Universität Zürich[UZH]

Is the $(g-2)_\mu$ anomaly a threat to Lepton Flavor Conservation?

Julie Pagès

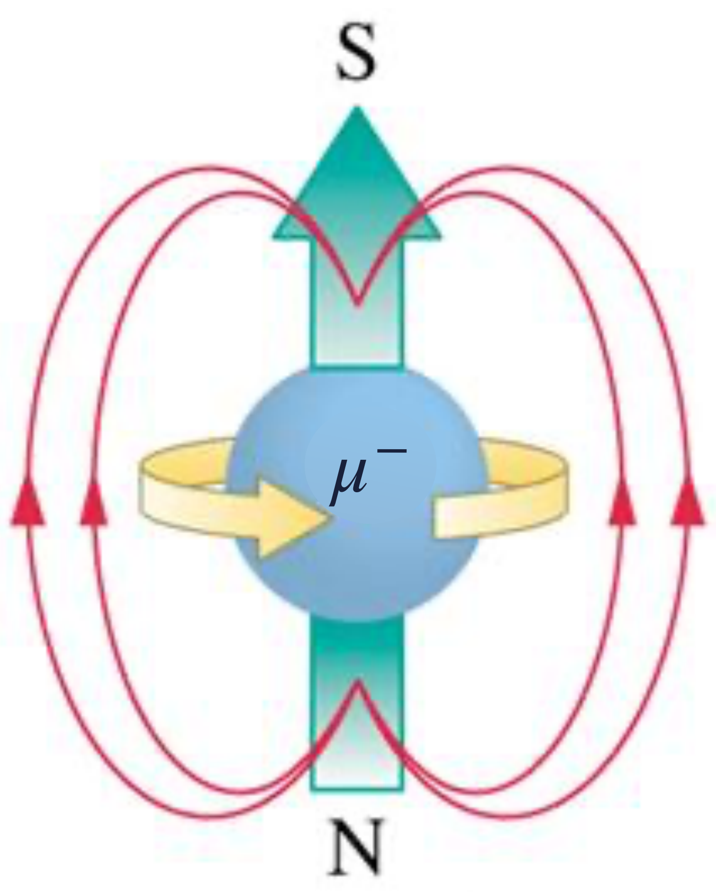
University of Zurich

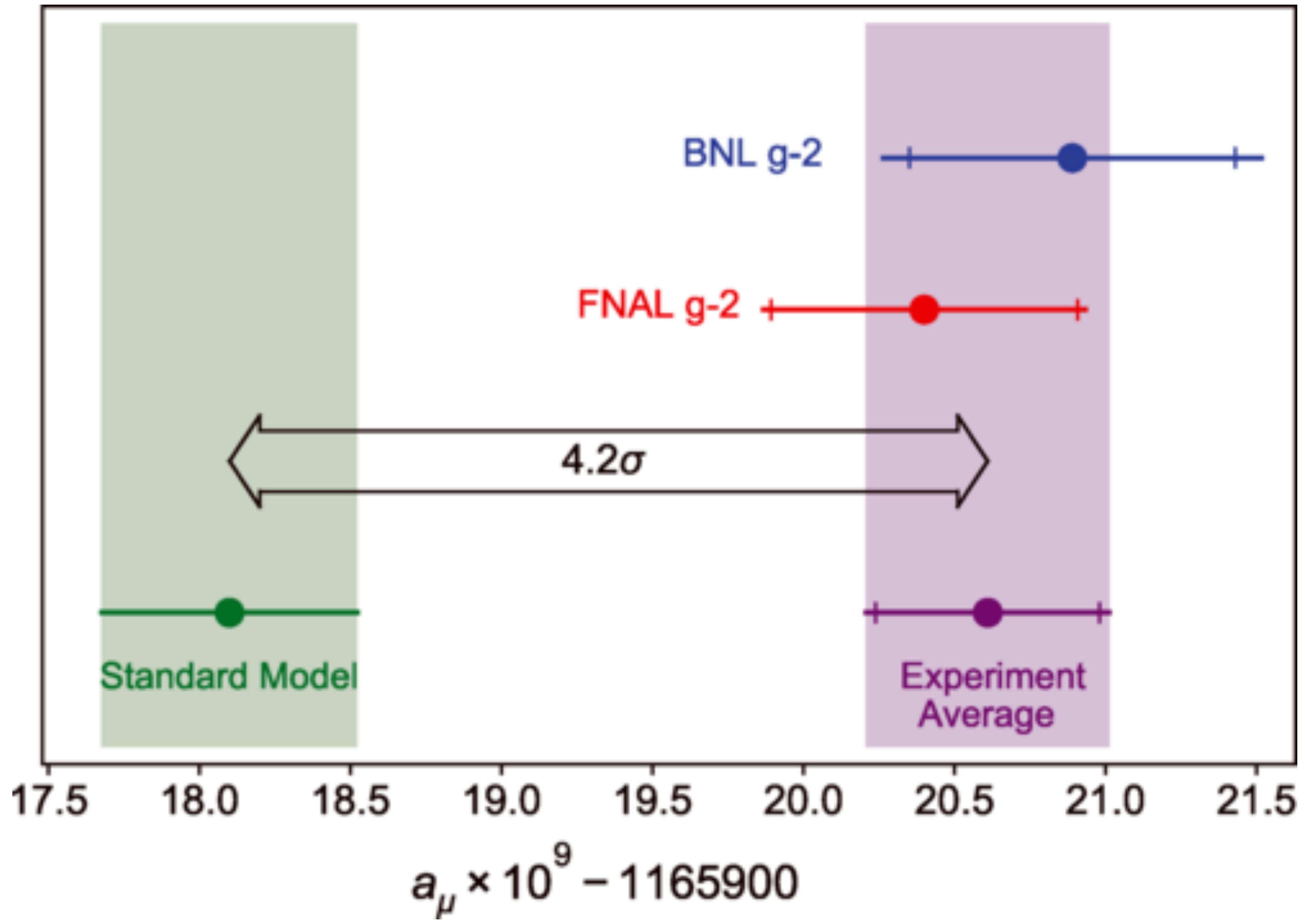
La Thuile, 9 March 2022 - Les Rencontres de Physique de la Vallée d'Aoste

Based on *G. Isidori, J. Pagès and F. Wilsch,* arXiv:2111.13724

Anomalies

in anomalous magnetic dipole moment of the muon $a_\mu \equiv \frac{(g-2)_\mu}{2}$





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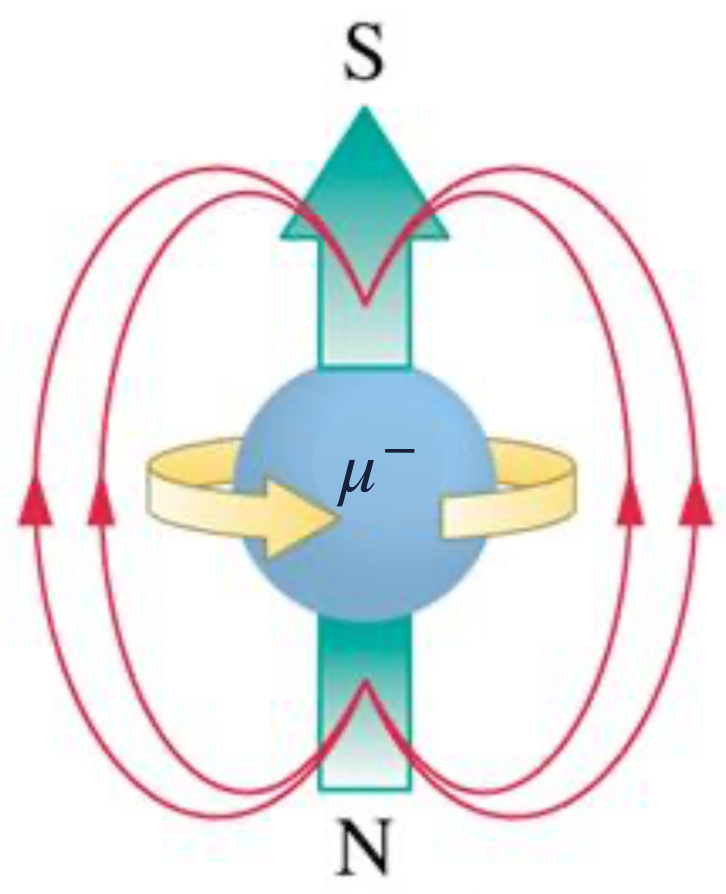

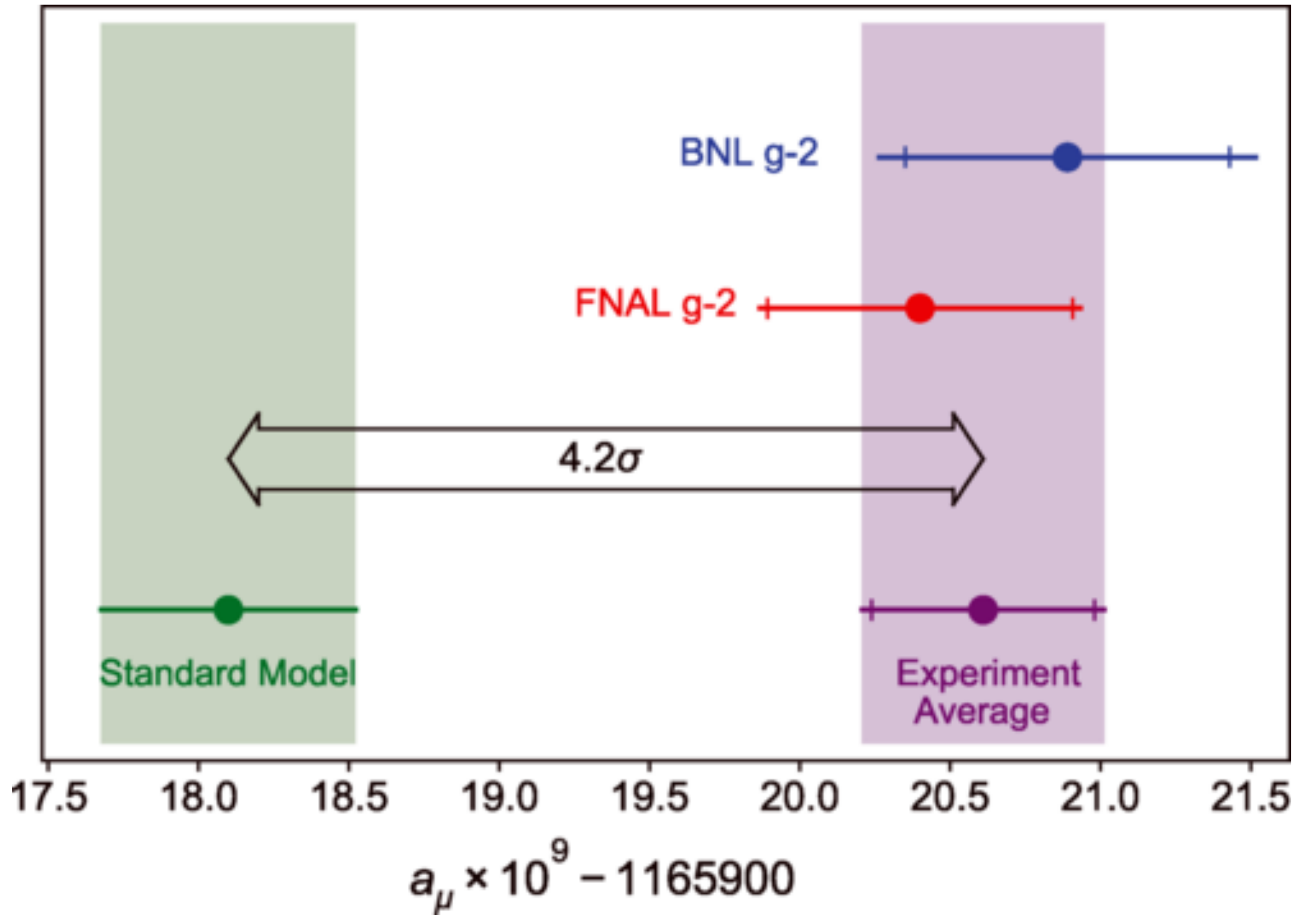

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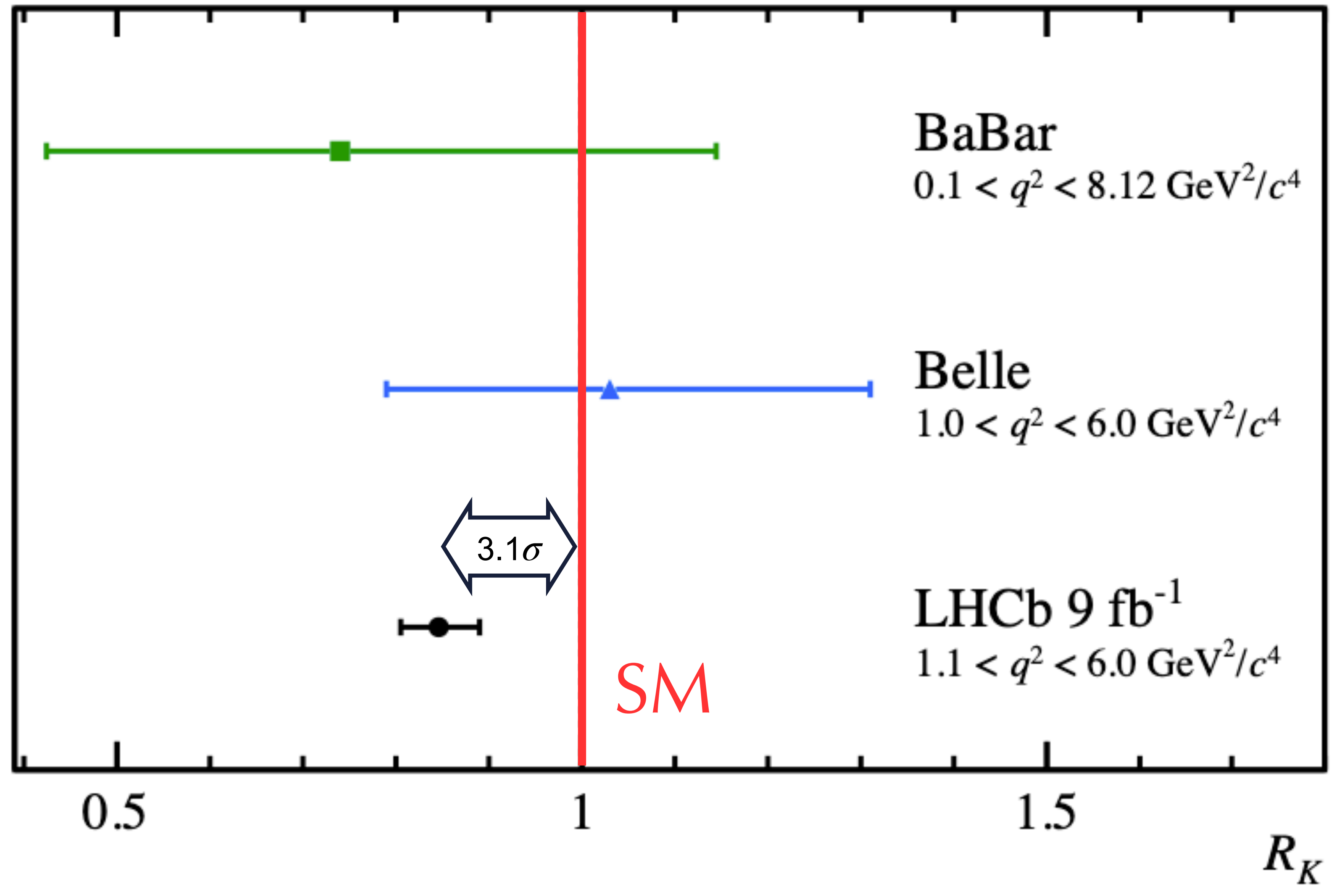
in semi-leptonic B-decays



