RECENT PROGRESS IN SOFT-COLLINEAR EFFECTIVE THEORY (SCET)

MATTHIAS NEUBERT

PRISMA+ CLUSTER OF EXCELLENCE & MAINZ INSTITUTE FOR THEORETICAL PHYSICS

JOHANNES GUTENBERG UNIVERSITY MAINZ

LA THUILE 2022 — LES RENCONTRES DE PHYSIQUE DE LA VALLÉE D'AOSTE

10 MARCH 2022

Cluster of Excellence PRISMA+

DFG EXC 2118/1

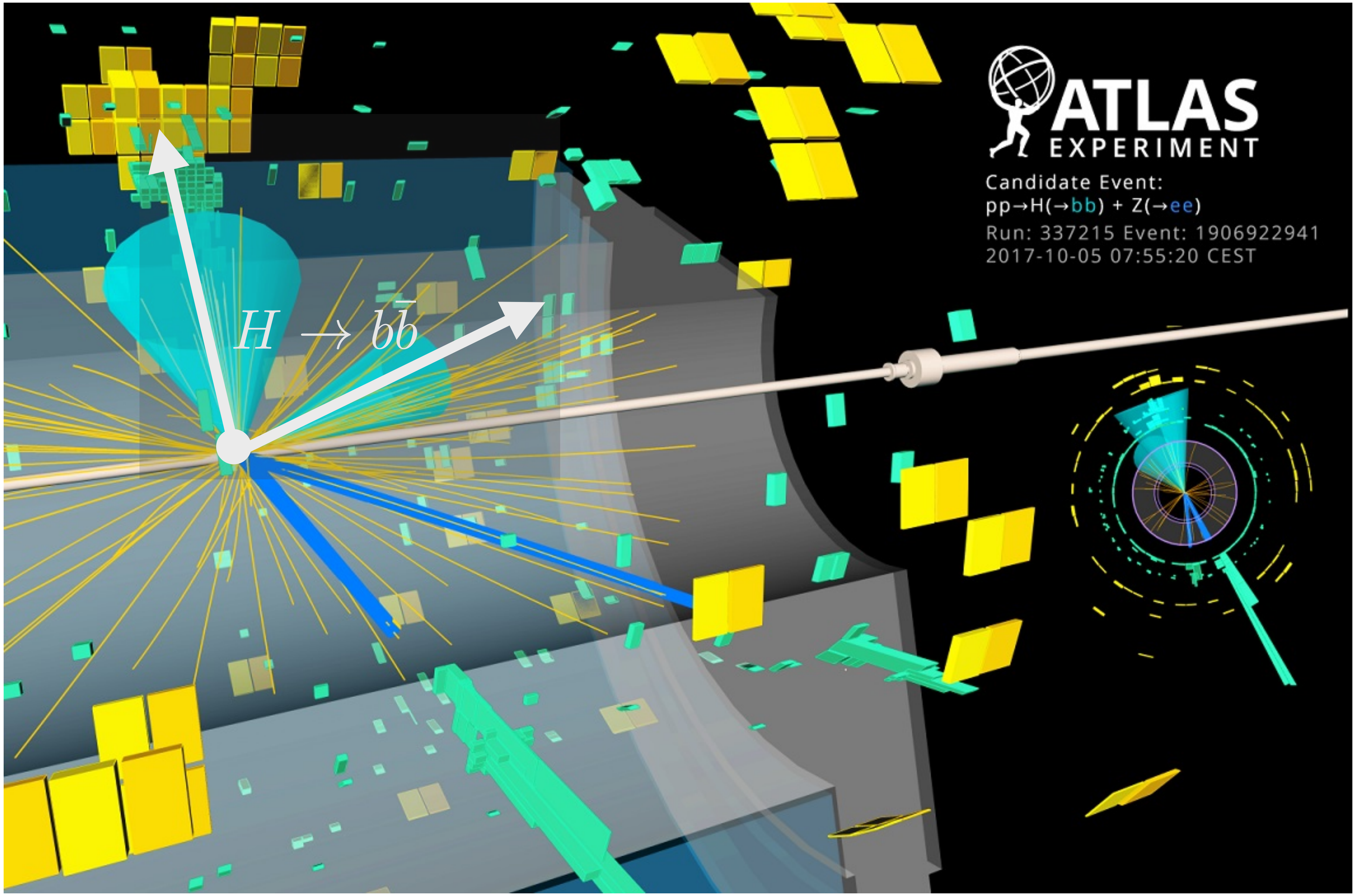
Precision Physics, Fundamental Interactions and Structure of Matter

