ON EXOTIC HADRONS COMPOSITION (THEORY OVERVIEW)

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Based on — Esposito, Maiani, Pilloni, ADP, Riquer *Phys. Rev. D* 105 (2022) 3, L031503
and — ADP, *Phys. Lett. B* 746 (2015) 248-250

INTRODUCTION

Since '03 more than 20 exotic hadron resonances have been discovered at e^+e^- and hadron colliders. They go under the name of tetraquarks, pentaquarks, hadron molecules, hybrids.

The problem about their composition is still debated, despite much progress has been made.

The first one of the series, and the most studied, is the $X(3872)$.

FEATURES OF $X(3872)$

1. $X(3872)$ has a mass precisely equal to $m_D + m_{D*}$
2. It is an extremely narrow state $\approx 1\text{MeV}$
3. Its strong decays in $J/\psi\,\rho$ and $J/\psi\,\omega$ violate isospin
4. It is produced in prompt hadron collisions with very high cross section and hard p_T cuts
5. It has been found in the X^0 neutral charge state only, for the moment (?)

Some interpretations given over the years:
Compact tetraquark, DD^* **hadron molecule (deuson)**, kinematical effect, hadrocharmonium, standard cahrmonium, Georgi's unparticle!

DEUTERONS & `DEUSONS`

Is there a way to tell from data if the deuteron is **elementary** (compact six quarks) or **composite** (a pn molecule)?

The *effective range* from np scattering amplitude is the discriminating observable [Weinberg '65].

For the $X(3872)$ mesonic deuteron there is no way of performing $D\bar{D}^*$ scatterings, but the resonance lineshape is well studied experimentally, and it encodes r_0.

ELEMENTARY AND COMPOSITE DEUTERON

$$H_0|\alpha\rangle = E|\alpha\rangle$$

(Continuum spectrum)

Let $|\alpha\rangle$ represent np pairs — c.o.m. at rest — with some relative momentum and with $J^P = 1^+$ (the deuteron).

$$H_0|m\rangle = E_m|m\rangle$$

(Bare elementary particle discrete spectrum)

$$\sum_m |m\rangle\langle m| + \int d\alpha\, |\alpha\rangle\langle\alpha| = 1$$

(Completeness relation)

See Weinberg Phys. Rev. 137, B672 (1965)

ELEMENTARY AND COMPOSITE DEUTERON

The physical normalized $|d\rangle$ deuteron state is

$$\underbrace{|d\rangle}_{\Psi} = \underbrace{\sqrt{Z}\,|\mathfrak{d}\rangle}_{\Psi_Q} + \underbrace{\int d\beta\, C_\beta\, |\beta\rangle}_{\Psi_P}$$

We assumed there is one elementary deuteron state $|\mathfrak{d}\rangle$, among the $|m\rangle$ states.

$$\int d\beta\, |C_\beta|^2 = 1 - Z$$

ELEMENTARY AND COMPOSITE DEUTERON: THE TWO EXTREME CASES

1) $Z = 0$; molecular case.

Include a potential V binding n with p such that

$$(H_0 + V_{np})\,|\,d\rangle = (M_{np} + \frac{\mathbf{k}^2}{2\mu} + V_{np})\,\Psi_P = \underbrace{(M_{np} - B)}_{M_d}\,\Psi_P$$

2) $Z = 1$; would name it fully elementary deuteron

$$H_0\,|\,d\rangle = H_0\,|\,\mathfrak{d}\rangle = M_d\,|\,d\rangle$$

(deuteron at rest)

ELEMENTARY AND COMPOSITE DEUTERON: MIXING

$$\begin{pmatrix} H_0 + V_{np} & H_{PQ} \\ H_{QP} & H_0 \end{pmatrix} \begin{pmatrix} \Psi_P \\ \Psi_Q \end{pmatrix} = M_d \begin{pmatrix} \Psi_P \\ \Psi_Q \end{pmatrix}$$

$Z \to 1 \Rightarrow \Psi_P = 0$ and requires $H_{PQ} = 0$.
The (fully) compact deuteron would not couple to pn!

$Z \to 0 \Rightarrow \Psi_Q = 0$ and requires $H_{PQ} = 0$.
The deuteron is a molecule and simply there is no compact state to couple to.

THE MEANING OF Z

The case $Z = 1$ is somewhat singular and the statement made about an elementary deuteron is

> … we have an elementary $\mathfrak{d}$ for **every** value of Z such that $0 < Z < 1$, the $Z = 0$ case being **the only** molecular case.

Is it possible then to extract Z from data?

See Weinberg Phys. Rev. 137, B672 (1965)

WEINBERG & DEUTERON (1965)

Weinberg finds, for *shallow bound states*, a relation between Z and a quantity known in low energy scattering theory as the *effective range* r_0 (Schwinger)

$$r_0 = -\frac{Z}{1-Z}R + O(\frac{1}{m_\pi})$$

$$R = \frac{1}{\sqrt{2mB}}$$

The "molecule" has $Z = 0$ thus $r_0 = O(1/m_\pi)$.

What is the sign of the unknown corrections?

See Weinberg Phys. Rev. 137, B672 (1965)

A THEOREM ON SHALLOW BOUND STATES IN QM

BETHE ('49), LANDAU-SMORODINSKY ('48)

$$\mathbf{r_0 > 0}$$

(indeed $r_0 = +1.74$ fm for deuteron)

This is a general theorem, together with case by case analyses (see e.g. Blatt & Weisskopf).

This agrees with original Weinberg's $Z = 0$ molecule:

"…an elementary deuteron would have $0 < Z < 1$"

"The true token that the deuteron is composite is an r_0 small and positive rather than large and negative "

"…an elementary deuteron would entail a large and negative r_0"

Esposito et al. 2108.11413

BETHE-LANDAU-SMORODINSKY

$$r_0 = -\frac{Z}{1-Z}R + O(\frac{1}{m_\pi})$$

Fix the sign of corrections by the exact relation

$$r_0 = 2\int_0^\infty (\psi_0^2 - u_0^2)\,dr > 0$$

Where u_0 is the solution to the S. equation with V and $k = 0$, whereas ψ_0 is the solution to the free equation, with $k = 0$.

For a proof see Esposito et al. 2108.11413

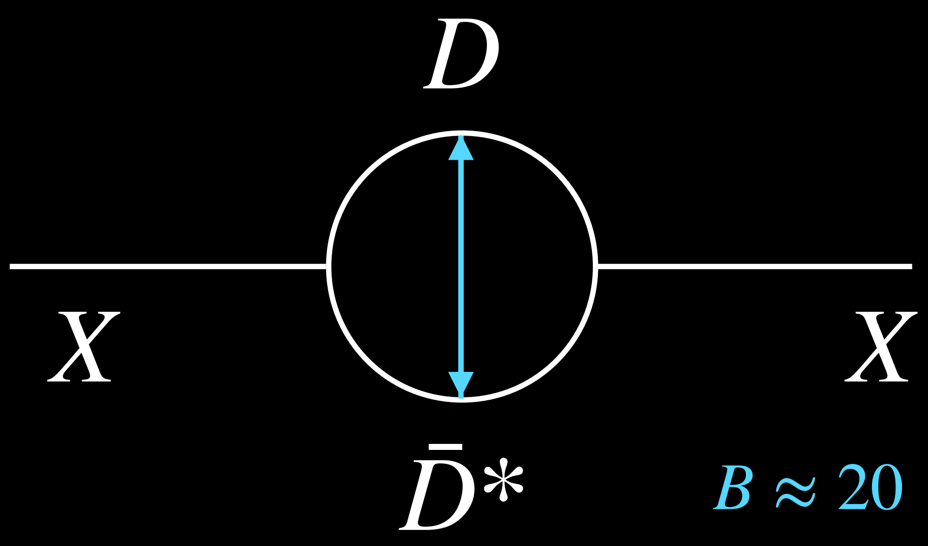
LHCB (2020)

arXiv:2005.13419

Allows to compute the effective range r_0 for the $X(3872)$. This was dubbed as the "deuson", a $D\bar{D}^*$ mesonic molecule analogue of deuteron: a viable option **iff** $Z = 0$ or $r_0 > 0$ and $O(1/m_\pi)$.

