# Overview SQMS Algorithms: attivita' italiana

- Leonardo Banchi – UniFi / INFN Firenze

- SQMS Algotithm Thrust
  - Simulation of condensed matter / quantum field theories
  - Device benchmarking
  - Control methods
  - Open source libraries

Florence:

- Quantum Simulations of Condensed Matter Systems
  - Slides coutesy of Laura Gentini

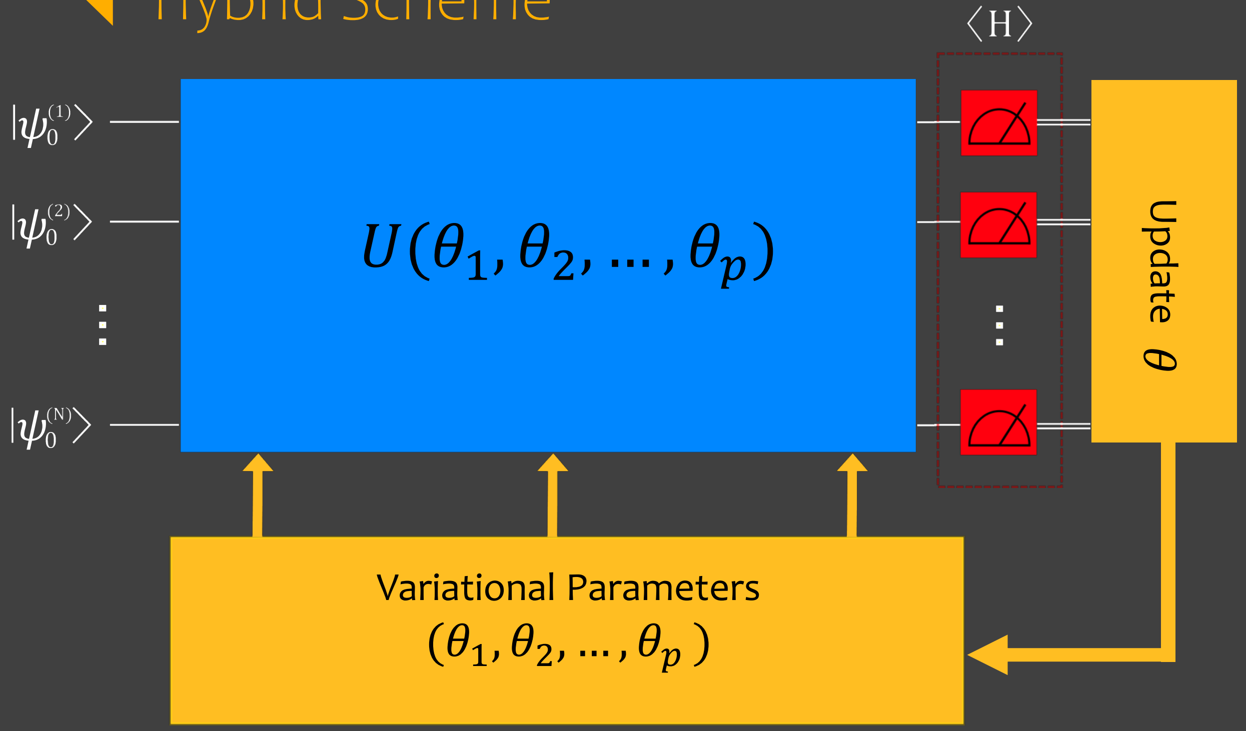
- Quantum Learning for Hardware Characterization
  - Slides courtesy of Paolo Braccia

# Variational method

Find ground state of an Hamiltonian, $\hat{H}$

- *Ansatz* → parametrize $|\psi(\theta)\rangle$
- Compute the energy $C(\theta) = \langle\psi(\theta)|\hat{H}|\psi(\theta)\rangle$
- Minimize $C(\theta)$ → find $\theta^{opt}$ | $C(\theta^{opt}) = \min_{\theta} C(\theta)$
- $|\psi(\theta^{opt})\rangle$ best approximation of the ground state

# Hybrid Scheme




## Quantum

- $|\psi(\theta)\rangle = U(\theta)|\psi_0\rangle$
- $C(\theta) = \langle\psi(\theta)|\hat{H}|\psi(\theta)\rangle$

## Classical

- Perform a optimization step
- Pass new parameters as feedback

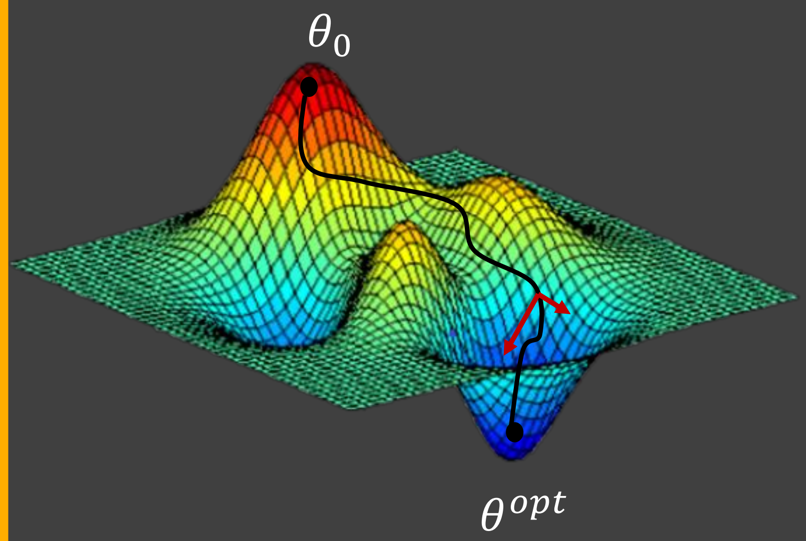

Find $\theta^{opt}$!

# Exploring the landscape

- $\theta$ define a parameter space
- $C(\theta) \rightarrow$ landscape
- At each optimization step we move a little

$$\theta^{(i+1)} = \theta^{(i)} - \alpha_i \nabla_\theta C(\theta^i)$$

- Guided by the gradient
- The algorithm defines a *path*



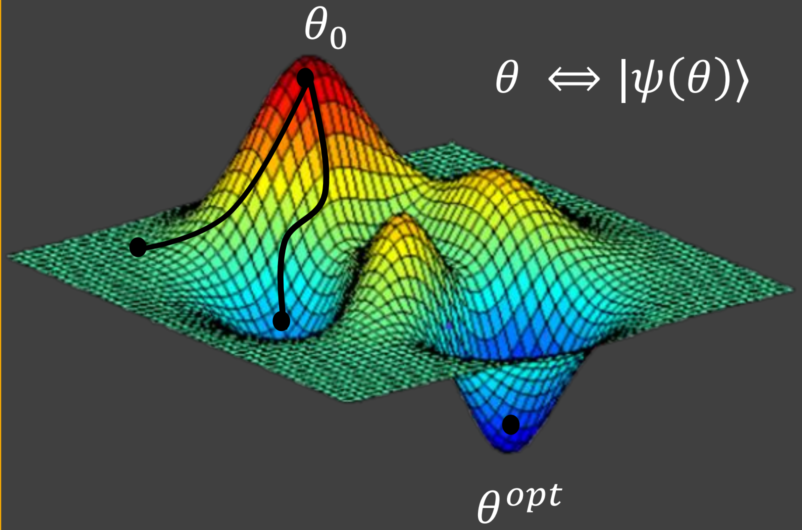
# Exploring the landscape

- Path full of traps
- Local minima or Plateaus

$$\nabla_\theta C = 0$$

- We can't take any step further
- but we do not have the solution

Stochastic outcomes & Noise → useful?




Convergence
Role of Noise and outcomes
} geometrical point of view?

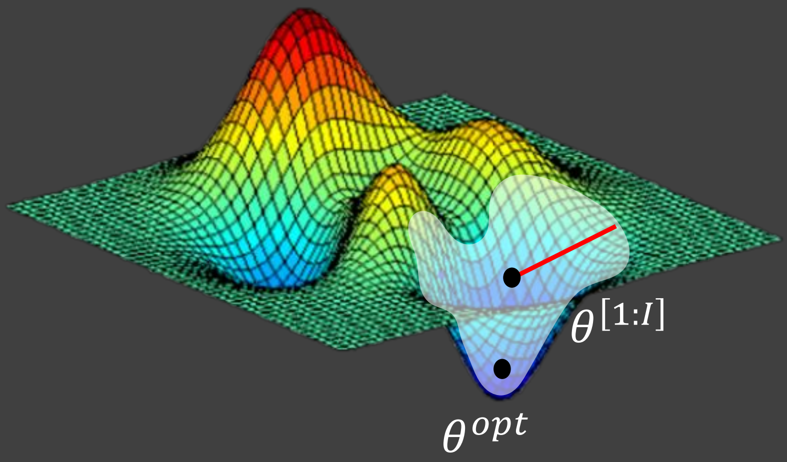
# On device gradient evaluation

$$\nabla C(\theta) = \langle \psi(\theta) | \hat{F} | \psi(\theta) \rangle$$

$$\nabla C(\theta) = \mathbb{E}_{z \sim q(z|\theta)}[\, g(\theta, z) \,]$$

$g\colon \mathbb{R}^p \to \mathbb{R}^p$ sampling $z$ from $q(z|\theta) \neq p(y|\theta)$

$$\theta^{(i+1)} = \theta^{(i)} - \alpha_i \, g(\theta^{(i)})$$




If $\mathbb{E}[\|g(\theta)\|^2] \leq G^2$

After $I$ iterations $\rightarrow$ $C(\theta^{[1:I]}) - C(\theta^{opt}) \leq R \frac{G}{\sqrt{I}}$

$$\alpha_i = \frac{R}{G\sqrt{I}} \qquad \theta^{[1:I]} = \frac{1}{I} \sum_i^I \theta_i$$

# Taking Noise into account

CPTP map $|\psi(\theta)\rangle = U(\theta)|\psi_0\rangle \;\rightarrow\; \rho(\theta) = \mathcal{E}(\theta)[\rho_0]$

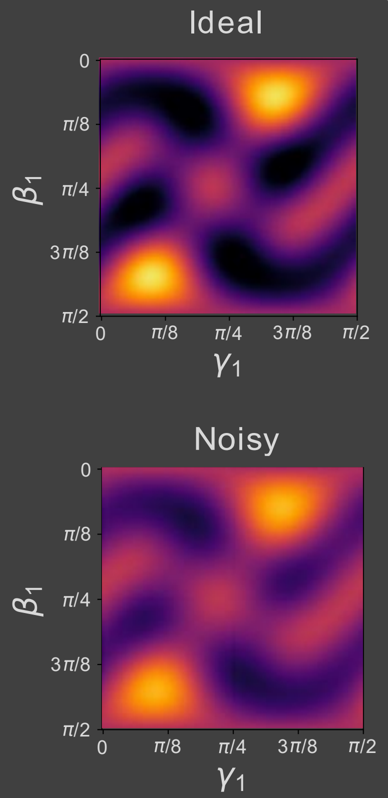
$\rho(\theta) = \mathcal{E}_p^{\theta_p} \circ \ldots \circ \mathcal{E}_1^{\theta_1}[\rho_0]$ $\mathcal{E}_j^{\theta_j}$ CPTP map

$C_{noisy}(\theta) = Tr[\rho(\theta)H] \rightarrow C_{noisy}(\vartheta^{opt})$ with $\vartheta^{opt} \neq \theta^{opt}$