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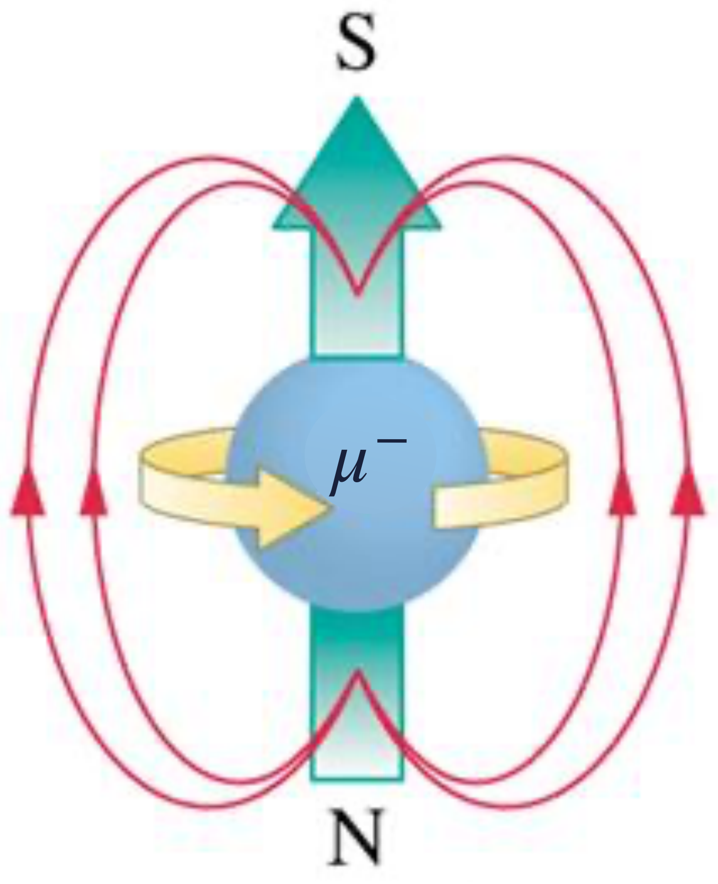

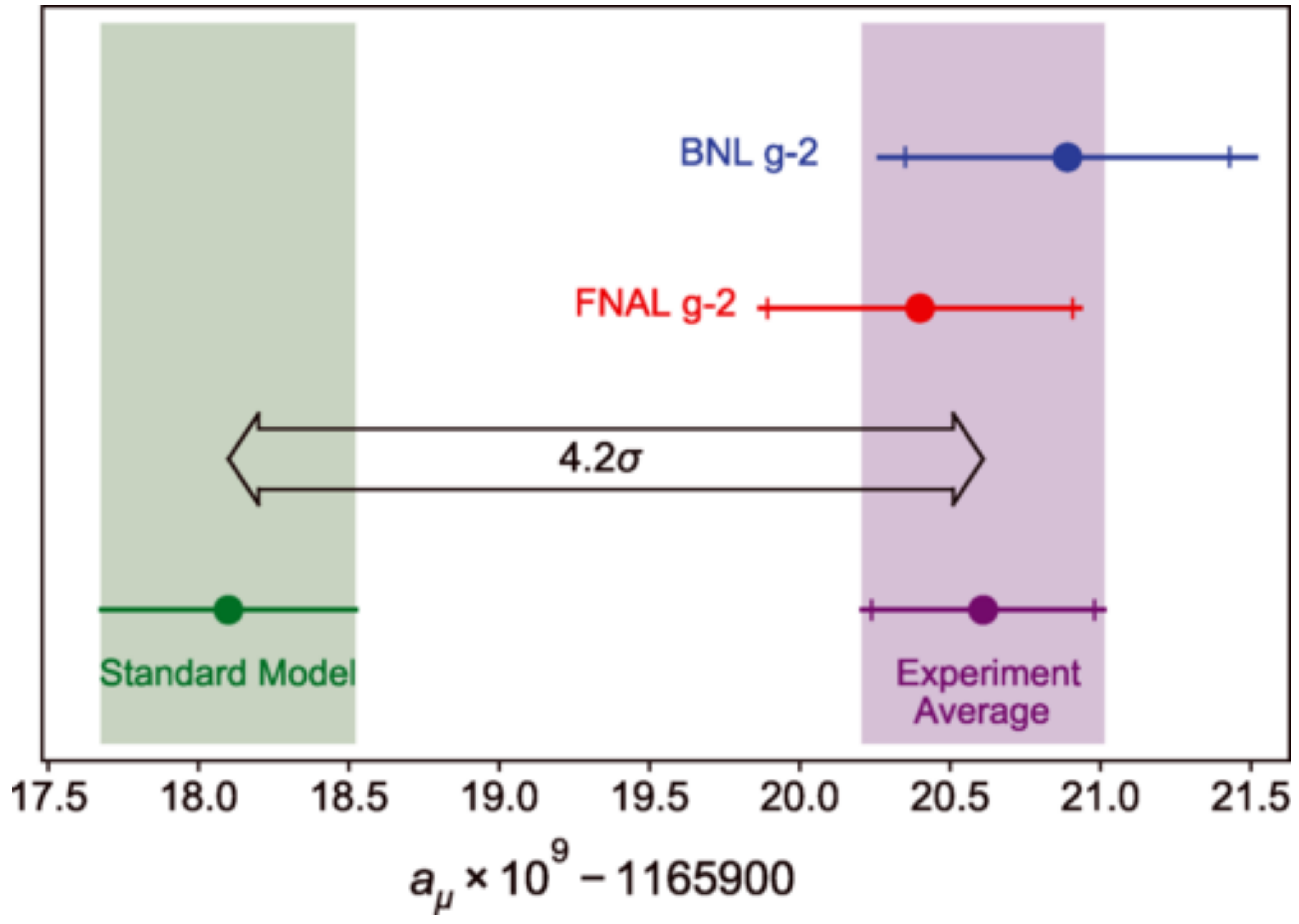

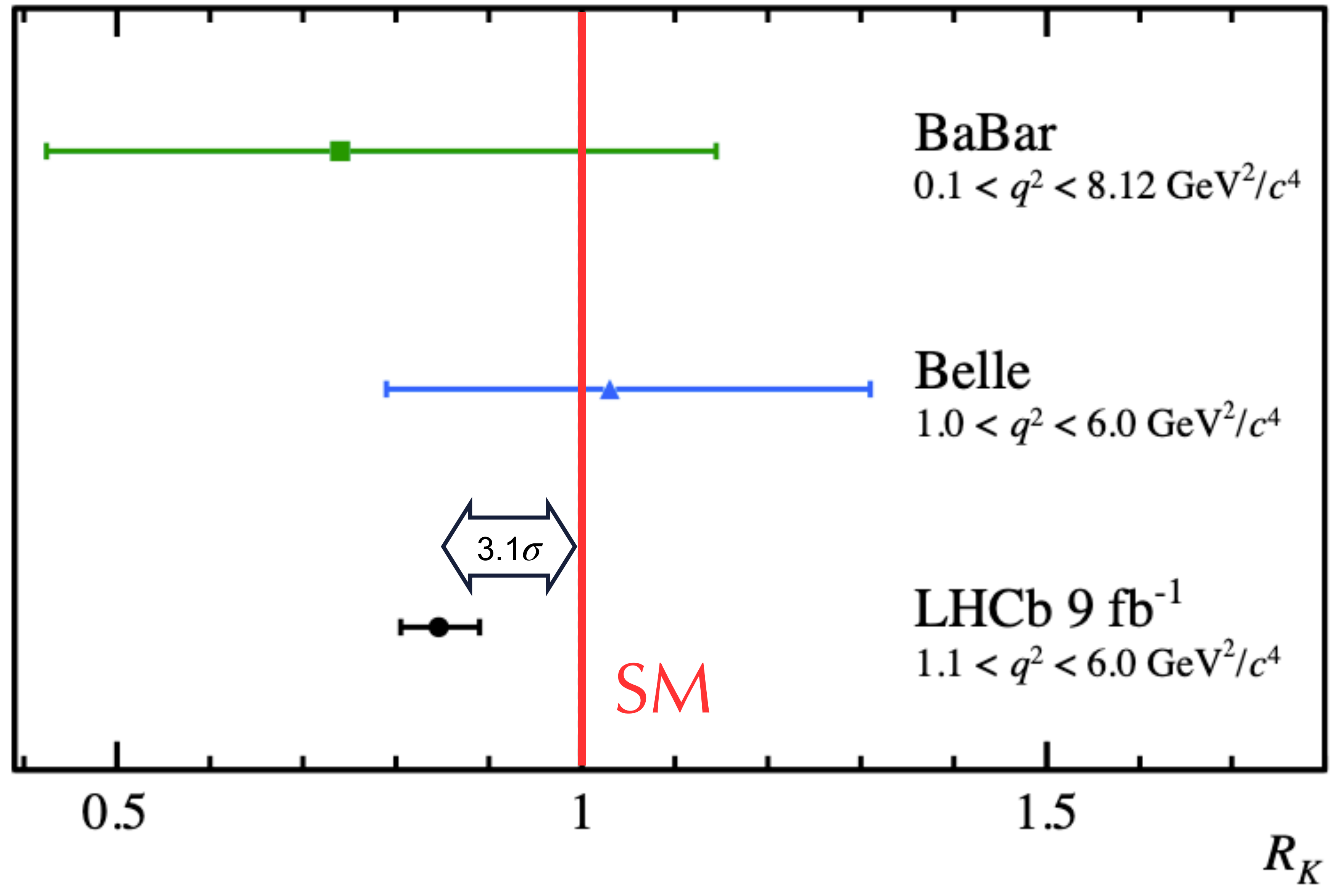

see talk by Massimo

see talks by Vitalii, Luca, Claudia

both suggest

Lepton Flavor Universality Violation (LFUV) → What about Lepton Flavor Violation (LFV)?

see talk by Andreas

$(g-2)_\mu$ and LFV

SMEFT describe heavy New Physics (NP) effects on low energy observables i.e. $m_{\rm NP} \gtrsim \mathcal{O}(100~\text{GeV})$

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \sum_i \mathscr{C}_i \mathcal{O}_i$$

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Same operator mediate $(g-2)_\mu$ and $\mu \to e\gamma$

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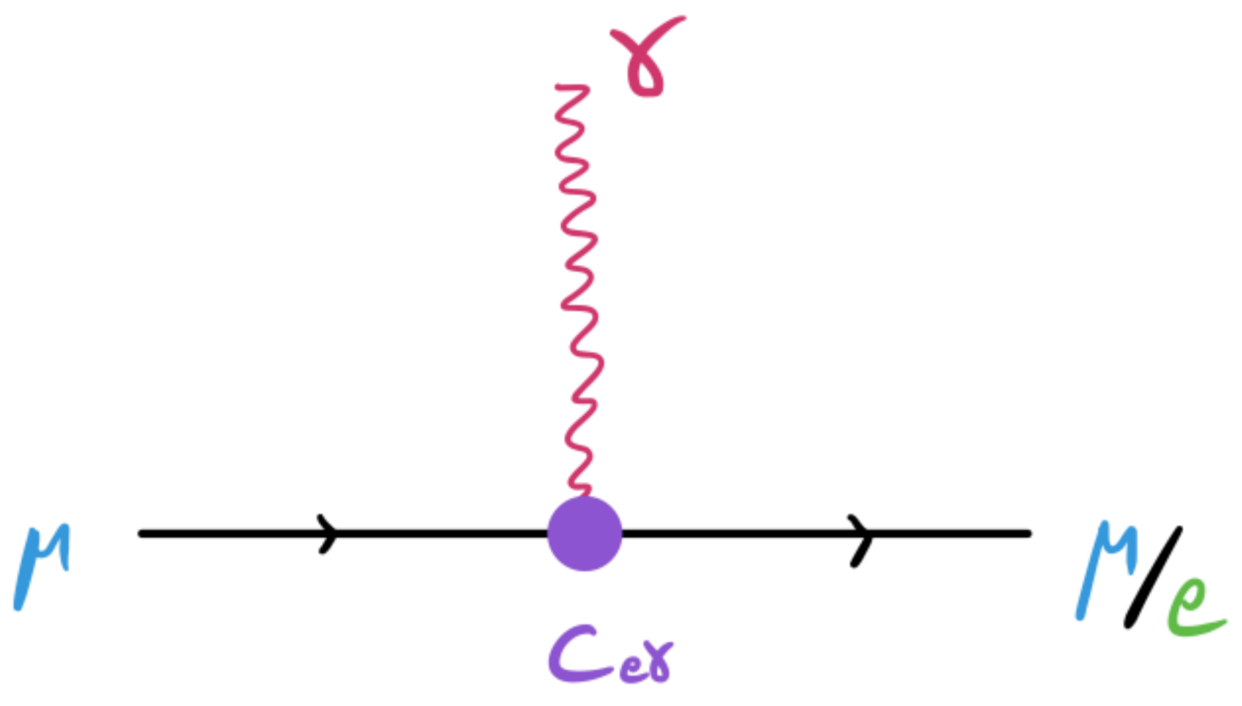