However we find $r_0 = -5.43$ fm and $|r_0| > 1/m_\pi$!

"…an elementary deuteron would entail a large and negative r_0"



$B \approx 20$ KeV

Esposito et al. 2108.11413

SCATTERING AMPLITUDE

$$f = \frac{1}{k\cot\delta(k) - ik} = \frac{1}{-1/a + \frac{1}{2}r_0k^2 - ik + \dots}$$

Compares with NR-BW formula

$$f = -\frac{\frac{1}{2}g_{\text{BW}}^2}{E - m_{\text{BW}} + \frac{i}{2}g_{\text{BW}}^2 k}$$

$$g_{\text{BW}}^2 = -\frac{2}{\mu r_0} \quad m_{\text{BW}} = \frac{1}{a\mu r_0} \quad E = \frac{k^2}{2\mu}$$

Esposito et al. [2108.11413]()

A FORMULA FOR r_0

We find

$$r_0 = -\frac{2}{\mu g_{\text{Flatte}}} - \sqrt{\frac{\mu'}{2\mu^2\delta}} = -5.34 \text{ fm}$$

Where μ' is the reduced mass of the charged open charm pair, μ of the neutral and δ is

$$\delta = m_{D^+} + m_{D^{*-}} - m_{D^0} - m_{\bar{D}^{*0}}$$

Esposito et al. 2108.11413

FINITE $Z \gtrsim 10\,\%$

Measuring a finite, although ``small`` Z

$\Rightarrow$

the state is elementary — the only conclusion that can be safely formulated.

The meson component is the `dressing` of the state.

Esposito et al. 2108.11413

THIS CONCLUSIONS IS DEBATED.

This conclusion has been disputed in a recent paper by C. Hanhart and collaborators 2110.07484.

Their approach is: we measure a `small` Z (even if for X and the new tetraquark this means $14\% \div 30\,\%$). That means that the state is essentially a molecule and marginally a compact quark state.

$\pi-$EXCHANGE IN X

Given that the potential is the FT of the propagator in the no-recoil approximation

$$-\frac{4g^2}{f_\pi^2}\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2-\mu^2-i\epsilon}\, d^3q = -\frac{4g^2}{3f_\pi^2}\left(\delta^3(r)+\mu^2\frac{e^{i\mu r}}{r}\right)\delta_{ij}$$

to be contracted with polarizations $e_i^{(\alpha)}(p_1)\ \bar{e}_j^{(\beta)}(k_2)$

$$\mu^2 = (m_{D*}-m_D)^2 - m_\pi^2 \approx 40 \text{ MeV}$$

[Esposito, Glioti, ADP, Rattazzi, work in progress]

FIELD THEORY DESCRIPTION

In the field theory description the $\delta^3(r)$ potential (which might have bound states) corresponds to

$$\lambda(\phi^\dagger\phi)^2$$

where ϕ are the fields for $D^0, \bar{D}^{*0}$ particles, whereas H_{PQ} corresponds to

$$g(\psi^\dagger\phi^2 + (\phi^\dagger)^2\psi)$$

coupling to the elementary field ψ of the X.

See T. Kinugawa and T. Hyodo 2112.00249

The negative r_0 originates in the coupling to the (bare) field ψ
What is the role of the complex potential in $D\bar{D}^*$?

FIELD THEORY DESCRIPTION

There could also be another 4-linear term contributing to r_0

$$\rho \nabla(\phi^\dagger \phi) \cdot \nabla(\phi^\dagger \phi)$$

however in the NR limit, dimensional analysis tells

$$g \sim \frac{1}{\sqrt{v}} \quad \lambda \sim v \quad \rho \sim v^3$$

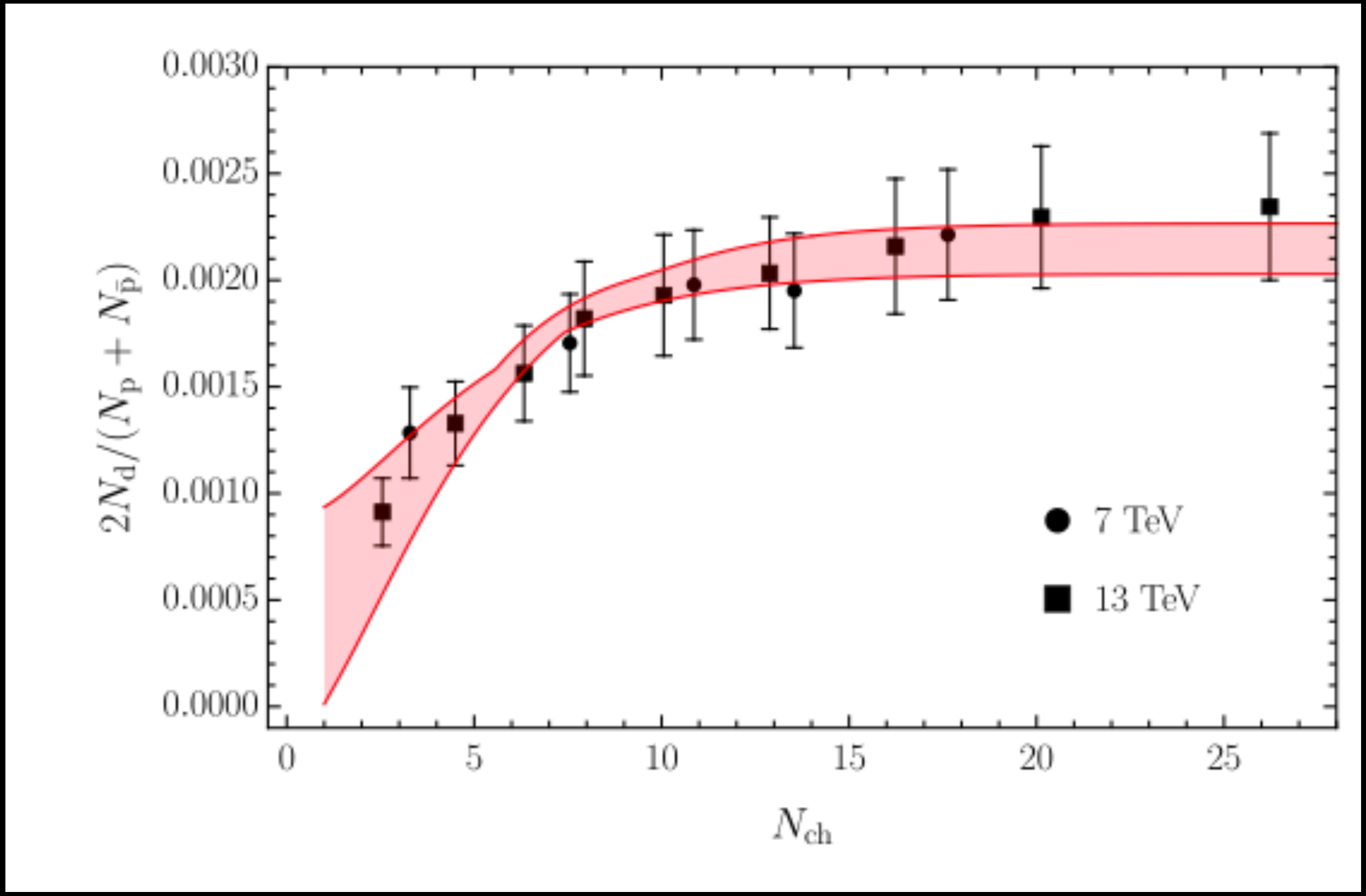
with $v \to 0$

See T. Kinugawa and T. Hyodo 2112.00249

DOES THE X BEHAVE LIKE A DEUTERON?

