Electroweak flavour unification

Joe Davighi, University of Zurich
2201.07245 with Joseph Tooby-Smith

La Thuile, 11th March 2022

Why a unification-based theory of flavour?

The SM is a very complicated QFT.

- 3 gauge couplings, non-abelian and abelian forces
- 5 (6 inc. υ_R) fermions in “weird” representations (for one generation)
- 3 generations; Yukawa structure
- Higgs mechanism in electroweak sector gives weak, short-range forces
- Confinement of QCD in the IR

Unification of forces and/or matter attempts to explain all (or part of) this structure as a consequence of something simpler at high energies.

There are two GUTs (one gauge coupling) that don't require extra fermions:

- SU(5) $\Psi \sim \mathbf{5} \oplus \overline{\mathbf{10}} \oplus \mathbf{1}$ Georgi, Glashow, 1974
- SO(10) $\Psi \sim \mathbf{16}$ Georgi, 1975, and Fritzsch, Minkowski, 1975

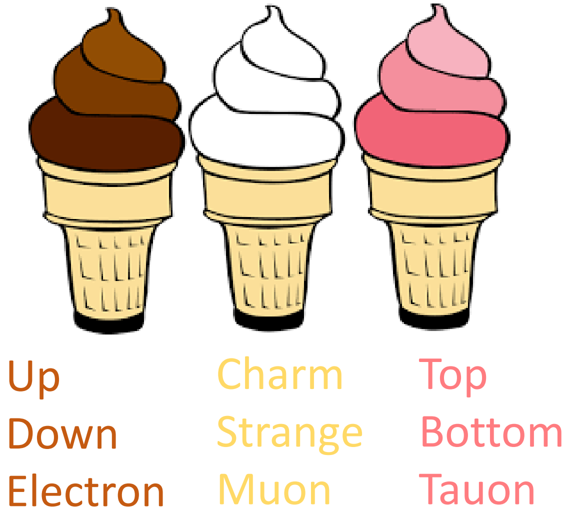
There are two GUTs (one gauge coupling) that don't require extra fermions:

- SU(5) $\Psi \sim [\mathbf{5} \oplus \overline{\mathbf{10}} \oplus \mathbf{1}]^{\oplus 3}$
- SO(10) $\Psi \sim [\mathbf{16}]^{\oplus 3}$

Georgi, Glashow, 1974

Georgi, 1975, and Fritzsch, Minkowski, 1975

But these say nothing about flavour



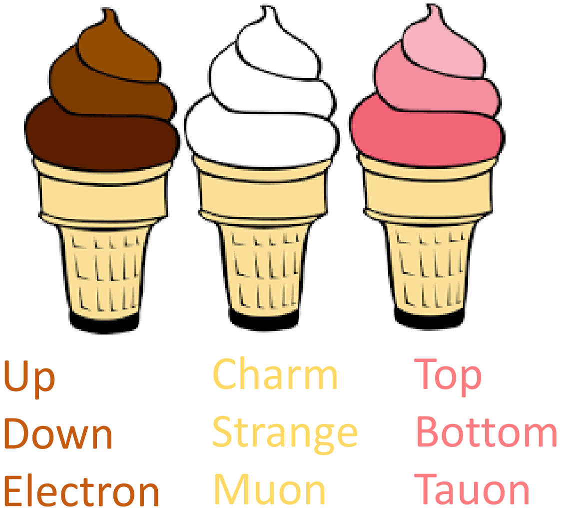
There are two GUTs (one gauge coupling) that don't require extra fermions:

- SU(5) $\Psi \sim [\mathbf{5} \oplus \overline{\mathbf{10}} \oplus \mathbf{1}]^{\oplus 3}$
- SO(10) $\Psi \sim [\mathbf{16}]^{\oplus 3}$

Georgi, Glashow, 1974

Georgi, 1975, and Fritzsch, Minkowski, 1975

Q: can we **unify** either SU(5) or SO(10) **with flavour**, thereby explaining the origin of three generations?



There are two GUTs (one gauge coupling) that don't require extra fermions:

- SU(5) $\Psi \sim [\mathbf{5} \oplus \overline{\mathbf{10}} \oplus \mathbf{1}]^{\oplus 3}$
- SO(10) $\Psi \sim [\mathbf{16}]^{\oplus 3}$

Georgi, Glashow, 1974

Georgi, 1975, and Fritzsch, Minkowski, 1975

Q: can we **unify** either SU(5) or SO(10) **with flavour**, thereby explaining the origin of three generations?

A: No! (at least not without extra fermions)

Up	Charm	Top
Down	Strange	Bottom
Electron	Muon	Tauon

A provocative claim:

“If we want to unify the three generations of matter, we must forgo the complete unification of forces.”

If we want to unify gauge and flavour symmetries, it turns out all roads go through Pati-Salam

Pati, Salam, 1974

$$PS = SU(4) \times SU(2) \times SU(2) \times G_F$$

$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$$

1. Colour flavour unification:

$$SU(12) \times SU(2) \times SU(2)$$
$$\Psi_L \sim (\mathbf{12}, \mathbf{2}, \mathbf{1}),\ \Psi_R \sim (\mathbf{12}, \mathbf{1}, \mathbf{2})$$

$G_F = SU(3)$

$$PS = SU(4) \times SU(2) \times SU(2) \times G_F$$

$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$$

Reminder:
The Lie group $Sp(6)$ is a subgroup of $SU(6)$:
$Sp(6) = \{U \in SU(6) | U^T \Omega U = \Omega\}$, where $\Omega = \begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}$

1. Colour flavour unification:

$$SU(12) \times SU(2) \times SU(2)$$
$$\Psi_L \sim (\mathbf{12}, \mathbf{2}, \mathbf{1}),\ \Psi_R \sim (\mathbf{12}, \mathbf{1}, \mathbf{2})$$

2. Electroweak flavour unification:

$$SU(4) \times Sp(6) \times Sp(6)$$ [1]
$$SU(4) \times Sp(6) \times SO(6)$$
$$\Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1}),\ \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6})$$

$$G_F = SO(3) \times SO(3)$$

$$PS = SU(4) \times SU(2) \times SU(2) \times G_F$$

$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$$

[1]This gauge group was in fact written down by Kuo and Nakagawa in 1984

These are the *only* gauge-flavour unified groups with just 2 Ψs, assuming no BSM Weyls

See Allanach, Gripaios, Tooby-Smith, 2104.14555

1. Colour flavour unification:

$$SU(12) \times SU(2) \times SU(2)$$
$$\Psi_L \sim (\mathbf{12}, \mathbf{2}, \mathbf{1}),\ \Psi_R \sim (\mathbf{12}, \mathbf{1}, \mathbf{2})$$

2. Electroweak flavour unification:

$$SU(4) \times Sp(6) \times Sp(6)$$
$$SU(4) \times Sp(6) \times SO(6)$$
$$\Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1}),\ \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6})$$

$$PS = SU(4) \times SU(2) \times SU(2) \times G_F$$

$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$$

These are the *only* gauge-flavour unified groups with just 2 Ψs, assuming no BSM Weyls

See Allanach, Gripaios, Tooby-Smith, 2104.14555

1. Colour flavour unification:

$$SU(12) \times SU(2) \times SU(2)$$
$$\Psi_L \sim (\mathbf{12}, \mathbf{2}, \mathbf{1}),\ \Psi_R \sim (\mathbf{12}, \mathbf{1}, \mathbf{2})$$

2. Electroweak flavour unification:

$$SU(4) \times Sp(6) \times Sp(6)$$
$$SU(4) \times Sp(6) \times SO(6)$$
$$\Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1}),\ \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6})$$

E.g. Lepton-flavoured gauge symmetries that stabilize the proton
(See Admir's talk)

$$PS = SU(4) \times SU(2) \times SU(2) \times G_F$$
$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$$

JD, Greljo, Eller Thomsen, 2202.05275

These are the *only* gauge-flavour unified groups with just 2 Ψs, assuming no BSM Weyls

See Allanach, Gripaios, Tooby-Smith, 2104.14555

1. Colour flavour unification:

$$SU(12) \times SU(2) \times SU(2)$$
$$\Psi_L \sim (\mathbf{12}, \mathbf{2}, \mathbf{1}),\ \Psi_R \sim (\mathbf{12}, \mathbf{1}, \mathbf{2})$$

2. Electroweak flavour unification:

This talk $\boxed{SU(4) \times Sp(6) \times Sp(6)}$

$$SU(4) \times Sp(6) \times SO(6)$$
$$\Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1}),\ \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6})$$

$$PS = SU(4) \times SU(2) \times SU(2) \times G_F$$
$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$$

Summarize our motivations:

1. Unification of quarks & leptons
2. Unification of 3 generations

3. **The challenge**: can we also explain the peculiar structure in fermion masses and mixings?

Let’s build a model of flavour.

