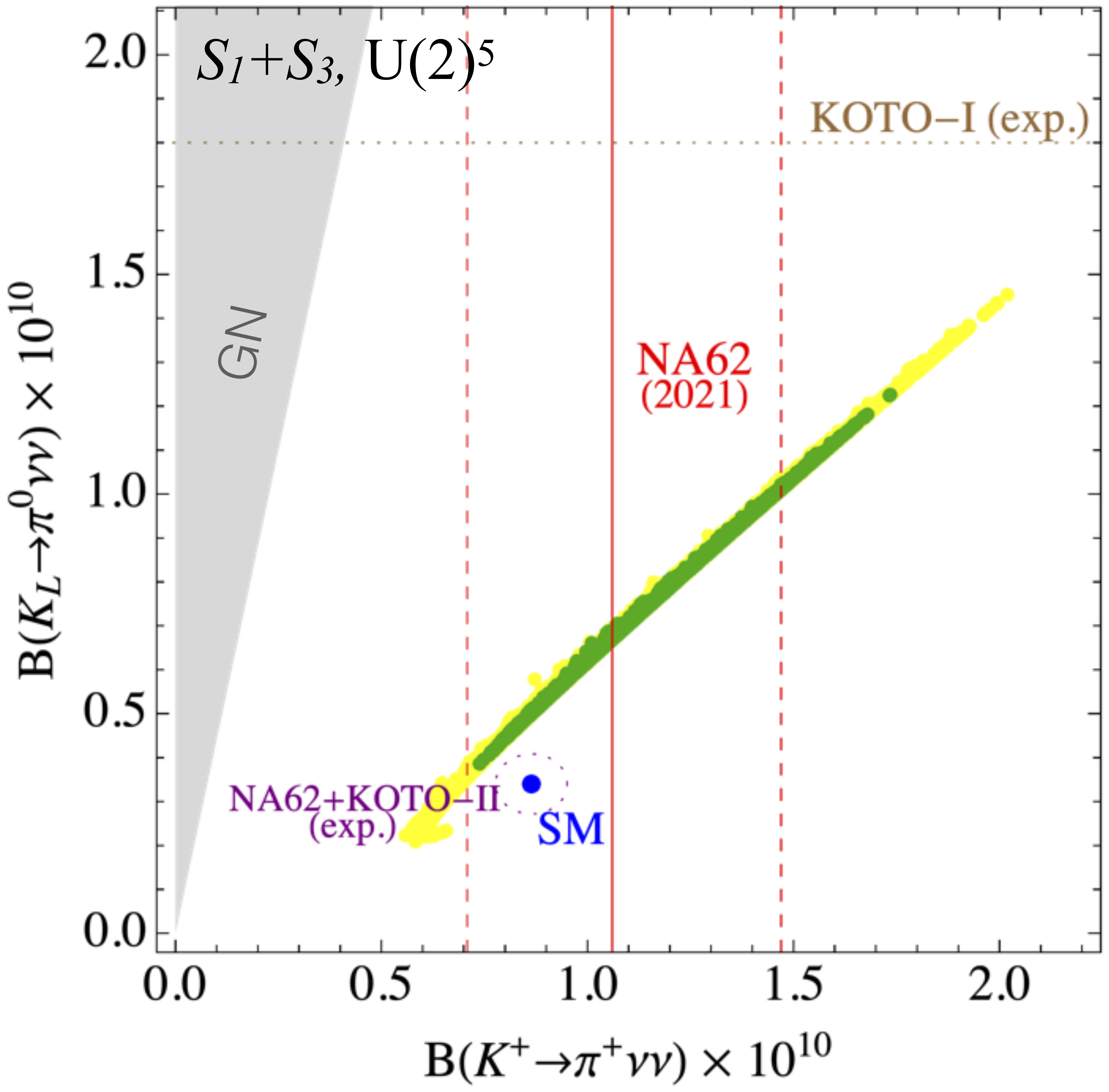


R(D(*)) $\quad \frac{\Delta R(D^{(*)})}{R(D^{(*)})_{\mathrm{SM}}} \approx v^2 \left(1.09 \frac{|\lambda^1|^2(1-x_q^{1*}V_{tb}^*)}{2M_1^2} - 1.02 \frac{|\lambda^3|^2(1-x_q^{3*}V_{tb}^*)}{2M_3^2}\right)$

K⁺→π⁺ νν $\quad C_{sd\nu_\tau\nu_\tau} \approx V_{td}^* V_{ts} \left(\frac{|\lambda^1|^2|x_q^1|^2}{2M_1^2} + \frac{|\lambda^3|^2|x_q^3|^2}{2M_3^2}\right)$

Z→ττ $\quad 10^3 \delta g^Z_{\tau_L} \approx 0.59 \frac{|\lambda^1|^2}{M_1^2/\,\mathrm{TeV}^2} + 0.80 \frac{|\lambda^1|^2}{M_1^2/\,\mathrm{TeV}^2}$

Leading effect in Kaon physics



$K \to \pi\, \nu\, \nu$

Dominated by **tau neutrinos**, due to largest couplings.

The **NA62** bound is already very constraining for this setup, future updated will put even more tension with R(D$^{(*)}$), or eventually a signal could be observed.

The correlation in the full model is stronger than just in EFT.

[see: Bordone, Buttazzo, Isidori, Monnard [1705.10729], Borsato, Gligorov, Guadagnoli, Martinez Santos, Sumensari [1808.02006], Fajfer, Kosnik, Vale-Silva [1802.00786]

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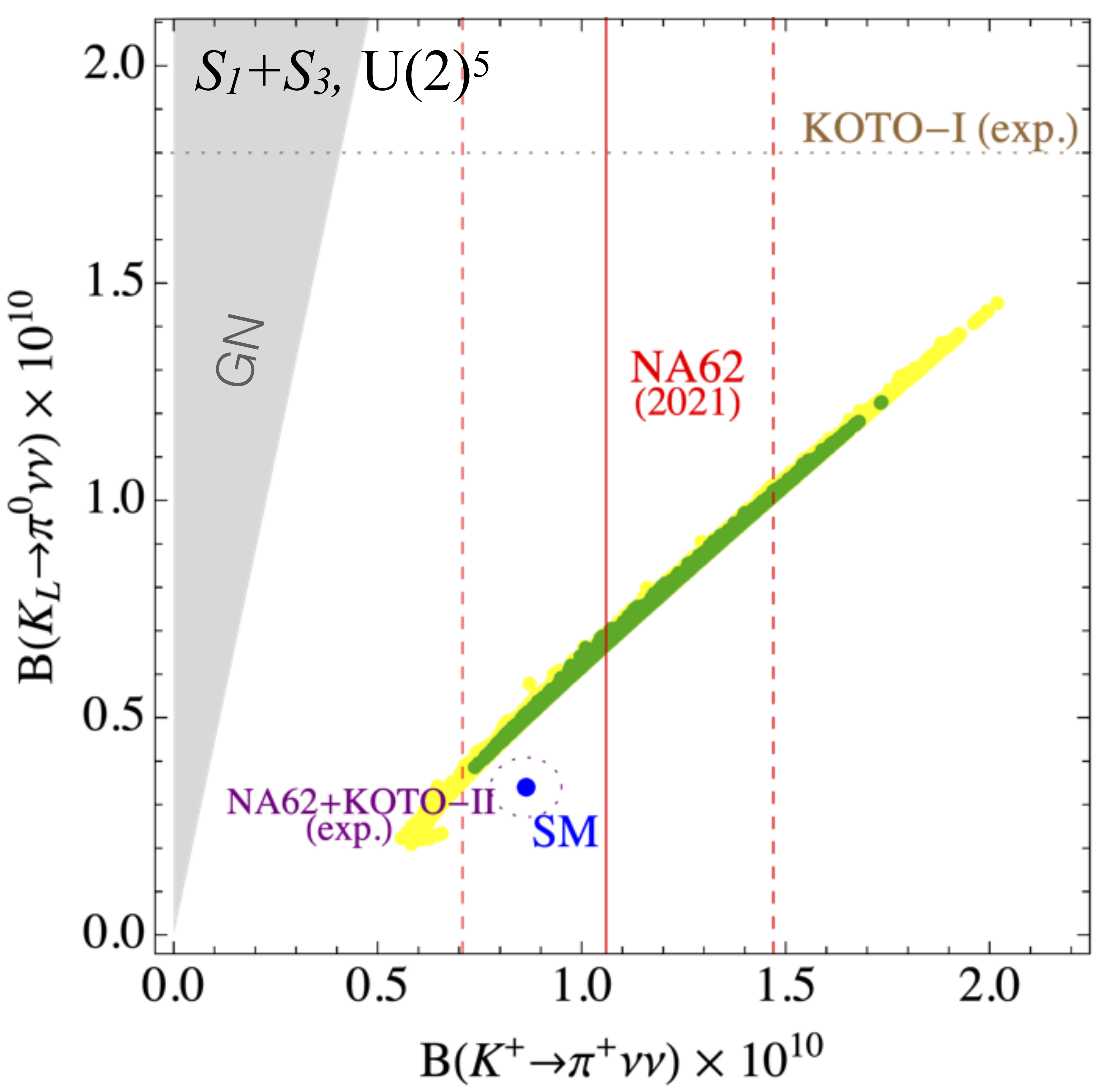

