Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare

Exact mixed NNLO QCD-EW corrections to the Drell-Yan processes

Alessandro Vicini
University of Milano, INFN Milano

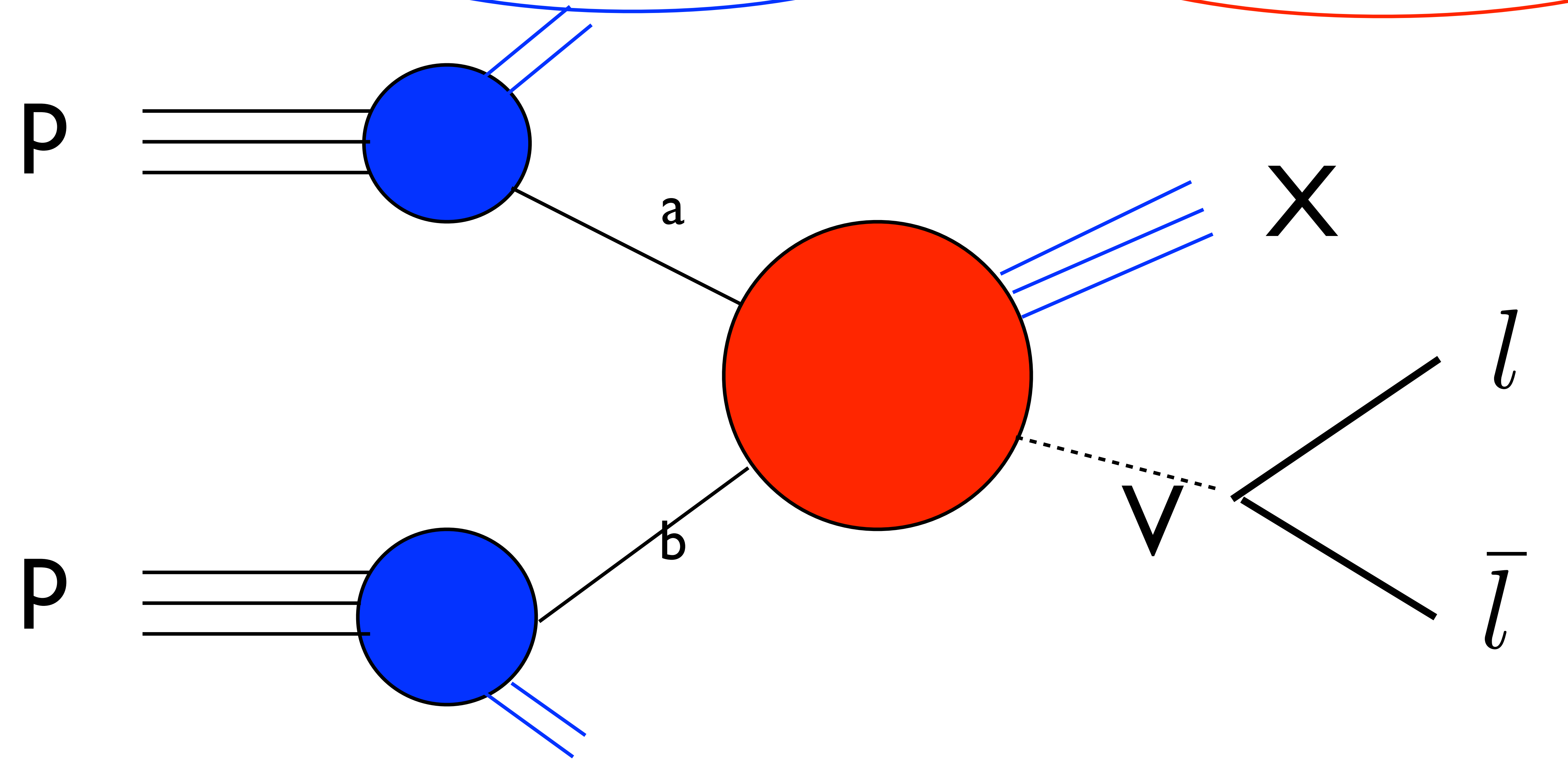
XXXV Rencontres de Physique de la Vallée d'Aoste, La Thuile, March 10th 2022

in collaboration with: T.Armadillo, R.Bonciani, F.Buccioni, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano,

Drell-Yan results: arXiv:2106.11953, arXiv:2201.01754 on-shell Z results arXiv:2007.06518, arXiv:2111.12694

Lepton-pair Drell-Yan production at hadron colliders

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F)\, \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$

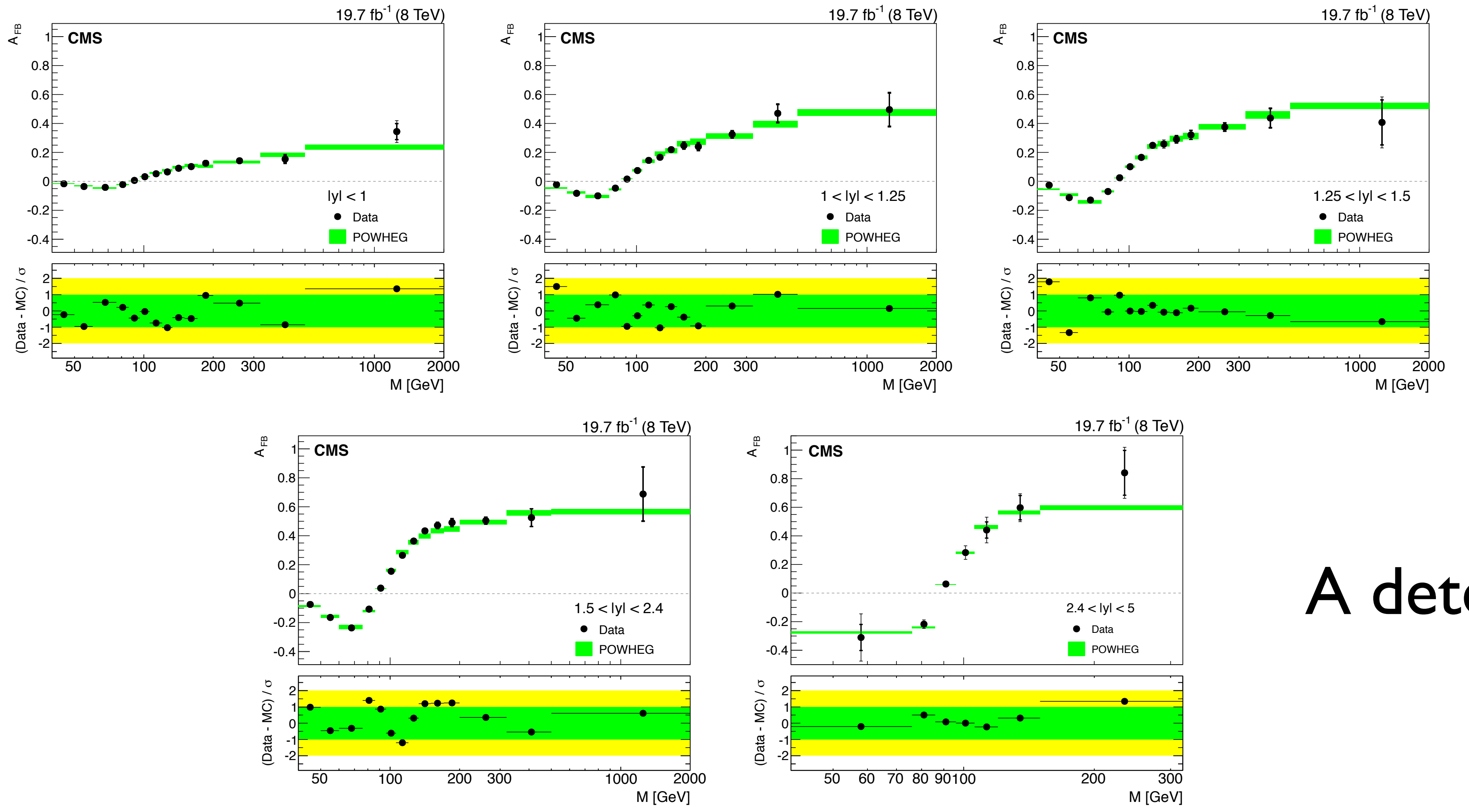


The factorisation theorems guarantee the validity of the above picture up to power correction effects

The interplay of QCD and EW interactions appears both in the partonic cross section and in the proton PDFs

Motivations: towards per mille physics 😲 at the LHC ?

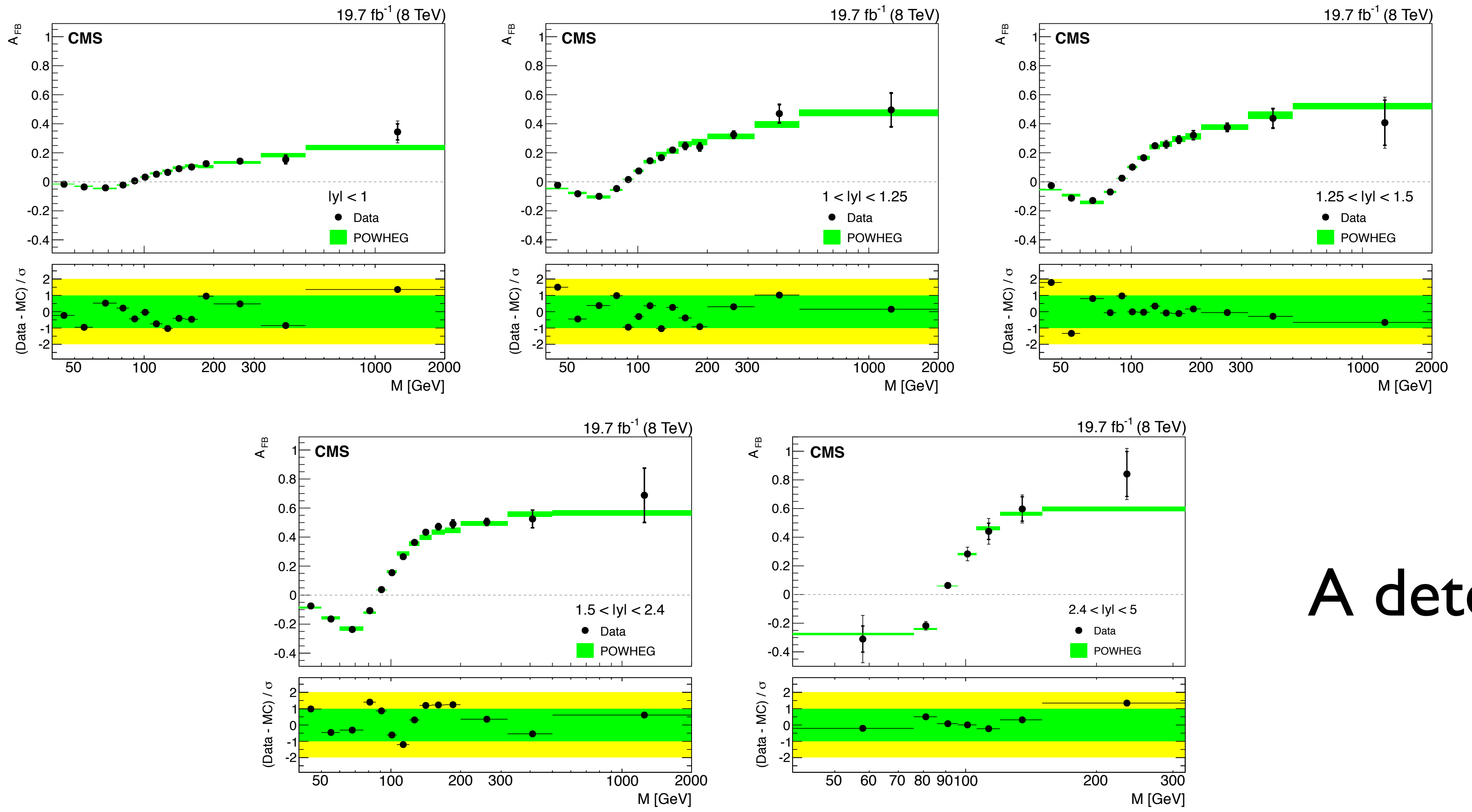
Motivations: towards per mille physics 😲 at the LHC ?



Channel	Not constraining PDFs	Constraining PDFs
Muons	0.23125 ± 0.00054	0.23125 ± 0.00032
Electrons	0.23054 ± 0.00064	0.23056 ± 0.00045
Combined	0.23102 ± 0.00057	0.23101 ± 0.00030

A determination of $\sin^2\theta^{lep}_{eff}$ competitive with the LEP results (0.23152(16)) is becoming possible

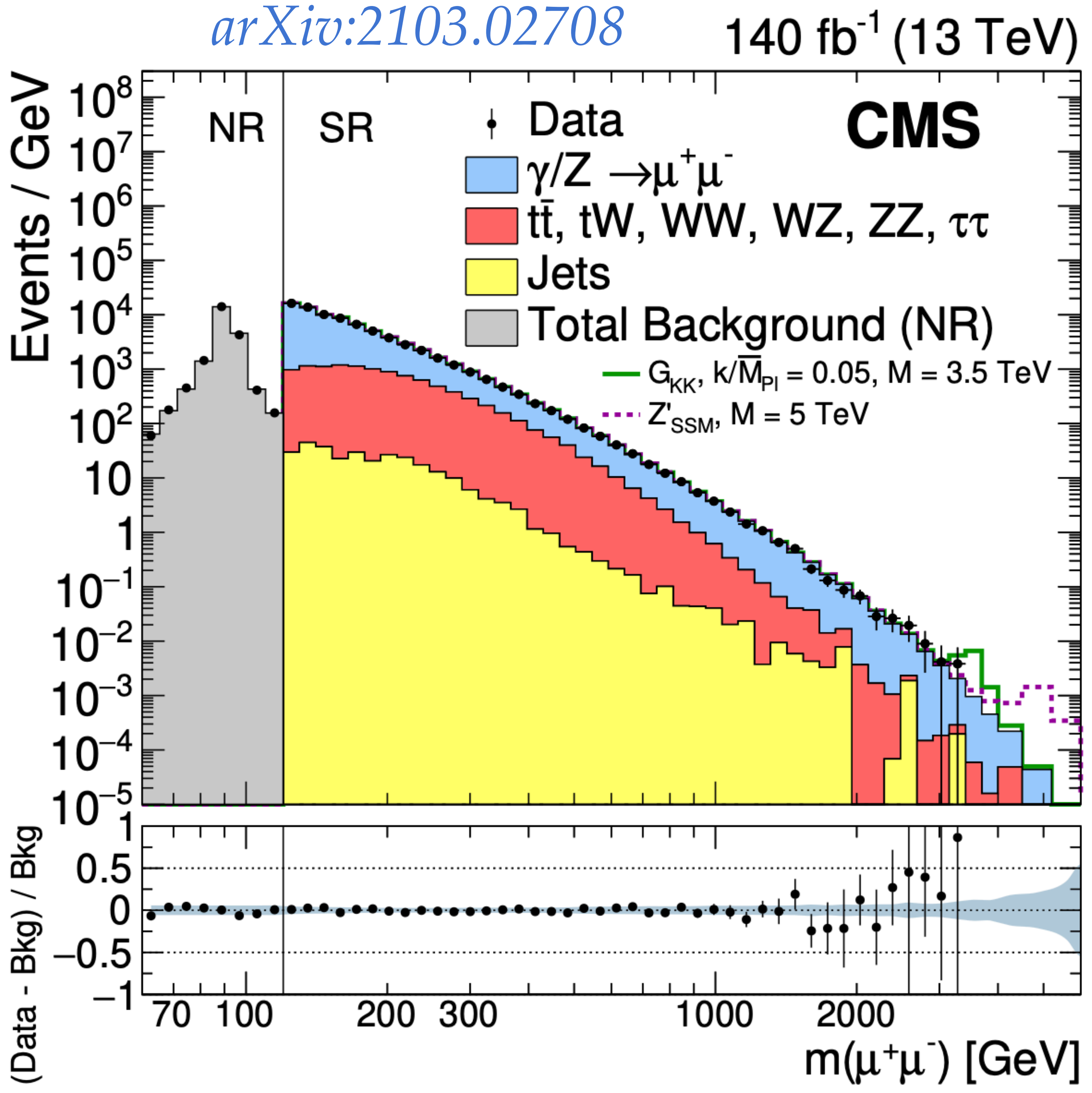
Motivations: towards per mille physics 😲 at the LHC ?



Channel	Not constraining PDFs	Constraining PDFs
Muons	0.23125 ± 0.00054	0.23125 ± 0.00032
Electrons	0.23054 ± 0.00064	0.23056 ± 0.00045
Combined	0.23102 ± 0.00057	0.23101 ± 0.00030

A determination of $\sin^2\theta_{eff}^{lep}$ competitive with the LEP results (0.23152(16)) is becoming possible

arXiv:2103.02708

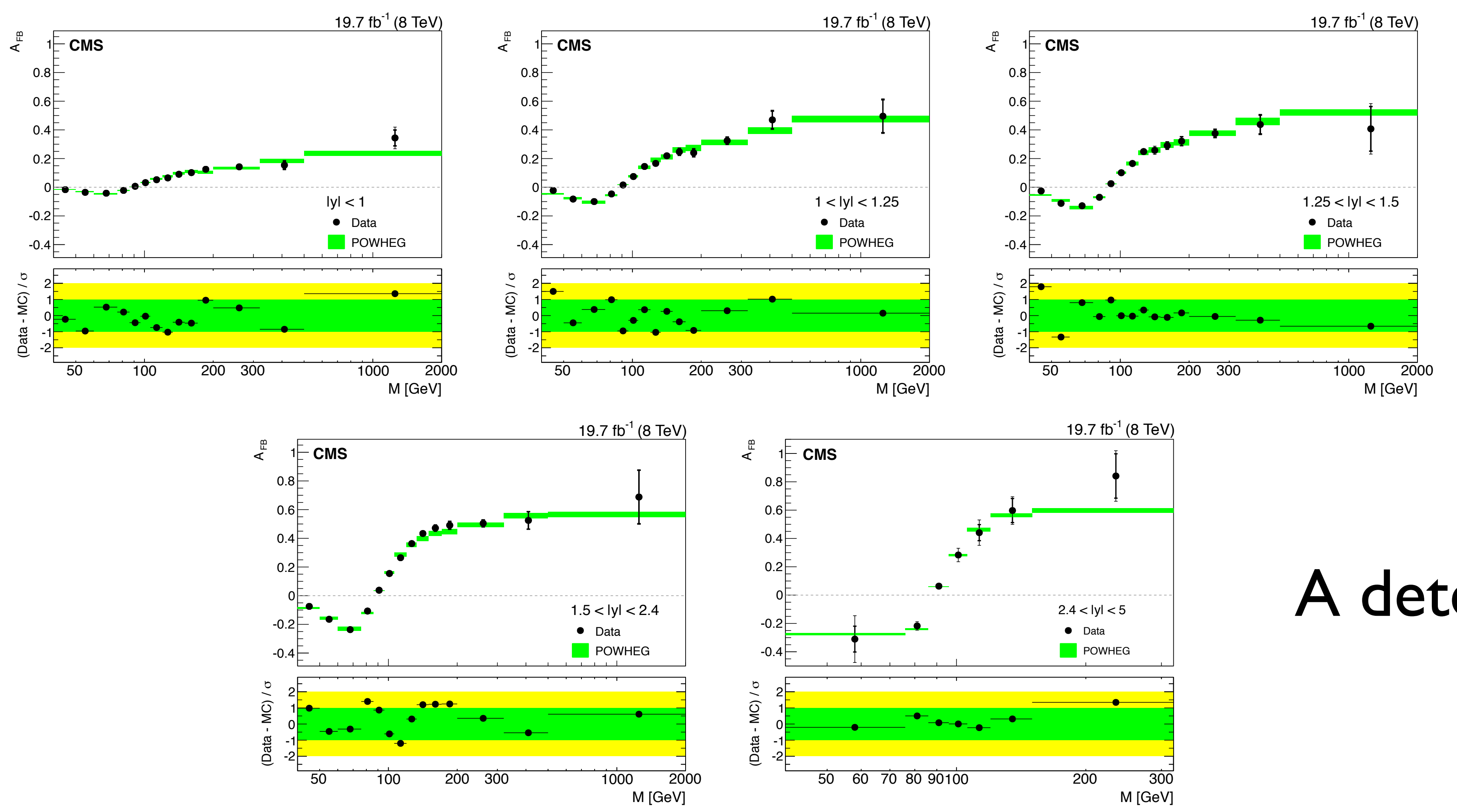


mass window [GeV]	stat. unc. 140fb^{-1}	stat. unc. 3ab^{-1}
600<$m_{\mu\mu}$<900	1.4%	0.2%
900<$m_{\mu\mu}$<1300	3.2%	0.6%

A deviation from the SM prediction can point towards New Physics

Is the SM prediction under control at the O(0.5%) level in the TeV region of the $m_{\ell\ell}$ distribution ?

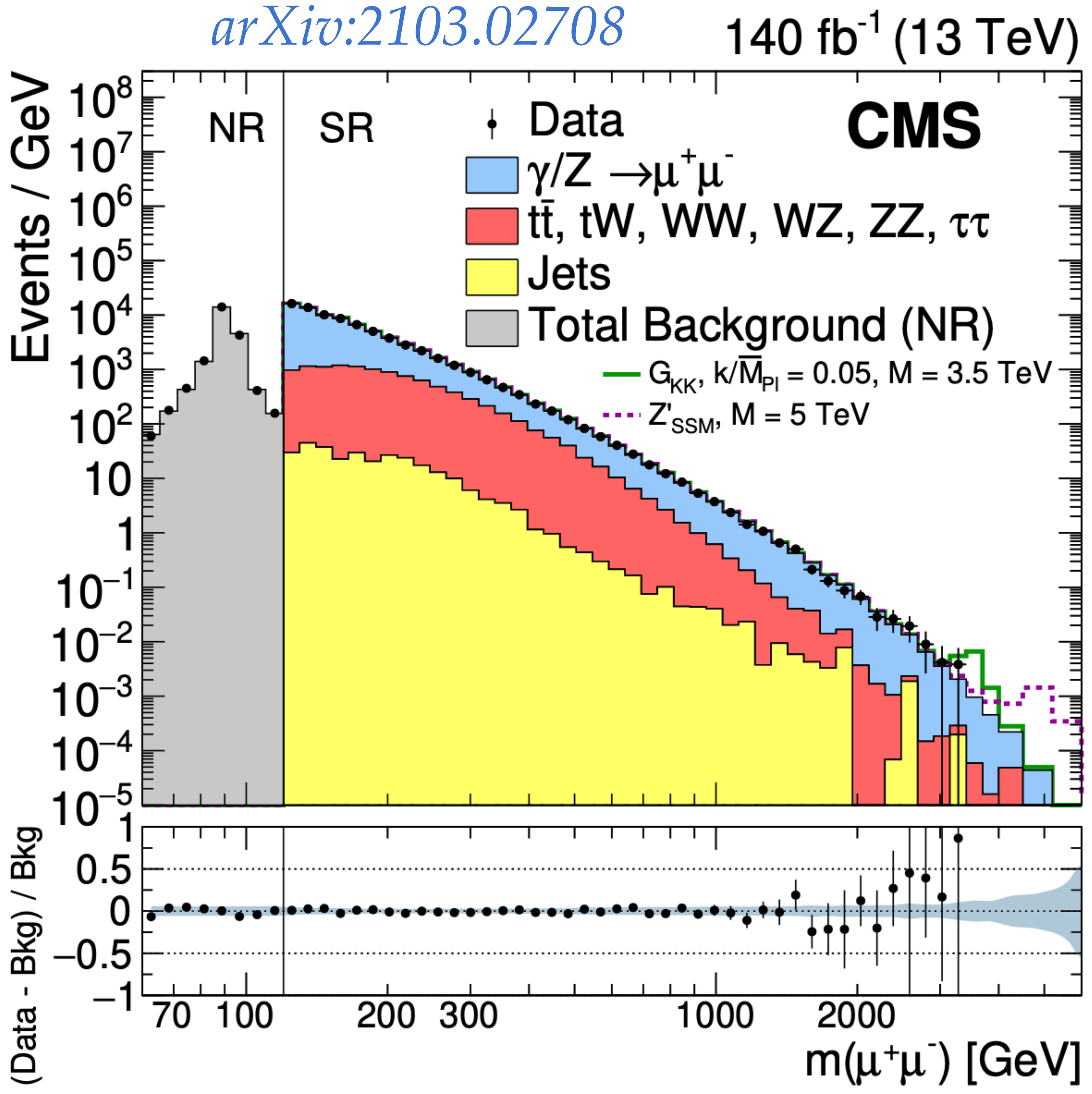
Motivations: towards per mille physics 😲 at the LHC ?