DEUTERON FROM ALICE

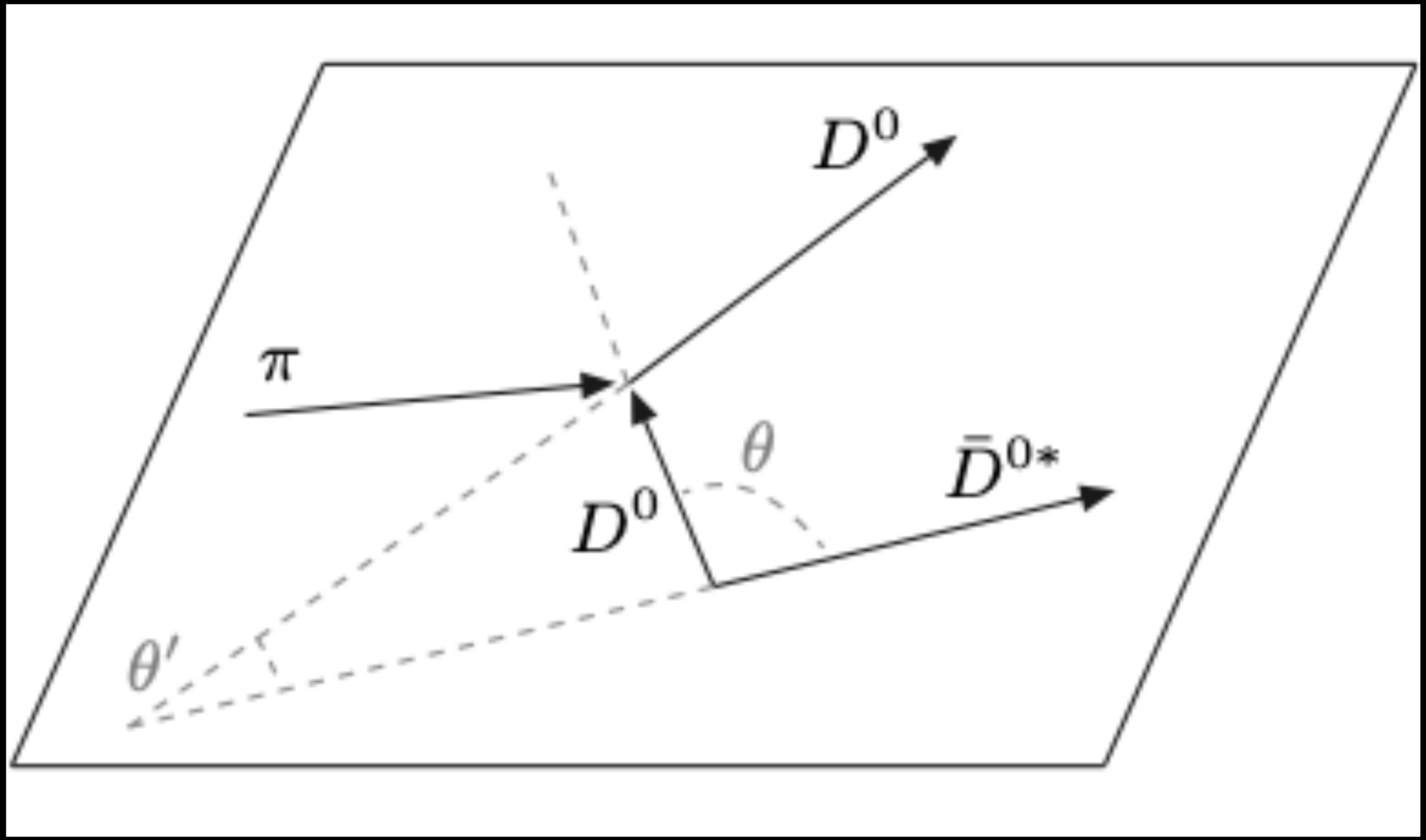
ALICE: 1902.09290; 2003.03184



Esposito, Ferreiro, Pilloni, ADP, Salgado *Eur. Phys. J. C* 81 (2021) 669

Numbers of molecules as a function of multiplicity, computed with Boltzmann eq. in a coalescence model.

COALESCENCE MODEL

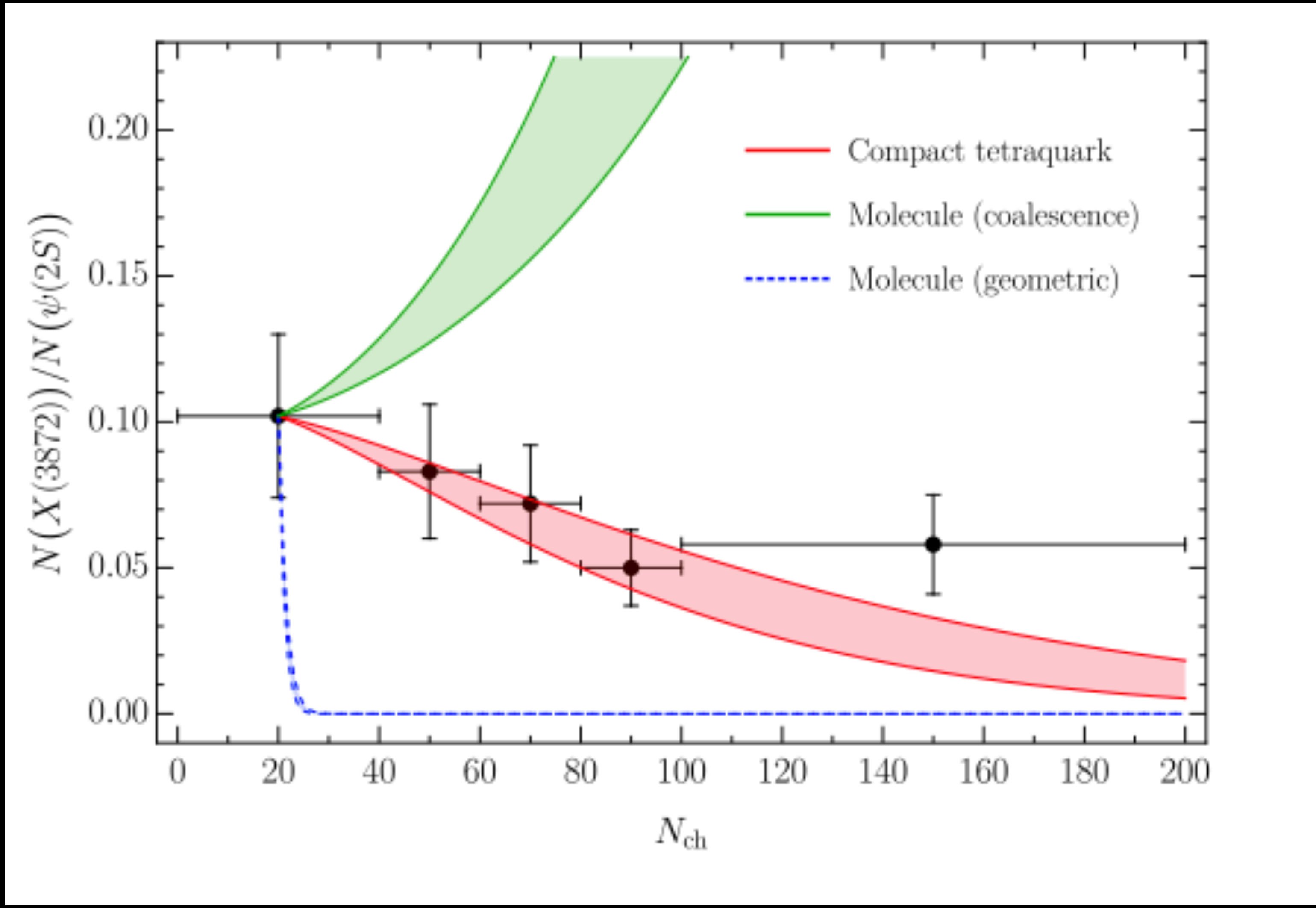


Esposito, Piccinini, Pilloni, ADP, *J. Mod. Phys.* 4 (2013) 1569-1573
Guerrieri, Piccinini, Pilloni, ADP, *Phys. Rev. D* 90 (2014) 3, 034003

In final states of hadron collisions, the would-be molecule constituents have large $(k > \Lambda)$ relative momenta and, after an interaction with a GeV comovers, the prob. of falling within $k < \Lambda$ is small. On the other hand a bound pair is most likely broken.