RESUMMATION OF SUPER-LEADING LOGARITHMS

SOLVING A 16-YEAR OLD QCD PROBLEM

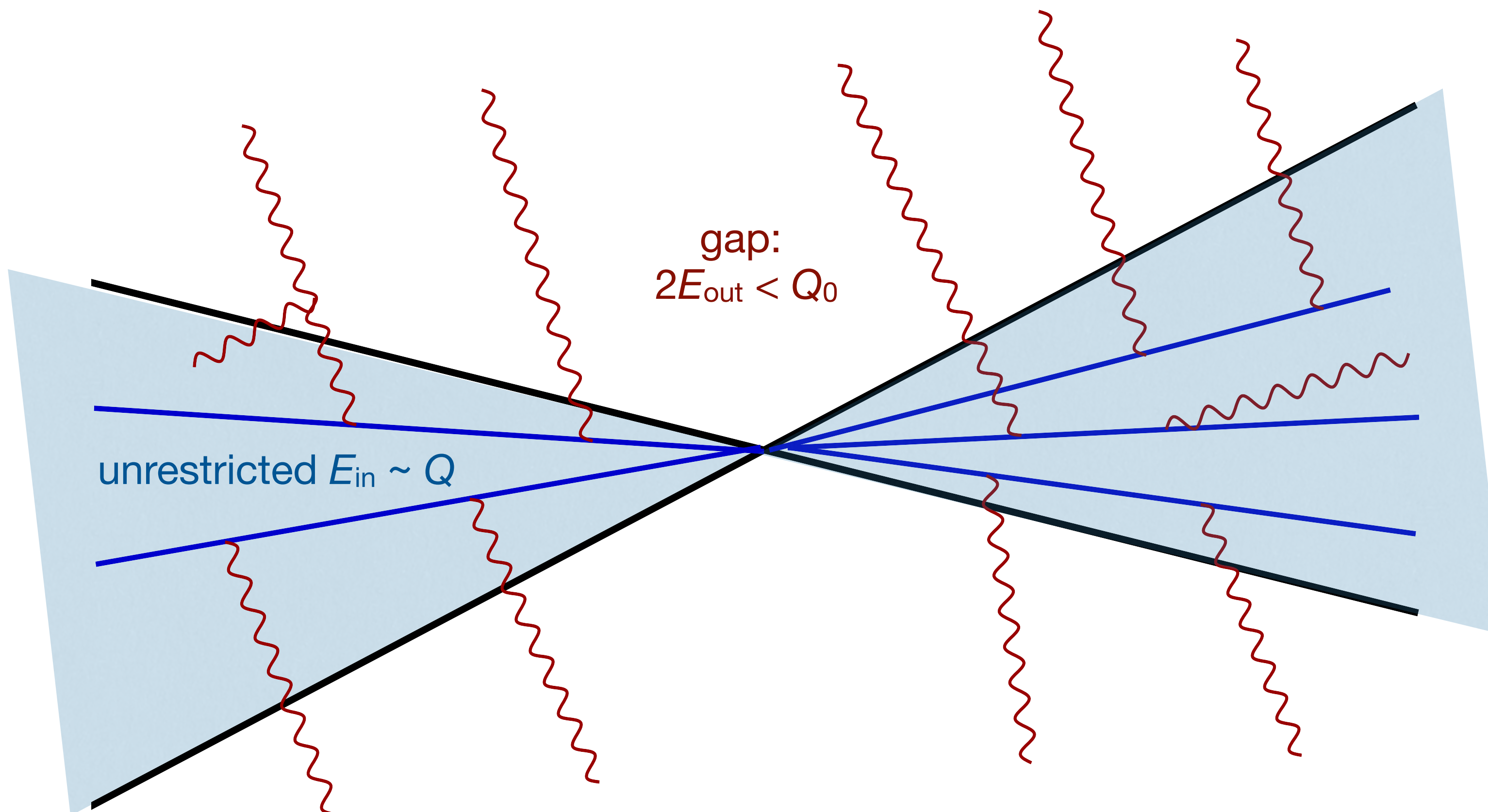
based on: T. Becher, MN, D.Y. Shao
Phys. Rev. Lett. 127 (2021) 212002