Channel	Not constraining PDFs	Constraining PDFs
Muons	0.23125 ± 0.00054	0.23125 ± 0.00032
Electrons	0.23054 ± 0.00064	0.23056 ± 0.00045
Combined	0.23102 ± 0.00057	0.23101 ± 0.00030

A determination of $\sin^2\theta^{lep}_{eff}$ competitive with the LEP results (0.23152(16)) is becoming possible

arXiv:2103.02708

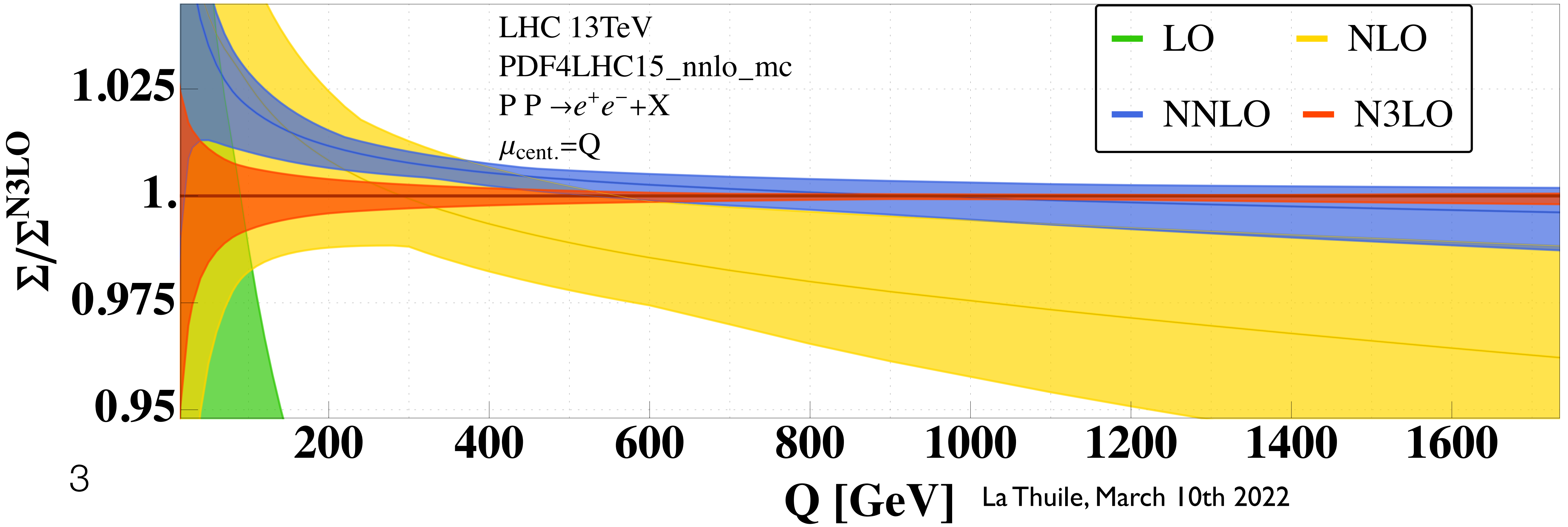


mass window [GeV]	stat. unc. 140fb^{-1}	stat. unc. 3ab^{-1}
600<$m_{\mu\mu}$<900	1.4%	0.2%
900<$m_{\mu\mu}$<1300	3.2%	0.6%

A deviation from the SM prediction can point towards New Physics

Is the SM prediction under control at the O(0.5%) level in the TeV region of the $m_{\ell\ell}$ distribution ?

C.Duhr, B.Mistlberger, arXiv:2111.10379



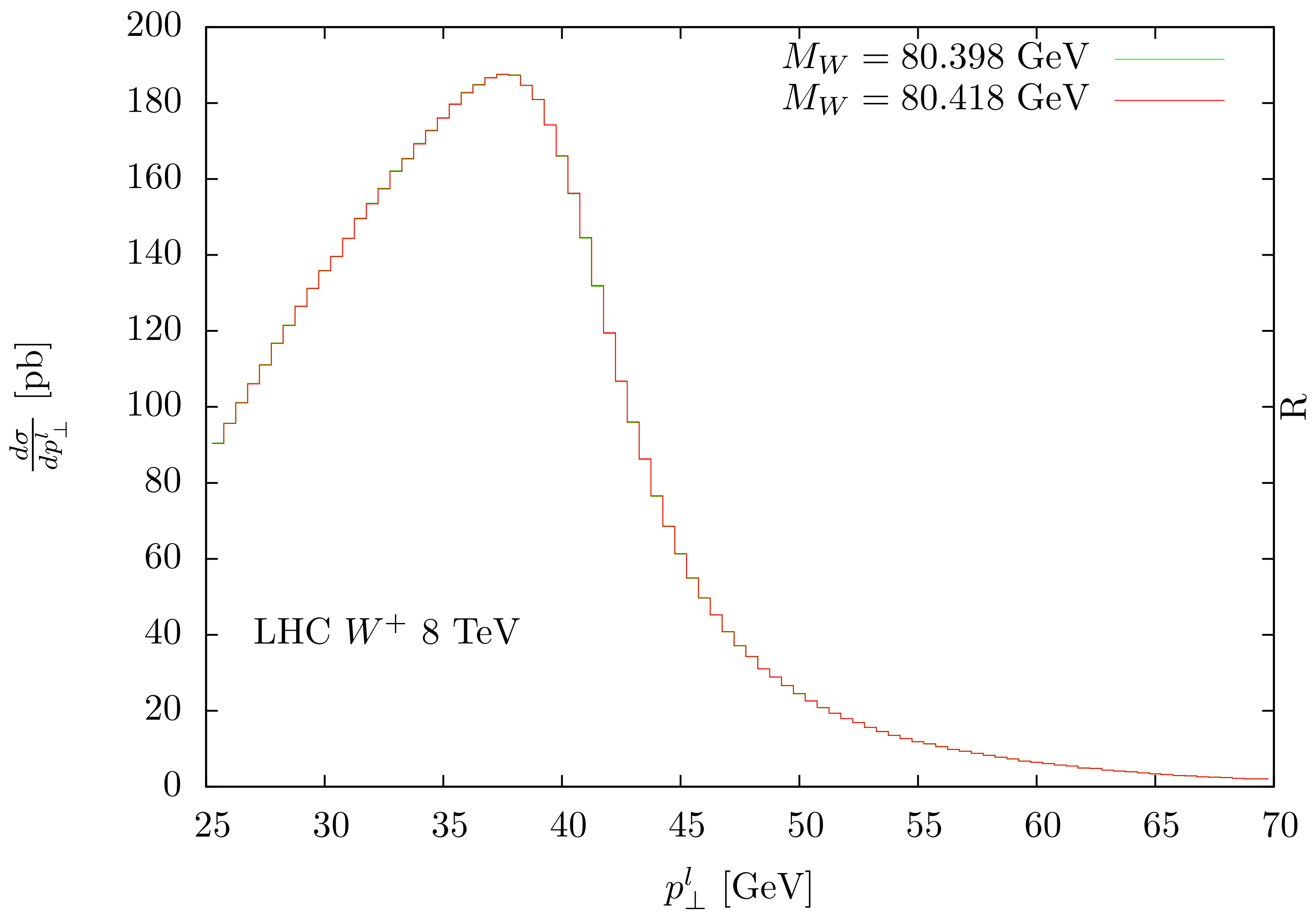
High precision determination of the SM parameters at the LHC

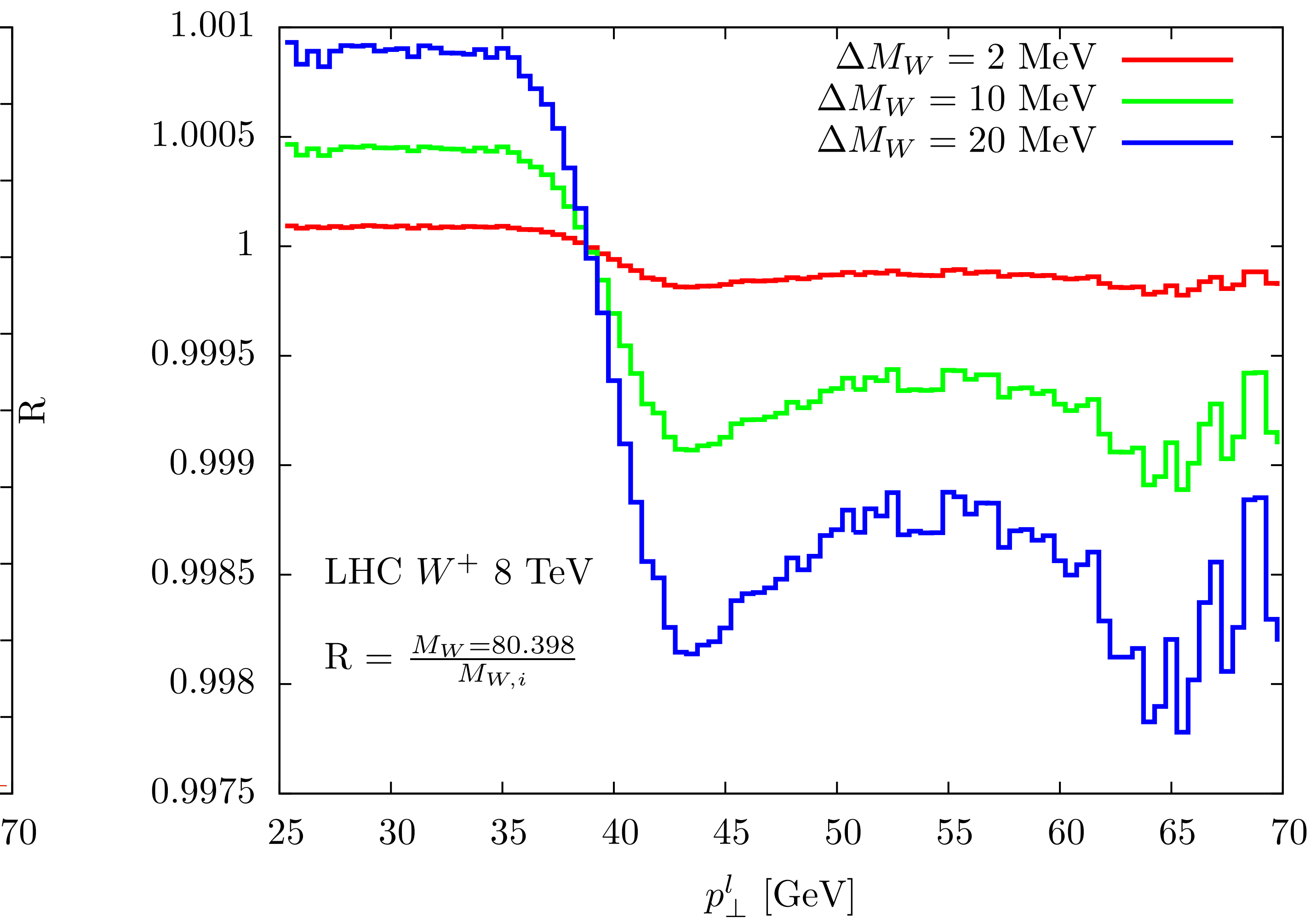
The SM parameters are extracted from the data via template fitting. Templates = theoretical histograms of the kinematical distr.
The template theoretical uncertainties propagate as systematic errors on the determination of $(\alpha, G_\mu, m_W, m_Z, \sin^2\theta_{eff}, \ldots)$

Given the very high precision goal

$$\delta m_W / m_W \sim 1 \cdot 10^{-4}, \qquad \delta \sin^2\theta_{eff} / \sin^2\theta_{eff} \sim 1 \cdot 10^{-3}$$

control on the shape of the distributions
at the sub-percent level is needed,
at a hadron collider…

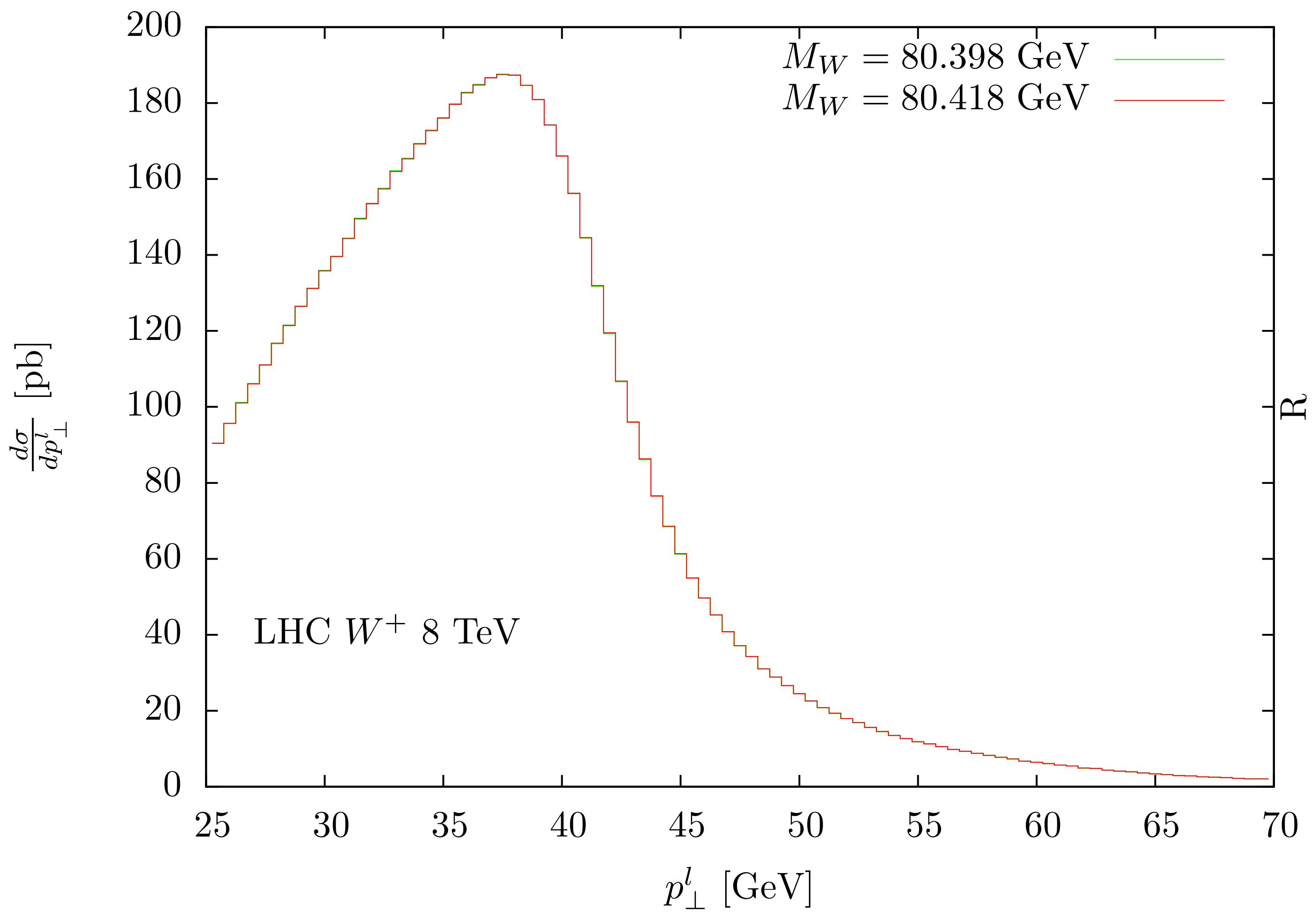


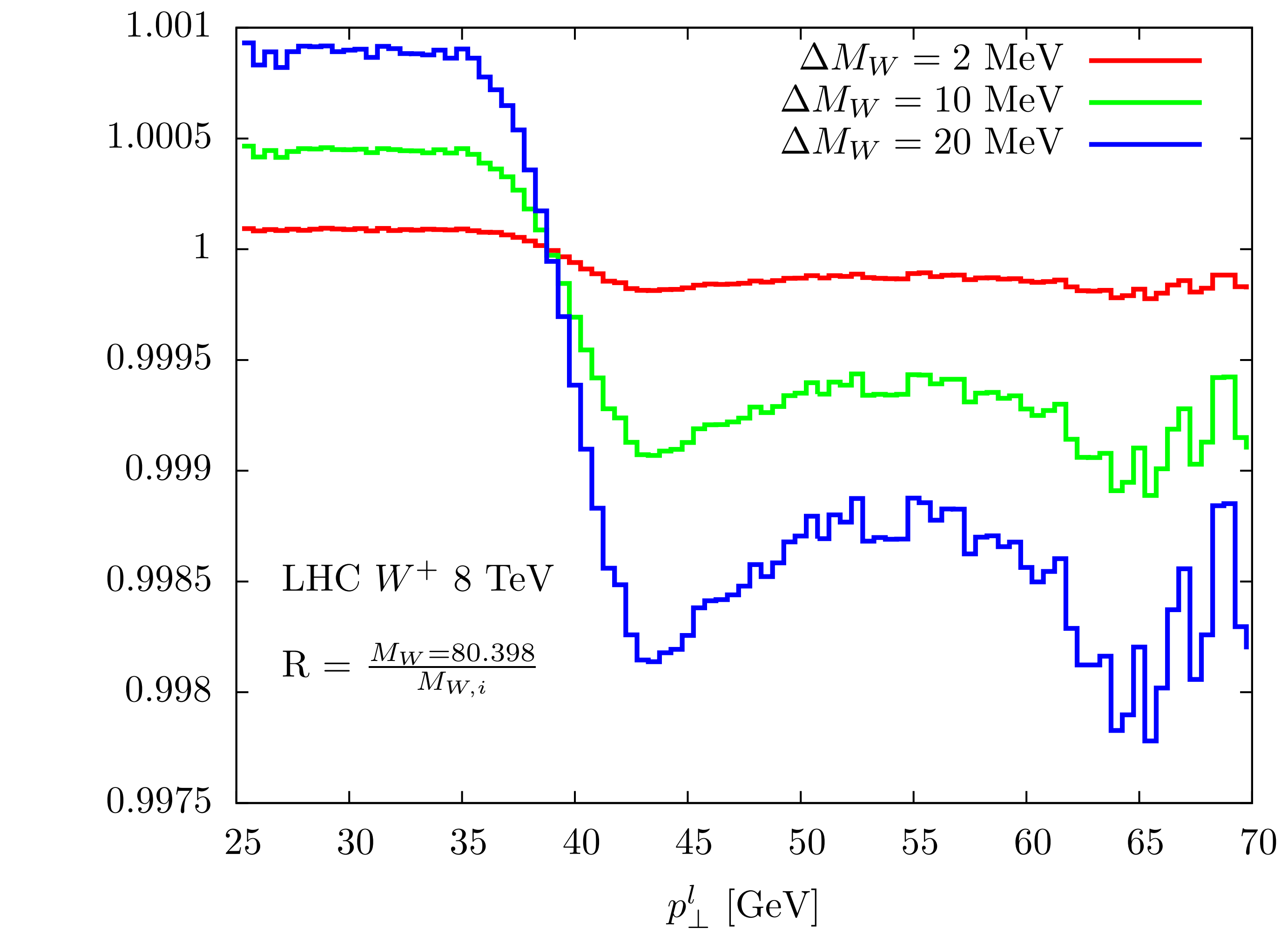


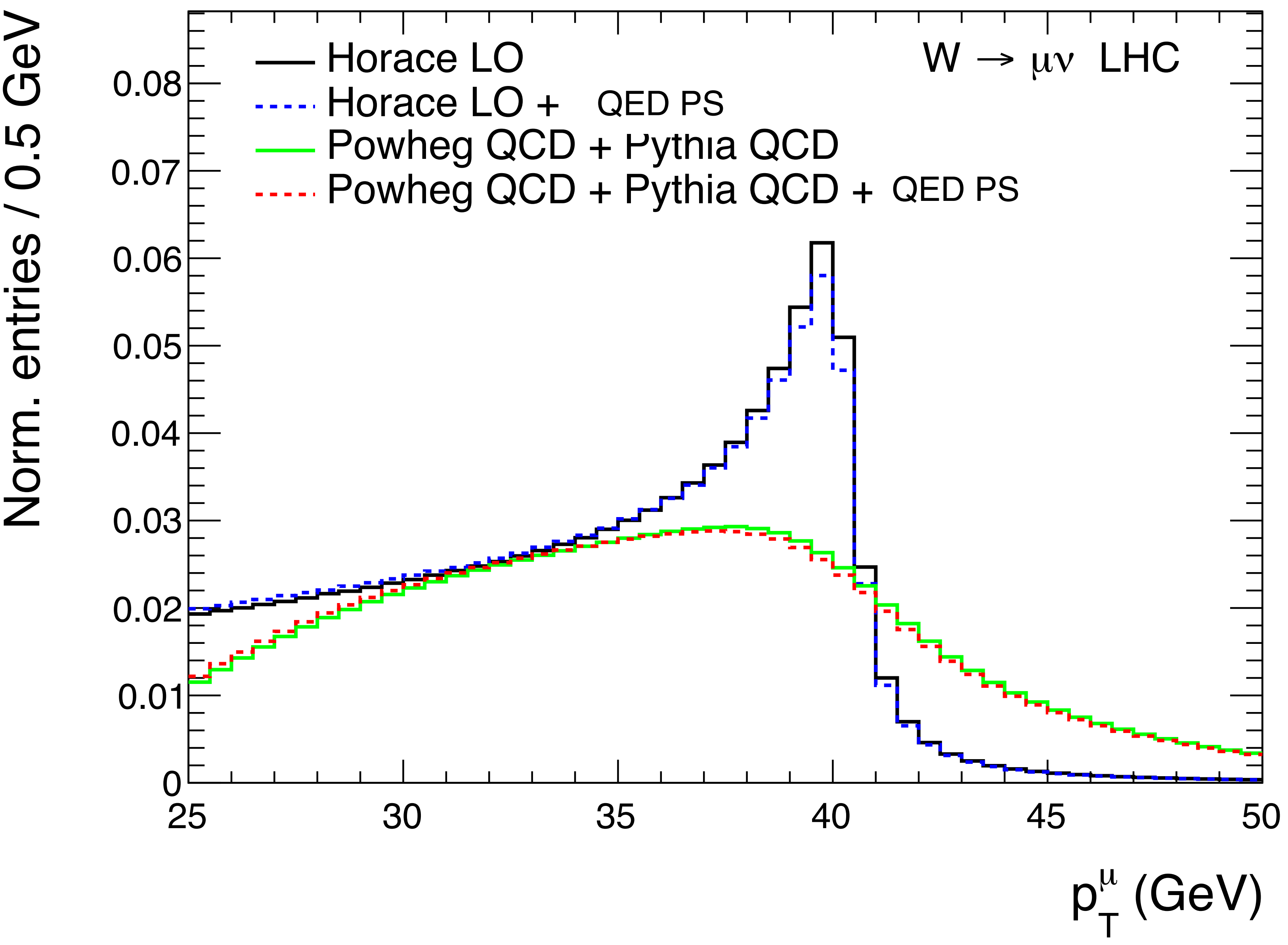
High precision determination of the SM parameters at the LHC