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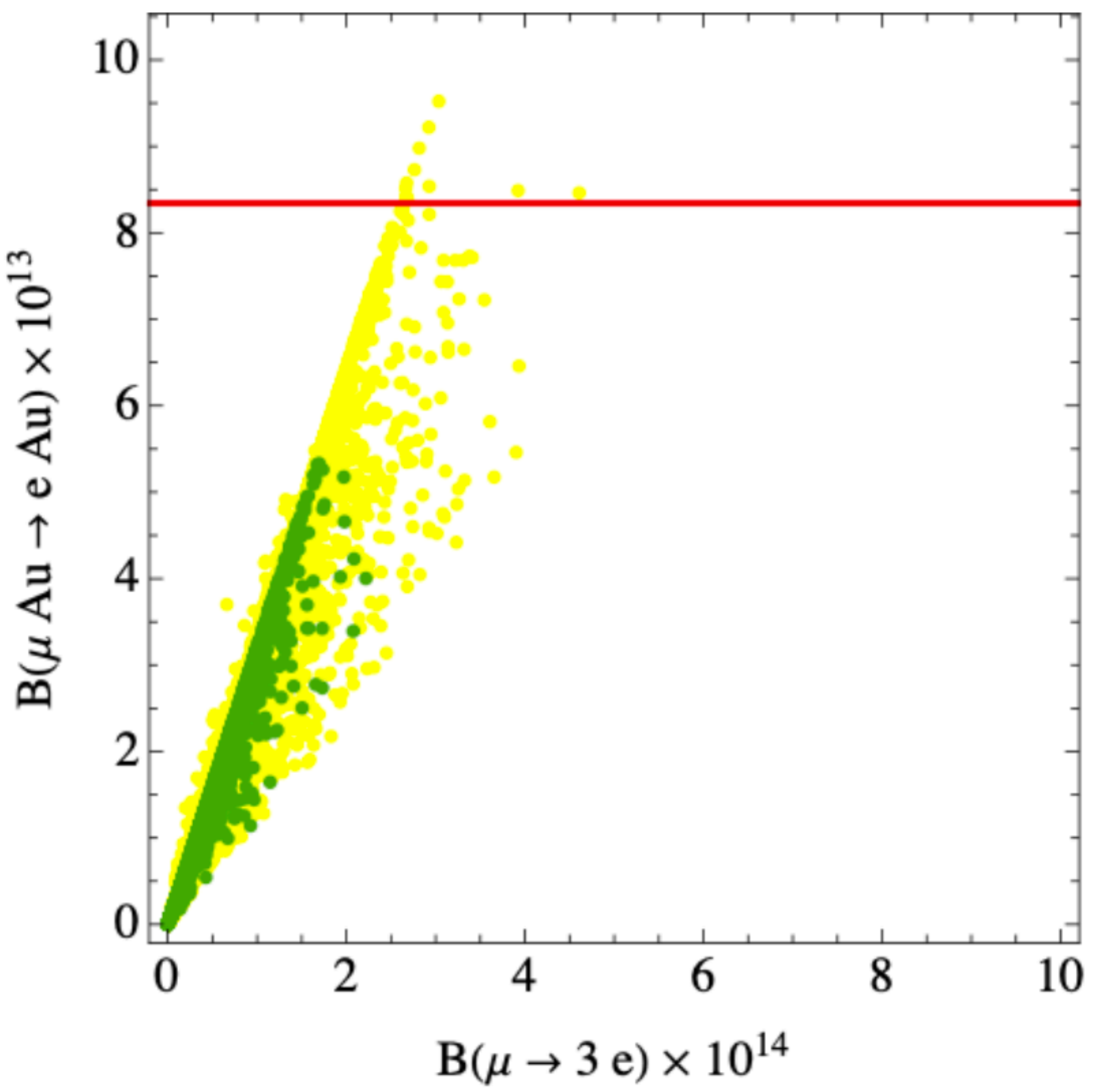
The **phase of NP** contribution is **fixed** to be SM-like:

$$C_{sd\nu_\tau\nu_\tau}\,(\bar{\nu}_\tau \gamma_\mu \nu_\tau)(\bar{d}_L \gamma^\mu s_L)$$

$$C_{sd\nu_\tau\nu_\tau} \approx V_{td}^* V_{ts} \left(\frac{|\lambda^1|^2 |x_q^1|^2}{2M_1^2} + \frac{|\lambda^3|^2 |x_q^3|^2}{2M_3^2} \right)$$

As consequence, the $K_L \to \pi^0$ mode is fully correlated and below the KOTO stage-I final sensitivity.

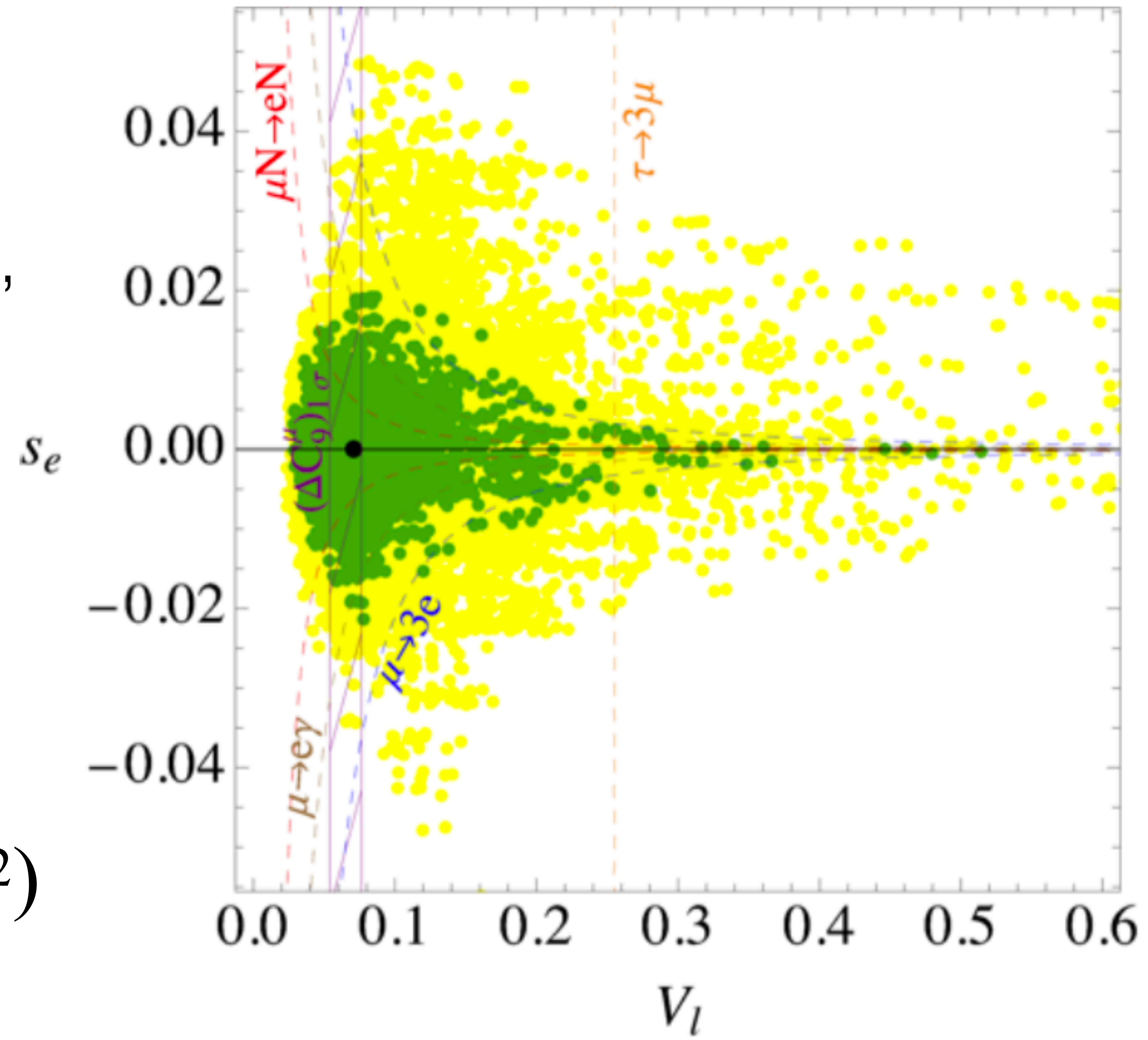
μ → e conversion



μ→e conversion in gold nuclei sets the **strongest constraint on s_e**.