LARGE LOGARITHMS IN JET PROCESSES



CERN Document Server, ATLAS-PHOTO-2018-022-6

LARGE LOGARITHMS IN JET PROCESSES



$$L = \ln(Q/Q_0) \gg 1$$

Perturbative expansion:

$$\sigma \sim \sigma_{\text{Born}} \times \left\{ 1 + \alpha_s L + \alpha_s^2 L^2 \right\}$$

state-of-the-art: 2-loop order

LARGE LOGARITHMS IN JET PROCESSES

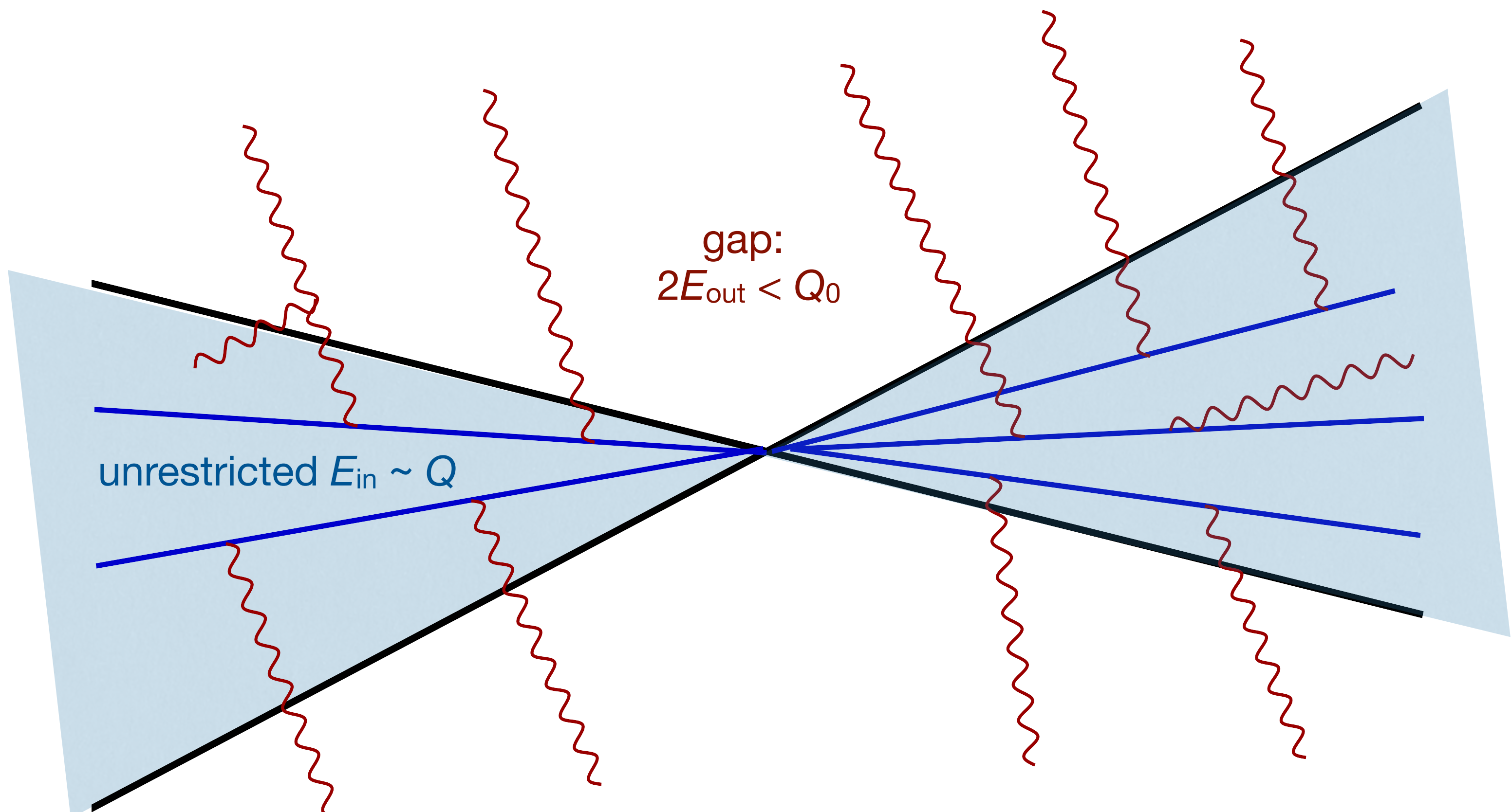
Non-global logarithms at lepton colliders

- high-energetic radiation restricted to certain regions (inside jets)
- large logarithms not contained in parton showers
- single-logarithmic effects $\sim (\alpha_s L)^n$ at lepton colliders
- resummation in large-N_c limit using BMS integral equation

J. Banfi, G. Marchesini, G. Smye: JHEP 08 (2002) 006

At hadron colliders, non-global logarithms take on more intricate form, and no generalization of BMS equation exists!