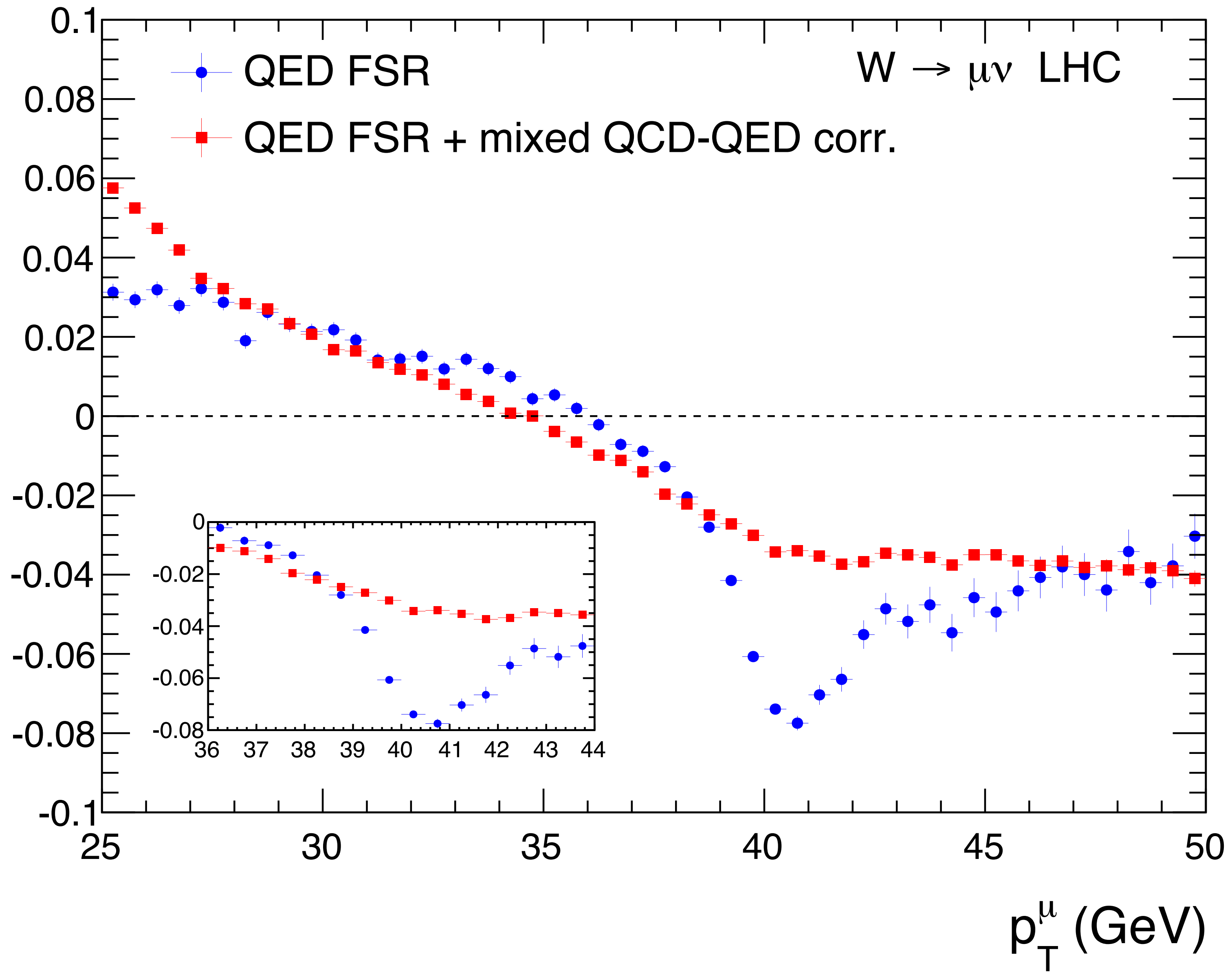
The SM parameters are extracted from the data via template fitting. Templates = theoretical histograms of the kinematical distr.
The template theoretical uncertainties propagate as systematic errors on the determination of $(\alpha, G_\mu, m_W, m_Z, \sin^2\theta_{eff}, \dots)$

Given the very high precision goal
$$\delta m_W/m_W \sim 1 \cdot 10^{-4}, \qquad \delta \sin^2\theta_{eff}/\sin^2\theta_{eff} \sim 1 \cdot 10^{-3}$$
control on the shape of the distributions
at the sub-percent level is needed,
at a hadron collider…









The convolution of QED-FSR with QCD catches the bulk of the radiative effects.

Mixed effects are still very large

No uncertainty is assigned to this combination

→ need of full NNLO QCD-EW results

Available tools and results

fixed order expansion

Drell-Yan (1970)

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} +$$

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

Altarelli, Ellis, Martinelli (1979)

$$\alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} +$$

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

$$\alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} +$$

still missing
Sudakov high-energy approximations

$$\alpha_s^3\,\sigma^{(3,0)} + \ldots$$

C.Duhr, B.Mistlberger, arXiv:2111.10379

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, arXiv:2201.01754

Available tools and results

fixed order expansion

Drell-Yan (1970)

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

Altarelli, Ellis, Martinelli (1979)

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} + \alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} + \alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} + \alpha_s^3\,\sigma^{(3,0)} + \ldots$$

still missing
Sudakov high-energy approximations

C.Duhr, B.Mistlberger, arXiv:2111.10379

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, arXiv:2201.01754

resummation of logarithmically enhanced contributions

Monte Carlo: different matching algorithms at NLO-QCD +QCD-PS, NNLO-QCD +QCD-PS, NLO-EW +QED-PS
MC@NLO, POWHEG, MiNLO, MiNNLOPS, Geneva, HORACE, POWHEG QCD+EW

Integrators: different resummation formalisms for qt-resummation up to N3LL-QCD
ResBos, DYTurbo, RadIsh, SCETlib

most of the experimental analyses based on the “plug-and-play” convolution of NLO QCD and QED-FSR results
the uncertainties of this combination are not available and observable dependent
→ requires a formal and a phenomenological discussions and the development of combined QCD and QED resummation

Progress towards Drell-Yan simulations at NNLO QCD-EW

Strong boost of the activities in the theory community in the last 2 years!

→ mathematical and theoretical developments and computation of universal building blocks

- 2-loop virtual Master Integrals with internal masses

U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193, R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581, M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491, S.Hasan, U.Schubert, arXiv:2004.14908, M.Long,R,Zhang,W.Ma,Y,Jiang,L.Han,,Z.Li,S.Wang, arXiv:2111.14130

- Altarelli-Parisi splitting functions including QCD-QED effects

D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612

- renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

→ on-shell Z and W production as a first step towards full Drell-Yan

- pole approximation of the NNLO QCD-EW corrections

S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016

- analytical total cross section including NNLO QCD-QED and NNLO QED corrections

D. de Florian, M.Der, I.Fabre, arXiv:1805.12214

- ptZ distribution including QCD-QED analytical transverse momentum resummation

L. Cieri, G. Ferrera, G. Sborlini, arXiv:1805.11948

- fully differential on-shell Z production including exact NNLO QCD-QED corrections

M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428

- total Z production cross section in fully analytical form including exact NNLO QCD-EW corrections

R. Bonciani, F. Buccioni, R.Mondini, AV, arXiv:1611.00645, R. Bonciani, F. Buccioni, N.Rana, I.Triscari, AV, arXiv:1911.06200, R. Bonciani, F. Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

- fully differential on-shell Z and W production including exact NNLO QCD-EW corrections

F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221, A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671,

→ complete Drell-Yan

- neutrino-pair production including NNLO QCD-QED corrections

L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315

- 2-loop amplitudes

M.Heller, A.von Manteuffel, R.Schabinger, arXiv:2012.05918

- NNLO QCD-EW corrections to charged-current DY including leptonic decay (2-loop contributions in pole approximation).

L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539

General structure of the inclusive cross section and the q_T-subtraction formalism in Matrix

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of massive final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore,Grazzini, Tramontano 2019.)

General structure of the inclusive cross section and the q_T-subtraction formalism

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of massive final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore, Grazzini, Tramontano 2019.)

$$\int d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} c_i \ln^i r_{cut} + c_0 + \mathcal{O}(r_{cut}^m) \qquad \rightarrow \qquad \int \left(d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right) \sim c_0 + \mathcal{O}(r_{cut}^m)$$

The counterterm removes the IR sensitivity to the cutoff variable
→ we need small values of the cutoff and explicit numerical tests to quantify the bias induced by the cutoff choice
we can fit the r_{cut} dependence and extrapolate in the $r_{cut} \rightarrow 0$ limit
(cfr. Buonocore, Kallweit, Rottoli, Wiesemann, arXiv:2111.13661, Camarda, Cieri, Ferrera, arXiv:2111.14509)

General structure of the inclusive cross section and the q_T-subtraction formalism

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)}\right]_{q_T/Q > r_{cut}}$$

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of massive final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore,Grazzini,Tramontano 2019.)

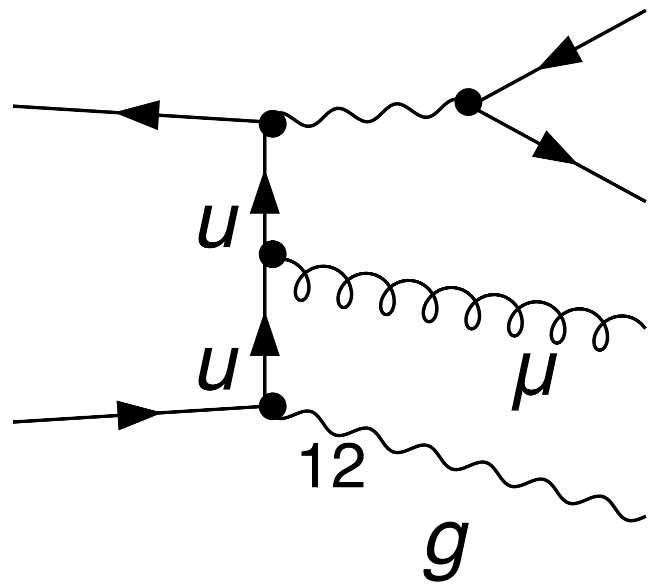
$$\int d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} c_i \ln^i r_{cut} + c_0 + \mathcal{O}(r_{cut}^m) \qquad \rightarrow \qquad \int \left(d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)}\right) \sim c_0 + \mathcal{O}(r_{cut}^m)$$

The counterterm removes the IR sensitivity to the cutoff variable
→ we need small values of the cutoff and explicit numerical tests to quantify the bias induced by the cutoff choice
we can fit the r_{cut} dependence and extrapolate in the $r_{cut} \rightarrow 0$ limit
(cfr. Buonocore, Kallweit, Rottoli, Wiesemann, arXiv:2111.13661, Camarda, Cieri, Ferrera, arXiv:2111.14509)

$$\mathscr{H}^{(1,1)} = H^{(1,1)} C_1 C_2 \qquad 2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}^{(1,1)} \rangle = \sum_{k=-4}^{0} \varepsilon^k f_i(s,t,m) \qquad | \mathscr{M}_{fin} \rangle \equiv (1-I) | \mathscr{M} \rangle \qquad H \propto \langle \mathscr{M}_0 | \mathscr{M}_{fin} \rangle$$

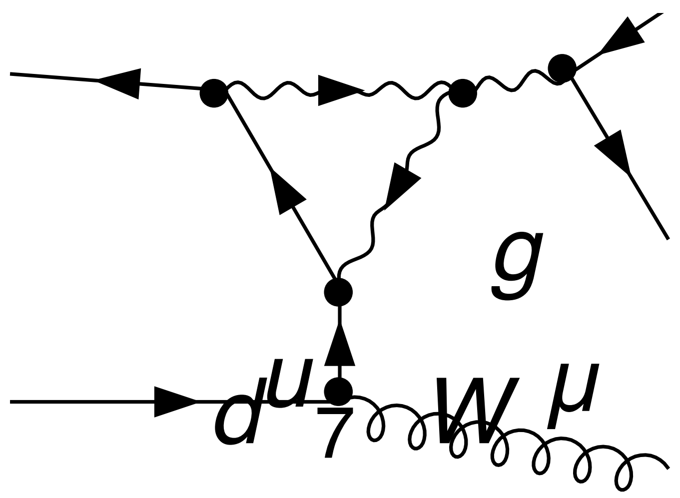
The IR poles are removed from the full 2-loop amplitude by means of a subtraction procedure (matching the real radiation one)

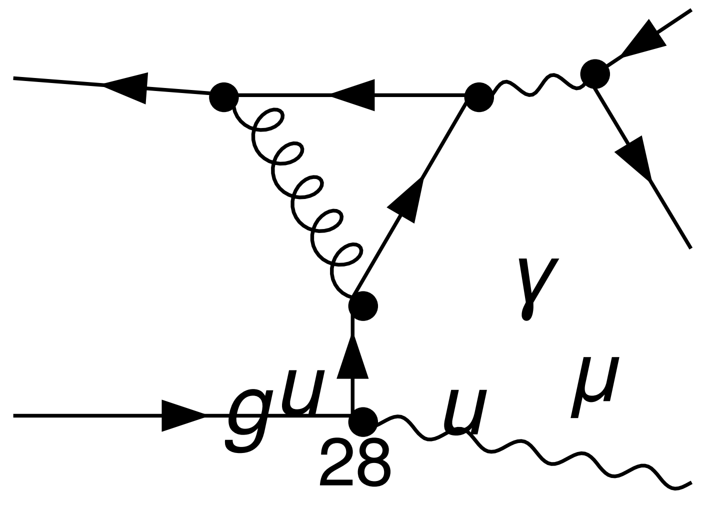
Different kinds of contributions at $\mathcal{O}(\alpha\alpha_s)$ and corresponding problems



double-real contributions

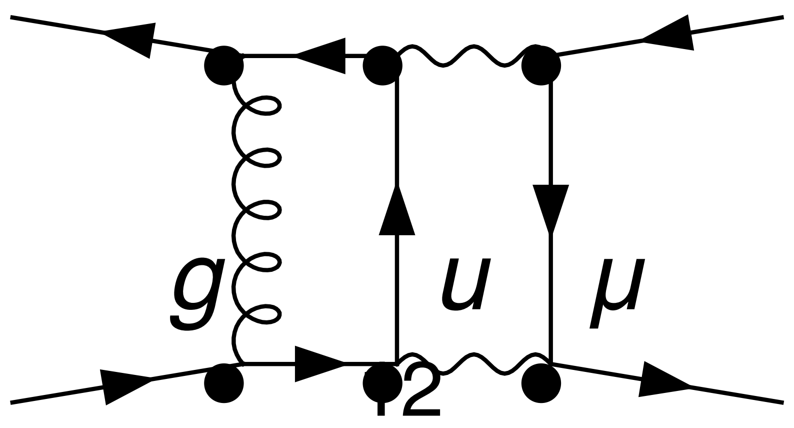
amplitudes are easily generated with OpenLoops
IR subtraction
care about the numerical convergence when aiming at 0.1% precision

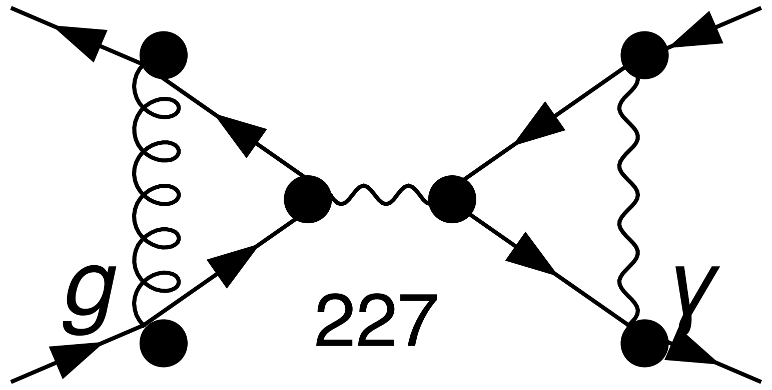




real-virtual contributions

amplitudes are easily generated with OpenLoops or Recola
1-loop UV renormalisation and IR subtraction
care about the numerical convergence when aiming at 0.1% precision





double-virtual contributions

generation of the amplitudes
γ_5 treatment
2-loop UV renormalization
subtraction of the IR divergences
solution and evaluation of the Master Integrals
numerical evaluation of the squared matrix element

The double-real and real-virtual corrections already known from studies of the large transverse momentum lepton pair final state

A.Denner, S.Dittmaier, T.Kasprzik, A.Muck, arXiv:1103.0914, A.Denner, S.Dittmaier, M.Hecht, C.Pasold, arXiv:1510.08742 J.Lindert et al., arXiv:1705.04664

Now we can consider the inclusive spectrum, also in the $q_T \to 0$ limit

The double virtual amplitude: reduction to Master Integrals

$$2\mathrm{Re}\left(\mathcal{M}^{(1,1)}(\mathcal{M}^{(0,0)})^\dagger\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathcal{T}_i(s,t,m;\varepsilon)$$

The coefficients c_i are rational functions of the invariants, masses and of ε
The size of the total expression can rapidly "explode"
→ careful work to identify the patterns of recurring subexpressions keeping the total size in the O(1-10 MB) range

The complexity of the MIs depends on the number of energy scales

MIs relevant for the QCD-QED corrections, with massive final state

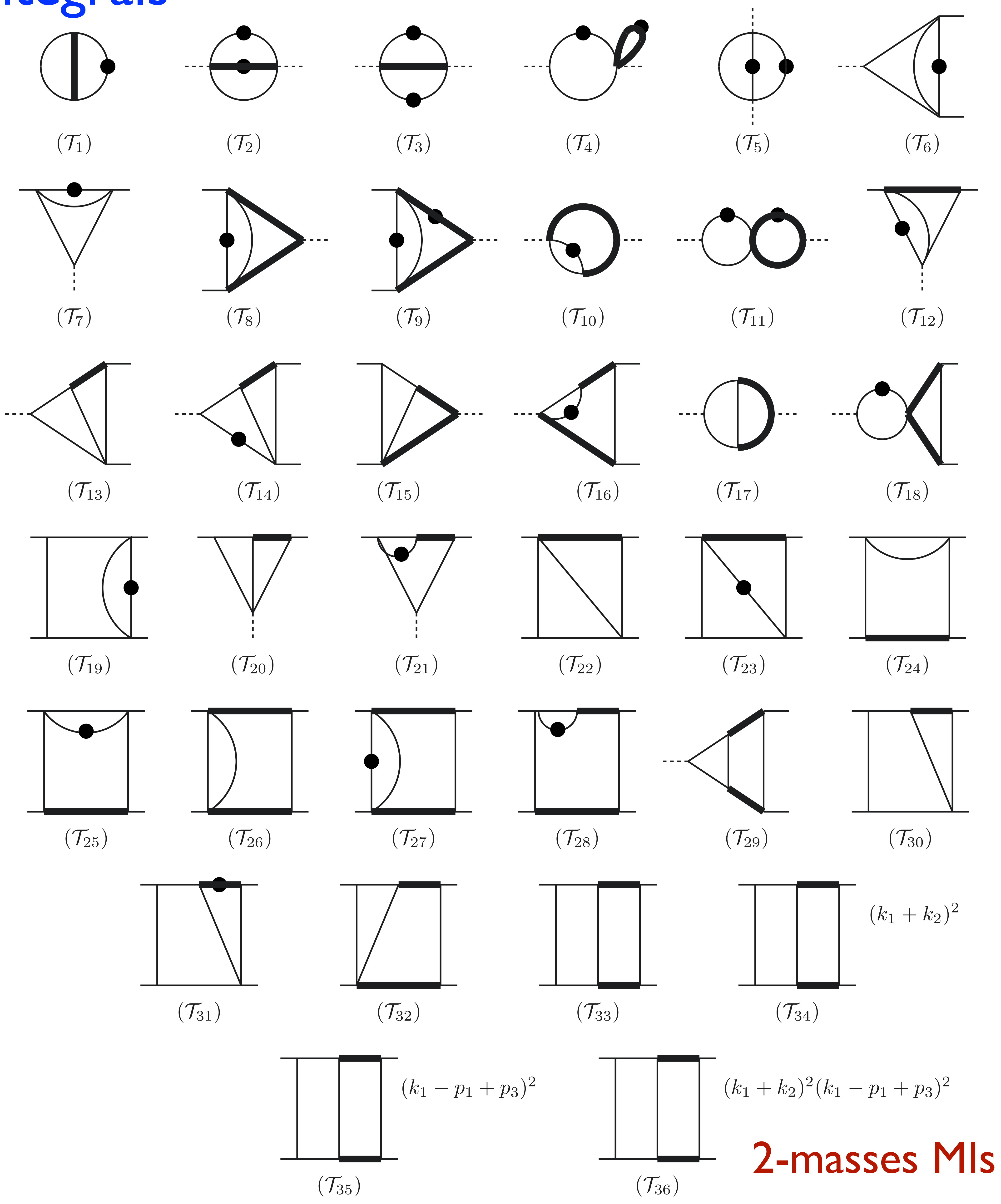
Bonciani, Ferroglia, Gehrmann, Maitre, Studerus., arXiv:0806.2301, 0906.3671

MIs with 1or 2 internal mass relevant for the EW form factor

Aglietti, Bonciani, hep-ph/0304028, hep-ph/0401193

31 MIs with 1 mass and 36 MIs with 2 masses including boxes, relevant for the QCD-weak corrections to the full Drell-Yan

Bonciani, Di Vita, Mastrolia, Schubert., arXiv:1604.08581



In the 2-mass case, 5 integrals can not be expressed in terms of GPLs
→ need an alternative strategy

cfr. also Heller, von Manteuffel, Schabinger, arXiv:1907.00491

Evaluation of the Master Integrals by series expansions

The Master Integrals satisfy a system of differential equations.