RECENT IMPLICATIONS

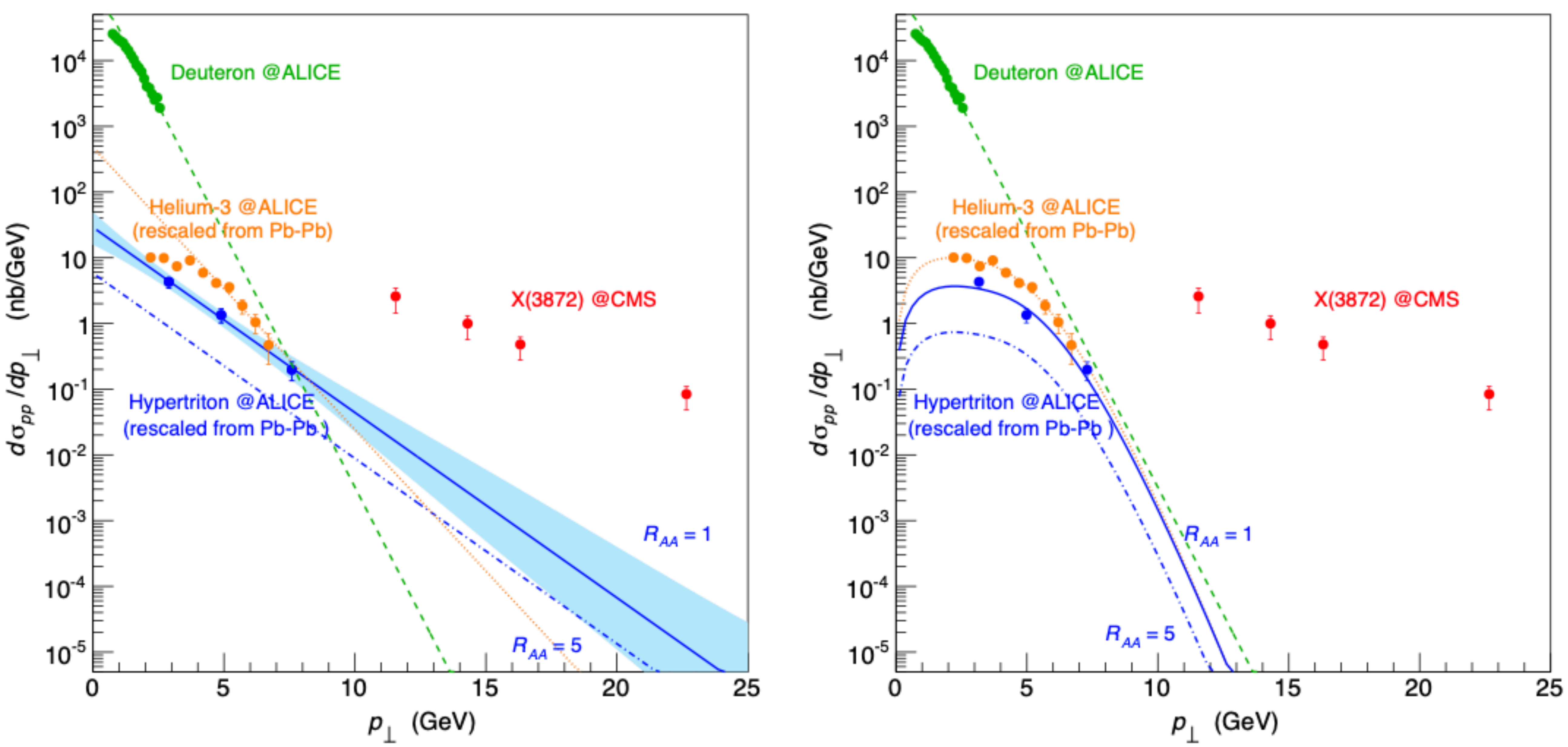
LHCb:2009.06619



Esposito, Ferreiro, Pilloni, ADP, Salgado *Eur. Phys. J. C* 81 (2021) 669

The coalescence picture predicts a behavior (green band) qualitatively different from data.

FROM MULTIPLICITY TO PT



Esposito, Guerrieri, Maiani, Piccinini, Pilloni, ADP, Riquer, *Phys. Rev. D* 92 (2015) 3, 034028

THE MOST RECENT TETRAQUARKS

DIQUARK-ANTIDIQUARK

$$X(1^{++}) = [cq][\bar{c}\bar{q}] = \frac{1}{\sqrt{2}}(|1,0\rangle_1 + |0,1\rangle_1) \qquad X(3872)$$

$$X(1^{+-}) = [cq][\bar{c}\bar{q}] = \frac{1}{\sqrt{2}}(|1,0\rangle_1 - |0,1\rangle_1) \qquad Z(3900)$$

$$X'(1^{+-}) = [cq][\bar{c}\bar{q}] = \frac{1}{2\sqrt{2}}(|1,1\rangle_1) \qquad Z(4020)$$

Here $q = u, d$. What about strange quarks?

Maiani, Piccinini, ADP, Riquer, Phys. Rev. D71, 014028 (2005); D89 114010 (2014); PLB778, 247 (2018)