LARGE LOGARITHMS IN JET PROCESSES



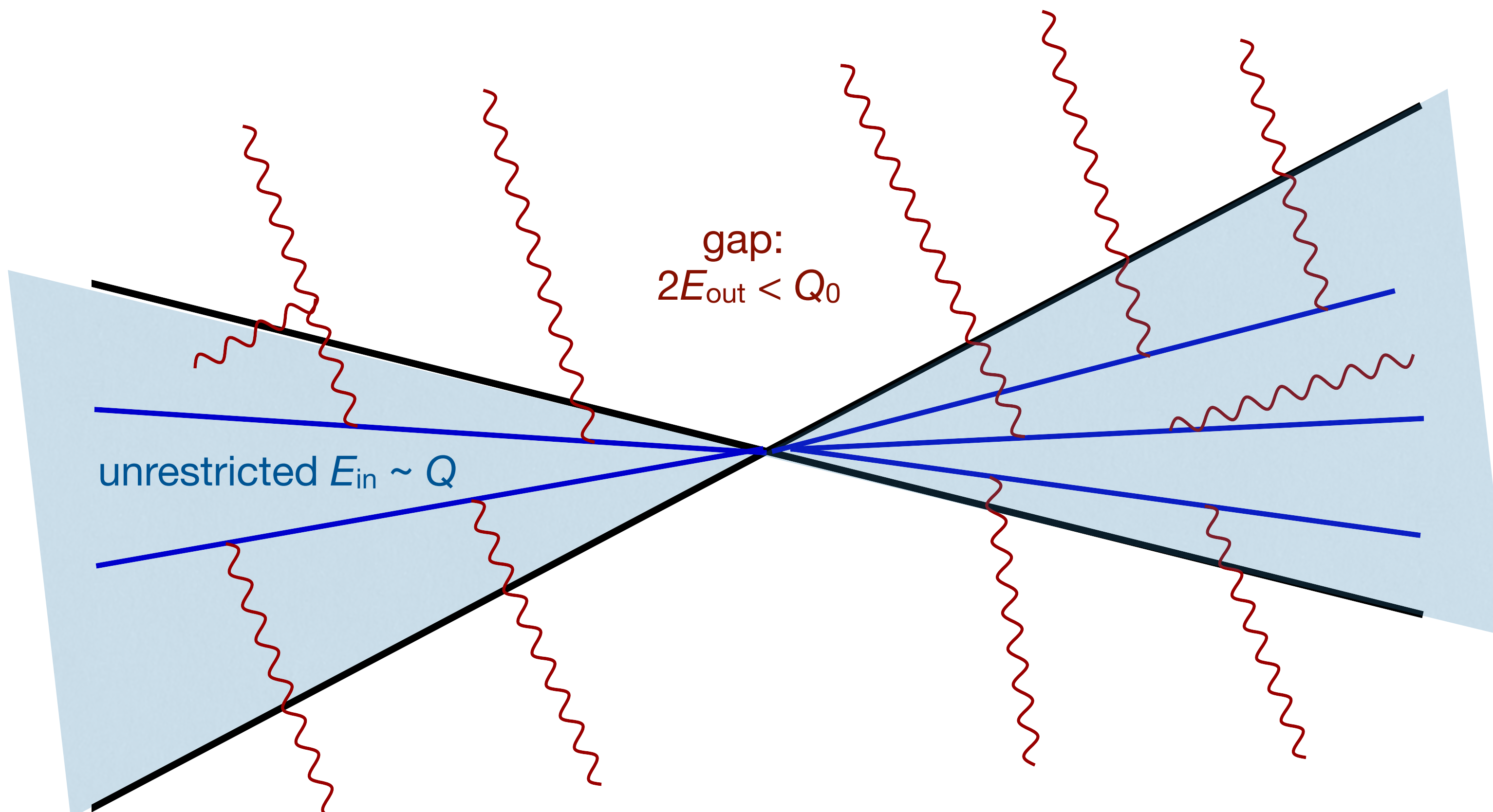
$$L = \ln(Q/Q_0) \gg 1$$

Perturbative expansion including “super-leading” logarithms:

$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4\, L^5 + \alpha_s^5\, L^7 + \dots}_{\text{formally larger than } O(1)}\}$$