The MIs are replaced by formal series with unknown coefficients → eqs for the unknown coefficients of the series.

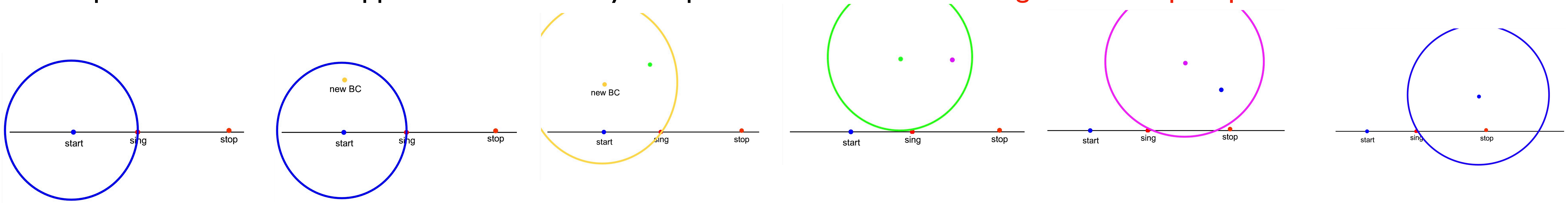
The package `DiffExp` by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.

But we need complex-valued masses of W and Z bosons (unstable particles) → we wrote a new package (`SeaFire`)

Evaluation of the Master Integrals by series expansions

The Master Integrals satisfy a system of differential equations.
The MIs are replaced by formal series with unknown coefficients → eqs for the unknown coefficients of the series.
The package `DiffExp` by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.
But we need complex-valued masses of W and Z bosons (unstable particles) → we wrote a new package (`SeaFire`)

We implemented the same approach, for arbitrary complex-valued masses, working in the complex plane of each kin var.

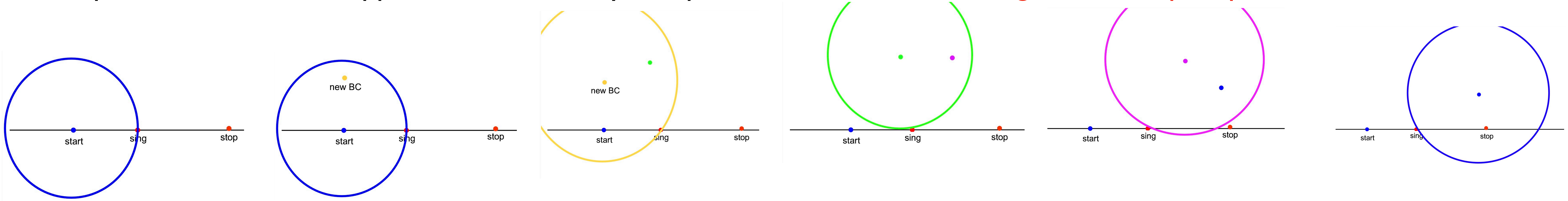


Complete knowledge about the singular structure of the MI can be read directly from the differential equation matrix
The solution can be computed with an arbitrary number of significant digits, but not in closed form → semi-analytical

Evaluation of the Master Integrals by series expansions

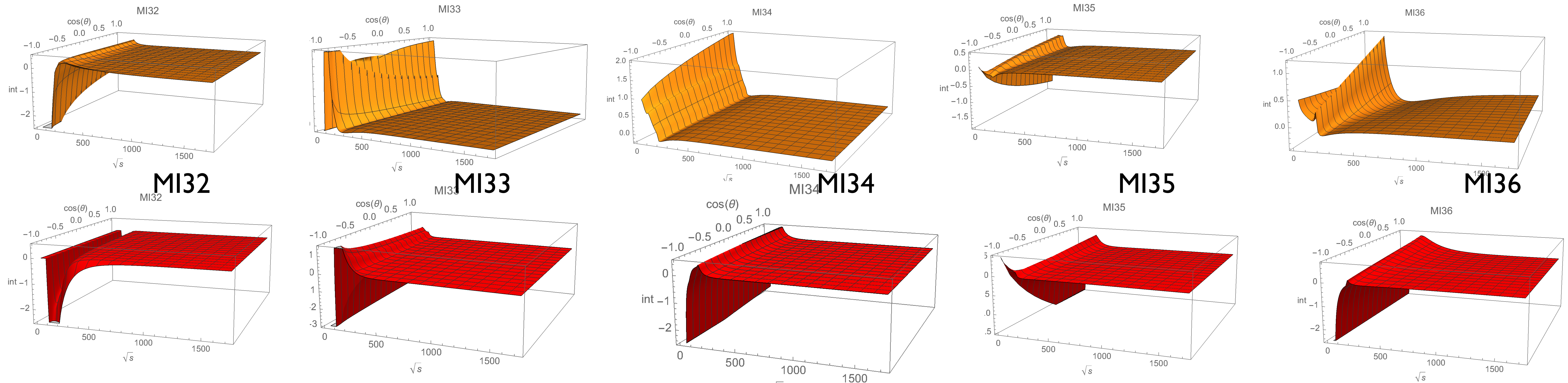
The Master Integrals satisfy a system of differential equations.
The MIs are replaced by formal series with unknown coefficients → eqs for the unknown coefficients of the series.
The package `DiffExp` by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.
But we need complex-valued masses of W and Z bosons (unstable particles) → we wrote a new package (`SeaFire`)

We implemented the same approach, for arbitrary complex-valued masses, working in the complex plane of each kin var.



Complete knowledge about the singular structure of the MI can be read directly from the differential equation matrix
The solution can be computed with an arbitrary number of significant digits, but not in closed form → semi-analytical

Re
Im



Numerical evaluation of the hard coefficient function

The interference term $2\mathrm{Re}\langle \mathscr{M}^{(1,1),fin} | \mathscr{M}^{(0,0)} \rangle$ contributes to the hard function $H^{(1,1)}$

After the subtraction of all the universal IR divergences, it is a finite correction

It has been published in arXiv:2201.01754 and is available as a Mathematica notebook

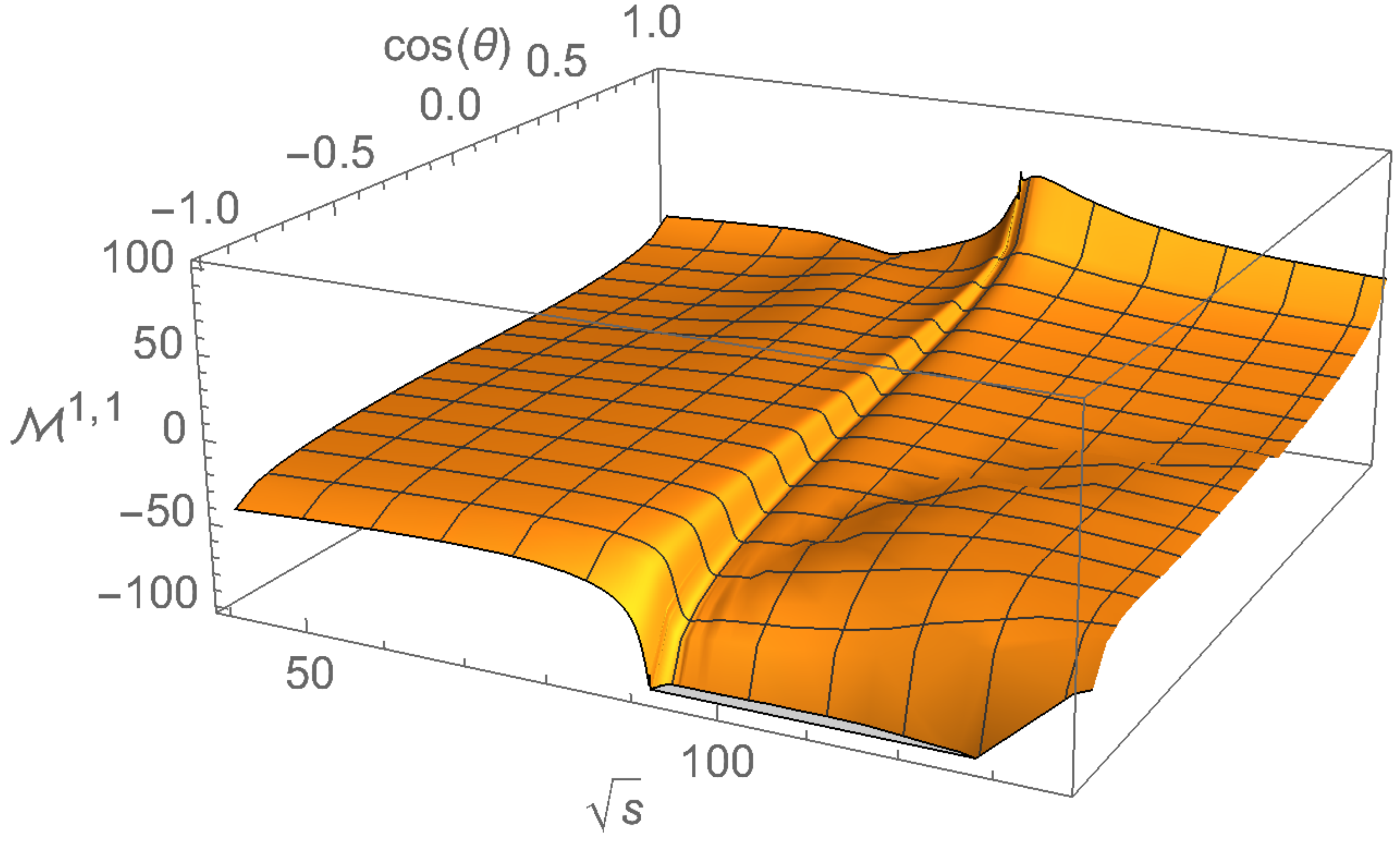
Several checks of the MIs performed with Fiesta and PySecDec

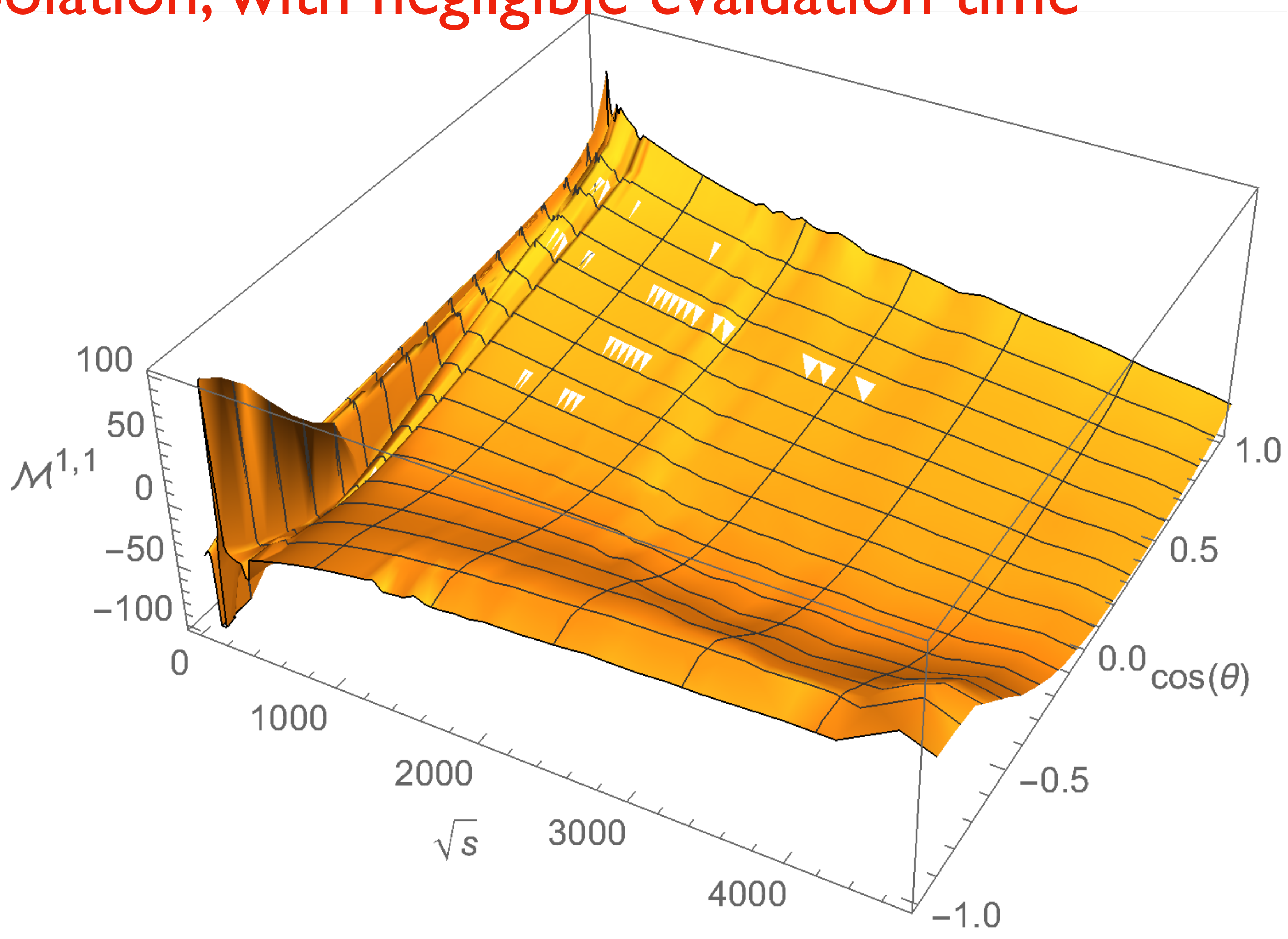
A numerical grid has been prepared for all the 36 MIs, with GiNaC and SeaFire (T.Armadillo et al, in preparation), covering the whole $2 \to 2$ phase space in (s,t), in O(9 h) on one 32-cores machine

→ a numerical grid for $2\mathrm{Re}\langle \mathscr{M}^{(1,1),fin} | \mathscr{M}^{(0,0)} \rangle$ has been prepared

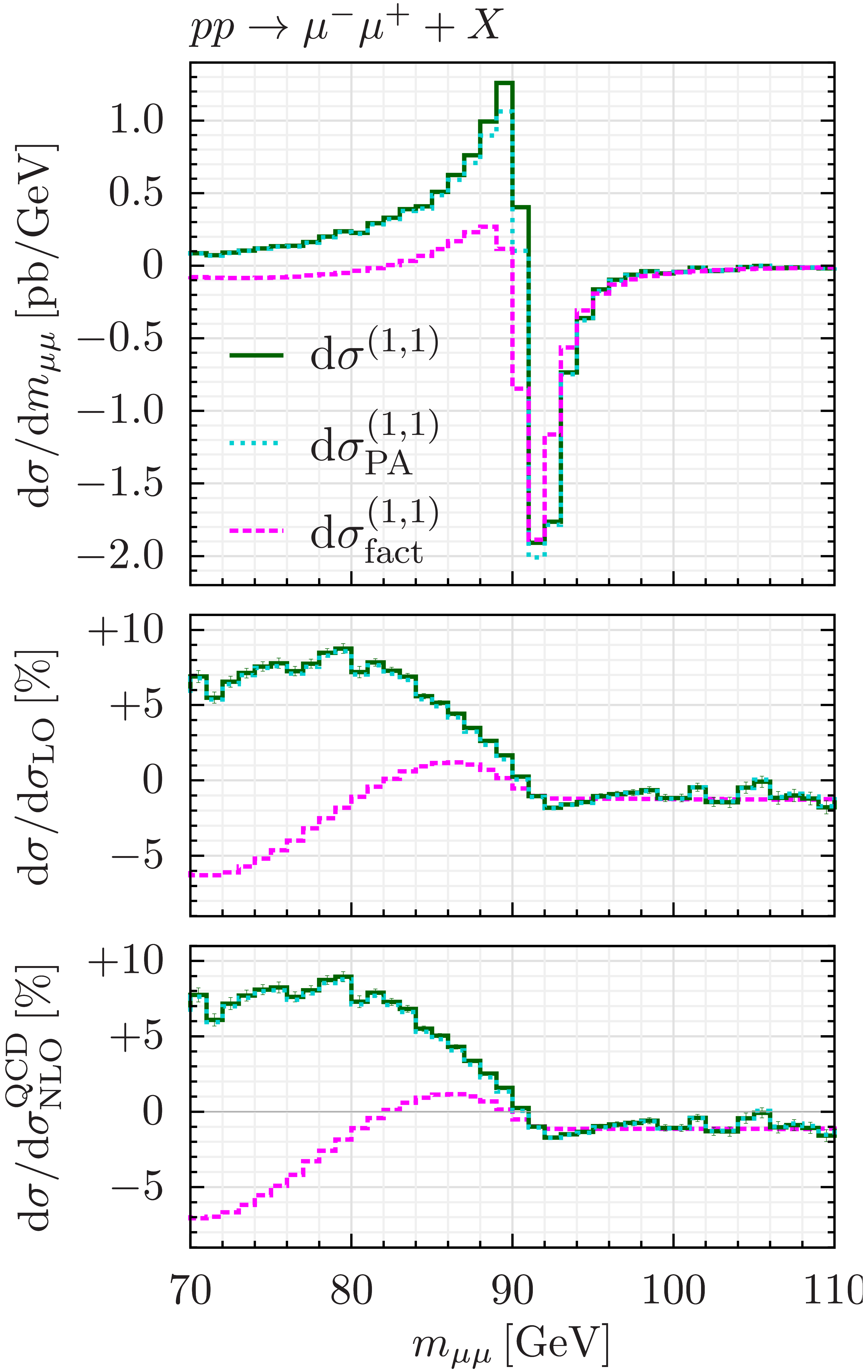
values at arbitrary phase space points with excellent accuracy via interpolation, with negligible evaluation time

in units $\frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi}\sigma_0$





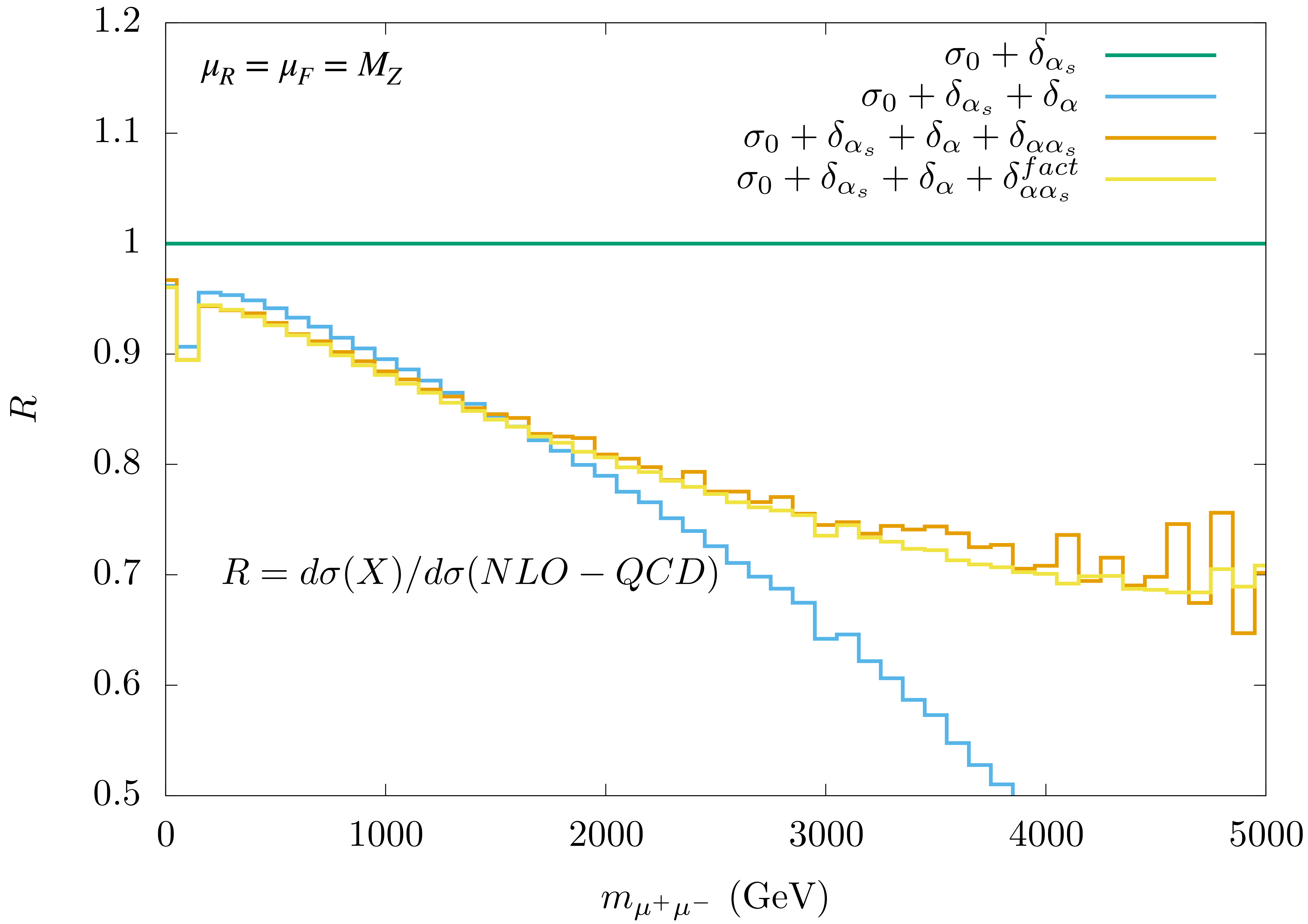
The lepton-pair invariant mass distribution: QCD-EW corrections



below the Z resonance,
QCD and QED effects do not factorise
- a proper treatment of kinematics is needed (cfr. e.g. POWHEG QCD+EW)
- the full calculation has a non-trivial correction

the pole approximation provides an excellent description of the full calculation confirming S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016

