Muon g-2: the SM prediction

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La Thuile 2022
Les Rencontres de Physique de la Vallée d'Aoste
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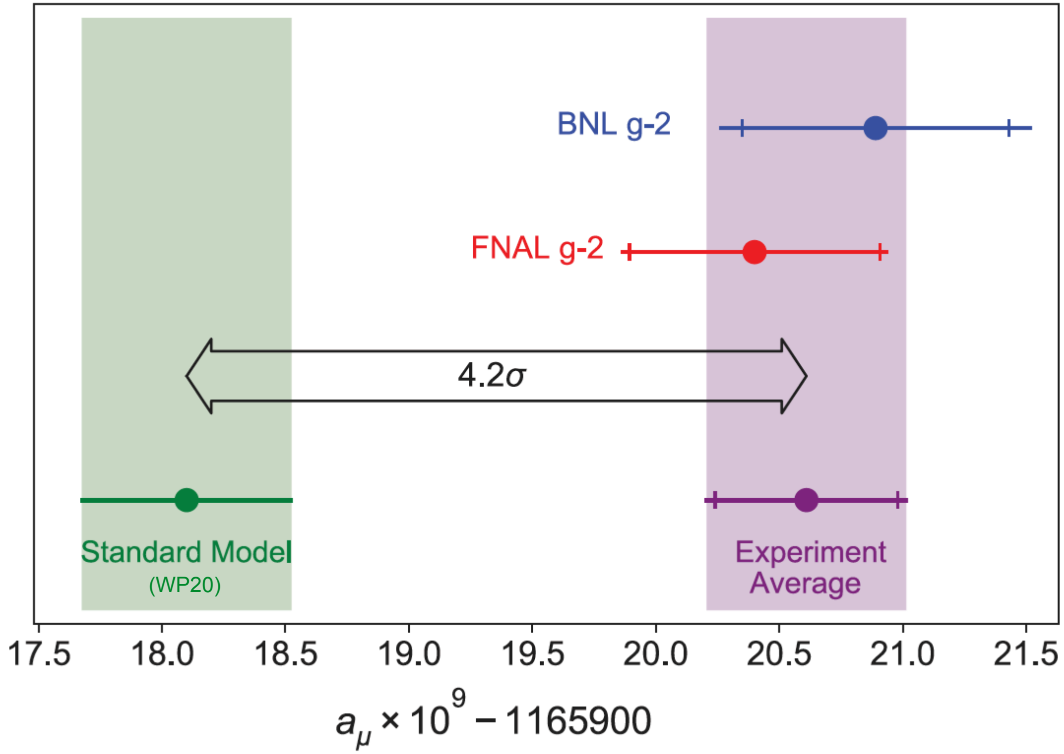
Alberto Sirlin (1930 - 2022)

A brilliant physicist, a superb mentor, and a dear friend

Muon g-2: FNAL confirms BNL



3.7 σ

3.3 σ

4.2 σ

$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821

$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1

$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

- **FNAL aims at 16 x 10^{-11}. First 4 runs completed, 5th in progress.**
- **Muon g-2 proposal at J-PARC: Phase-1 with ~ BNL precision.**

Muon g-2: the Standard Model prediction

WP20 = White Paper of the Muon g-2 Theory Initiative: arXiv:2006.04822

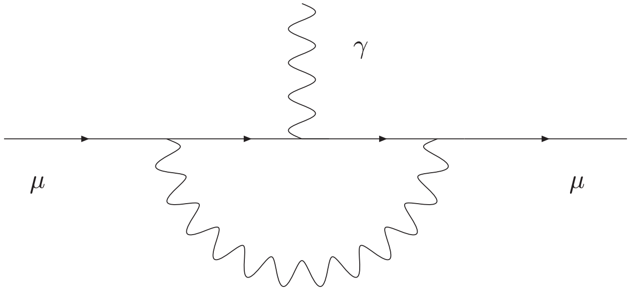
The beginning

- 1948: Schwinger, using Quantum ElectroDynamics (QED), predicts

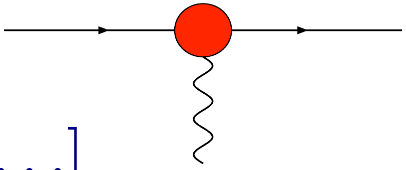
a = (g-2)/2 = α/(2π) = 0.00116

in perfect agreement with Kusch & Foley's measurement

a = (g-2)/2 = 0.00119(5)



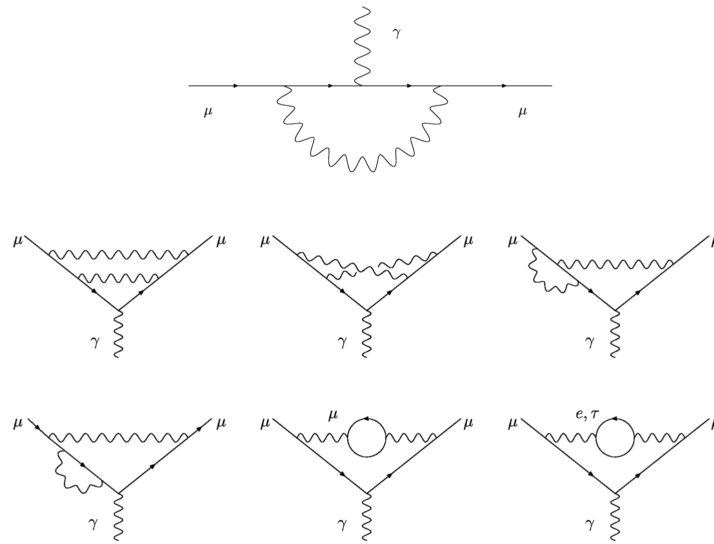
- Tremendous quantitative triumph for relativistic QFT (QED).
- Today we keep studying the lepton-photon vertex:



$$\Gamma^\mu = ie[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2) + \ldots]$$

$$\boxed{F_1(0) = 1 \qquad F_2(0) = a}$$

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948



$+\ 0.765857426\ (16)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann ’57; Elend ’66; MP ’04

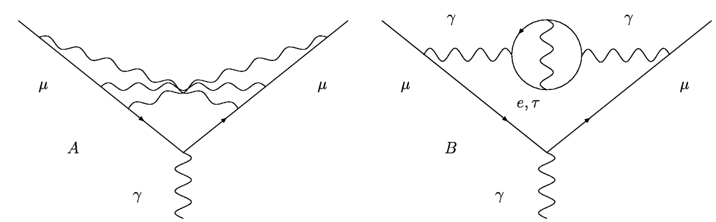
$+\ 24.05050988\ (28)\ (\alpha/\pi)^3$

Remiddi, Barbieri, Laporta … ; Czarnecki, Skrzypek ’99; MP ’04;
Friot, Greynat & de Rafael ’05, Ananthanarayan, Friot, Ghosh 2020

$+\ 130.8780\ (60)\ (\alpha/\pi)^4$

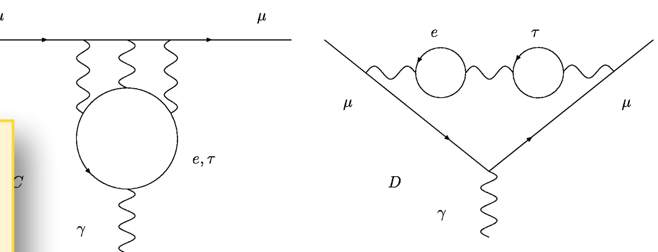
Kinoshita & Lindquist ’81, … , Kinoshita & Nio ’04, ’05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
Laporta, PLB 2017 (mass independent term) COMPLETED²!

$+\ 750.86\ (88)\ (\alpha/\pi)^5$ COMPLETED!



Kinoshita et al. ‘90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012, 2015, 2017 & 2019.
Volkov 1909.08015: $A_1^{(10)}$[no lept loops] at variance, but negligible $\delta a_\mu \sim 6\times10^{-14}$

Adding up, we get:



$a_\mu^{QED} = 116584718.931\ (19)(100)(23) \times 10^{-11}$

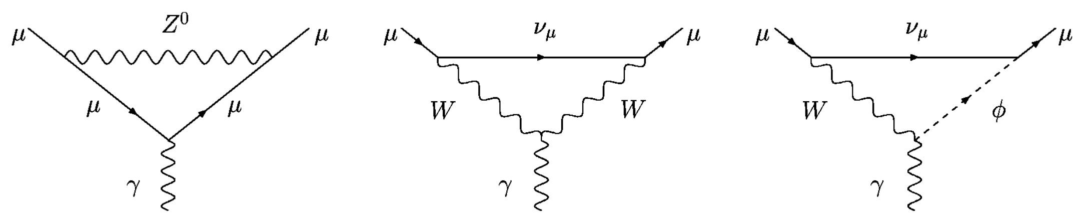
mainly from 4-loop coeff. unc. 6-loop from α(Cs)

α = 1/137.035999046(27) [0.2ppb] Parker et al 2018 WP20 value

Shift down in $a_\mu^{QED} \sim 1.4 \times 10^{-12}$ with new LKB Paris α(Rb) value (Morel et al 2020)

The electroweak contribution

- **One-loop term:**



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

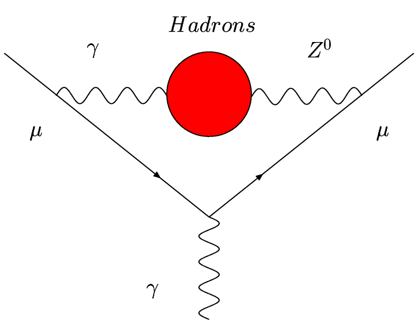
- **One-loop plus higher-order terms:**

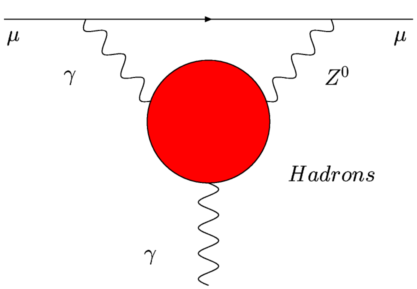
$\boxed{a_\mu^{\rm EW} = 153.6\ (1.0) \times 10^{-11}}$

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.