J. R. Forshaw, A. Kyrieleis, M. H. Seymour: JHEP 08 (2006) 031

LARGE LOGARITHMS IN JET PROCESSES



$$L = \ln(Q/Q_0) \gg 1$$

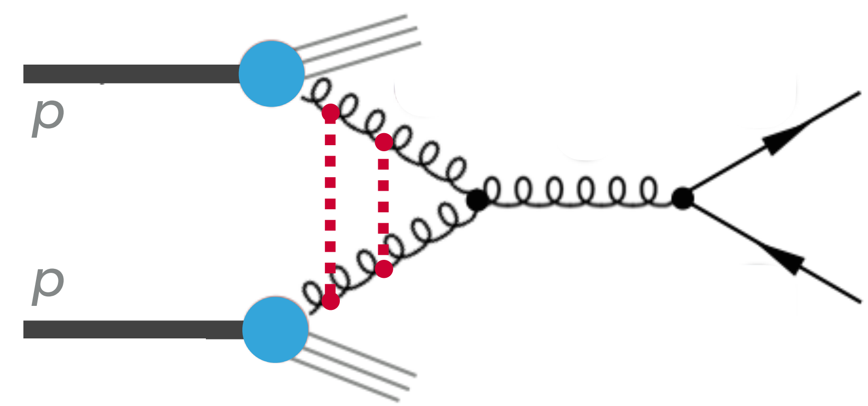
Really, double logarithmic series starting at 3-loop order:

$$\sigma \sim \sigma_{\text{Born}} \times \left\{1 + \alpha_s L + \alpha_s^2 L^2 + (\alpha_s \underbrace{\pi^2}_{(\Im\text{m}\, L)^2}) \underbrace{\left[\alpha_s^2 L^3 + \alpha_s^3 L^5 + \dots\right]}_{\text{formally larger than } O(1)}\right\}$$

COULOMB PHASES BREAK COLOR COHERENCE

Super-leading logarithms

- breakdown of color coherence due to a subtle quantum effect: soft gluon exchange between initial-state partons



- soft anomalous dimension:

$$\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = \sum_{(ij)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s^3)$$

where $s_{ij} > 0$ if particles i and j are both in initial or final state

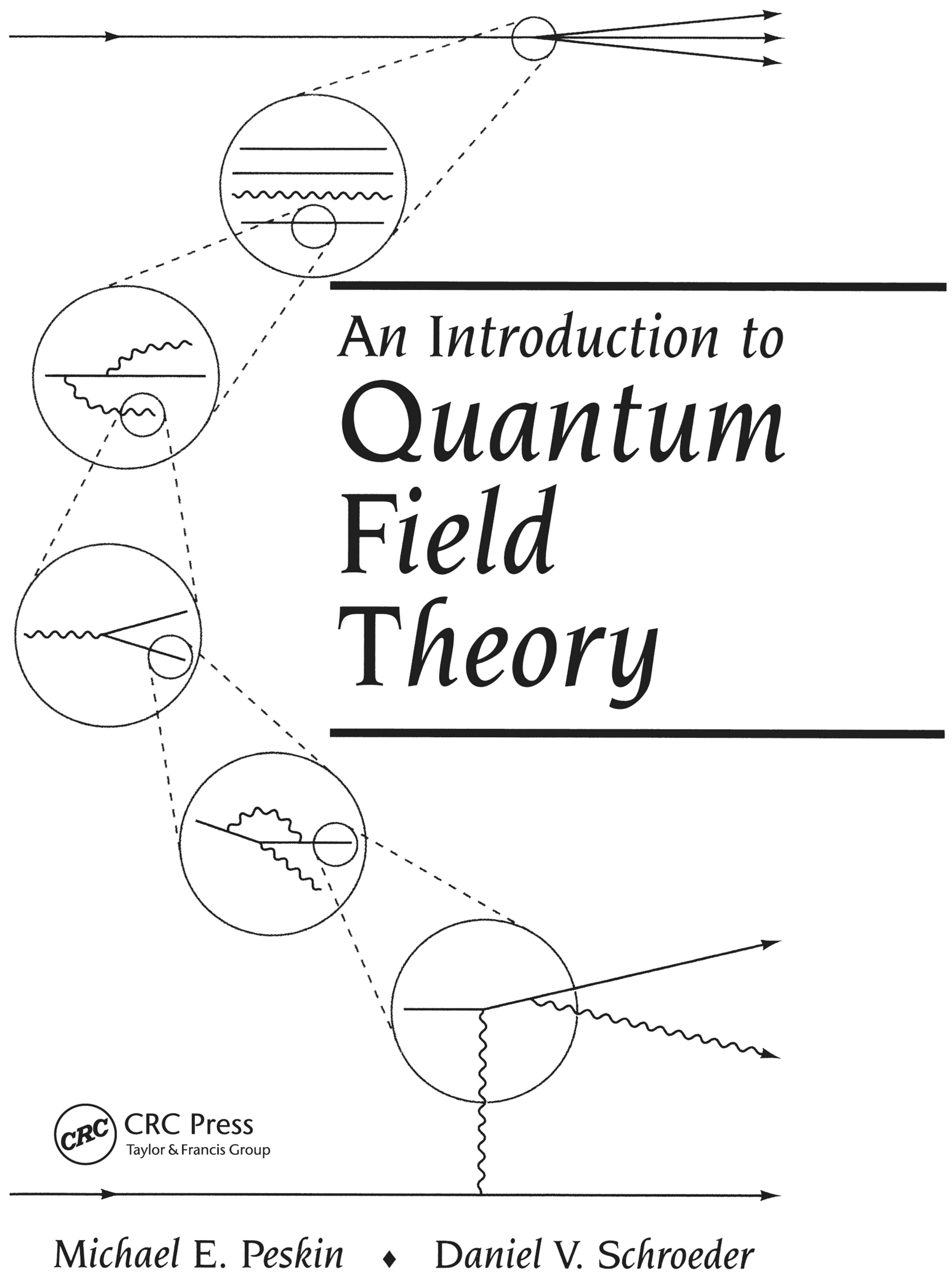
- imaginary part (only at hadron colliders):