The lepton-pair invariant mass distribution: QCD-EW corrections



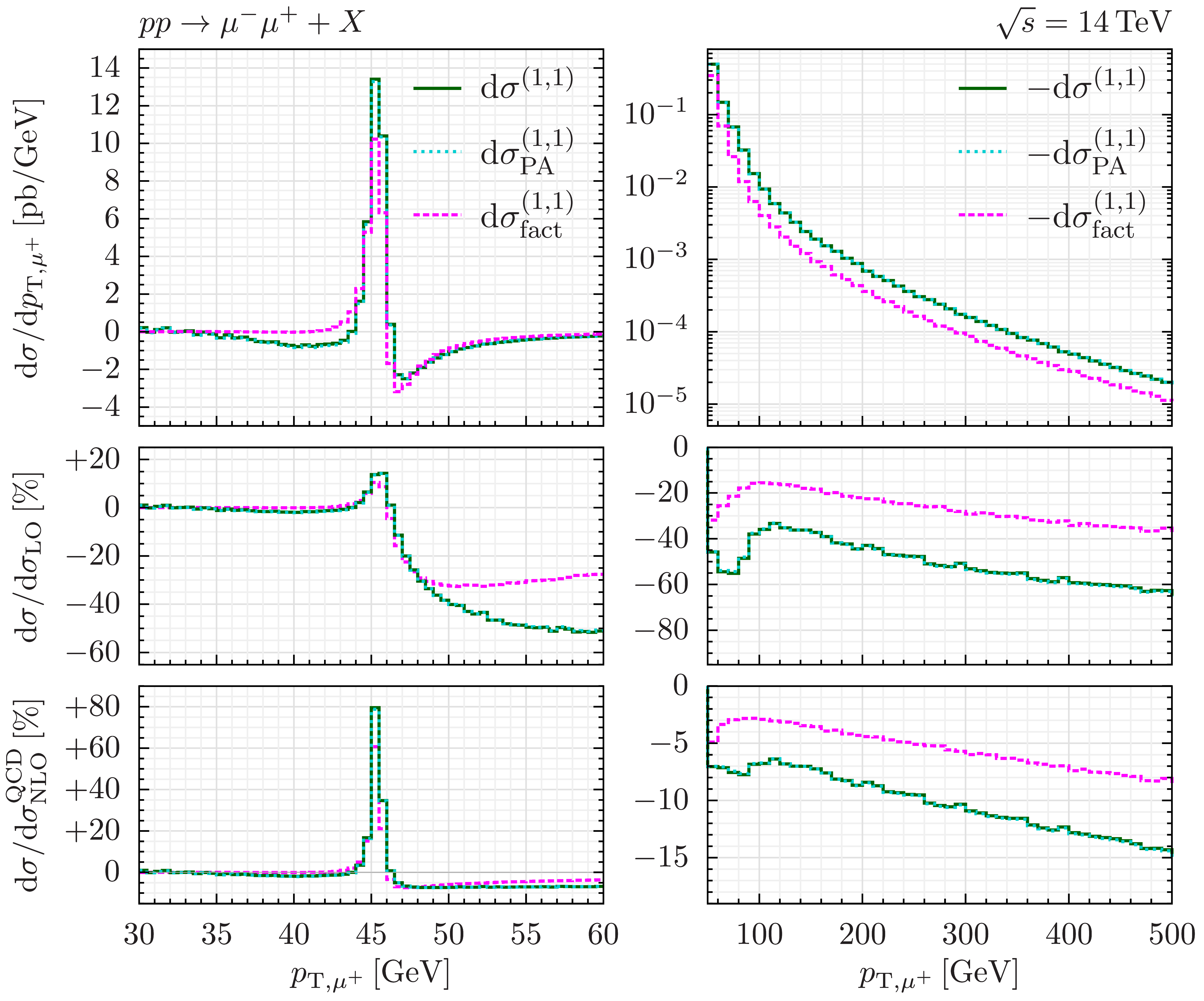
in the very high invariant mass range,
QCD-EW effects are large and positive

at 3 TeV they are O(+10%), still comparable
with the statistical uncertainty at the end of HL-LHC

the factorised approximation
catches the bulk of the QCD-EW correction
but
a residual O(1%) non-factorisable effect emerges
(but more statistics is needed)

using a dynamical choice for μ_R and μ_F
causes a non-trivial change in the size and shape
of the corrections

The lepton transverse momentum distribution: QCD-EW corrections



In the jacobian peak region the xsec is very much affected by the fixed-order nature of the results

In the large transverse momentum tail
the dominant topology is Z+jet (qg-induced)
which receives negative and large EW corrections

The exact calculation deviates from the factorised approximation because
the EW correction $d\sigma^{(0,1)}$ applies correctly to the $p_T^Z = 0$ bin
but
misses the large Sudakov logs which develop at $p_T^Z \gg 0$

The pole approximation on the 2-loop virtual corrections
affects only the $q\bar{q}$ process with $p_T^Z = 0$
which is negligible at large p_T^μ

Important impact on the Z+jet and W+jet generators

Estimate of the residual uncertainties: total cross section

The impact of the NNLO QCD-EW corrections is twofold: more accurate predictions (additional higher orders)
reduced uncertainties (scale, inputs, matching)

Ongoing phenomenological studies for full NC DY

A representative example from the results for the on-shell Z production total cross section

R.Bonciani, F.Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

→ dependence on the EW input-scheme choice

comparison of (G_μ, M_W, M_Z) and $(\alpha(0), M_W, M_Z)$ (very conservative choice that maximises the spread of the results)

order	G_μ	$\alpha(0)$	$\delta(G_\mu - \alpha(0))$ (%)
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

the LO + NLO-EW result would suffer of only 0.55% spread;
the NLO-QCD and NNLO-QCD corrections are only LO-EW and reintroduce a dependence (→0.88%)
which is reduced by the NNLO QCD-EW (→0.23%)

Estimate of the residual uncertainties: total cross section

The impact of the NNLO QCD-EW corrections is twofold: more accurate predictions (additional higher orders)
reduced uncertainties (scale, inputs, matching)

Ongoing phenomenological studies for full NC DY

A representative example from the results for the on-shell Z production total cross section

R.Bonciani, F.Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

→ dependence on the EW input-scheme choice

comparison of (G_μ, M_W, M_Z) and $(\alpha(0), M_W, M_Z)$ (very conservative choice that maximises the spread of the results)

order	G_μ	$\alpha(0)$	$\delta(G_\mu - \alpha(0))$ (%)
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

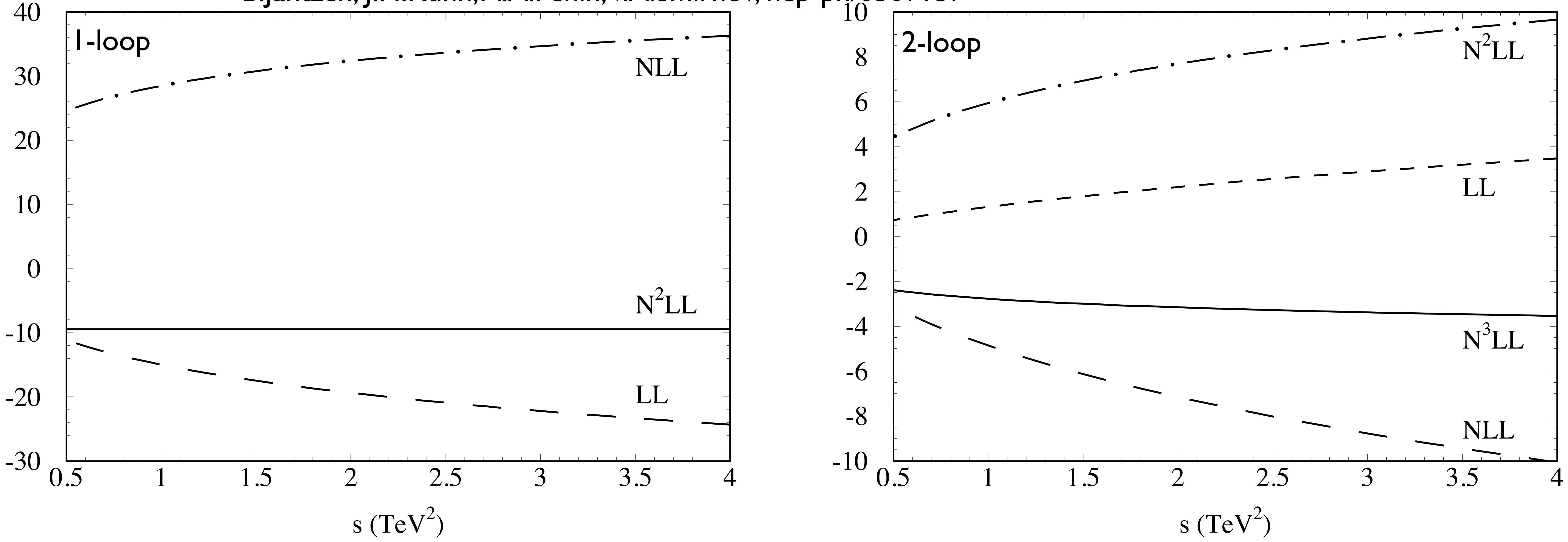
the LO + NLO-EW result would suffer of only 0.55% spread;
the NLO-QCD and NNLO-QCD corrections are only LO-EW and reintroduce a dependence (→0.88%)
which is reduced by the NNLO QCD-EW (→0.23%)

The availability of N3LO-QCD and NNLO QCD-EW results can bring the study of EW gauge bosons in the per mille arena !!!
Is the full NNLO-EW calculation negligible at this level ?

Towards the NNLO-EW calculation ?

The NNLO-EW corrections could modify in a non-trivial way the large-mass/momentum tails of the distributions
Large logarithmic corrections (EW Sudakov logs) appear in the virtual corrections
At two-loop level, we have up to the fourth power of $\log(s/m_V^2)$,
the different corrections are comparable in size and with alternate signs
→ how can we estimate the constant term ?

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157

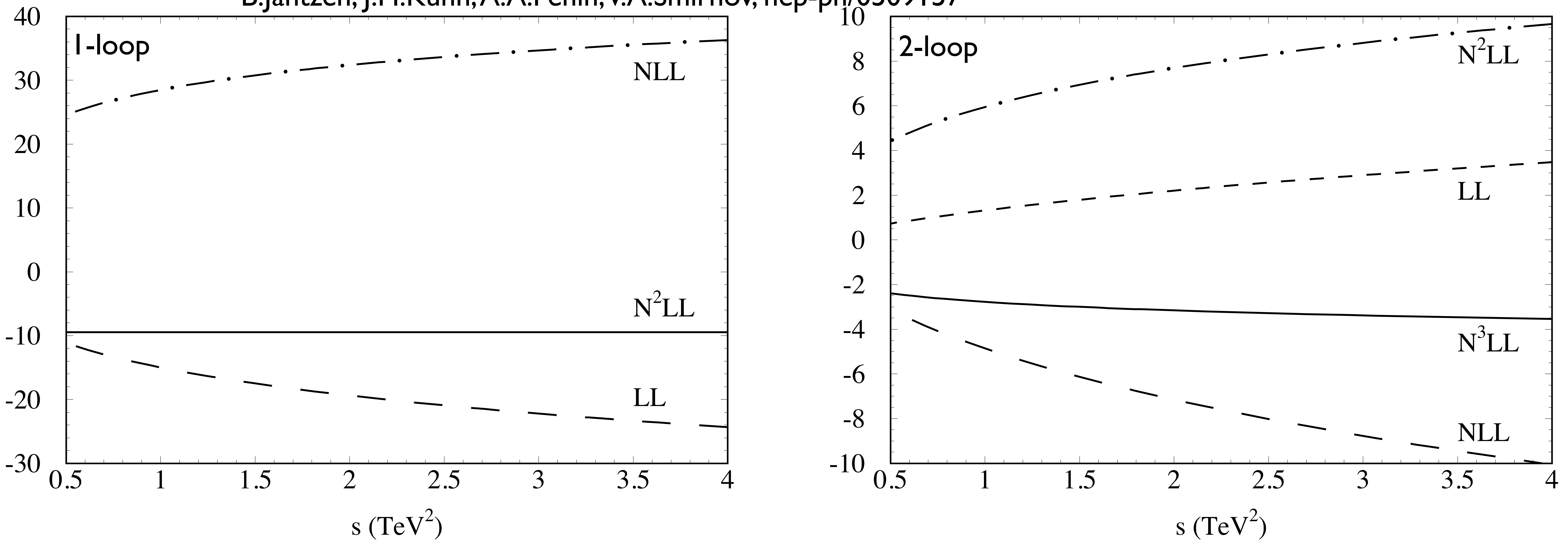


corrections to $e^+e^- \to q\bar{q}$
due to EW Sudakov logs

Towards the NNLO-EW calculation ?

The NNLO-EW corrections could modify in a non-trivial way the large-mass/momentum tails of the distributions
Large logarithmic corrections (EW Sudakov logs) appear in the virtual corrections
At two-loop level, we have up to the fourth power of $\log(s/m_V^2)$,
the different corrections are comparable in size and with alternate signs
→ how can we estimate the constant term ?

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157



corrections to $e^+e^- \to q\bar{q}$
due to EW Sudakov logs

The NNLO-EW corrections will require an extra step compared to the mixed QCD-EW case
- for the number of additional Master Integrals (→ automation)
- for the complexity of the amplitudes (size problems? large cancellations?)
- for the conceptual problems (γ_5?, complex-mass scheme at two-loop?)

but the discussion has started

Conclusions

The evaluation of the NNLO QCD-EW corrections is not yet a "pressing-just-one-button" game
but
the main obstacles to compute the 2-loop virtual corrections have been understood and solved for NC DY

- amplitude manipulation
- IR structure
- evaluation of Master Integrals with 2 internal masses

The complete set of corrections has been combined in the Matrix framework to compute phenomenological predictions

The systematic automation of the progresses in the 2-loop virtual section is ongoing and will
allow the study of NNLO QCD-EW corrections to other scattering processes
be the starting point for the evaluation of NNLO-EW corrections

The phenomenological impact of mixed NNLO QCD-EW corrections is not negligible
in the precision physics program at the LHC

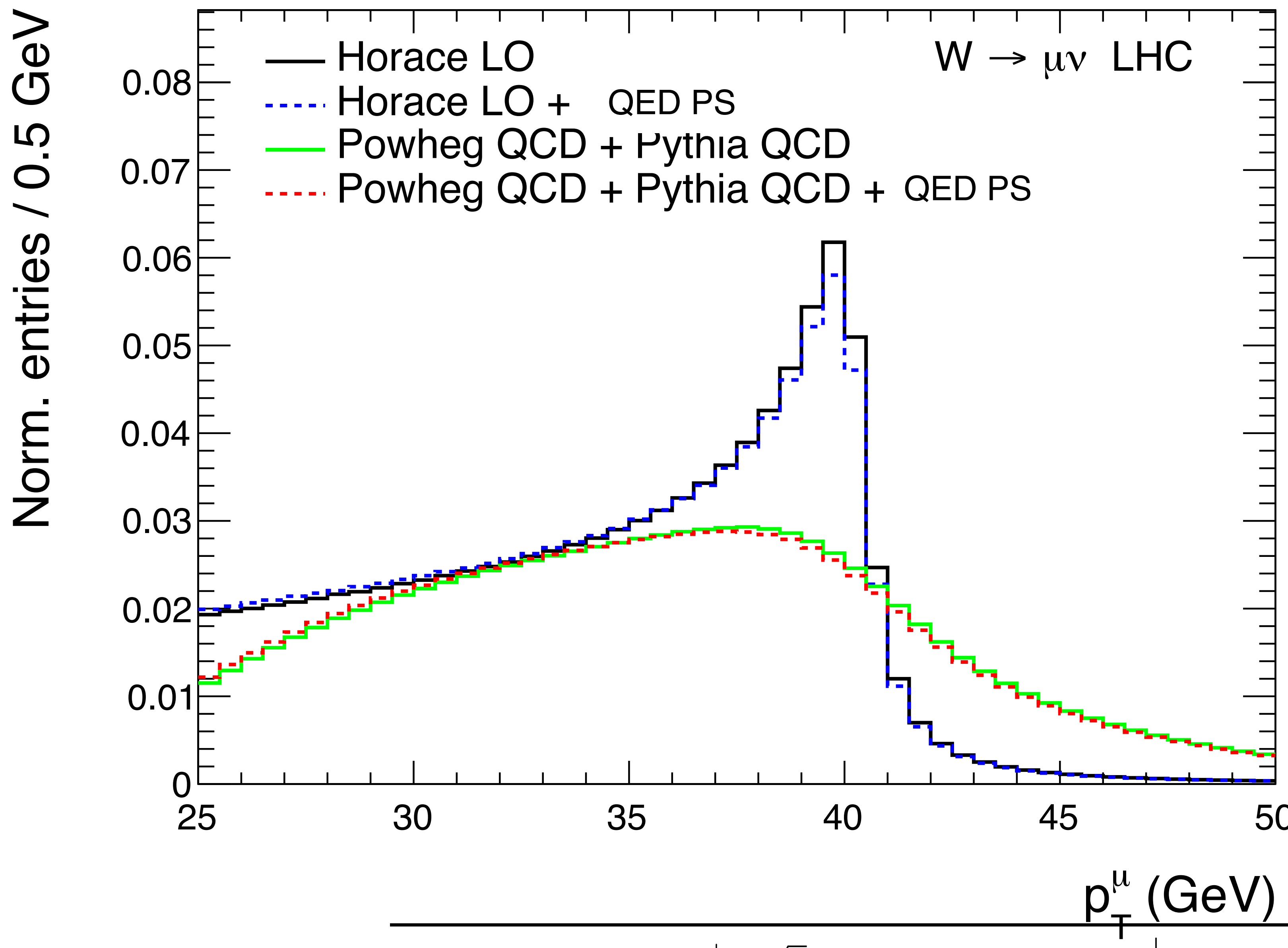
A precise SM prediction is the mandatory starting point for any SMEFT study or search in a UV-complete model

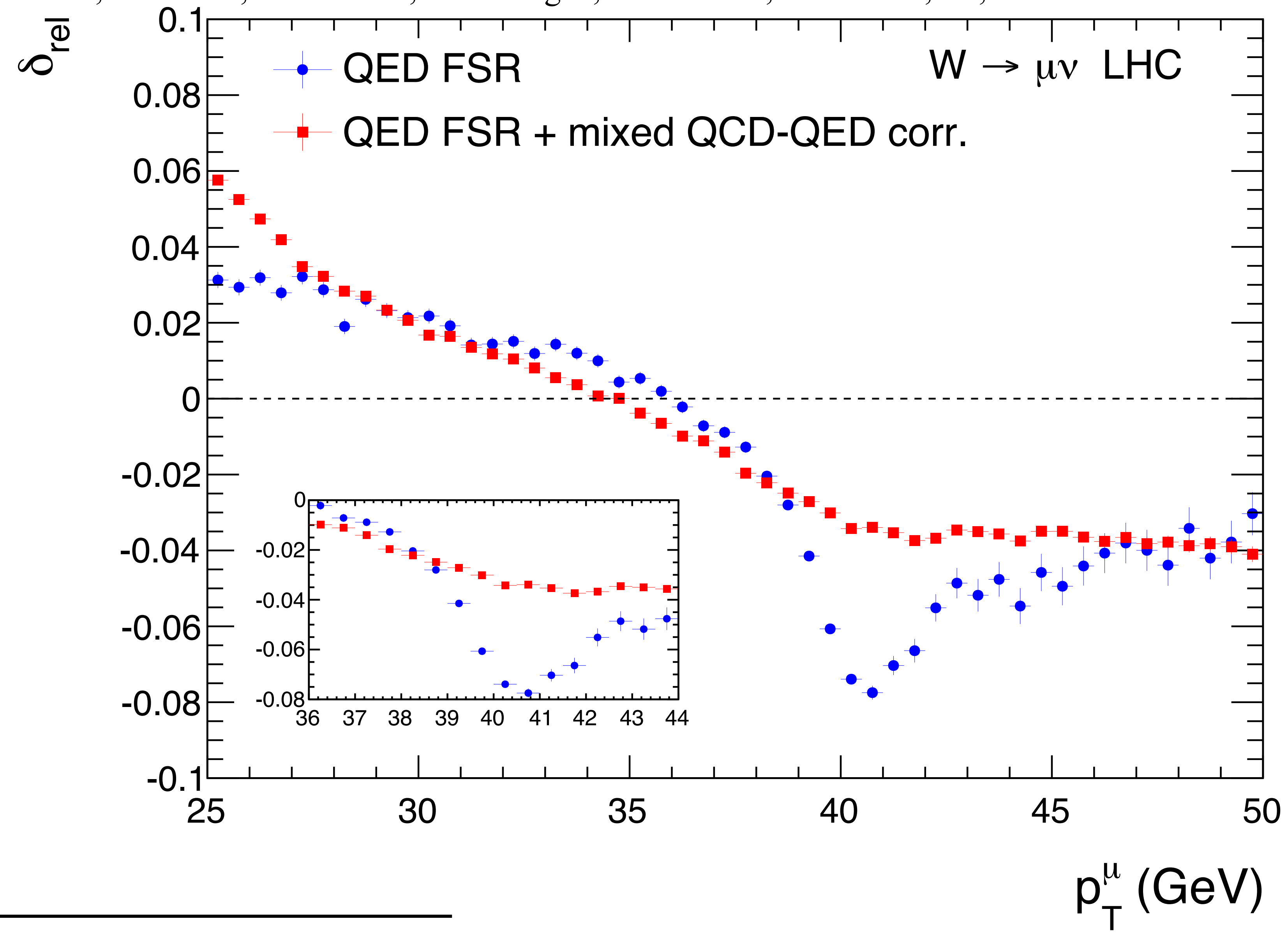
Few per mille precision of the theoretical predictions at large masses/momenta is
assuming a perfect proton PDF → a severe constraint on New Physics models
assuming the SM validity → a strong constraint in the proton PDF fit

Back-up

Combined QCD-EW simulation tools: impact of QED-FSR on MW

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841





$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo–data accuracy	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$ M_T	 p_T^ℓ
1 Horace only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2 Horace FSR-LL	-89±1	-97±1	-179±1	-195±1
3 Horace NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4 Horace FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5 Photos FSR-LL	-92±1	$\boxed{-100±2}$	-182±1	-199±2

$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: NLO-QCD+QCD$_{PS}$ Pseudodata accuracy	 QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 p_T^ℓ
1 NLO-QCD+(QCD+QED)$_{PS}$	Pythia	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2 NLO-QCD+(QCD+QED)$_{PS}$	Photos	-88.0±0.6	$\boxed{-368±2}$	-38.4±0.6	-150±3
3 NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	Pythia	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4 NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	Photos	-88.6±0.6	-370±3	-39.2±0.6	-159±2

the impact on MW of the mixed QCD QED-FSR corrections strongly depends on the underlying QCD shape/model

given that the bulk of the corrections is included in the analyses

- what is the associated uncertainty ?
- what happens if we change the underlying QCD model ?

can we constrain the formulation, for the $\alpha\alpha_s$ contribution ?

The Neutral Current Drell-Yan cross section in the SM: perturbative expansion

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} + \alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} + \alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} + \alpha_s^3\,\sigma^{(3,0)} + \ldots$$

$$\sigma(h_1 h_2 \to l\bar{l} + X) = \sum_{i,j=q\bar{q},g,\gamma} \int dx_1\, dx_2\, f_i^{h_1}(x_1,\mu_F)\, f_j^{h_2}(x_2,\mu_F)\, \hat{\sigma}(ij \to l\bar{l} + X)$$

$\sigma^{(1,1)}$ requires the evaluation of the xsecs of the following processes, including photon-induced

0 additional partons — $q\bar{q} \to l\bar{l},\ \gamma\gamma \to l\bar{l}$ — including virtual corrections of $\mathcal{O}(\alpha_s), \mathcal{O}(\alpha), \mathcal{O}(\alpha\alpha_s)$

1 additional parton — $q\bar{q} \to l\bar{l}g,\ qg \to l\bar{l}q$ — including virtual corrections of $\mathcal{O}(\alpha)$

1 additional parton — $q\bar{q} \to l\bar{l}\gamma,\ q\gamma \to l\bar{l}q$ — including virtual corrections of $\mathcal{O}(\alpha_s)$

2 additional partons — $q\bar{q} \to l\bar{l}g\gamma, qg \to l\bar{l}q\gamma, q\gamma \to l\bar{l}qg, g\gamma \to l\bar{l}q\bar{q}$

$q\bar{q} \to l\bar{l}q\bar{q},\ q\bar{q} \to l\bar{l}q'\bar{q}',\ qq' \to l\bar{l}qq',\ q\bar{q}' \to l\bar{l}q\bar{q}',\ qq \to l\bar{l}qq$ — at tree level

Computational framework

The complete calculation has been included in the Munich/Matrix framework

- fully automatic generation and bookkeeping of all the double-real and real-virtual contributions based on an interface with OpenLoops and Recola/Collier
- the 2-loop virtual corrections are separately computed and provided in fast-evaluation format

In this specific framework, main compatibility requirement to include the double-virtual corrections:
the q_T-subtraction formalism to handle the IR singularities (Catani, Grazzini, 2007)

Upon inclusion of the appropriate scheme-dependent subtraction term,
the double virtual corrections can be used with any other simulation code

General structure of the inclusive cross section and the q_T-subtraction formalism

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore, Grazzini, Tramontano 2019.)