COMET and *Mu2e* will push this bound to ~**10^{-16}**, while *Mu3e* at PSI will push the limit on **Br(μ→3e) to ~10^{-16}**.

These will set much stronger **bounds on s_e**, or could see a New Physics effect.



Naive expectation would be $s_e \sim \sqrt{(m_e/m_\mu)} \sim O(10^{-2})$

Conclusions

- **Flavor anomalies** still require data (and theory) to give us a definitive picture.
  This could potentially be our **threshold to an unexpected New Physics sector**!

- **S_1+S_3 scalar leptoquarks** offer good solutions to **B anomalies** (and **$(g\text{-}2)_\mu$**),
  > simplified model is **fully calculable**
  > possible UV origin from a **Composite Higgs model** as pNGB partners of the Higgs.

- In order to understand the **underlying flavour structure**
  we need to **connect B-anomalies with other observables**.
  > Rare **Kaon decays** and **$\mu \to e$** probes stand out and offer exceptional prospects.

- The **minimally broken $U(2)^5$ flavor symmetry** is creates **tension** between **B-anomalies**
  and **the present NA62 bound on $K^+ \to \pi^+ \nu\nu$**.

Thank you!

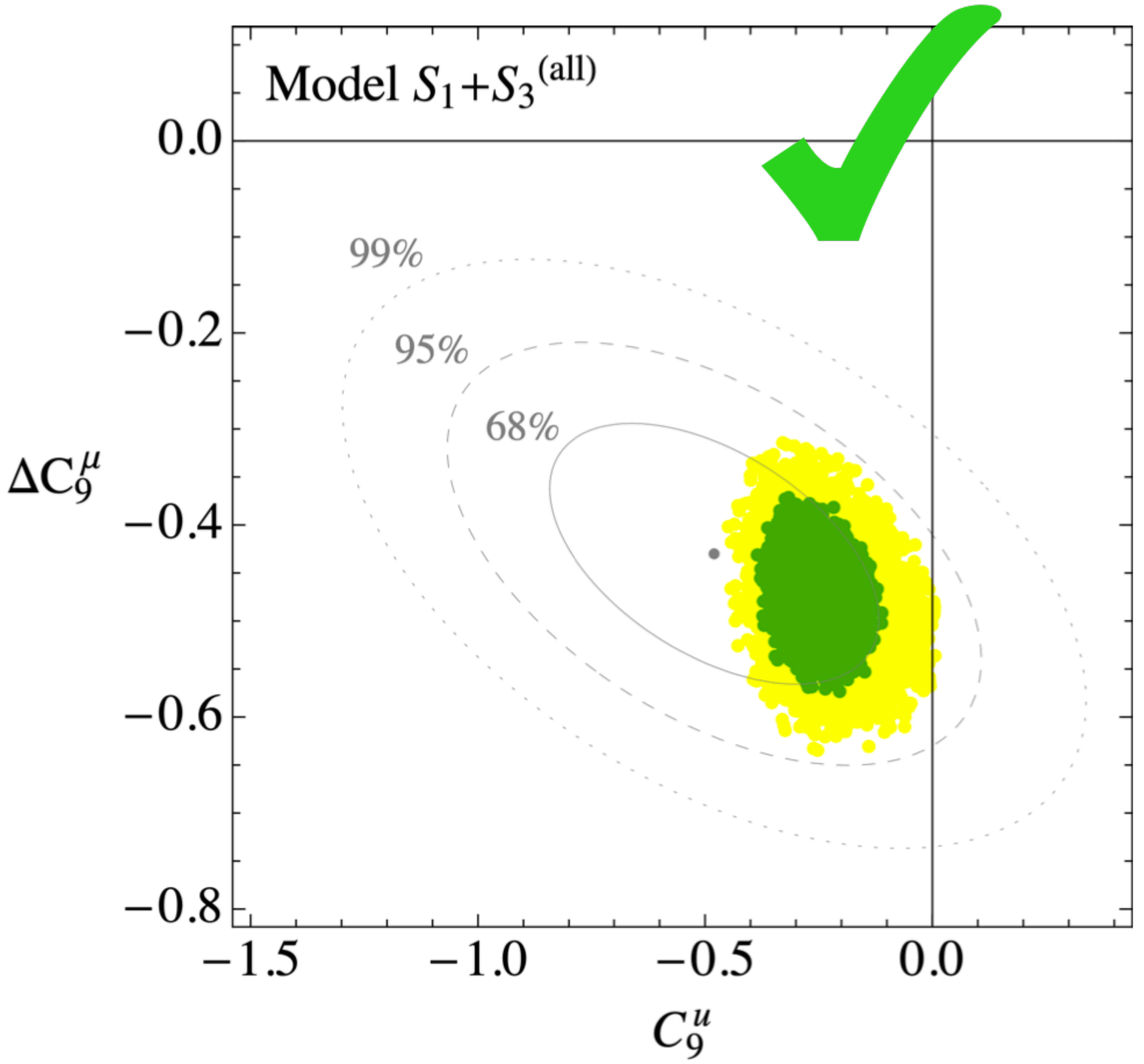
Backup

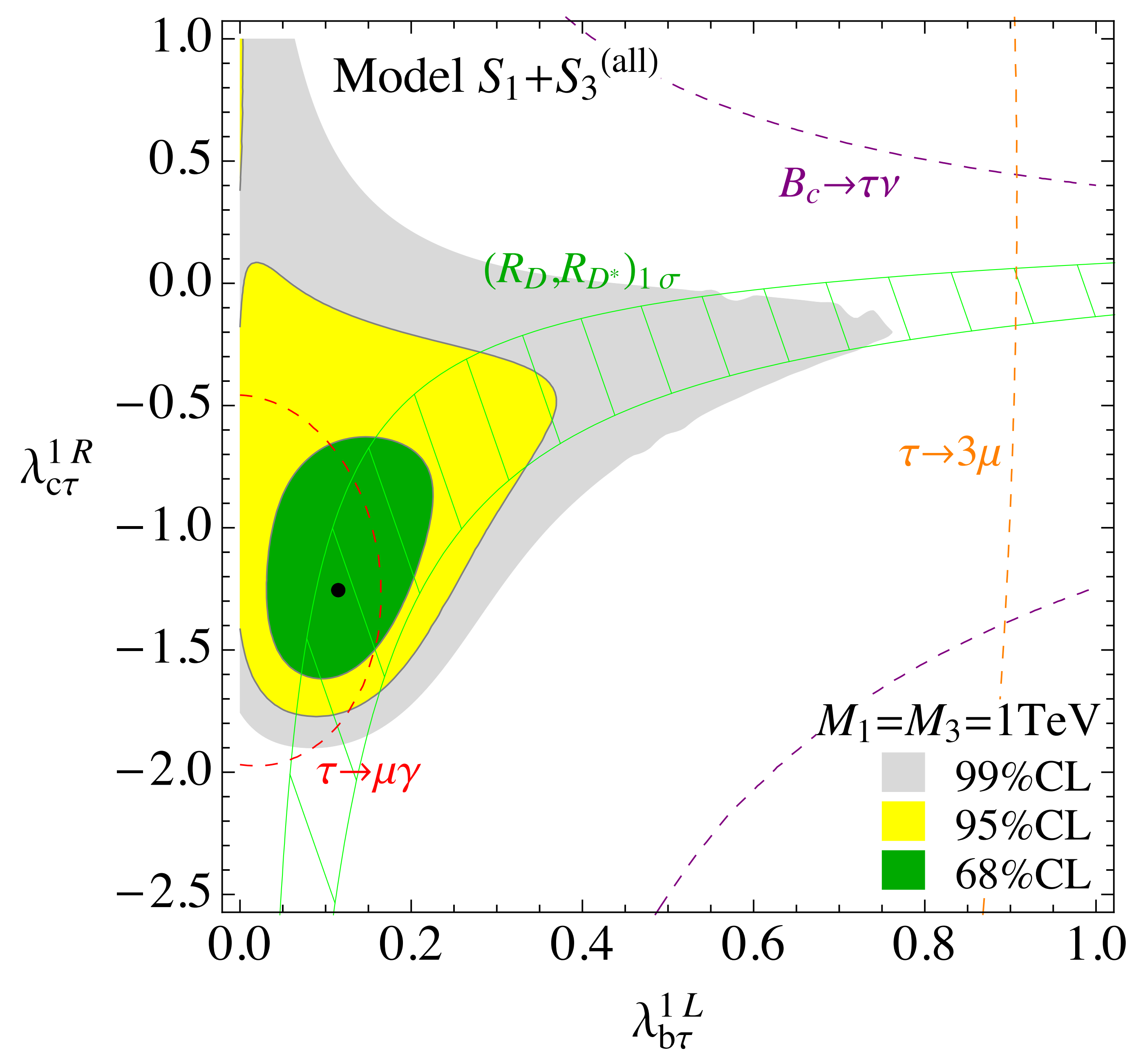
S_1 and S_3 : $R(K^{(*)})$ + $R(D^{(*)})$ + $(g-2)_\mu$