$G = SU(4) \times Sp(6)_L \times Sp(6)_R$

Embedding the SM fields

Embed SM chiral fermions in 2 fields:

$$\Psi_L \sim (\mathbf{4},\mathbf{6},\mathbf{1}) \sim \begin{pmatrix} u_1^r & u_2^r & u_3^r & d_1^r & d_2^r & d_3^r \\ u_1^g & u_2^g & u_3^g & d_1^g & d_2^g & d_3^g \\ u_1^b & u_2^b & u_3^b & d_1^b & d_2^b & d_3^b \\ \upsilon_1 & \upsilon_2 & \upsilon_3 & e_1 & e_2 & e_3 \end{pmatrix}, \qquad \Psi_R \sim (\mathbf{4},\mathbf{1},\mathbf{6}) \sim \text{ similar}$$

Embed SM Higgs in $H_1 \sim (\mathbf{1},\mathbf{6},\mathbf{6})$ and $H_{15} \sim (\mathbf{15},\mathbf{6},\mathbf{6})$,

c.f. Pati-Salam model

with Yukawa couplings:

$$\mathcal{L} = y_1 \mathrm{Tr}\,[\overline{\Psi_L}\Omega H_1 \Omega \Psi_R] + y_{15} \mathrm{Tr}\,[\overline{\Psi_L}\Omega H_{15} \Omega \Psi_R] + \bar{y}_1 \mathrm{Tr}\,[\overline{\Psi_L}\Omega H_1^* \Omega \Psi_R] + \bar{y}_{15} \mathrm{Tr}\,[\overline{\Psi_L}\Omega H_{15}^* \Omega \Psi_R]$$

Recall $\Omega = \begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}$

The Pati-Salam Higgs fields have become flavoured

$$G = SU(4) \times Sp(6)_L \times Sp(6)_R$$

We must break $G \longrightarrow \cdots \longrightarrow SM$

We do so using an (almost) minimal set of scalars

Nothing else will be needed to generate realistic fermion masses and quark mixings

Type	Field	G_{EWF} irrep
SM fermions	Ψ_L	$(\mathbf{4}, \mathbf{6}, \mathbf{1})$
	Ψ_R	$(\mathbf{4}, \mathbf{1}, \mathbf{6})$
Higgs	H_1	$(\mathbf{1}, \mathbf{6}, \mathbf{6})$
	H_{15}	$(\mathbf{15}, \mathbf{6}, \mathbf{6})$
SSB scalars	S_L	$(\mathbf{1}, \mathbf{14}, \mathbf{1})$
	S_R	$(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{6})$
	Φ_L	$(\mathbf{1}, \mathbf{14}, \mathbf{1})$
	Φ_R	$(\mathbf{1}, \mathbf{1}, \mathbf{14})$

Overview: generation of light Yukawas

$$SU(4)\times Sp(6)_L\times Sp(6)_R$$

$\Lambda_L,\ \Lambda_R$ $\quad \langle S_L\rangle \sim (\mathbf{1},\mathbf{14},\mathbf{1}) \quad \langle S_R\rangle \sim (\bar{\mathbf{4}},\mathbf{1},\mathbf{6})$

Λ_H $\quad$ [Heavy Higgses integrated out]

Flavour non-universal intermediate gauge group $\quad \boxed{SU(3)\times \prod_i SU(2)_{L,i}\times Sp(4)_{R,12}\times U(1)_R}$

$\epsilon\Lambda_H$ $\quad \langle \Phi_L\rangle \sim (\mathbf{1},\mathbf{14},\mathbf{1}) \quad \langle \Phi_R\rangle \sim (\mathbf{1},\mathbf{1},\mathbf{14})$

$$SU(3)\times SU(2)_L\times U(1)_Y$$

Generate EFT operators for light Yukawas $\mathcal{O}\sim\left(\frac{\Phi_{L/R}}{\Lambda_H}\right)^n \overline{\psi_L} H \psi_R$

Match onto SM Yukawas

Step 1. Deconstruction of electroweak symmetry

At a high scale, break $Sp(6)_L \to SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3}$ via a scalar $S_L \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$

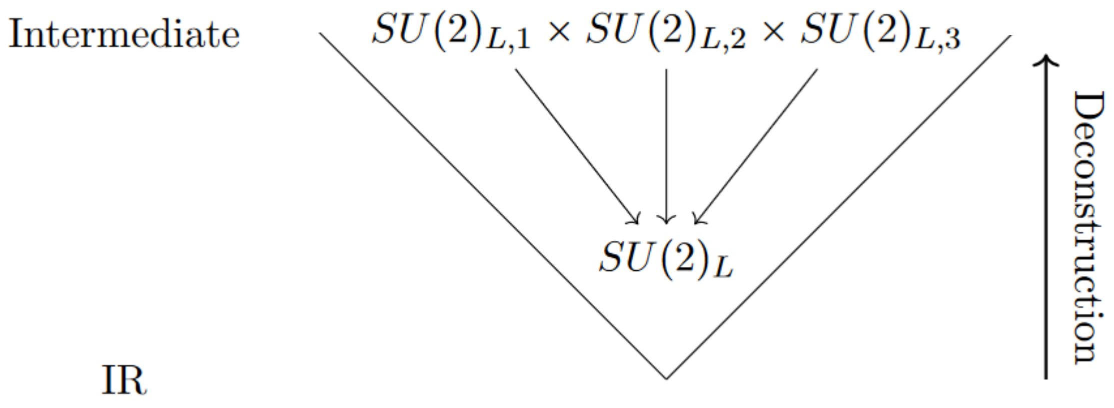
$$\begin{pmatrix} 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \\ 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \end{pmatrix}$$

See also Kuo, Nakagawa, 1984

We do something similar for right-sector, with $S_R \sim (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{6})$ breaking

$$SU(4) \times Sp(6)_R \to SU(3) \times Sp(4)_{R,12} \times U(1)_R$$

Aside:

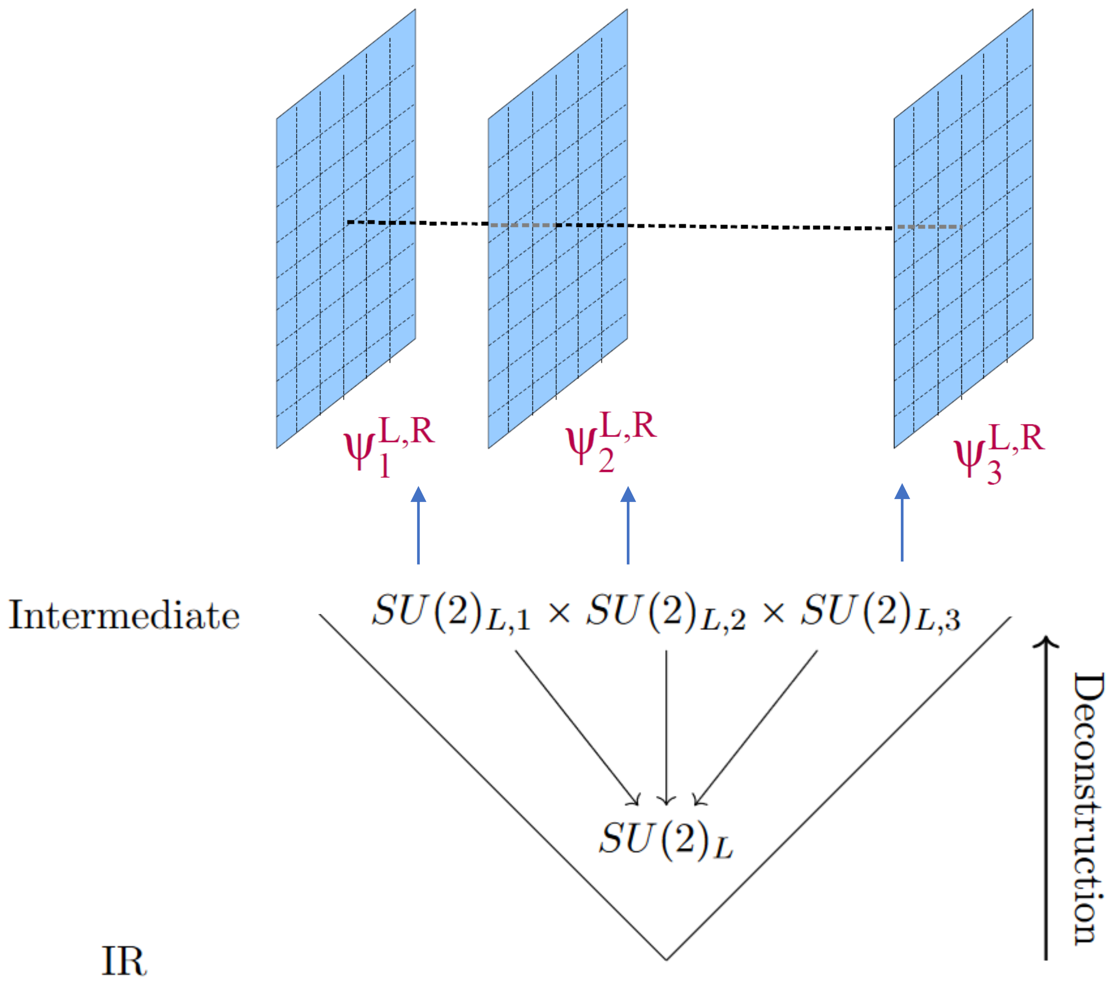


Deconstructed gauge groups have been used in flavour model building e.g. $G = \prod_i^3 PS_i$ for B-anomalies + fermion masses.