General structure of the inclusive cross section and the q_T-subtraction formalism

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathcal{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

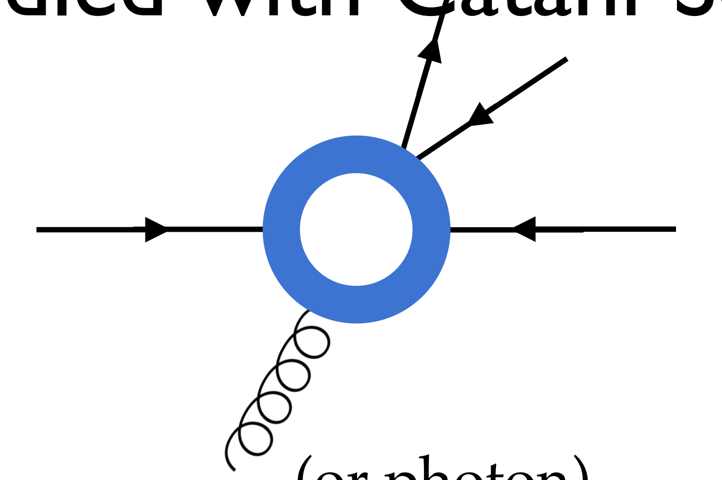
IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore, Grazzini, Tramontano 2019.)

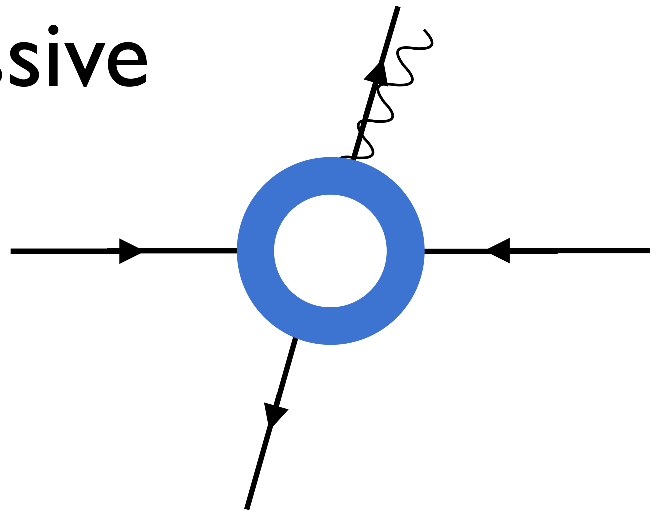
the gauge-boson phase space is split into $q_T = 0$ and $q_T > 0$ regions

$$r_{cut} = q_T^{cut}/Q$$

for ISR, if $q_T > 0$ the emitted parton is always resolved
and the process under study receives only NLO corrections
which can be handled with Catani-Seymour dipoles



in the FSR case, with $q_T > 0$,
the emitted parton is always resolved
only if the emitter is massive



the final state consists of a pair of massive leptons (treated as bare) to regulate the collinear (mass) singularities

The q_T-subtraction and the residual cut-off dependency

$$\boxed{d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}}$$

When $q_T/Q > r_{cut}$ the double-real and the real-virtual contributions, subtracted with CS dipoles, are finite

$d\sigma_{CT}^{(1,1)}$ is obtained by expanding to fixed order the q_T resummation formula

The q_T-subtraction and the residual cut-off dependency

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

When $q_T/Q > r_{cut}$ the double-real and the real-virtual contributions, subtracted with CS dipoles, are finite

$d\sigma_{CT}^{(1,1)}$ is obtained by expanding to fixed order the q_T resummation formula

Logarithmic sensitivity on r_{cut} in the double unresolved limit $\int d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} c_i \ln^i r_{cut} + c_0 + \mathcal{O}(r_{cut}^m)$

The counterterm removes the IR sensitivity to the cutoff variable $\int \left(d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right) \sim c_0 + \mathcal{O}(r_{cut}^m)$

→ we need small values of the cutoff

→ explicit numerical tests to quantify the bias induced by the cutoff choice (cfr. Buonocore, Kallweit, Rottoli, Wiesemann, arXiv:2111.13661 Camarda, Cieri, Ferrera, arXiv:2111.14509)

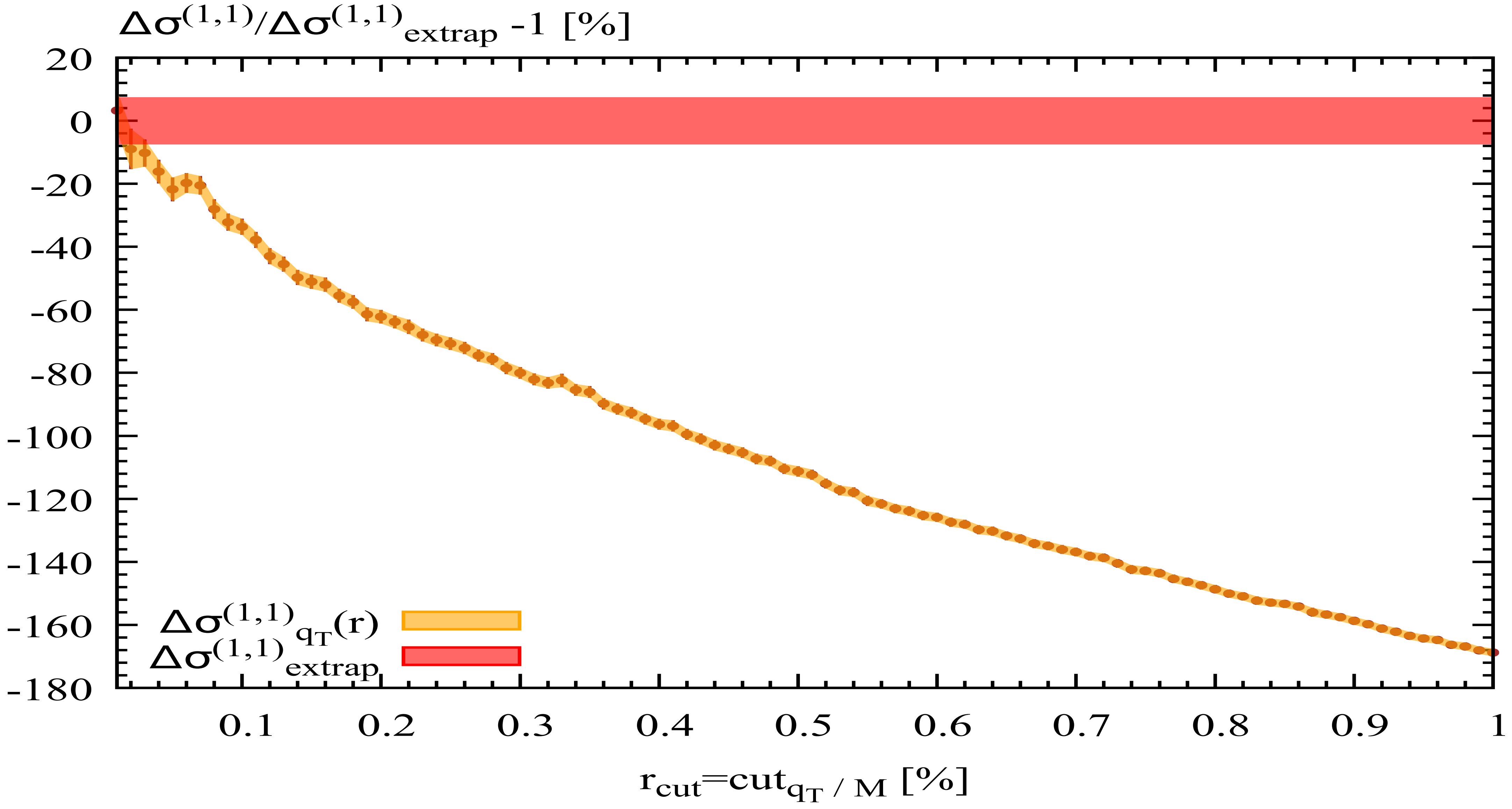
we can fit the r_{cut} dependence and extrapolate in the $r_{cut} \to 0$ limit

Dependence on r_{cut} of the NNLO QCD-EW corrections to NC DY

courtesy of S.Kallweit

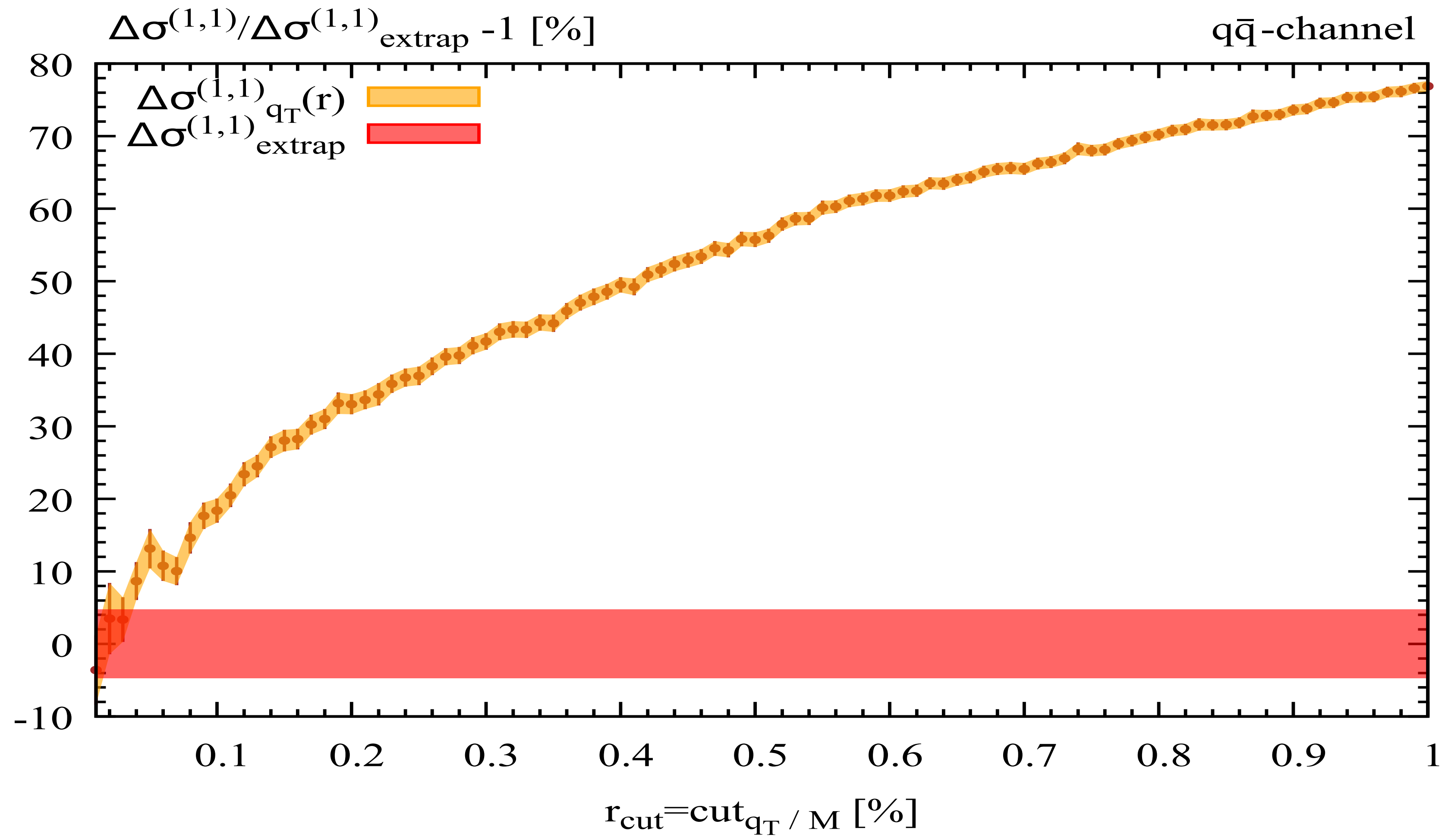
Symmetric-cut scenario

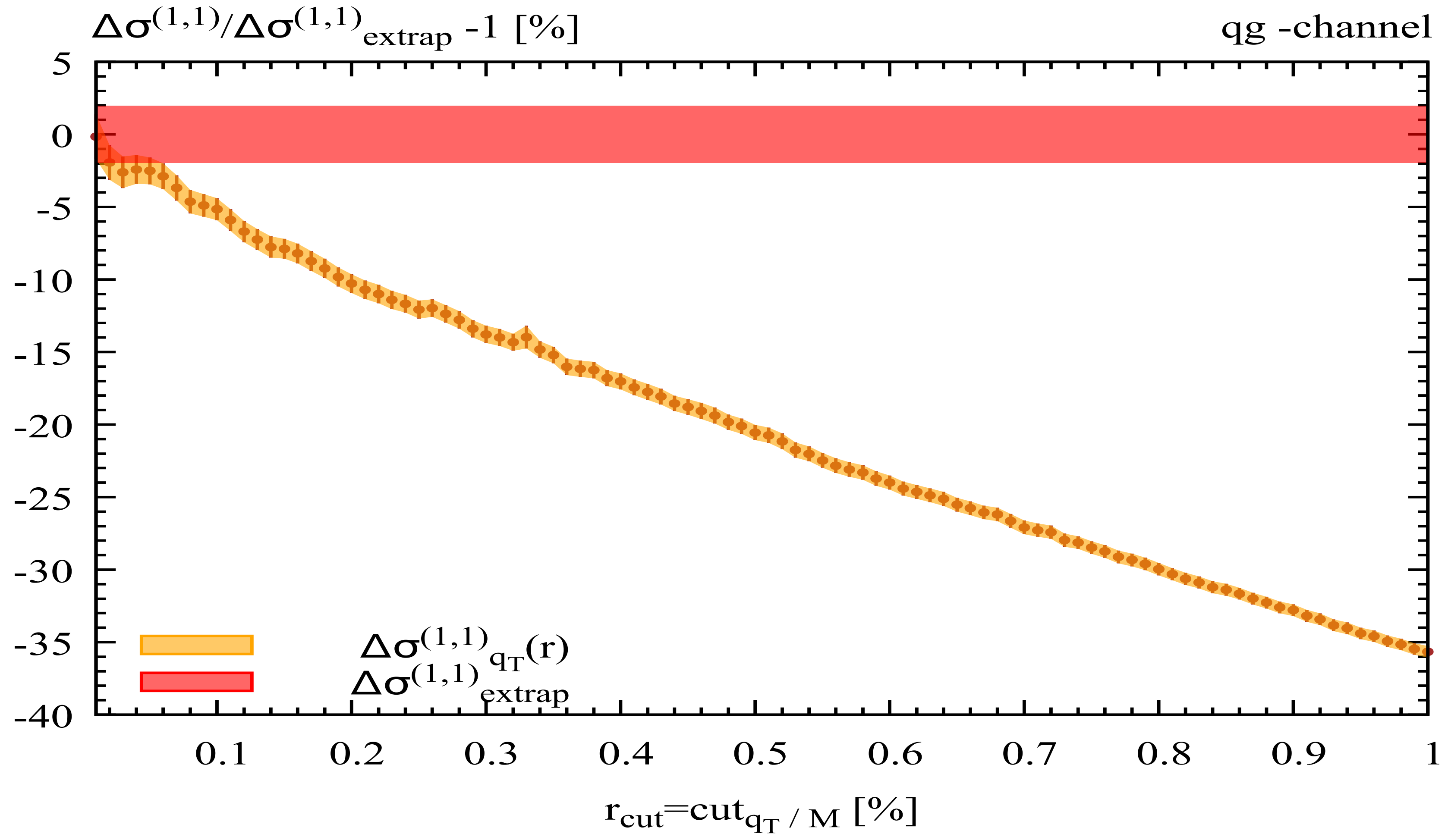
$p_{T,\ell^\pm} > 25$ GeV $y_{\ell^\pm} < 2.5$ $m_{\ell\ell} > 50$ GeV



- **large power corrections in r_{cut} for mixed corrections**
  - explained by overall small size of corrections, and in parts also by cancellation between partonic channels
- **by far less dramatic dependence at level of cross sections**
  - better than permille precision at inclusive level

Splitting into partonic channels





Subtraction of the IR divergences from the 2-loop amplitude

we identify QCD-QED (poles up to $1/\varepsilon^4$) and QCD-weak (poles up to $1/\varepsilon^2$ with cumbersome coefficients) diagrams

$$|\mathcal{M}^{(1,0),fin}\rangle = |\mathcal{M}^{(1,0)}\rangle - \mathcal{I}^{(1,0)}|\mathcal{M}^{(0)}\rangle\,,$$

standard NLO-QCD subtraction

$$|\mathcal{M}^{(0,1),fin}\rangle = |\mathcal{M}^{(0,1)}\rangle - \mathcal{I}^{(0,1)}|\mathcal{M}^{(0)}\rangle\,.$$

NLO-EW subtraction, with massive leptons

$$|\mathcal{M}^{(1,1),fin}\rangle = |\mathcal{M}^{(1,1)}\rangle - \mathcal{I}^{(1,1)}|\mathcal{M}^{(0)}\rangle - \tilde{\mathcal{I}}^{(0,1)}|\mathcal{M}^{(1,0),fin}\rangle - \tilde{\mathcal{I}}^{(1,0)}|\mathcal{M}^{(0,1),fin}\rangle\,.$$

$$\mathcal{I}^{(1,0)} = \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{s}{\mu^2}\right)^{-\epsilon} C_F\left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(3+2i\pi) + \zeta_2\right)\,,$$

$$\mathcal{I}^{(0,1)} = \left(\frac{\alpha}{4\pi}\right)\left(\frac{s}{\mu^2}\right)^{-\epsilon}\left[Q_u^2\left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(3+2i\pi) + \zeta_2\right) + \frac{4}{\epsilon}\Gamma_l^{(0,1)}\right],$$

$$\Gamma_l^{(0,1)} = Q_u Q_l \log\left(\frac{2p_1.p_3}{2p_2.p_3}\right) + \frac{Q_l^2}{2}\left(-1 - \frac{1+x_l^2}{1-x_l^2}\log(x_l)\right).$$

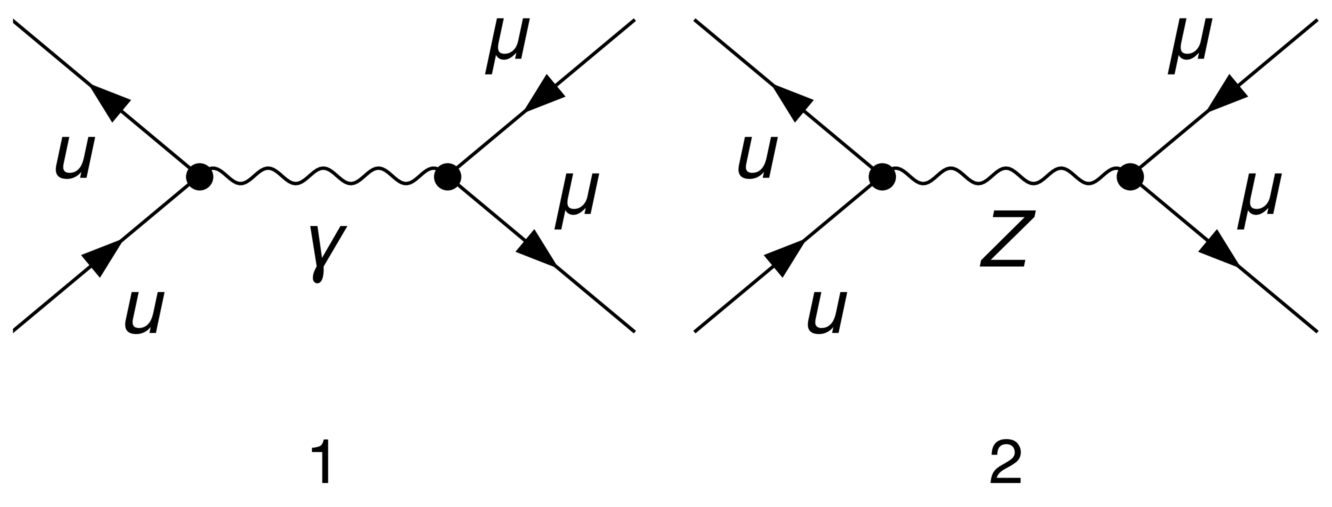
$$\mathcal{I}^{(1,1)} = \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha}{4\pi}\right)\left(\frac{s}{\mu^2}\right)^{-2\epsilon} C_F Q_u^2\left(\frac{4}{\epsilon^4} + \frac{1}{\epsilon^3}(12+8i\pi) + \frac{1}{\epsilon^2}(9 - 28\zeta_2 + 12i\pi) + \frac{1}{\epsilon}\left(-\frac{3}{2} + 6\zeta_2 - 24\zeta_3 - 4i\pi\zeta_2\right)\right).$$

$2\mathrm{Re}\langle\mathscr{M}^{(0,0)}|\mathscr{M}^{(1,1),fin}\rangle$ is free of any singularity

the analytical check of the cancellation of the IR poles in the QCD-weak sector is one very demanding test of the calculation

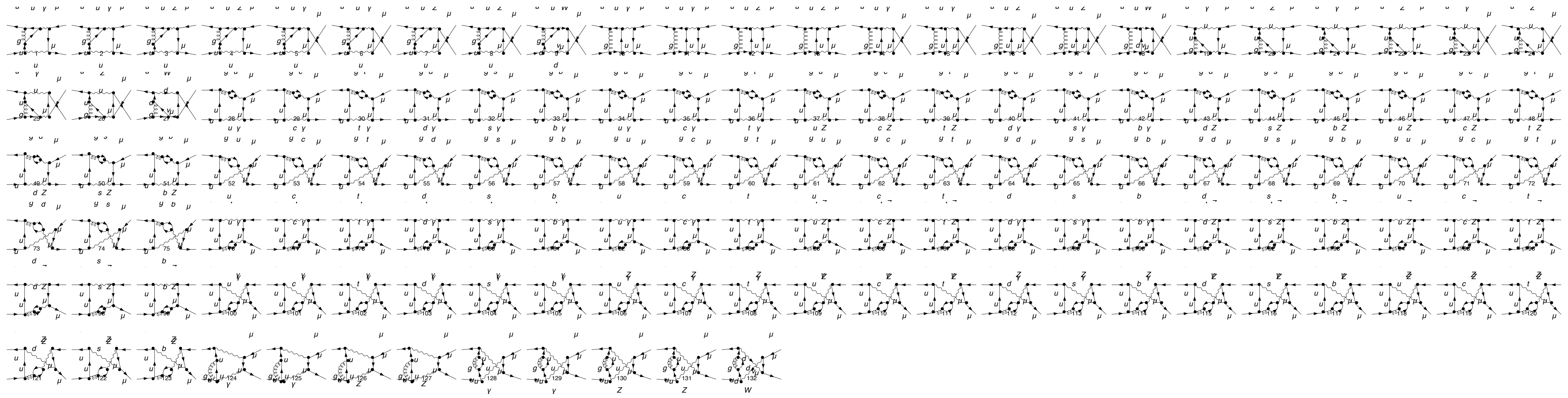
The double virtual amplitude: generation of the amplitude