No a-priori flavour structure imposed

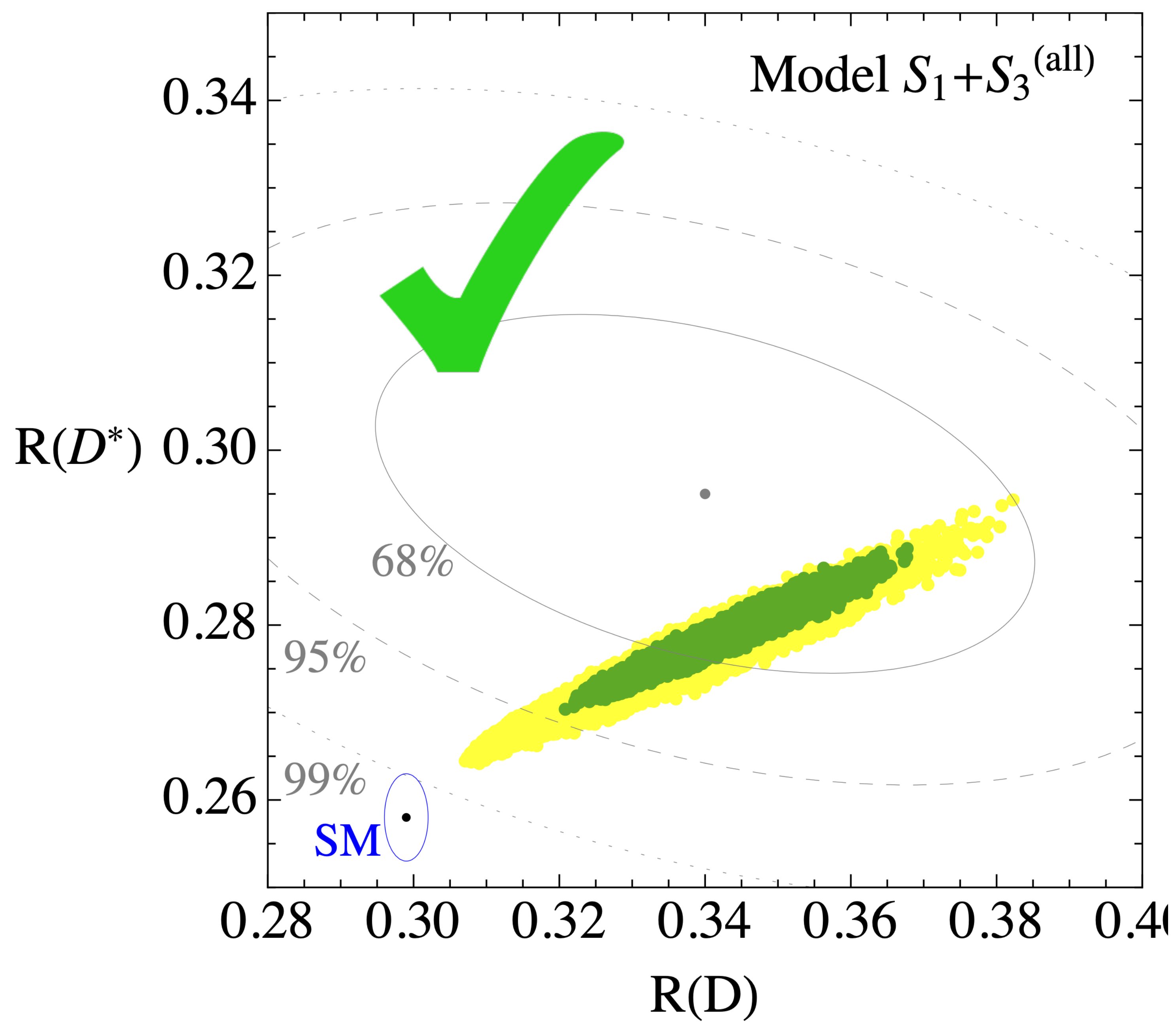
$$\lambda^{1L} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & s\tau \\ 0 & b\mu & b\tau \end{pmatrix} \qquad \lambda^{3L} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s\mu & s\tau \\ 0 & b\mu & b\tau \end{pmatrix}$$

$$\lambda^{1R} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & c\tau \\ 0 & t\mu & t\tau \end{pmatrix}$$