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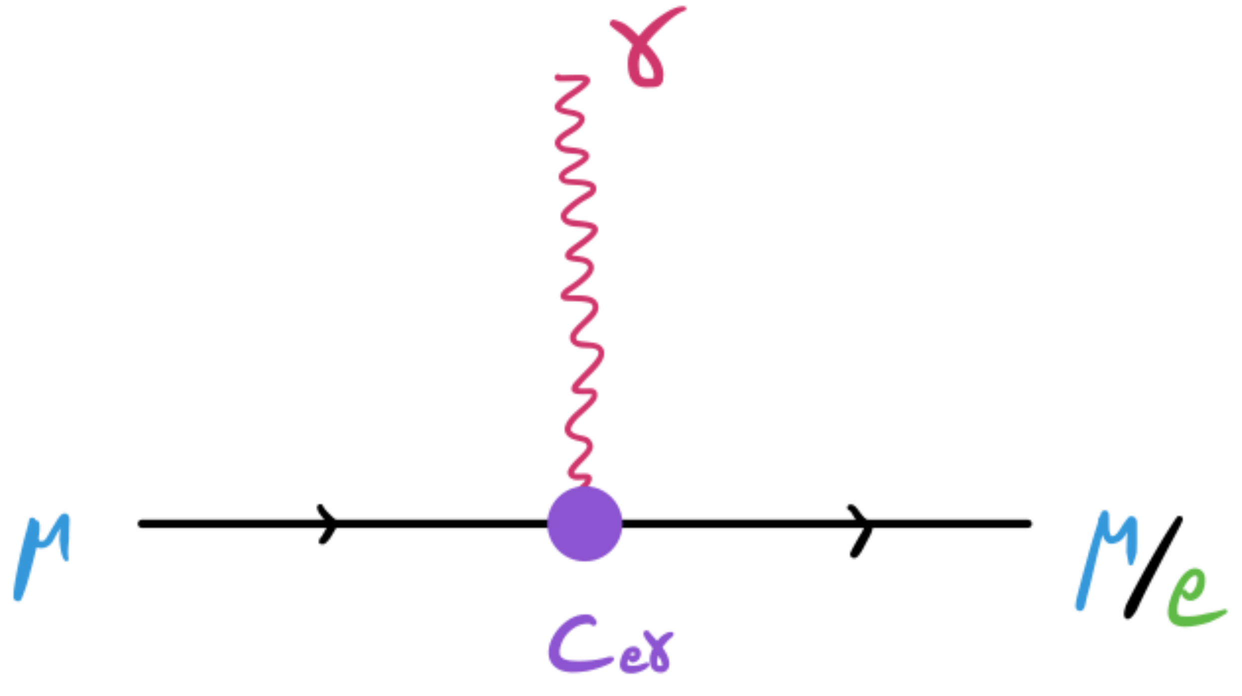

Tree-level contribution to Δa_μ

$$\Delta a_\mu = \frac{4 m_\mu v}{e\sqrt{2}} \text{Re}\, \mathscr{C}'_{\substack{e\gamma\\22}}$$

Tree-level contribution to $\mu \to e\gamma$

$$\mathscr{B}(\mu \to e\gamma) = \frac{m_\mu^3 v^2}{8\pi\Gamma_\mu} \left(|\mathscr{C}'_{\substack{e\gamma\\12}}|^2 + |\mathscr{C}'_{\substack{e\gamma\\21}}|^2 \right)$$

Observables and Constraints on NP

$(g-2)_\mu$

$\mu \to e\gamma$

Combined FNAL and BNL result:
[hep-ex/0602035, 2104.03281, 2006.04822]

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

Evidence of

$$\text{Re}\, \mathscr{C}'_{\substack{e\gamma \\ 22}} = 1 \times 10^{-5}\ \text{TeV}^{-2}$$

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Branching ratio measured by MEG:
[1605.05081]

$$\mathscr{B}(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13}\ [90\,\%\ \text{C.L.}]$$

Upper bound on

$$|\mathscr{C}'_{\underset{12}{e\gamma}}| < 2 \times 10^{-10}\ \text{TeV}^{-2}$$

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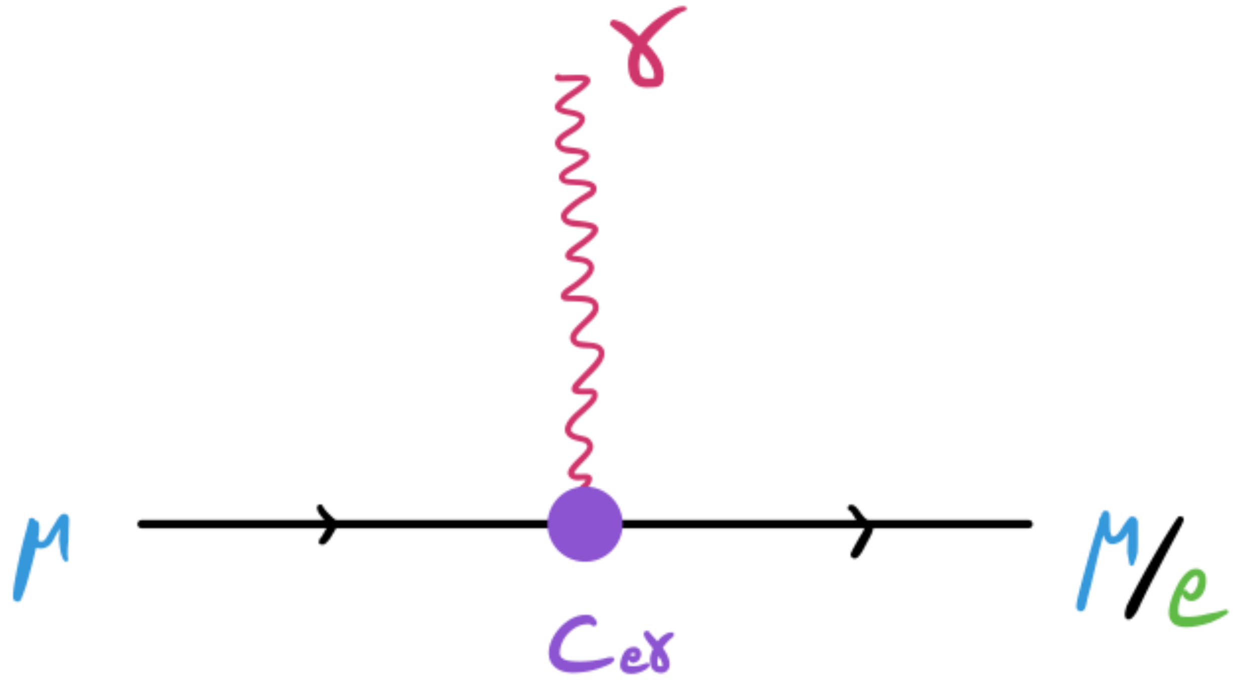
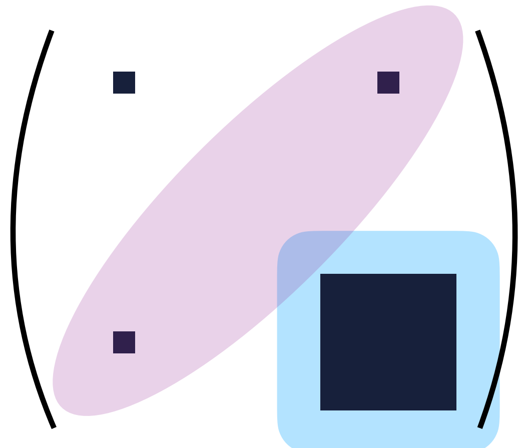
The SMEFT operator



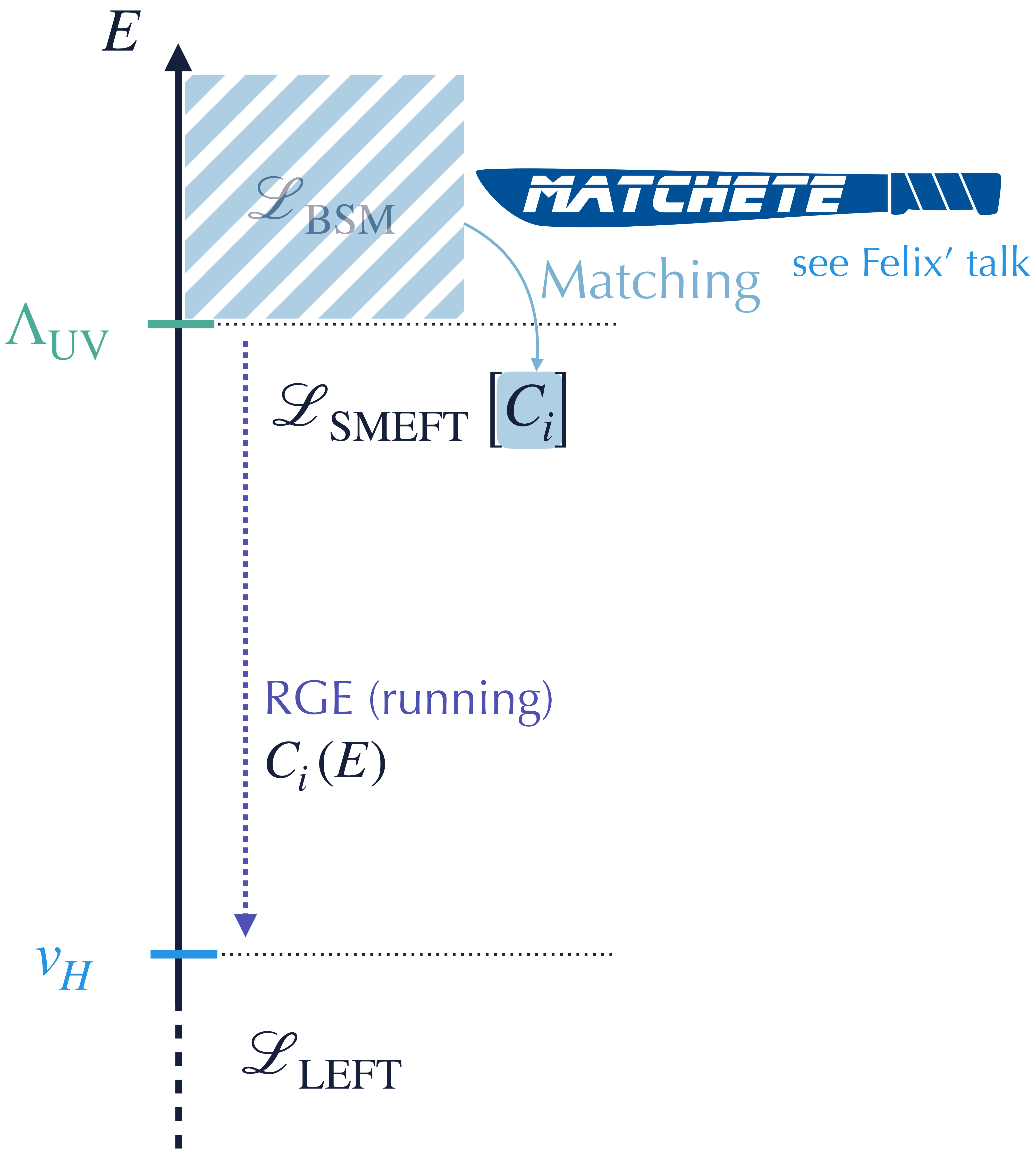
has a specific flavor structure

$\mathscr{C}'_{\underset{ij}{e\gamma}} =$ 

with strong flavor alignment

$$\epsilon^L_{12} \equiv \frac{\mathscr{C}'_{\underset{12}{e\gamma}}}{\mathscr{C}'_{\underset{22}{e\gamma}}} < 2 \times 10^{-5}$$

SMEFT Renormalization Group Evolution



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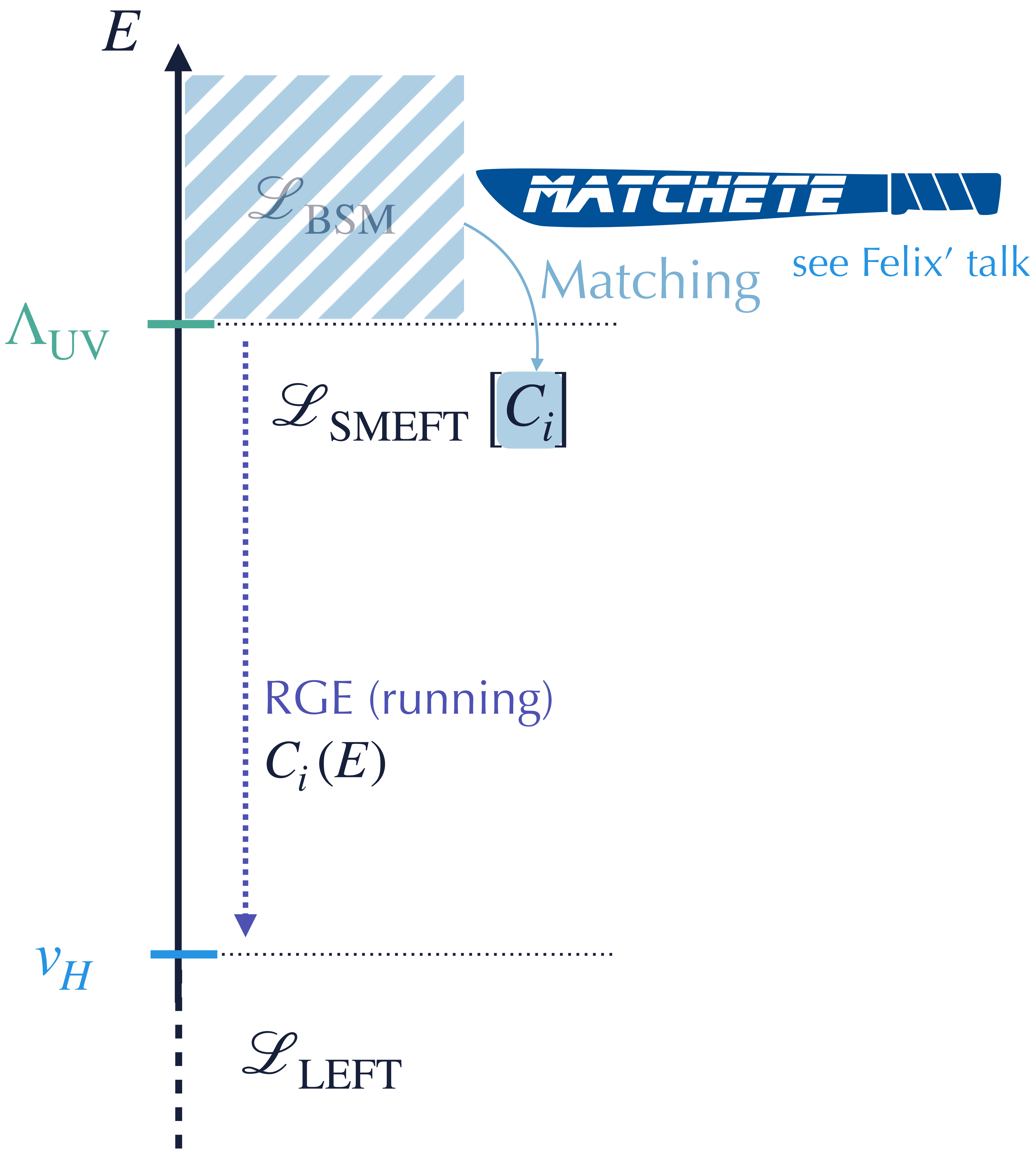

From high-scale, 2 sources can spoil alignment at low-scale:

1. Operators mix through Renormalization Group Evolution

   Jenkins et al. [1308.2627, 1310.4838, 1312.2014]

$$\mu \frac{d}{d\mu} C_i = \frac{1}{16\pi^2} \beta_i \qquad \text{where} \quad \beta_i = \sum_j \gamma_{ij} C_j$$

with solution
$$C_i(\mu_L) = C_i(\mu_H) + \underbrace{\frac{1}{16\pi^2} \log\left(\frac{\mu_L}{\mu_H}\right)}_{-\hat{L}} \beta_i$$

