OTTIMIZZAZIONE DI MISURE, SQUEEZING METROLOGY E ATTIVITÀ TEORICHE

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Outline

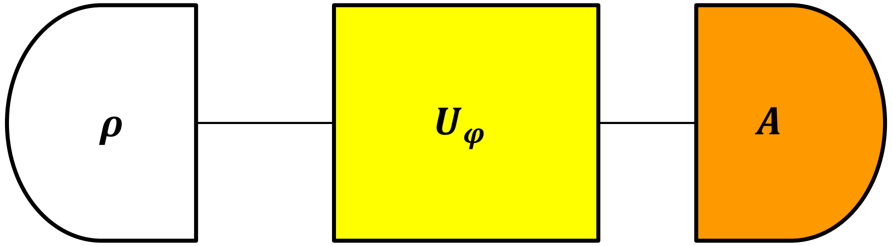
- **Quantum Metrology**
- **Squeezing Metrology**
- **Applications in SQMS**

Lorenzo Maccone and A. Riccardi:
Squeezing metrology: a unified framework, Quantum 4, 292 (2020).

Quantum metrology

Goal: to estimate an **unknow parameter** $\boldsymbol{\varphi}$ → φ is encoded into **a probe state** $\boldsymbol{\rho}$ through a unitary transformation $\boldsymbol{U_{\varphi} = e^{i\varphi H}}$,being H the Hamiltonian that generates the interaction.

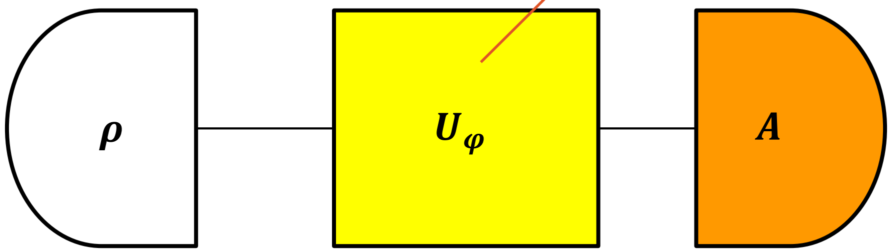
→ The state is then tested by measuring an **observable** $\boldsymbol{A}$**.**



The same scheme applies if we want to verify whether a transformation occured $(\varphi \neq 0)$.

Quantum metrology

$U_\varphi = e^{i\varphi H}$: H cannot be changed!



The precision of this estimation, which has a degree of statistical uncertainty, is characterized by its error $\Delta\varphi$:

Error: $\Delta\varphi = \dfrac{\Delta A}{|\langle [A,H] \rangle|}$

Strategies that can be adopted in order to minimize $\Delta\varphi$:

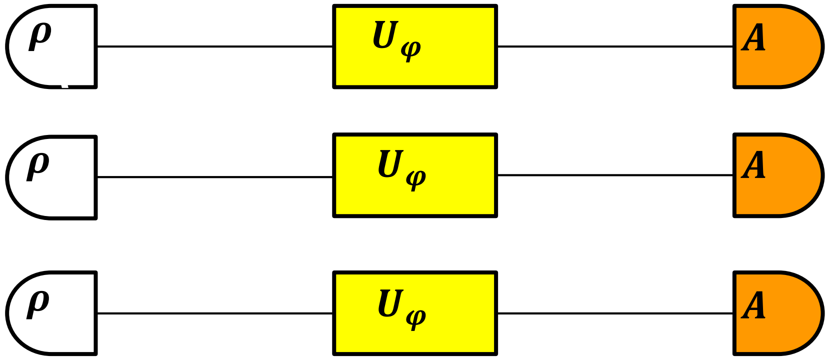
- a more sensitive probe ρ
- a different observable A ⟹ Optimal: Mix of them
- more resources *(e.g. number of probes)*

Quantum metrology

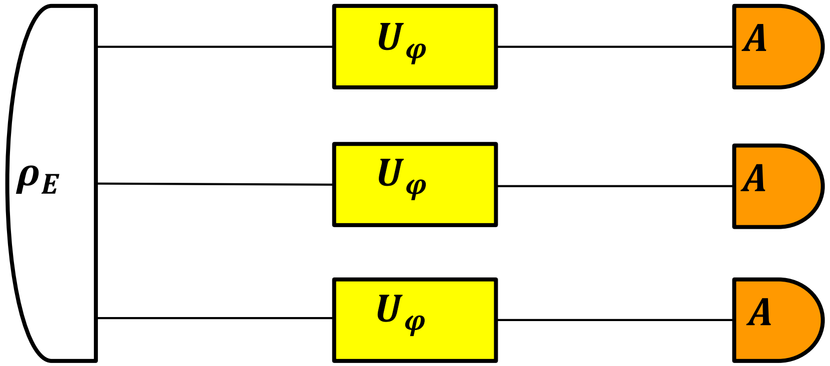
H and ***A*** are fixed in this context!