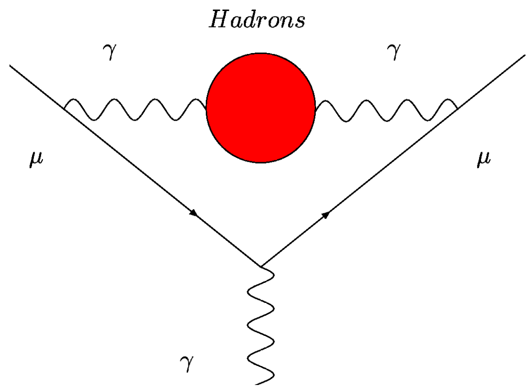
Hadronic loop uncertainties (and 3-loop nonleading logs).

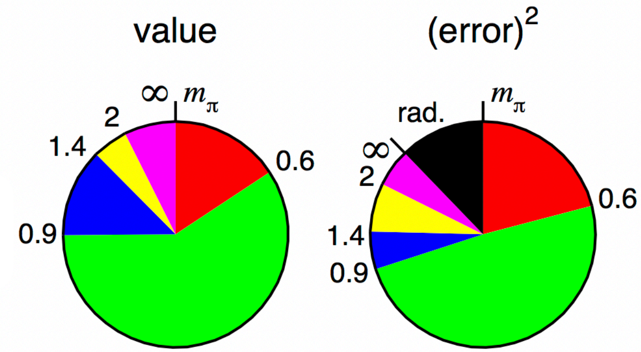
WP20 value





The hadronic LO contribution





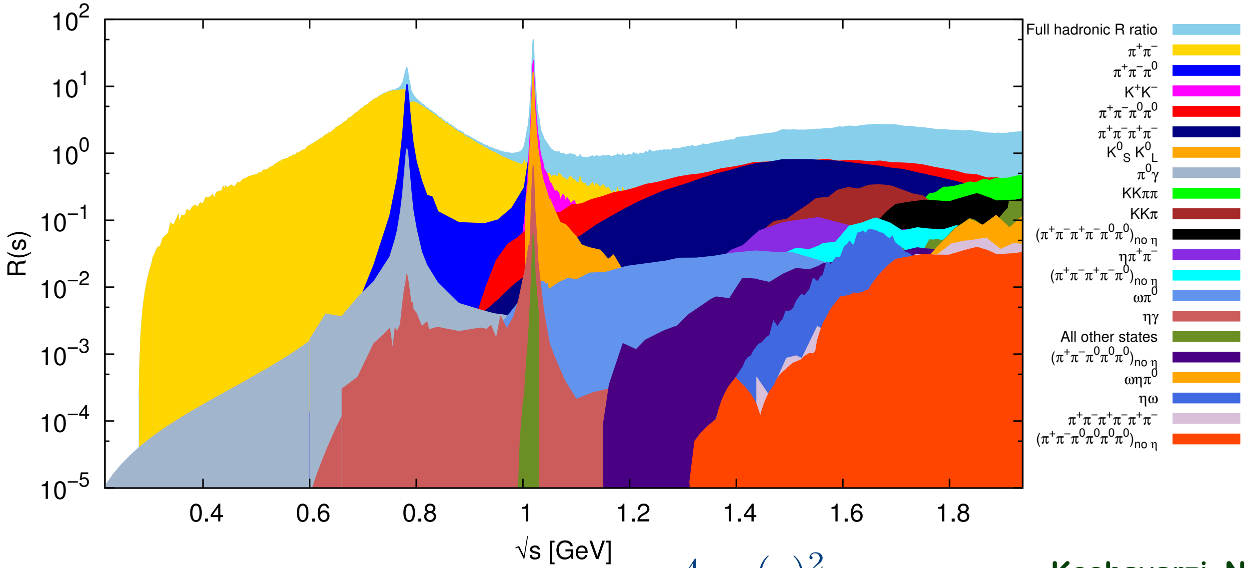
Keshavarzi, Nomura, Teubner 2018

$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}_{\rm had}(s) \qquad K(s) = \int_0^1 dx \frac{x^2\,(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

$a_\mu^{\rm HLO}$ = 6895 (33) x 10^{-11}	F. Jegerlehner, arXiv:1711.06089
= 6939 (40) x 10^{-11}	Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921
= 6928 (24) x 10^{-11}	Keshavarzi, Nomura, Teubner, arXiv:1911.00367
= 6931 (40) x 10^{-11} (0.6%)	WP20 value

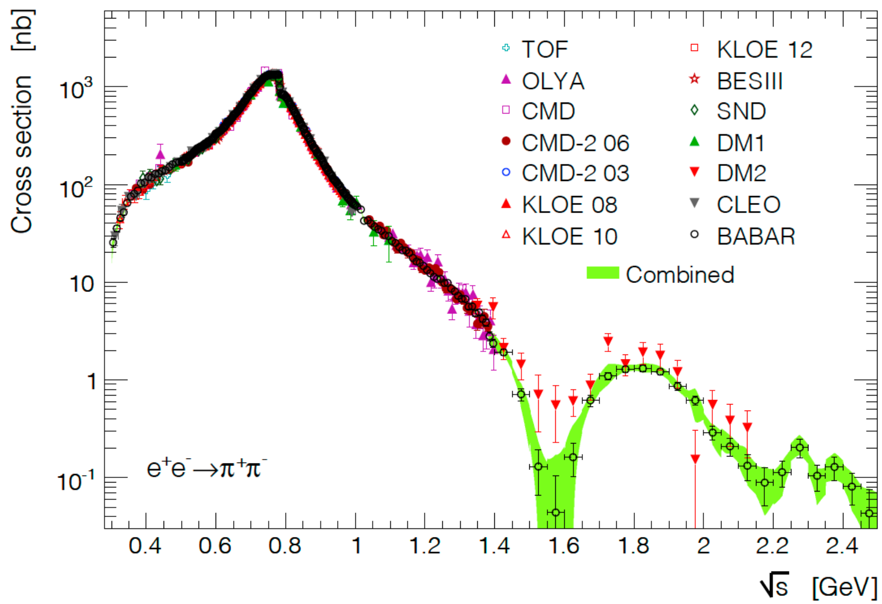
- **WP20 value obtained merging conservatively DHMZ + KNT + constraints from CHHKS**
  Colangelo, Hoferichter, Hoid, Kubis, Stoffer 2018-19
- **Radiative Corrections to σ(s) are crucial.**
  S. Actis et al, Eur. Phys. J. C66 (2010) 585

