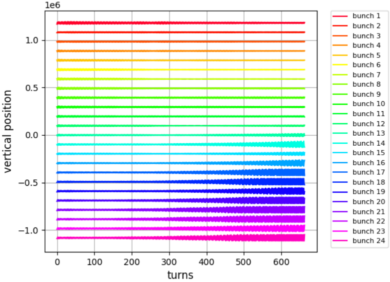
A novel Vlasov approach for modeling electron cloud instabilities

ECLOUD 22

Sofia Johannesson, Giovanni Iadarola, Beams Department CERN

Outline

Instabilities driven by e-cloud.



- Electron clouds can drive transverse instabilities

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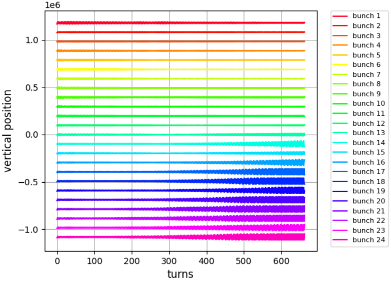

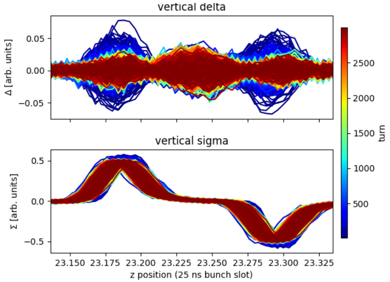

LHC measurements, June 2022

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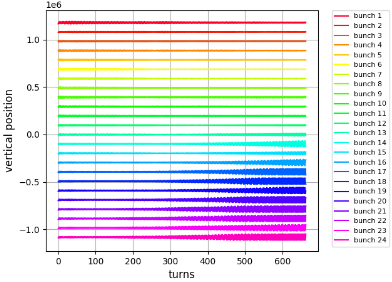

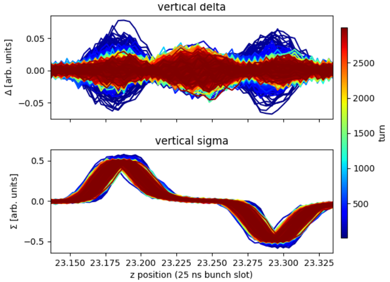

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- Conventional simulations using macroparticle tracking together with the PIC method for e-cloud beam interaction, are very computationally heavy. [7] G. Iadarola. et al., 2017, *Evolution of Python Tools for the Simulation of Electron Cloud Effects*

Instabilities driven by electron cloud

- Instabilities driven by impedance effects have been modelled using the linearised Vlasov Equation, which identifies the Instability growth rate and betatron frequency shift for each instability mode. [8] N. Mounet, 2017, *Vlasov Solvers and Macroparticle Simulations*

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Previous attempts of using the Vlasov method to model e-cloud driven instabilities have not included these points together. [10] K. Ohmi et al, 2001, *Wake-Field and Fast Head-Tail Instability Caused by an Electron Cloud.*, [11] E. Perevedentsev, 2002, *Head-Tail Instability Caused by Electron Cloud*

Outline

General description of e-cloud forces

Begin by describing the dipolar e-cloud forces:

[2] G. Iadarola, et. al. 2020, *Linearized method for the study of transverse instabilities driven by electron clouds*

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Choose a set of sinusoid beam distortions, $h_n(z)$ where z is the position along the bunch. The sinusoid test functions satisfy the orthogonality condition: $\int h_n(z)h_{n'}(z) = H_n^2\delta_{n,n'}$

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Each distortion, h_n, corresponds to a response function k_n calculated from the interaction with e-cloud using single-pass PIC simulations.

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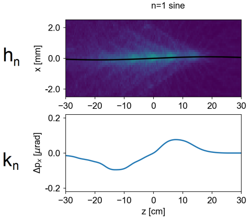

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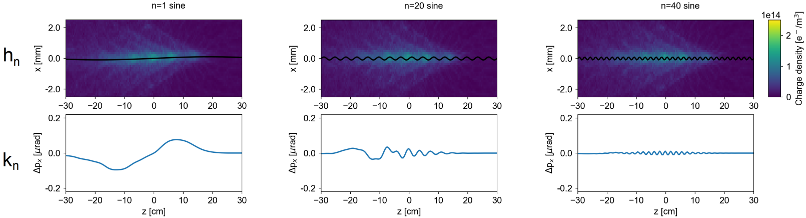

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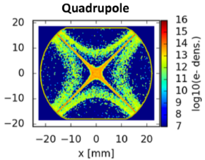

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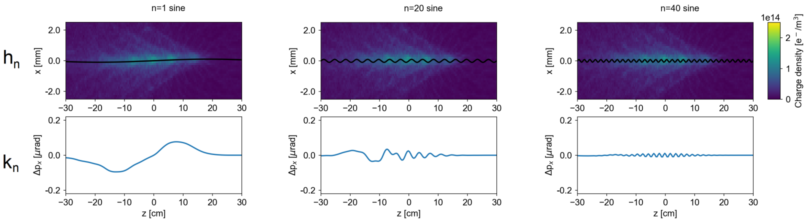

These calculations use the e-cloud in the superconducting quadrupoles of the LHC for a beam energy of 450GeV.