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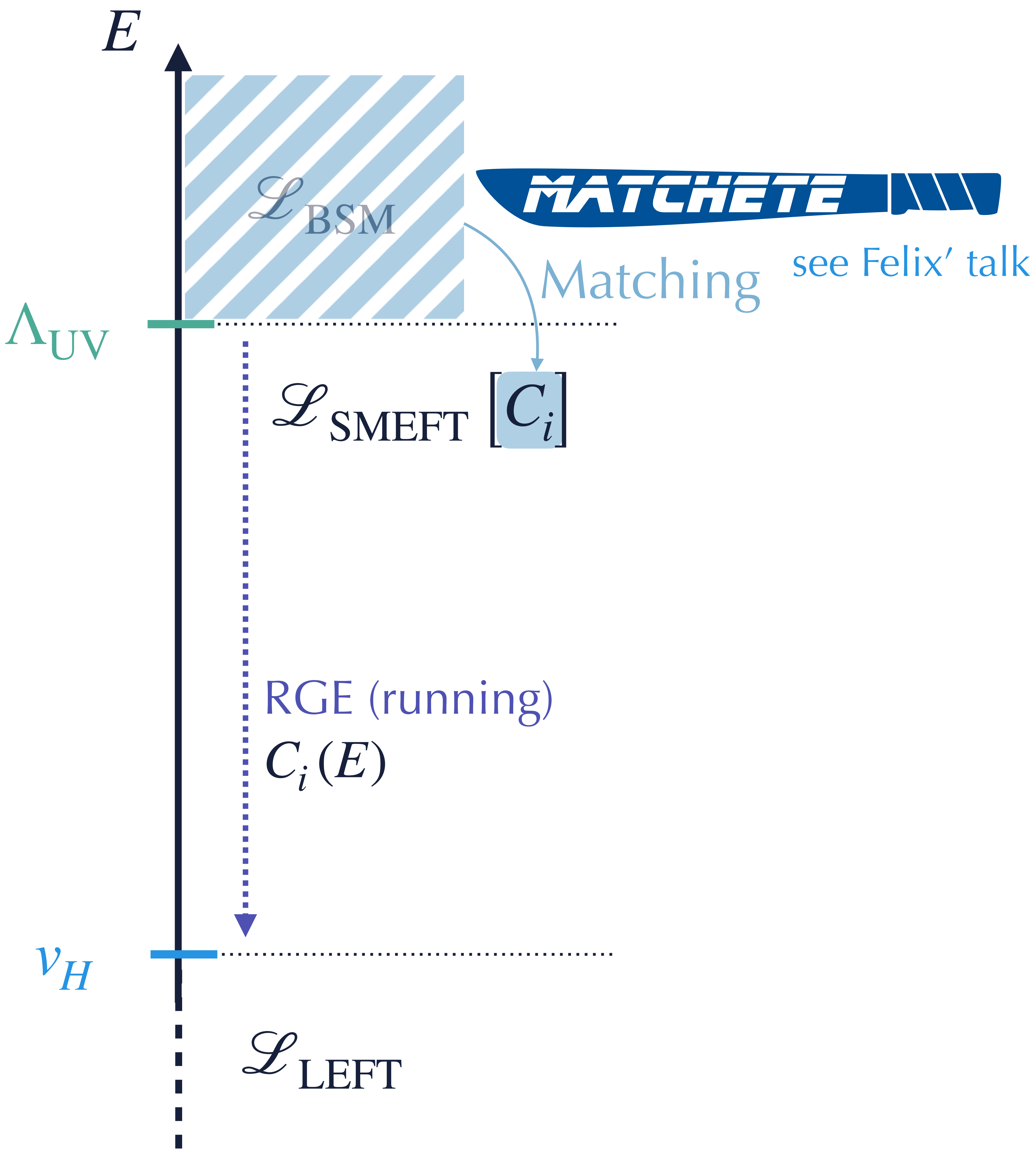

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2. Rotation to the mass basis:

$$\Theta^{\mathscr{Y}}_{L(R)} = -\left.\frac{\left[\mathscr{Y}_e\right]_{12(21)}}{\left[\mathscr{Y}_e\right]_{22}}\right|_{\mu_L}$$

$\Rightarrow$ Dipole in the mass basis:

$$\mathscr{C}'_{\underset{12}{e\gamma}}\left(\mu_L\right) = \mathscr{C}_{\underset{12}{e\gamma}}\left(\mu_L\right) + \Theta^{\mathscr{Y}}_L\,\mathscr{C}_{\underset{22}{e\gamma}}\left(\mu_L\right)$$

$$\mathscr{C}'_{\underset{22}{e\gamma}}\left(\mu_L\right) \approx \mathscr{C}_{\underset{22}{e\gamma}}\left(\mu_L\right)$$

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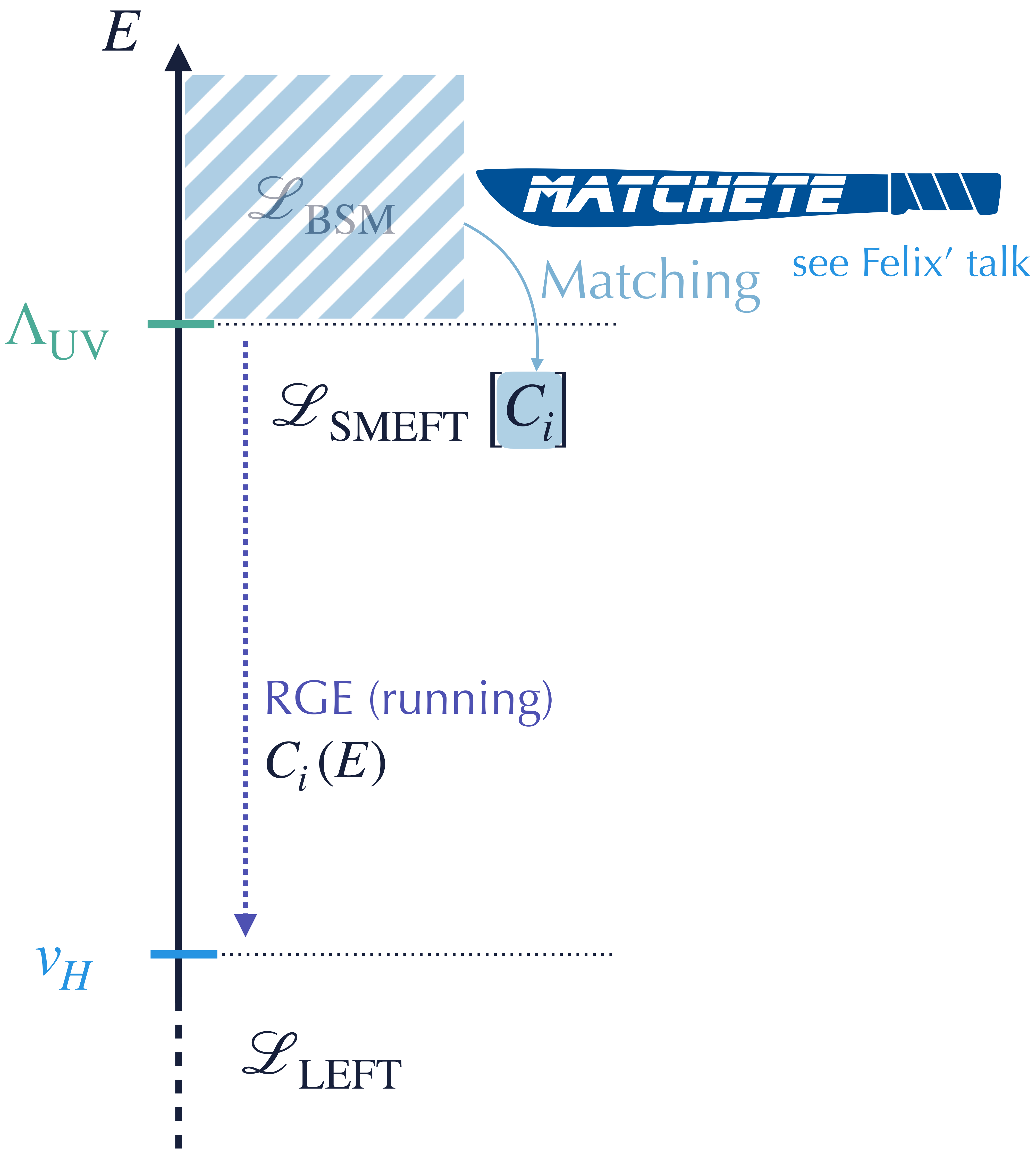

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$\neq 0$ for Δa_μ

$$\mathscr{C}'_{\underset{22}{e\gamma}}(\mu_L) \approx \mathscr{C}_{\underset{22}{e\gamma}}(\mu_L)$$

Definitions of Operators

Operators in the broken phase

$$\underset{rs}{\mathcal{O}_{e\gamma}} = \frac{v}{\sqrt{2}} \bar{e}_{Lr} \sigma^{\mu\nu} e_{Rs} F_{\mu\nu}$$

$$\underset{rs}{\mathcal{O}_{eZ}} = \frac{v}{\sqrt{2}} \bar{e}_{Lr} \sigma^{\mu\nu} e_{Rs} Z_{\mu\nu}$$

$$\underset{rs}{\mathcal{O}_{\mathcal{Y}_e}} = \frac{v}{\sqrt{2}} \bar{e}_{Lr} e_{Rs}$$

$$\underset{rs}{\mathcal{O}_{\mathcal{Y}_{he}}} = \frac{h}{\sqrt{2}} \bar{e}_{Lr} e_{Rs}$$

Definitions of Operators

Operators in the broken phase	Operators in the unbroken phase
$\mathcal{O}_{\substack{e\gamma\\rs}} = \frac{v}{\sqrt{2}} \bar{e}_{Lr} \sigma^{\mu\nu} e_{Rs} F_{\mu\nu}$	$O_{\substack{eB\\rs}} = \bar{\ell}_{Lr} \sigma^{\mu\nu} e_{Rs} H\, B_{\mu\nu}$
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weak mixing angle

$$\begin{pmatrix} \mathcal{C}_{e\gamma} \\ \mathcal{C}_{eZ} \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ -s_\theta & -c_\theta \end{pmatrix} \begin{pmatrix} C_{eB} \\ C_{eW} \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{Y}_e \\ \mathcal{Y}_{he} \end{pmatrix} = \begin{vmatrix} 1 & -\frac{1}{2} \\ 1 & -\frac{3}{2} \end{vmatrix} \begin{pmatrix} Y_e \\ v^2 C_{eH} \end{pmatrix}$$

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4-fermions operators for RGE mixing

$$O^{(3)}_{\underset{prst}{lequ}} = (\bar{\ell}^j_{Lp} \sigma^{\mu\nu} e_{Rr}) \, \epsilon_{jk} \, (\bar{q}^k_{Ls} \sigma^{\mu\nu} u_{Rt})$$

$$O^{(1)}_{\underset{prst}{lequ}} = (\bar{\ell}^j_{Lp} e_{Rr}) \, \epsilon_{jk} \, (\bar{q}^k_{Ls} u_{Rt})$$

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Assumptions:

- $g_i^2, \lambda \to 0$
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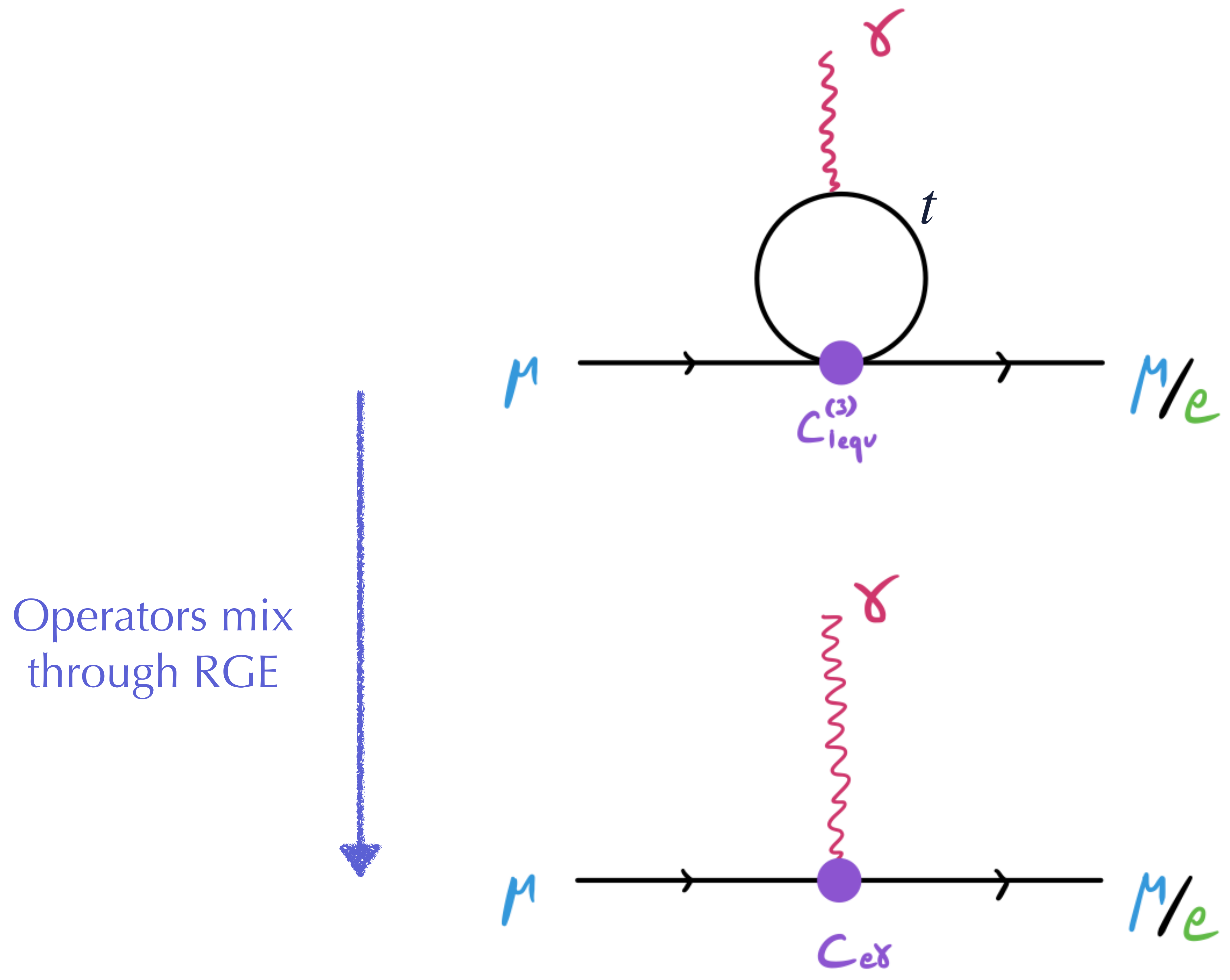
Flavor phases defined as

$$\theta_X = \left.\frac{C_{\underset{12}{X}}}{C_{\underset{22}{X}}}\right|_{\mu_H}$$