The low-energy hadronic cross section



$$R(s) = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$

Keshavarzi, Nomura Teubner PRD 2018

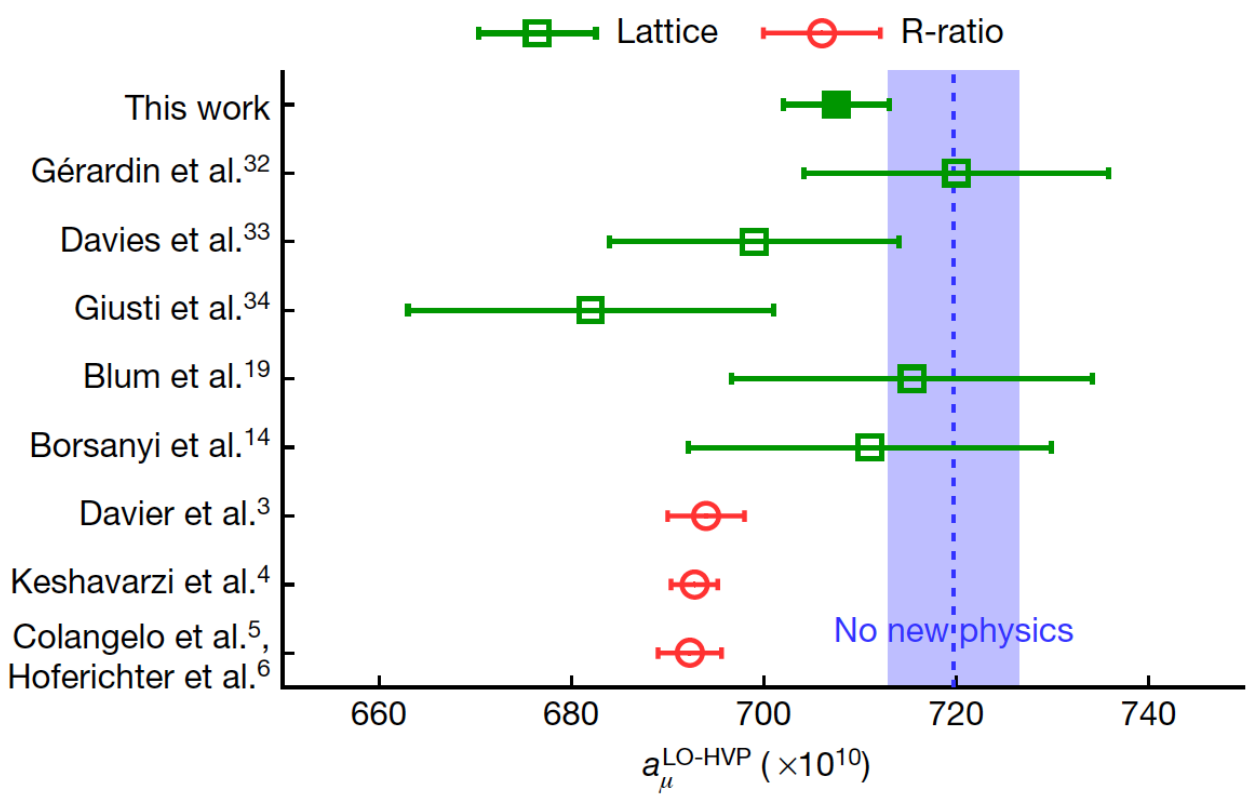


Davier, Hoecker, Malaescu, Zhang EPJC 2020

- **Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:**

$$a_\mu^{HLO} = 7075(23)_{stat}(50)_{syst}\ [55]_{tot} \times 10^{-11}$$

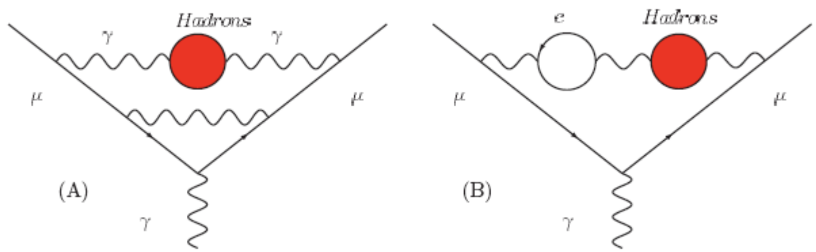
- **2–2.5σ tension with the dispersive evaluations.** BMW collaboration 2021



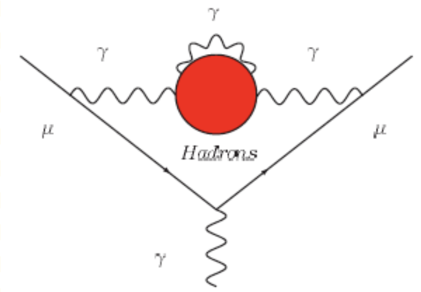
Borsanyi et al (BMWc), Nature 2021

The hadronic HO VP contribution

- **O(α³) contributions of diagrams containing HVP insertions:**



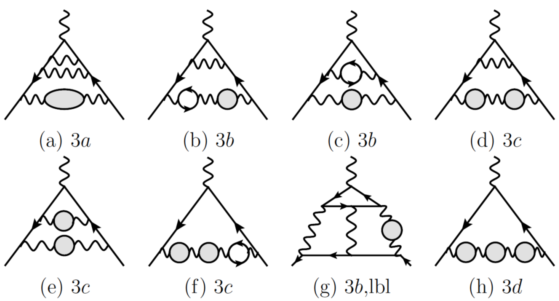
Already included in a_μ^{HLO}



$$a_\mu^{HNLO}(vp) = -98.3\ (7) \times 10^{-11}$$

Krause '96; Keshavarzi, Nomura, Teubner 2019; WP20.

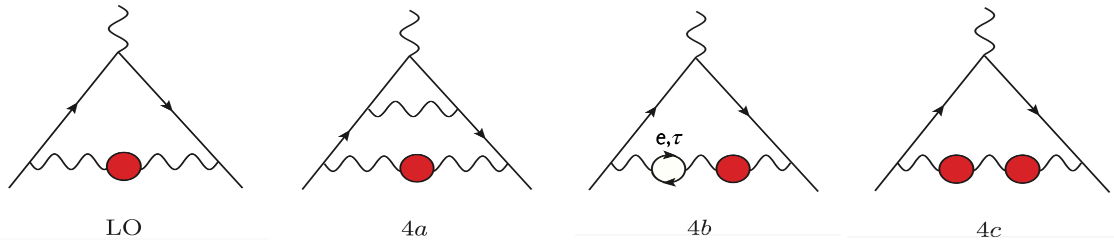
- **O(α⁴) contributions of diagrams containing HVP insertions:**



$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

The O(α² & α³) hadronic VP contributions — space-like approach



$$a_\mu^{\rm LO} = \frac{\alpha}{\pi}\int_0^1 {\rm d}x\, \kappa^{(2)}(x)\, \Delta\alpha_{\rm had}(t(x)), \qquad \kappa^{(2)}(x) = 1-x \qquad t(x) = \frac{{m_\mu}^2 x^2}{x-1}$$

$$a_\mu^{(4a)} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^1 {\rm d}x\, \kappa^{(4)}(x)\, \Delta\alpha_{\rm had}(t(x)), \qquad \kappa^{(4)}(x) = \frac{2\,(2-x)}{x\,(x-1)}\, F^{(4)}(x-1)$$

$$\begin{aligned} F^{(4)}(y) = {} & R_1(y) + R_2(y)\ln(-y) \\ & + R_3(y)\ln(1+y) + R_4(y)\ln(1-y) \\ & + R_5(y)\big[4{\rm Li}_2(y) + 2{\rm Li}_2(-y) \\ & \quad + \ln(-y)\ln\big((1-y)^2(1+y)\big)\big], \end{aligned}$$

$$R_1 = \frac{23y^6 - 37y^5 + 124y^4 - 86y^3 - 57y^2 + 99y + 78}{72(y-1)^2 y(y+1)},$$

$$R_2 = \frac{12y^8 - 11y^7 - 78y^6 + 21y^5 + 4y^4 - 15y^3 + 13y + 6}{12(y-1)^3 y(y+1)^2},$$

$$R_3 = \frac{(y+1)\left(-y^3 + 7y^2 + 8y + 6\right)}{12y^2},$$

$$R_4 = \frac{-7y^4 - 8y^3 + 8y + 7}{12y^2},$$

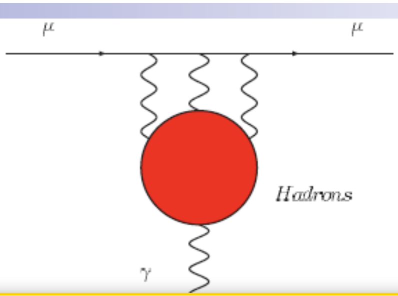
$$R_5 = -\,\frac{3y^4 + 5y^3 + 7y^2 + 5y + 3}{6y^2}$$

$$a_\mu^{(4b)} = \frac{\alpha}{\pi}\int_0^1 {\rm d}x\, \kappa^{(2)}(x)\, \Delta\alpha_{\rm had}(t(x)) \times 2\left[\Delta\alpha_e^{(2)}(t(x)) + \Delta\alpha_\tau^{(2)}(t(x))\right],$$

$$a_\mu^{(4c)} = \frac{\alpha}{\pi}\int_0^1 {\rm d}x\, \kappa^{(2)}(x)\, \left[\Delta\alpha_{\rm had}(t(x))\right]^2$$

- **These results allow to precisely compute $a_\mu^{\rm HVP}$ at NLO on the lattice & at MUonE → consistent comparisons, through NLO, with "time-like" results.**

Balzani, Laporta, MP, 2112.05704, A.V. Nesterenko, 2112.05009



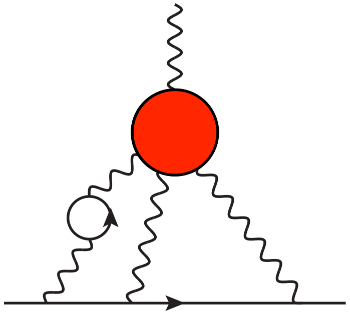
- **Hadronic light-by-light at $O(\alpha^3)$**
  - **This term had a troubled life! But nowadays:**

$a_\mu^{HNLO}(lbl) = 80\ (40) \times 10^{-11}$ Knecht & Nyffeler '02
$= 136\ (25) \times 10^{-11}$ Melnikov & Vainshtein '03
$= 105\ (26) \times 10^{-11}$ Prades, de Rafael, Vainshtein '09
$= 100\ (29) \times 10^{-11}$ Jegerlehner, arXiv:1705.00263
$= 92\ (19) \times 10^{-11}$ WP20 (phenomenology)

  - **Significant improvements due to data-driven dispersive approach.**
    Colangelo, Hoferichter, Kubis, Procura, Stoffer, 2014–17; Pauk, Vanderhaeghen 2014.
  - **Lattice: RBC: $82(35) \times 10^{-11}$** 1911.08123 **Mainz: $110(15) \times 10^{-11}$** 2104.02632

- **Hadronic light-by-light at $O(\alpha^4)$**

$$a_\mu^{HNNLO}(lbl) = 2\ (1) \times 10^{-11}$$



Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; WP20

- Comparing the SM prediction with the measured muon g-2 value:

$a_\mu^{EXP} = 116592061\ (41) \times 10^{-11}$ BNL+FNAL

$a_\mu^{SM} = 116591810\ (43) \times 10^{-11}$ WP20

$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = 251\ (59) \times 10^{-11}$ 4.2 σ

If BMW 2021 HLO instead of WP20, EXP & SM differ only by 1.6σ

Is Δa_μ due to new physics beyond the SM?