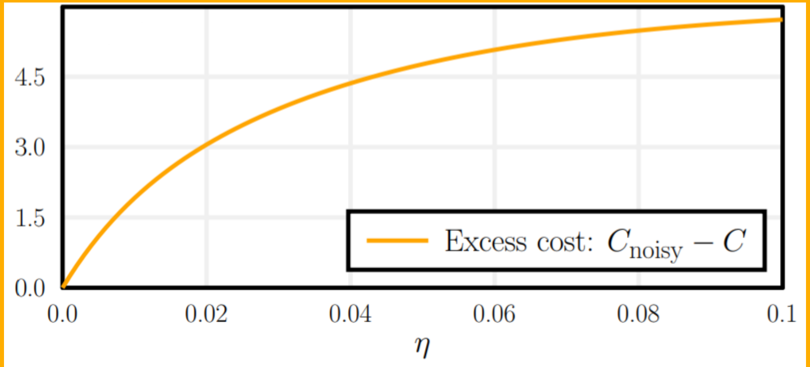
After $I$ iterations $\rightarrow$ $C_{noisy}\left(\theta^{[1:I]}\right) - C_{noisy}(\vartheta^{opt}) \leq R\frac{G_{noisy}}{\sqrt{I}}$

$$C_{noisy}\left(\theta^{[1:I]}\right) - C(\theta^{opt}) \leq Err(\theta^{opt}, \vartheta^{opt}) + R\frac{G_{noisy}}{\sqrt{I}}$$

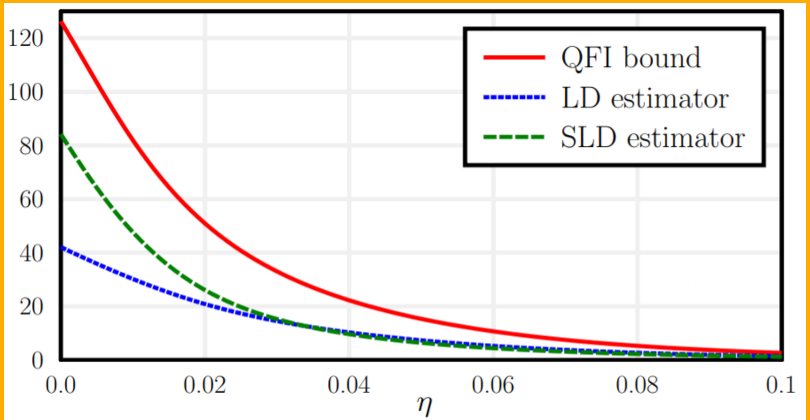
$$Err(\theta, \vartheta) \doteq C_{noisy}(\vartheta) - C(\theta)$$



## Increases with noise strength




## Decreases with noise strength




## Accuracy after $I$ iterations

$$C_{noisy}\left(\theta^{[1:I]}\right) - C(\theta^{opt})$$

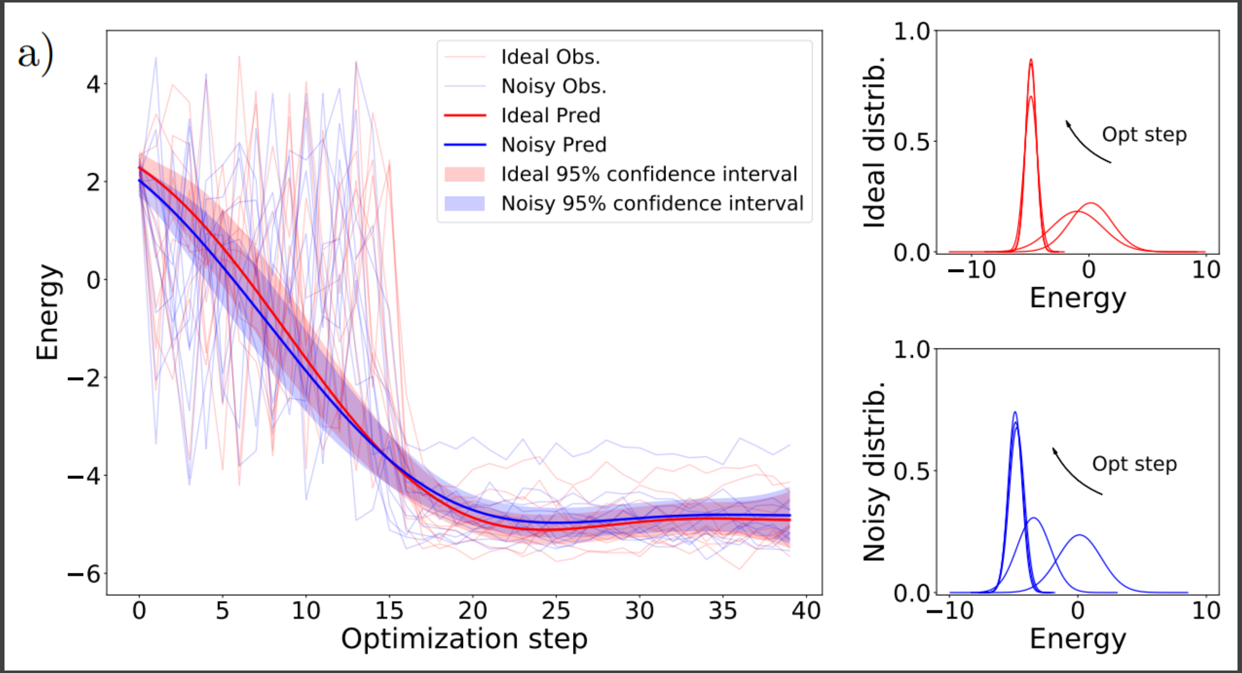
$$\leq$$

$$Err(\theta^{opt}, \vartheta^{opt})$$

$$+$$

$$\frac{R\sqrt{p}\|H\|_\infty}{\sqrt{I}} \max_{j,\theta} \sqrt{\mathrm{QFI}_j(\theta)}$$

# Numerical Experiment



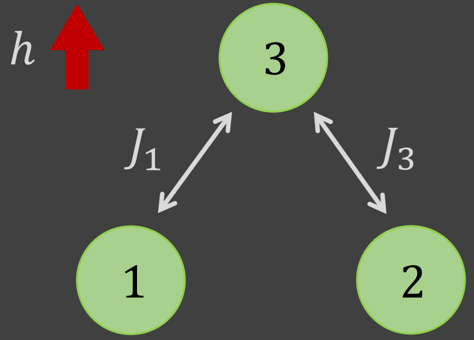
# Noise Resilient Variational Quantum Classical Optimization

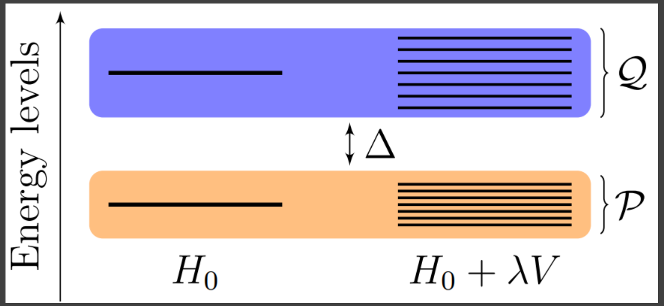
https://doi.org/10.1103/PhysRevA.102.052414
Gentini, Cuccoli, Pirandola, Verrucchi, Banchi

# Variational Approximation of Low-Energy Hamiltonians on Real Quantum Hardware

Gentini, Cuccoli, Banchi

# Low energy approximation




$$H_\lambda = H_0 + \lambda V$$

$$H_\lambda = h\sigma_3^z + \lambda(J_1\sigma_1^x\sigma_3^x + J_3\sigma_2^x\sigma_3^x)$$

$$J_1, J_3 \ll h$$

Hilbert Space

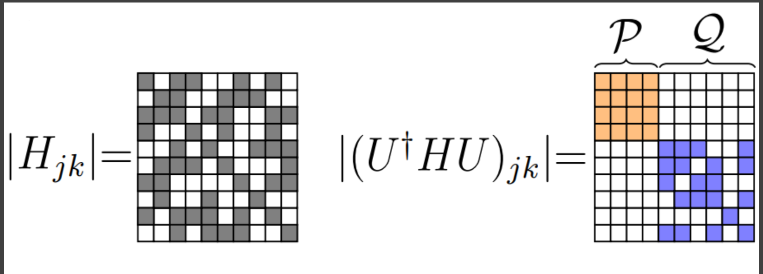
$\mathcal{P}$ : Low energy sector
$E \sim -h$
$|\psi_{\mathcal{P}}\rangle = |\psi_{12}\rangle|0_3\rangle$

$\mathcal{Q}$ : High energy sector
$E \sim +h$
$|\psi_{\mathcal{Q}}\rangle = |\psi_{12}\rangle|1_3\rangle$

Effective interaction of qubits 1 and 2

Effective Hamiltonian $\langle\psi_{\mathcal{P}}|H_{eff}|\psi_{\mathcal{Q}}\rangle = 0$

# Variational low energy approximation




Block diagonalize the Hamiltonian

=

Find a Low energy effective Hamiltonian

$$U^\dagger HU = D_{\mathcal{P}\mathcal{Q}}$$

$$U(\theta) \ ?$$

- No trivial cost function for "Block Diagonalization" (or even Diagonalization…)
- Very difficult to create a sufficiently complex ansatz → Need a physical motivated structure