THE EQUAL SPACING RULE

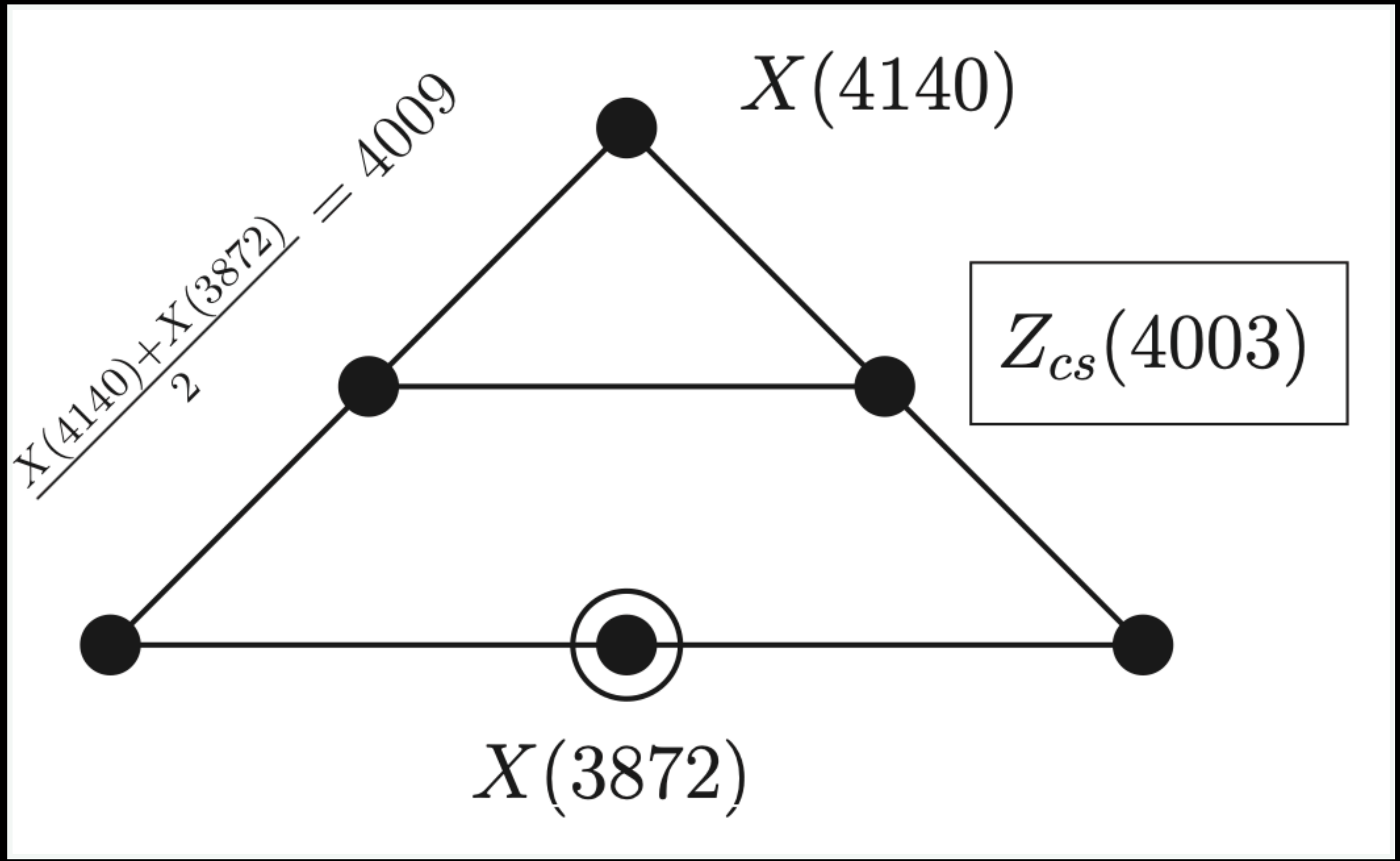
$$K^* \approx (\phi + \rho)/2$$

$$X(1^{++}) = [cs][\bar{c}\bar{s}] \qquad X(4140)$$

To first order of SU(3) flavor symmetry breaking we predict

$$Z_{cs} \stackrel{!}{=} (X(4140) + X(3872))/2 = 4009 \text{ MeV}$$

Spacing = 275 MeV
wrt 244 MeV for $\phi - \rho$



$[cs][\bar{c}\bar{q}] \vee [cq][\bar{c}\bar{s}]$

Maiani, ADP, Riquer
arXiv:2103.08331
Sci. Bulletin 66, 1616 (2021)

LHCB (2021)

The $Z_{cs}(4003)$ was observed by LHCb in the decay

$$B^+ \rightarrow \phi + Z_{cs}^+(4003) \rightarrow \phi + K^+ + J/\psi$$

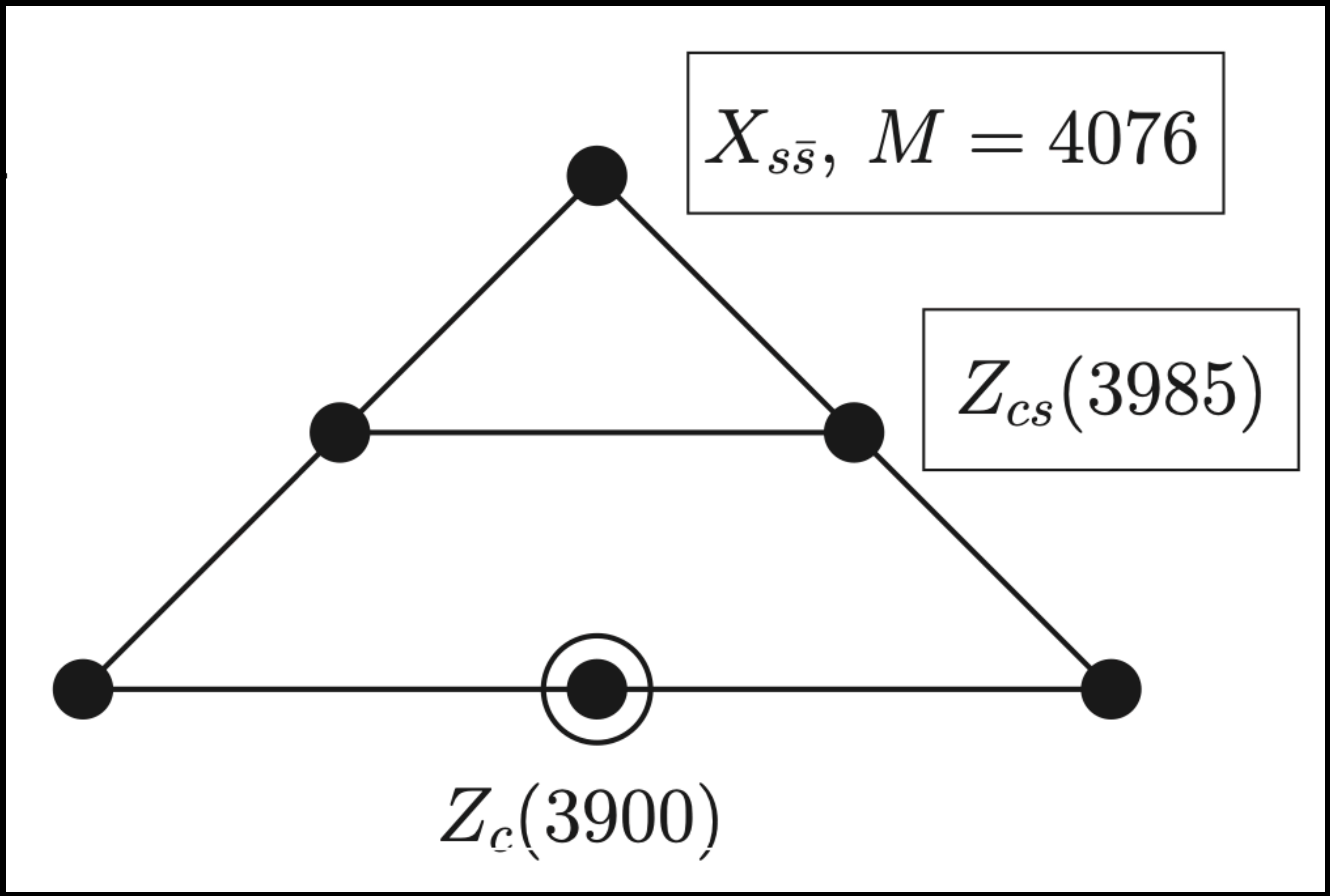
At a mass very close to that given by the equal spacing rule

NEGATIVE CHARGE CONJUGATION

The diquark-antidiquark model requires the quasi-degeneracy $M(X(1^{++})) = M(Z(1^{+-}))$ and we expect a similar multiplet for 1^{+-}

Spacing = 188 MeV
wrt 200 MeV for $f_2' - a_2$

Maiani, ADP, Riquer
arXiv:2103.08331
Sci. Bulletin 66, 1616 (2021)



Prediction

$[cs][\bar{c}\bar{q}] \vee [cq][\bar{c}\bar{s}]$

BESIII recently observed $e^+e^- \to K^+Z_{cs}^-(3985) \to K^+(D_s^{*-}D^0 + D_s^-D^{*0})$

CONCLUSIONS

- It is always mentioned that symmetry arguments predict 'too many' states. However a particle zoo was identified — not expected in molecular models. Recently a new kind of tetraquark $[cc][qq']$ has been reported.