Contributions to dipole operator



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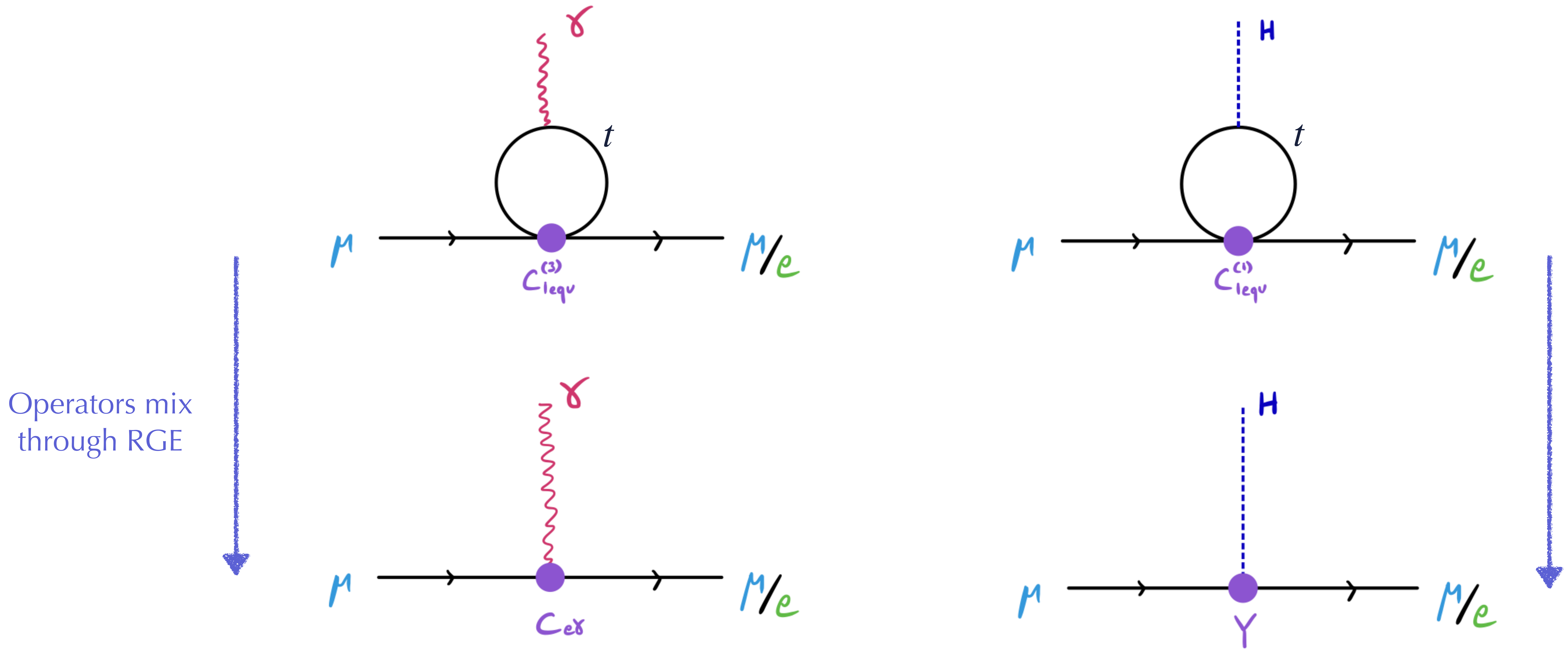
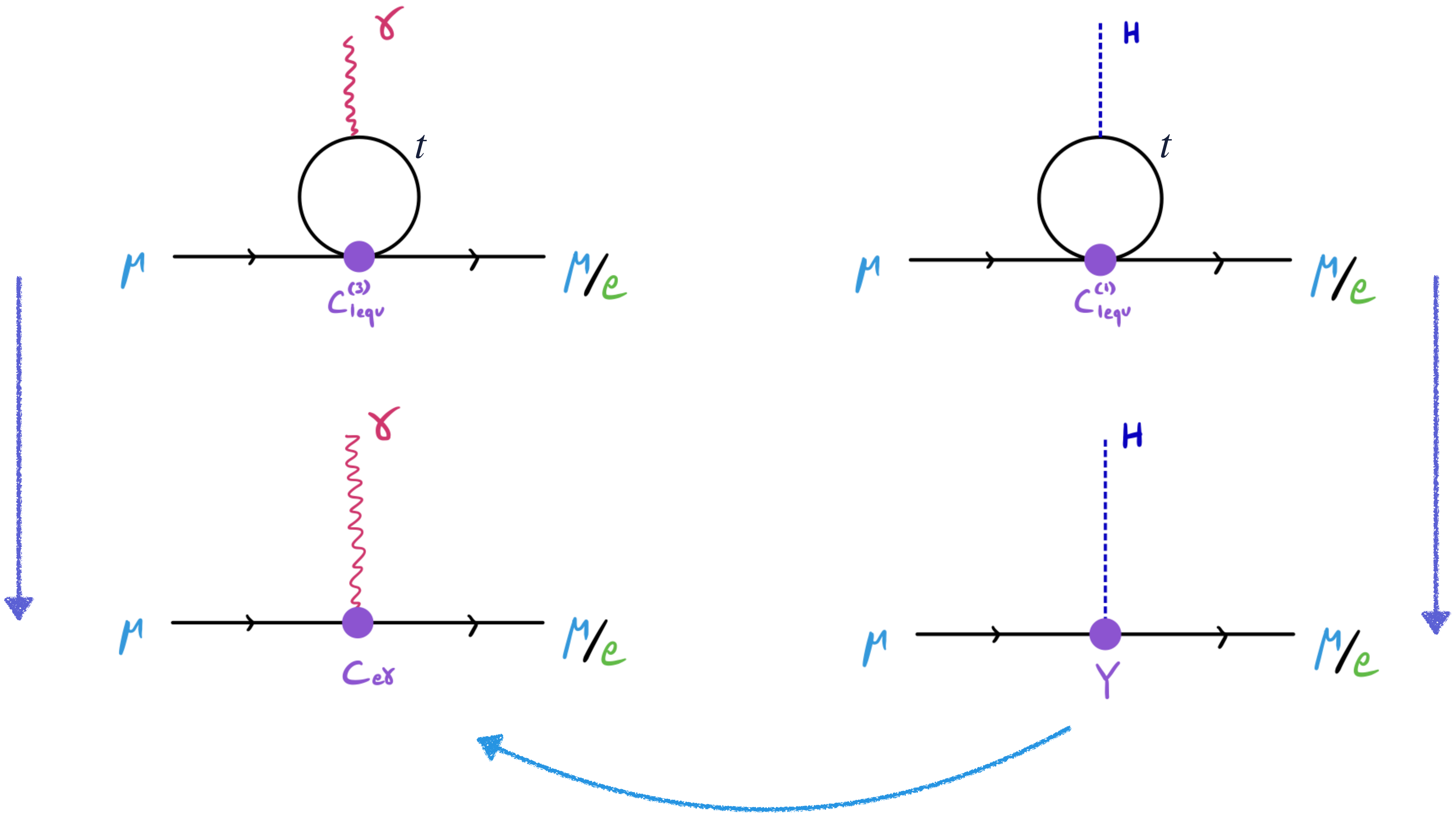


Contributions to dipole operator

Operators mix through RGE



Rotation to mass basis can reintroduce
Lepton Flavor Violation

Alignment of New Physics

Alignment master formula:

$$\epsilon^L_{12} \equiv \left.\frac{\mathscr{C}'_{e\gamma\,12}}{\mathscr{C}'_{e\gamma\,22}}\right|_{\mu_L} = (\theta_{e\gamma} - \theta_Y) + (\theta_{lequ^{(3)}} - \theta_{e\gamma})\,\Delta_3 + (\theta_{lequ^{(1)}} - \theta_Y)\,\Delta_1 \quad < 2 \times 10^{-5}$$

with $\Delta_3 = \dfrac{-16\hat{L}ey_t C^{(3)}_{lequ\,2233}(\mu_H)}{\mathscr{C}_{e\gamma\,22}(\mu_L)}$ and $\Delta_1 = \dfrac{-6\hat{L}y_t^3 v^2 C^{(1)}_{lequ\,2233}(\mu_H)}{[\mathscr{Y}_e]_{22}(\mu_L)}$

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How can we reach this alignment?

- Dynamical alignments
- Flavor symmetries

Dynamical Alignments

$$\epsilon^L_{12} = (\theta_{e\gamma} - \theta_Y) + (\theta_{lequ^{(3)}} - \theta_{e\gamma})\,\Delta_3 + (\theta_{lequ^{(1)}} - \theta_Y)\,\Delta_1$$

Dynamical Alignments

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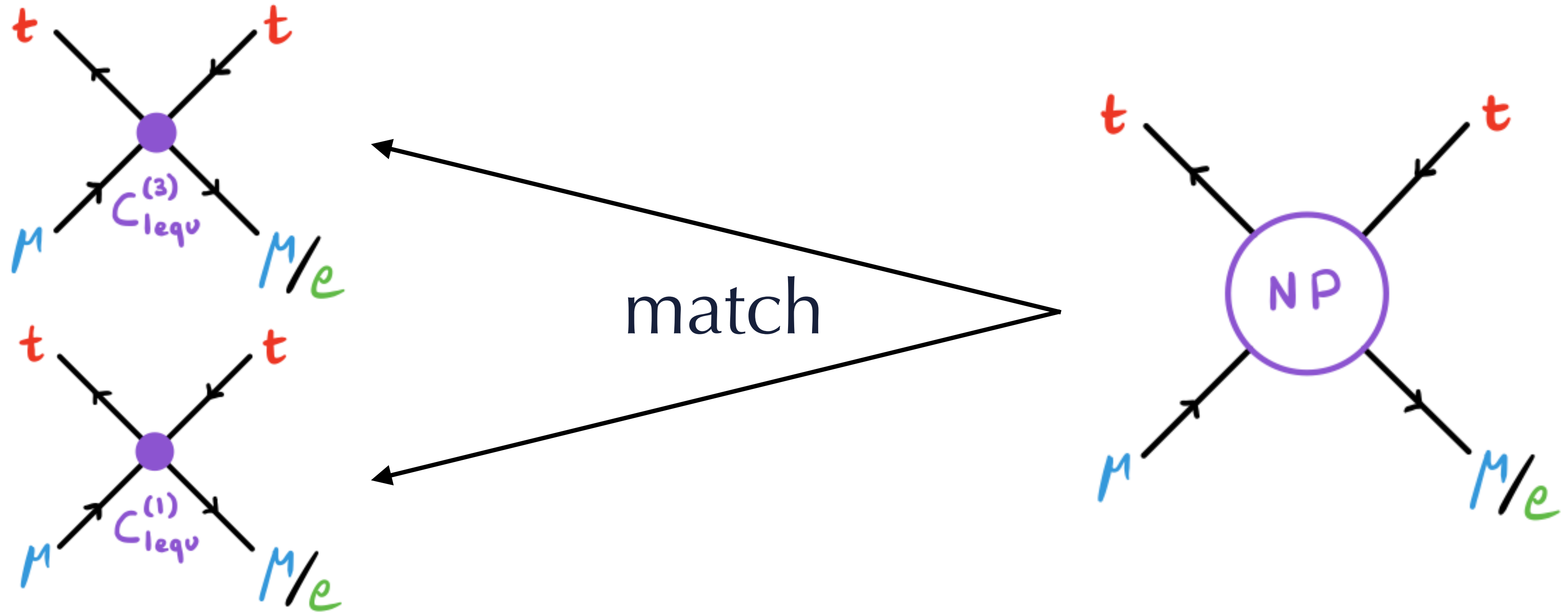
1) Same NP for $C^{(1),(3)}_{lequ}$ & radiative Dipole and Yukawa



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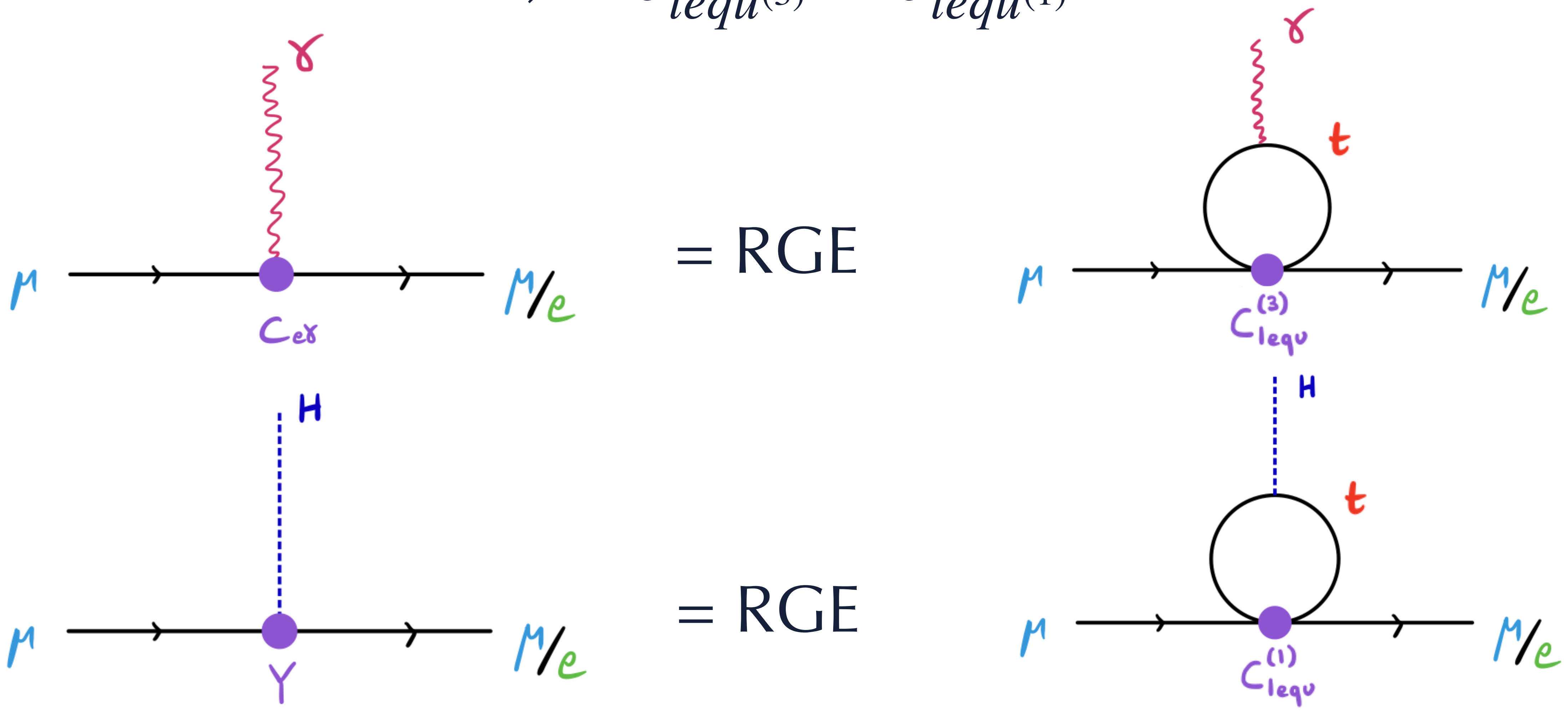

$$\Rightarrow \quad \theta_{e\gamma} = \theta_Y = 0 \quad \text{and} \quad \Delta_3 = -\Delta_1 = 1$$