# Adiabatic Gauge Potential

## Adiabatic state preparation

$$H_\lambda = H_0 + \lambda V$$

$$\mu \in [0 \ldots \lambda]$$

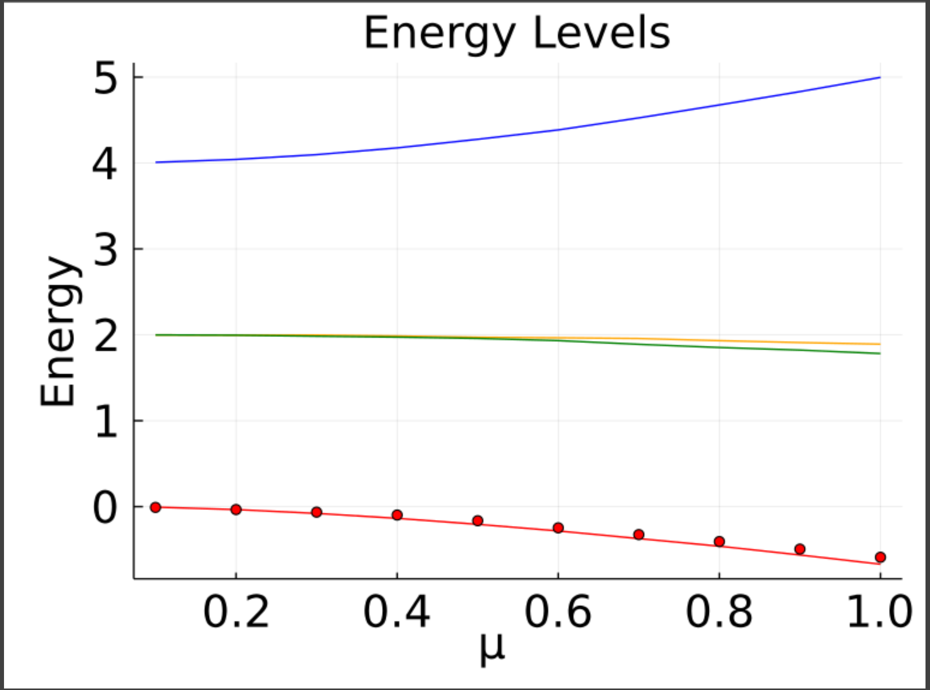
$\mu$ ↓

$|g_0\rangle \qquad H(\mu = 0) = H_0$

$|g_\lambda\rangle \qquad H(\mu = \lambda) = H_\lambda$

Only ground state

Level crossing → Failure



# Adiabatic Gauge Potential

$$H_\lambda = H_0 + \lambda V \qquad \mu \in [0 \ldots \lambda]$$

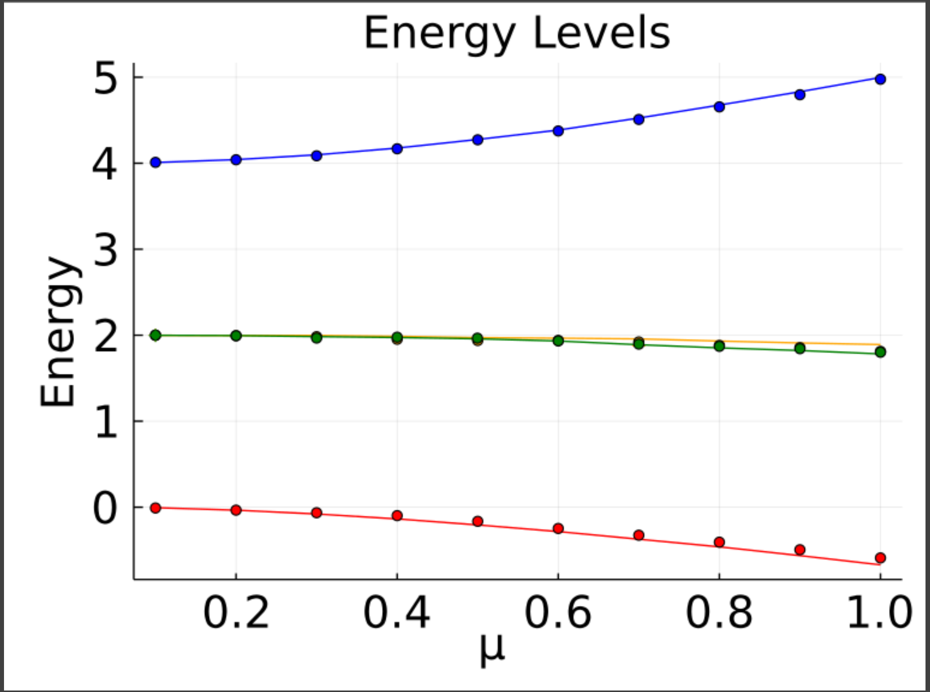
$$U_\mu = \mathcal{T}_\nu \; e^{\left(-i \int_0^\mu A_\nu \, d\nu\right)}$$

$$A_\mu = i\left(\partial_\mu U_\mu\right) U_\mu^\dagger$$

$$\widetilde{H}_\mu := U_\mu^\dagger H_\mu U_\mu$$

↓

$\widetilde{H}_\mu$ is diagonal at every step
In the eigenbasis of $H_0$



# Adiabatic Gauge Potential

$$A_\mu = i(\partial_\mu U_\mu)U_\mu^\dagger$$

$$\widetilde{H}_\mu := U_\mu^\dagger H_\mu U_\mu$$

$d/d\mu$

$$\widetilde{H}_\mu = \sum_n E_n(\mu)|n(0)\rangle\langle n(0)|$$

$$|n(0)\rangle \equiv |n(\mu = 0)\rangle$$

$$\|G_\mu\| = \|V + i\,[A_\mu, H_\mu]\| = 0$$

$$C_\mu(\boldsymbol{\alpha}) = \|G_\mu(\boldsymbol{\alpha})\| = \|V + i\,[A_\mu(\boldsymbol{\alpha}), H_\mu]\|$$

$$\boldsymbol{\alpha}^{opt} \quad | \quad C_\mu(\boldsymbol{\alpha}^{opt}) = \min_{\boldsymbol{\alpha}} C_\mu(\boldsymbol{\alpha})$$

# Approximating the AGP

$$A_\mu(\boldsymbol{\alpha}) = \sum_{i=1}^{L} \alpha_i^\mu \, B_i$$

$B_i$'s are Local Operators → Approximation!

Efficiently reproduce the EXACT AGP between sectors

$$\|\mathcal{P}\left(V + i\left[A_\mu(\boldsymbol{\alpha}), H_\mu\right]\right)\mathcal{Q}\| \coloneqq \|\mathcal{P}\left(G_\mu\right)\mathcal{Q}\| = 0$$

Fail in reproducing the EXACT AGP inside each sector

$$\|\mathcal{P}\left(V + i\left[A_\mu(\boldsymbol{\alpha}), H_\mu\right]\right)\mathcal{P}\| \coloneqq \|\mathcal{P}\left(G_\mu\right)\mathcal{P}\| \neq 0$$

$$\tilde{\mathrm{H}}_\mu = U_\mu^\dagger(\boldsymbol{\alpha}) H_\mu U_\mu(\boldsymbol{\alpha})$$

Block diagonalized the Hamiltonian

Variational Schrieffer-Wolff Transformations for Quantum Many-Body Dynamics J.Wurtz, P.W. Claeys, and A. Polkovnikov
10.1103/PhysRevB.101.014302

# Embedding in Quantum Computing framework

$$U_\mu(\boldsymbol{\alpha}) = \prod_{i=1}^{L} e^{-i\alpha_i^\mu B_i} \qquad g_i^\mu = e^{-i\alpha_i^\mu B_i}$$

Single qubit or two qubit parametrized gate

$$\mu \in [0 \dots \lambda] \rightarrow \quad \mu_t = \delta\mu \, t$$

$$\delta\mu = \frac{\lambda}{T}, \qquad t \in [0, T] \in \mathbb{N}$$

$$C_\mu(\boldsymbol{\alpha}_t) \rightarrow C_t\,(\boldsymbol{\alpha}_{t+1} - \boldsymbol{\alpha}_t)$$

$$X_t \left( \frac{\boldsymbol{\alpha}_{t+1} - \boldsymbol{\alpha}_t}{\delta\mu} \right) = Y_t$$

Analytic Minimization
solving a linear system

$t = 0\,, \mu = 0,\; H = H_0$ is diagonal

$$t = 0 \qquad C_0\,(\boldsymbol{\alpha}_1 - \mathbf{0}) \longrightarrow X_0 \left( \frac{\boldsymbol{\alpha}_1}{\delta\mu} \right) = Y_0$$

$$t = 1 \qquad C_1\,(\boldsymbol{\alpha}_2 - \boldsymbol{\alpha}_1^{opt}) \longrightarrow X_1 \left( \frac{\boldsymbol{\alpha}_2}{\delta\mu} \right) = Y_1$$

$$t = 2 \qquad C_1\,(\boldsymbol{\alpha}_3 - \boldsymbol{\alpha}_2^{opt}) \qquad \dots$$

$$t = T \quad (\mu = \lambda) \qquad \boldsymbol{\alpha}_T^{opt}$$

# Embedding in Quantum Computing framework

$$X_t\left(\frac{\boldsymbol{\alpha}_{t+1}}{\delta\mu}\right) = Y_t$$

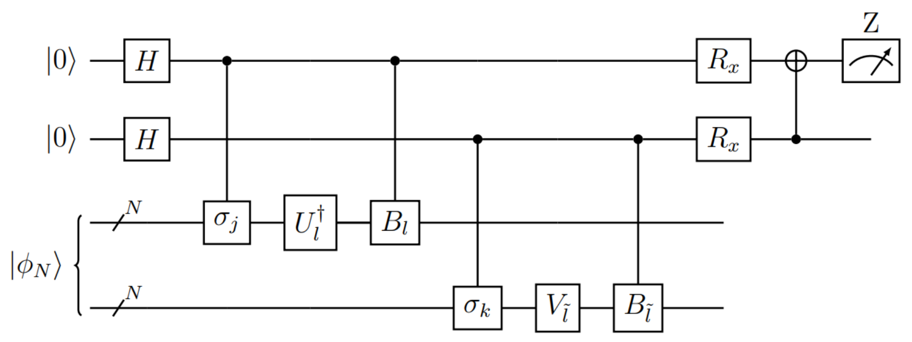
$$H_\mu = \sum_k h_k \sigma_k$$

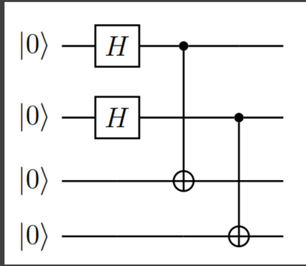
$$X_{\tilde{l}l} = \sum_{jk} h_j\, h_k\, Tr(i[U_l\, B_l\, U_l^\dagger, \sigma_j]\;\; i[U_{\tilde{l}} B_{\tilde{l}} U_{\tilde{l}}^\dagger, \sigma_k])$$

$$U_l(\boldsymbol{\alpha}[1{:}\,l]) = \prod_{i=1}^{l} e^{-i\alpha_i B_i}$$

$p(0) \propto X_{\tilde{l}l}$ Repeat $S$ times

$$|\phi_N\rangle \rightarrow Tr(AB) = 2^N \langle\phi_N| A^T \otimes B\, |\phi_N\rangle$$



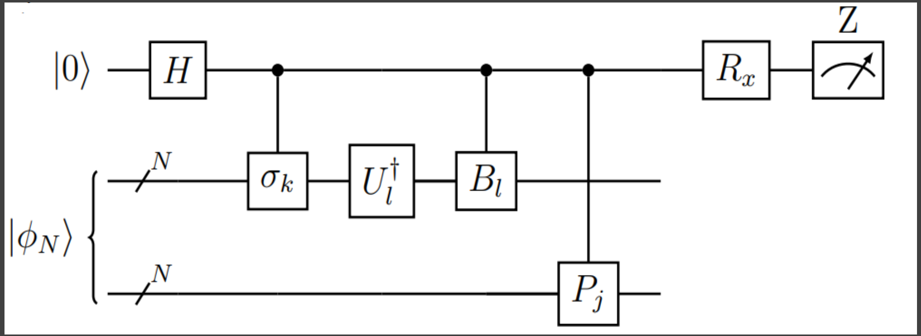



# Embedding in Quantum Computing framework

$$X_t\left(\frac{\boldsymbol{\alpha}_{t+1}}{\delta\mu}\right) = Y_t$$

$$Y_l = -\sum_{jk} v_j\, h_k\, Tr\left(P_j\, i\left[U_l\, B_l\, U_l^\dagger, \sigma_k\right]\right)$$

$p(0) \propto Y_l$ Repeat $S$ times




$$H_\mu = \sum_k h_k \sigma_k$$

$$U_l(\boldsymbol{\alpha}[1:l]) = \prod_{i=1}^{l} e^{-i\alpha_i B_i}$$

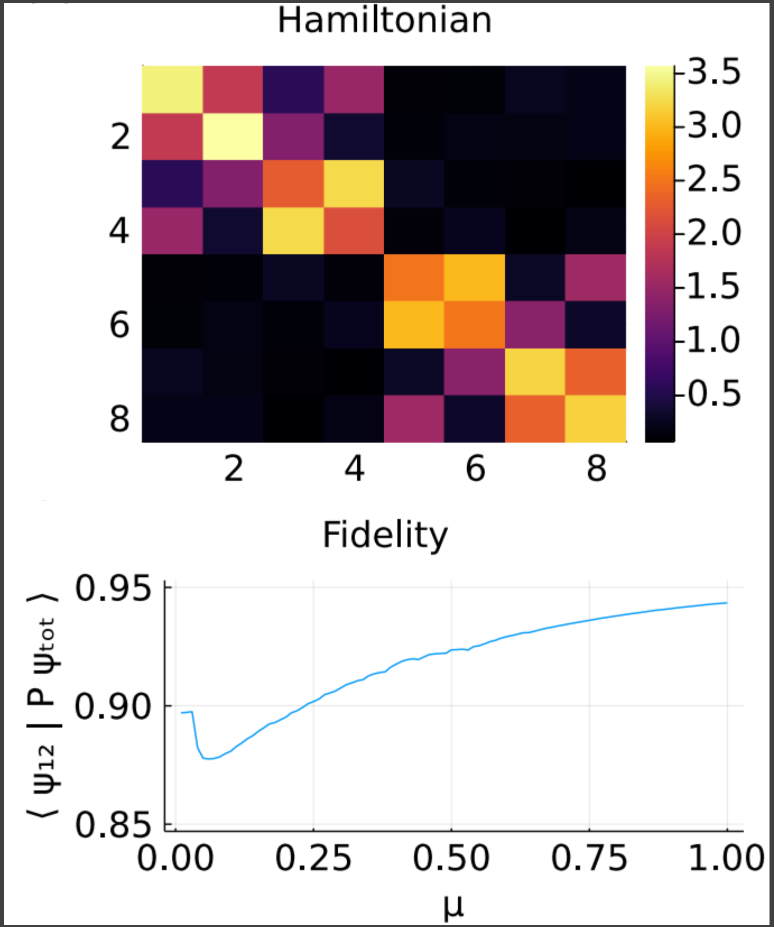
$$V = \sum_k v_k P_k$$

$$\mathcal{O}(N^{2\gamma} L^2 T)$$

$\gamma = 1$ $H_\mu$ is 1-local or 2-local

$\gamma = 2$ $H_\mu$ all couples interactions

...