- In quark models the vicinity to threshold, from below and from above, is natural — not for molecules.

- Vicinity to threshold does not necessarily mean loosely bound state of hadrons.

- **Independent measurements of r_0 in $X(3872)$ would be crucial to address the compositeness `vexata questio`.**

CONCLUSIONS

$X(3872)$	$Z_c^{0\pm}(3900)$	$Z_c^{0\pm}(4020)$	$Z_b^{0\pm}(10610)$	$Z_b^{0\pm}(10650)$
$D^0\bar{D}^{*0}$	$D^0\bar{D}^{*0\pm}$	$D^{*0}\bar{D}^{*0\pm}$	$B^0\bar{B}^{*0\pm}$	$B^{*0}\bar{B}^{*0\pm}$
$\delta \approx 0$	$+7.8$	$+6.7$	$+2.7$	$+1.8$

(MeV)

ADDITIONAL SLIDES

ELEMENTARY AND COMPOSITE DEUTERON

Introduce projection operators $P^2 = P$ and $Q^2 = Q$, $PQ = QP = 0$, $P + Q = 1$ and $P\Psi = \Psi_P$, $Q\Psi = \Psi_Q$

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad Q = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Psi = \begin{pmatrix} \Psi_P \\ \Psi_Q \end{pmatrix}$$

ELEMENTARY AND COMPOSITE DEUTERON

$$H\Psi = M_d\Psi$$

$$H(\Psi_P + \Psi_Q) = HP\Psi_P + HQ\Psi_Q$$

so that if $H_{PP} \equiv PHP\ldots$

$$\boxed{\begin{aligned} H_{PP}\Psi_P + H_{PQ}\Psi_Q &= M_d\Psi_P \\ H_{QP}\Psi_P + H_{QQ}\Psi_Q &= M_d\Psi_Q \end{aligned}}$$

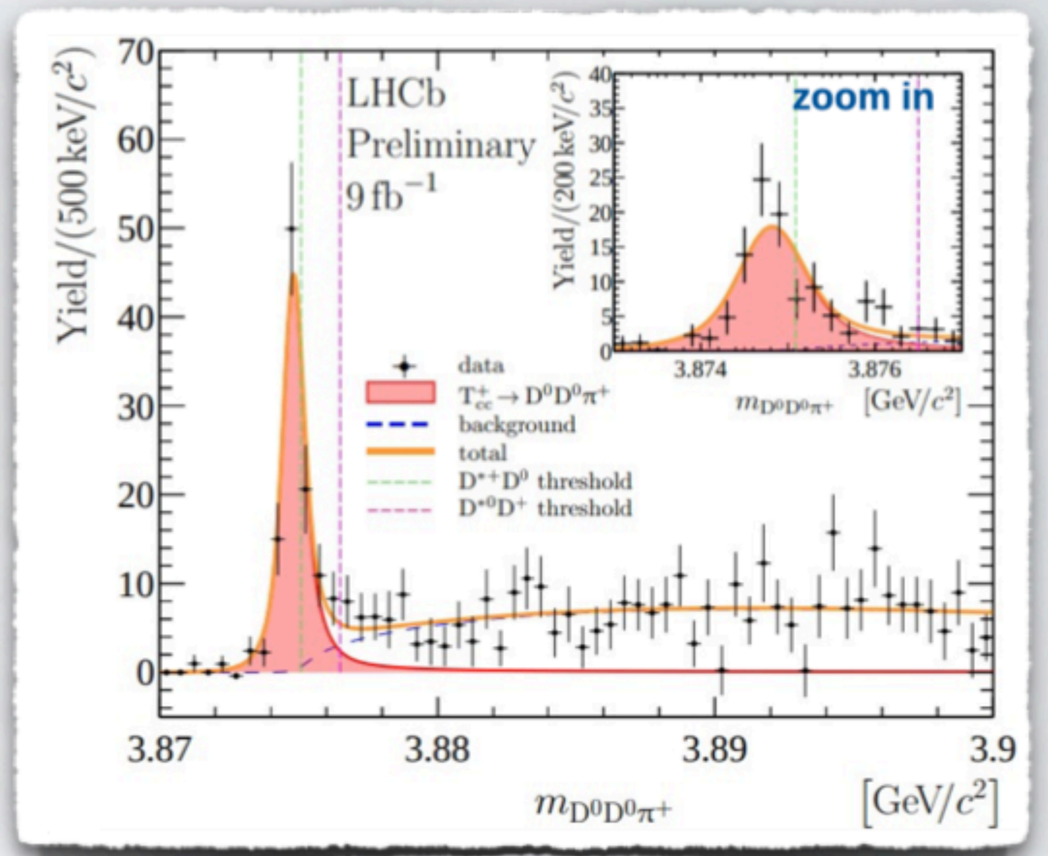
AN EMERGING PATTERN

Doubly charmed

- LHCb observed the first doubly charmed tetraquark $T^+_{cc}(cc\bar{u}\bar{d}) \to D^0 D^0 \pi^+$

- These states have been predicted/studied in several works

[see e.g. del Fabbro et al. — PRD (2005), hep-ph/0408258; Carames et al. — PLB (2011); Hyodo et al. — PLB (2013), 1209.6207; AE et al. — PRD (2013), 1307.2873]



[LHCb preliminary — talk presented at EPS-HEP 2021]

- Important for two reasons:

  1. Should be paired with a T_{bb} state far away from threshold

  2. $SU(3)$ multiplet contains doubly-charged states ⟶ cannot be meson molecules because of Coulomb repulsion

LINESHAPE OF THE $X(3872)$

The LHCb data

- To apply the composite criterion we must set $\Gamma_\rho^0 = \Gamma_\omega^0 = \Gamma_0^0 = 0$:

$$f\big(X \to J/\psi \pi^+\pi^-\big) \simeq -\frac{N\frac{2}{g_{LHCb}}}{\frac{2}{g_{LHCb}}\big(E - m_X^0\big) - \sqrt{2\mu_+\delta} + E\sqrt{\frac{\mu_+}{2\delta}} + ik}$$

- From here we extract

$$\boxed{r_0 = -\frac{2}{\mu g_{LHCb}} - \sqrt{\frac{\mu_+}{2\mu^2\delta}} \simeq -5.34 \text{ fm}}$$

[Esposito, Maiani, Pilloni, Polosa, Riquer — 2108.11413]

- The effective range is negative and well beyond m_π^{-1} $\longrightarrow$ the dynamics can only be explained if the X is an elementary, interacting tetraquark

POINTLIKE/COMPOSITE PARTICLES

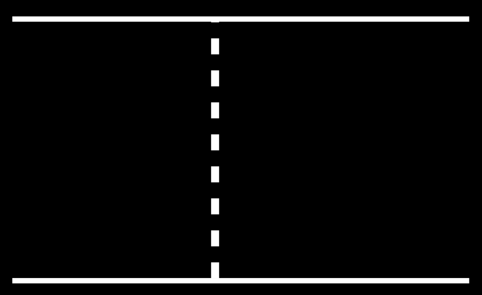
If a pointlike particle is hit by another pointlike particle, the only effect is that its momentum will change and the strength of the interaction is insensitive to the exchanged momentum.

