Preparing for MUonE experiment — what can we learn from lattice and dispersive data?

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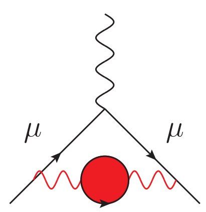
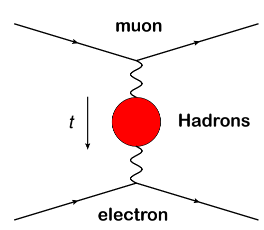
STRONG 2020 Virtual Workshop: Spacelike and Timelike determination of the Hadronic Leading Order contribution to the Muon g-2

24-26 November 2021

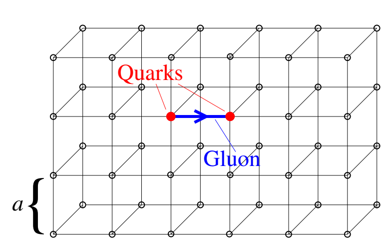
Outline

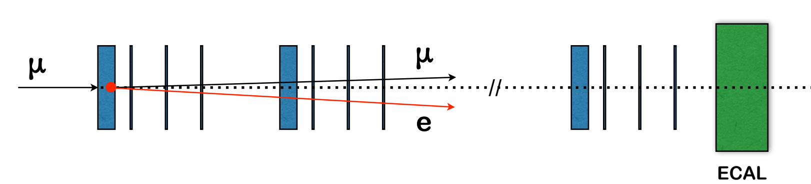
1. **Timelike** vs. **spacelike** determinations of the **HVP**
   Lattice QCD vs. **MUonE**