$$H_\lambda = H_0 + \lambda V$$

$$H_\lambda = h\sigma_3^z + \lambda(J_1\sigma_1^x\sigma_3^x + J_3\sigma_2^x\sigma_3^x)$$

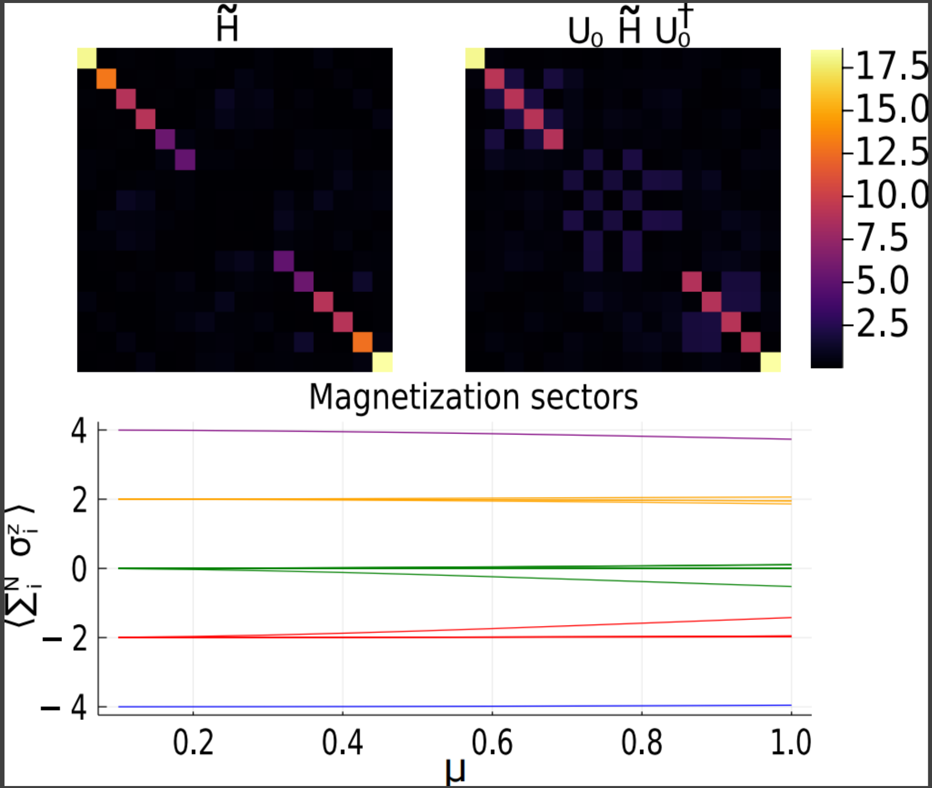
$$J_1 = J_3 = 1, \qquad h = 4.5$$

$$T = 100, \qquad L = 54$$

$$|\widetilde{H}_\lambda| = |U_\lambda^\dagger H_\lambda U_\lambda|$$

$$|\psi_{12}(t=1)\rangle = e^{-i\mathcal{P}\widetilde{H}_\mu\mathcal{P}}|\psi_{12}(t=0)\rangle$$

$$\mathcal{P}\,|\psi_{tot}(t=1)\rangle = \mathcal{P}\,e^{-i\widetilde{H}_\mu}\,|\psi_{12}(t=0)\rangle \otimes |0_3\rangle$$



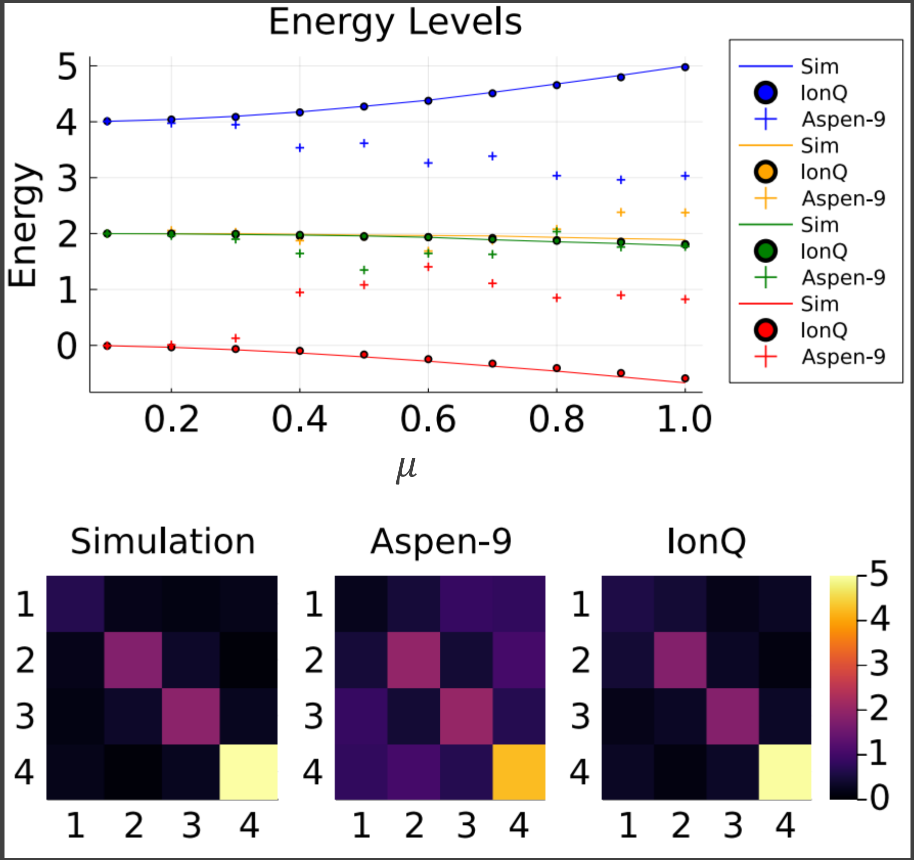

$$H_\lambda = \sum_{i=1}^{N} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + h\,\sigma_i^z) + \lambda \sum_{i=1}^{N} \sigma_i^x$$

$$m = \langle \sum_i^N \sigma_i^z \rangle$$

$N = 4, \quad T = 100,$

$L = 210, \; h = 4.5, \; \lambda = 1$

$$|\widetilde{H}_\lambda| = |U_\lambda^\dagger H_\lambda U_\lambda|$$




$$H_0 = 2\mathbb{I} + \sigma_1^z + \sigma_2^z$$

$$\mathrm{V} = \mathrm{v}_1\, \sigma_1^x + \mathrm{v}_2\, \sigma_2^x + \mathrm{v}_3 \sigma_1^y + \\ + \mathrm{v}_4\, \sigma_2^y + \mathrm{v}_5 \sigma_1^x \otimes \sigma_2^x + \mathrm{v}_6 \sigma_1^x \otimes \sigma_2^y \\ + \mathrm{v}_7 \sigma_1^y \otimes \sigma_2^x + \mathrm{v}_8 \sigma_1^y \otimes \sigma_2^y$$

$v_i \in (0,1)$ randomly chosen

$T = 100, \qquad S = 100, \qquad L = 15$

No sectors & Universal Ansatz

↓

Diagonalization

## Noise Resilient Variational Quantum Classical Optimization

- convergence speed of variational hybrid quantum-classical optimization algorithms
- Bounded by two competing terms
- Accuracy may actually be higher in some noisy regimes
- Open question: obtain Fisher efficient estimators of the optimal parameters

https://doi.org/10.1103/PhysRevA.102.052414
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## Variational Approximation of Low-Energy Hamiltonians on Real Quantum Hardware

- Diagonalization or Block diagonalization
- Scalable and NISQ suitable
- Tested it on 3 models, one of them on quantum Hardware, concluding that connectivity is crucial
- Outcomes: Apply this method to complex many body Hamiltonians

Gentini, Cuccoli, Banchi

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Gentini, Cuccoli, Banchi

# Enhancing Quantum Generative Adversarial Networks for Noise Sensing Applications

# Outline

- Generative Adversarial Networks
- Quantum Generative Adversarial Networks
- Training QGANs with noisy information
- SuperQGANs for noise-sensing

"How to enhance quantum generative adversarial learning of noisy information." Braccia, Caruso, Banchi *New Journal of Physics* 23.5 (2021): 053024.

"Quantum Noise Sensing by generating Fake Noise." Braccia, Banchi, Caruso. *arXiv preprint arXiv:2107.08718*.

# Generative Adversarial Networks (GANs)

# Generative Adversarial Networks (GANs)

- https://thispersondoesnotexist.com/

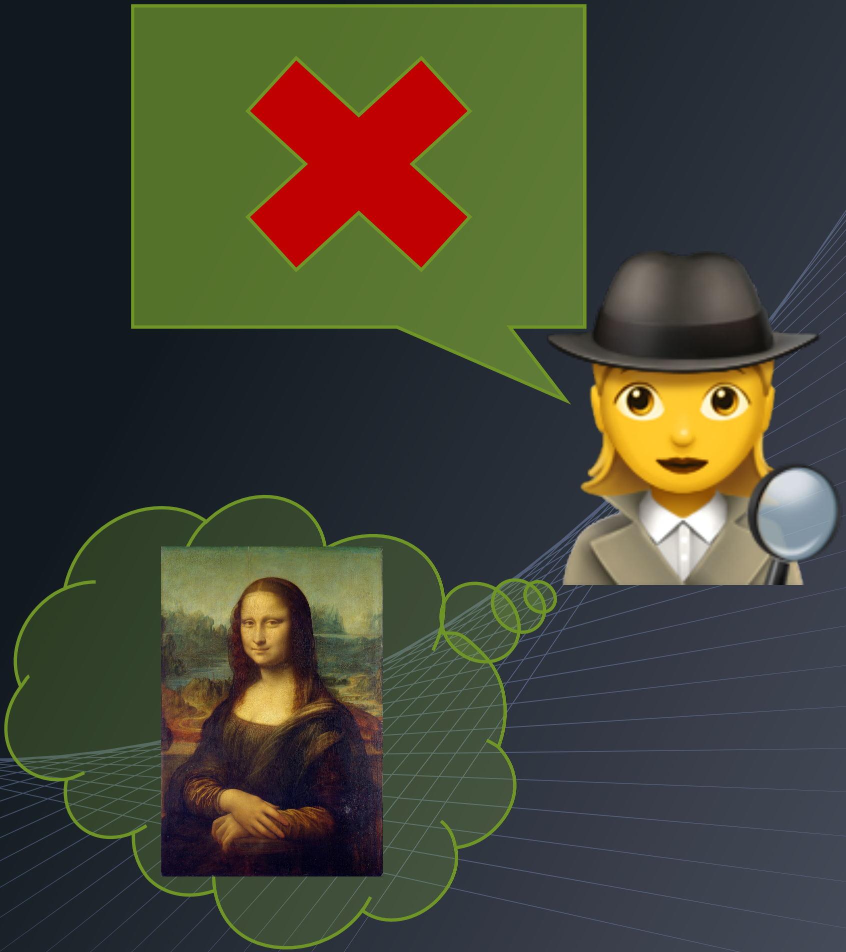
# How do GANs work?