However if the target is composite, a charge distribution in space may result, and the coupling is $\mathbf{k}$ dependent differently from the constant e coupling

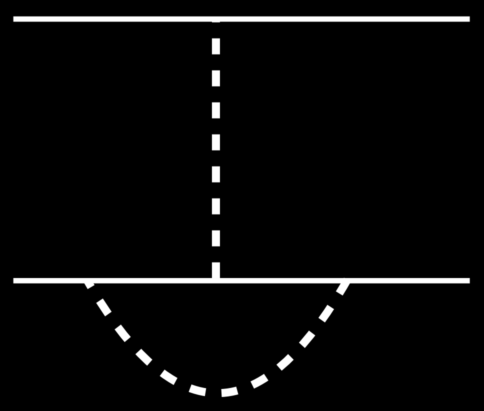
$$\delta\phi(P) = \int d^3k\, e^{i\mathbf{k}\cdot\mathbf{r}} \frac{F(\mathbf{k})}{\mathbf{k}^2}$$

with $F(\mathbf{k})$ in place of e (which is obtained if $\rho(\mathbf{r}) = e\delta^3(\mathbf{r})$)

QUANTUM LOOPS FORM FACTORS



The interaction strength depends only on g (cubic coupling)



The interaction strength is a function of the exchanged momentum as a result of the loop. A *composite system of virtual* particles.

Particles are composite if their interaction with probes depends on momentum.

Then, due to quantum fluctuations, all particles are composite!

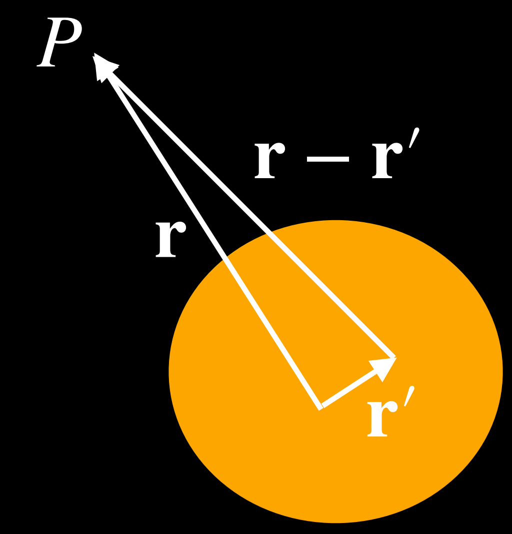
What is the meaning of elementary?

FORM FACTORS & COMPOSITENESS

$$\delta\phi(P) = \int d^3r' \frac{\rho(\mathbf{r}')}{4\pi|\mathbf{r}-\mathbf{r}'|} = \int d^3r' \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\int d^3k\, e^{i\mathbf{k}\cdot\mathbf{r}'} F(\mathbf{k})$$

$$= \int d^3k\, F(\mathbf{k}) \int d^3\mathbf{r}' \frac{e^{i\mathbf{k}\cdot\mathbf{r}'}}{4\pi|\mathbf{r}-\mathbf{r}'|} = \int d^3k\, e^{i\mathbf{k}\cdot\mathbf{r}} \frac{F(\mathbf{k})}{\mathbf{k}^2}$$

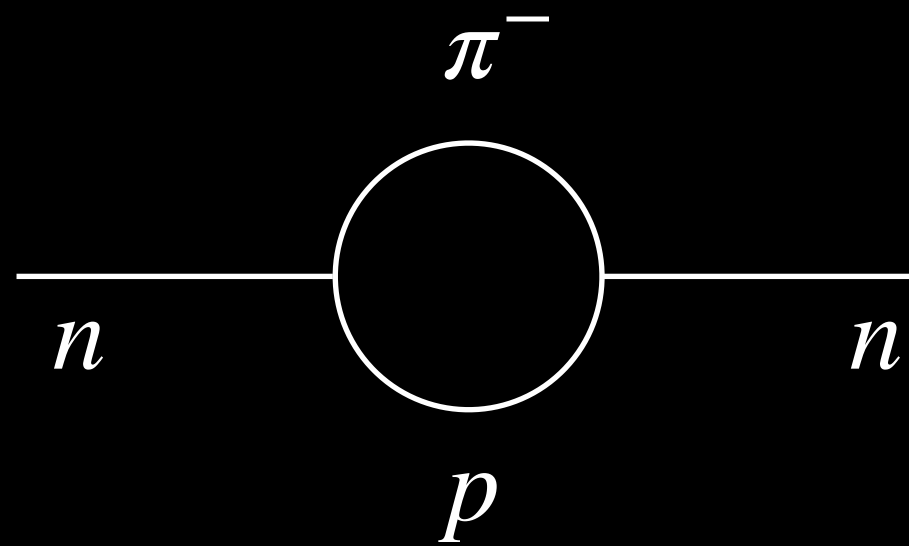
$$= \int d^4x'\, i\Delta(x-x')\, J(x') \text{ with } J(x') \equiv \rho(\mathbf{r}')$$



$F(\mathbf{k})$ in place of $e \Rightarrow$

the strength of the inter. depends on $\mathbf{k}$

THE $\pi^- p$ MOLECULE EXAMPLE



$$|n, \mathrm{in}\rangle = \sqrt{Z}\,|n, \mathrm{bare}\rangle + \int_{\mathbf{k}} \Psi_\pi(\mathbf{k})\,|p\,\pi^-(\mathbf{k}), \mathrm{bare}\rangle$$