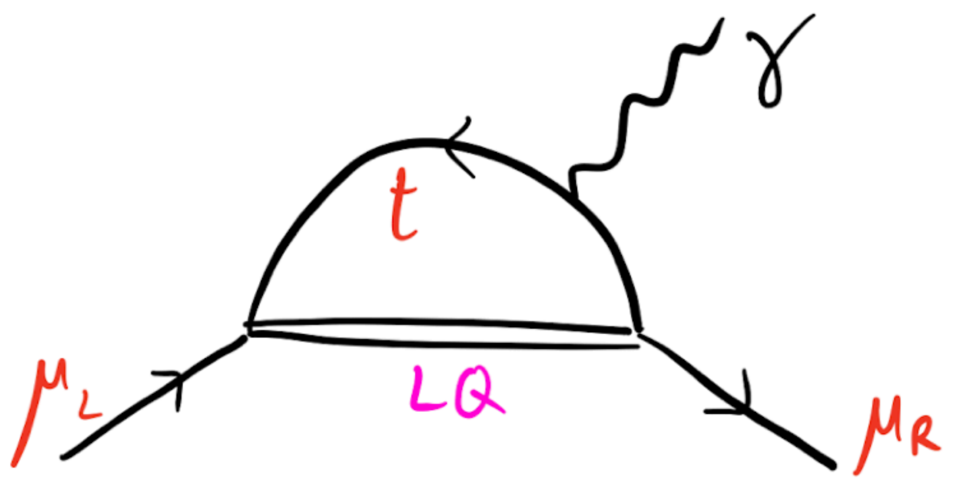


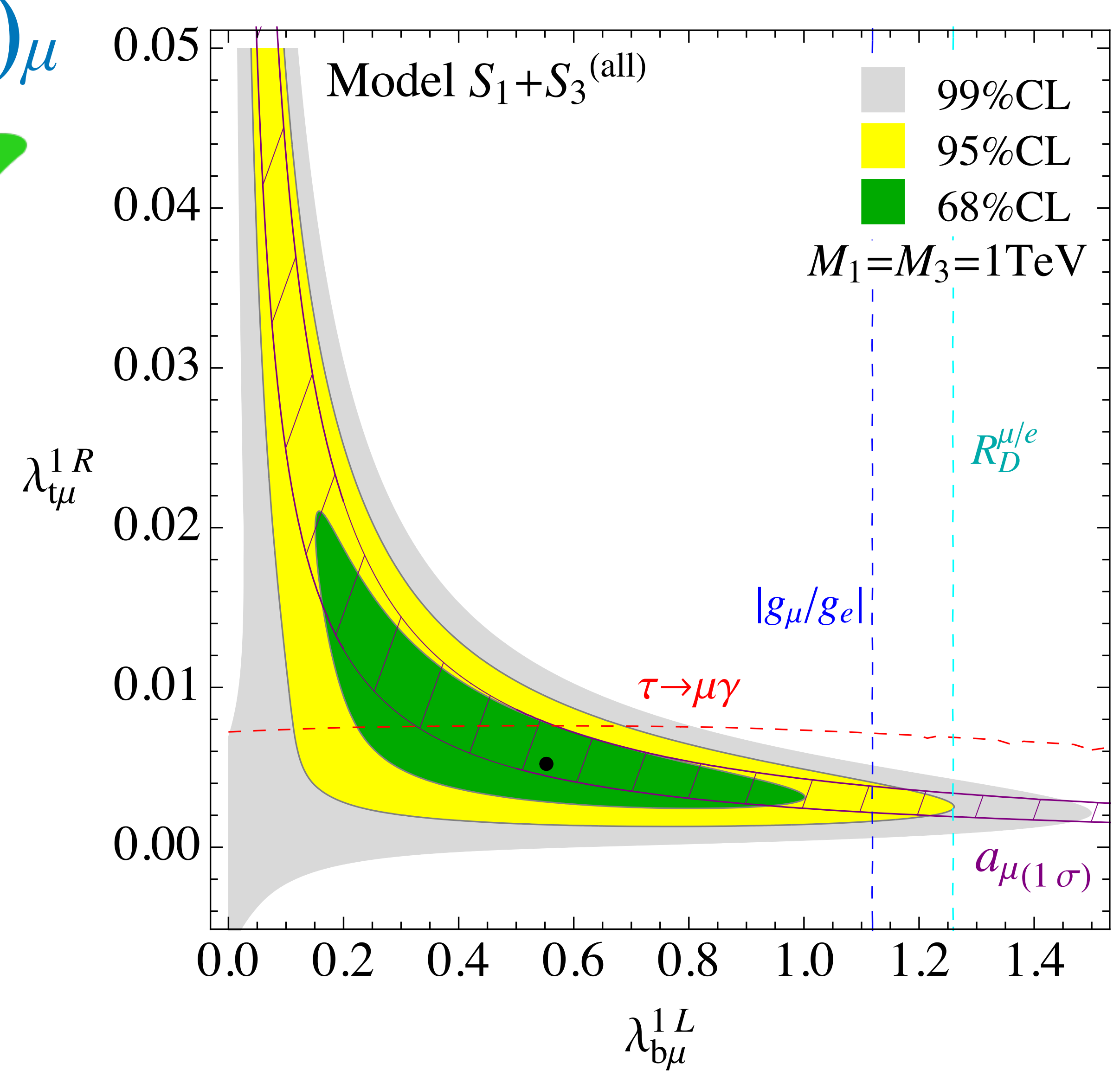


$R(D^{(*)})$



$(g-2)_\mu$



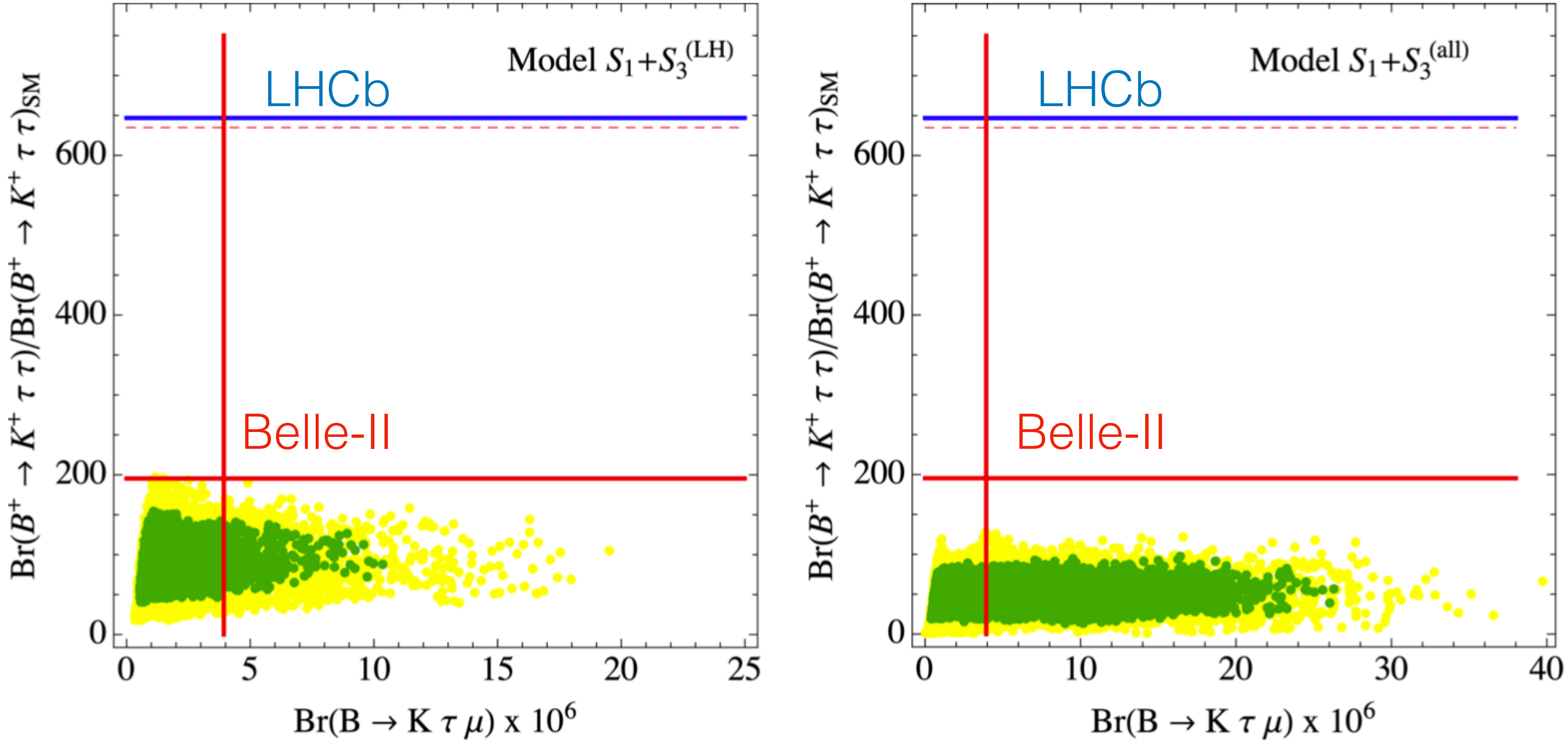


Very good fit of all anomalies!

Predictions

The large couplings to τ imply signatures in **DY tails of pp→ τ τ**, deviations in **τ LFU** tests and **τ → μ LFV** tests (Belle-II).