$$\boxed{\mathrm{Im}\,\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = +2\pi\,\gamma_{\rm cusp}(\alpha_s)\, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2} + (\dots)\,\mathbf{1}$$

irrelevant

THEORY OF NON-GLOBAL LHC OBSERVABLES

Factorization



THEORY OF NON-GLOBAL LHC OBSERVABLES

Novel factorization theorem from SCET

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu) \rangle$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002

high scale

low scale

Renormalization-group equation:

$$\mu \frac{d}{d\mu} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\},Q,\mu) = -\sum_{m\le l} \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)\, \boldsymbol{\Gamma}_{ml}^H(\{\underline{n}\},Q,\mu)$$

operator in color space and in the infinite space of parton multiplicities

All-order summation of large logarithmic corrections, including the super-leading logarithms!

THEORY OF NON-GLOBAL LHC OBSERVABLES

Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- low-energy matrix element:

$$\boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a/p}(x_1)\, f_{b/p}(x_2)\, \mathbf{1} + \mathcal{O}(\alpha_s)$$

- hard-scattering functions:

$$\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\}, Q, \mu_s) = \sum_{l \le m} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\}, Q, Q)\, \mathbf{P} \exp\left[\int_{\mu_s}^{Q} \frac{d\mu}{\mu}\, \boldsymbol{\Gamma}^H(\{\underline{n}\}, Q, \mu)\right]_{lm}$$

- expanding the solution in a power series generates arbitrarily high parton multiplicities starting from the $2 \to M$ Born process

THEORY OF NON-GLOBAL LHC OBSERVABLES

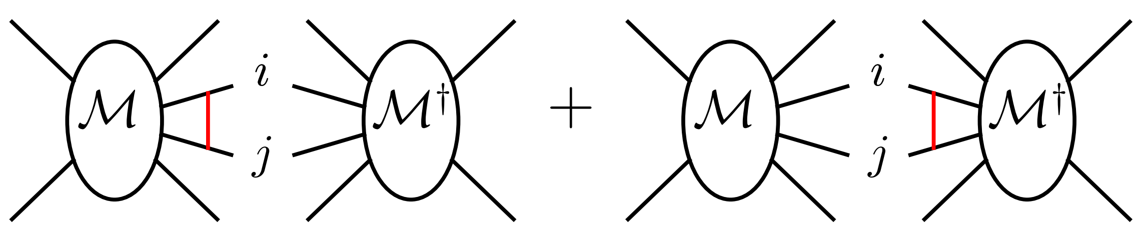
Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- anomalous-dimension matrix:

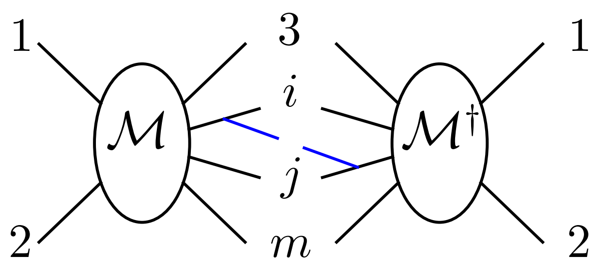
$$\boldsymbol{\Gamma}^H = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}_4 & \boldsymbol{R}_4 & 0 & 0 & \cdots \\ 0 & \boldsymbol{V}_5 & \boldsymbol{R}_5 & 0 & \cdots \\ 0 & 0 & \boldsymbol{V}_6 & \boldsymbol{R}_6 & \cdots \\ 0 & 0 & 0 & \boldsymbol{V}_7 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

- action on hard functions:

$$\boldsymbol{\mathcal{H}}_m \boldsymbol{V}_m = \sum_{(ij)}$$



$$\boldsymbol{\mathcal{H}}_m \boldsymbol{R}_m = \sum_{(ij)}$$



RESUMMATION OF SUPER-LEADING LOGARITHMS

Extract all-order series of super-leading logarithmic (SLL)

- infinite series starting at 3-loop order:

$$\sigma_{\rm SLL} = \sigma_{\rm Born} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^{n+3} L^{2n+3}\, \frac{(-4)^n\, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r\,(r!)^2}\, C_{rn}$$

- with ten basic color structures: T. Becher, M. Neubert, D. Y. Shao: in preparation

$$C_{rn} = -256\pi^2\,(4N_c)^{n-r} \left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{O}_i^{(j)} \right\rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{S}_i^{(r)} \right\rangle \right]$$

- kinematic information contained in $(M+1)$ angular integrals:

$$J_j = \int \frac{d\Omega(n_k)}{4\pi} \left(W_{1j}^k - W_{2j}^k\right) \Theta_{\rm veto}(n_k)\,; \quad \text{with} \quad W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\, n_j\cdot n_k}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

Extract all-order series of super-leading logarithmic (SLL) terms

- master formula: T. Becher, M. Neubert, D. Y. Shao: in preparation

$$C_{rn} = -256\pi^2 \,(4N_c)^{n-r} \left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{O}_i^{(j)} \right\rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\, \boldsymbol{S}_i \right\rangle \right]$$

- example color structures:

$$\boldsymbol{O}_1^{(j)} = f_{abe} f_{cde}\, \boldsymbol{T}_2^a \,\{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\}\, \boldsymbol{T}_j^d - (1 \leftrightarrow 2)$$

$$\boldsymbol{S}_3 = d_{ade} d_{bce} \left[\boldsymbol{T}_2^a \left(\boldsymbol{T}_1^b \boldsymbol{T}_1^c \boldsymbol{T}_1^d \right)_+ + (1 \leftrightarrow 2) \right]$$

- coefficient functions:

$$c_1^{(r)} = 2^{r-1} \left[(3N_c+2)^r + (3N_c-2)^r \right]$$

$$d_3^{(r)} = 2^{r-1} N_c \left[\frac{(3N_c+2)^r}{N_c+2} + \frac{(3N_c-2)^r}{N_c-2} - \frac{(2N_c)^{r+1}}{N_c^2-4} \right]$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

Simplifications for (anti-)quark-initiated processes

- in fundamental representation, symmetrized products of color generators can be reduced ($\sigma_i = \pm 1$ for (anti-)quarks):

$$\{\boldsymbol{T}_i^a, \boldsymbol{T}_i^b\} = \frac{1}{N_c}\,\delta_{ab} + \sigma_i\, d_{abc}\, \boldsymbol{T}_i^c$$

- simple results in terms of three non-trivial color structures:

$$C_{rn} = -2^{8-r}\pi^2\,(4N_c)^n \left\{ \sum_{j=3}^{M+2} J_j \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \left[(\boldsymbol{T}_1 - \boldsymbol{T}_2)\cdot \boldsymbol{T}_j - 2^{r-1} N_c\,(\sigma_1 - \sigma_2)\, d_{abc}\, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \right] \right\rangle \right.$$
$$\left. - \, 2\,(1-\delta_{r0})\, J_2 \;\left\langle \boldsymbol{\mathcal{H}}_{2\to M} \left[C_F\, \boldsymbol{1} + (2^r - 1)\, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \right] \right\rangle \right\}$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002

RESUMMATION OF SUPER-LEADING LOGARITHMS

Summation of super-leading logarithms for $qq \to qq$ scattering:

$$\sigma_{\rm SLL}^{(S)} = -\sigma_{\rm Born} \boxed{\frac{16\alpha_s L}{81\pi}} \Delta Y \left(\frac{3\alpha_s}{\pi}\,\pi^2\right) w\, {}_2F_2\Big(1,1;2,\frac{5}{2};-w\Big)$$