$$\int_{\mathbf{k}} |\Psi_\pi(\mathbf{k})|^2 = 1 - Z$$

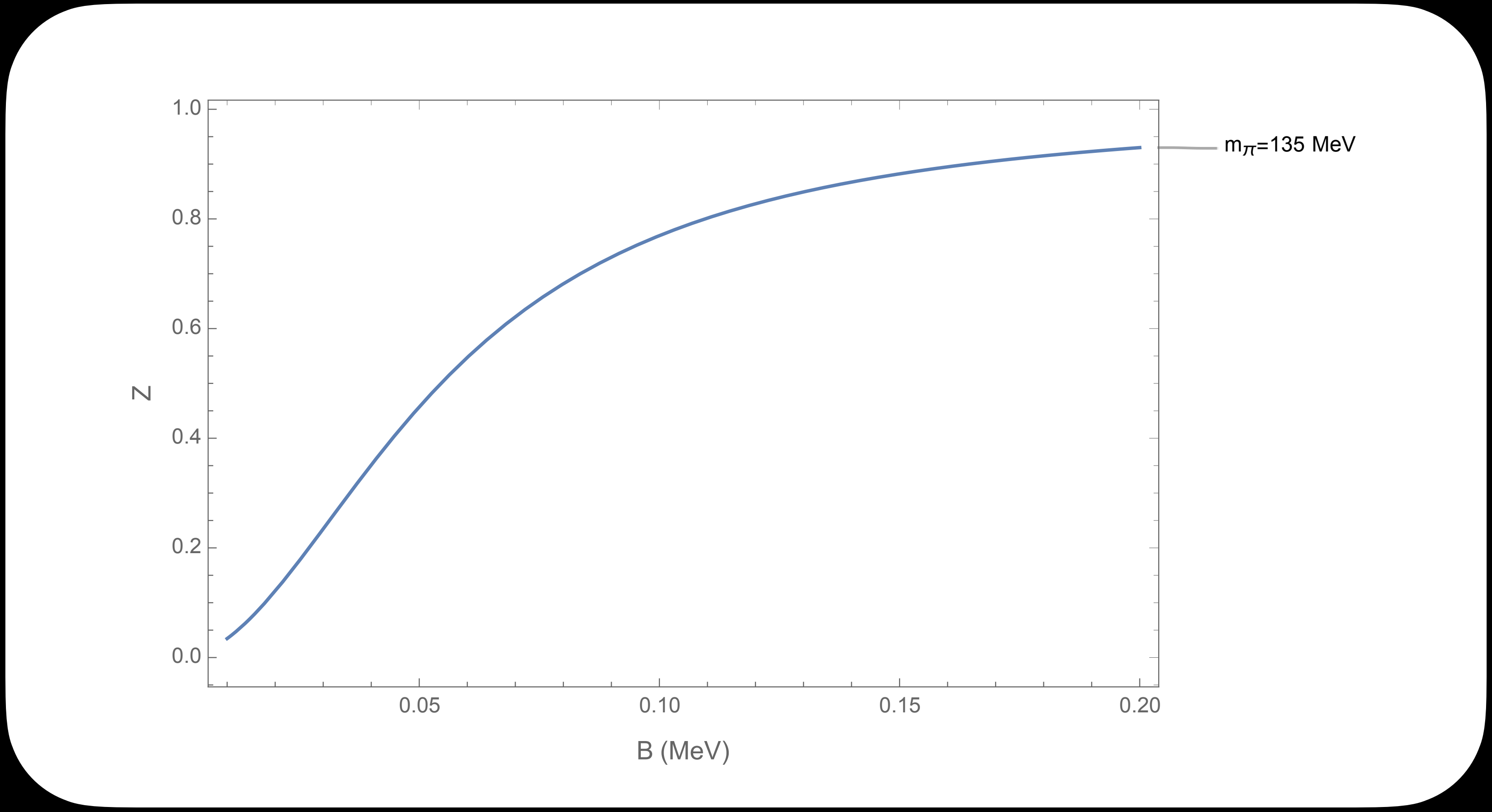
Same equations as Weinberg's

See the "Lee-model" ('54) in Henley & Thirring, Elementary Quantum Field Theory, McGraw-Hill
T.D. Lee, Phys. Rev. 95, 1329 (1954)

THE $\pi^- p$ MOLECULE EXAMPLE

$$Z^{-1} = 1 + \left(\frac{M_p}{F_\pi} g_A\right)^2 \frac{1}{2\pi^2} \int_0^\infty \frac{k^2}{(\sqrt{2m_\pi})^2 (B + \frac{k^2}{2(M_p + m_\pi)})^2 (1 + \frac{k^2}{M_A^2})^4} dk$$

Tune the mass of the neutron, i.e. tune B



See the "Lee-model" ('54) in Henley & Thirring, Elementary Quantum Field Theory, McGraw-Hill

T.D. Lee, Phys. Rev. 95, 1329 (1954)

NB: THE DERIVATION OF $r_0(Z)$ REQUIRES AT SOME STEP THAT $\mathscr{E} = -B$ (STRICTLY VALID IF $Z = 0$ OR $H_{PQ} = 0$).

$$|d\rangle = \sqrt{Z}\,|\mathfrak{d}\rangle + \int d\beta\, C_\beta\, |\beta\rangle \text{ with } \int d\beta\, |C_\beta|^2 = 1 - Z$$

$$\Rightarrow \int d\alpha\, |\langle\alpha|d\rangle|^2 = 1 - Z$$

then use

$$(H_0 + V)\,|d\rangle = \mathscr{E}\,|d\rangle$$

$$\Rightarrow \int d\alpha\, \frac{|\langle\alpha|V|d\rangle|^2}{E(\alpha) + B} = 1 - Z \text{ if } \mathscr{E} = -B$$

From here obtain a formula for $g \equiv |\langle\alpha|V|d\rangle|$;

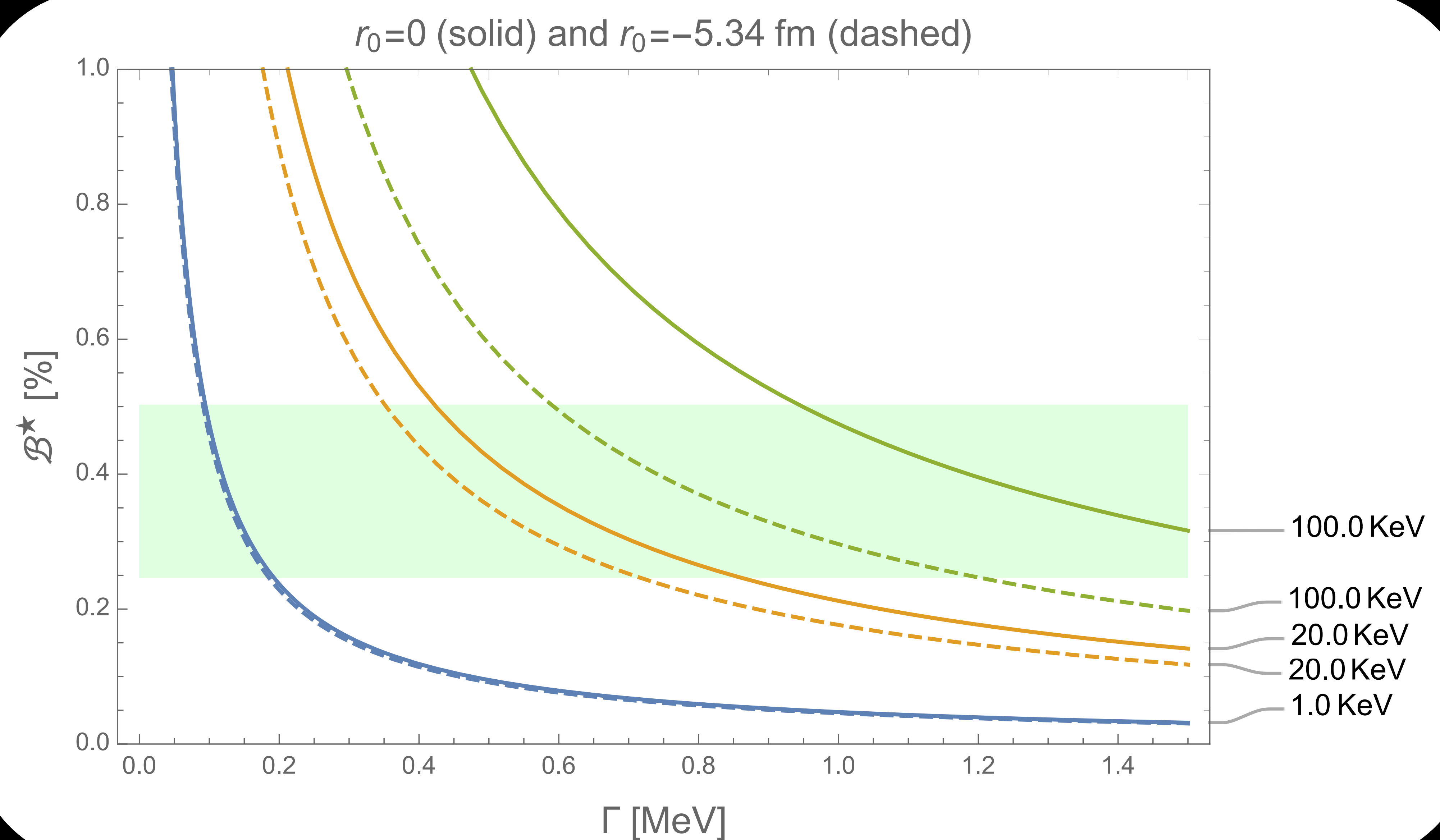
$g = 0$ if $Z = 1$ (no cubic interaction!)

THE LANDAU COUPLING & $X \to DD\pi$

$$g^2 = 16\pi\sqrt{\frac{2B}{m}}\,\frac{(m_a+m_b)^2}{1-r_0\sqrt{2mB}}$$

To compute $\Gamma(X \to DD\pi)$

ADP, 1505.03083



FURTHER IMPLICATIONS

— BESIII observed $e^+e^- \to K^+Z_{cs}^-(3985) \to K^+(D_s^{*-}D^0 + D_s^-D^{*0})$

$\uparrow$

$Z_{cs}(4003)$ should appear here too

— Viceversa $Z_{cs}(3985)$ should be observed by LHCb

— A multiplet associated to $Z(4020)$ should also be observed