Classical strategy (no additional quantum resource involved): we may prepare N identical probes, independent to each other, run the estimation scheme N times and average the results.
The error, due to the central limit theorem, reduces to $\Delta\varphi/\sqrt{N}$

Quantum strategy (quantum correlations): we can use ***N entangled probes***, that allow to decrease the error to $\Delta\varphi/N$, ***attaining a quadratic gain*** with respect the *classical* strategy.



$$\Delta\varphi \propto \frac{1}{\sqrt{N}}$$



$$\Delta\varphi_{ent} \propto \frac{1}{N}$$

Entanglement allows a quadratic gain!

V. Giovannetti, S. Lloyd, L. Maccone, Phys. Rev. Lett. 96, 010401 (2006).

Alternative strategies?

H and ***A*** are fixed in this context!

In order to minimize the error, we may change the probe ρ. How?

A suggestion is given by the Heinseberg-Robertson's uncertainty relation (UR): $\boldsymbol{\Delta A \Delta H \geq \frac{|\langle [A,H] \rangle|}{2}}$

Minimum uncertainty (MU) states: $\Delta A \Delta H = \frac{|\langle [A,H] \rangle|}{4}$

For **minimum uncertainty states**:

$$\Delta\varphi = \frac{\Delta A}{|\langle [A,H] \rangle|} = \frac{1}{2\Delta H}$$

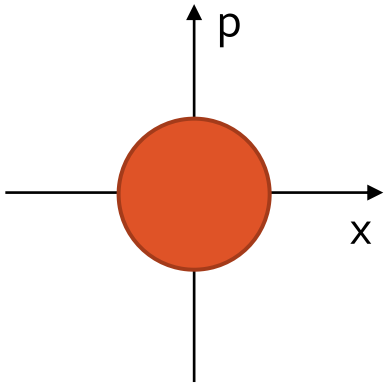
- Increase ΔH!
- Decrease ΔA! ⟹ **Squeezed states**
- MU state.