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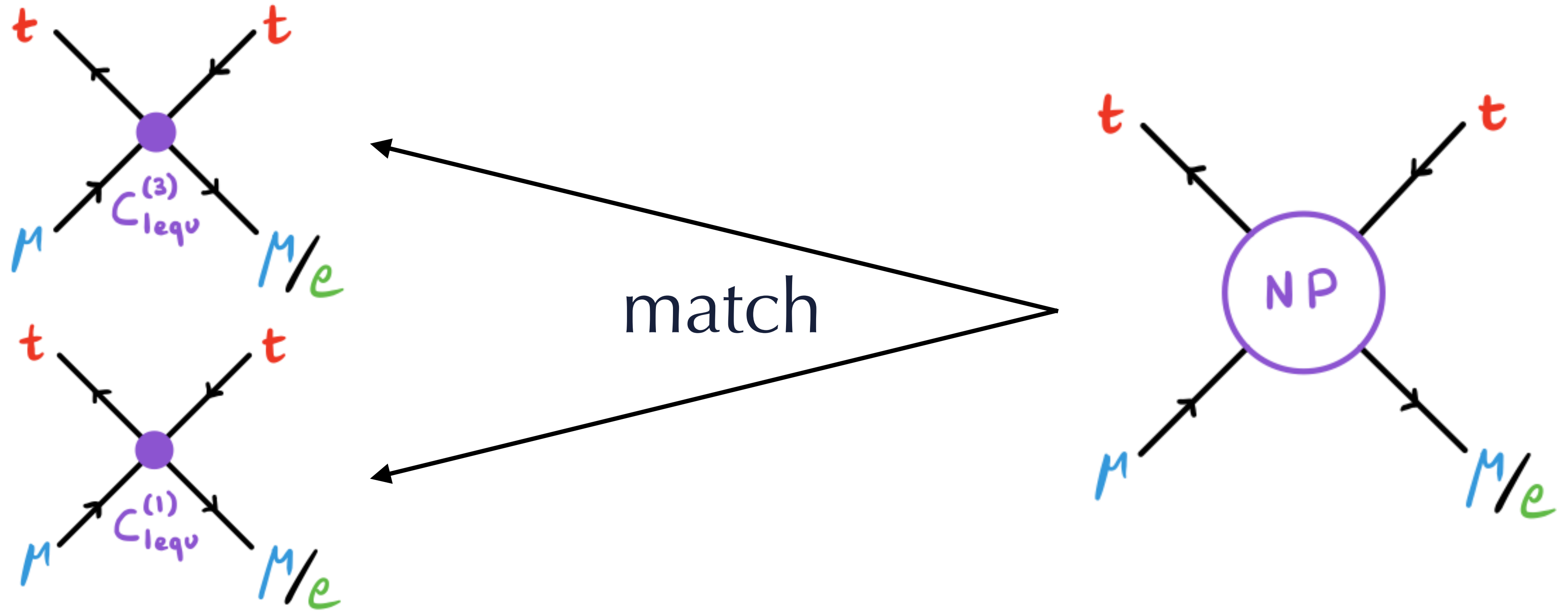

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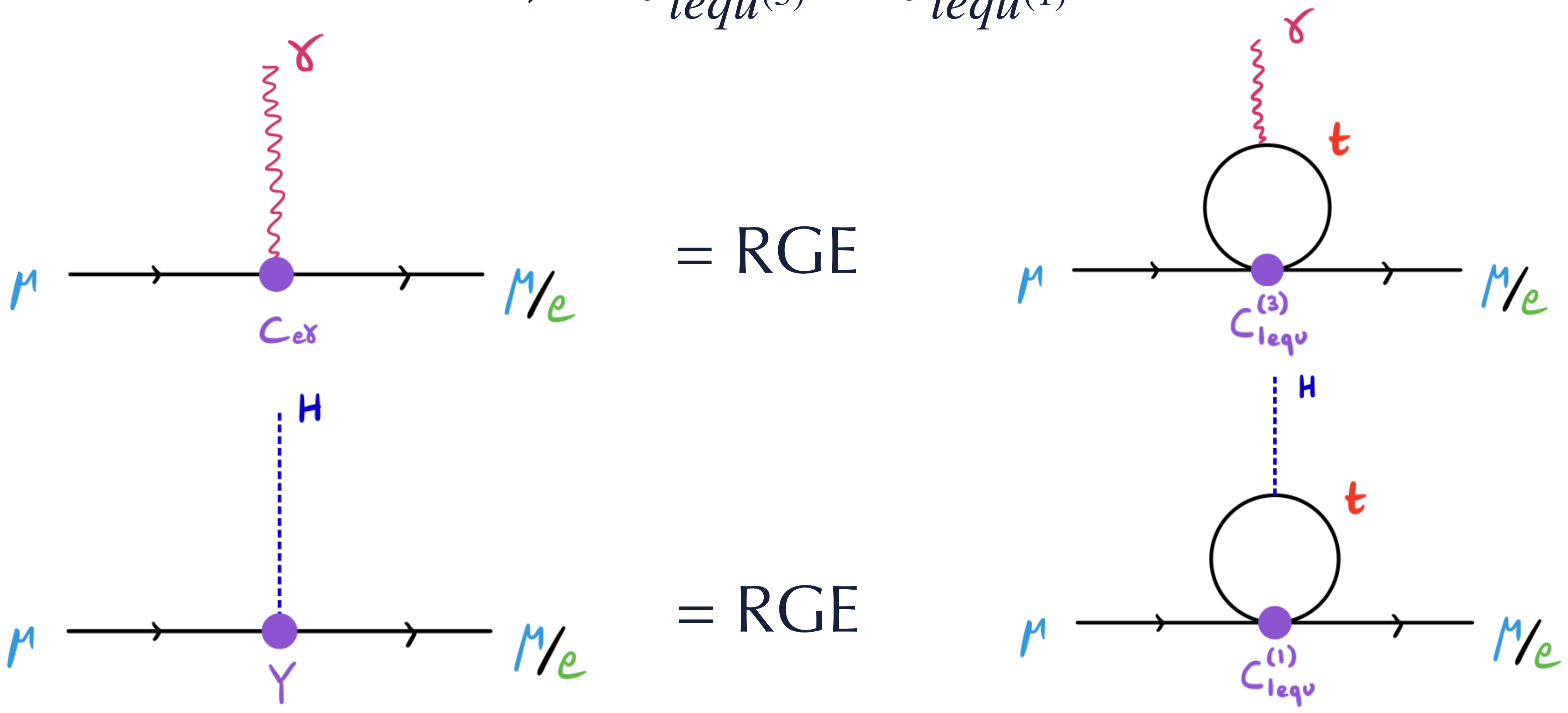

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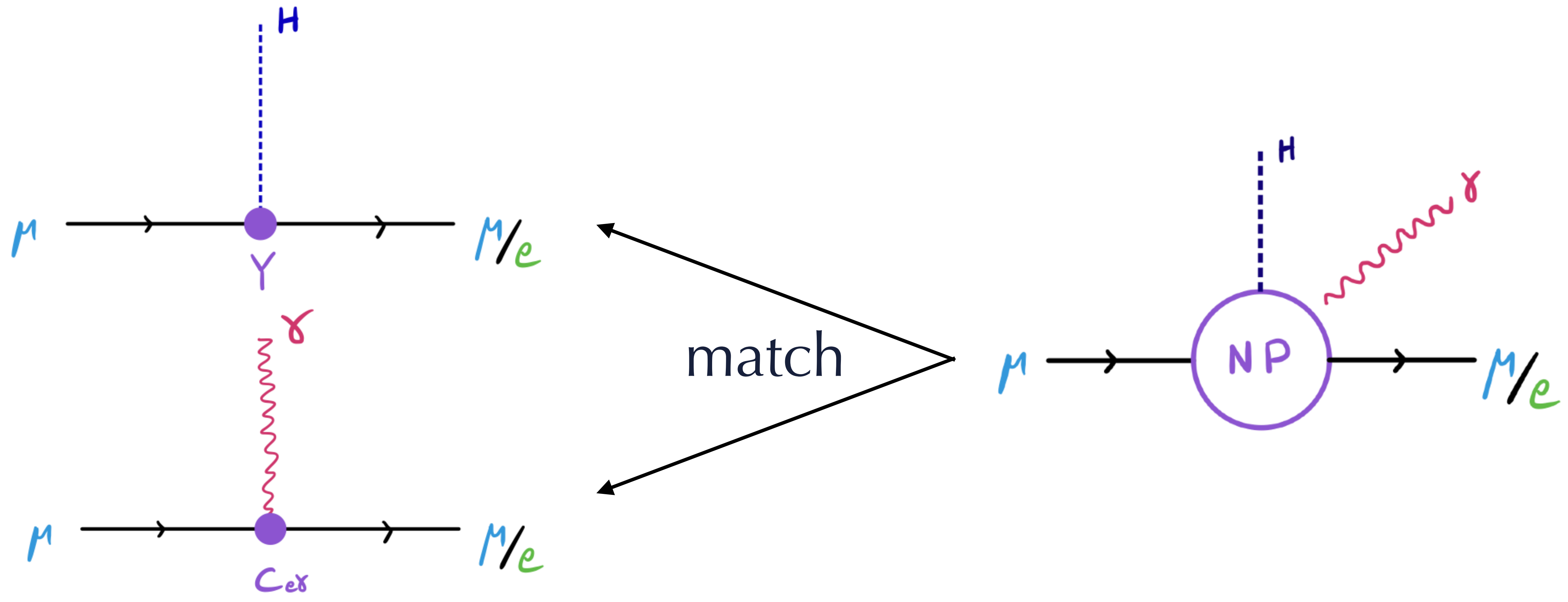

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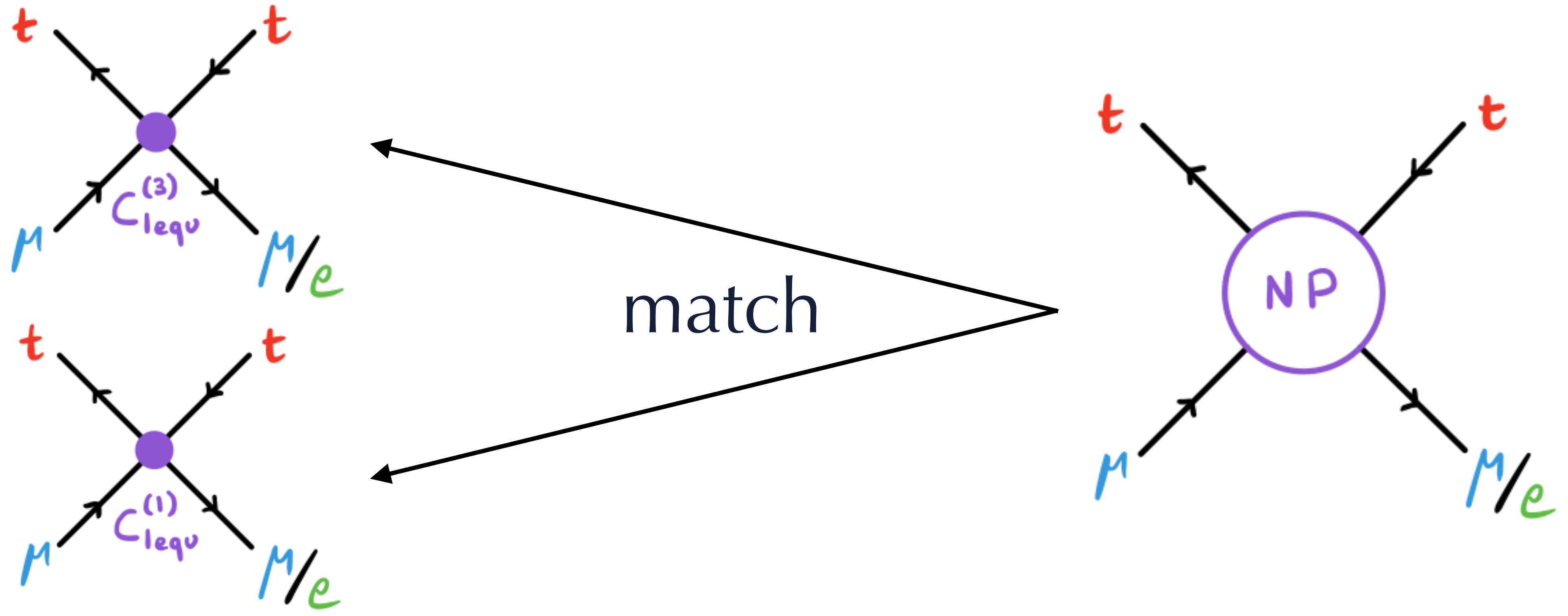

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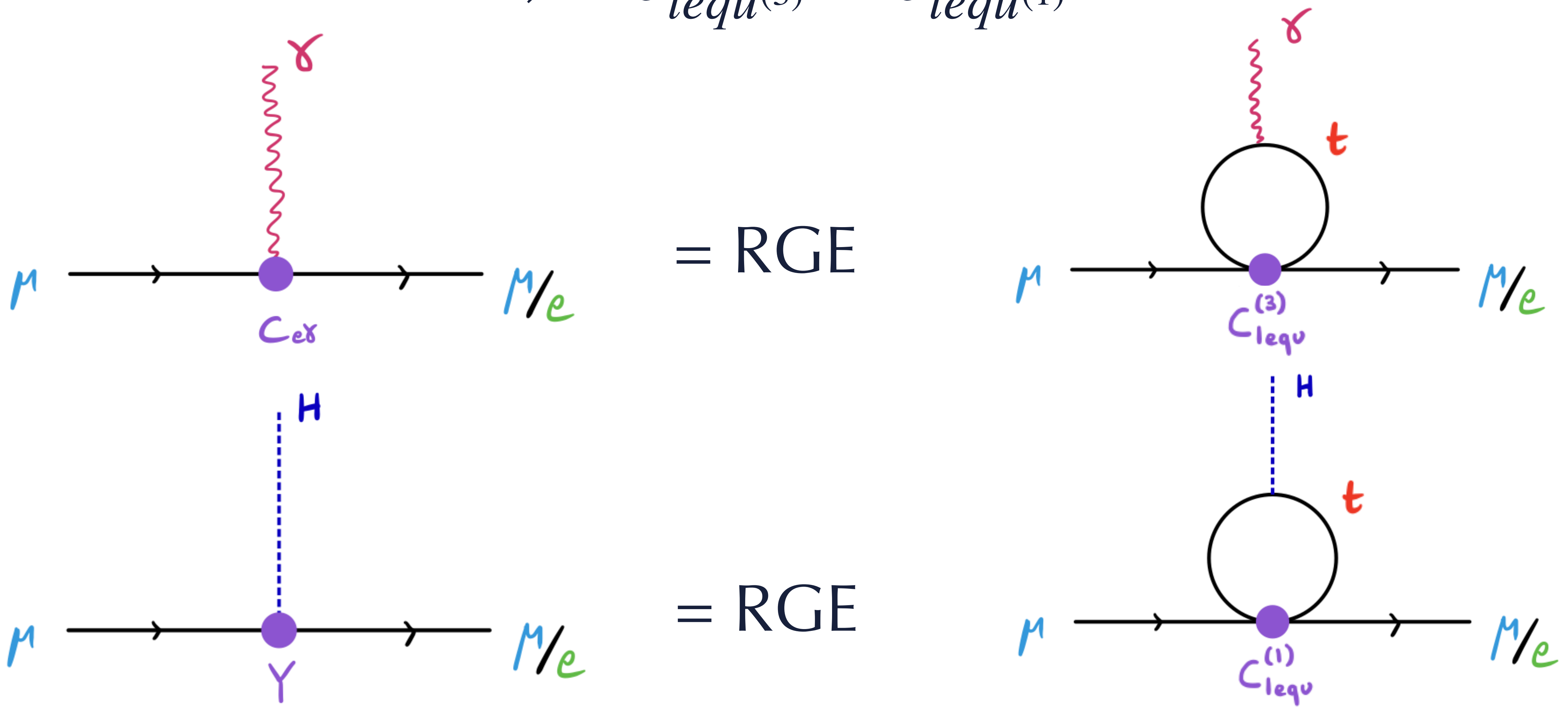