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Linear model of e-cloud - Dipolar Forces

Describe the transverse centroid along the bunch, $\bar{x}(z)$, as a linear combination of test functions h_n:

$$\bar{x}(z) = \sum_{n=0}^{\infty} a_n h_n(z); \quad a_n = \frac{1}{H_n^2} \int \bar{x}(z) h_n(z) dz \tag{1}$$

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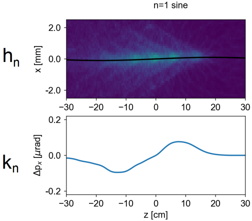
k_n is the resulting electron cloud kick from a bunch distortion h_n.



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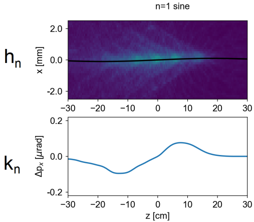
From simulation we verify linear behaviour such that the kick, $\Delta x'$, of arbitrary distribution $\bar{x}(z)$ is:

$$\Delta x'(z) = \sum_{n=0}^{\infty} a_n k_n(z) \qquad (2)$$



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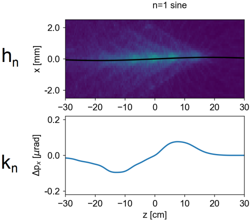

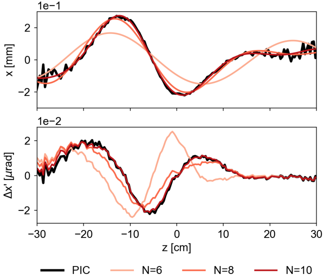

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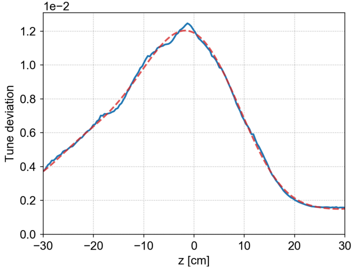
Linear model of electron clouds - Quadrupolar forces

Model detuning using a polynomial

$$\Delta Q(z) = \sum_{n=0}^{N_p} A_n z^n \qquad (3)$$



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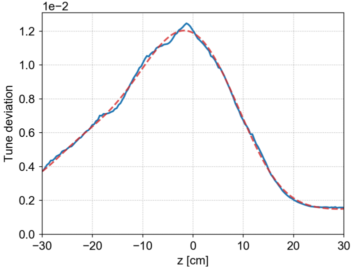

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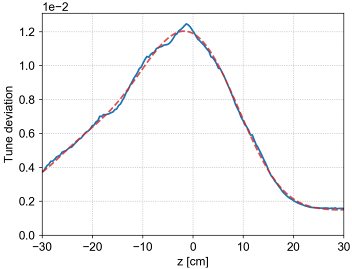

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Including only linear chromaticity:

$$\Delta Q(z,\delta) = Q'\delta + \sum_{n=0}^{N_p} A_n z^n \qquad (5)$$

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$$\frac{\partial \Delta\psi}{\partial t} + [\Delta\psi, H_0] = -[\psi_0, \Delta H] \tag{6}$$

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- The electron cloud forces are contained in $\Delta\mathbf{H}$
- The distortion $\Delta\psi$ is the impact of the perturbation and the **unknown**

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Factorize unpertubed bunch distribution, $\psi_0 = \frac{\eta v}{\omega_0} f_0(J_x) g_0(r)$

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$$\frac{\partial \Delta\Psi}{\partial t} - \omega_0 (Q_{x0} + \Delta Q(r,\phi)) \frac{\partial \Delta\Psi}{\partial \theta_x} + \omega_s \frac{\partial \Delta\Psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma} \frac{df_0}{dJ_x} \sqrt{\frac{2 J_x R}{Q_{x0}}} \sin\theta_x F_x^{coh}(z,t)$$

reds are unknowns and greens come from e-cloud forces

E-cloud in the Vlasov Equation - Dipolar Forces

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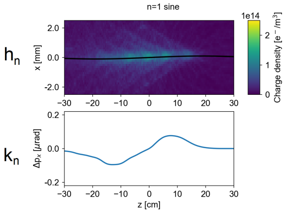
$\Delta x'$ is a linear combination of the responses k_n using the coefficients calculated from the projection of h_n on $\bar{x}(x,t)$, the average transverse position at longitudinal position z.

$$\Delta x' = \sum_{n=0}^{N} a_n k_n(z)\,;\; a_n = \frac{1}{H_n^2} \int \bar{x}(z,t) h_n(z) dz \tag{8}$$

Note that $\Delta\psi$ is used to calculate $\bar{x}(z,t)$



Solving the Vlasov Equation - Ansatz of $\Delta\psi$

Equation to solve:

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First ansatz: $\Delta\psi(J_x, \theta_x, r, \phi, t) = e^{j\Omega t} \Delta\psi(J_x, \theta_x, r, \phi)$.

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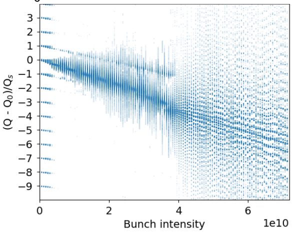
$(\frac{Re(\Omega)}{\omega_0} - Q_0)/Q_s$ is the tune shift



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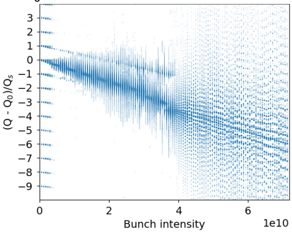
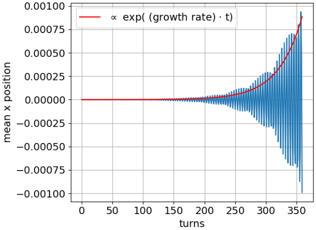
First ansatz: $\Delta\psi(J_x, \theta_x, r, \phi, t) = e^{j\Omega t}\Delta\psi(J_x, \theta_x, r, \phi)$.