2. **Lattice QCD + MUonE**

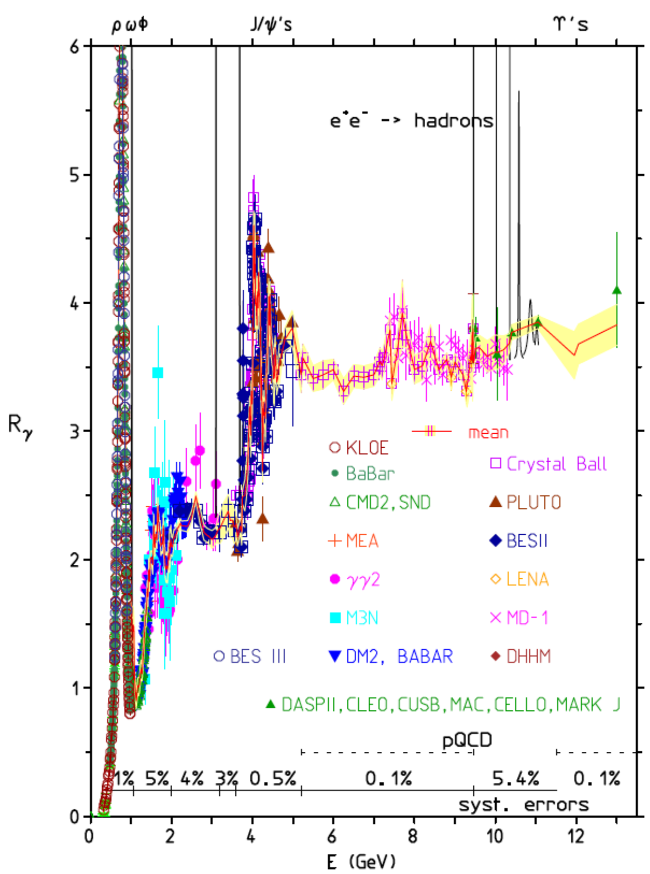




3. Fitting procedure and its systematics on an example of **I=1 HVP**

Timelike vs spacelike evaluation of $a_\mu^{had,\,LO}$

Time-like



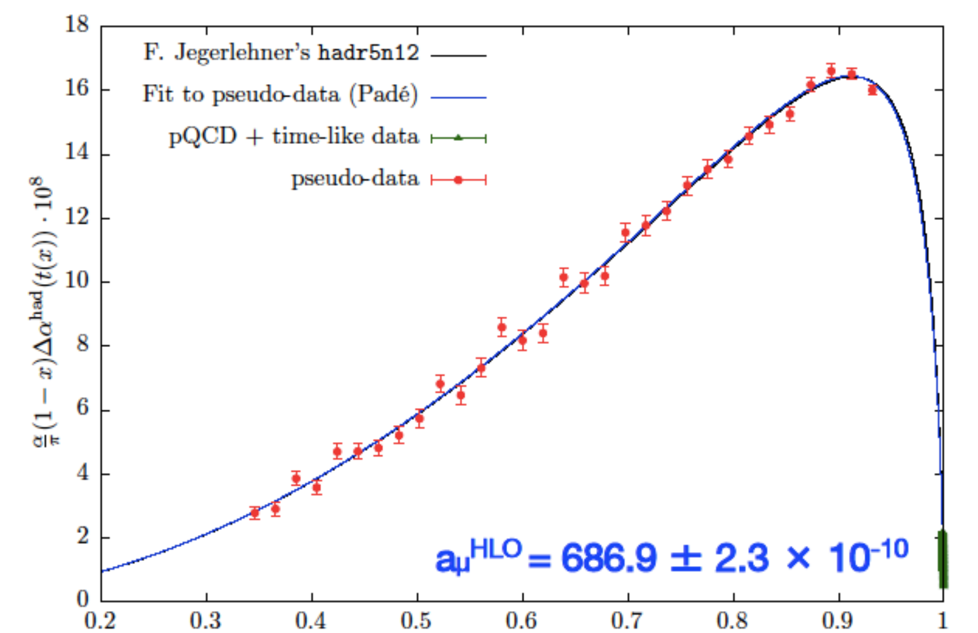
[Credit: F. Jegerlehner]

[Lautrup, Peterman de Rafael '72]

[T. Blum '03]

Space-like

Statistical reach of MUonE on a_μ^{HLO}
(2 years of data taking at 1.3 x10⁷μ/s)



A **0.3%** stat error can be achieved on a_μ^{HLO} in 2 years of data taking with ~10^7 μ/s (4×10^{14}μ total)

[Credit: G. Venanzoni, G. Abbiendi]

[see slides by S. Laporta, Fri. 14.00]

[see slides by U. Marconi, Fri. 14.20]

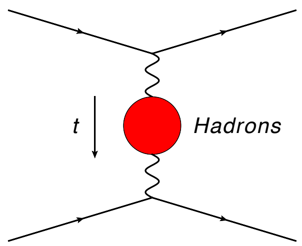
[see slides by G. Abbiendi, Fri. 14.40]

MUonE: theoretical framework

[MUonE Collaboration https://web.infn.it/MUonE/]

[For details, see LoI: SPSC-I-252]
https://cds.cern.ch/record/2677471

[see slides by U. Marconi, Fri. 14.20]
[see slides by G. Abbiendi, Fri. 14.40]



- Utilise the running of the fine-structure constant $\alpha(t)$:

$$a_\mu^{had,LO} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[Q^2(x)]$$

[Lautrup, Peterman de Rafael '72]

➡ In space-like (Euclidean) momenta region:

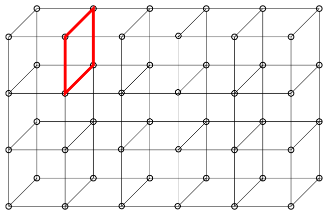
$$Q^2 = \frac{x^2 m_\mu^2}{1-x}$$

➡ Measuring the **Q^2** - dependent fine-structure constant:

$$\alpha(Q^2) = \frac{\alpha(O)}{1-\Delta\alpha(Q^2)}$$

[Phys.Lett. B746 (2015) 325-329 by Carloni, Passera,Trentadue, Venanzoni] **@KLOE2**
[Eur.Phys.J. C77 (2017) no.3, 139 by Abbiendi et al.] **@CERN**

Spacelike determination(s) combined

MUonE +  + P.T.