See Claudia's talk

Bordone, Cornella, Fuentes-Martín, Isidori, 1712.01368
Bordone, Cornella, Fuentes-Martín, Isidori, 1805.09328
Fuentes-Martín, Isidori, Pagès, Stefanek, 2012.10492
Fuentes-Martín, Isidori, Lizana, Selimovic, Stefanek, 2203.01952

Aside:



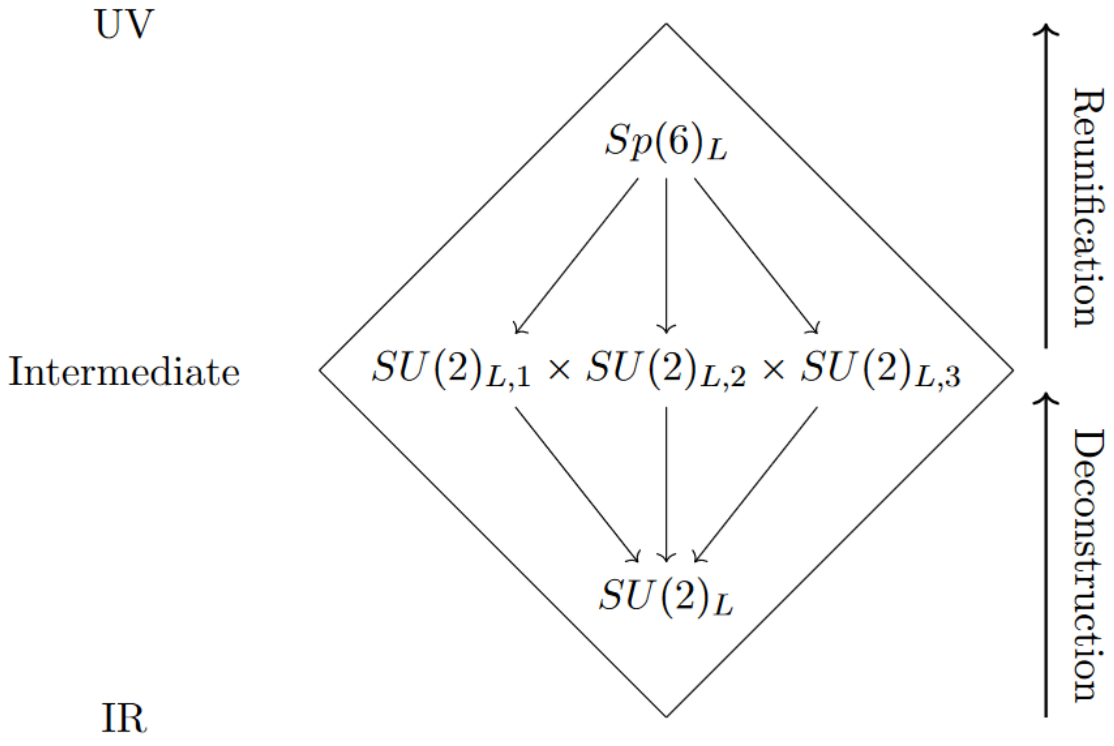
Deconstructed gauge groups have been used in flavour model building e.g. $G = \prod_i^3 PS_i$ for B-anomalies + fermion masses.

A relic of 5d physics?

See Ben's talk

Bordone, Cornella, Fuentes-Martín, Isidori, 1712.01368
Bordone, Cornella, Fuentes-Martín, Isidori, 1805.09328
Fuentes-Martín, Isidori, Pagès, Stefanek, 2012.10492
Fuentes-Martín, Isidori, Lizana, Selimovic, Stefanek, 2203.01952

Aside:



Deconstructed gauge groups have been used in flavour model building e.g. $G = \prod_i^3 PS_i$ for B-anomalies + fermion masses.

Here, “gauge-flavour unification” provides a natural 4d explanation of such a flavour-deconstructed gauge symmetry.

Step 2. Flavoured Higgses

Under the deconstruction $SU(4) \times Sp(6)_L \times Sp(6)_R \ \rightarrow SU(3) \times \prod_{i=1}^{3} SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R$, the Higgs fields split into **flavoured** components:

$$
\begin{aligned}
H_{1,15} \mapsto & [\mathbf{1},(\mathbf{2},\mathbf{1},\mathbf{1}),\mathbf{1}]_{-3} \oplus [\mathbf{1},(\mathbf{1},\mathbf{2},\mathbf{1}),\mathbf{1}]_{-3} \oplus \underline{[\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{2}),\mathbf{1}]_{-3}} \\
& \oplus [\mathbf{1},(\mathbf{2},\mathbf{1},\mathbf{1}),\mathbf{1}]_{3} \oplus [\mathbf{1},(\mathbf{1},\mathbf{2},\mathbf{1}),\mathbf{1}]_{3} \oplus \underline{[\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{2}),\mathbf{1}]_{3}} \\
& \oplus [\mathbf{1},(\mathbf{2},\mathbf{1},\mathbf{1}),\mathbf{4}]_{0} \oplus [\mathbf{1},(\mathbf{1},\mathbf{2},\mathbf{1}),\mathbf{4}]_{0} \oplus [\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{2}),\mathbf{4}]_{0} \\
& \oplus \{SU(3) \text{ triplets and octets for } H_{15}\}\,.
\end{aligned}
$$

SM Higgs

- Reasonable for the Higgs vev to fall into a small number of these family-aligned components.
- This **picks out one family to be heavy, defining the third family.**
- Other fermions **massless** at renormalizable level.

We assume the **other Higgs components are heavy**, and integrated out at a high scale Λ_H

Step 3. Breaking to the SM

Last two scalars Φ_L and Φ_R break $SU(3) \times \prod_{i=1}^{3} SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R$ to SM. The 2-index **14** reps provide link fields:

$$\Phi_L \to \mathbf{1}^{\oplus 2} \oplus \underbrace{(\mathbf{2},\mathbf{2},\mathbf{1})}_{\Phi_L^{12}} \oplus (\mathbf{2},\mathbf{1},\mathbf{2}) \oplus \underbrace{(\mathbf{1},\mathbf{2},\mathbf{2})}_{\Phi_L^{23}}$$

$$\langle \Phi_L^{12} \rangle = \epsilon_L^{12} \Lambda_H, \langle \Phi_L^{23} \rangle = \epsilon_L^{23} \Lambda_H : SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \longrightarrow SU(2)_L$$

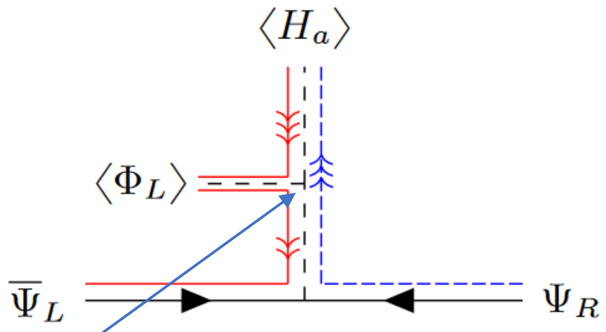
$\langle \Phi_R \rangle = \Lambda_H(\ldots)$ more complicated… also decomposes as [12] + [23] "link fields": break $Sp(4)_{R,12} \times U(1)_R \to U(1)_Y$

Type	Field	G_{EWF} irrep
SM fermions	Ψ_L	$(\mathbf{4}, \mathbf{6}, \mathbf{1})$
	Ψ_R	$(\mathbf{4}, \mathbf{1}, \mathbf{6})$
Higgs	H_1	$(\mathbf{1}, \mathbf{6}, \mathbf{6})$
	H_{15}	$(\mathbf{15}, \mathbf{6}, \mathbf{6})$
SSB scalars	S_L	$(\mathbf{1}, \mathbf{14}, \mathbf{1})$
	S_R	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{6})$
	Φ_L	$(\mathbf{1}, \mathbf{14}, \mathbf{1})$
	Φ_R	$(\mathbf{1}, \mathbf{1}, \mathbf{14})$