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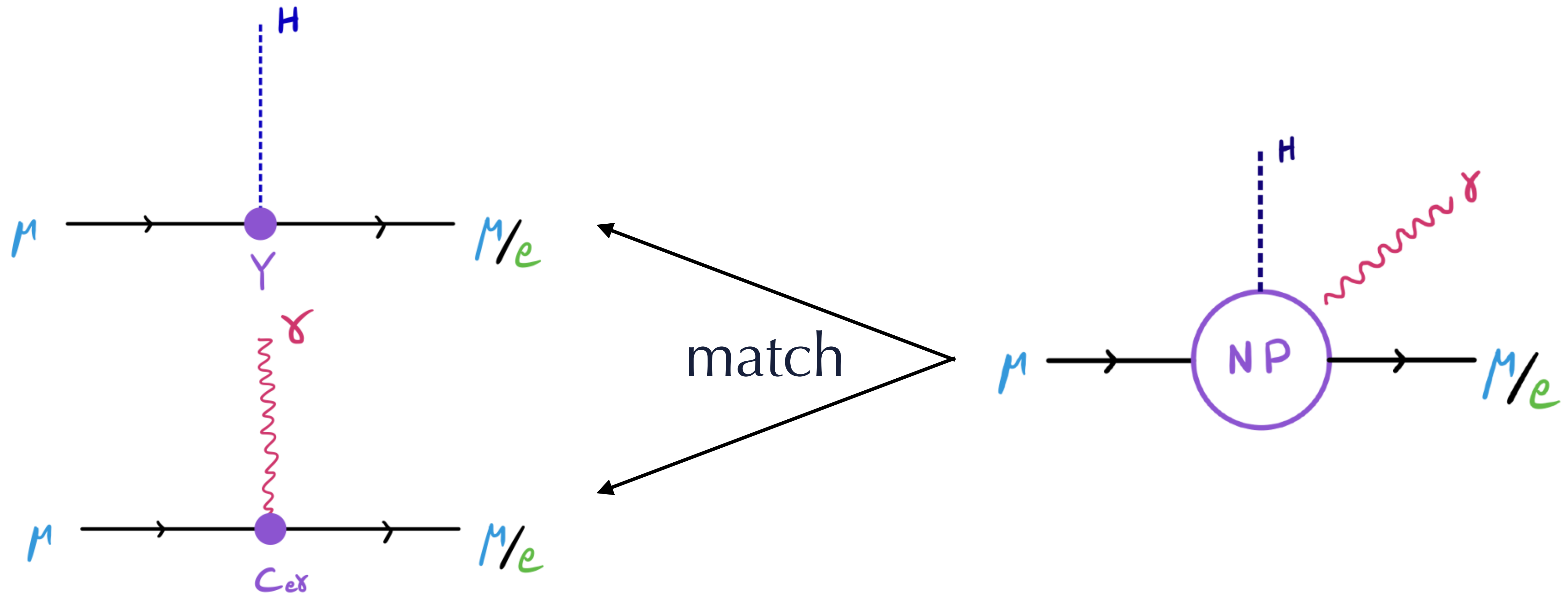


$$\Rightarrow \quad \Delta_3 = \Delta_1 = 0 \quad \text{and} \quad \theta_{e\gamma} = \theta_Y$$

3) Same NP for all operators

$$\Rightarrow \quad \theta_{e\gamma} = \theta_Y = \theta_{lequ^{(3)}} = \theta_{lequ^{(1)}}$$

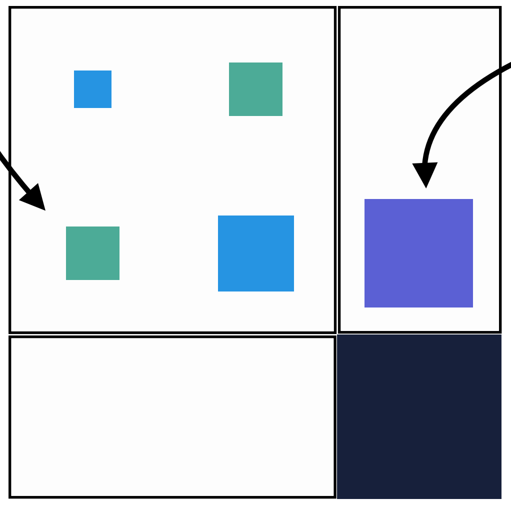
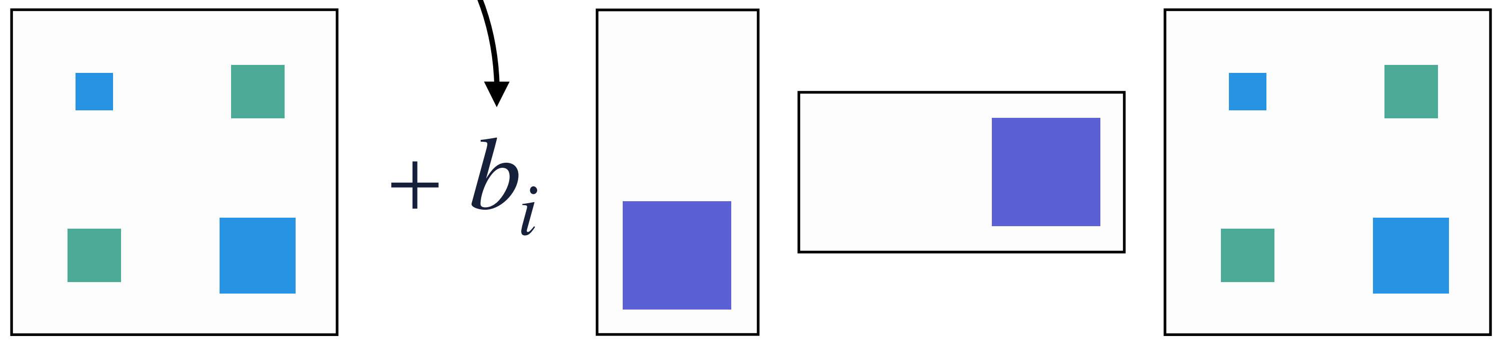
Flavor Symmetries

1) $U(2)_{\ell_L} \times U(2)_{e_R}$ symmetry [Barbieri, Isidori, Jones-Pérez, Lodone, Straub, 1105.2296]

natural expectation $\mathcal{O}\left(\sqrt{m_e/m_\mu}\right)$

$\mathcal{O}\left(10^{-1}\right)$ for B anomalies

$\mathcal{O}(1)$ coefficients

Spurions: $Y_e = y_\tau$  $\Rightarrow$ operators flavor structure in 1-2 sector = a_i 

Flavor phase: $\theta_i \approx \left(1 - \frac{b_i}{a_i}\right)$ Alignment: $\theta_i - \theta_j = \left(\frac{b_j}{a_j} - \frac{b_i}{a_i}\right) < 2 \times 10^{-5}$ $\Rightarrow$ unnatural tuning

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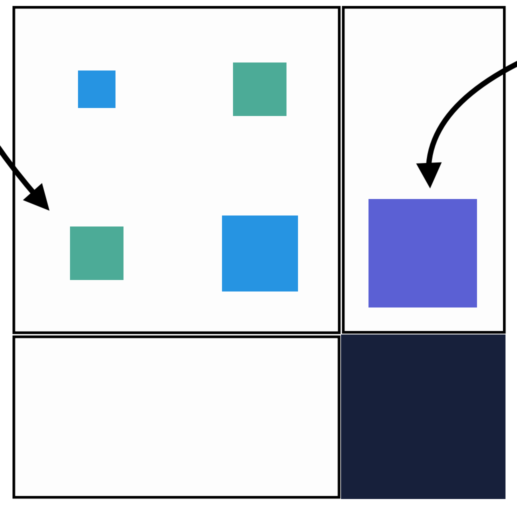
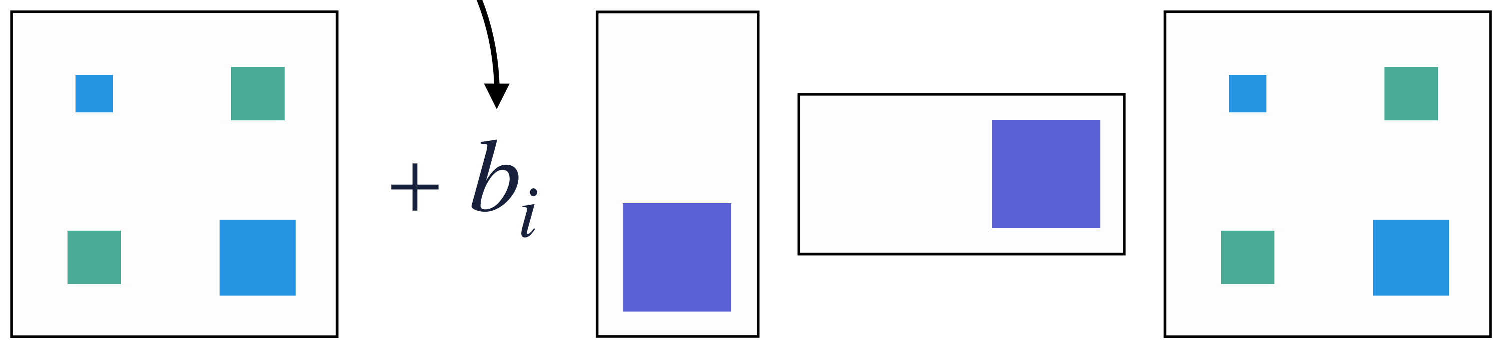
Spurions: $Y_e = y_\tau$  $\Rightarrow$ operators flavor structure in 1-2 sector $= a_i$ 

Flavor phase: $\theta_i \approx ■ \left(1 - \frac{b_i}{a_i} ■^2\right)$ Alignment: $\theta_i - \theta_j = ■ ■^2 \left(\frac{b_j}{a_j} - \frac{b_i}{a_i}\right) < 2 \times 10^{-5}$ $\Rightarrow$ unnatural tuning

2) $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$ symmetry

see talk by Admir

Same as 1) with ■ $=$ ■ $= 0$ $\Rightarrow$ protect completely from LFV

For the 1-2 sector any combination of $U(1)_{a\,L_\mu + b\,L_\tau}$ is enough

Example of alignment in explicit NP Model

UV theory: scalar leptoquark $S_1 \sim (\bar{3},1)_{1/3}$ additional Higgs $\Phi \sim (1,2)_{1/2}$

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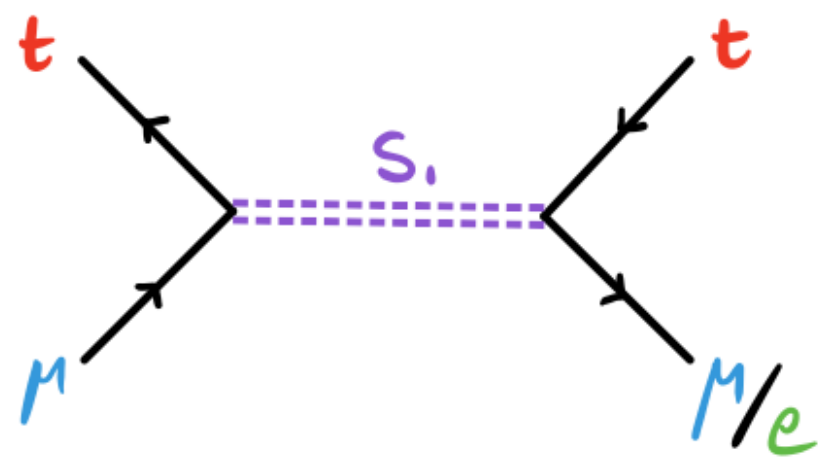

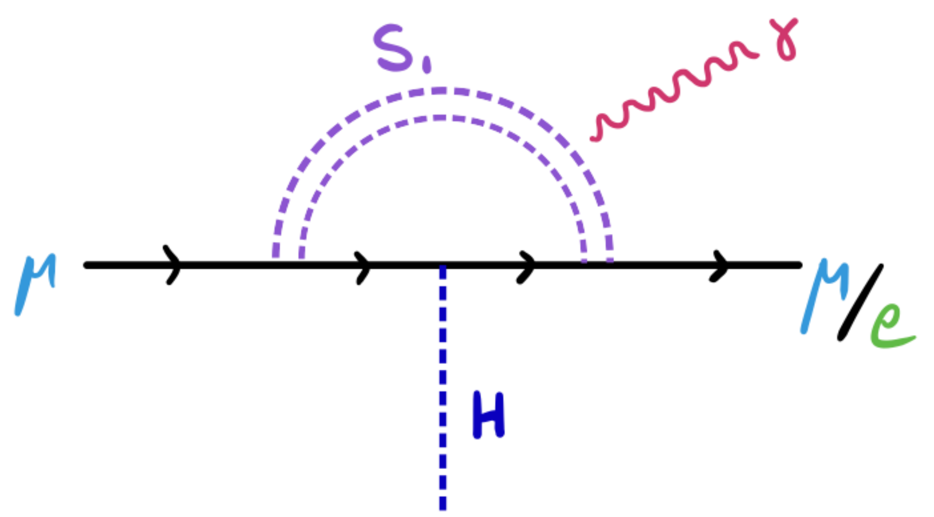

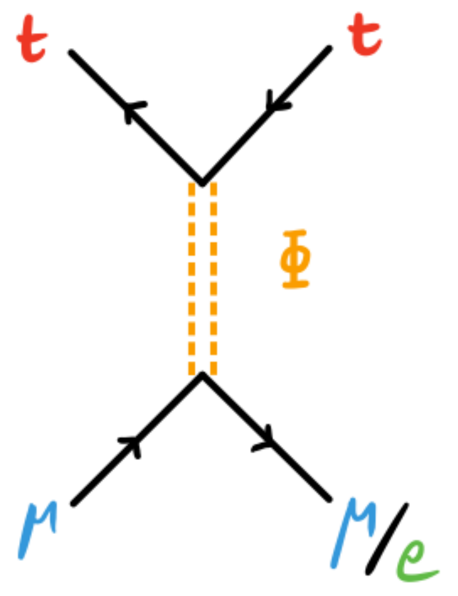

Match to: $C_{lequ}^{(1)}, C_{lequ}^{(3)}$ $C_{e\gamma}$ $C_{lequ}^{(1)}$

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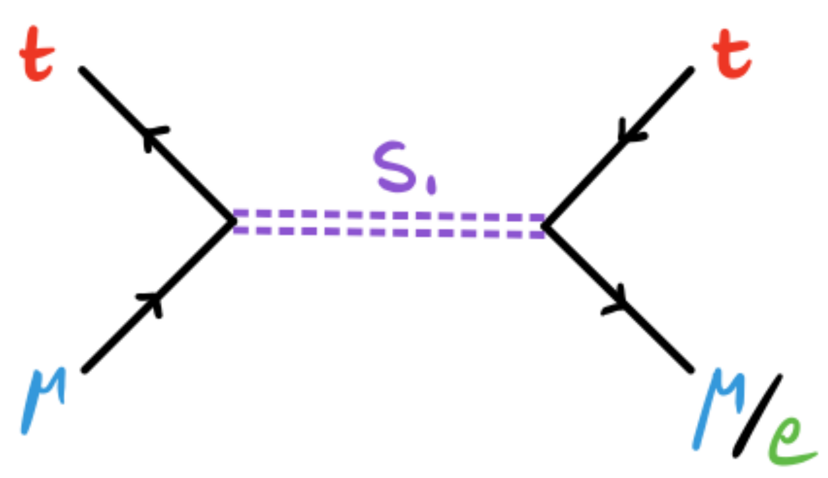