Ex: position and momentum

Heisenberg-Robertson's UR: $\Delta X \Delta P \geq \frac{1}{2}$

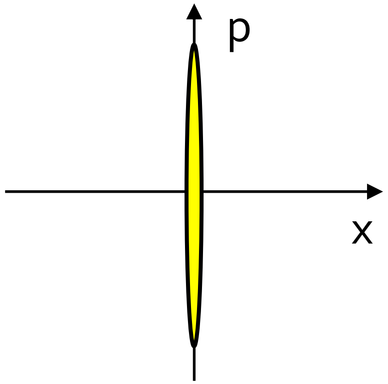
Minimum uncertainty states

Coherent states

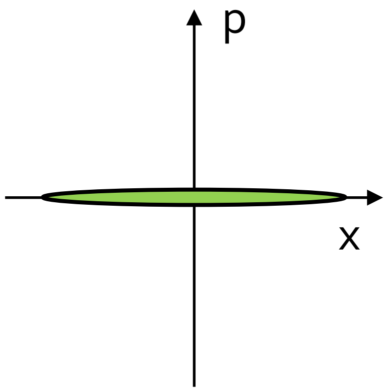


$\Delta X = \Delta P$

Squeezed states



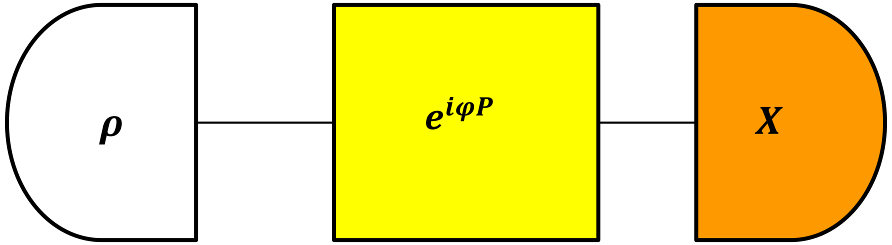
$\Delta X \ll \Delta P$



$\Delta X \gg \Delta P$

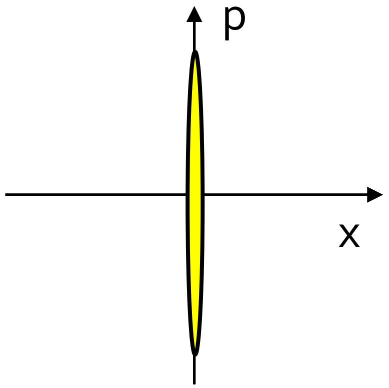
Ex: position and momentum

Minimum uncertainty state



$$\Delta\varphi = \Delta X = \frac{1}{2\Delta P}$$

Optimal state



$\Delta P \gg \Delta X$

The more the state is squeezed in X the better is the resolution!

Squeezing the probe requires energy. A constraint is neeeded to avoid unphysical situations, such as infinite squeezing which leads to a better resolution but also an infinite amount of this resource.

Resource: $\boldsymbol{E = \langle s|H|s\rangle}$
The energy of the squeezed state $|s\rangle$ with respect to the Hamiltonian H

Squeezing metrology

Squeezing can enhance the precision of some estimation protocols, leading to **quadratic resolution gains.**

Position Measurement: H.P. Yuen, Phys. Rev. Lett. 51, 719 (1976).
Optical interferometry: S. L. Braunstein et. Al. Ann. Phys. 247, 135-173 (1996).
L. Pezzé et. al. Phys. Rev. Lett. 100, 073601 (2008).
C. Brif, A. Mann, Phys. Rev. A 54, 4505 (1996).

Spin squeezing: D. J. Wineland et. al. Phys. Rev. A 46, R6797 (1992).

Many examples but no general theory of metrology based on squeezing for arbitrary quantum system.

Reminder: squeezing the probe requires some energy!

$$\boldsymbol{E = \langle s|H|s\rangle}$$

D.A. Trifonov, J. Math. Phys. 35, 2297 (1994).
D.A. Trifonov, JOSA A 17, 2486 (2000).

Generalized squeezed states

For any uncertainty relation (UR) is important to characterize which states achieve the lower bounds. For the product of variances UR the minimum uncertainty states are connected to the intelligent states.

INTELLIGENT STATES: Eigenstates of the operator $\boldsymbol{L(\lambda) = \lambda A + iH}$

λ complex number

⇓⇑

Minimum uncertainty states $\Delta^2 A \Delta^2 H = \frac{|\langle [A,H] \rangle|^2}{4} + (\Delta AH)^2$

$C = -i[A,H]$

$$\Delta^2 A = \frac{|\langle C \rangle|}{2\, Re\lambda}$$

$$\Delta^2 H = \frac{|\lambda|^2 |C|}{2\, Re\lambda}$$

⟹

Coherent states

$$\Delta^2 A_{coh} = \Delta^2 H_{coh}$$

$$|\lambda| = 1$$

Squeezed states in A

$$\Delta^2 A_{sq} < \Delta^2 H_{sq}$$

$$|\lambda| > 1$$

Squeezing metrology

For a fixed amount of energy E, we compare:

(1) The strategy where all the energy is used to squeezing the probe → $\boldsymbol{\Delta\varphi}_{\mathrm{sq}}.$

(2) The strategy where the energy is used to produce N coherent states → $\boldsymbol{\Delta\varphi(N)_{coh}}$