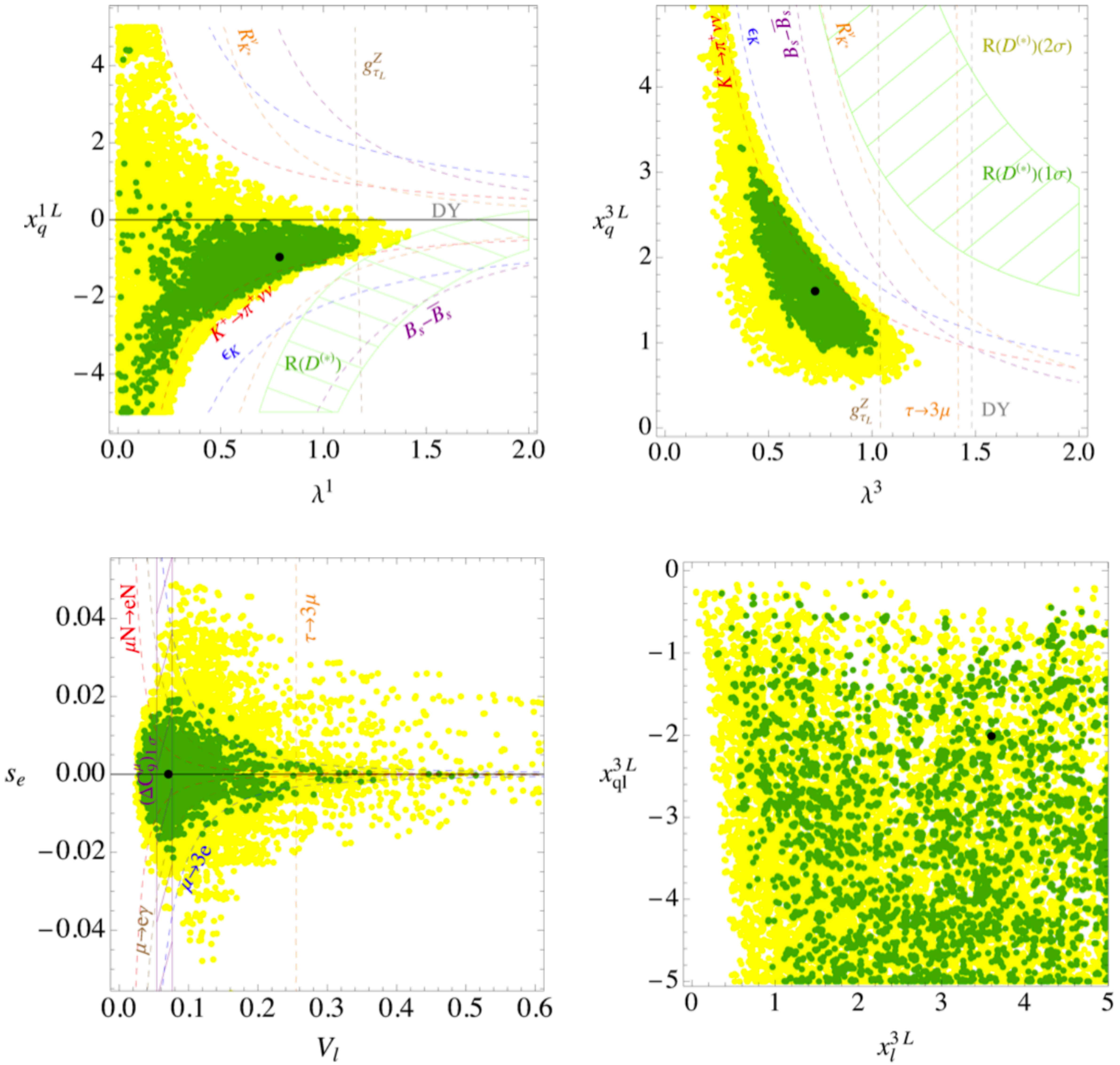
Large effects are also expected in **b → s τ τ** and **b → s τ μ** transitions:



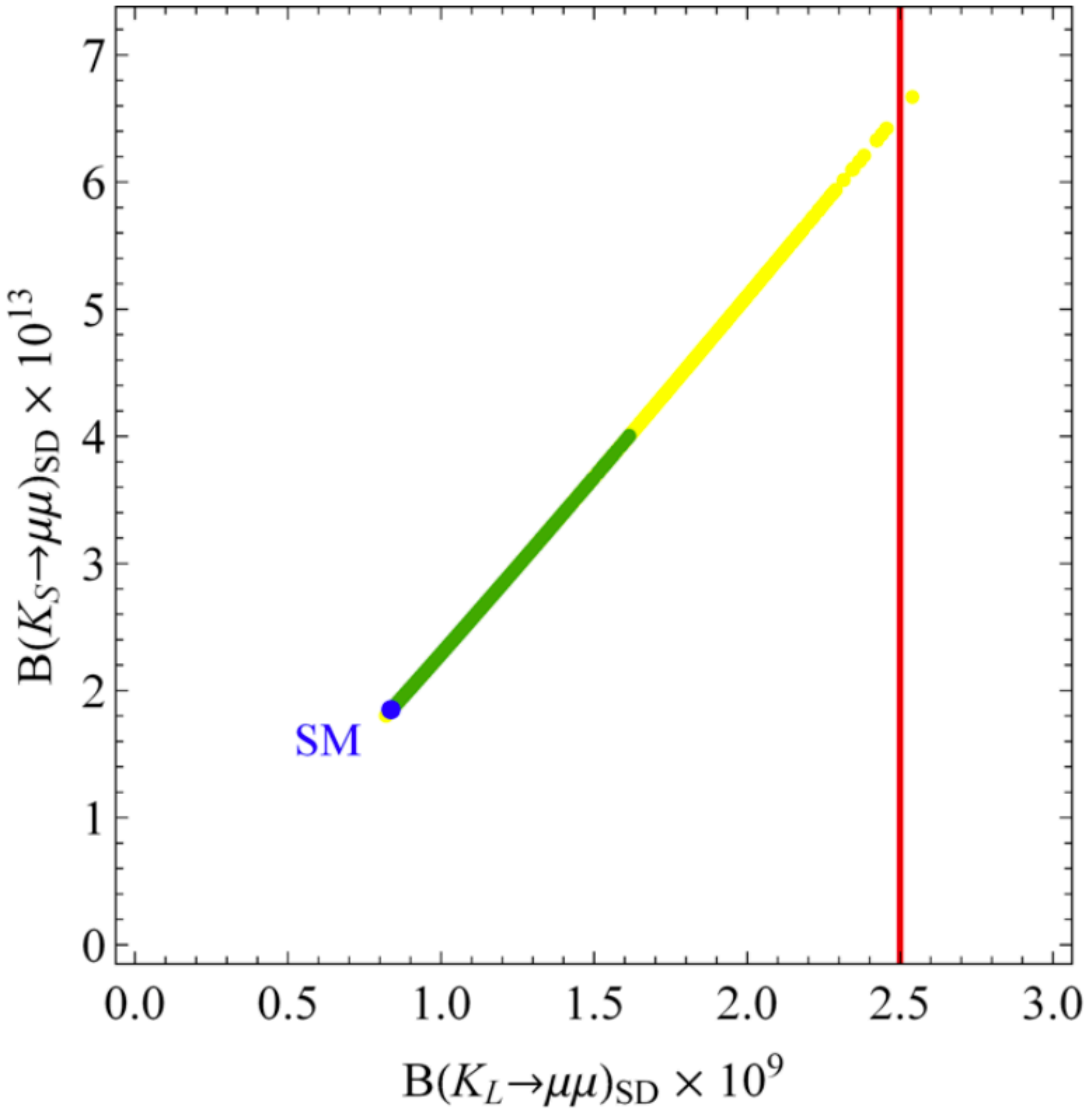
From B to K with LQ and U(2)^5

S. Trifinopoulos, E. Venturini, D.M. [2106.15630]

We perform a global fit in the U(2)^5 flavour structure. $M_1 = M_3 = 1.1$ TeV



Leading effects in Kaon physics



Also in this case **the phase of NP contribution is fixed** to be SM-like

$$\Delta C_9^{sd\mu\mu} = -\Delta C_{10}^{sd\mu\mu} \approx \frac{\pi V_{ts}^* V_{td}}{\sqrt{2} G_F \alpha} \frac{|\lambda^3|^2 |V_\ell|^2 |x_{q\ell}^3|^2}{M_3^2}$$

The two channels are fully correlated.

- In **K_L** the model **saturates the present bound**

Isidori, Unterdorfer [hep-ph/0311084]

- in **K_S** the effect is ~ **10^{-13}**, below the SM long-distance contribution (~5×10^{-12}).

D'Ambriosio, Kitahara [1707.06999]

About other Kaon decays:

We also obtain $Br(K_L \to \mu e) \sim 10^{-15}$ and $Br(K^+ \to \pi^+ \mu e) \sim 10^{-18}$.