See Andreas Crivellin and Admir Greljo talks

- Can Δa_μ be due to missing contributions in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$\boxed{\begin{aligned} a &= \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), & f(s) &= \frac{K(s)}{4\pi^3},\ s_u < M_Z^2, \\ b &= \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), & g(s) &= \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}}$$

and the increase

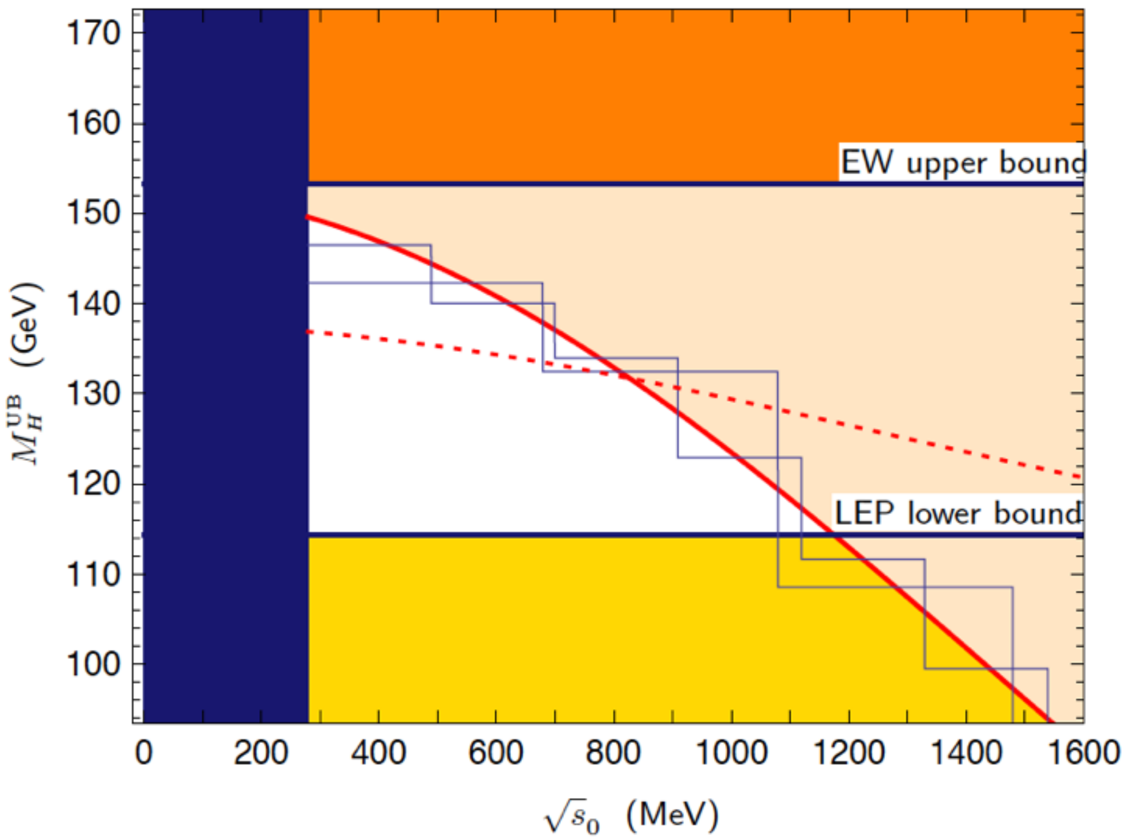
$$\Delta\sigma(s) = \epsilon\sigma(s)$$

ε>0, in the range:

$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2] \longrightarrow$$

Marciano, MP, Sirlin, PRD 2008

How much does the M_H upper bound from the EW fit change when we shift up $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to fix Δa_μ ?



Marciano, MP, Sirlin, PRD 2008

Muon g-2: connection with the SM Higgs mass (update) $\Delta\alpha$

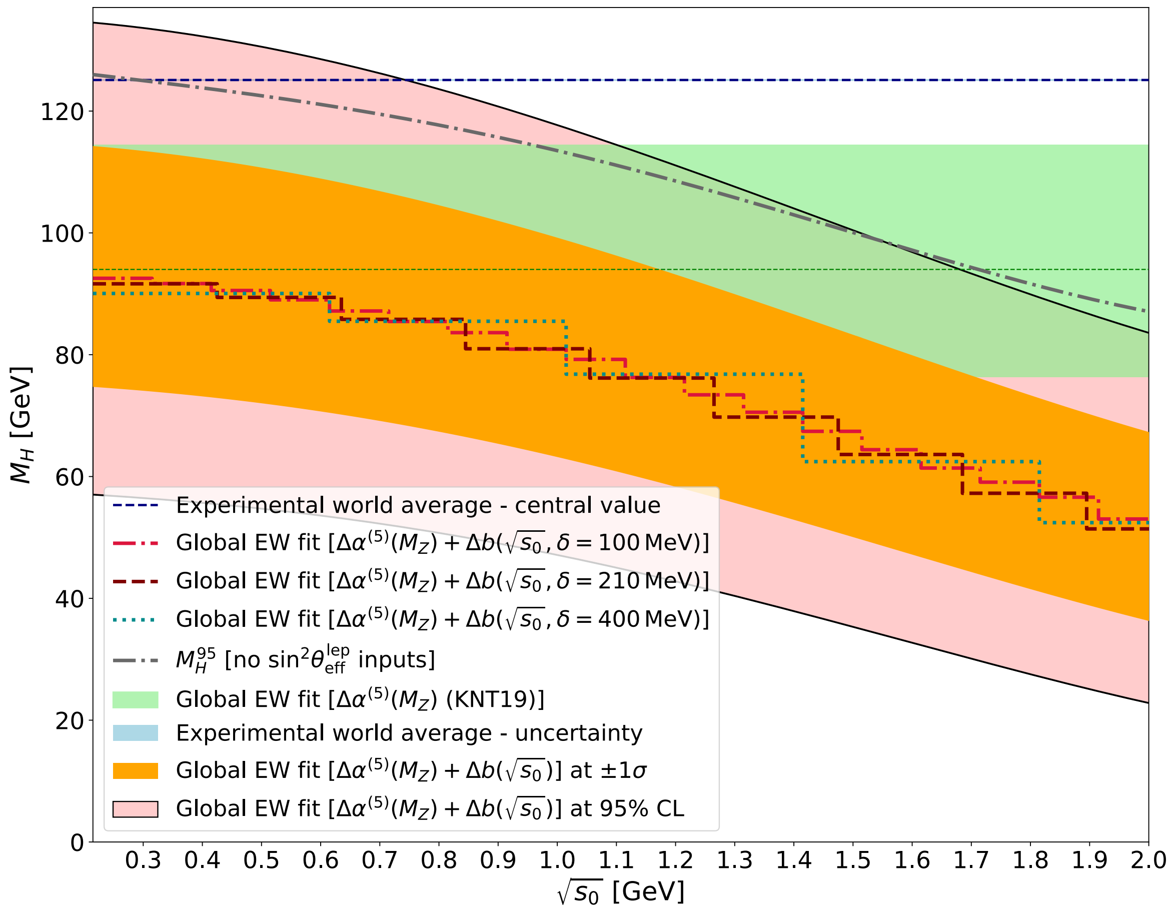
Major update: Higgs discovered, improved EW observables (M_W, $\sin^2\theta$, M_{top}, …), updates to $\sigma(s)$, theory improvements, global fit, …

Parameter	Input value	Reference	Fit result	Result w/o input value
M_W (GeV)	80.379(12)	[5]	80.359(3)	80.357(4)(5)
M_H (GeV)	125.10(14)	[5]	125.10(14)	94^{+20+6}_{-18-6}
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \times 10^4$	276.1(1.1)	[23]	275.8(1.1)	272.2(3.9)(1.2)
m_t (GeV)	172.9(4)	[5]	173.0(4)	…
$\alpha_s(M_Z^2)$	0.1179(10)	[5]	0.1180(7)	…
M_Z (GeV)	91.1876(21)	[5]	91.1883(20)	…
Γ_Z (GeV)	2.4952(23)	[5]	2.4940(4)	…
Γ_W (GeV)	2.085(42)	[5]	2.0903(4)	…
$\sigma^0_{\rm had}$ (nb)	41.541(37)	[108]	41.490(4)	…
R^0_l	20.767(25)	[108]	20.732(4)	…
R^0_c	0.1721(30)	[108]	0.17222(8)	…
R^0_b	0.21629(66)	[108]	0.21581(8)	…
$\bar{m}_c$ (GeV)	1.27(2)	[5]	1.27(2)	…
$\bar{m}_b$ (GeV)	$4.18^{+0.03}_{-0.02}$	[5]	$4.18^{+0.03}_{-0.02}$	…
$A^{0,l}_{\rm FB}$	0.0171(10)	[108]	0.01622(7)	…
$A^{0,c}_{\rm FB}$	0.0707(35)	[108]	0.0737(2)	…
$A^{0,b}_{\rm FB}$	0.0992(16)	[108]	0.1031(2)	…
A_ℓ	0.1499(18)	[75,108]	0.1471(3)	…
A_c	0.670(27)	[108]	0.6679(2)	…
A_b	0.923(20)	[108]	0.93462(7)	…
$\sin^2\theta^{\rm lep}_{\rm eff}(Q_{\rm FB})$	0.2324(12)	[108]	0.23152(4)	0.23152(4)(4)
$\sin^2\theta^{\rm lep}_{\rm eff}$(Had Coll)	0.23140(23)	[100]	0.23152(4)	0.23152(4)(4)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (using Gfitter)

Muon g-2: connection with the SM Higgs mass (2020)