[M.K.M, LATTICE 2018]

[M.K.M, Cardoso, 2019]

[Aoyama et al, 2020]

$$a_\mu^{had,LO} = \underbrace{\frac{\alpha}{\pi}\int_0^{0.93\ldots} dx(1-x)\Delta\alpha_{had}[Q^2(x)]}_{I_0} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{0.14}^{Q^2_{max}} dQ^2 f(Q^2)\times\hat{\Pi}(Q^2)}_{I_1} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{Q^2_{max}}^{\infty} dQ^2 f(Q^2)\times\hat{\Pi}_{pert.}(Q^2)}_{I_2}$$

- lattice QCD
- R-ratios

(1) I_1 contribution to the HVP from the lattice

(2) I_1 contribution to the HVP from R-ratio data

(3) Full-range HVP, by interpolating between high-quality MUonE data and P.T. (I_2)

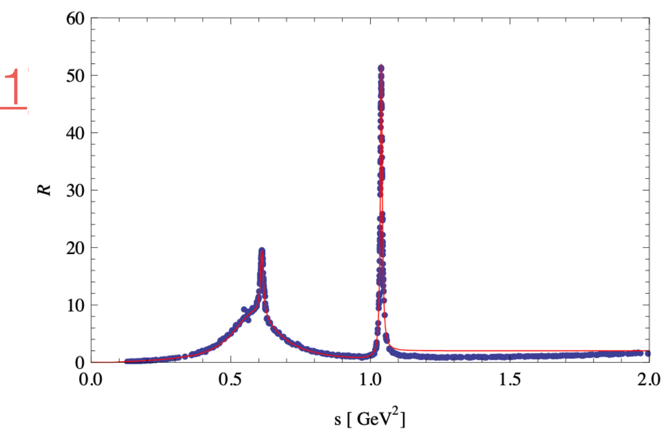
[see slides by G. Abbiendi, Fri. 14.40]

$$\Delta\alpha_{had}(t) = k\left\{-\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\log\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

What can dispersive/lattice approach tell us about the systematics of the full-range HVP fits in (3)?