1-loop factor

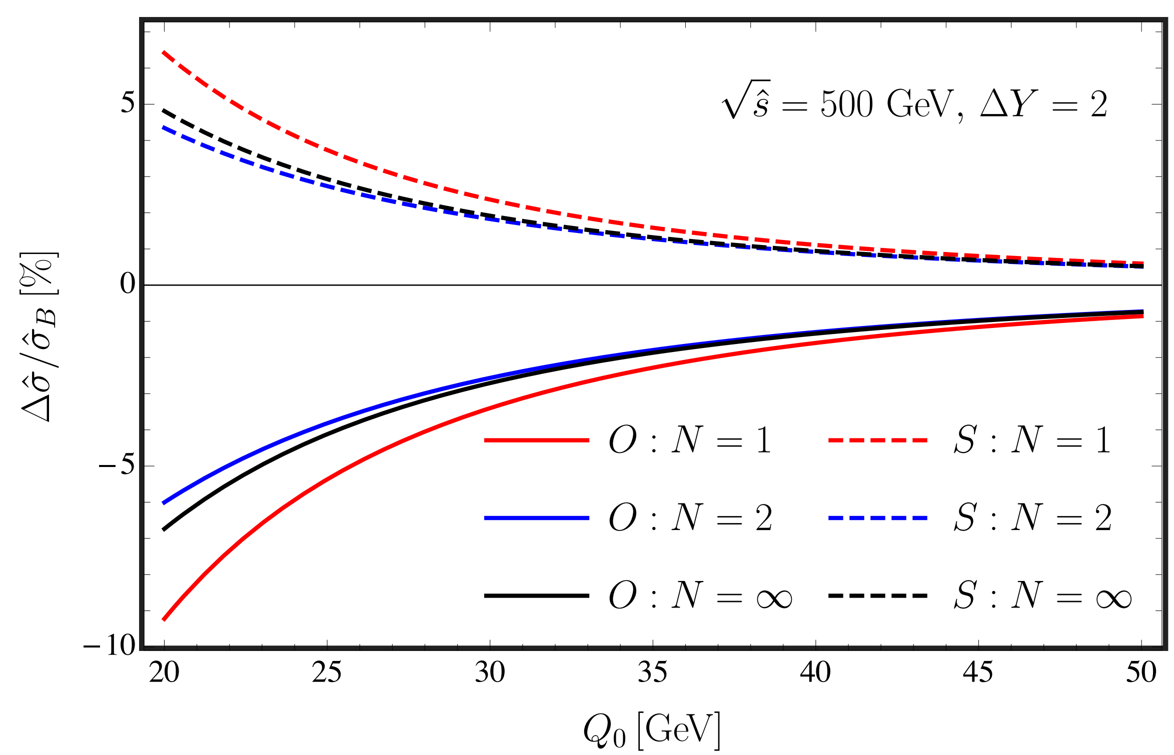
$$w = \frac{3\alpha_s}{\pi}\,L^2$$

T. Becher, M. Neubert, D. Y. Shao: Phys. Rev. Lett. 127 (2021) 212002

- asymptotic behavior for $L \to \infty$:

$$w\, {}_2F_2\Big(1,1;2,\frac{5}{2};-w\Big) \to \frac{3}{2}\Big[\ln(4w) + \gamma_E - 2\Big]$$

- expect larger effects for gluon-initiated processes



$$\sum_{n=1}^{N} \dots$$

SLLs only!
($\geq$ 4 loops)

IMPORTANT REMARKS

- SCET-based approach solves 16-year old QCD problem, extending existing results to all orders of perturbation theory and to arbitrary $2 \to M$ hard-scattering processes
- master formula also applies to cases where $M = 1$ or even $M = 0$, which were not considered before (SLLs start at 4- and 5-loop order, respectively)
- relevant for both SM phenomenology (e.g. $pp \to h + \text{jet}$) and New-Physics searches (e.g. WIMP searches in $pp \to \text{jet} + \not{E}_T$)

CONCLUSIONS

Complete theory of LHC jet processes

- powerful new factorization theorem derived using SCET
- in future, extension to massive final-state partons and calculations beyond leading logarithms
- detailed study of low-energy matrix elements using SCET with Glauber gluons will offer *ab initio* understanding of violations of conventional factorization
- new levels of theoretical precision in predictions for important LHC processes and future improvements of parton showers