Lattice calculation of the short and intermediate time windows contributing to the leading-order HVP term of the muon g - 2 using twisted mass fermions.

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(On behalf of the ETM Collaboration)

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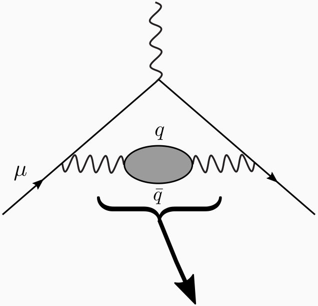
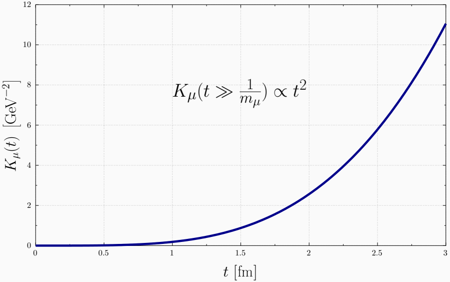
Hadron Vacuum Polarization from Lattice QCD





$$\Pi_{\mu\nu}(Q) = \int d^4x \, e^{iQ\cdot x} \left\langle J_\mu(x) J_\nu(0) \right\rangle = \left(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right) \Pi(Q^2)$$

$$\Downarrow$$

$$a_\ell^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \, \frac{1}{m_\ell^2} f\left(\frac{Q^2}{m_\ell^2}\right) \cdot \left(\Pi(Q^2) - \Pi(0)\right).$$

Time-Momentum representation (Bernecker & Meyer, 2011)

$$a_\ell^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dt \; K_\ell(t) \, V(t), \qquad V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \left\langle J_i(\vec{x}, t) J_i(0) \right\rangle \, .$$

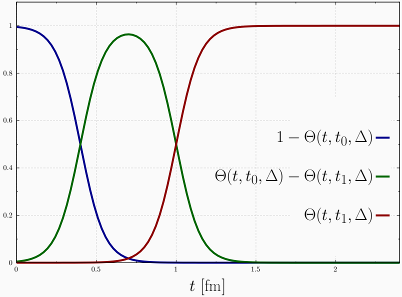
RBC/UKQCD windows

a_μ^{HVP} decomposed as a sum of three contributions that probe different (Euclidean) time regions:

$$a_\mu^{SD} = 4\alpha_{em}^2 \int_0^\infty dt\ K_\mu(t)\, V(t) \cdot \left[1 - \Theta\left(t, t_0, \Delta\right)\right]$$

$$a_\mu^{W} = 4\alpha_{em}^2 \int_0^\infty dt\ K_\mu(t)\, V(t) \cdot \left[\Theta\left(t, t_0, \Delta\right) - \Theta\left(t, t_1, \Delta\right)\right]$$

$$a_\mu^{LD} = 4\alpha_{em}^2 \int_0^\infty dt\ K_\mu(t)\, V(t) \cdot \left[\Theta\left(t, t_1, \Delta\right)\right]$$



$$a_\mu^{\mathrm{HVP}} = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

$$\Theta\left(t, t', \Delta\right) = \frac{1}{1 + e^{-2(t-t')/\Delta}}$$

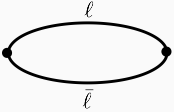
$$t_0 = 0.4\ \mathrm{fm}, \quad t_1 = 1\ \mathrm{fm}, \quad \Delta = 0.15\ \mathrm{fm}$$

What has been computed

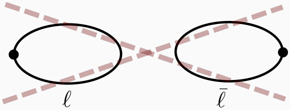
We compute the light-quark, fermionically connected contribution to a_μ^{SD} and a_μ^{W} in the iso-symmetric limit $m_u{=}m_d$, neglecting α_{em} effects.

$$J_\mu(x) = \underbrace{(q_u+q_d)\;\bar\psi_\ell(x)\gamma_\mu\psi_\ell(x)}_{J^\ell_\mu} + \sum_{f\neq u,d}\underbrace{q_f\;\bar\psi_f(x)\gamma_\mu\psi_f(x)}_{\text{not considered here}}$$

$$V^\ell(t) \equiv \frac{1}{3}\sum_{i=1,2,3}\int d\vec{x}\langle J^\ell_i(\vec{x},t)J^\ell_i(0)\rangle = (q_u^2+q_d^2)\times$$



$$-\,(q_u+q_d)^2\times$$



Relevance of the intermediate window

Tension in previous lattice determinations of $a_\mu^W(light, conn., isoQCD)$.

3.7σ discrepancy between BMW and $R-$ratio in the total a_μ^W.

In the SD and intermediate windows the analysis of the systematics is facilitated as the tail of the correlator $V_\ell(t)$ does not contribute.