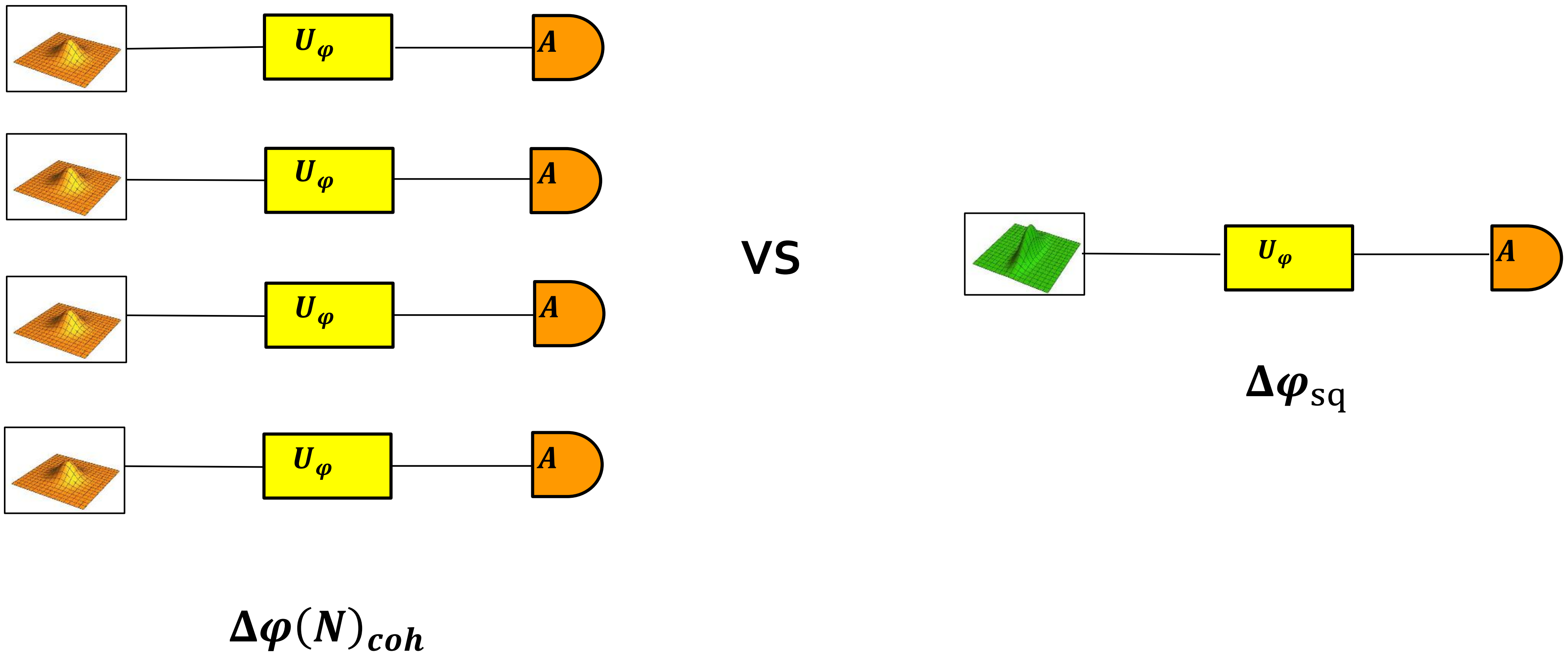
Number of coherent probes: $\boldsymbol{N = \frac{\langle s|H|s\rangle - E_0}{\langle c|H|c\rangle - E_0}}$

Number of probes that we can produce with that fixed energy

$\langle c|H|c\rangle$ energy of a single coherent probe

E_0 ground state energy

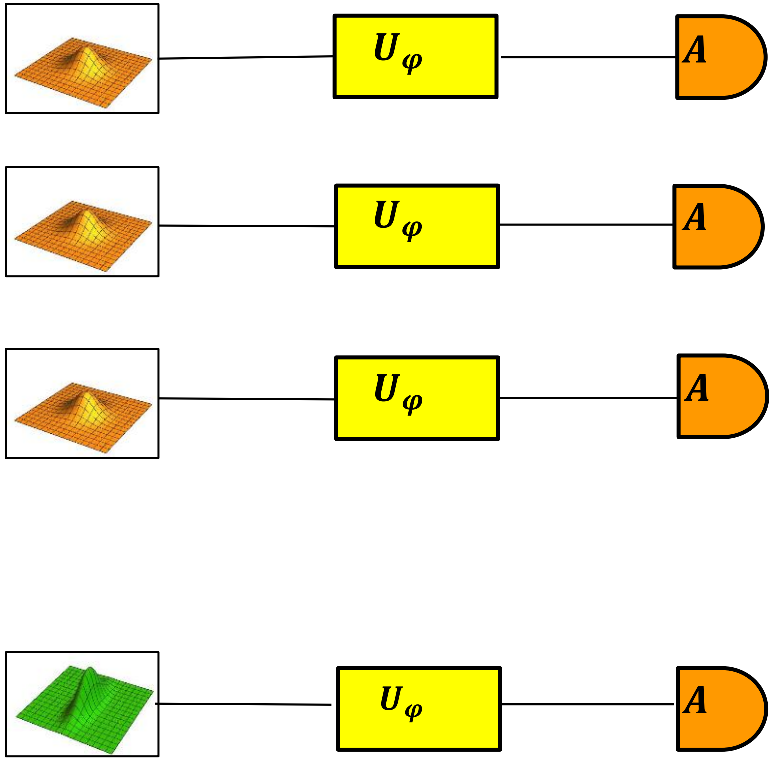
Squeezing metrology



Squeezing metrology

From a metrological point of view, given an amount of energy, **it is more convenient to squeeze a single probe instead of producing, by using the same energy, N coherent probes.**