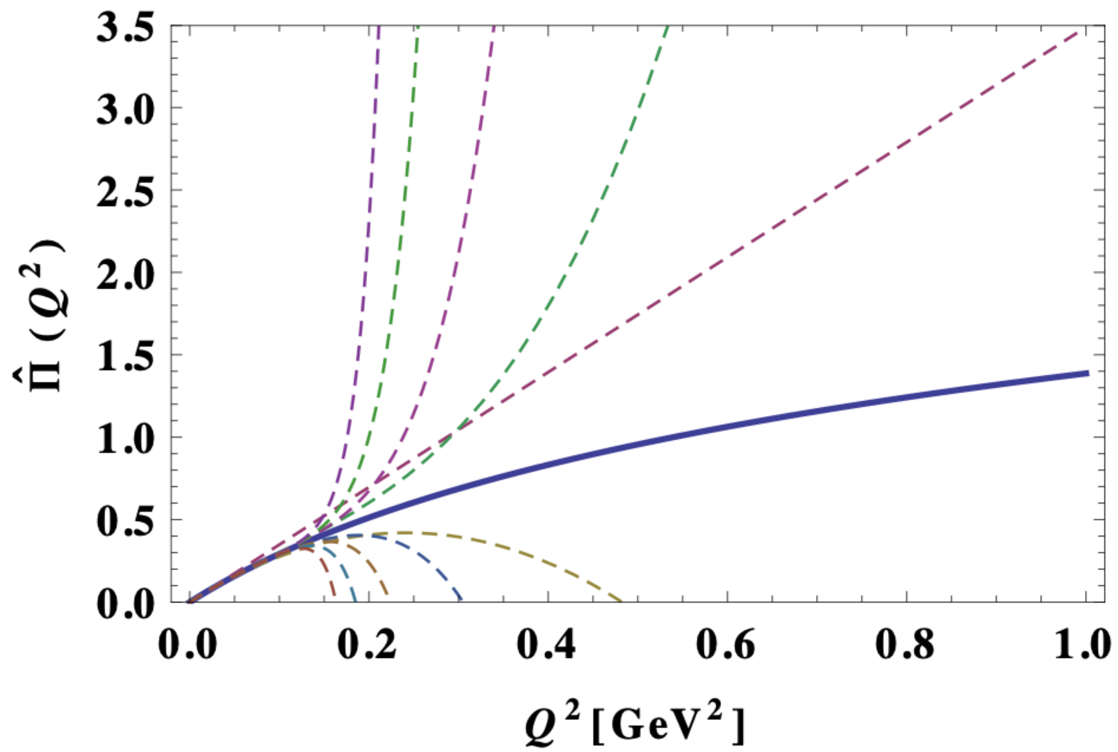
Phenomenological model of HVP

- Phenomenological R ratio ($e^+e^- \longrightarrow \text{hadrons}$) [Bernecker, Meyer, 2011]



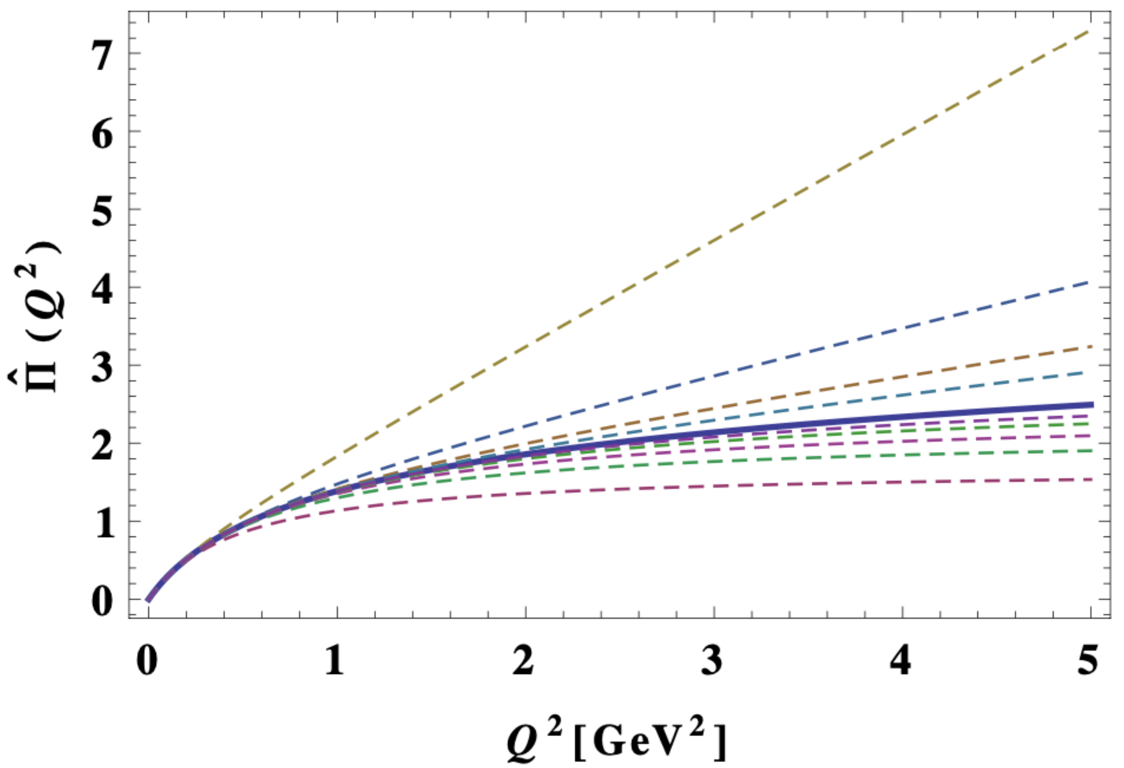
- $$\Pi(Q^2) - \Pi(0) = Q^2 \int_0^\infty ds \frac{\rho(s)}{s(s+Q^2)};$$