Simulation details

Four ($\simeq$) physical point ensembles, with $a \in [0.058 \text{ fm} - 0.082 \text{ fm}]$.
$L \sim 5.2$ fm and $L \sim 7.8$ fm to control Finite Size Effects (FSEs).

ensemble	β	V/a^4	a (fm)	$a\mu_\ell$	M_π (MeV)	L (fm)	$M_\pi L$
cB211.072.64	1.778	$64^3 \times 128$	0.0816 (3)	0.00072	136.8 (0.5)	5.22	3.62
cB211.072.96	1.778	$96^3 \times 192$	0.0816 (3)	0.00072	136.7 (0.5)	7.83	5.43
cC211.060.80	1.836	$80^3 \times 160$	0.0694 (3)	0.00060	134.3 (0.5)	5.55	3.78
cD211.054.96	1.900	$96^3 \times 192$	0.0577 (2)	0.00054	138.9 (0.5)	5.53	3.90

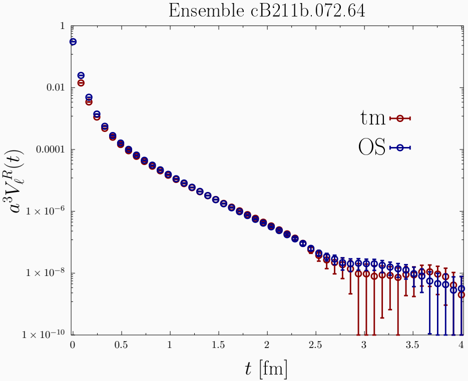
ensemble	Z_V	Z_A
cB211.072.64	0.709932 (7)	0.71403 (77)
cB211.072.96	0.709950 (5)	0.71577 (35)
cC211.060.80	0.728477 (5)	0.73803 (47)
cD211.054.96	0.746595 (5)	0.76134 (27)

- Renormalization Constants at sub-permille precision.
- $N_f = 2 + 1 + 1$ flavors.

- Iwasaki action for gluons.
- Wilson-clover twisted mass fermions at maximal twist for quarks (automatic $\mathcal{O}(a)$ improvement).

Extended Twisted Mass Collaboration

The vector-vector correlator $V_\ell(t)$



- Light-quark propagator evaluated using 10^3 stochastic sources.
- Signal visible up to $t \sim 3$ fm.
- Careful treatment of the tail only needed for a_μ^{LD},

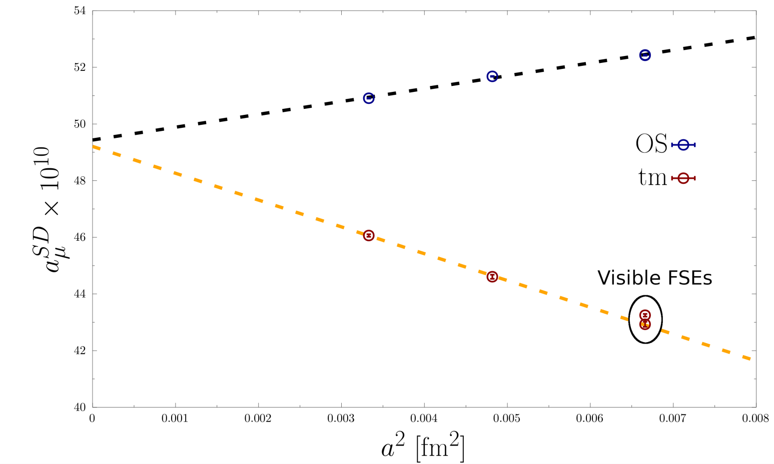
Two different ways to approach the continuum limit

$J_\mu^{\ell,OS} \propto \bar{\psi}_\ell^+ \gamma_\mu \psi_\ell^+$ (RC: Z_V).

$J_\mu^{\ell,tm} \propto \bar{\psi}_\ell^+ \gamma_\mu \psi_\ell^-$ (RC: Z_A).

- The two (renormalized) currents differ by $\mathcal{O}(a^2)$ lattice artifacts, and possibly $\mathcal{O}(a^2)$ FSEs.
- $\pm$ is the sign of the twisted Wilson parameter.

Short-distance contribution



- Good agreement between OS and tm after extrapolating with a simple linear fit in a^2 only data points with ($\simeq$) same $M_\pi L$.
- FSEs on the OS determination of a_μ^{SD} absent within accuracy.