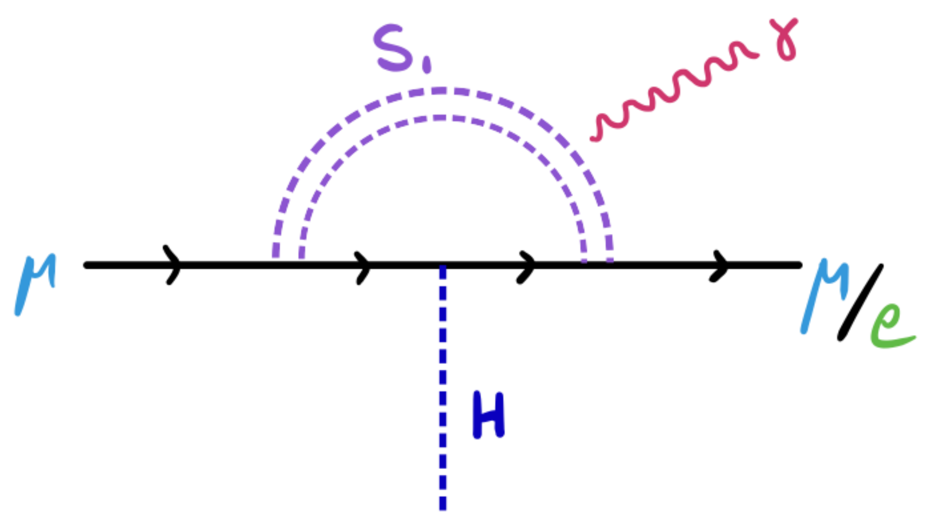

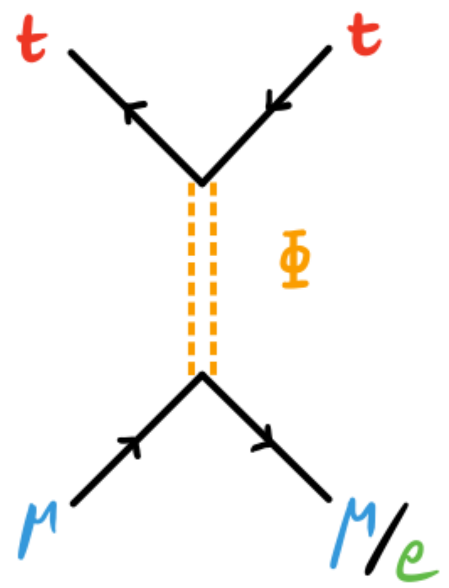

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Assumptions to obtain alignment:

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- coupling to left-handed top dominant $\rightarrow$ $\theta_{e\gamma} = \theta_{lequ^{(3)}}$
- $U(2)^2$ for Yukawa coupling? $\rightarrow$ $\theta_Y - \theta_{e\gamma} \approx$ ■ $< 2 \times 10^{-5}$

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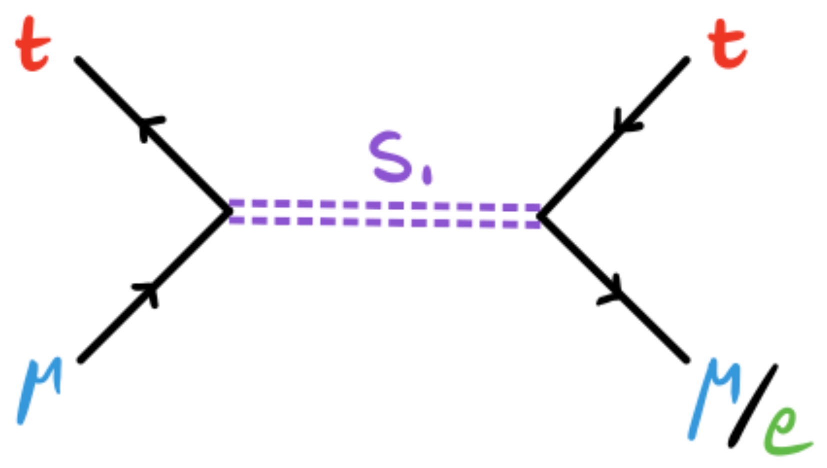

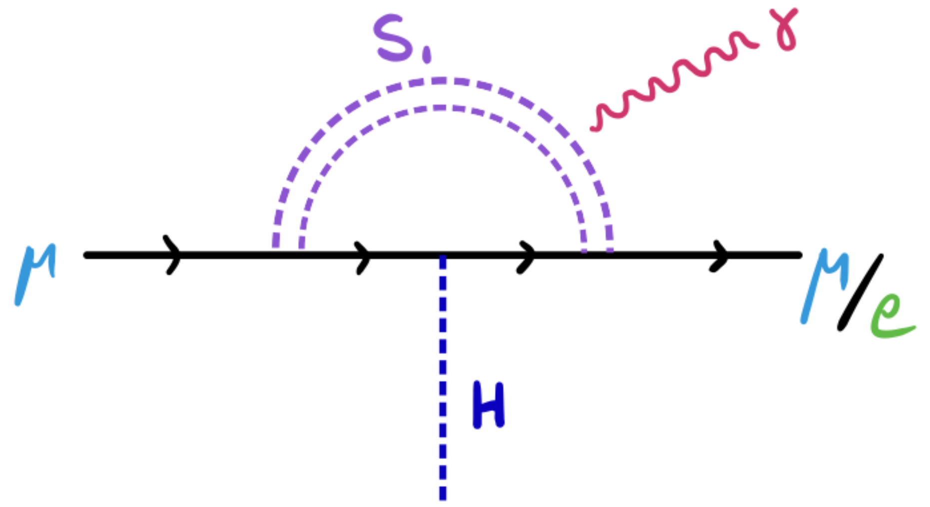

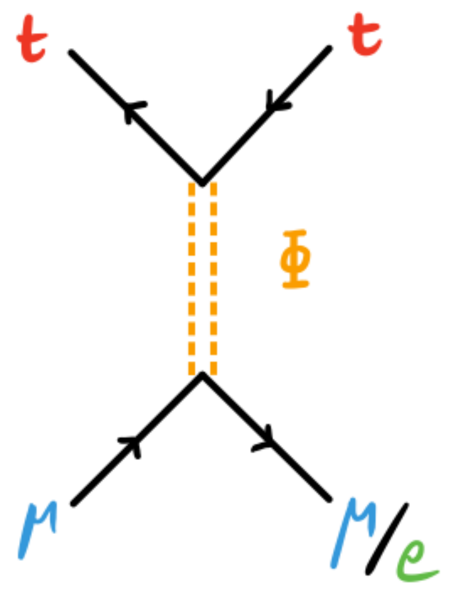

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Tension in aligning: $\theta_Y \longleftrightarrow \theta_{e\gamma} \longleftrightarrow \theta_{lequ^{(3)}}$

Conclusion

- Tight bound on flavor alignment in dipole operator in SMEFT from $(g-2)_\mu$ and ~~$\mu \to e\gamma$~~

- Misaligned NP at high-scale can spoil alignment at low-scale
  - through direct RGE contribution to the dipole $C^{(3)}_{lequ}$
  - through rotation to the mass basis Y_e , $C^{(1)}_{lequ}$ *provided* $(g-2)_\mu$ *is confirmed*

$\rightarrow$ impose constraints on some 4-fermion operators

- Flavor symmetries and/or Dynamical mechanism can help explain flavor alignment
  - $U(2)_{\ell_L} \times U(2)_{e_R}$ ∿
  - $U(1)_{a L_\mu + b L_\tau}$ ✓
  - $\theta_{lequ^{(3)}} = \theta_{lequ^{(1)}}$ from matching & RG dipole and Yukawa
  - $\theta_{e\gamma} = \theta_Y$ from matching & $C^{(1)}_{lequ}$, $C^{(3)}_{lequ}$ not generated
  - $\theta_{e\gamma} = \theta_Y = \theta_{lequ^{(3)}} = \theta_{lequ^{(1)}}$ from matching

$\Rightarrow$ If $(g-2)_\mu$ anomaly is a sign of NP, quark sector $\neq$ lepton sector beyond the SM

Thank you for your attention!

Back-up slides

Dipole 12 element in mass basis after RGE

$O_{\substack{ledq\\prst}} = (\bar{\ell}^j_{Lp} e_{Rr})(\bar{d}_{Rs} q_{Ltj})$

RGE for dipole and mass Yukawa

$\hat{L} = \frac{1}{16\pi^2}\log\left(\frac{\mu_H}{\mu_L}\right)$

$$\mathcal{C}_{\substack{e\gamma\\ij}}(\mu_L) = \left[1 - 3\hat{L}\left(y_t^2 + y_b^2\right)\right]\mathcal{C}_{\substack{e\gamma\\ij}}(\mu_H) - \left[16\hat{L}y_t e\right] C^{(3)}_{\substack{lequ\\ij33}}(\mu_H)\,,$$

$$[\mathcal{Y}_e]_{ij}(\mu_L) = [Y_e]_{ij}(\mu_H) - \frac{v^2}{2}C_{\substack{eH\\ij}}(\mu_H) + 6v^2\hat{L}\left[y_t^3 C^{(1)}_{\substack{lequ\\ij33}} - y_b^3 C_{\substack{ledq\\ij33}} + \frac{3}{4}(y_t^2 + y_b^2)C_{\substack{eH\\ij}}\right]_{\mu_H}$$

LFV Dipole in terms of high-scale quantities

$$\begin{aligned}
\mathcal{C}'_{\substack{e\gamma\\12}}(\mu_L) \;=\;& (\theta_L^{e\gamma} - \theta_L^{Y})\mathcal{C}_{\substack{e\gamma\\22}}(\mu_L) + (\theta_L^{e\gamma} - \theta_L^{u_3})(16\hat{L}e y_t)C^{(3)}_{\substack{lequ\\2233}}(\mu_H)\\
&+ \left[(\theta_L^{Y} - \theta_L^{u_1})(6y_t^3)C^{(1)}_{\substack{lequ\\2233}}(\mu_H) + (\theta_L^{d} - \theta_L^{Y})(6y_b^3)C_{\substack{ledq\\2233}}(\mu_H)\right]\frac{1}{[\mathcal{Y}_e]_{22}(\mu_L)}\hat{L}v^2\mathcal{C}_{\substack{e\gamma\\22}}(\mu_L)\\
&+ (\theta_L^{eH} - \theta_L^{Y})\frac{1 - 9(y_t^2 + y_b^2)\hat{L}}{2}C_{\substack{eH\\22}}(\mu_H)\frac{1}{[\mathcal{Y}_e]_{22}(\mu_L)}v^2\mathcal{C}_{\substack{e\gamma\\22}}(\mu_L)\,.
\end{aligned}$$

Explicit NP model Lagrangian and flavor phases

$S_1 \sim (\bar{3},1)_{1/3}$ scalar leptoquark + $\Phi \sim (1,2)_{1/2}$ scalar doublet

UV Lagrangian:

$$\mathcal{L}_{S_1} = \mathcal{L}_{SM} + (D_\mu S_1)^\dagger (D^\mu S_1) - M_{S_1}^2 S_1^\dagger S_1 - \left[\lambda^L_{i\alpha}(\bar{q}^c_i \epsilon \ell_\alpha) S_1 + \lambda^R_{i\alpha}(\bar{u}^c_i e_\alpha) S_1 + \text{h.c.}\right]$$
$$+ (D_\mu \Phi)^\dagger (D^\mu \Phi) - M_\Phi^2 \Phi^\dagger \Phi - \left[\lambda^e_{\alpha\beta}(\bar{\ell}_\alpha e_\beta)\Phi + \lambda^u_{ij}(\bar{q}_i u_j)\tilde{\Phi} + \text{h.c.}\right]$$

After matching:

$$\theta_L^{u_1} = \frac{\lambda^{L\,*}_{31}\lambda^R_{32} + 2\lambda^e_{12}\lambda^u_{33} M^2_{S_1}/M^2_\Phi}{\lambda^{L\,*}_{32}\lambda^R_{32} + 2\lambda^e_{22}\lambda^u_{33} M^2_{S_1}/M^2_\Phi}$$

$$\theta_L^{u_3} = \frac{\lambda^{L\,*}_{31}}{\lambda^{L\,*}_{32}}$$

$$\theta_L^{e\gamma} = \frac{(Y_e)_{1\alpha}\lambda^{R\,*}_{i\alpha}\lambda^R_{i2} + \lambda^{L\,*}_{i1}\lambda^L_{i\alpha}(Y_e)_{\alpha 2} - 14\, y_t \lambda^{L\,*}_{31}\lambda^R_{32}}{(Y_e)_{2\alpha}\lambda^{R\,*}_{i\alpha}\lambda^R_{i2} + \lambda^{L\,*}_{i2}\lambda^L_{i\alpha}(Y_e)_{\alpha 2} - 14\, y_t \lambda^{L\,*}_{32}\lambda^R_{32}}$$

Alignment in the 2-3 sector

Flavor Alignment from $\tau \to \mu\gamma$

$$\mathcal{B}(\tau^\pm \to \mu^\pm\gamma) < 4.4 \times 10^{-8} \ (90\% \ \text{CL}) \qquad \Rightarrow \qquad |\mathcal{C}'_{\underset{23(32)}{e\gamma}}| < 2.7 \times 10^{-6}\,\text{TeV}^{-2}$$

Flavor alignment in 2-3: $$|\epsilon^L_{23}|\,,\ |\epsilon^R_{23}| < 1.6 \times 10^{-2} \times \left| \frac{y_\tau\, \mathcal{C}'_{\underset{22}{e\gamma}}}{y_\mu\, \mathcal{C}'_{\underset{33}{e\gamma}}} \right|$$