- True nature of the vacuum polarisation; Stieltjes function argument [Aubin, Blum, Golterman, Peris 2012]



Taylor expansions for $N = 1, \cdots, 10$

$\longrightarrow$

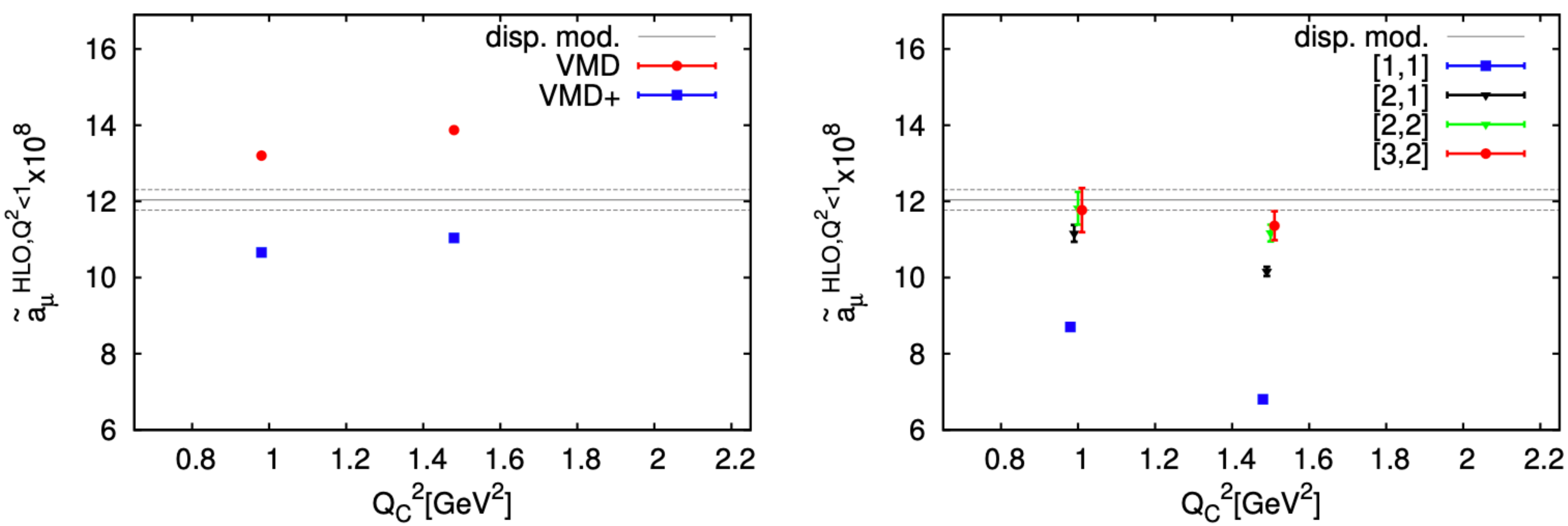


$[N, N-1]$ & $[N, N]$ Padé's: $[1, 1] \rightarrow [5, 5]$

[Credit: L. Lellouch]

Phenomenological model of HVP [Golterman, Maltman, Peris 1309.2153]

- A method to quantitatively examine the systematics of lattice computations
- Dispersive τ-based $I=1$ model: $\hat{\Pi}^{I=1}(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} ds \frac{\rho^{I=1}(s)}{s(s+Q^2)}$
- "Fake" lattice data for $\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$; then compared with the true answer from model