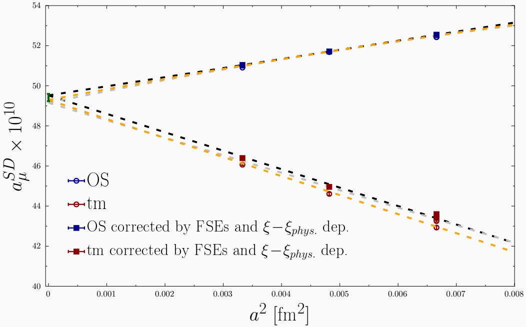
Extrapolation of the SD contribution (PRELIMINARY)

Fitting Ansatz $\quad [\xi \equiv (M_\pi / 4\pi f_\pi)^2]$

$$a_\mu^{SD;tm}(L, a, \xi) = a_\mu^{SD}(phys.) \times \left(1 + A_m(\xi - \xi_{phys}) + D_1^{tm} \cdot a^2\right) \times$$

$$\times \left(1 + a^2 \xi \cdot D_L \cdot \frac{1}{(M_\pi L)^{3/2}} e^{-M_\pi L}\right) \;,$$

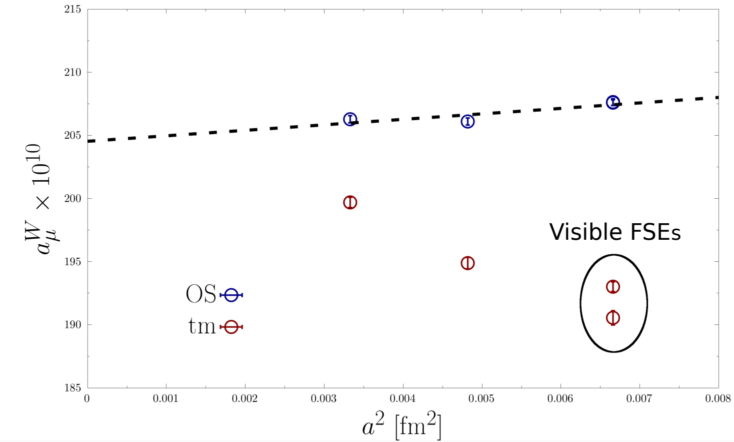
$$a_\mu^{SD;OS}(L, a, \xi) = a_\mu^{SD}(phys.) \times \left(1 + A_m(\xi - \xi_{phys}) + D_1^{OS} \cdot a^2\right)$$



- Included fits with a^4 or $a^2 \alpha_s^n(1/a)$ terms to evaluate systematics.
- Best fits with $n = 0$.
- Fits combined using AIC.

$$a_\mu^{SD} = 49.41\ (18) \times 10^{-10}$$

Intermediate window

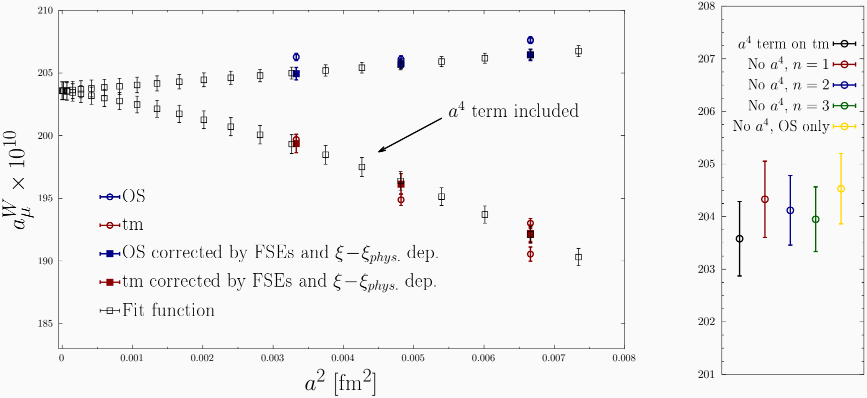


- Approximate a^2 scaling on the OS determination.
- No FSEs on OS. Visible FSEs on tm and large discretization effects.

Data must be corrected for FSEs, M_π-dependence. a^4 corrections to scaling must be included.

Extrapolation of the intermediate window (PRELIMINARY)

Same Ansatz used for a_μ^{SD} to perform continuum/thermodynamic and physical point extrapolation of a_μ^W.

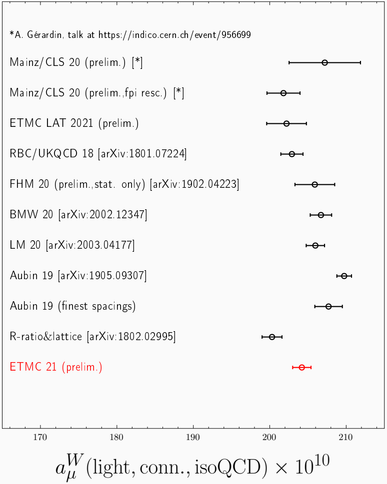


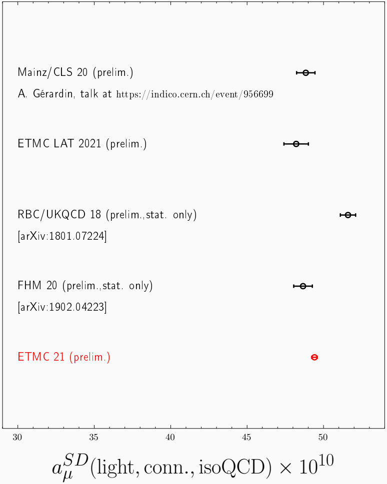
- Lattice data well fitted including an a^4 term on the tm determination of a_μ^W.
- Also fits with $a^2\alpha_s^n(1/a)$ terms.