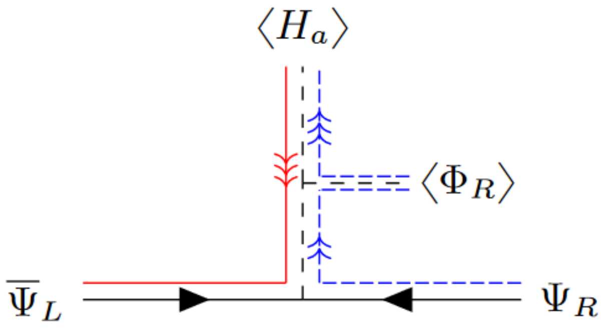
EFT: light fermion Yukawas

Dimension 5: $\mathcal{O} \sim \overline{\psi_L} H \psi_R \phi$

$$\begin{pmatrix} & & \times \\ & \times & \end{pmatrix}$$



$Y_{23}^{u,d,e}$



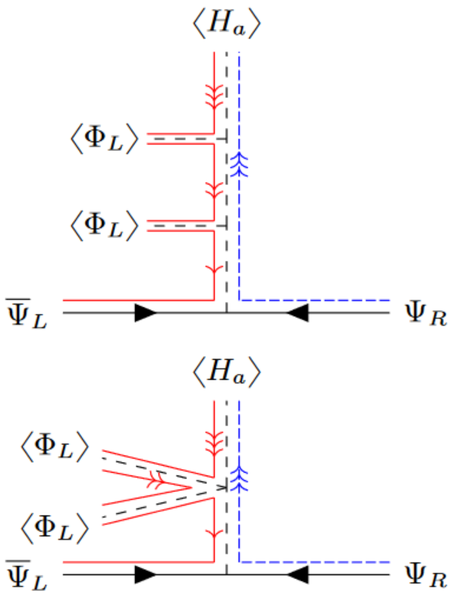
$Y_{32}^{u,d,e}$

Scalar potential $\supset \mathrm{Tr}\left(\Omega^T H_1^\dagger \Omega \Phi_L \Omega H_1\right) + \cdots$

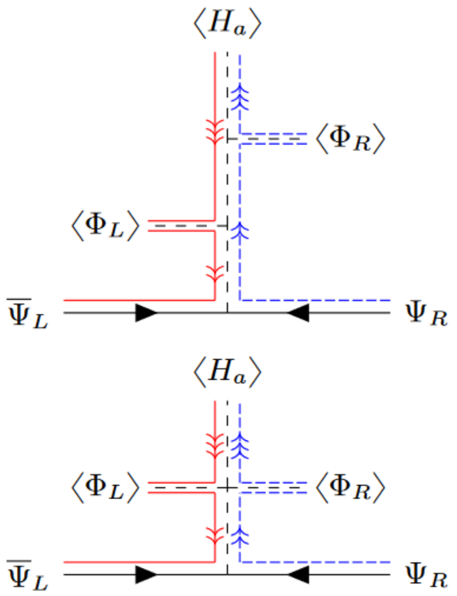
EFT: light fermion Yukawas

Dimension 6: $\mathcal{O} \sim \overline{\psi_L} H \psi_R \phi^2$

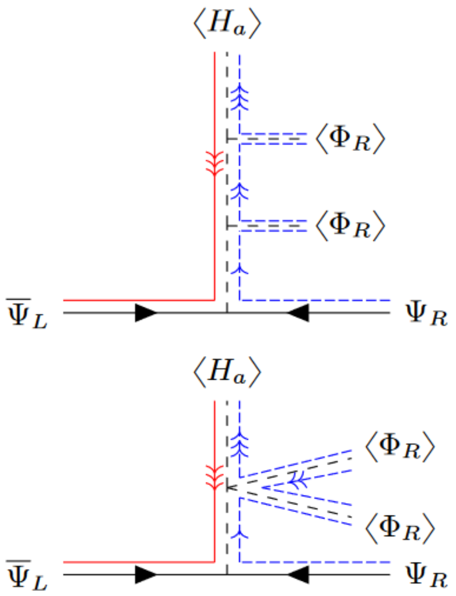
$$\begin{pmatrix} & & \times \\ & \times & \\ \times & & \end{pmatrix}$$



$Y_{13}^{u,d,e}$



$Y_{22}^{u,d,e}$

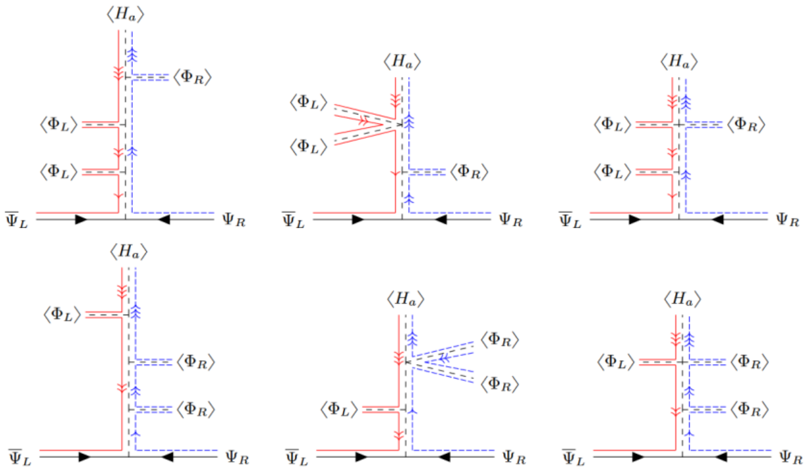


$Y_{31}^{u,d,e}$

EFT: light fermion Yukawas

Dimension 7: $\mathcal{O} \sim \overline{\psi_L} H \psi_R \phi^3$

$$\begin{pmatrix} & \times & \\ \times & & \\ & & \end{pmatrix}$$

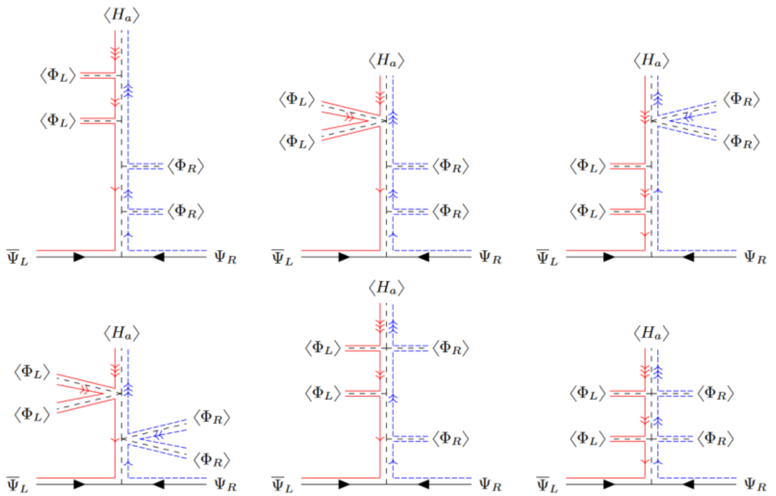


$Y_{12}^{u,d,e}$

$Y_{21}^{u,d,e}$

Dimension 8: $\mathcal{O} \sim \overline{\psi_L} H \psi_R \phi^4$

$$\begin{pmatrix} \times & & \\ & & \\ & & \end{pmatrix}$$



$Y_{11}^{u,d,e}$

Quark masses and mixings

Yukawa matrices have the hierarchical structure

$$\frac{M^f}{v} \sim \begin{pmatrix} \epsilon_L^{12}\epsilon_L^{23}\epsilon_R^{12}\epsilon_R^{23} & \epsilon_L^{12}\epsilon_L^{23}\epsilon_R^{23} & \epsilon_L^{12}\epsilon_L^{23} \\ \epsilon_L^{23}\epsilon_R^{12}\epsilon_R^{23} & \epsilon_L^{23}\epsilon_R^{23} & \epsilon_L^{23} \\ \epsilon_R^{12}\epsilon_R^{23} & \epsilon_R^{23} & 1 \end{pmatrix}$$

for $f \in u, d, e$.

Quark masses and mixings

Extract observables using matrix perturbation theory:

Mass eigenvalues:

$$y_{u,d,e} \approx \left|\frac{\det(\mathbf{h}^{u,d,e})}{\mathbf{k}_{11}^{u,d,e}}\right| \epsilon_L^{12}\epsilon_R^{12}\epsilon_L^{23}\epsilon_R^{23},$$

$$y_{c,s,\mu} \approx \left|\frac{\mathbf{k}_{11}^{u,d,e}}{\mathbf{h}_{33}^{u,d,e}}\right| \epsilon_L^{23}\epsilon_R^{23},$$

$$y_{t,b,\tau} \approx \left|\mathbf{h}_{33}^{u,d,e}\right|,$$

The $\mathbf{h}_{ij}^{u,d,e}$ and $\mathbf{k}_{ij}^{u,d,e}$ are combinations of our EFT coefficients.