$(\frac{Re(\Omega)}{\omega_0} - Q_0)/Q_s$ is the tune shift



$-Im(\Omega)$ is the instability growth rate



Solving the Vlasov Equation - Ansatz of $\Delta\psi$

Equation to solve:

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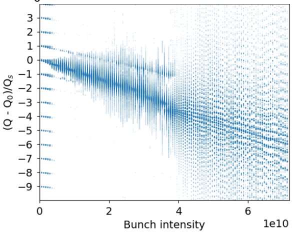

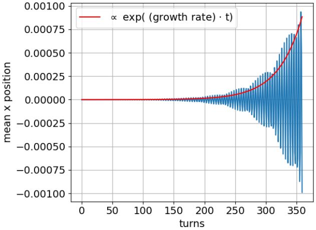

Beam instabilities are characterized by these qualities.

ω_0 is the angular revolution frequency, Q_0 is the unperturbed tune and Q_s is the synchrotron frequency

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[2] G. Iadarola, et. al. 2020, *Linearized method for the study of transverse instabilities driven by electron clouds*

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Expand ansatz:

$$\Delta\psi(J_x,\theta_x,r,\phi,t) = e^{j(\Omega t - \Delta\Phi(r,\phi))} f_1(J_x) e^{j\theta_x} \sum_{l=-\infty}^{\infty} e^{-jl\phi} W_l(r) \sum_{m=0}^{+\infty} b_{lm} f_{lm}(r)$$

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Phase shift term to be chosen

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Choose $\Delta\Phi$ to have the constraint:

$$\frac{\partial \Delta\Phi}{\partial \phi} = -\frac{\omega_0}{\omega_s} \Delta Q_\Phi(r,\phi) \quad (10)$$

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Detuning from e-cloud can be divided into a detuning w longitudinal amplitude and a head-tail phase shift

$$\Delta Q(z,\delta) = \Delta Q(r,\phi) = Q'\delta + \sum_{n=0}^{N_p} A_n z^n = \Delta Q_R(r) + \Delta Q_\Phi(r,\phi) \tag{11}$$

[2] G. Iadarola, et. al. 2020, *Linearized method for the study of transverse instabilities driven by electron clouds*

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The linerized Vlasov Equation now becomes an **eigenvalue problem**:

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Unkowns in red and terms including electron cloud forces in green

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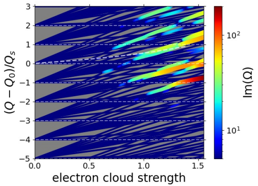

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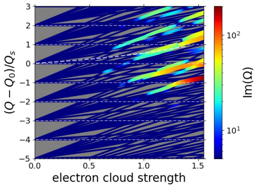

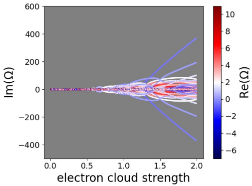

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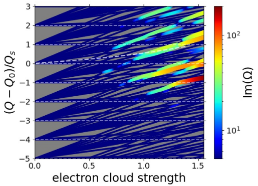

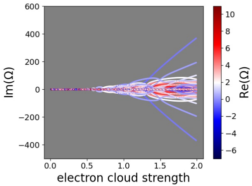

The e-cloud strength is a factor that multiply the e-cloud forces.

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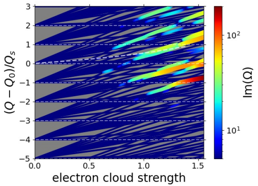

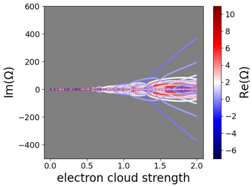

The e-cloud strength is a factor that multiply the e-cloud forces.
The Vlasov equation is solved for one e-cloud strength at a time yielding a set of Ω which corresponds to a vertical line in the plots.

Outline

Benchmark against macro-particle simulations

- Use PyHEADTAIL, a conventional macro-particle simulation code, to benchmark the Vlasov approach.

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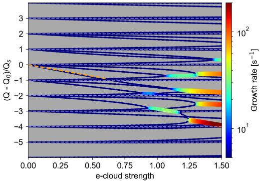
Will now compare the results from macro-particle tracking and the Vlasov approach, both using the linear model of e-cloud forces.