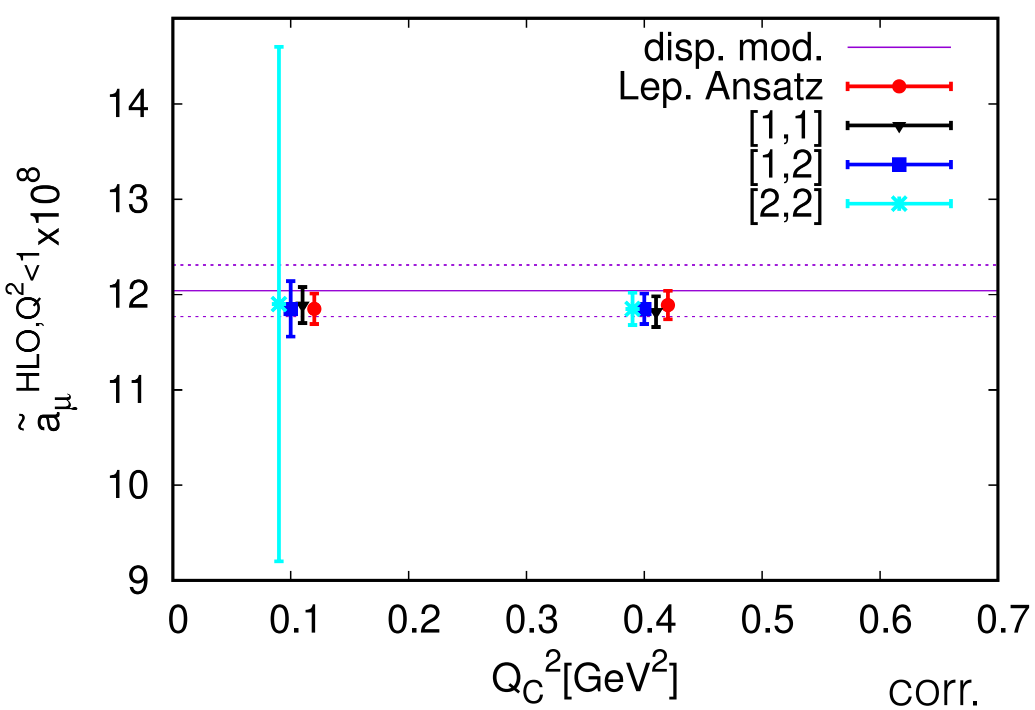


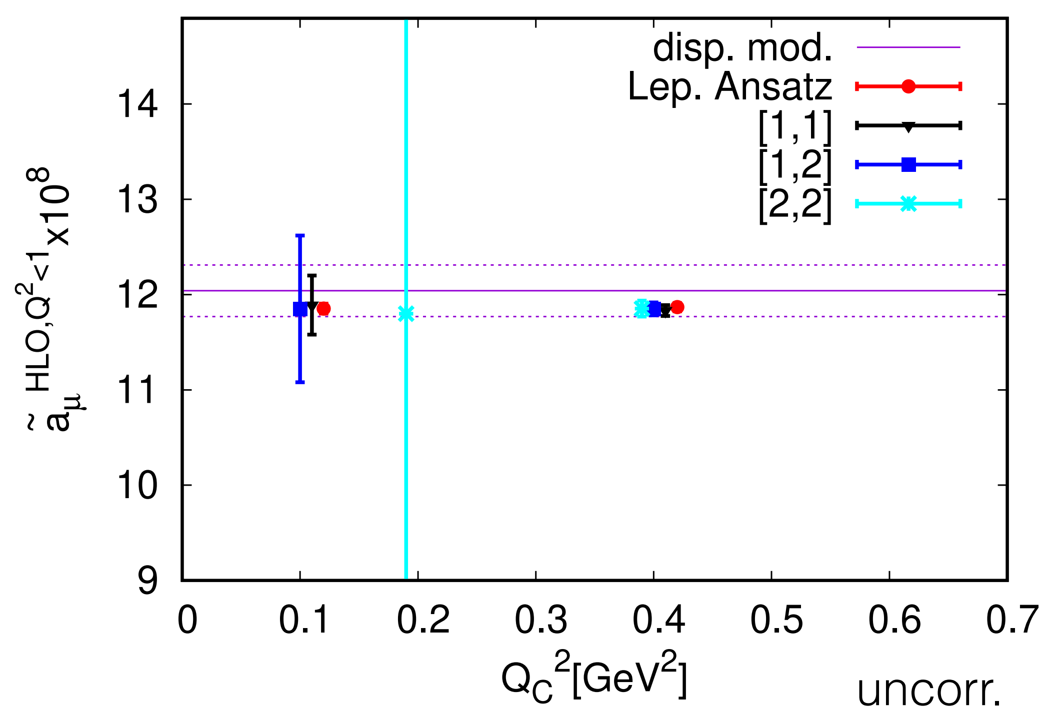
el $\tilde{a}_\mu^{HLO,Q^2<1} = 12.04(27) \times 10^{-8}$