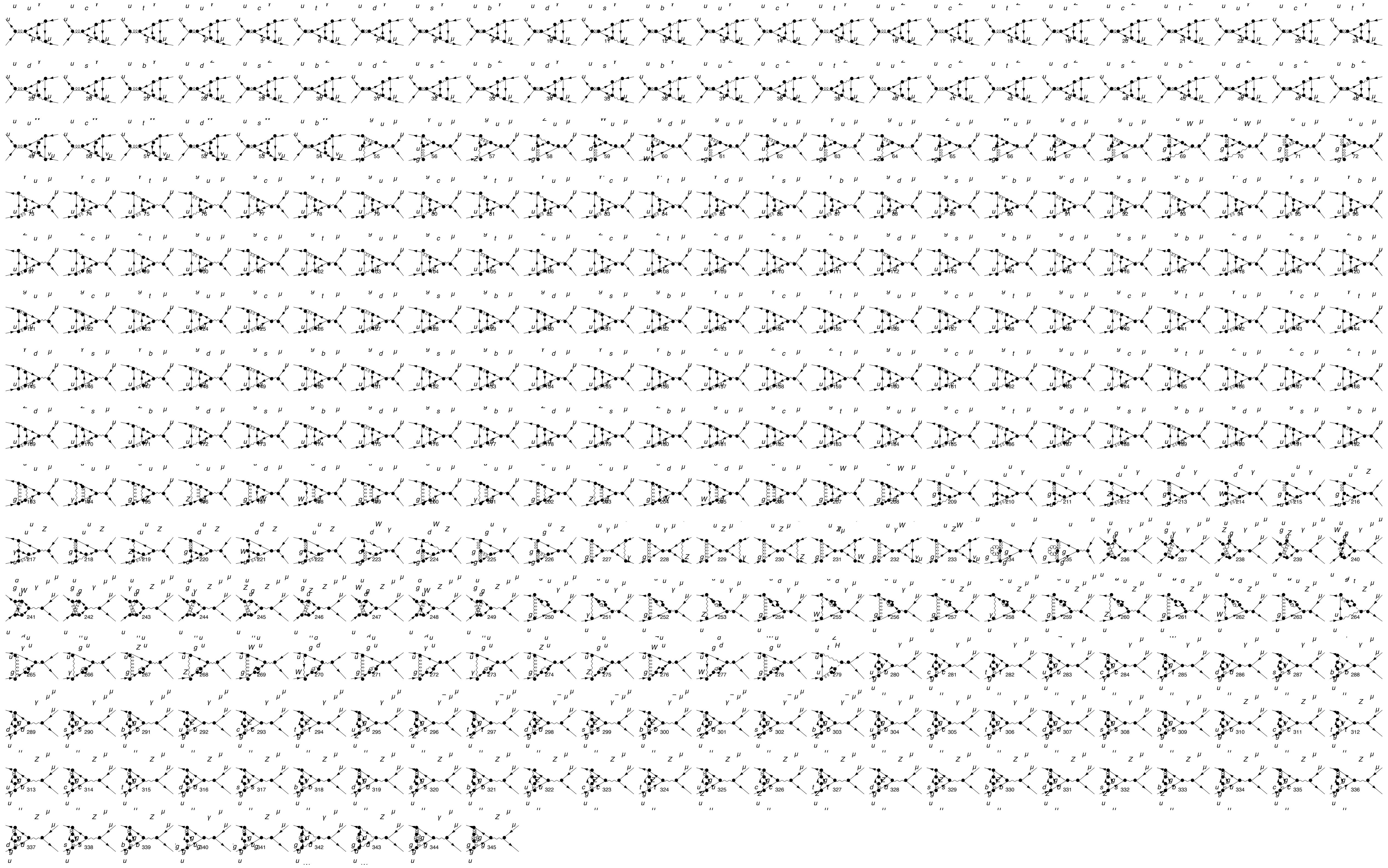
$\mathscr{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$



$\mathscr{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$ O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop

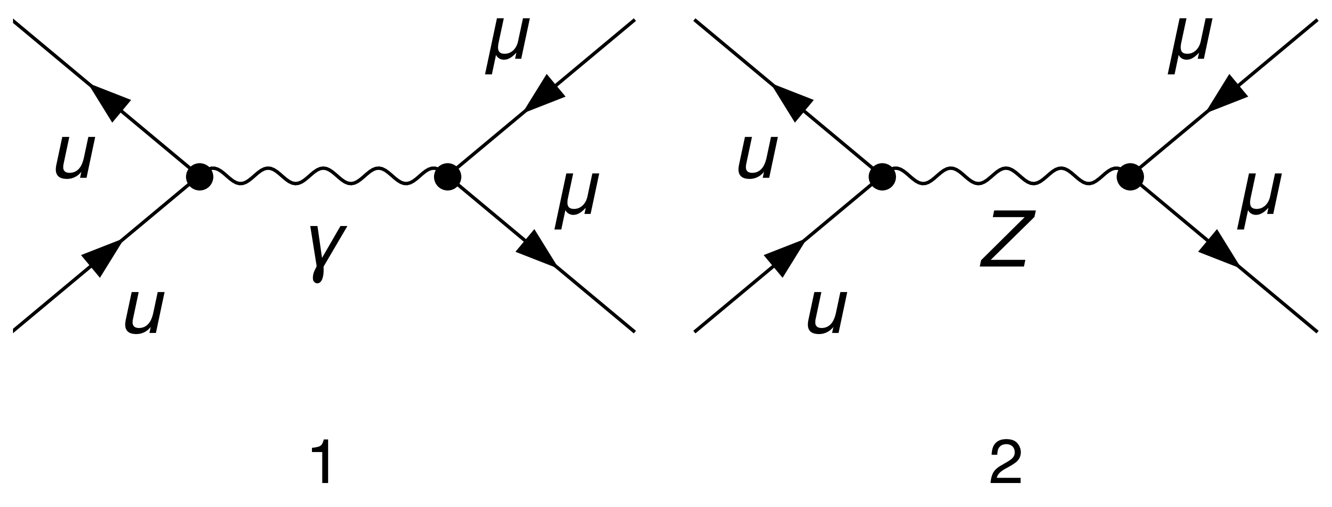
(before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)





The double virtual amplitude: generation of the amplitude

$$\mathscr{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$$



$$\mathscr{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$$

O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop
(before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)

Two independent calculations based on QGraf and FeynArts in the EW Background Field Gauge

The BFG choice guarantees the validity of EW Ward identities for the initial state vertex → additional technical checks

- UV finiteness when combining 2-loop vertex and quark WF in the full EW SM → that combination has only IR poles
- UV renormalisation is confined to the gauge-boson propagators sector, where IR divergences are absent

The 1-loop check of the gauge-parameter independence identifies those subsets of diagrams yielding the cancellation.

The 2-loop calculation is organised splitting the total amplitude in the combination of different subsets, according to their EW charges (# of Ws, Zs, γs)

The double virtual amplitude: UV renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

Complex mass scheme

$$\mu_{W0}^2 = \mu_W^2 + \delta\mu_W^2, \quad \mu_{Z0}^2 = \mu_Z^2 + \delta\mu_Z^2, \quad e_0 = e + \delta e$$

$$\frac{\delta s^2}{s^2} = \frac{c^2}{s^2}\left(\frac{\delta\mu_Z^2}{\mu_Z^2} - \frac{\delta\mu_W^2}{\mu_W^2}\right)$$

the mass counterterms are defined at the complex pole of the propagator

the weak mixing angle is complex valued $c^2 \equiv \mu_W^2/\mu_Z^2$

BFG EW Ward identity → cancellation of the UV divergences combining vertex and fermion WF corrections

The bare couplings of Z and photon to fermions in the (G_μ, μ_W, μ_Z) input scheme are given by

$$\frac{g_0}{c_0} = \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left[1 - \frac{1}{2}\Delta r + \frac{1}{2}\left(2\frac{\delta e}{e} + \frac{s^2 - c^2}{c^2}\frac{\delta s^2}{s^2}\right)\right] \equiv \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left(1 + \delta g_Z^{G_\mu}\right)$$

$$g_0 s_0 = \sqrt{4\sqrt{2}G_\mu\mu_W^2 s^2}\left[1 + \tfrac{1}{2}\left(-\Delta r + 2\tfrac{\delta e}{e}\right)\right] \equiv e_{ren}^{G_\mu}\left(1 + \delta g_A^{G_\mu}\right)$$

Gauge boson renormalised propagators

$$\Sigma_{R,T}^{AA}(q^2) = \Sigma_T^{AA}(q^2) + 2\,q^2\,\delta g_A$$
$$\Sigma_{R,T}^{ZZ}(q^2) = \Sigma_T^{ZZ}(q^2) - \delta\mu_Z^2 + 2\,(q^2 - \mu_Z^2)\,\delta g_Z$$

$$\Sigma_{R,T}^{AZ}(q^2) = \Sigma_T^{AZ}(q^2) - q^2\,\frac{\delta s^2}{sc}$$
$$\Sigma_{R,T}^{ZA}(q^2) = \Sigma_T^{ZA}(q^2) - q^2\,\frac{\delta s^2}{sc},$$

After the UV renormalisation, the singular structure is entirely due to IR soft and/or collinear singularities

The double virtual amplitude: γ_5 treatment

The absence of a consistent definition of γ_5 in $n = 4 - 2\varepsilon$ dimensions yields a practical problem

The trace of Dirac matrices and γ_5 is a polynomial in ε
The UV or IR divergences of Feynman integrals appear as poles $1/\varepsilon$

$$\mathrm{Tr}(\gamma_\alpha \ldots \gamma_\mu \gamma_5) \times \int d^n k \frac{1}{[k^2 - m_0^2][(k+q_1)^2 - m_1^2][(k+q_2)^2 - m_2^2]} \sim (a_0 + a_1 \varepsilon + \ldots) \times \left(\frac{c_{-2}}{\varepsilon^2} + \frac{c_{-1}}{\varepsilon} + c_0 + \ldots \right)$$

If a_1 is evaluated in a non-consistent way,
then poles might not cancel and the finite part of the xsec might have a spurious contribution

The double virtual amplitude: γ_5 treatment

The absence of a consistent definition of γ_5 in $n = 4 - 2\varepsilon$ dimensions yields a practical problem

The trace of Dirac matrices and γ_5 is a polynomial in ε
The UV or IR divergences of Feynman integrals appear as poles $1/\varepsilon$

$$Tr(\gamma_\alpha \ldots \gamma_\mu \gamma_5) \times \int d^n k \frac{1}{[k^2 - m_0^2][(k+q_1)^2 - m_1^2][(k+q_2)^2 - m_2^2]} \sim (a_0 + a_1 \varepsilon + \ldots) \times \left(\frac{c_{-2}}{\varepsilon^2} + \frac{c_{-1}}{\varepsilon} + c_0 + \ldots \right)$$

If a_1 is evaluated in a non-consistent way,
then poles might not cancel and the finite part of the xsec might have a spurious contribution

- 't Hooft-Veltman treat γ_5 (anti)commuting in (4) $n-4$ dimensions preserving the cyclicity of the traces (one counterterm is needed)
- Kreimer treats γ_5 anticommuting in n dimensions, abandoning the cyclicity of the traces (→ need of a starting point)

- Heller, von Manteuffel, Schabinger verified that the IR-subtracted squared matrix element are identical in the two approaches

- we adopted the naive anticommuting prescription (Kreimer); we use $\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$ to compute traces with one γ_5
  - we computed the 2-loop amplitude and, independently, the IR subtraction term; both depend on the prescription chosen
  - the cancellation of all the lowest order poles is checked (and non trivial)
  - absence of fermionic triangles because of colour conservation

The double virtual amplitude: solution and evaluation of the Master Integrals

The system of first-order linear differential equations satisfied by the 36 QCD-weak Master Integrals chosen by Bonciani et al. can be written in dlog form

$$d\mathbf{I} = \epsilon \, d\mathbb{A} \, \mathbf{I} \,, \qquad d\mathbb{A} = \sum_{i=1}^{n} \mathbb{M}_i \; d\log \eta_i \,.$$

The letters η_i provide the complete information about the singular structure of the amplitude

Boundary Conditions

The BCs have been evaluated outside the physical phase space and are expressed in exact form

Master Integrals 1-31

When the letters have a rational (linear) expression, it is possible to integrate the system in terms of GPLs

The appearance, for kinematical reasons, of four square roots among the letters is handled with a change of variables that makes all the new letters linear, leading to a GPL solution in the new variables

Master Integrals 32-36

The appearance of another distinct square root among the letters, makes it impossibile to linearise the weights
→ the equations are formally solved with a Chen-Goncharov iterated representation

- the poles of these MIs contain Chen-Goncharov functions, but the latter cancel in the physical amplitude
- in the finite part, the Chen-Goncharov functions remain → problems to evaluate the amplitude in the physical region

Total cross section in the fiducial region

$G_\mu = 1.1663787 \times 10^{-5}\,\text{GeV}^{-2},\ M_W = 80.358\ \text{GeV}, \Gamma_W = 2.084\ \text{GeV}, M_Z = 91.1535\ \text{GeV}, \Gamma_Z = 2.4943\ \text{GeV}$

$M_H = 125.09\ \text{GeV},\ m_t = 173.07\ \text{GeV}$ NNPDF31_nnlo_as_0118_luxqed

$p_T^{\mu^\pm} > 25\,\text{GeV}, \quad |\eta^{\mu^\pm}| < 2.5, \quad m_{\mu^+\mu^-} > 50\,\text{GeV}, \quad \mu_R = \mu_F = M_Z$

σ [pb]	σ_{LO}	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	809.56(1)	191.85(1)	−33.76(1)	49.9(7)	−4.8(3)
qg	—	−158.08(2)	—	−74.8(5)	8.6(1)
$q(g)\gamma$	—	—	−0.839(2)	—	0.084(3)
$q(\bar{q})q'$	—	—	—	6.3(1)	0.19(0)
gg	—	—	—	18.1(2)	—
$\gamma\gamma$	1.42(0)	—	−0.0117(4)	—	—
tot	810.98(1)	33.77(2)	−34.61(1)	−0.5(9)	4.0(3)
$\sigma^{(m,n)}/\sigma_{\text{LO}}$		+4.2 %	−4.3 %	∼ 0 %	+0.5 %

Accidental cancellation of NLO-QCD and NLO-EW, small contribution from NNLO-QCD

→ the NNLO QCD-EW is comparable (or larger) in size than the combination of the previous orders

Differential distributions: exact vs approximated predictions

The exact $\mathcal{O}(\alpha\alpha_s)$ corrections allow to test the validity of different recipes based on NLO-QCD and NLO-EW results

factorised Ansatz

$$\frac{d\sigma_{fact}}{dX} = \frac{d\sigma^{(0,0)}}{dX}\left[1 + \frac{d\sigma^{(1,0)}}{dX}\left(\frac{d\sigma^{(0,0)}}{dX}\right)^{-1}\right] \times \left[1 + \frac{d\sigma^{(0,1)}}{dX}\left(\frac{d\sigma^{(0,0)}}{dX}\right)^{-1}\right]$$

$$\simeq \frac{d\sigma^{(0,0)}}{dX} + \frac{d\sigma^{(1,0)}}{dX} + \frac{d\sigma^{(0,1)}}{dX} + \frac{d\sigma^{(0,1)}}{dX}\frac{d\sigma^{(1,0)}}{dX}\left(\frac{d\sigma^{(0,0)}}{dX}\right)^{-1}$$

- the last term is absent in a purely additive formulation
- Factorisation is expected to work when both QCD and EW corrections factorise w.r.t. the gauge boson production
- the giant K-factors (qg and qγ processes) should not be applied to photon-induced channels

pole approximation

in the hard coefficient, the 2-loop virtual is approximated by $H_{PA}^{(1,1)} = \dfrac{2\mathrm{Re}(\mathcal{M}^{(1,1)}\mathcal{M}^{(0,0)*})_{PA}}{|\mathcal{M}_{PA}^{(0,0)}|^2}$

the 2-loop virtual corrections are evaluated in pole approximation
- on-shell Z boson form factor
- the resonant contributions of the γZ box diagrams cancel

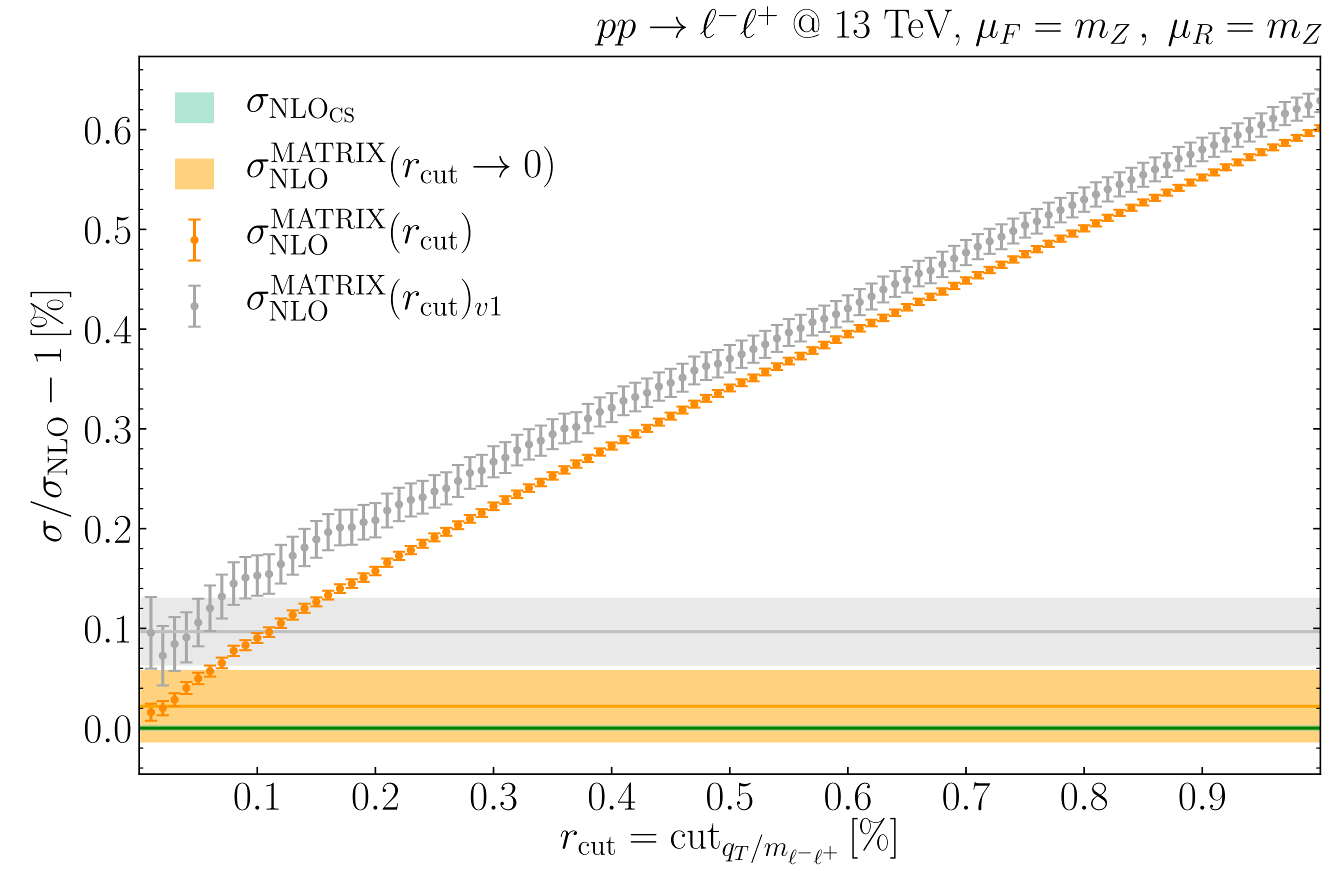
The q_T-subtraction and the residual cut-off dependency in different acceptance setups

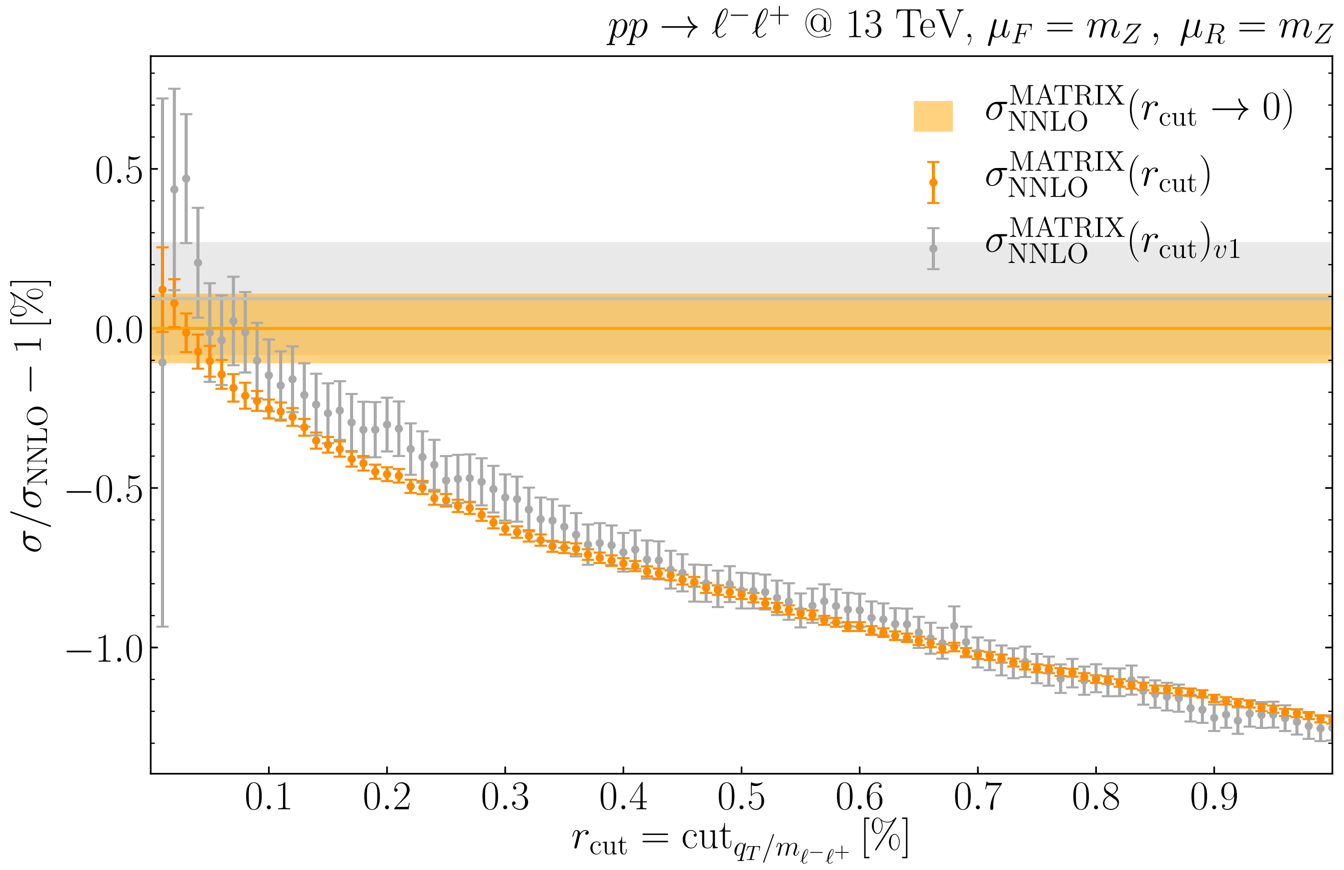
courtesy of S.Kallweit

(cfr. Buonocore, Kallweit, Rottoli, Wiesemann, 2111.13661)