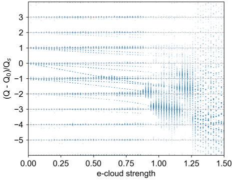
Results with zero chromaticity - Phase Shift

$\Delta Q_R = 0, \Delta Q_\Phi = 0$

Vlasov



linear macro-particle



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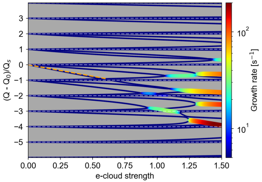

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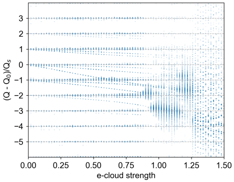

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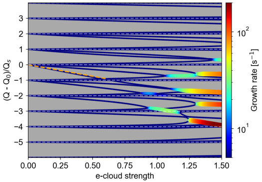

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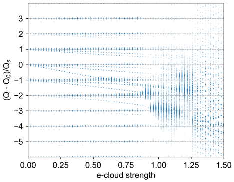

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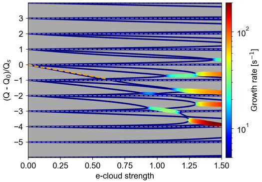

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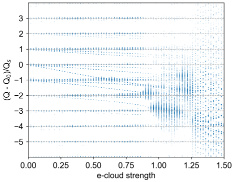

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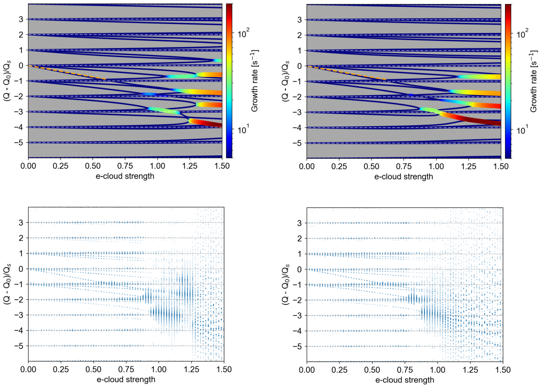
- The same modes are visible in the results from the Vlasov approach and the macro-particle tracking.

Results with zero chromaticity - Phase Shift

$\Delta Q_R = 0, \Delta Q_\Phi = 0$ | $\Delta Q_R = 0, \Delta Q_\Phi \neq 0$

Vlasov

linear macro-particle



Now the head-tail phase shift ΔQ_Φ, is included.

Results with zero chromaticity - Phase Shift

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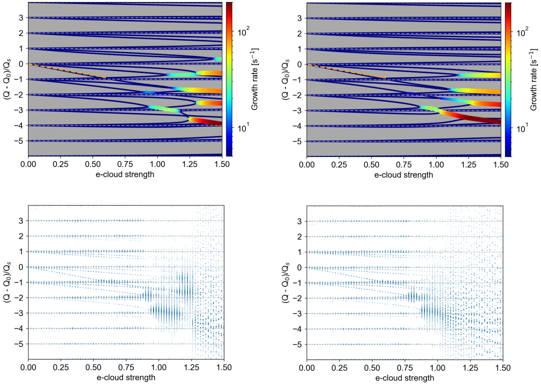
Vlasov

linear macro-particle



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The same modes are visible in both plots!

Results with zero chromaticity - Phase Shift

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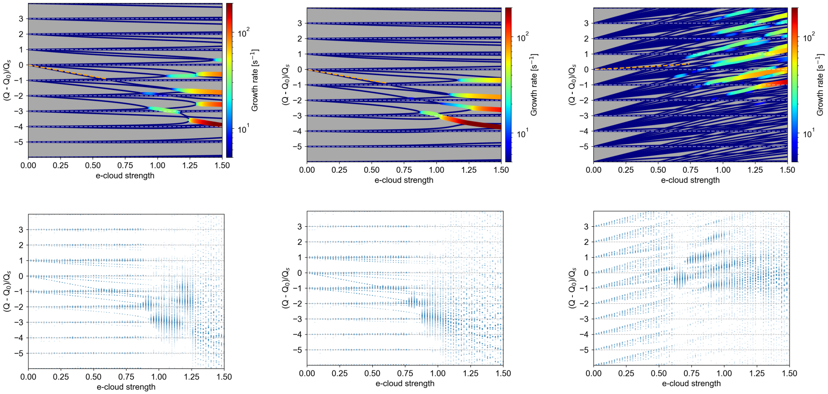
Vlasov

linear macro-particle



Include also detuning with longitudinal amplitude.

Results with zero chromaticity - Phase Shift

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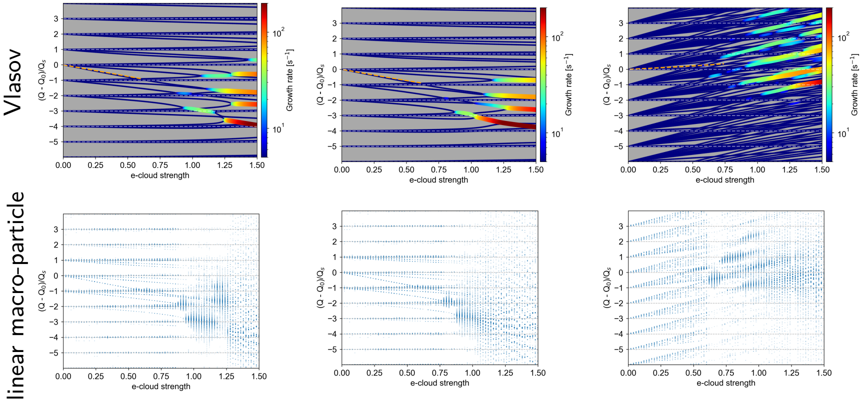
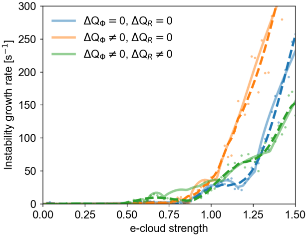


Include also detuning with longitudinal amplitude.