$$N = \frac{\langle s|H|s\rangle - E_0}{\langle c|H|c\rangle - E_0}$$



$$\Delta\varphi(N)_{coh} \propto \frac{1}{\sqrt{N}}$$

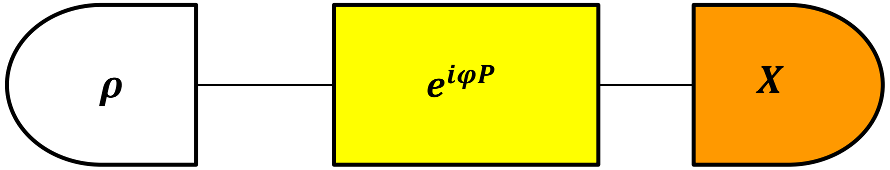
$$\frac{\Delta\varphi_{\mathrm{sq}}}{\Delta\varphi(N)_{coh}} \propto \frac{1}{\sqrt{N}}$$

$$\Delta\varphi_{sq} \propto \frac{1}{N}$$

Squeezing allows a quadratic gain!

Ex: Position measurement

Minimum uncertainty state



$$\Delta\varphi = \Delta X = \frac{1}{2\Delta P}$$

System: quantum harmonic oscillator
m: mass
ω: angular frequency

$$a = \sqrt{\frac{m\omega}{2}}X + i\sqrt{2m\omega}P$$

$$\boldsymbol{L(\lambda) = \lambda X + iP}$$

$\lambda = 1$ Coherent states

Squeezing operator

$$\mathbf{S}(\boldsymbol{\mu}, \boldsymbol{\nu}) = \boldsymbol{\mu a} + \boldsymbol{\nu a}^{\dagger}$$

$\mu = \frac{\lambda+1}{\sqrt{4\lambda}}$ and $\nu = \frac{\lambda-1}{\sqrt{4\lambda}}$

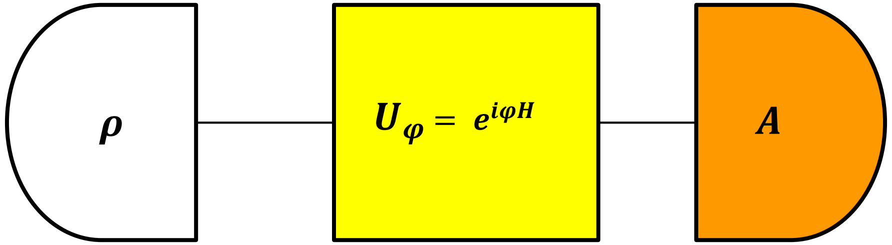
$$\Delta X(\lambda) = \sqrt{\frac{1}{2m\omega\lambda}}$$

$$\Delta P(\lambda) = \sqrt{\frac{m\omega\lambda}{2}}$$

$$\mathrm{N} = \frac{\langle P\rangle_{sq}}{\langle P\rangle_{coh}} = \frac{\Delta P_{sq}}{\Delta P_{coh}} = \sqrt{\lambda} \quad \text{then} \quad \frac{\Delta\varphi_{sq}}{\Delta\varphi_{coh}} = \frac{\Delta X_{sq}}{\Delta X_{coh}} = \frac{1}{\sqrt{\lambda}} = \frac{1}{N}$$

Single coherent state. If we consider N coherent states then $\Delta\varphi_{coh}$ is reduced by a factor $\sqrt{N}$

Experimental squeezing metrology recipe



1. Identify H, which generates the interaction, and the available measurements according to the experimental setup.

2. Find the eigenstates of the non-hermitian operator $L(\lambda) = \lambda A + iH$:

$$\boldsymbol{L(\lambda)|z,\lambda\rangle = z|z,\lambda\rangle}$$

Theoretically hard task

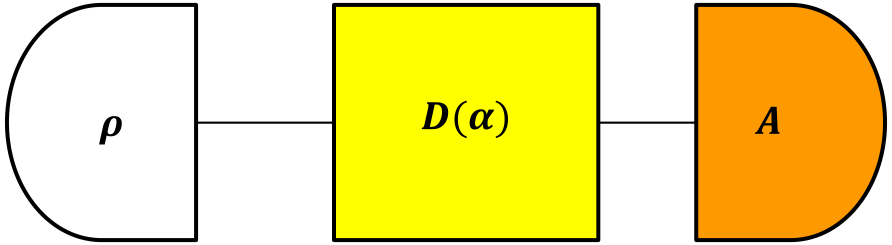
3. Among the eigenstates, select those with $|\lambda| > 1$.