Goldstone Bosons

D.M. 1803.10972

$$G = SU(10)_L \times SU(10)_R \times U(1)_V \xrightarrow{\langle \bar\Psi_i \Psi_j \rangle = -B_0 f^2 \delta_{ij}} H = SU(10)_V \times U(1)_V$$

Like QCD pions, the pNGB are composite states of HC-fermion bilinears: $\bar\Psi\Psi$

In terms of SM representations

Two Higgs doublets: $H_{1,2} \sim (\mathbf{1,2})_{1/2}$

Singlet and Triplet LQ: $S_1 \sim (\mathbf{3,1})_{-1/3} \quad + \quad S_3 \sim (\mathbf{3,3})_{-1/3}$

Three singlets: $\eta_{1,2,3} \sim (\mathbf{1,1})_0$

Other electroweak states: $\omega \sim (\mathbf{1,1})_1 \quad + \quad \Pi_{L,Q} \sim (\mathbf{1,3})_0$

Other coloured states: $R_2 \sim (\mathbf{3,2})_{1/6} \quad + \quad T_2 \sim (\mathbf{3,2})_{-5/6}$

$\tilde\pi_1 \sim (\mathbf{8,1})_0 \quad + \quad \tilde\pi_3 \sim (\mathbf{8,3})_0$

H and LQ are close partners!!

$$H_1 \sim i\sigma^2(\bar\Psi_L \Psi_N)$$
$$H_2 \sim (\bar\Psi_E \Psi_L)$$
$$S_1 \sim (\bar\Psi_Q \Psi_L)$$
$$S_3 \sim (\bar\Psi_Q \sigma^a \Psi_L)$$

For energies $E \ll \Lambda_{HC}$ the theory is described by a weakly coupled effective chiral Lagrangian.
Structure driven by the symmetries and spurions.

Yukawas & LQ couplings

Coupling with SM fermions from 4-Fermi operators

$$\mathcal{L}_{4-\mathrm{Fermi}} \sim \frac{c_{\psi\Psi}}{\Lambda_t^2} \bar{\psi}_{\mathrm{SM}} \psi_{\mathrm{SM}} \bar{\Psi}\Psi \quad \xrightarrow{E \lesssim \Lambda_{HC}} \quad \sim y_{\psi\phi}\, \bar{\psi}_{\mathrm{SM}} \psi_{\mathrm{SM}}\, \phi + \ldots$$

$$\Lambda_t \gtrsim \Lambda_{HC}$$

SM Yukawas + LQ couplings arise from the same UV dynamics

A new sector responsible for these operators is necessary (as Extended Technicolor)

Scalar operators allowed by gauge-invariance

Higgses Yukawas

$$\left(\bar{q}_L u_R + \bar{d}_R q_L + \bar{e}_R l_L\right)\left(\bar{\Psi}_N \Psi_L\right)$$

$$\left(\bar{q}_L u_R + \bar{d}_R q_L + \bar{e}_R l_L\right)\left(\bar{\Psi}_L \Psi_E\right)$$

S_1 and S_3 couplings

$$\left(\bar{q}^c_L l_L + \bar{e}^c_R u_R\right)\left(\bar{\Psi}_Q \Psi_L\right)$$

$$\left(\bar{q}^c_L \sigma^a l_L\right)\left(\bar{\Psi}_Q \sigma^a \Psi_L\right)$$

S_1 coupling to diquark

$$\cancel{\left(\bar{q}^c_L q_L + \bar{u}^c_R d_R\right)\left(\bar{\Psi}_L \Psi_Q\right)}$$

ω coupling to dilepton

$$\cancel{\left(\bar{l}^c_L l_L\right)\left(\bar{\Psi}_E \Psi_N\right)}$$

$\tilde{R}_2$ coupling

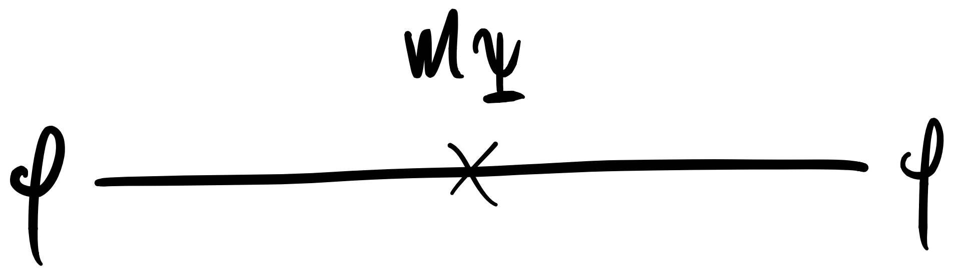
$$\cancel{\left(\bar{d}_R l_L\right)\left(\bar{\Psi}_E \Psi_Q\right)}$$

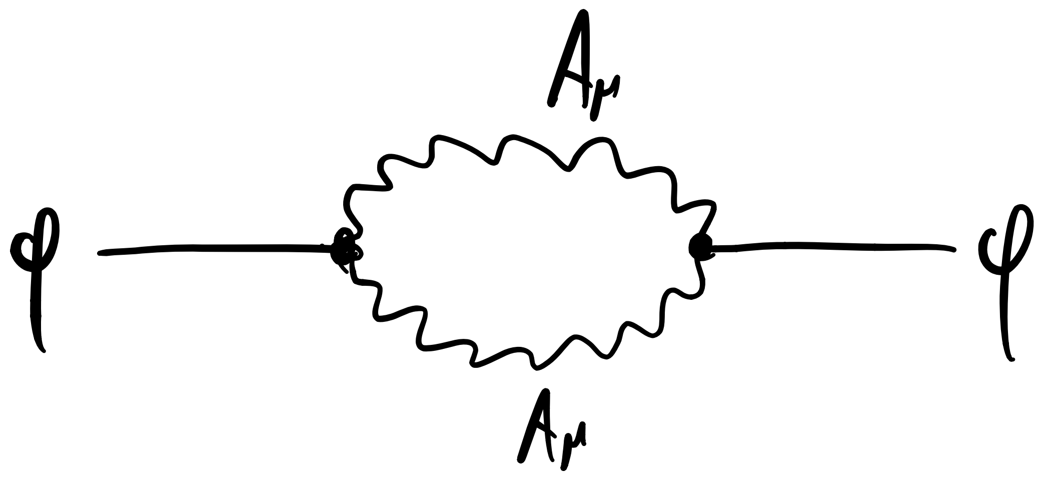
Assuming conservation of this symmetry $F_+ = 3B + L$ so that Yukawas and LQ coupl. allowed all other couplings are forbidden.

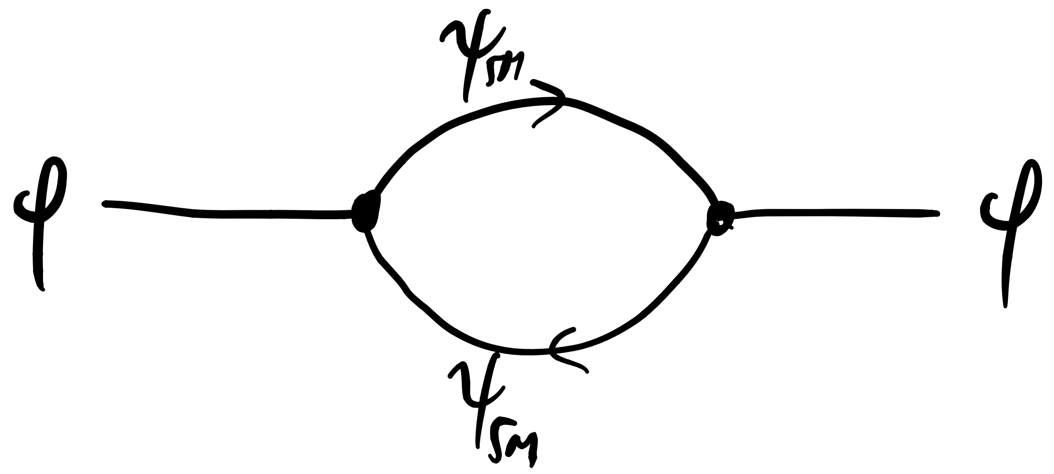
$$F_+(\Psi_L) = F_+(\Psi_N) = F_+(\Psi_E) = F_L\,, \qquad F_+(\Psi_Q) = F_L + 2$$