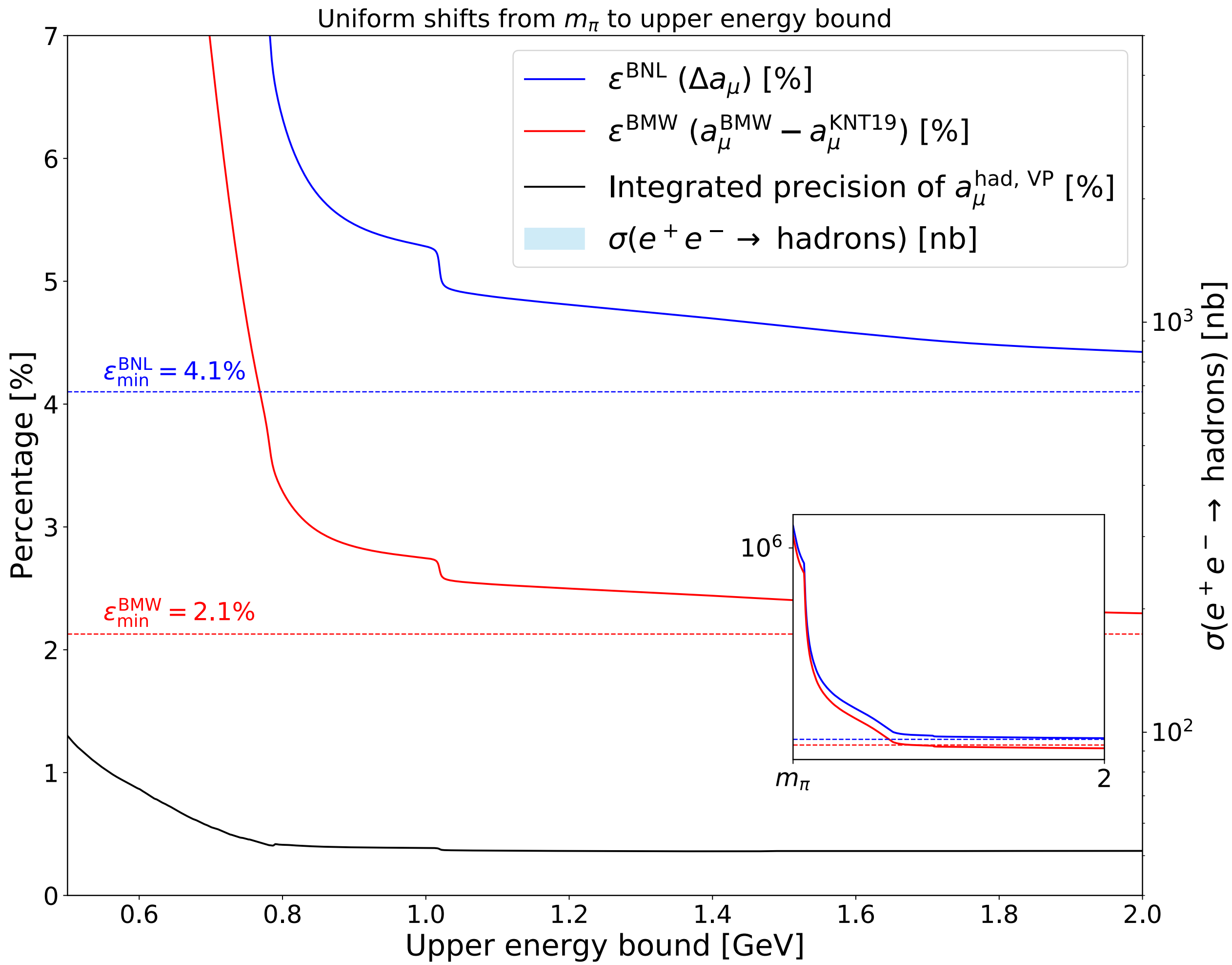
Δα



Shifts $\Delta\sigma(s)$ to fix Δa_μ are possible, but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

How large are the required shifts $\Delta\sigma(s)$?



Shifts below ~1 GeV conflict with the quoted exp. precision of $\sigma(s)$

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (updated 2021)

$a_\mu \Longleftrightarrow \Delta\alpha$: related literature

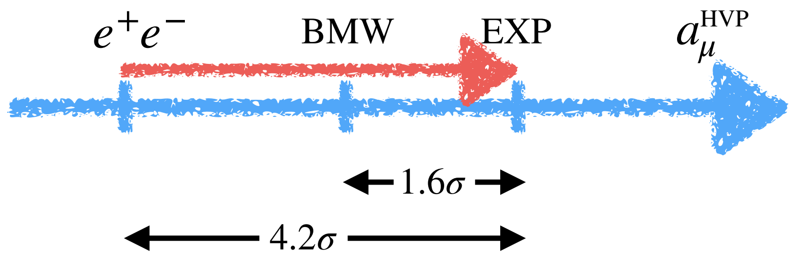
- Crivellin, Hoferichter, Manzari and Montull, "Hadronic vacuum polarization: $(g\text{-}2)_\mu$ versus global electroweak fits," arXiv:2003.04886. See Andreas' talk.

- Eduardo de Rafael, "On Constraints Between $\Delta\alpha_{had}(M_Z^2)$ and $(g_\mu\text{-}2)_{HVP}$,'' arXiv:2006.13880.

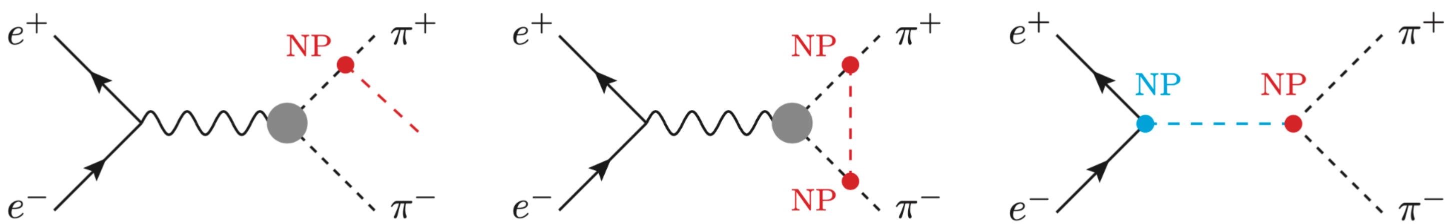
- Malaescu and Schott, "Impact of correlations between a_μ and α_{QED} on the EW fit," arXiv:2008.08107.

- Colangelo, Hoferichter and Stoffer, "Constraints on the two-pion contribution to hadronic vacuum polarization," arXiv:2010.07943.

A new muon g-2 puzzle: a_μ^{HLO} dispersive vs BMW



- Can this puzzle be solved by NP in the hadronic cross section $\sigma(s)$?
- NP may couple only to hadrons (small effects) or both to hadrons & electrons:



- If NP enters $\sigma(s)$ at tree-level, then σ in the dispersive integral should be replaced by $\sigma - \Delta\sigma^{NP}$, with $\Delta\sigma^{NP} < 0$ needed to increase the dispersive a_μ^{HLO}
- This sizeable negative NP interference with the SM is excluded by a number of experimental constraints.

Di Luzio, Masiero, Paradisi, MP, 2112.08312

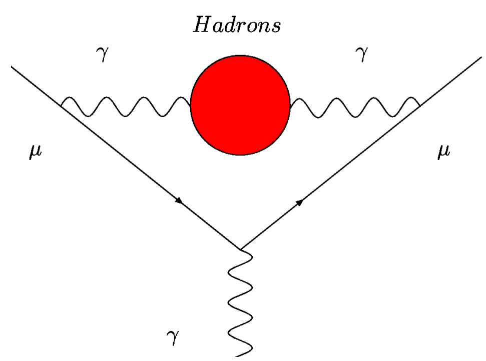
- NP in e^+e^- & $\mu^+\mu^-$ final states scatterings?

Darmé, Grilli di Cortona, Nardi, 2112.09139

The MUonE project

The spacelike method for a_μ^{HLO}

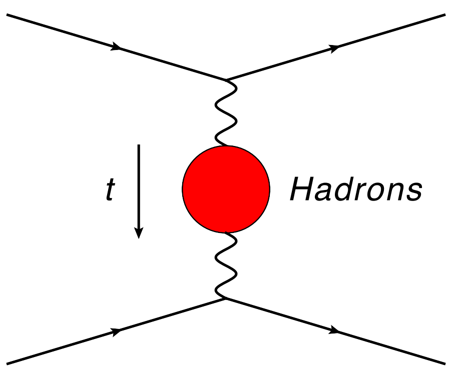
- **Leading hadronic contribution computed via the usual dispersive (timelike) formula:**



$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}_{\mathrm{had}}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2\,(1-x)}{x^2 + (1-x)\left(s/m_\mu^2\right)}$$

- **Alternatively, simply exchanging the x and s integrations:**



$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

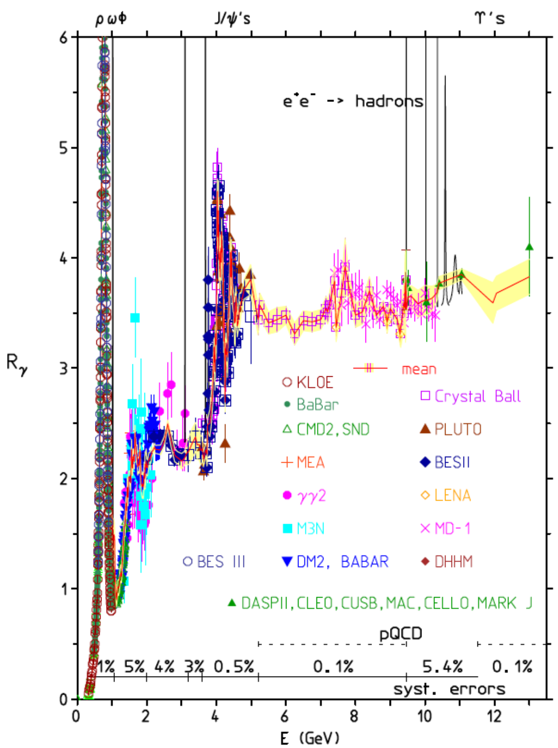
Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{had}$(t) is the hadronic contribution to the space-like running of α: proposal to measure a_μ^{HLO} via scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 2015