4. Prepare the probes accordingly and run the experiment.

Experimentally hard task

Different measurements can lead to completely different classes of states.

Applications in SQMS: dark matter axion



The dark matter axion field is presumably in a coherent state with high occupation number.
Its effect on the electromagnetic field (e.g. in an optical or microwave cavity) is then to act as a **displacement**

$$D(\alpha) = exp(\alpha a^{\dagger} - \alpha^{*} a)$$

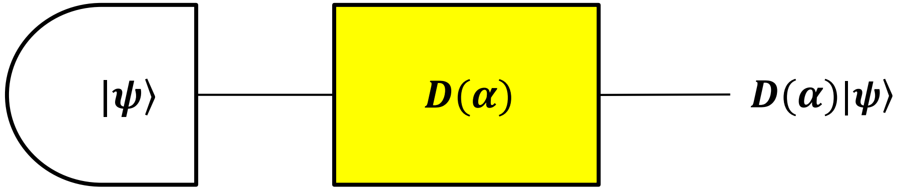
a is the cavity field annihilation parameter and the displacement parameter α is typically very small (due to the fact that the coupling between the axion and the electromagnetic field is small).

$|\alpha|^2$ is the mean number of photons in the cavity after the interaction.

The phase of the complex number is random, since the phase of the axion field is unknown.

Collaboration with Caterina Braggio's group in Padova

Applications in SQMS: dark matter axion



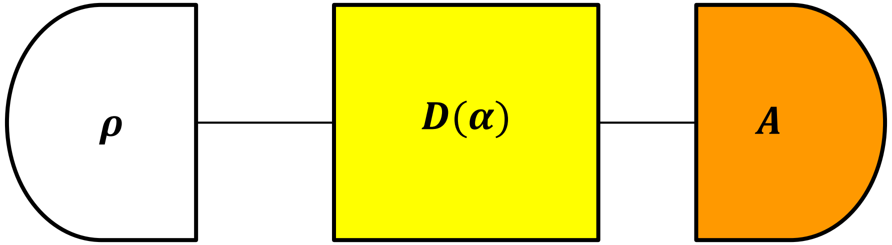
$\boldsymbol{\alpha}$ is typically very small

First approach: state sensibility → **Evaluate** $\langle\psi|D(\alpha)|\psi\rangle$

Possible probes:

1. Vacuum state
2. (common) Coherent states: no advantage over the vacuum
3. (common) Squeezed states: the performance are strongly dependent on the displacement phase (which is random), i.e. poor results if the phase is aligned to the direction of the squeezing.
4. N-cat states: superpositions of coherent states with equispaced phases. GOOD RESULTS
5. Modular states and sub-planckian states. **WORK IN PROGRESS**

Applications in SQMS: dark matter axion



$$D(\alpha) = exp(\alpha a^{\dagger} - \alpha^{*} a)$$

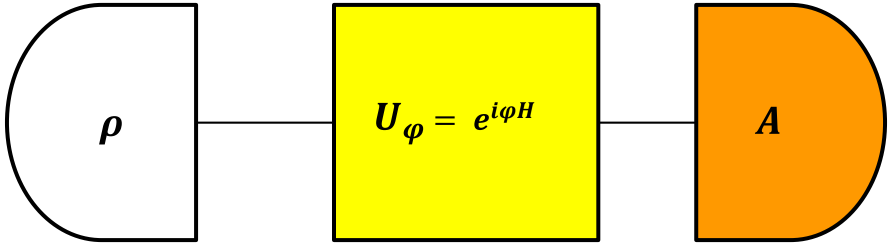
Second approach: squeezing metrology

1. Identify which measurements can be performed according to the experimental setup.

2. Among those available, (try to) study the operator $L(\lambda) = \lambda A + iH$ and its eigenstate equation:

$$L(\lambda)|z,\lambda\rangle = z|z,\lambda\rangle$$

Perspectives



1. Improve the detection of dark matter axions.

2. Multi-parameter squeezing metrology.

3. Discarding information metrology: optimal strategies when we are interested in estimate only partially the parameter (e.g. discarding the phase as for the dark matter axion)

4. Study of modular states and sub-plackian states: applications to metrology and to complementarity