**Generator**
(Counterfeiter)

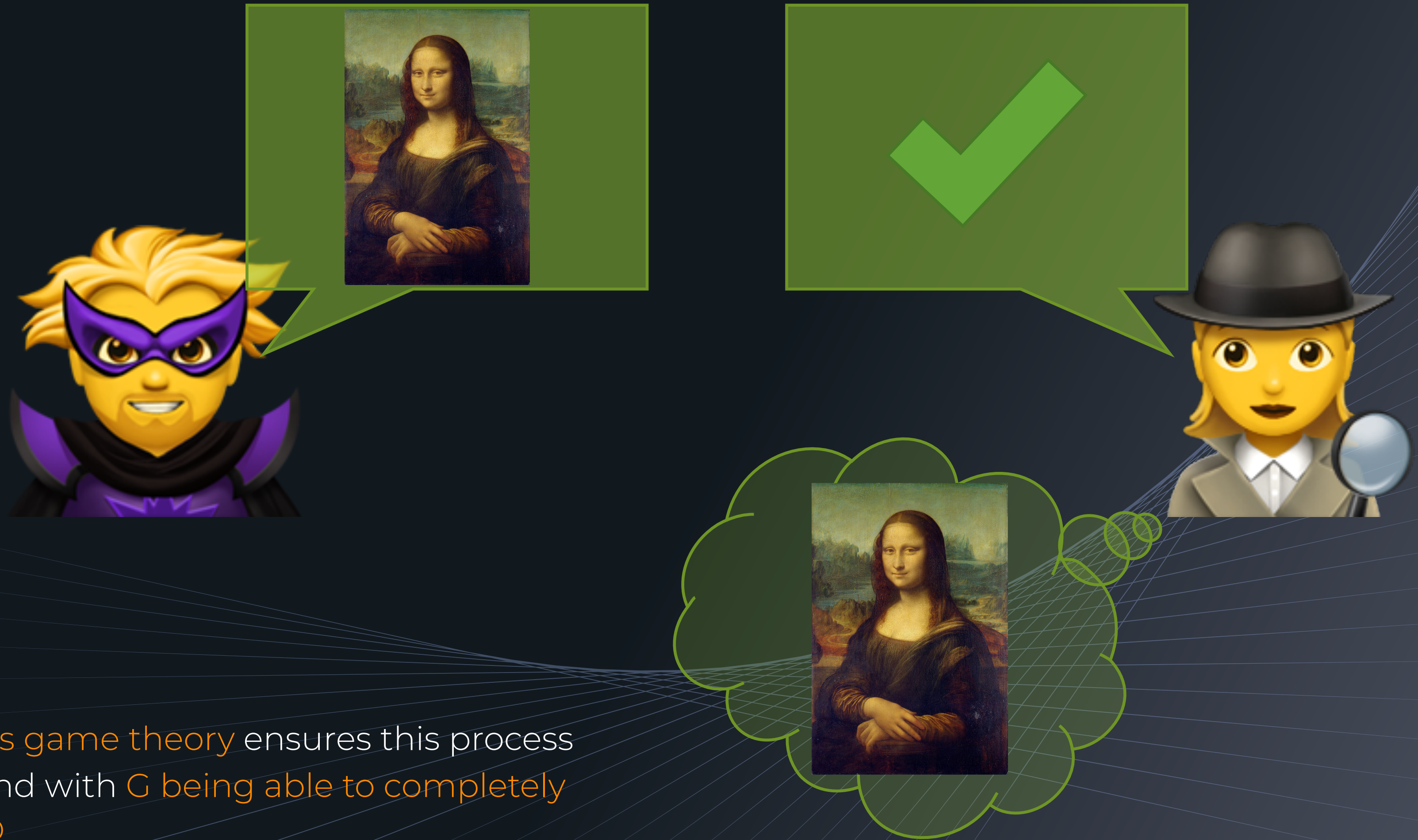
- Generator (G) and Discriminator (D) play a turn based game against each other.
- Each turn G produces a fake copy of some target object (say an image), and D compares it to the real one deciding the probability of it being actually real.
- After each turn G uses D's feedback to improve its counterfeiting ability, and D uses G's fake samples to improve its discrimination strategy
- Nash's game theory ensures this process will end with G being able to completely fool D

**Discriminator**
(Detective)

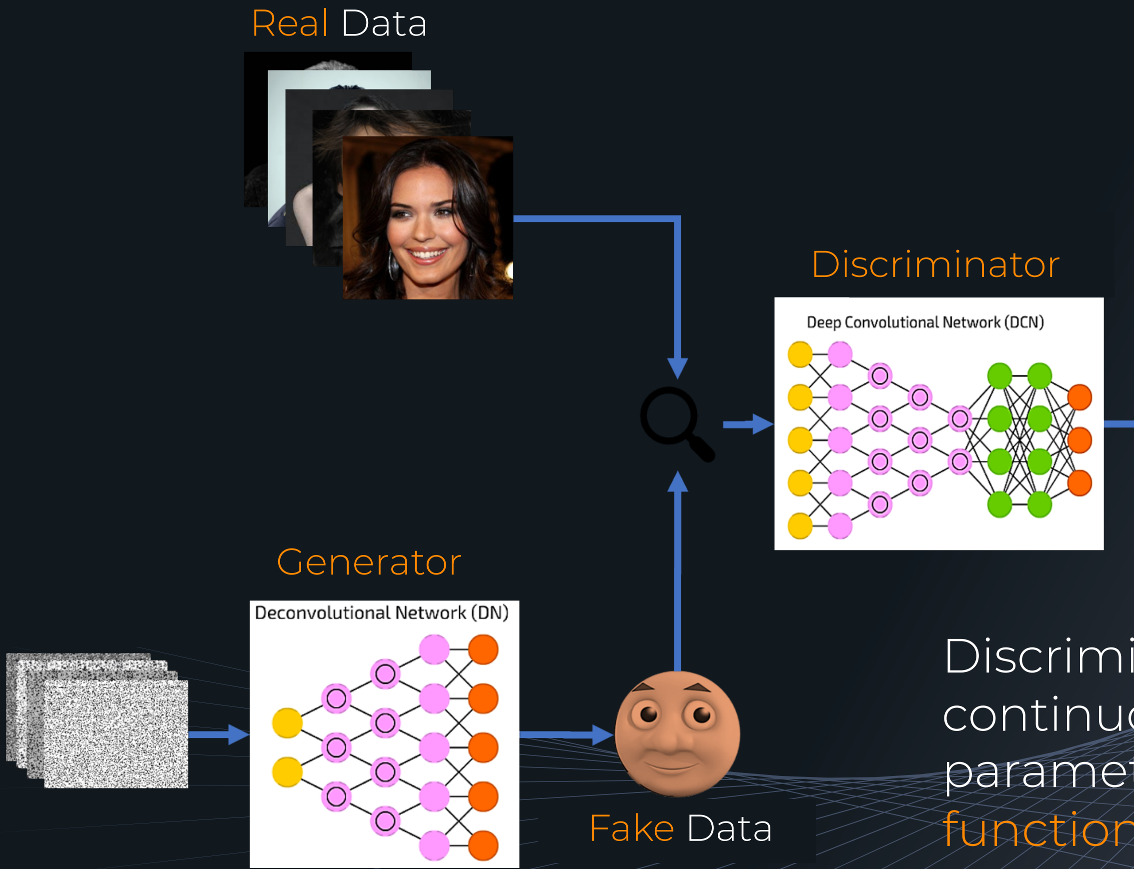
Each turn G produces a fake copy of some target object (say an image), and D compares it to the real one deciding the probability of it being actually real.

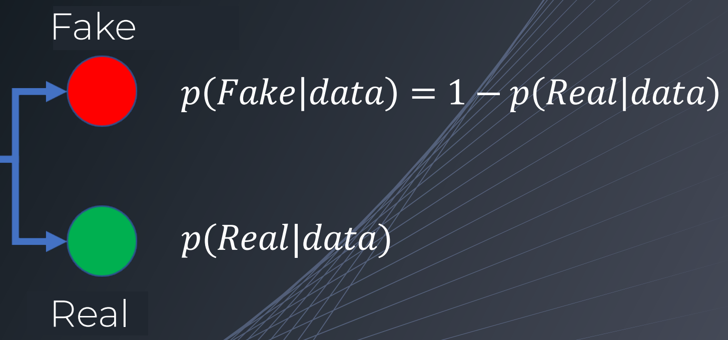


After each turn G uses D’s feedback to improve its counterfeiting ability, and D uses G’s fake samples to improve its discrimination strategy



Nash’s game theory ensures this process will end with G being able to completely fool D

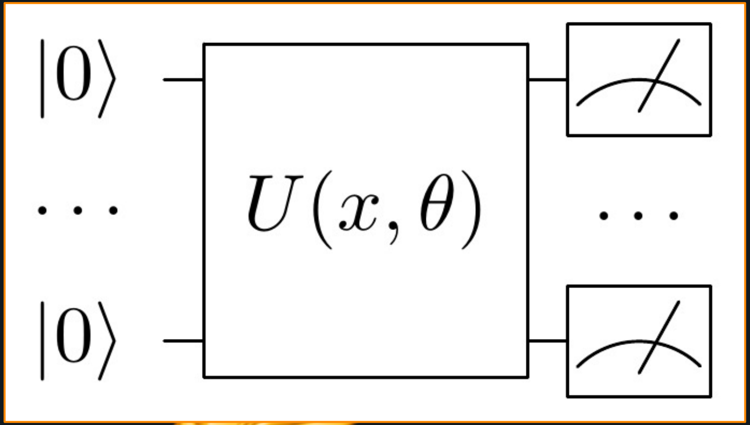


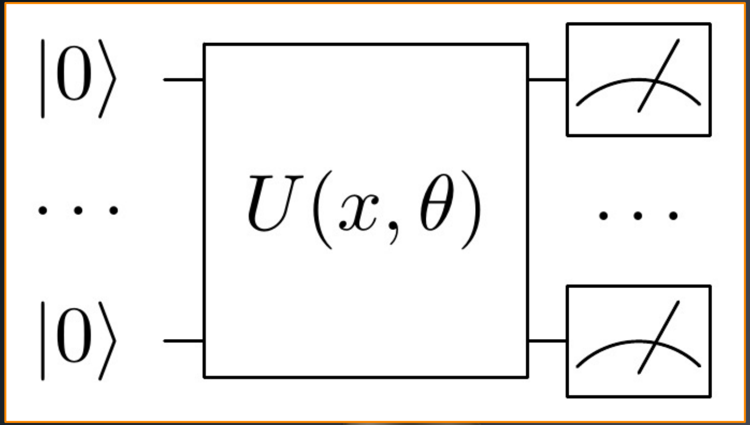

Discriminator's output is a continuous function of the agents' parameters, use it to build a score function

We can differentiate w.r.t. them and apply backpropagation algorithms.