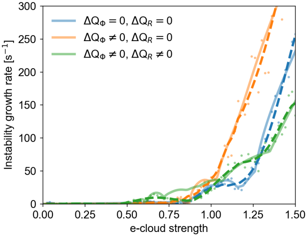
Modes agree when full detuning from e-cloud are included!

Results with zero chromaticity - Growth Rate



- The instability growth rate of macro-particle simulations is reached by doing an exponential fit on the mean x position (dashed lines)

Results with zero chromaticity - Growth Rate



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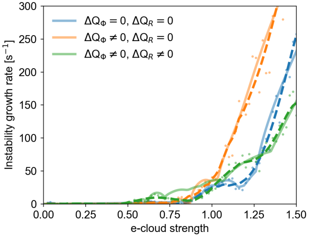
- The Vlasov mode with the highest instability growth rate, the worst mode, is plotted as the whole lines.

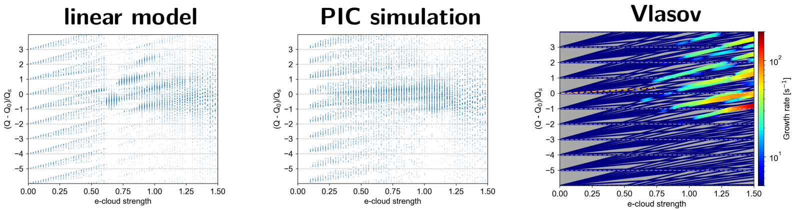
Results with zero chromaticity - Growth Rate



- The instability growth rate of macro-particle simulations is reached by doing an exponential fit on the mean x position (dashed lines)
- The Vlasov mode with the highest instability growth rate, the worst mode, is plotted as the whole lines.

The instability growth rates agree well!

Benchmark linear model with PIC simulations



The two left plots are made with macro-particle simulations.

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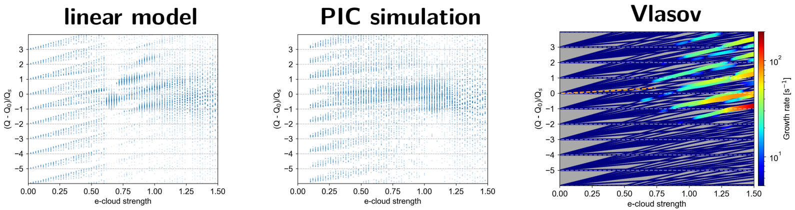

The two left plots are made with macro-particle simulations.

All simulations methods have fans of modes for low e-cloud strengths.

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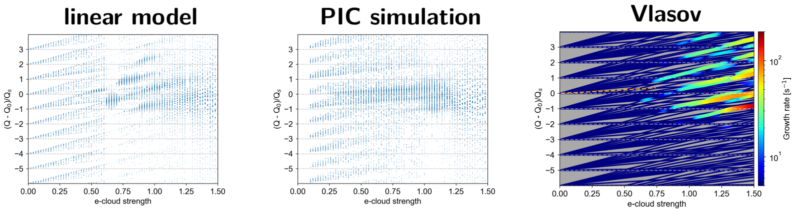


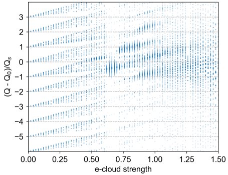
The two left plots are made with macro-particle simulations.

All simulations methods have fans of modes for low e-cloud strengths. Non-linear effects from e-cloud are present in middle plot which have a stabilizing effect.

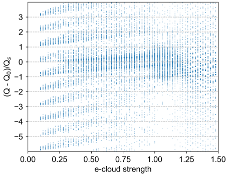
Modes with a tune shift $\Delta Q/Q_s > 0$ are not visible in the PIC simulations.

Benchmark linear model with PIC simulations

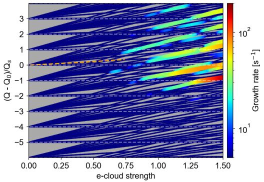
linear model



PIC simulation



Vlasov



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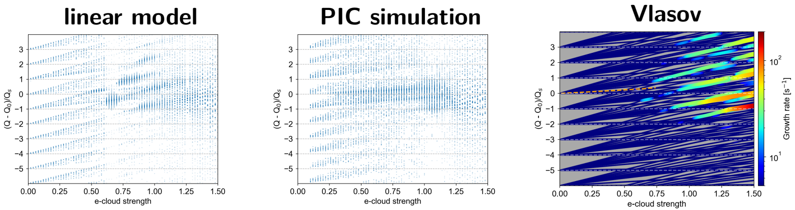
All simulations methods have fans of modes for low e-cloud strengths. Non-linear effects from e-cloud are present in middle plot which have a stabilizing effect.

Modes with a tune shift $\Delta Q/Q_s > 0$ are not visible in the PIC simulations.

For high e-cloud strength (>1.25), both macro-particle simulation methods give similar results

Benchmark linear model with PIC simulations



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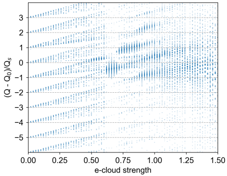
Modes with a tune shift $\Delta Q/Q_s > 0$ are not visible in the PIC simulations.

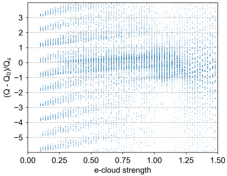
For high e-cloud strength ($>$1.25), both macro-particle simulation methods give similar results and the worst Vlasov mode also have a tune shift between $-1 < (Q - Q_0)/Q_s < 0$.