- Outcome:
  - ➡ With lattice uncertainties: fitting until high Q^2 dangerous, unless higher order Pade approximants used
  - ➡ Better focus on low-Q^2 region needed

Various fit ansaetze for the τ-based I=1 model

- A method to quantitatively examine the systematics of lattice/MUonE fits in the spacelike
- Dispersive τ-based $I = 1$ model: $\hat{\Pi}^{I=1}(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} ds \frac{\rho^{I=1}(s)}{s(s+Q^2)}$
- Different fit forms applied on the compared with the true answer from model

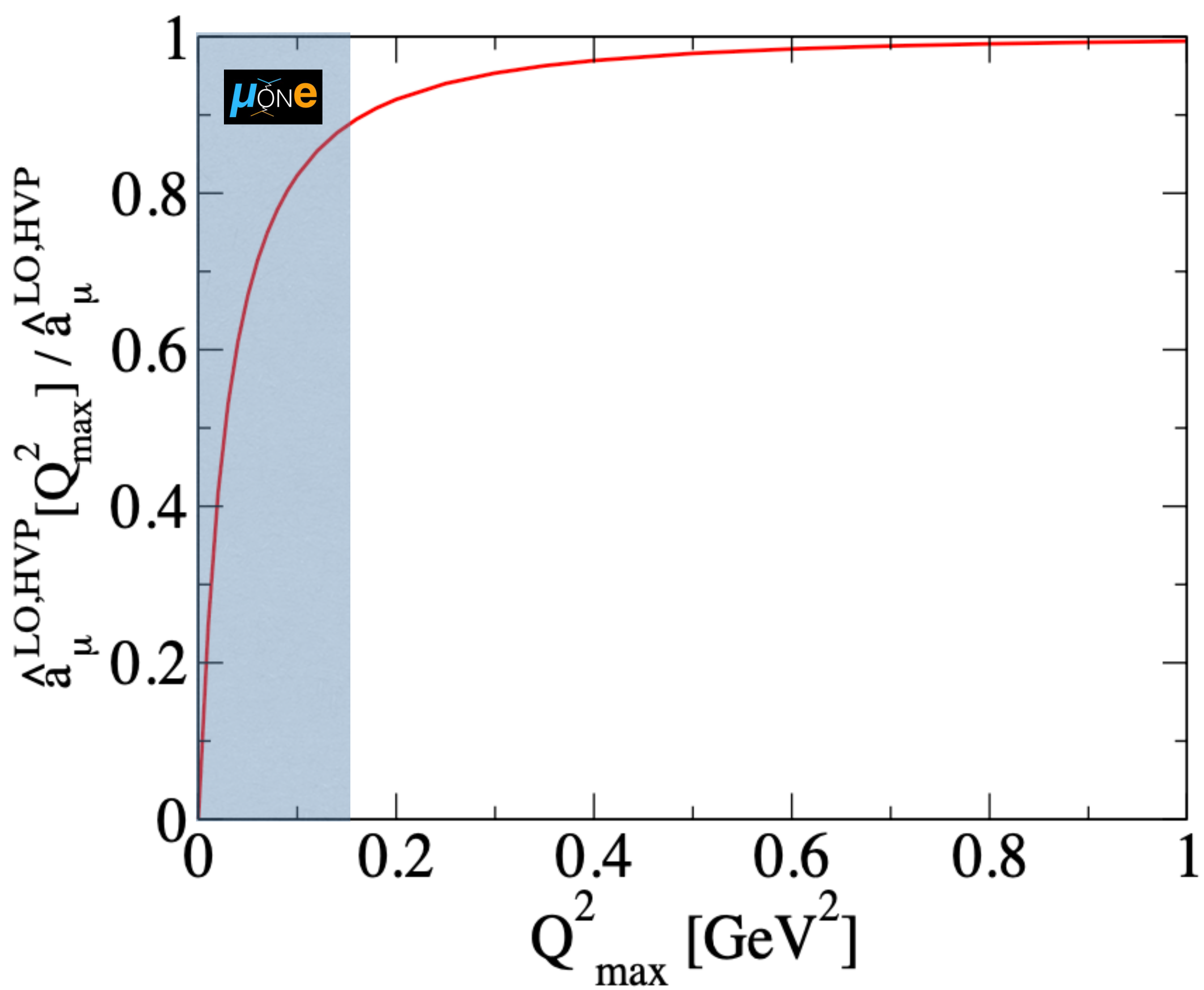




- Remarks:
  - ➡ Preliminary results obtained with original ALEPH τ-decay covariances (curtesy of Kim Maltman)
  - ➡ Next steps: use lattice and MUonE projected uncertainties

Choice of upper fit boundary [Golterman, Maltman, Peris 1309.2153]

- Goal: use this as a method to quantitatively examine the systematics of MUonE fits, and space-like in gen.
- More than 80% of the a_μ^{HLO} is accumulated below $Q^2_{max} = 0.1\ \mathrm{GeV}^2$
- More than 90% below $Q^2_{max} = 0.2\ \mathrm{GeV}^2$



- Thus low cuts in our tests ($Q^2_C = 0.1\mathrm{GeV}^2$ and $Q^2_C = 0.4\mathrm{GeV}^2$) justified
  - ➡ capture the majority of the HVP; directly measured MUonE region (0.3<x<0.932): $\approx 84\ \%$ [Abbiendi et. al]