# Quantum GANs (QGANs)




PQCs

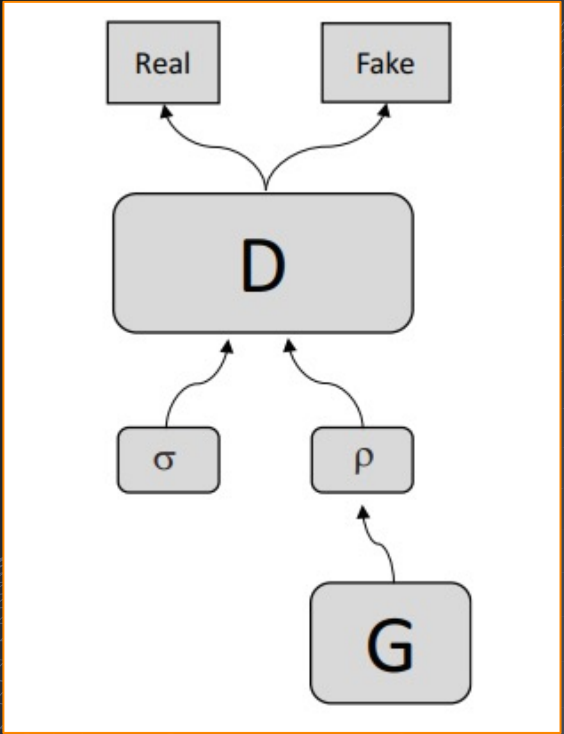



# Quantum Generative Adversarial Games

- D: two-outcomes POVM $\{T, F = I - T\}$
- G: quantum state generator $\rho = \rho_F$
- Real data: quantum state $\sigma = \rho_R$

Strategies

- D: maximize $p(T|R) - p(T|F) = Tr[T(\rho_R - \rho_F)]$
- G: maximize $p(T|F) = Tr[T\rho_F]$




Min-Max Problem

$$\min_{\rho_F} \max_{T} S(T, \rho_F) = \min_{\rho_F} \max_{T} Tr\,[T(\rho_R - \rho_F)]$$
$$= \min_{\rho_F} \max_{T} p(T|R) - p(T|F)$$

"Quantum Generative Adversarial Learning" S. Lloyd, C. Weedbrook. *Physical review letters*, *121*(4), 040502

# **Quantum** Generative Adversarial Games

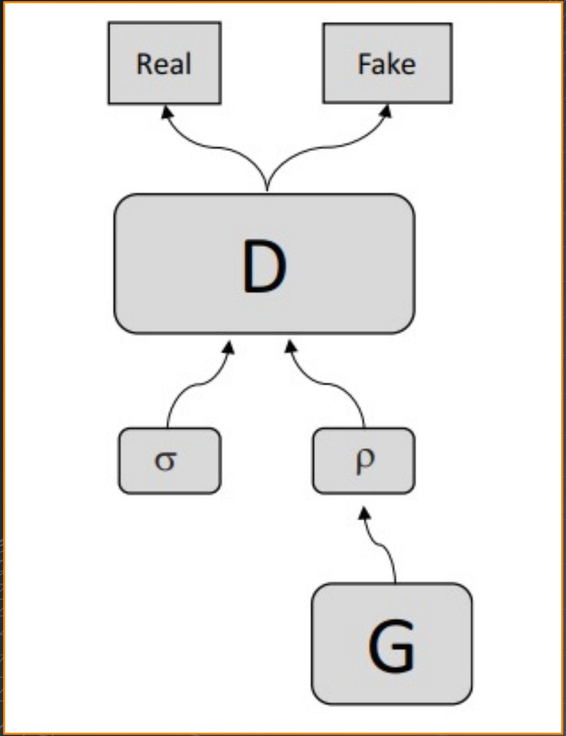
Min-Max Problem

$$\min_{\rho_F} \max_{T} Tr\, S(T, \rho_F) = \min_{\rho_F} \max_{T} Tr\, [T(\rho_R - \rho_F)]$$
$$= \min_{\rho_F} \max_{T} p(T|R) - p(T|F)$$

Quantum States (density matrices)

Positive Operator s.t. $\|T\|_1 \leq 1$

Compact and Covex Sets

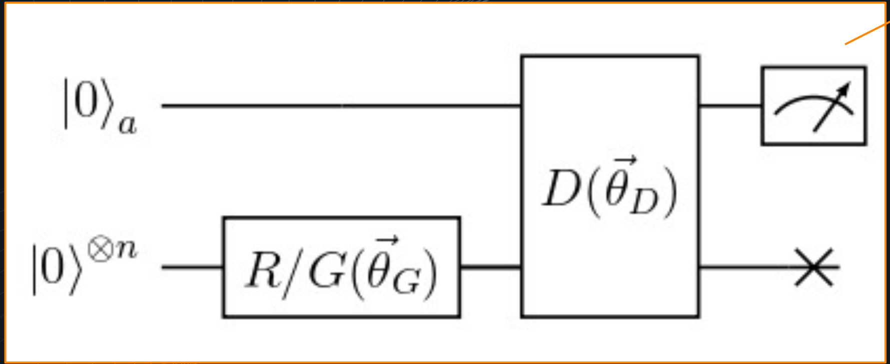



"Quantum Generative Adversarial Learning" S. Lloyd, C. Weedbrook. *Physical review letters*, *121*(4), 040502

# Possible Quantum Advantages

- Quantum processors can compress $N$ dimensional feature vectors over $\log N$ qubits
- If the classical optimization algorithm takes $O(poly(N))$ time to run, the quantum version would ideally require only $O(poly(\log(N)))$ so.
- For classical models such as neural networks, learning quantum distributions of data is exponentially costly, whereas quantum ones can learn them natively.

# Pure states QGAN




$$p(T|R/G) = \frac{\langle Z_a\rangle + 1}{2}$$

POVM measurement via Naimark Theorem

$$\mathrm{T} = Tr_a\big[\,|0\rangle_a\langle 0| \otimes I_S[D(|0\rangle_a\langle 0| \otimes I_S)D^\dagger]\big]$$

$$|0\rangle_a\langle 0| = \frac{Z_a + I_a}{2}$$

Two-outcomes POVM, single ancilla qubit!

$$f(\theta) = \langle 0|U^\dagger(\theta)\hat{O}U(\theta)|0\rangle$$

$$\partial_\theta f = \frac{f\left(\theta + \frac{\pi}{2}\right) - f\left(\theta - \frac{\pi}{2}\right)}{2}$$

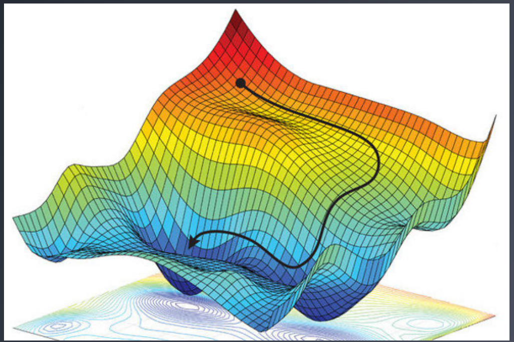
s.t. all gates appearing in $U$ are generated by Pauli matrices

Parameter Shift Rule

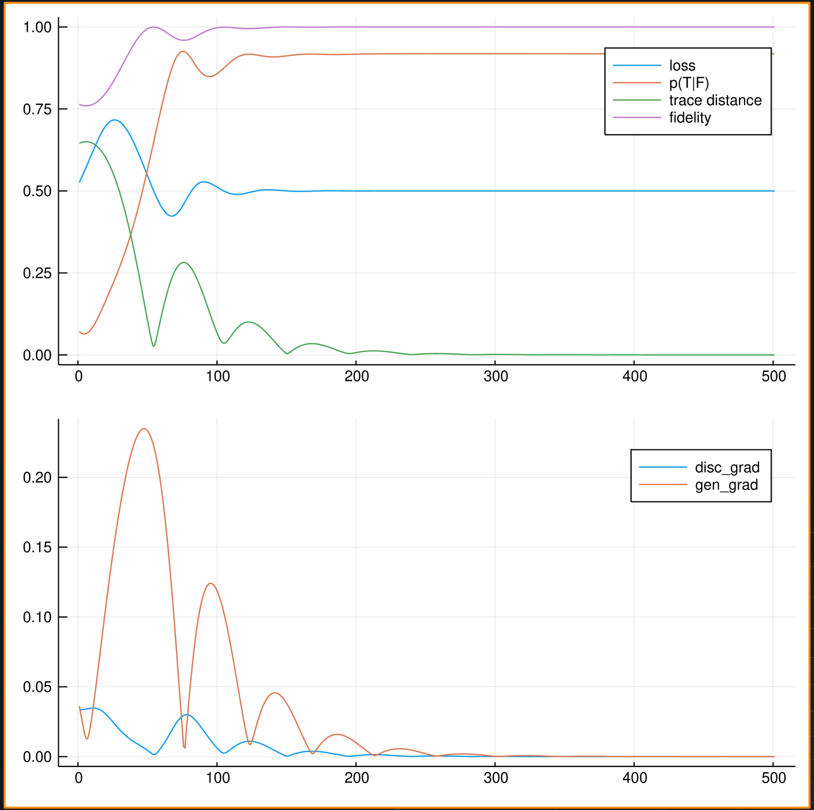
Schuld, Maria, et al. "Evaluating analytic gradients on quantum hardware." *Physical Review A* 99.3 (2019): 032331.

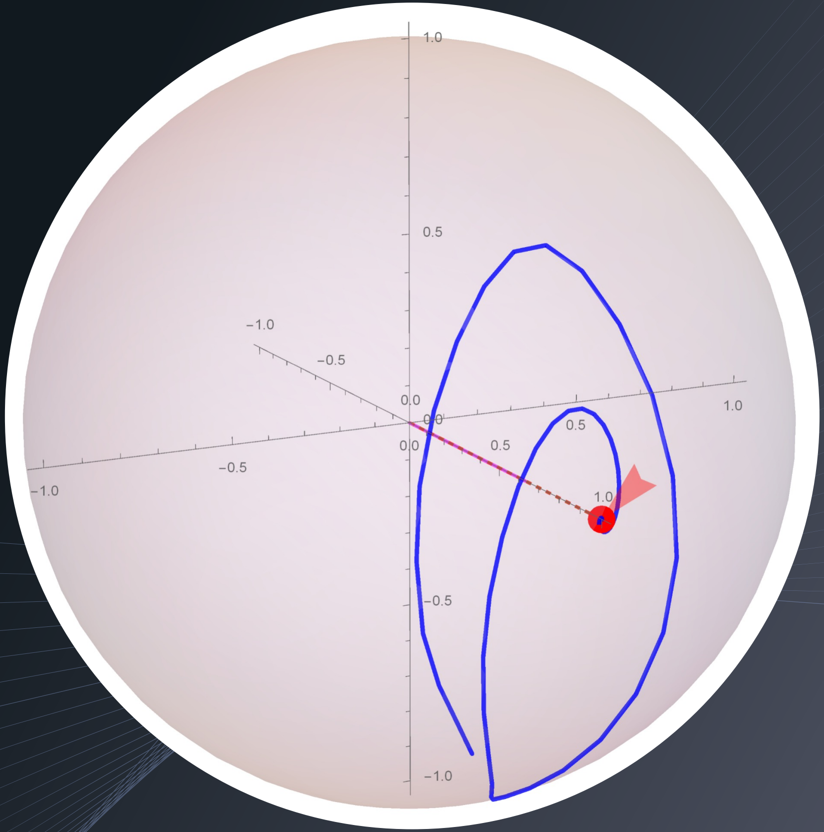
Gradient Descent (GD)

$$\vec{\theta} \leftarrow \vec{\theta} - \eta \nabla_{\vec{\theta}} f$$



# Everything goes fine!



# What is up with mixed states?

Recall mixed states are described by density matrices

$$|\psi\rangle \rightarrow \rho \in Lin^{+}(H)\ s.t.\ Tr(\rho) = 1$$

Qubit: the interior of the Bloch ball is accessible

$$\rho = \frac{1}{2}(I + \vec{r} \cdot \vec{\sigma}), \qquad 0 \leq |\vec{r}| \leq 1$$