Benchmark linear model with PIC simulations

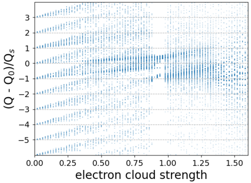
linear model



PIC simulation



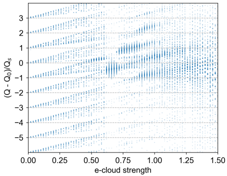
linear model +
static non-linear map



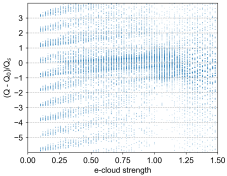
A static non-linear map, independent on z, can be made by removing the linear forces from the field map of the electron pinch and averaging along z.

Benchmark linear model with PIC simulations

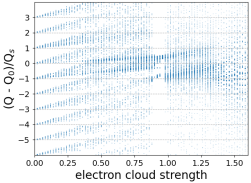
linear model



PIC simulation



linear model + static non-linear map



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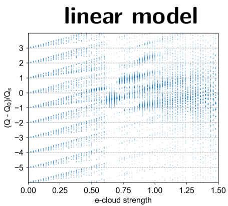
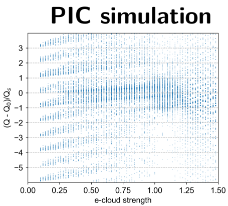
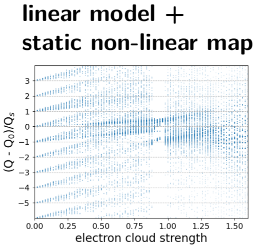
This map is then added to the linear e-cloud model.

Benchmark linear model with PIC simulations





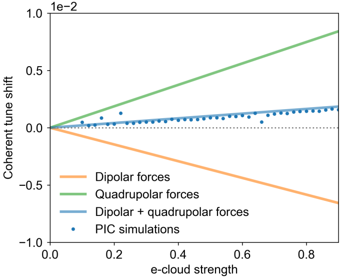


A static non-linear map, independent on z, can be made by removing the linear forces from the field map of the electron pinch and averaging along z.

This map is then added to the linear e-cloud model.

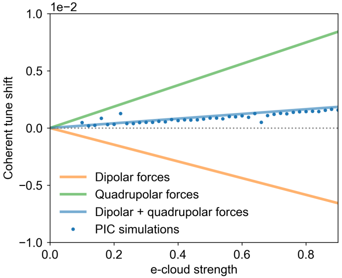
The results using the linear model + a static non-linear map are similar to the results using PIC: modes with $\Delta Q/Q_s > 0$ are stabilized.

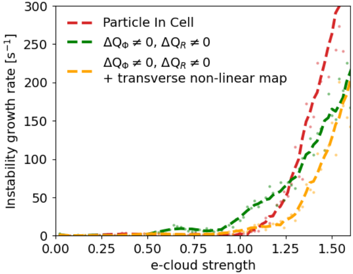
Benchmark linear model with PIC simulations



An excellent agreement of tune shift between the Vlasov approach using **both** dipolar and quadrupolar forces and the PIC method.

Benchmark linear model with PIC simulations





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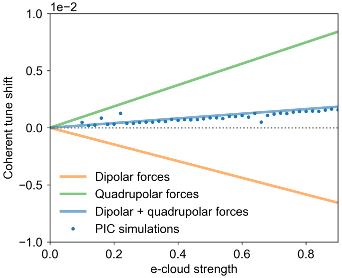
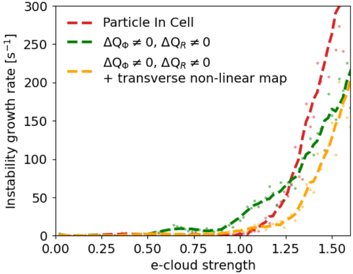
Benchmark linear model with PIC simulations





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The static non-linear map has a stabilizing effect on the instability growth rate for low e-cloud strengths and the behaviour is *slightly* more similar to the PIC instability growth rate.

Outline

Introduction

Simulation Model

Results with zero chromaticity

Results with chromaticity

Chromaticity in Vlasov

Chromaticity is included in Vlasov together with the detuning caused by e-cloud forces:

$$\Delta Q(z,\delta) = Q'\delta + \sum_{n=0}^{N_p} A_n z^n \tag{13}$$

which is included in the phase shift term $\Delta\Phi$ used in the ansatz of the distortion $\Delta\psi$:

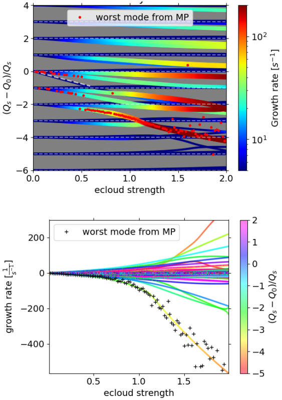
$$\Delta\psi(J_x,\theta_x,r,\phi,t) = e^{j\Omega t}\sum_{p=-\infty}^{\infty} f^p(J_x)e^{jp[\theta_x-\Delta\Phi(r,\phi)]}\sum_{l=-\infty}^{\infty} R_l^p(r)e^{-jl\phi} \tag{14}$$

Equation to solve:

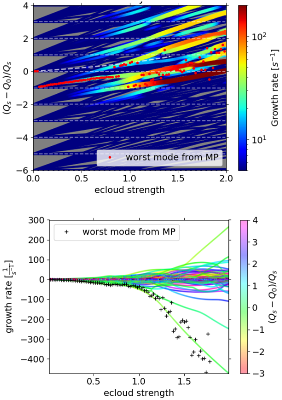
$$\frac{\partial\Delta\Psi}{\partial t} - \omega_0(Q_{x0}+\Delta Q(r,\phi))\frac{\partial\Delta\Psi}{\partial\theta_x} + \omega_s\frac{\partial\Delta\Psi}{\partial\phi} = -\frac{\eta g_0(r)}{\omega_s m_0\gamma}\frac{df_0}{dJ_x}\sqrt{\frac{2J_xR}{Q_{x0}}}\sin\theta_x F_x^{coh}(z,t) \tag{15}$$

Chromaticity = -5

$\Delta Q_R = 0, \Delta Q_\Phi = 0$



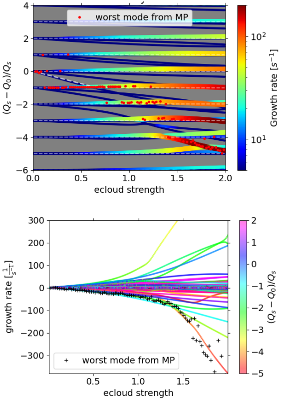
$\Delta Q_R \neq 0, \Delta Q_\Phi \neq 0$



The macro-particle simulation results follow the behaviour of the worst mode from Vlasov.

Chromaticity = 5

$\Delta Q_R = 0, \Delta Q_\Phi = 0$



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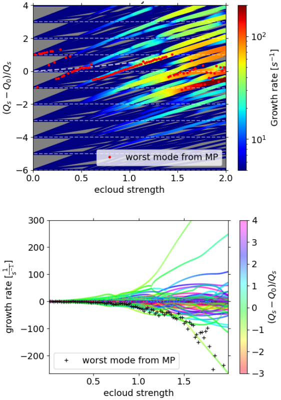
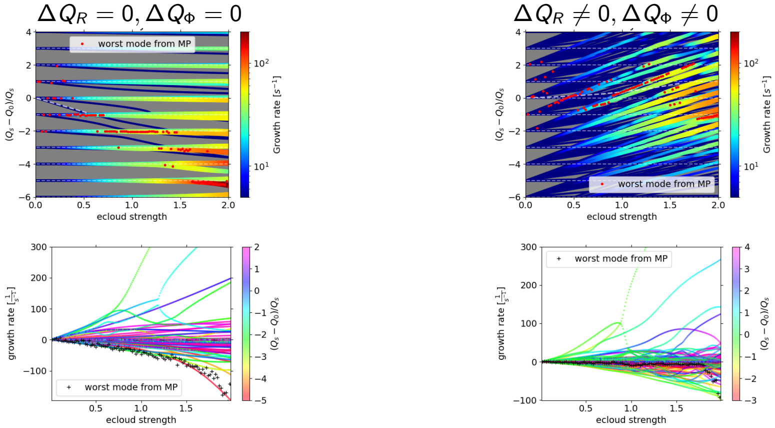


Good agreement also for low and positive chromaticity.
The instability growth rates are lower compared to chromaticity -5

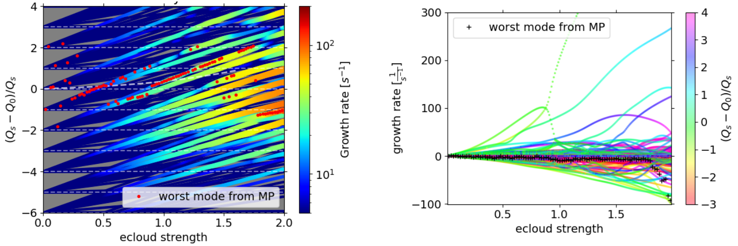
Chromaticity $= 15$



Good agreement when only dipolar forces are included

Chromaticity = 15

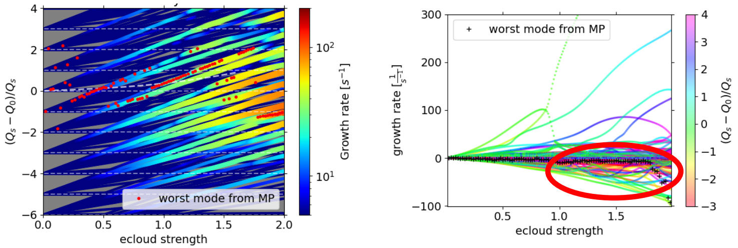
$$\Delta Q_R \neq 0, \Delta Q_\Phi \neq 0$$



The tune shift from macro-particle simulations does **not** follow the worst mode, **but always follow an existing Vlasov mode**.

Chromaticity $= 15$

$$\Delta Q_R \neq 0, \Delta Q_\Phi \neq 0$$



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Instabilities in the macro-particle simulations for e-cloud strenghts <1.8 are damped.

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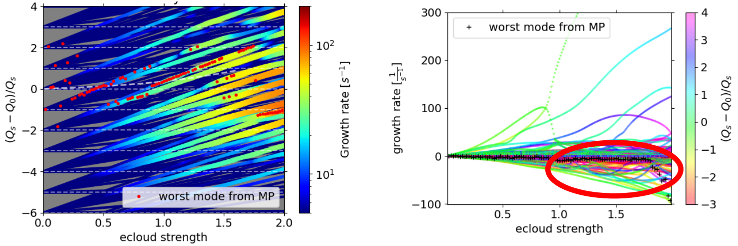

The tune shift from macro-particle simulations does **not** follow the worst mode, **but always follow an existing Vlasov mode**.