$$\boxed{a_\mu^W = 204.2\ (1.2) \times 10^{-10}}$$

Summary

	$a_\mu^{SD} \times 10^{10}$	$a_\mu^{W} \times 10^{10}$
light, connected, isoQCD	49.41 (18)	204.2 (1.2)





Conclusions

- We performed a first-principle evaluation of the isoQCD light-connected contribution to the short-distance and intermediate time windows.
- Thanks to our dedicated simulations at the ($\simeq$) physical point, and to a high-statistics computation of the V$-$V correlator, we achieved a relative precision of $\simeq 0.5\%$ on a_μ^{SD} and a_μ^{W}.
- Our preliminary result for a_μ^{W} is between the determination of RBC/UKQCD (agreement at $\sim 0.7\sigma$) and the one from BMW (agreement at $\sim 1.4\sigma$).

Work in progress

- Analysis of the isoQCD light-disconnected contribution to a_μ^{SD}, a_μ^{W}.
- Inclusion of the strange and charm contributions.
- Evaluation of a_μ^{LD}.
- In the future: strong IB, α_{em} contributions.

Thanks for your attention

Backup slides

Effective lepton mass/window trick

We introduce effective (lattice) values for both the muon mass m_μ^{eff}, and for the window parameters $t_0^{eff}, t_1^{eff}, \Delta^{eff}$.

$$m_\mu^{eff} \equiv m_\mu \left(\frac{aX}{\bar{X}} \right), \; t_0^{eff} \equiv t_0 \left(\frac{aX}{\bar{X}} \right), \; t_1^{eff} \equiv t_1 \left(\frac{aX}{\bar{X}} \right), \; \Delta^{eff} \equiv \Delta \left(\frac{aX}{\bar{X}} \right).$$

- aX is an hadronic quantity extracted from lattice correlators.
- $\bar{X}$ can be either $X^{phys.}$ or the ensemble average $\langle X \rangle$.

$$\boxed{\begin{aligned} a_\mu^W(m_\mu^{eff}, t_0^{eff}, t_1^{eff}, \Delta^{eff}) = 4\alpha_{em}^2 \sum_{t/a=1}^{T} \overbrace{\tilde{K}_\mu(t/a, m_\mu^{eff})}^{\text{dimensionless}} \cdot \left(a^3 V_\ell(t/a)\right) \cdot \\ \cdot \left[\Theta(t/a, t_0^{eff}, \Delta^{eff}) - \Theta(t/a, t_1^{eff}, \Delta^{eff})\right] \end{aligned}}$$

- Advantage: Insensitive to the uncertainty on scale setting.
- We use the ELMW trick for a_μ^W, with $X = M_\pi, \bar{X} = \langle M_\pi \rangle$.

Hadronic determination of Z_A

We define:

$$R_A(t) = 2\mu_\ell \frac{C_{PP}^{OS}(t)}{\partial_t C_{AP}^{OS}(t)}$$

P and A are pseudoscalar and axial local bare current in C_{AP} and C_{PP}.

In the large time limit $t/a \gg 1$ ($X = tm, OS$):

$$C_{PP}^{X}(t) \to |G_\pi^X|^2 \frac{e^{-M_\pi^X t} + e^{-M_\pi^X (T-t)}}{2M_\pi^X}, \; R_A(t) \to 2a\mu_\ell \frac{Z_A}{f_\pi^{OS}} \frac{G_\pi^{OS}}{M_\pi^{OS} \sinh\left(aM_\pi^{OS}\right)}$$

We impose (true up to $\mathcal{O}(a^2)$ lattice artifacts):

$$\boxed{f_\pi^{OS} = f_\pi^{tm} = 2a\mu_\ell \frac{G_\pi^{tm}}{M_\pi^{tm} \sinh\left(aM_\pi^{tm}\right)}}$$

and using $\frac{Z_P}{Z_S} = \frac{G_\pi^{OS}}{G_\pi^{tm}}$, we can extract Z_A through

$$\boxed{\bar{R}_A(t) \equiv R_A(t) \frac{M_\pi^{OS} \sinh\left(aM_\pi^{OS}\right)}{M_\pi^{tm} \sinh\left(aM_\pi^{tm}\right)} \frac{Z_S}{Z_P} \to Z_A}$$

$\bar{R}_A$ on the three physical point ensembles at ($\simeq$) same $M_\pi L$