Hierarchies in mixing angles:
Choose $\epsilon_L^{12} \sim \lambda$ (Cabibbo), $\epsilon_L^{23} \sim |V_{cb}| \sim \lambda^2$

Hierarchies in mass ratios:
Choose $\epsilon_R^{12} \sim \lambda^2$, $\epsilon_R^{23} \sim \lambda$

CKM matrix $V_{\text{CKM}} = V_L^u V_L^{d*} \approx$

$$\begin{pmatrix} 1-\left(\left|\frac{\mathbf{k}_{21}^d}{\mathbf{k}_{11}^d}\right|^2+\left|\frac{\mathbf{k}_{21}^u}{\mathbf{k}_{11}^u}\right|^2-2\frac{\mathbf{k}_{21}^{u*}\mathbf{k}_{21}^d}{\mathbf{k}_{11}^{u*}\mathbf{k}_{11}^d}\right)\frac{(\epsilon_L^{12})^2}{2} & \left(\frac{\mathbf{k}_{21}^d}{\mathbf{k}_{11}^d}-\frac{\mathbf{k}_{21}^{u*}}{\mathbf{k}_{11}^{u*}}\right)\epsilon_L^{12} & \left(\frac{\mathbf{k}_{31}^{u*}}{\mathbf{k}_{11}^{u*}}+\frac{\mathbf{h}_{13}^d}{\mathbf{h}_{33}^d}-\frac{\mathbf{h}_{23}^d}{\mathbf{h}_{33}^d}\frac{\mathbf{k}_{21}^{u*}}{\mathbf{k}_{11}^{u*}}\right)\epsilon_L^{12}\epsilon_L^{23} \\ \left(\frac{\mathbf{k}_{21}^u}{\mathbf{k}_{11}^u}-\frac{\mathbf{k}_{21}^{d*}}{\mathbf{k}_{11}^{d*}}\right)\epsilon_L^{12} & 1-\left(\left|\frac{\mathbf{k}_{21}^d}{\mathbf{k}_{11}^d}\right|^2+\left|\frac{\mathbf{k}_{21}^u}{\mathbf{k}_{11}^u}\right|^2-2\frac{\mathbf{k}_{21}^u\mathbf{k}_{21}^{d*}}{\mathbf{k}_{11}^u\mathbf{k}_{11}^{d*}}\right)\frac{(\epsilon_L^{12})^2}{2} & \left(\frac{\mathbf{h}_{23}^d}{\mathbf{h}_{33}^d}-\frac{\mathbf{h}_{23}^{u*}}{\mathbf{h}_{33}^{u*}}\right)\epsilon_L^{23} \\ \left(\frac{\mathbf{k}_{31}^{d*}}{\mathbf{k}_{11}^{d*}}+\frac{\mathbf{h}_{13}^u}{\mathbf{h}_{33}^u}-\frac{\mathbf{h}_{23}^u}{\mathbf{h}_{33}^u}\frac{\mathbf{k}_{21}^{d*}}{\mathbf{k}_{11}^{d*}}\right)\epsilon_L^{12}\epsilon_L^{23} & \left(\frac{\mathbf{h}_{23}^u}{\mathbf{h}_{33}^u}-\frac{\mathbf{h}_{23}^{d*}}{\mathbf{h}_{33}^{d*}}\right)\epsilon_L^{23} & 1 \end{pmatrix}$$

Our CKM matches onto the Wolfenstein parametrization

Wolfenstein, 1983

Quark masses and mixings

Extract observables using matrix perturbation theory:

Mass eigenvalues:

$$y_{u,d,e} \approx \left|\frac{\det(\mathbf{h}^{u,d,e})}{\mathbf{k}_{11}^{u,d,e}}\right| \epsilon_L^{12}\epsilon_R^{12}\epsilon_L^{23}\epsilon_R^{23},$$

$$y_{c,s,\mu} \approx \left|\frac{\mathbf{k}_{11}^{u,d,e}}{\mathbf{h}_{33}^{u,d,e}}\right| \epsilon_L^{23}\epsilon_R^{23},$$

$$y_{t,b,\tau} \approx \left|\mathbf{h}_{33}^{u,d,e}\right|,$$

The $\mathbf{h}_{ij}^{u,d,e}$ and $\mathbf{k}_{ij}^{u,d,e}$ are combinations of our EFT coefficients.

Hierarchies in mixing angles:
Choose $\epsilon_L^{12} \sim \lambda$ (Cabibbo), $\epsilon_L^{23} \sim |V_{cb}| \sim \lambda^2$

Hierarchies in mass ratios:
Choose $\epsilon_R^{12} \sim \lambda^2$, $\epsilon_R^{23} \sim \lambda$

... And there is **enough freedom** in the EFT coefficients to fit all the data ☑

CKM matrix $V_{\mathrm{CKM}} = V_L^u V_L^{d*} \approx$

$$\begin{pmatrix} 1 - \left(\left|\frac{\mathbf{k}_{21}^d}{\mathbf{k}_{11}^d}\right|^2 + \left|\frac{\mathbf{k}_{21}^u}{\mathbf{k}_{11}^u}\right|^2 - 2\frac{\mathbf{k}_{21}^{u*}\mathbf{k}_{21}^d}{\mathbf{k}_{11}^{u*}\mathbf{k}_{11}^d}\right)\frac{(\epsilon_L^{12})^2}{2} & \left(\frac{\mathbf{k}_{21}^d}{\mathbf{k}_{11}^d} - \frac{\mathbf{k}_{21}^{u*}}{\mathbf{k}_{11}^{u*}}\right)\epsilon_L^{12} & \left(\frac{\mathbf{k}_{31}^{u*}}{\mathbf{k}_{11}^{u*}} + \frac{\mathbf{h}_{13}^d}{\mathbf{h}_{33}^d} - \frac{\mathbf{h}_{23}^d}{\mathbf{h}_{33}^d}\frac{\mathbf{k}_{21}^{u*}}{\mathbf{k}_{11}^{u*}}\right)\epsilon_L^{12}\epsilon_L^{23} \\ \left(\frac{\mathbf{k}_{21}^u}{\mathbf{k}_{11}^u} - \frac{\mathbf{k}_{21}^{d*}}{\mathbf{k}_{11}^{d*}}\right)\epsilon_L^{12} & 1 - \left(\left|\frac{\mathbf{k}_{21}^d}{\mathbf{k}_{11}^d}\right|^2 + \left|\frac{\mathbf{k}_{21}^u}{\mathbf{k}_{11}^u}\right|^2 - 2\frac{\mathbf{k}_{21}^{u}\mathbf{k}_{21}^{d*}}{\mathbf{k}_{11}^{u}\mathbf{k}_{11}^{d*}}\right)\frac{(\epsilon_L^{12})^2}{2} & \left(\frac{\mathbf{h}_{23}^d}{\mathbf{h}_{33}^d} - \frac{\mathbf{h}_{23}^{u*}}{\mathbf{h}_{33}^{u*}}\right)\epsilon_L^{23} \\ \left(\frac{\mathbf{k}_{31}^{d*}}{\mathbf{k}_{11}^{d*}} + \frac{\mathbf{h}_{13}^u}{\mathbf{h}_{33}^u} - \frac{\mathbf{h}_{23}^u}{\mathbf{h}_{33}^u}\frac{\mathbf{k}_{21}^{d*}}{\mathbf{k}_{11}^{d*}}\right)\epsilon_L^{12}\epsilon_L^{23} & \left(\frac{\mathbf{h}_{23}^u}{\mathbf{h}_{33}^u} - \frac{\mathbf{h}_{23}^{d*}}{\mathbf{h}_{33}^{d*}}\right)\epsilon_L^{23} & 1 \end{pmatrix}$$

Our CKM matches onto the Wolfenstein parametrization

Wolfenstein, 1983

Our EWFU model explains

- The origin of 3 generations
- The hierarchical structure of fermion masses and quark mixing angles

in terms of a flavour-enriched version of Pati—Salam unification

Our EWFU model explains

- The origin of 3 generations
- The hierarchical structure of fermion masses and quark mixing angles

in terms of a flavour-enriched version of Pati—Salam unification

Protons are stable in this UV model. So the scales of EWFU can be brought low…

How low can you go?

We have the following heavy gauge bosons in our model:

		Heavy scales ($\Lambda_{L,R}$)	Intermediate scale ($\epsilon\Lambda_H$)
Name	$G_{\rm SM}$ representation	Number (origin)	Number (origin)
Charged Z'	$(\mathbf{1},\mathbf{1})_6$	3 (S_R)	3 (Φ_R)
U_1 leptoquark	$(\bar{\mathbf{3}},\mathbf{1})_{-4}$	1 (S_R)	—
(W', Z') triplet	$(\mathbf{1},\mathbf{3})_0$ ($\mathbb{R}$)	3 (S_L)	2 (Φ_L)
Real Z'	$(\mathbf{1},\mathbf{1})_0$ ($\mathbb{R}$)	3 (S_L), 5 (S_R)	4 (Φ_R)

Heavy

The light states – all flavoured versions of the EW gauge bosons

How low can you go?