Instabilities in the macro-particle simulations for e-cloud strenghts <1.8 are damped.

This is not true for the Vlasov modes.

Summary and Conclusions

- A linear model of e-cloud forces including both dipolar and quadrupolar forces have been developed.

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- Currently checks with impedance driven instabilities are being done to further study this behaviour.

References

[1] G. Iadarola et al, 2018, *Electron Cloud Effects*
[2] G. Iadarola, et. al. 2020, *Linearized method for the study of transverse instabilities driven by electron clouds*
[3] N. Mounet, 2018, *Direct Vlasov Solvers*
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[10] K. Ohmi et al, 2001, *Wake-Field and Fast Head-Tail Instability Caused by an Electron Cloud.*
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Thank you for your attention!

e-cloud in Vlasov Equation - quadrupolar forces

Anstatz of $\Delta\psi$:

- $\rightarrow$ Assume time dependence contained in $e^{j\Omega t}$
- $\rightarrow$ transverse Fourier decomposition (angle θ_x)
- $\rightarrow$ extract phase shift term $e^{-jp\Delta\Phi(r,\phi)}$
- $\rightarrow$ longitudinal Fourier decomposition (angle ϕ)

$$\Delta\psi(J_x,\theta_x,r,\phi,t) = e^{j\Omega t}\sum_{p=-\infty}^{\infty} f^p(J_x) e^{jp[\theta_x-\Delta\Phi(r,\phi)]}\sum_{l=-\infty}^{\infty} R_l^p(r) e^{-jl\phi} \quad (16)$$

where the unknown terms are Ω and $f^p(J_x)$ and $R_l^p(r)$.

Choose $\Delta\Phi$ in the phase shift term so that: $$\frac{\partial\Delta\Phi}{\partial\phi} = -\frac{\omega_0}{\omega_s}\Delta Q_\Phi(r,\phi) \quad (17)$$

The total detuning from e-cloud forces and chromaticity can be divided into transverse detuning with longitudinal amplitude, ΔQ_R, and head-tail phase shift, ΔQ_Φ:

$$\Delta Q(z,\delta) = \Delta Q(r,\phi) = Q'\delta + \sum_{n=0}^{N_p} A_n z^n = \Delta Q_R(r) + \Delta Q_\Phi(r,\phi)$$

Solving the linerized Vlasov Equation

$$\frac{\partial \Delta\Psi}{\partial t} - \omega_0(Q_{x0} + \Delta Q(r,\phi))\frac{\partial \Delta\Psi}{\partial \theta_x} + \omega_s \frac{\partial \Delta\Psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma}\frac{df_0}{dJ_x}\sqrt{\frac{2J_x R}{Q_{x0}}}\sin\theta_x F_x^{coh}(z,t) \tag{18}$$

Using the ansatz of the distortion:

$$\Delta\psi(J_x,\theta_x,r,\phi,t) = e^{j\Omega t}\sum_{p=-\infty}^{\infty} f^p(J_x) e^{jp[\theta_x - \Delta\Phi(r,\phi)]}\sum_{l=-\infty}^{\infty} R_l^p(r) e^{-jl\phi} \tag{19}$$

after an additional decomposition of the radial functions $R_l(r) = W_l(r)\sum_{m=0}^{\infty} b_{lm} f_{lm}$ where $f_{lm}(r)$ are orthogonal and $W_l(r)$ is arbitrary.

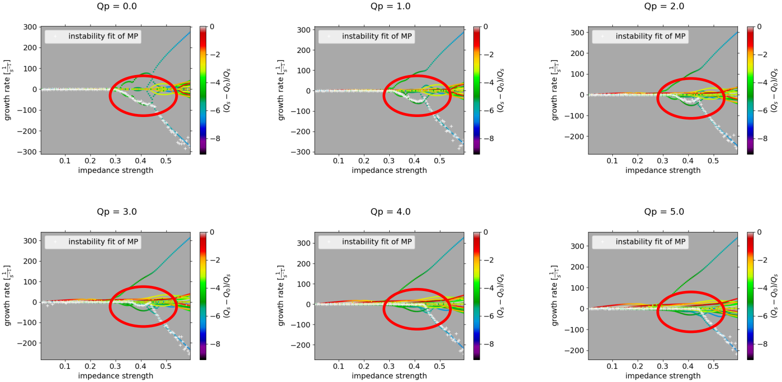
The linerized Vlasov Equation now becomes an **eigenvalue problem**:

$$b_{lm}(\Omega - Q_{x0}\omega_0 - l\omega_s) = \sum_{l'm'}(\mathbf{M}_{lm,l'm'} + \tilde{\mathbf{M}}_{lm,l'm'})b_{l'm'} \tag{20}$$

Unkowns in red and terms including electron cloud forces in green

Solved by computing the matrices $\mathbf{M}_{lm,l'm'}$ and $\tilde{\mathbf{M}}_{lm,l'm'}$ and solve for "eigenvalue" Ω and mode b_{lm} using standard linear algebra packets

Discrepancy between simulations results



As chromaticity increases the vlasov mode for medium impedance strength is no longer visible in the macroparticle simulations results.