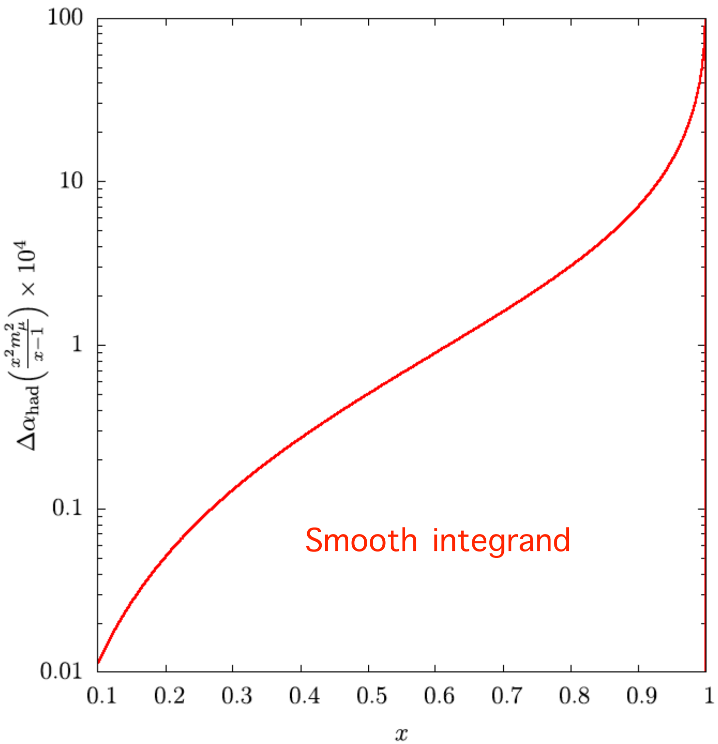
a_μ^{HLO} : timelike vs spacelike method

Timelike



F. Jegerlehner, arXiv:1511.04473

Spacelike



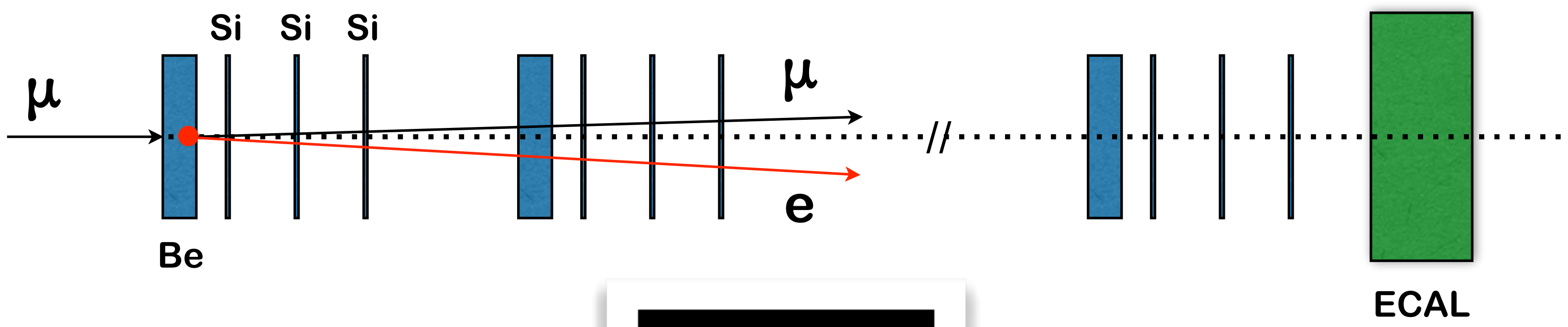
Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015

- ☑ **Inclusive measurement**
- ☑ **Smooth integrand**
- ☑ **Direct interplay with lattice QCD**

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.

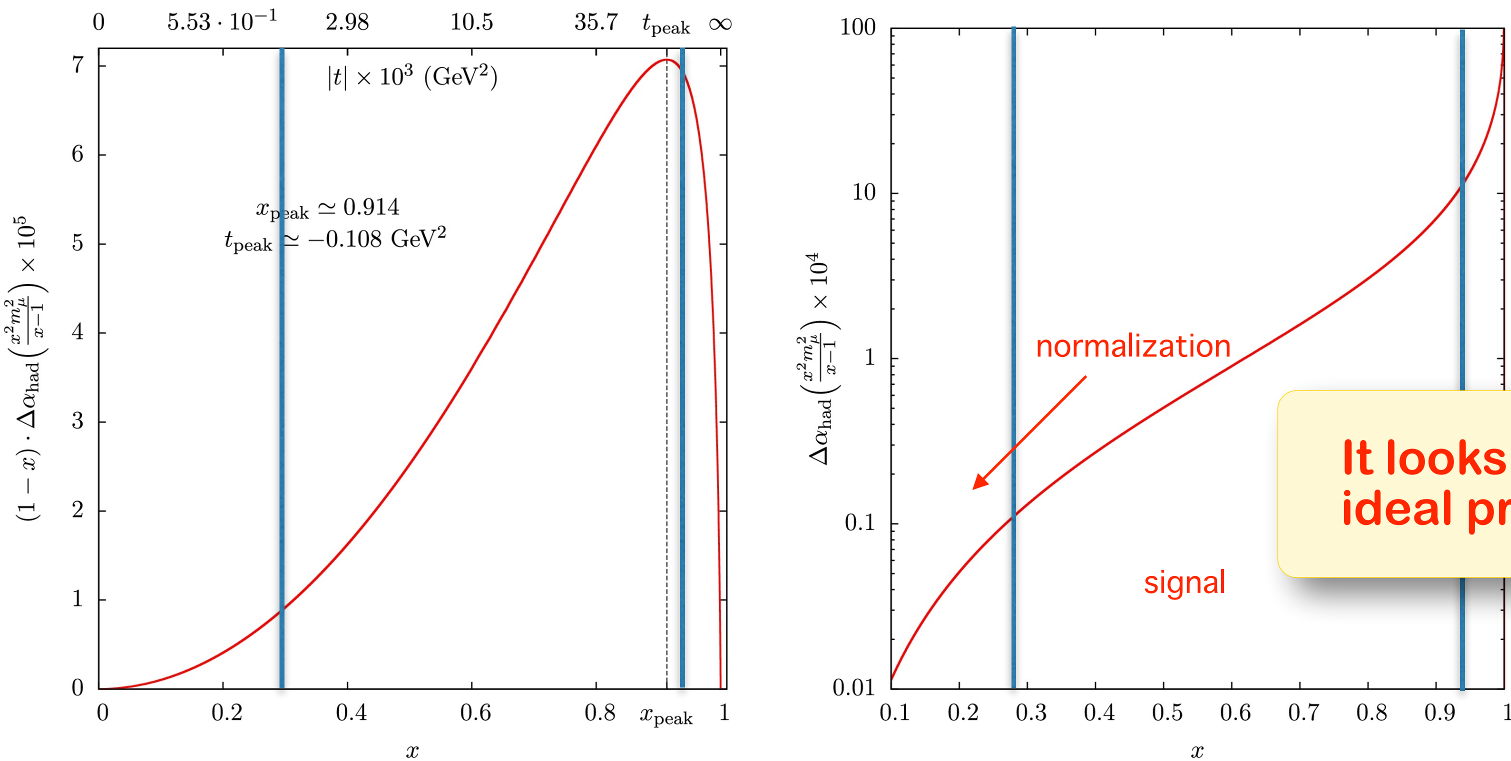
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.



Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni

EPJC 2017 - arXiv:1609.08987

- For a 150 GeV muon beam ($\sqrt{s}$~400 MeV), MUonE's scan region extends up to x=0.932, ie beyond the x=0.914 peak!



It looks like an ideal process!

MUonE — Status

- **Statistics**: With CERN's 150 GeV muon beam M2 (1.3×10^7 μ/s), incident on 40 15mm Be targets (total Be thickness: 60cm), 2-3 years of data taking (2×10^7 s/yr) → $\mathscr{L}_{\rm int} \sim 1.5 \times 10^7\,\mathrm{nb}^{-1}$.

- With this $\mathscr{L}_{\rm int}$ we estimate that measuring the shape of dσ/dt we can reach a statistical sensitivity of ~0.3% on a_μ^{HLO}, ie $\sim 20 \times 10^{-11}$.

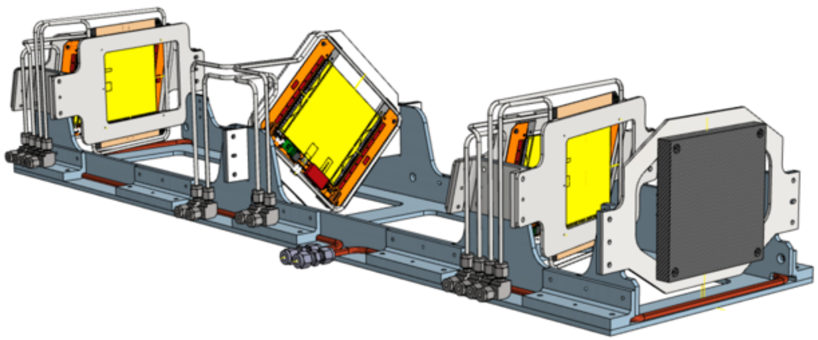
- **Systematic** effects must be known at ≲ 10ppm!