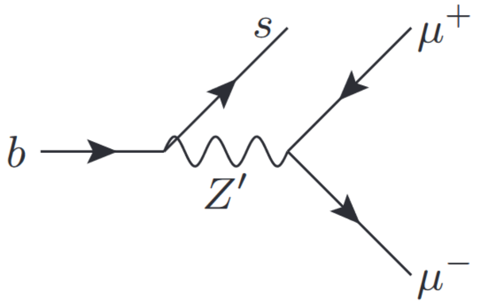
We have the following heavy gauge bosons in our model:

		Heavy scales ($\Lambda_{L,R}$)	Intermediate scale ($\epsilon\Lambda_H$)
Name	$G_{\rm SM}$ representation	Number (origin)	Number (origin)
Charged Z'	$(\mathbf{1},\mathbf{1})_6$	3 (S_R)	3 (Φ_R)
U_1 leptoquark	$(\bar{\mathbf{3}},\mathbf{1})_{-4}$	1 (S_R)	—
(W',Z') triplet	$(\mathbf{1},\mathbf{3})_0$ ($\mathbb{R}$)	3 (S_L)	2 (Φ_L)
Real Z'	$(\mathbf{1},\mathbf{1})_0$ ($\mathbb{R}$)	3 (S_L), 5 (S_R)	4 (Φ_R)

Heavy

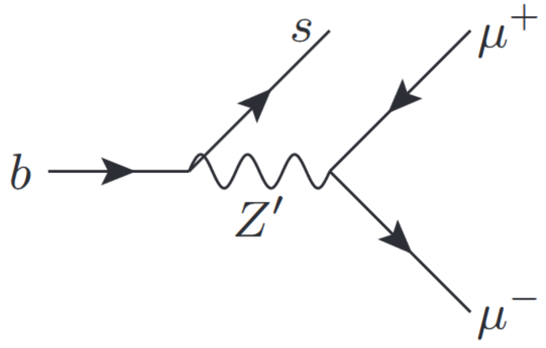
The light states – all flavoured versions of the EW gauge bosons

For example, consider the (W',Z') triplets from Φ_L.
The lightest Z' couples to $Q_{L,2}$, $Q_{L,3}$, $L_{L,2}$, $L_{L,3}$ …



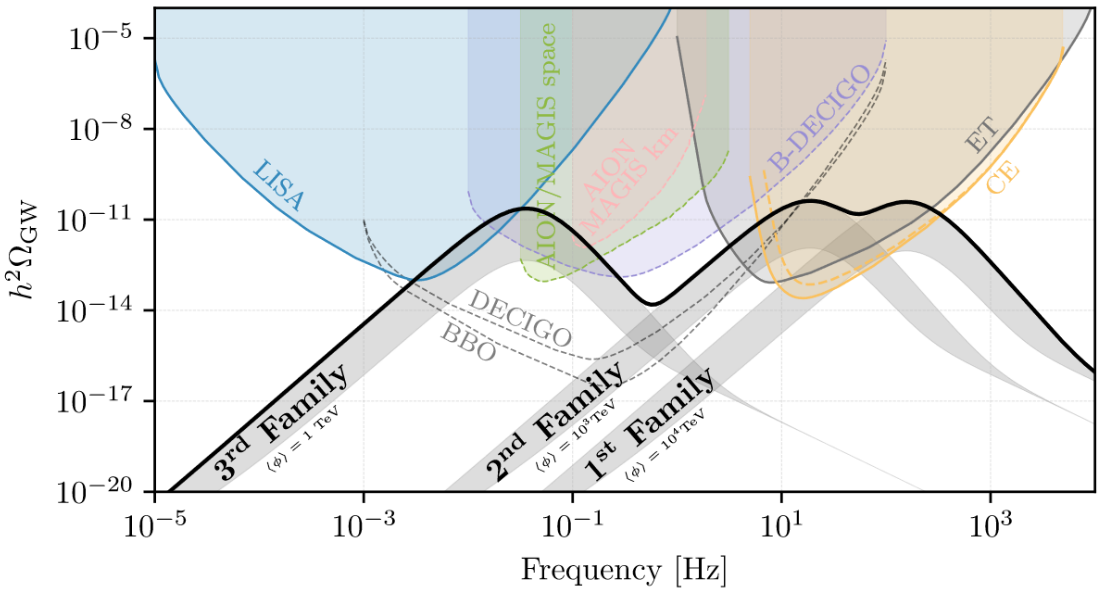
Some future directions

- Low scale EWFU
  - Flavour-dependent forces – B anomalies etc?
  - Phenomenological analysis: compute **lower bounds on scales**
  - How much tuning in scalar sector?



- Neutrino masses

- Cosmology
  - EWFU predicts **monopole** production. Dilute by taking $\Lambda_R > \Lambda_{\text{inflation}}$
  - **Gravitational wave** production in early Universe: stochastic multi-peaked GW signal. An alternative probe of EWFU, even if the SSB scales are very high



Greljo, Opferkuch, Stefanek, 2019

Backup slides

$$SU(4) \times Sp(6)_L \times Sp(6)_R$$

$\Lambda_L,\ \Lambda_R$ $\quad\langle S_L\rangle \sim (\mathbf{1},\mathbf{14},\mathbf{1}) \downarrow \quad \downarrow \langle S_R\rangle \sim (\bar{\mathbf{4}},\mathbf{1},\mathbf{6})$

Λ_H [Heavy Higgses integrated out]

$$SU(3) \times \prod_i SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R$$

$\epsilon\Lambda_H$ $\quad\langle \Phi_L\rangle \sim (\mathbf{1},\mathbf{14},\mathbf{1}) \downarrow \quad \downarrow \langle \Phi_R\rangle \sim (\mathbf{1},\mathbf{1},\mathbf{14})$

$$SU(3) \times SU(2)_L \times U(1)_Y$$

Arbitrary scale separation; can be small to reduce tuning / m_h^2 sensitivity

Small scale separation, $\mathcal{O}(10^{-1} - 10^{-2})$, fixed by Yukawa hierarchy

"Distance to New Physics": can be as low as experimentally allowed by flavour bounds

Step 1: quark-lepton breaking and deconstruction of $Sp(6)_R$ at Λ_R

Break $SU(4) \times Sp(6)_R \;\rightarrow SU(3) \times Sp(4)_{R,12} \times U(1)_R$ via $\mathbb{C}$ scalar $S_R \sim (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{6})$

Vev $\langle S_R \rangle = \Lambda_R a_4^* \otimes c_3$

$SU(4)$ (anti)-fundamental index

$Sp(6)_R$ fundamental index

$$\begin{pmatrix} \times & \times & \cdot & \times & \times & \cdot \\ \times & \times & \cdot & \times & \times & \cdot \\ \cdot & \cdot & \times & \cdot & \cdot & \cdot \\ \times & \times & \cdot & \times & \times & \cdot \\ \times & \times & \cdot & \times & \times & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \times \end{pmatrix}$$

$U(1)_R$ generated by
$X = t_{B-L} + (t_3^R)_{\text{3rd family}}$

$\Lambda_L,\ \Lambda_R$

Λ_H

$\epsilon\Lambda_H$

v

Step 1: quark-lepton breaking and deconstruction of $Sp(6)_R$ at Λ_R

Break $SU(4) \times Sp(6)_R \rightarrow SU(3) \times Sp(4)_{R,12} \times U(1)_R$ via $\mathbb{C}$ scalar $S_R \sim (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{6})$

Vev $\langle S_R \rangle = \Lambda_R a_4^* \otimes c_3$

$SU(4)$ (anti)-fundamental index

$Sp(6)_R$ fundamental index

$$\begin{pmatrix} \times & \times & . & \times & \times & . \\ \times & \times & . & \times & \times & . \\ . & . & \times & . & . & . \\ \times & \times & . & \times & \times & . \\ \times & \times & . & \times & \times & . \\ . & . & . & . & . & \times \end{pmatrix}$$

$U(1)_R$ generated by
$X = t_{B-L} + (t_3^R)_{\text{3rd family}}$

17 heavy gauge bosons decouple at $m \sim \Lambda_R$:

- $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$ leptoquark, flavour universal couplings
- x3 charged (complex) $Z' \sim (\mathbf{1}, \mathbf{1})_1$
- x5 neutral (real) $Z' \sim (\mathbf{1}, \mathbf{1})_0$

$\Lambda_L,\ \Lambda_R$

Λ_H

$\epsilon \Lambda_H$

v

Step 2: deconstruction of $Sp(6)_L$ at Λ_L

Break $Sp(6)_L \to SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3}$ via $\mathbb{R}$ scalar $S_L \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$

Vev $\langle S_L \rangle = \Lambda_L (b_1 \wedge b_4 - b_3 \wedge b_6)$

$Sp(6)_L$ fundamental index

Antisymmetrize

$$\begin{pmatrix} 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \\ 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \end{pmatrix}$$

See also Kuo, Nakagawa, 1984

Λ_L, Λ_R

Λ_H

$\epsilon \Lambda_H$

v

Step 2: deconstruction of $Sp(6)_L$ at Λ_L

Break $Sp(6)_L \rightarrow SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3}$ via $\mathbb{R}$ scalar $S_L \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$

Vev $\langle S_L \rangle = \Lambda_L(b_1 \wedge b_4 - b_3 \wedge b_6)$

$Sp(6)_L$ fundamental index

Antisymmetrize

$$\begin{pmatrix} 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \\ 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \end{pmatrix}$$

See also Kuo, Nakagawa, 1984

12 broken generators decouple at $m \sim \Lambda_L$:

- x3 (W', Z') triplets,
- x3 more Z's

Λ_L, Λ_R

Λ_H

$\epsilon\Lambda_H$

v

Breaking to the SM

Linearly realised gauge symmetry is at this point

$$G_{\mathrm{int}} := SU(3) \times \prod_i SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R$$

UV fermion	Rep	Intermediate fermion	Rep
Ψ_L	$(\mathbf{4},\mathbf{6},\mathbf{1})$	Q_1	$[\mathbf{3},(\mathbf{2},\mathbf{1},\mathbf{1}),\mathbf{1}]_1$
		Q_2	$[\mathbf{3},(\mathbf{1},\mathbf{2},\mathbf{1}),\mathbf{1}]_1$
		Q_3	$[\mathbf{3},(\mathbf{1},\mathbf{1},\mathbf{2}),\mathbf{1}]_1$
		L_1	$[\mathbf{1},(\mathbf{2},\mathbf{1},\mathbf{1}),\mathbf{1}]_{-3}$
		L_2	$[\mathbf{1},(\mathbf{1},\mathbf{2},\mathbf{1}),\mathbf{1}]_{-3}$
		L_3	$[\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{2}),\mathbf{1}]_{-3}$
Ψ_R	$(\mathbf{4},\mathbf{1},\mathbf{6})$	$\mathcal{Q}_{R,12}$	$[\mathbf{3},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{4}]_1$
		$\mathcal{L}_{R,12}$	$[\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{4}]_{-3}$
		U_3	$[\mathbf{3},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{1}]_4$
		D_3	$[\mathbf{3},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{1}]_{-2}$
		E_3	$[\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{1}]_{-6}$
		$\nu_{R,3}$	$[\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{1}]_0$

Just need 2 more scalars to break this to SM:

$\mathbb{R}$ scalar $\Phi_L \sim (\mathbf{1},\mathbf{14},\mathbf{1})$

$\mathbb{C}$ scalar $\Phi_R \sim (\mathbf{1},\mathbf{1},\mathbf{14})$

$$\Phi_L \mapsto [\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{1}]_0^{\oplus 2} \oplus [\underline{\mathbf{1},(\mathbf{2},\mathbf{2},\mathbf{1}),\mathbf{1}}]_0 \oplus [\underline{\mathbf{1},(\mathbf{2},\mathbf{1},\mathbf{2}),\mathbf{1}}]_0 \oplus [\underline{\mathbf{1},(\mathbf{1},\mathbf{2},\mathbf{2}),\mathbf{1}}]_0$$

$$\Phi_R \mapsto [\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{1}]_0 \oplus [\underline{\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{5}}]_0 \oplus [\underline{\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{4}}]_{-3} \oplus [\underline{\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{1}),\mathbf{4}}]_3$$

$\Lambda_L,\ \Lambda_R$

Λ_H

$\epsilon \Lambda_H$

v

Scalar vevs: breaking to the SM

$$\langle\Phi_L\rangle = \underbrace{\epsilon_L^{23}\Lambda_H\,(b_2\wedge b_6 + b_3\wedge b_5)}_{\langle\phi_L^{23}\rangle} + \underbrace{\epsilon_L^{12}\Lambda_H\,(b_1\wedge b_5 + b_2\wedge b_4)}_{\langle\phi_L^{12}\rangle}$$

breaks $SU(2)_{L,1}\times SU(2)_{L,2}\times SU(2)_{L,3}\longrightarrow SU(2)_L$

Gives x2 (W',Z') triplets;

one coupled to 1st and 2nd families, with $m_{12}\sim g_L\epsilon_L^{12}\Lambda_H$

one coupled to 2nd and 3rd families, with $m_{23}\sim g_L\epsilon_L^{23}\Lambda_H$ [more later]

$$\langle\Phi_R\rangle = \underbrace{\Lambda_H\epsilon_R^{23}w_{23}c_2\wedge c_6}_{\phi_R^{23}} + \underbrace{\Lambda_H\epsilon_R^{23}\overline{w}_{23}c_3\wedge c_5}_{\overline{\phi}_R^{23}} + \underbrace{\Lambda_H\epsilon_R^{12}\,(w_{12}c_1\wedge c_5 + \overline{w}_{12}c_2\wedge c_4)}_{\phi_R^{12}}$$

breaks $Sp(4)_{R,12}\times U(1)_R\longrightarrow U(1)_Y$

Gives 10 charged & neutral Z's, masses $m_R\sim g_R\epsilon_R^{ij}\Lambda_H$

$\Lambda_L,\ \Lambda_R$

Λ_H

$\epsilon\Lambda_H$

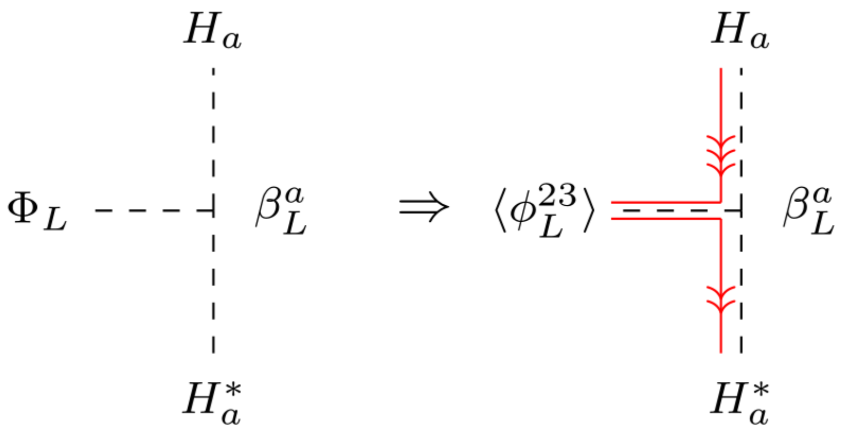
v

Terms in the scalar potential

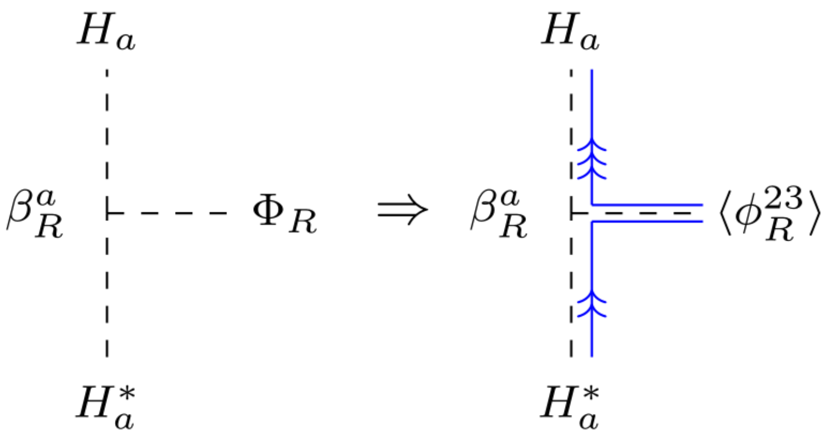
All the required EFT operators are already generated in our model, by integrating out the heavy components of $H_{1,15}$; if we include (renormalizable) interactions in the scalar potential.

Scalar Interactions

Cubics:



x1 $\mathbb{R}$ coupling (per a)



x1 $\mathbb{C}$ coupling (per a)

$\Lambda_L,\ \Lambda_R$

Λ_H

$\epsilon\Lambda_H$

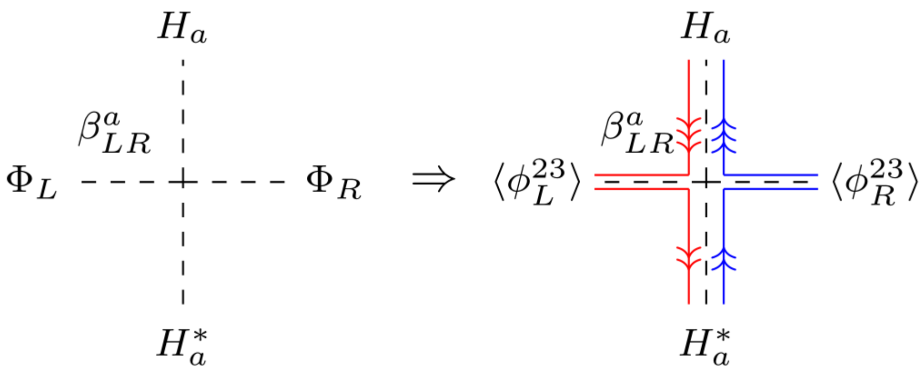
v

Terms in the scalar potential

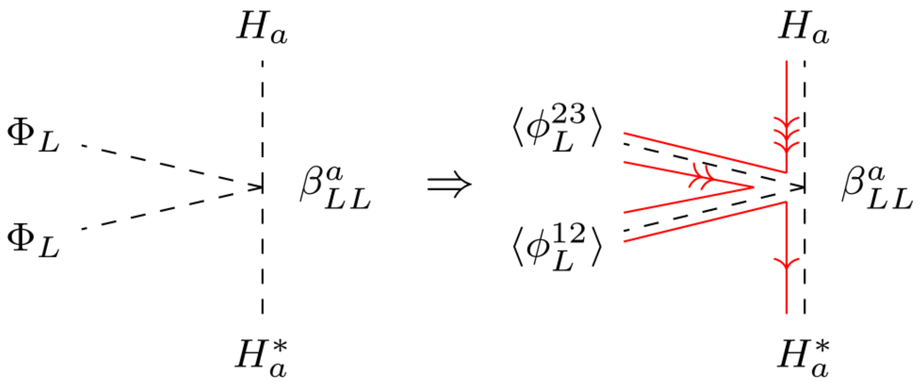
In fact, all the required EFT operators are already generated in our model, by integrating out the heavy components of $H_{1,15}$; if we include (renormalizable) interactions in the scalar potential.