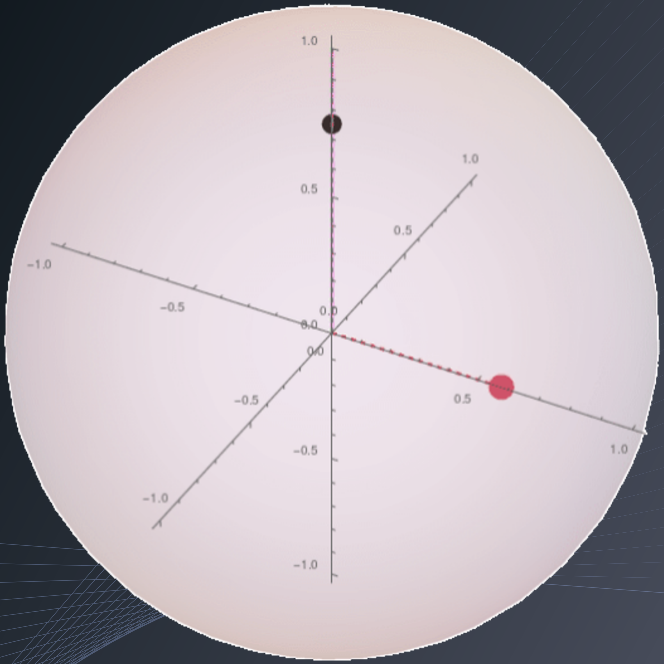
POVM elements can be described analogously

$$\Pi_D = \frac{1}{2}(d^0 I + \vec{d} \cdot \vec{\sigma}), \qquad d^0 \geq |\vec{d}|$$

QGAN Score Function reads:

$$S(\Pi_D, \rho_G) = \frac{\vec{d} \cdot (\vec{r} - \vec{g})}{2}$$

Quadratic function of Bloch vectors




GD, ADAM
LIMIT CYCLES !!!

"How to enhance quantum generative adversarial learning of noisy information." Braccia, Caruso, Banchi *New Journal of Physics* 23.5 (2021): 053024.

# All we lack is some OPTIMISM!

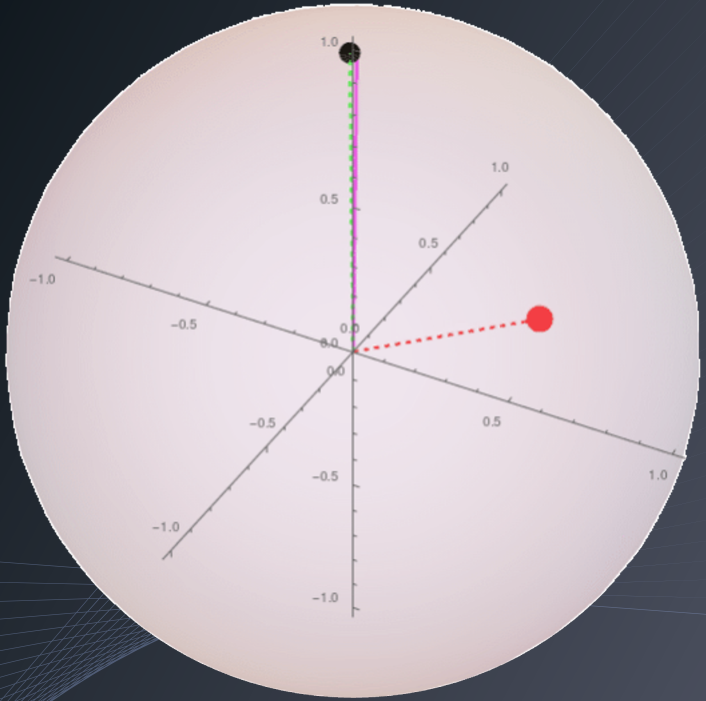
Daskalakis, Constantinos, et al. "Training gans with optimism." *arXiv:1711.00141* (2017).

## Optimistic Mirror Descent (OMD)

Players exploit knowledge of foe's strategy

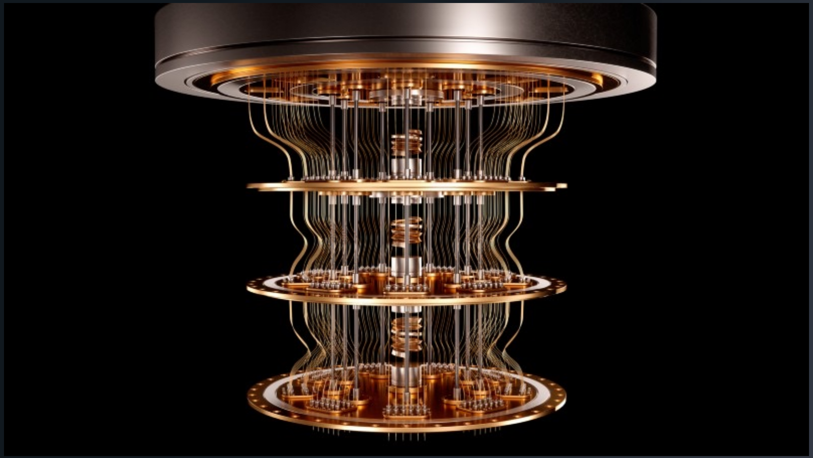
$$\vec{\theta}^{t+1} \leftarrow \vec{\theta}^{t} - 2\eta\nabla_{\vec{\theta}}^{t} S + \eta\nabla_{\vec{\theta}}^{t-1} S$$

No more convergence issues!




"How to enhance quantum generative adversarial learning of noisy information." Braccia, Caruso, Banchi *New Journal of Physics* 23.5 (2021): 053024.

# SuperQGANs for Noise Sensing



# NISQ devices

Nowadays quantum processors are Noisy (and Small)...

## *Expectation* vs *Reality*



$$|\psi_{in}\rangle \rightarrow |\psi_{out}\rangle = U|\psi_{in}\rangle$$

$$|\psi_{in}\rangle \rightarrow \rho_{out} = \Phi(|\psi_{in}\rangle\langle\psi_{in}|)$$

# Quantum Channels

CPTP maps $\Phi$

- Linear: $\sum_i p_i \rho_i \to \sum_i pi\Phi(\rho_i)$
- Trace Preserving
- Positive (preserve positivity of $\rho$)

Stinespring representation

$$\Phi(\rho) = \mathrm{Tr}_E[U_{SE}(\rho_S \otimes \omega_E)U_{SE}^\dagger]$$

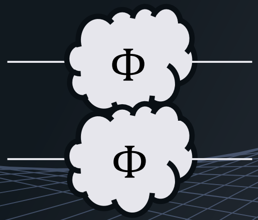
Kraus representation

$$\Phi(\rho) = \sum_i K_i \rho \mathrm{K}_i^\dagger, \qquad \sum_i K_i^\dagger K_i = I$$

Composition of channels:

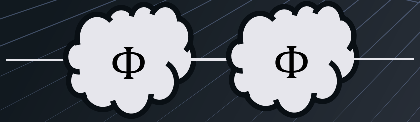
One would expect

- «Parallel», different systems pass through $n$ copies of the channel




$$= \Phi \otimes \cdots \otimes \Phi = \Phi^{\otimes n}$$

- «Series», the same system is processed $n$ times by the channel




$$= \Phi \circ \cdots \circ \Phi = \Phi^n$$

# Quantum Memory Channels

Most often instead


Memory channel with $n$ uses

$$\Phi^{(n)}$$

# Pauli Channels

Knill, Emanuel. "Quantum computing with realistically noisy devices." *Nature* 434.7029 (2005): 39-44.

## Random Unitary Channels

$$\Phi(\rho) = \sum_i p_i U_i \rho U_i^\dagger$$

System undergoes evolution $U_i$ with probability $p_i$

## Pauli Channels

Pauli Channels are exceptionally good at describing realistic noise models

$$\Phi^{(n)}(\rho) = \sum_{\vec{k}} p_{\vec{k}} \sigma_{\vec{k}} \rho \sigma_{\vec{k}}$$

Pauli word

$$\sigma_{\vec{k}} = \sigma_{k_0} \otimes \sigma_{k_1} \otimes \cdots \otimes \sigma_{k_n}$$

$$\sigma_{\vec{k}} = \sigma_{k_0} \circ \sigma_{k_1} \circ \cdots \circ \sigma_{k_n}$$

$$p_{\vec{k}} = p_{k_0} p_{k_1} \cdots p_{k_n} \rightarrow \text{"}\textit{memoryless channel}\text{"}$$

$$p_{\vec{k}} \neq p_{k_0} p_{k_1} \cdots p_{k_n} \rightarrow \text{"}\textit{memory (correlated) channel}\text{"}$$

This is the most common scenario, we want to characterize correlations!

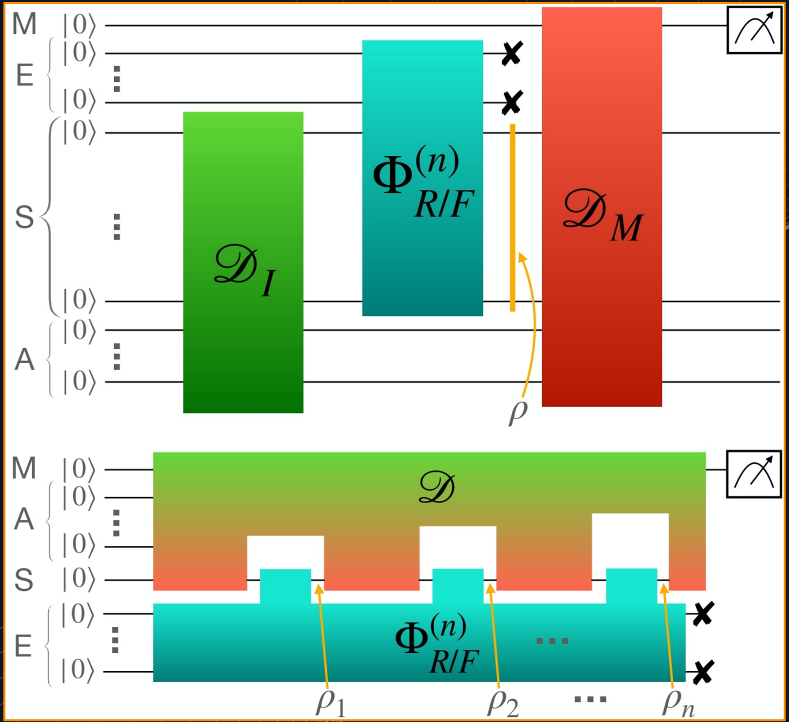
# SuperQGANs
## setup

Spatially Correlated Noise:

- $D_I$ prepares a probe state
- Real or Fake noisy channel $\Phi_{R/F}$ is applied
- $D_M$ implements measuring POVM

Temporally Correlated Noise:

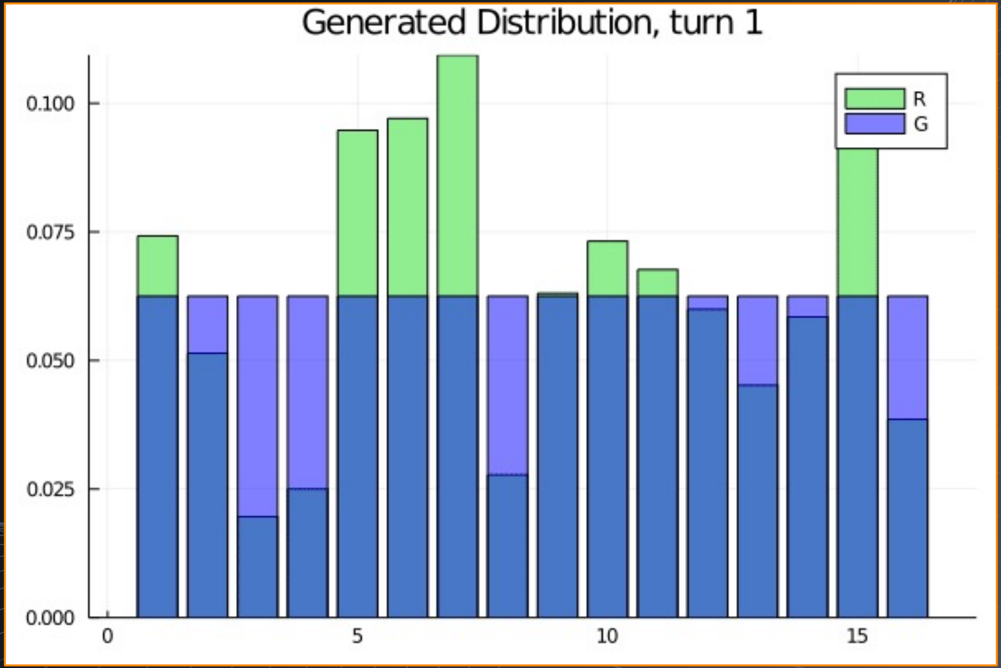
- General Quantum Comb scheme is applied
- Comb $D$ interacts with noisy Real/Fake channel at each of the $n$ intermediate times
- A final map entangles $D$'s comb with the measurement ancilla qubit




"Quantum Noise Sensing by generating Fake Noise." Braccia, Banchi, Caruso. *arXiv preprint arXiv:2107.08718.*

# SuperQGANs
## performance

- Since channel outputs are mixed, Optimistic strategy comes in handy
- Great advantage in using QCNN for the POVM-implementing PQC
- Method does not use any approximation, there is room for tweakings and speedups!



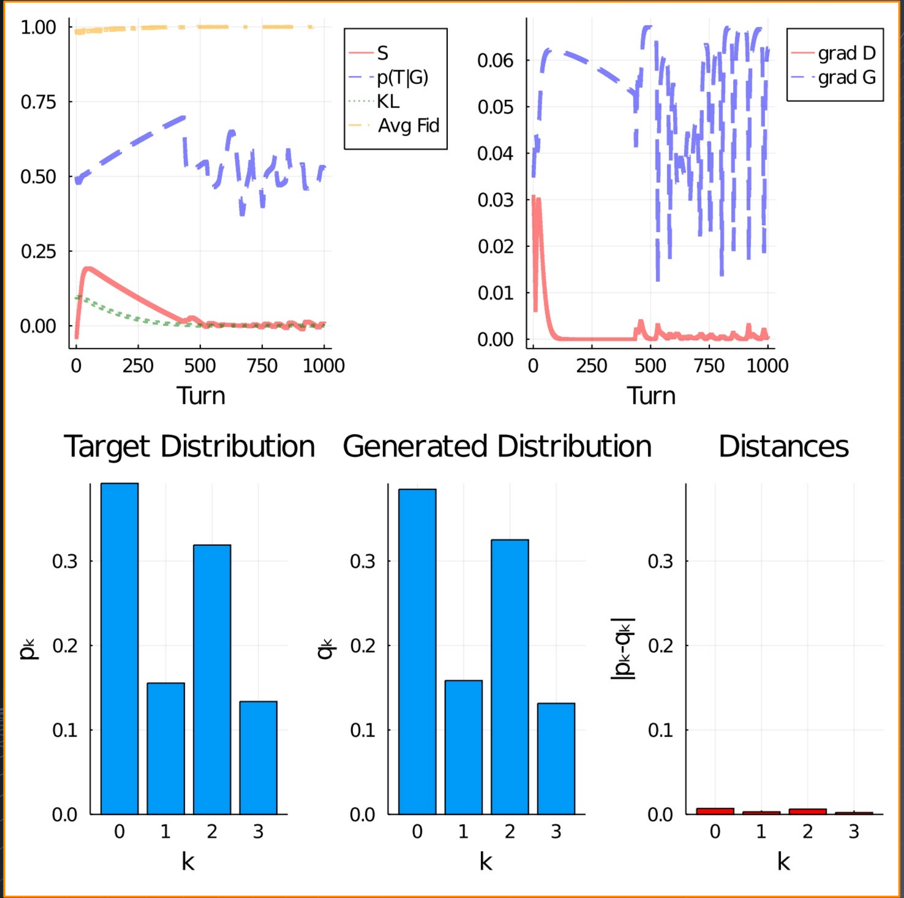

"Quantum Noise Sensing by generating Fake Noise." Braccia, Banchi, Caruso. *arXiv preprint arXiv:2107.08718*.

# SuperQGANs
## tackling correlations

Correlation model

$$p_{ij}^{(2)} = (1-\mu)p_i^{(1)}p_j^{(1)} + \mu p_i^{(1)}\delta_{ij}$$

First, learn single use channel $\Phi^{(1)}$



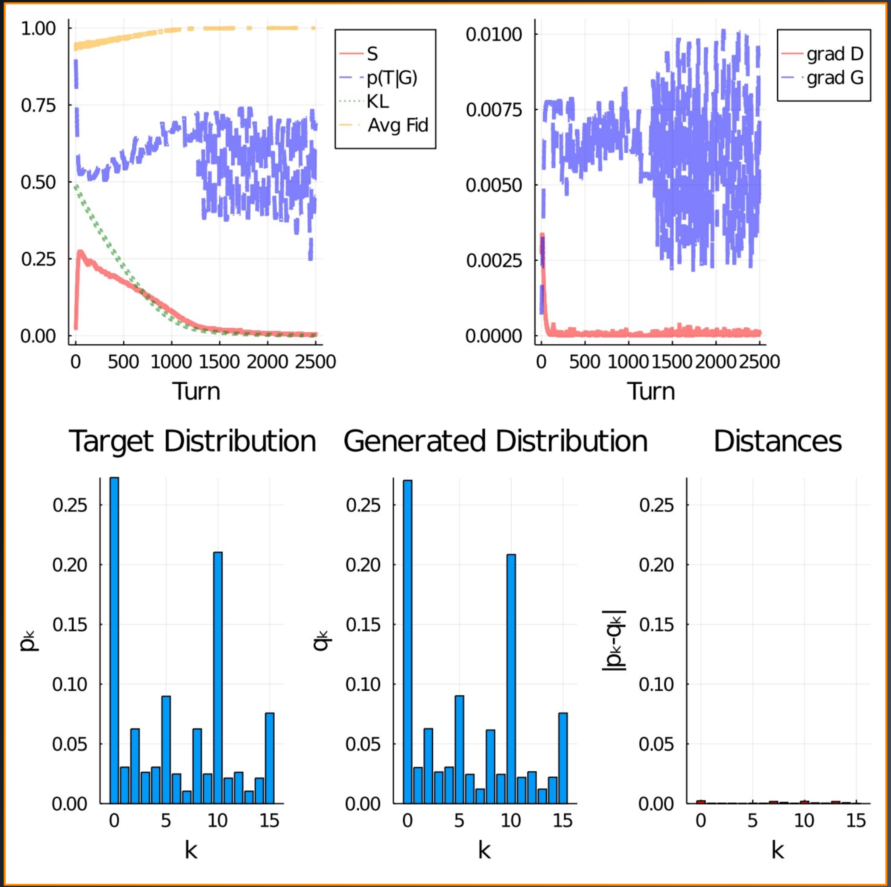

"Quantum Noise Sensing by generating Fake Noise." Braccia, Banchi, Caruso. *arXiv preprint arXiv:2107.08718*.

# SuperQGANs
## tackling correlations

### Correlation model

$$p_{ij}^{(2)} = (1-\mu)p_i^{(1)}p_j^{(1)} + \mu p_i^{(1)}\delta_{ij}$$

Then, use learnt «prior» $p_i^{(1)}$ to check wether $\Phi^{(2)} = \Phi^2$ (temporal), or $\Phi^{(2)} = \Phi^{\otimes 2}$ (spatial)

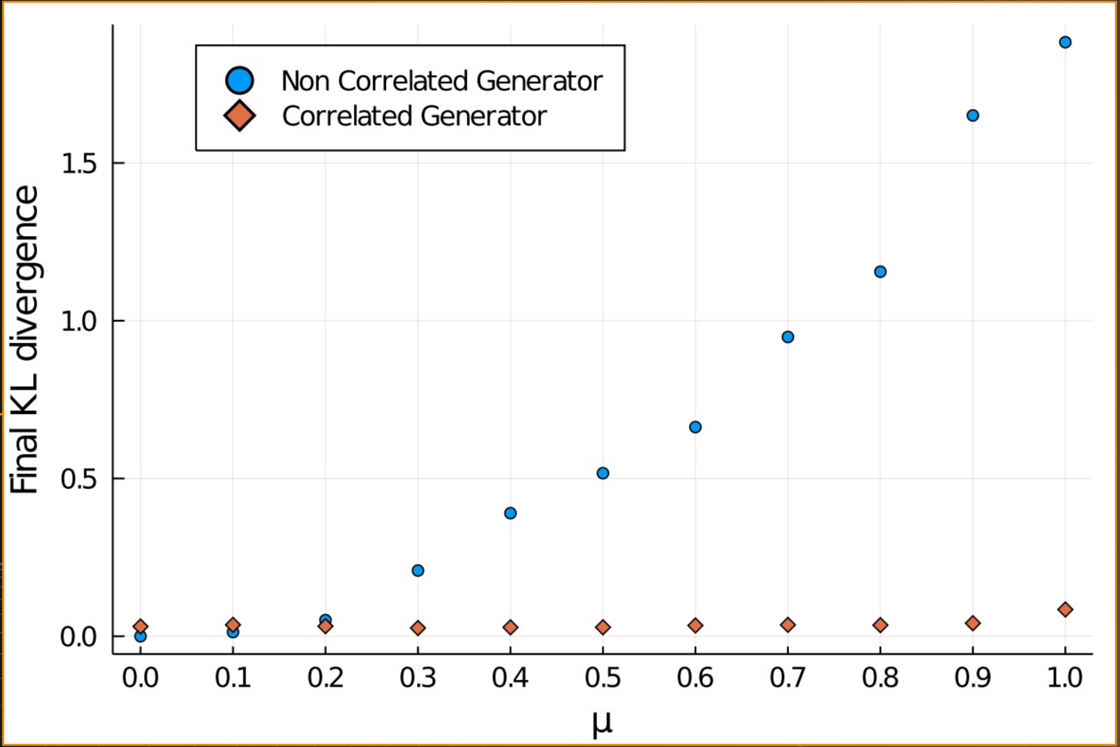



"Quantum Noise Sensing by generating Fake Noise." Braccia, Banchi, Caruso. *arXiv preprint arXiv:2107.08718*.

# SuperQGANs
## tackling correlations

Constraining G to control only factorized error-rates and monitoring convergence failure is another way to check for correlations

$$D_{KL}(P\|Q) = \sum_{x \in X} P(x)\log(\frac{P(x)}{Q(x)})$$




"Quantum Noise Sensing by generating Fake Noise." Braccia, Banchi, Caruso. *arXiv preprint arXiv:2107.08718*.

# Outlooks

Adversarial game paradigm can be shaped to perform implicit state/process tomography, what next?

- Find clever ways to exploit previously known structure of target state/noise

- Part from exact reconstruction and make QGAN/SuperQGAN scalable, e.g. by using Tensor Network representation of states/channels

# Thank you for your attention!