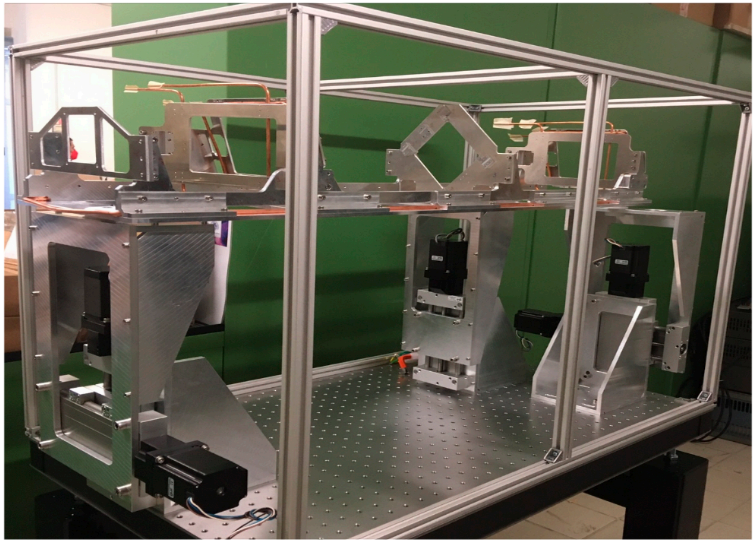
- Test beams performed at CERN in 2017 & 2018 arXiv:1905.11677, 2102.11111

- LoI submitted to CERN SPSC in 2019: Test run approved (2022).

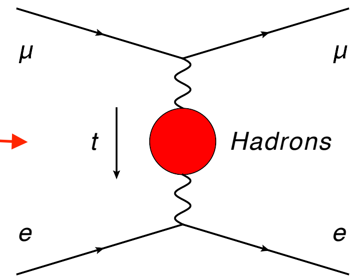
- If test run successful, intermediate run hopefully in 2023–24.

MUonE — Getting ready for the Test Run





- To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at $\lesssim$ 10ppm!



- **Fully differential fixed-order MC @ NLO ready** Pavia and PSI 2018-19
- **NNLO QED: All master integrals for 2-loop box diagrams computed. Full 2-loop amplitude completed! (m_e=0), Mastrolia et al.** Padova 2017 - present
- **Two MC built including partial subsets of the NNLO QED corrections due to electron and muon radiation** Pavia and PSI 2020 - present
- **NNLO hadronic effects computed** Padova, KIT, Pavia 2019 - present
- **Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes** PSI 2019 - present
- **No New Physics extracting $\Delta\alpha_{had}(t)$ at MUonE** Padova and Heidelberg 2020
- …

Theory for muon-electron scattering @ 10 ppm:
A report of the MUonE theory initiative. arXiv:2004.13663

MUonE — Theory workshops

Muon-electron scattering: Theory kickoff workshop

4-5 September 2017

Padova
Europe/Rome timezone

- **Overview**
- Venue
- Timetable
- Logistic
- Map

Support

MUonE theory workshops: Padova 2017, MITP Mainz 2018, Zurich 2019
Next MUonE theory workshop: MITP Mainz Nov 14-18 2022

Conclusions

- **Fermilab's Muon g-2 experiment confirms BNL's result: the discrepancy between experiment and SM increases to 4.2σ.**

- **The BMWc lattice QCD result weakens the exp-SM discrepancy. It must be confirmed or refuted by other lattice calculations.**

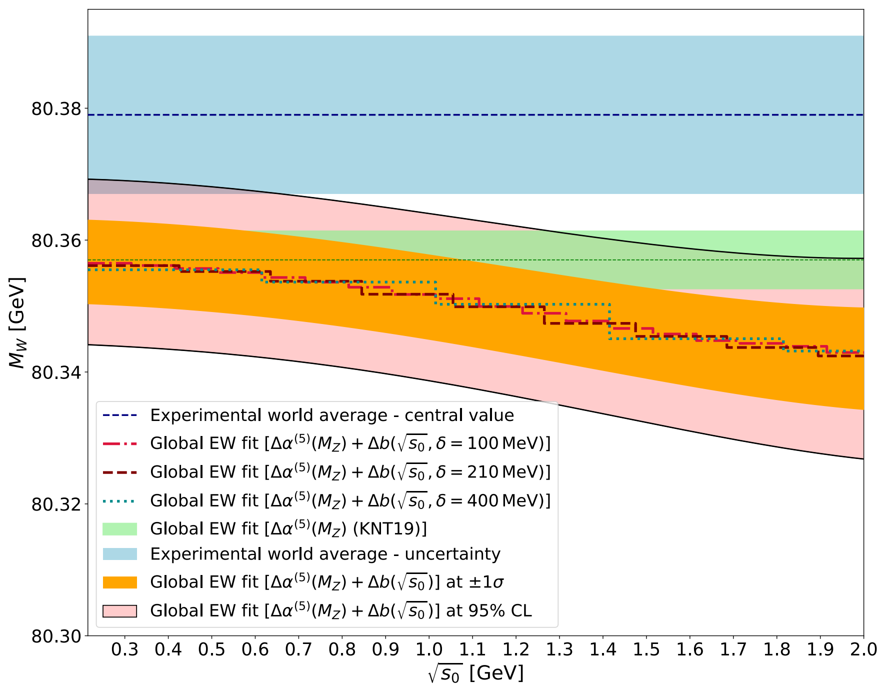
- **Is Δa_μ due to missed contributions in the hadronic cross section? Shifts above 1 GeV to fix Δa_μ conflict with the electroweak fit.**

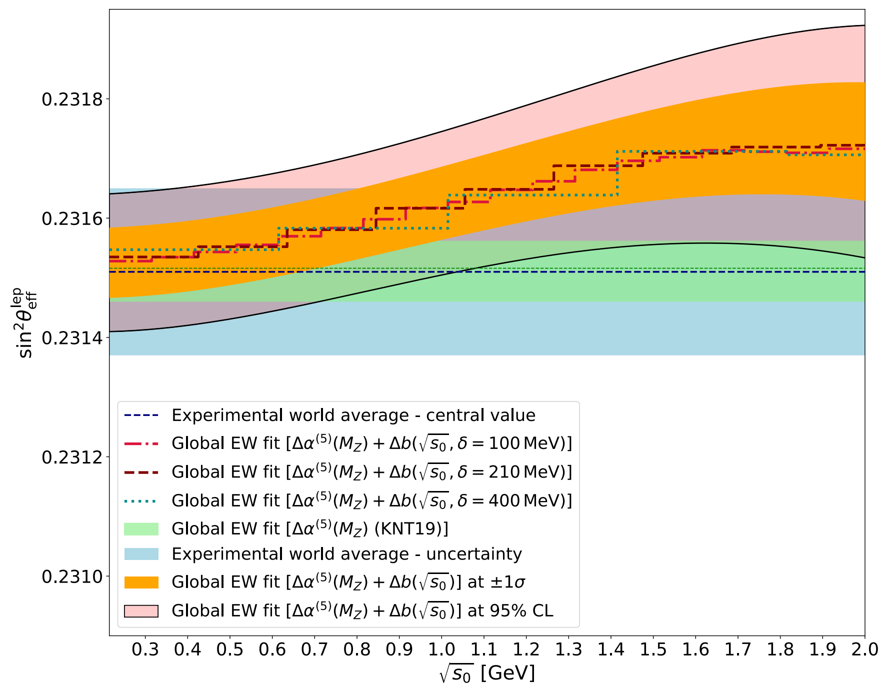
- **Leading hadronic contribution to a_μ: dispersive vs lattice. MUonE will provide a new independent & alternative determination.**