Scalar Interactions

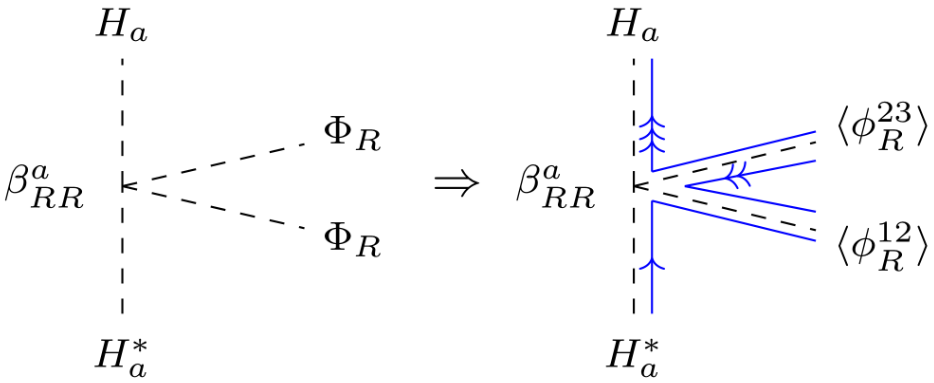
Quartics:



x1 $\mathbb{C}$ coupling (per a)



x1 $\mathbb{R}$ coupling (per a)



x1 $\mathbb{C}$ and x2 $\mathbb{R}$ couplings (per a)

$\Lambda_L,\ \Lambda_R$

Λ_H

$\epsilon\Lambda_H$

v

Properties of our CKM model

Our CKM is not a general unitary matrix. Like Wolfenstein, it satisfies

$$|V_{ud}| = |V_{cs}|, \qquad |V_{ts}| = |V_{cb}|, \qquad |V_{ud}| = 1 - \frac{1}{2}|V_{us}|^2$$

at leading order. Also, Jarlskog invariant satisfies

$$4J^2 = 2|V_{us}V_{cb}|^2(|V_{ub}|^2 + |V_{td}|^2) + 2|V_{ub}V_{td}|^2 - |V_{td}|^4 - |V_{ub}|^4 - |V_{us}V_{cb}|^4$$

which implies CP-violating phase $\delta_{13} \approx 1.25$ radians.

All these relations agree well with data.

Properties of our CKM model

Our CKM is not a general unitary matrix. Like Wolfenstein, it satisfies

$$|V_{ud}| = |V_{cs}|, \qquad |V_{ts}| = |V_{cb}|, \qquad |V_{ud}| = 1 - \frac{1}{2}|V_{us}|^2$$

at leading order. Also, Jarlskog invariant satisfies

$$4J^2 = 2|V_{us}V_{cb}|^2(|V_{ub}|^2 + |V_{td}|^2) + 2|V_{ub}V_{td}|^2 - |V_{td}|^4 - |V_{ub}|^4 - |V_{us}V_{cb}|^4$$

which implies CP-violating phase $\delta_{13} \approx 1.25$ radians.

All these relations agree well with data.

Also, $V_{td} = -V_{ub}^* + (V_{us}V_{cb})^*$.

<u>The upshot of these relations:</u>

If, in our model, we can fit V_{us}, V_{cb}, and V_{ub} to be arbitrary $\mathbb{C}$-numbers, then can freely fit $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $|V_{td}|$ to their central experimental values, and the rest of CKM is in close agreement.

Fitting quark masses and mixings

Indeed there is enough freedom in the model to freely fit the coefficients of [all as $\mathbb{C}$-numbers]

- x9 masses (quarks and charged leptons)
- V_{us}, V_{cb}, and V_{ub}

Fitting quark masses and mixings

Indeed there is enough freedom in the model to freely fit the coefficients of [all as $\mathbb{C}$-numbers]

- x9 masses (quarks and charged leptons)
- V_{us}, V_{cb}, and V_{ub}

<u>Sketch of how this works:</u>

1. Fit $\{m_t, m_b, m_\tau, V_{cb}\}$ from $\{y_1, y_{15}, \overline{y_1}, \overline{y_{15}}\}$, for any* values of $(\beta_L^1, \beta_L^{15})$

$$\begin{aligned} m_t &\approx (y_1 \overline{v}_1 + \overline{y}_1 v_1^*) + (y_{15}\overline{v}_{15} + \overline{y}_{15} v_{15}^*), \\ m_b &\approx (y_1 v_1 + \overline{y}_1 \overline{v}_1^*) + (y_{15} v_{15} + \overline{y}_{15}\overline{v}_{15}^*), \\ m_\tau &\approx (y_1 v_1 + \overline{y}_1 \overline{v}_1^*) - 3(y_{15} v_{15} + \overline{y}_{15}\overline{v}_{15}^*), \end{aligned}$$

$$\begin{aligned} V_{cb} =& \frac{\lambda^2}{2}\Big\{ \frac{\beta_L^1}{y_b}(y_1 v_1 + \overline{y}_1 \overline{v}_1^*) + \frac{\beta_L^{15}}{y_b}(y_{15} v_{15} + \overline{y}_{15}\overline{v}_{15}^*) \\ & - \frac{\beta_L^1}{y_t^*}(\overline{y}_1^* v_1 + y_1^* \overline{v}_1^*) - \frac{\beta_L^{15}}{y_t^*}(\overline{y}_{15}^* v_{15} + y_{15}^* \overline{v}_{15}^*)\Big\}, \end{aligned}$$

*Away from a small set of points

Fitting quark masses and mixings

Indeed there is enough freedom in the model to freely fit the coefficients of [all as $\mathbb{C}$-numbers]

- x9 masses (quarks and charged leptons)
- V_{us}, V_{cb}, and V_{ub}

<u>Sketch of how this works:</u>

1. Fit $\{m_t, m_b, m_\tau, V_{cb}\}$ from $\{y_1, y_{15}, \overline{y_1}, \overline{y_{15}}\}$, for any* values of $(\beta_L^1, \beta_L^{15})$

$$m_t \approx (y_1\overline{v}_1 + \overline{y}_1 v_1^*) + (y_{15}\overline{v}_{15} + \overline{y}_{15} v_{15}^*),$$
$$m_b \approx (y_1 v_1 + \overline{y}_1 \overline{v}_1^*) + (y_{15} v_{15} + \overline{y}_{15} \overline{v}_{15}^*),$$
$$m_\tau \approx (y_1 v_1 + \overline{y}_1 \overline{v}_1^*) - 3(y_{15} v_{15} + \overline{y}_{15} \overline{v}_{15}^*),$$

$$V_{cb} = \frac{\lambda^2}{2}\left\{\frac{\beta_L^1}{y_b}(y_1 v_1 + \overline{y}_1 \overline{v}_1^*) + \frac{\beta_L^{15}}{y_b}(y_{15} v_{15} + \overline{y}_{15} \overline{v}_{15}^*) - \frac{\beta_L^1}{y_t^*}(\overline{y}_1^* v_1 + y_1^* \overline{v}_1^*) - \frac{\beta_L^{15}}{y_t^*}(\overline{y}_{15}^* v_{15} + y_{15}^* \overline{v}_{15}^*)\right\},$$

2. Fit $\{m_c, m_s, m_\mu, V_{us}, V_{ub}\}$ from $\{\beta_R^1, \beta_{LR}^1, \beta_{LL}^1, \beta_{LL}^{15}, w_{23}, \overline{w_{23}}\}$ …
3. Fit $\{m_u, m_d, m_e\}$ from $\{\beta_{RR}^1, w_{12}, \overline{w_{12}}\}$

Recall

$$\langle\Phi_R\rangle = \underbrace{\Lambda_H \epsilon_R^{23} w_{23} c_2 \wedge c_6}_{\phi_R^{23}} + \underbrace{\Lambda_H \epsilon_R^{23} \overline{w}_{23} c_3 \wedge c_5}_{\overline{\phi}_R^{23}} + \underbrace{\Lambda_H \epsilon_R^{12} (w_{12} c_1 \wedge c_5 + \overline{w}_{12} c_2 \wedge c_4)}_{\phi_R^{12}}$$

The hierarchies are "in-built" from the dependence on $\epsilon_{L,R}^{12,23}$

*Away from a small set of points