Symmetric cuts

- $p_{T,\ell^\pm} > 25$ GeV

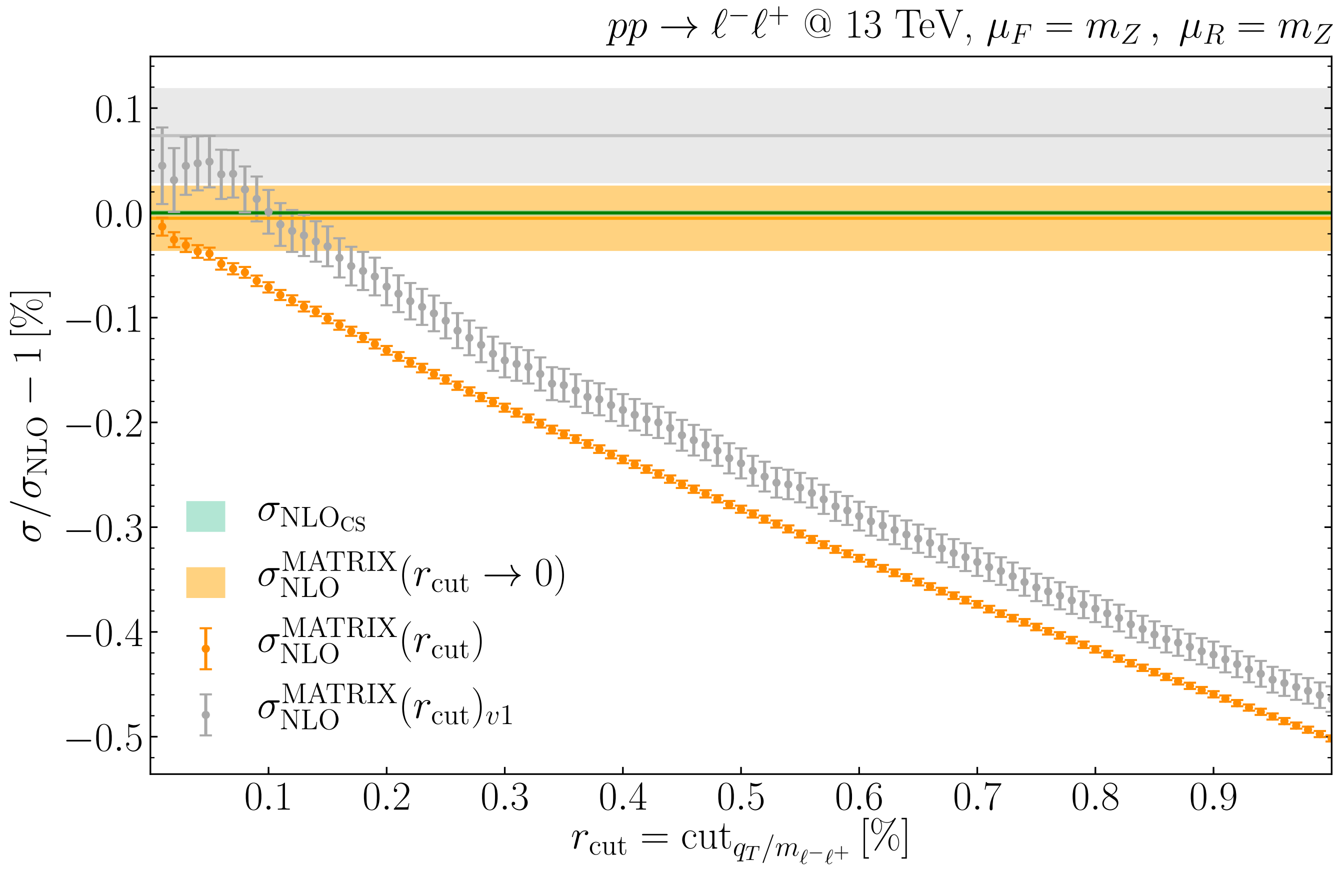


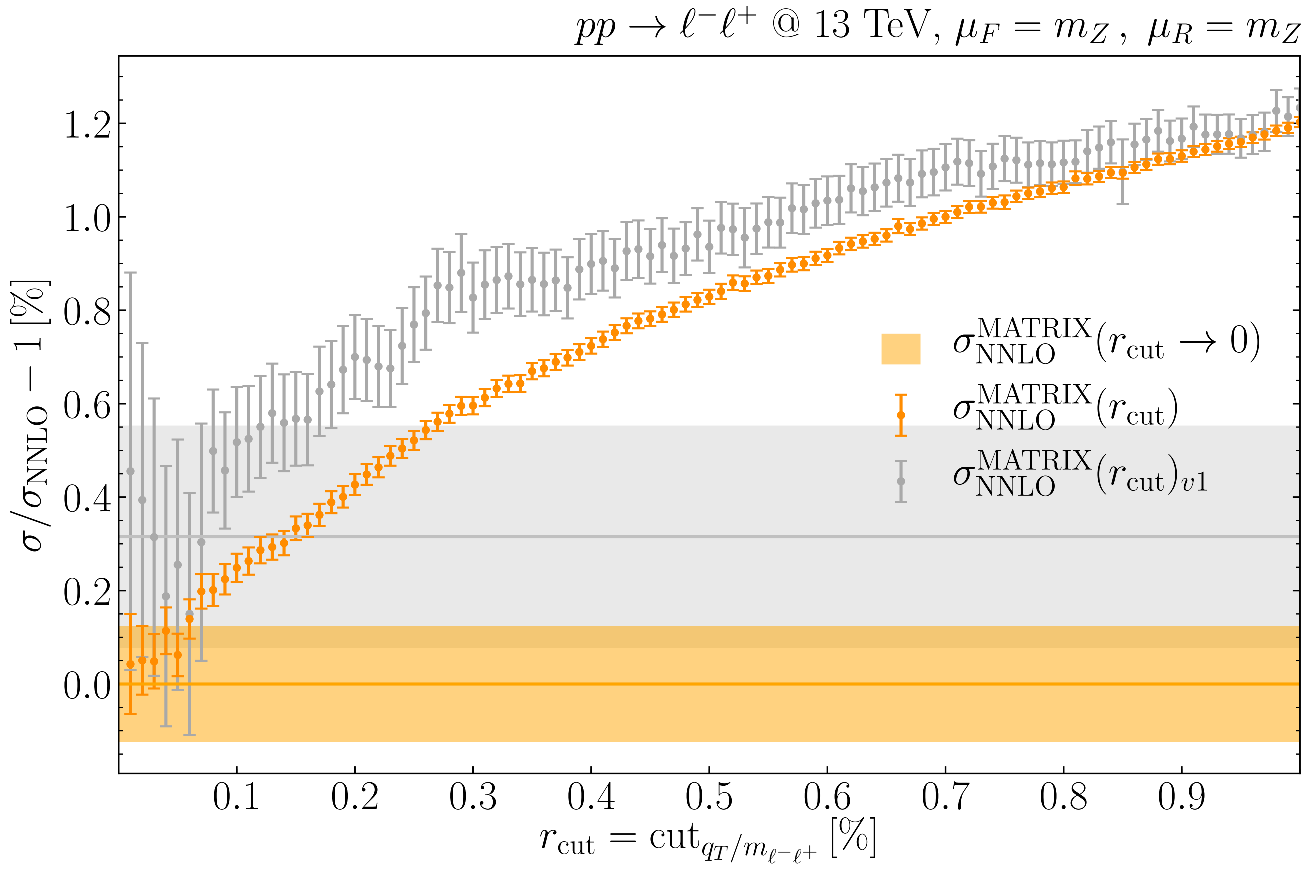


➡ large power corrections in r_{cut}

Asymmetric cuts on ℓ_1 and ℓ_2

- $p_{T,\ell_1} > 25$ GeV $\quad p_{T,\ell_2} > 20$ GeV

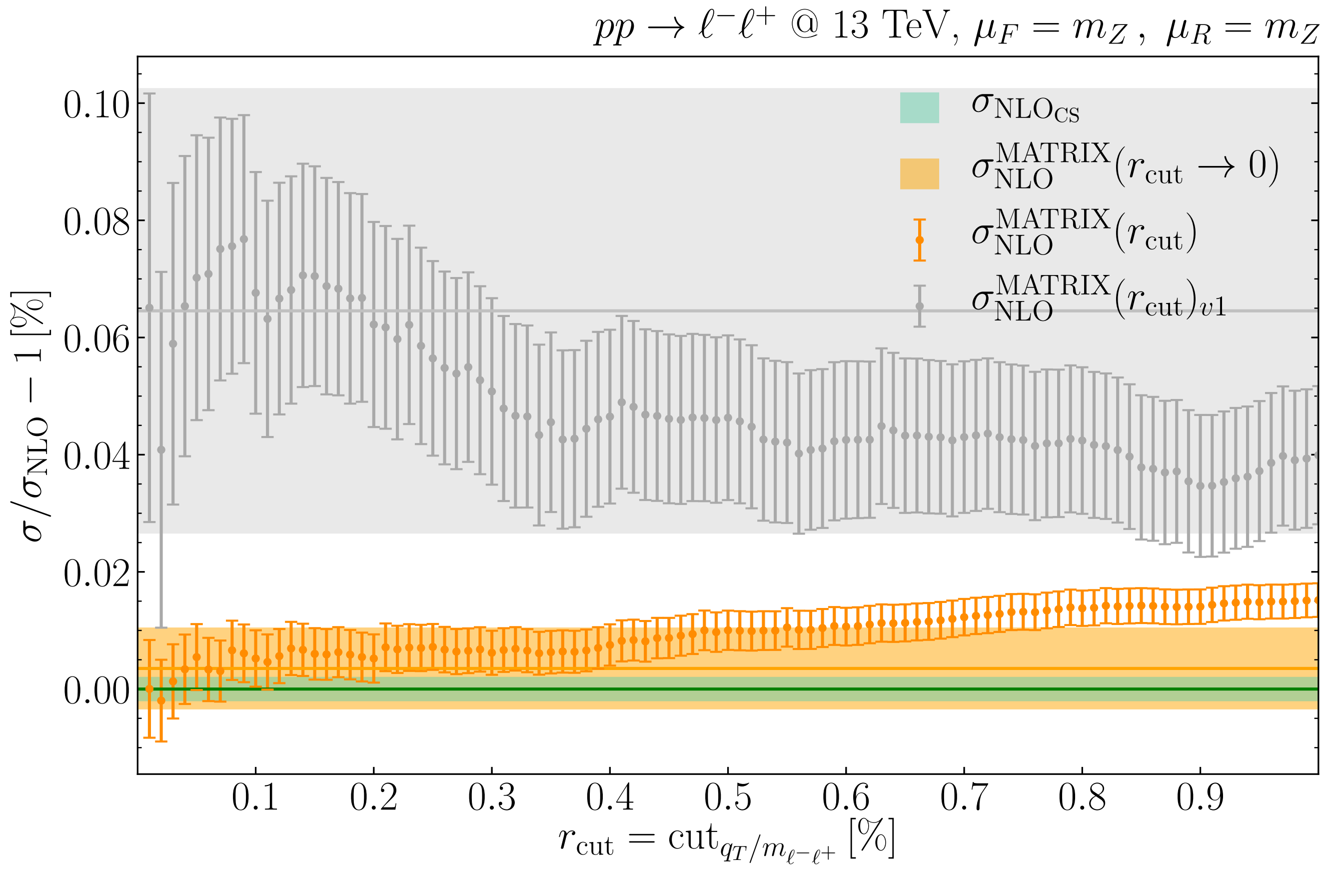


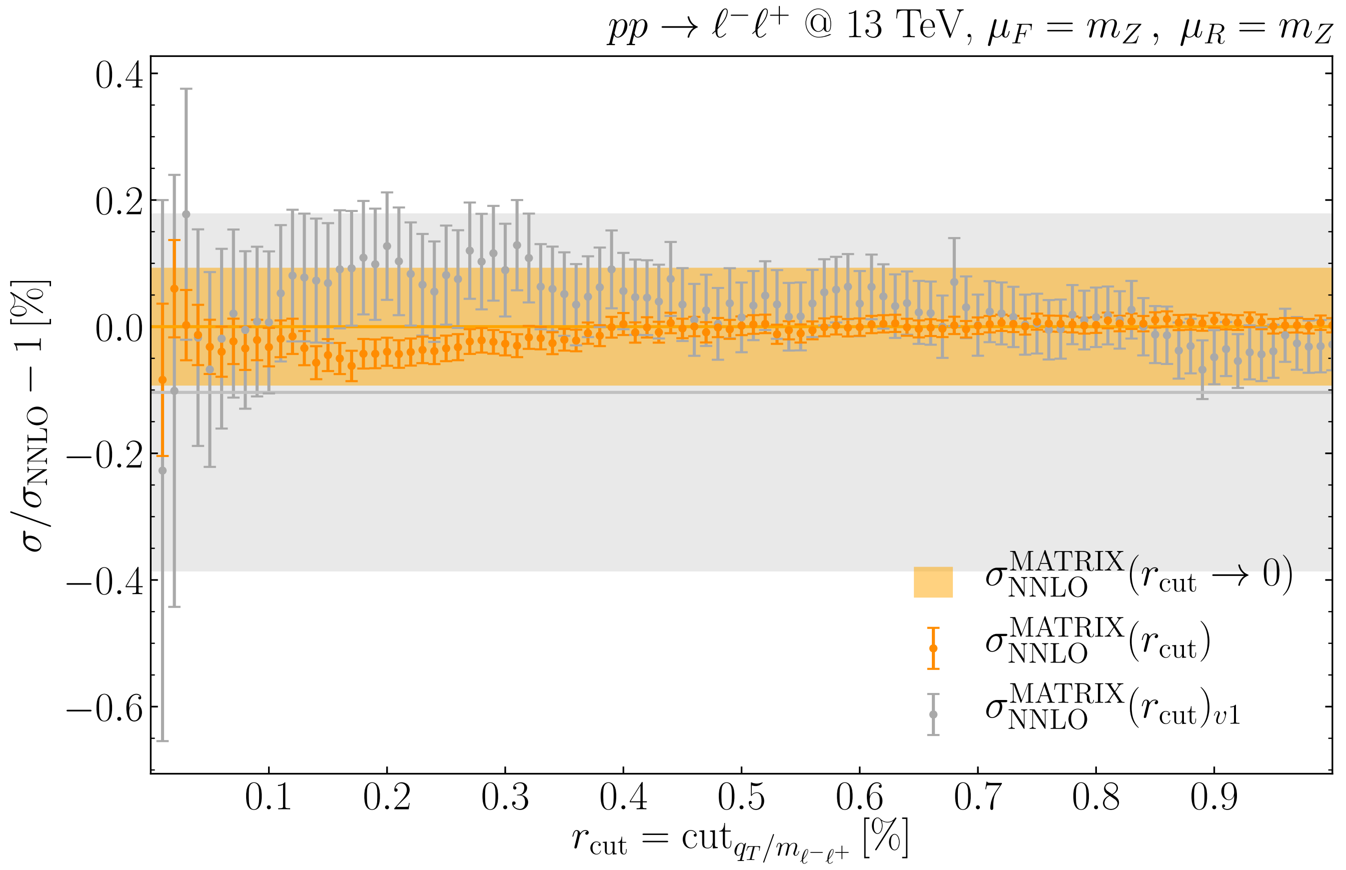


➡ large power corrections in r_{cut}

Asymmetric cuts on ℓ^+ and ℓ^-

- $p_{T,\ell^+} > 25$ GeV $\quad p_{T,\ell^-} > 20$ GeV



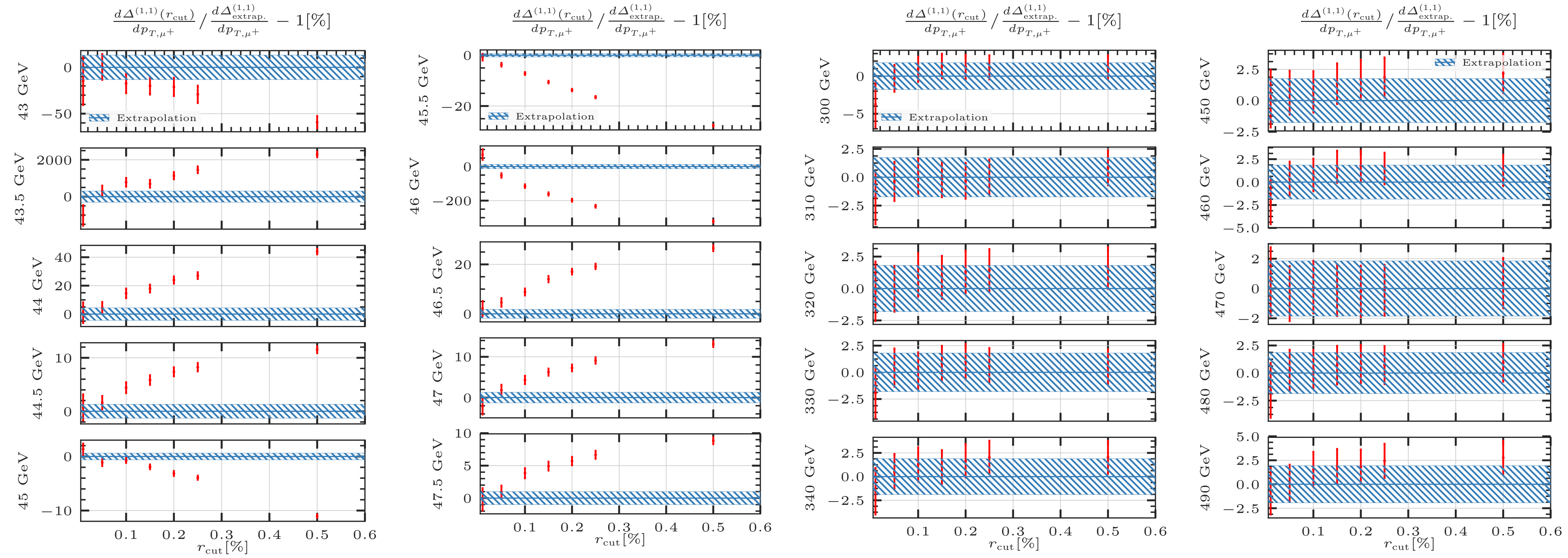


➡ no significant dependence on r_{cut}

Differential sensitivity to r_{cut}

Binwise $r_{\rm cut}$ dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

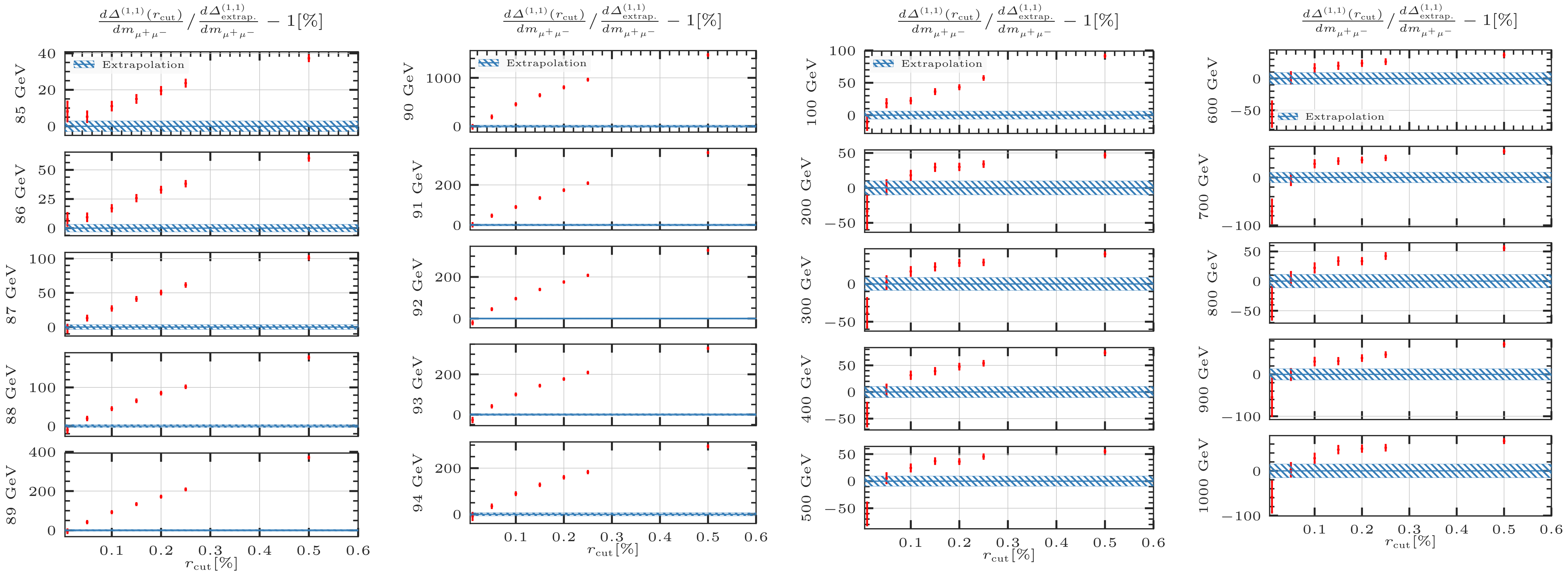
Differential distribution in p_{T,μ^+}: peak (left panels) and tail (right panels) regions



- large $r_{\rm cut}$ dependence in particular around the peak of the distribution, and typically precision of $\lesssim 3\%$ on the relative mixed QCD–EW corrections (artificially large where corrections are basically zero)

Binwise $r_{\rm cut}$ dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

Differential distribution in $m_{\mu^+\mu^-}$: peak (left panels) and tail (right panels) regions



- quite large $r_{\rm cut}$ dependence throughout, and lower numerical precision of $\lesssim 10\%$ on the relative mixed QCD–EW corrections (but still permille-level precision at the level of cross sections)