Scalar Potential: NDA + symmetry

The pNGB potential arises at 1-loop from all the explicit breaking terms







NDA + spurion analysis

$$m^2_{(\bar{\Psi}_i \Psi_j)} = B_0 (m_i + m_j)$$

$$V_{\mathcal{G}} = -\frac{3 f^2 \Lambda_{HC}^2}{16\pi^2} \sum_X c_X \mathrm{Tr}\left[\mathcal{G}_X^L U \mathcal{G}_X^R U^\dagger\right]$$

$$V_t = -\frac{c_t y_t^2 N_c \Lambda_{HC}^2}{16\pi^2} |H_1 - H_2|^2$$

$$V_{\mathrm{LQ}} = -\frac{(c_1 g_1^2 + c_1^u g_1^{u2}) \Lambda_{HC}^2}{8\pi^2} |S_1|^2 - \frac{c_3 g_3^2 \Lambda_{HC}^2}{8\pi^2} |S_3|^2$$

The gauge contribution is positive and is larger for colored states.
EW charges give subleading corrections.

$$\Delta m^2_\omega \approx (0.05 \Lambda_{HC})^2 \,, \quad \Delta m^2_{H_{1,2}} \approx (0.08 \Lambda_{HC})^2 \,, \quad \Delta m^2_{\Pi_{L,Q}} \approx (0.13 \Lambda_{HC})^2 \,,$$ ~ **1** of SU(3)c

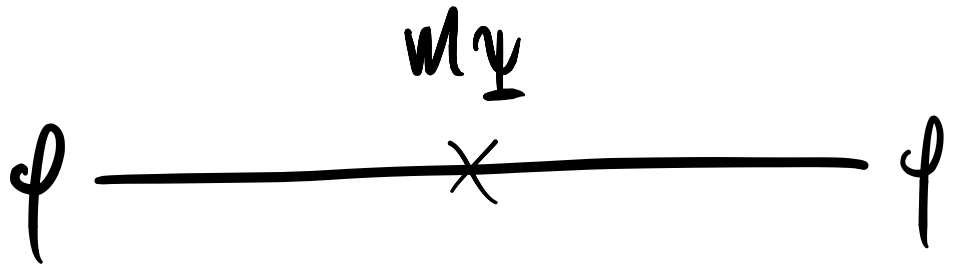
$$\Delta m^2_{S_1} \approx (0.17 \Lambda_{HC})^2 \,, \quad \Delta m^2_{S_3} \approx (0.21 \Lambda_{HC})^2 \,. \quad \Delta m^2_{\tilde{R}_2, T_2} \approx (0.19 \Lambda_{HC})^2 \,.$$ ~ **3** of SU(3)c

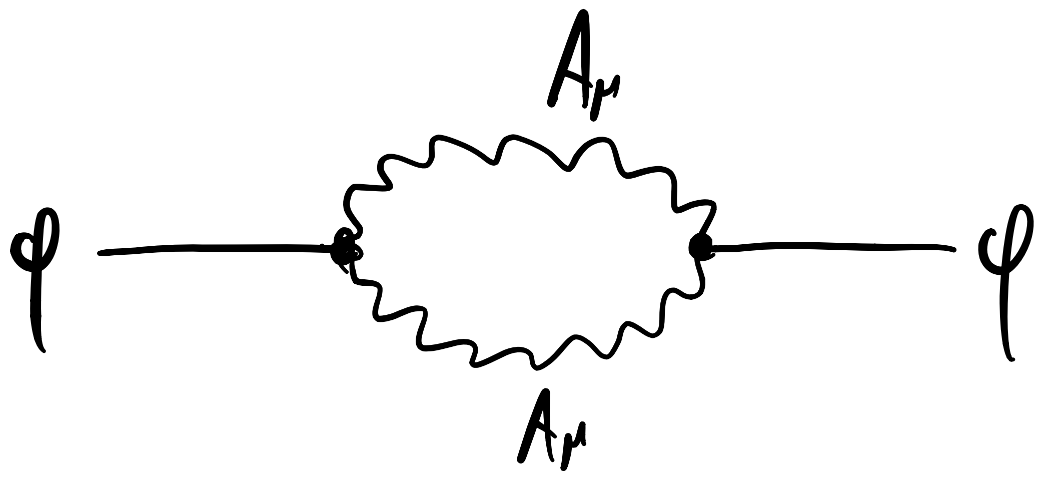
$$\Delta m^2_{\tilde{\pi}_1} \approx (0.26 \Lambda_{HC})^2 \,, \quad \Delta m^2_{\tilde{\pi}_3} \approx (0.28 \Lambda_{HC})^2 \,,$$ ~ **8** of SU(3)c

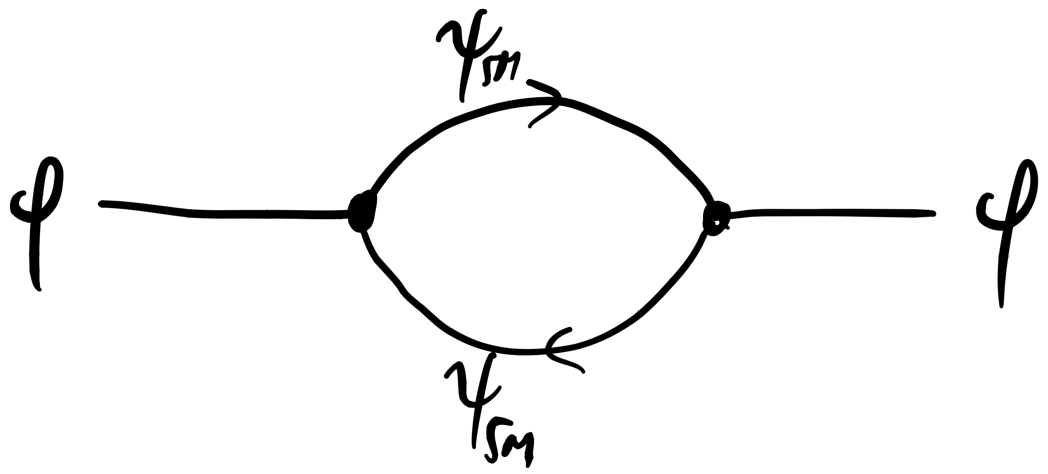
$\Lambda_{HC} \sim 4\pi f \gtrsim 10$ TeV

Scalar Potential: NDA + symmetry

The pNGB potential arises at 1-loop from all the explicit breaking terms







NDA + spurion analysis

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$$V_t = - \frac{c_t y_t^2 N_c \Lambda^2_{HC}}{16\pi^2} |H_1 - H_2|^2$$

$$V_{\mathrm{LQ}} = - \frac{(c_1 g_1^2 + c_1^u g_1^{u2}) \Lambda^2_{HC}}{8\pi^2} |S_1|^2 - \frac{c_3 g_3^2 \Lambda^2_{HC}}{8\pi^2} |S_3|^2$$

valence	irrep.	valence	irrep.
$H_1 \sim i\sigma^2 (\bar{\Psi}_L \Psi_N)$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$H_2 \sim (\bar{\Psi}_E \Psi_L)$	$(\mathbf{1}, \mathbf{2})_{1/2}$
$S_1 \sim (\bar{\Psi}_Q \Psi_L)$	$(\bar{\mathbf{3}}, \mathbf{1})_{1/3}$	$S_3 \sim (\bar{\Psi}_Q \sigma^a \Psi_L)$	$(\bar{\mathbf{3}}, \mathbf{3})_{1/3}$
$\omega^\pm \sim (\bar{\Psi}_N \Psi_E)$	$(\mathbf{1}, \mathbf{1})_{-1}$	$\Pi_L \sim (\bar{\Psi}_L \sigma^a \Psi_L)$	$(\mathbf{1}, \mathbf{3})_0$
$\tilde{R}_2 \sim (\bar{\Psi}_E \Psi_Q)$	$(\mathbf{3}, \mathbf{2})_{1/6}$	$T_2 \sim (\bar{\Psi}_Q \Psi_N)$	$(\bar{\mathbf{3}}, \mathbf{2})_{5/6}$
$\tilde{\pi}_1 \sim (\bar{\Psi}_Q T^A \Psi_Q)$	$(\mathbf{8}, \mathbf{1})_0$	$\tilde{\pi}_3 \sim (\bar{\Psi}_Q T^A \sigma^a \Psi_Q)$	$(\mathbf{8}, \mathbf{3})_0$
$\Pi_Q \sim (\bar{\Psi}_Q \sigma^a \Psi_Q)$	$(\mathbf{1}, \mathbf{3})_0$	$\eta_i \sim 3 \times c_i^a (\bar{\Psi}_a \Psi_a)$	$(\mathbf{1}, \mathbf{1})_0$

pNGB spectrum: example