Summary & Outlook

- **Hadronic contributions** (HVP and HLbL) dominating uncertainties in the muon g-2
- New lattice calculations and independent space-like calculation by MUonE
- **MUonE**: provide an **independent input for HVP**
- Massive experimental and theory efforts:

[see slides by U. Marconi, Fri. 14.20]
[see slides by G. Abbiendi, Fri. 14.40]

[see slides by S. Laporta, Fri. 14.00]
[see slides by M.K.Mandal, Fri. 14.55]
[see slides by T. Engel, Fri. 15.15] . . .
[see slides by E. Budassi, Fri. 15.45] . . .

- Systematics in lattice simulations: VMD, Pade approximants, Lepton-motivated ansatz
- $\tau-$based $I = 1$ phenomenological model: informs the choice of the fit function in low-Q^2 region

- Muon g-2 Theory Initiative as a framework for exchange between **lattice/data-driven/experimental** results
- Spacelike methods instrumental also for other NP searches (e.g. heavy flavour, EW fits)

Next workshop of Muon g-2 Theory Initiative:
5-9 September 2022, University of Edinburgh
https://muon-gm2-theory.illinois.edu/

Further discussions on g-2 phenomenology & flavour sector [Pauli Center Workshops]:
ZPW2022: 10-12 January 2022
LF(U)V Workshop: 12-14 January 2022