Backup

Muon g-2: connection with M_W and $\sin^2\theta$

$\Delta\alpha$

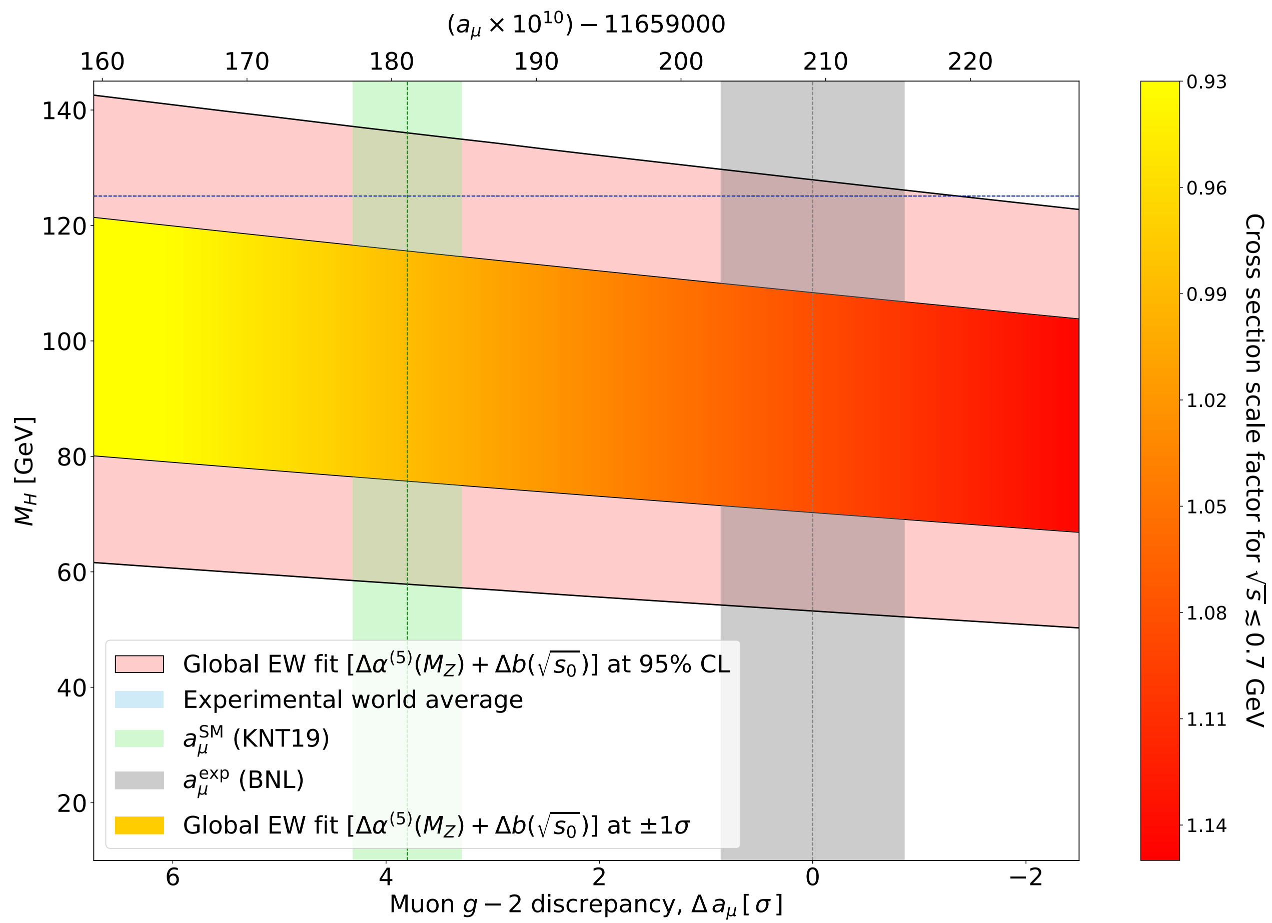




Keshavarzi, Marciano, MP, Sirlin, PRD 2020

The muon g-2: connection with the SM Higgs mass (2020) - II

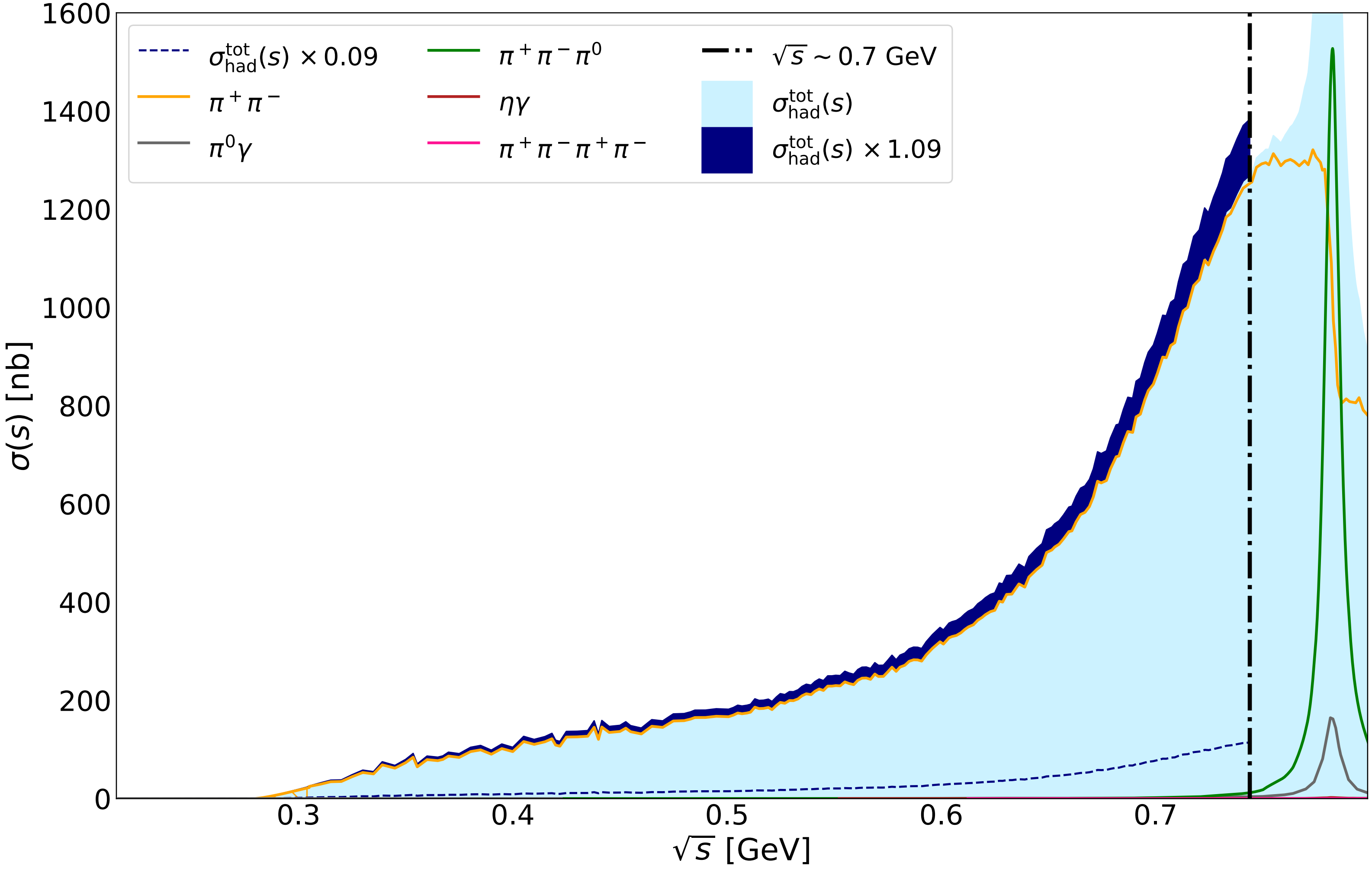
Δα



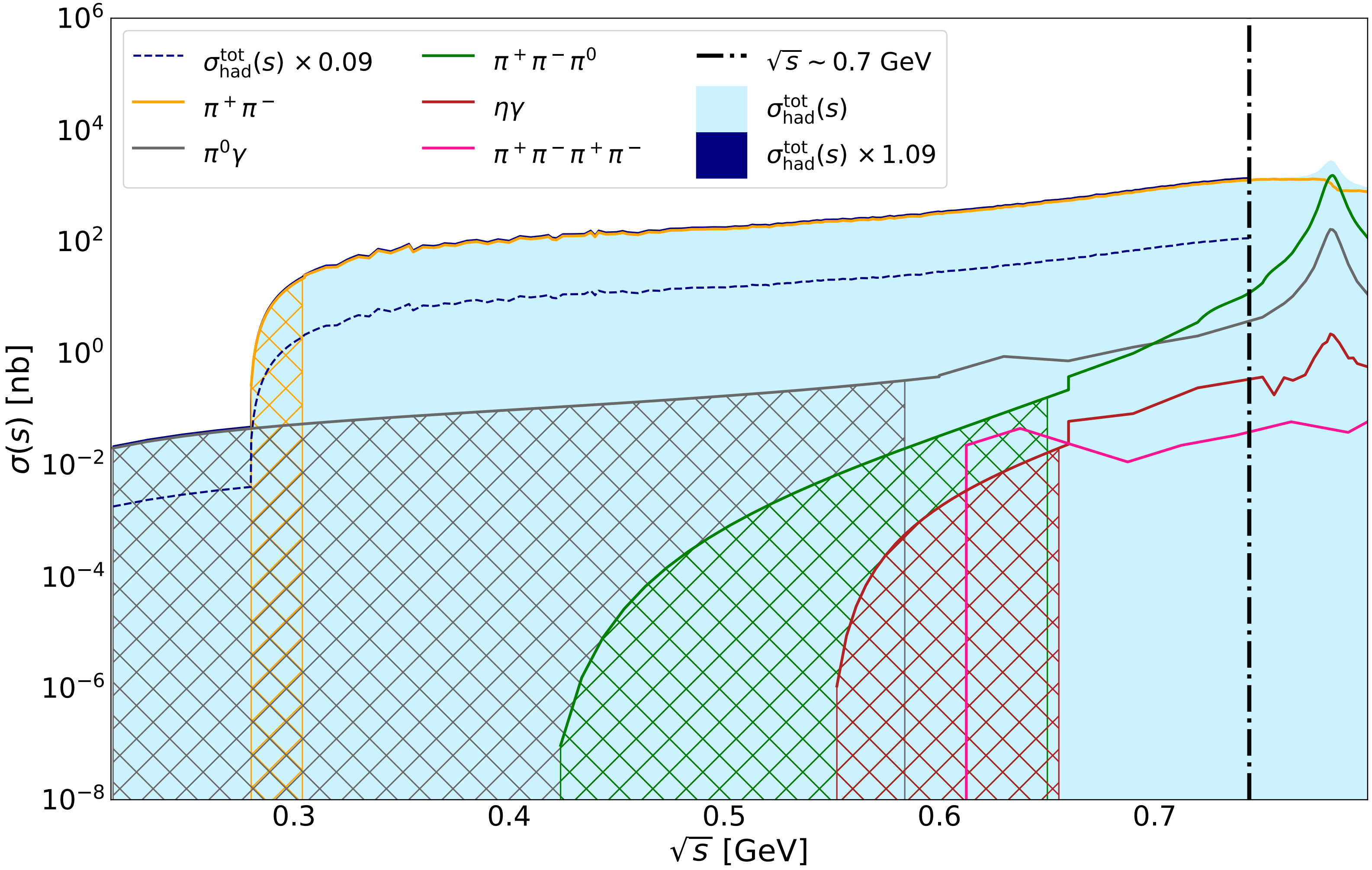
Uniform scaling of σ(s) below ~0.7 GeV? +9% required!

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Δα



Keshavarzi, Marciano, MP, Sirlin, PRD 2020



Keshavarzi, Marciano, MP, Sirlin, PRD 2020