Towards muon-electron scattering at NNLO

STRONG2020

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26/11/2021

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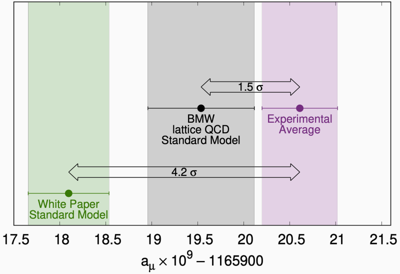
Dipartimento di Fisica

The muon $g-2$ & Spacelike approach

Starting point: $g_\mu - 2$ & Theoretical Approaches

$$a_\mu^{SM} \times 10^{11} = 116591810(43)$$
$$a_\mu^{COMB} \times 10^{11} = 116592061(41)$$



T. Aoyama *et al.* Phys.Rept. 887 (2020) 1-166

B. Abi *et al.* [Muon g-2], Phys. Rev. Lett. **126** (2021) no.14, 141801.

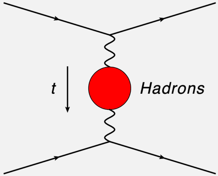
Borsanyi, S. *et al.* *Nature* **593**, 51–55 (2021).

Timelike approach:

$$a_\mu^{HLO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left[\int_{4m_\pi^2}^{\infty} ds \frac{K(s)R^{had}(s)}{s^2}\right]$$

Spacelike approach:

$$a_\mu^{HLO} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$



Muon-electron scattering at NLO for $g_\mu - 2$

MESMER, a MC event generator for $\mu e \to \mu e$ scattering

Theoretical predictions must achieve the 10ppm accuracy for meaningful extraction of $\Delta\alpha_{had}(t)$ from MUonE data with the required precision.

- A fully differential Monte Carlo event generator for MUonE simulations and data analysis is under active development

MESMER: Muon Electron Scattering with Multiple Electromagnetic Radiation

- Weak effects included, at the level of $\sim 10^{-5}$ (LO) and $\lesssim 10^{-6}$ (NLO).
- Full dependence on masses and Radiative Corrections have been studied with specific kinematical cuts that mimic the experimental setup of MUonE
  - **Basic acceptance cuts**: $\vartheta_e, \vartheta_\mu < 100$ mrad and $E_e > 1$ GeV
  - **Acoplanarity cut**: $\xi = |\pi - |\phi_e - \phi_\mu|| < \xi_c = 3.5$ mrad

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M. Alacevich, C. M. Carloni Calame, M. Chiesa, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 02 (2019) 155.

Towards muon-electron scattering at NNLO in QED

based on

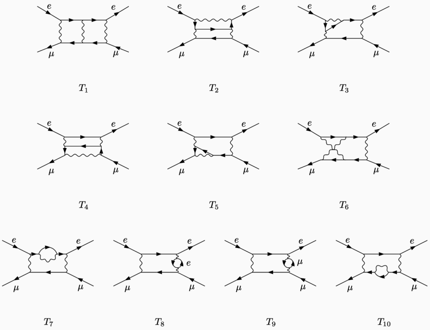
C.M. Carloni Calame *et al.*, Towards muon-electron scattering at NNLO, JHEP 11 (2020) 028

and

E. Budassi *et al.*, NNLO virtual and real leptonic corrections to muon-electron scattering, JHEP 11 (2021) 098

State of the art at NNLO

- NNLO virtual hadronic effects computed.
- Master Integrals for 2-loop box diagrams computed with $m_e = 0$ and $m_\mu \neq 0$.
- Full QED NNLO amplitude for $f^+f^- \to F^+F^-$ ($m_f = 0$) has been calculated: crossing symmetry connects to $\mu e \to \mu e$.



M. Fael and M. Passera, Phys. Rev. Lett.122(2019), no. 19 192001; M. Fael, JHEP02 (2019) 027,.

R. Bonciani, *et al.* [arXiv:2106.13179 [hep-ph]].

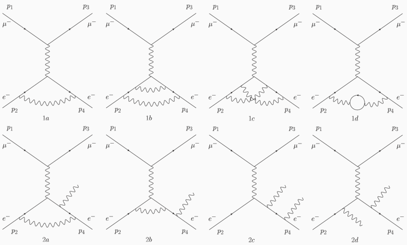
P. Mastrolia, M. Passera, *et al.* JHEP11 (2017) 198; S. Di Vita, S. Laporta, P. Mastrolia, *et al.*, JHEP09 (2018) 016

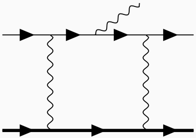
Towards muon-electron scattering at NNLO in QED

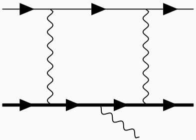
NNLO Photonic Corrections

Photonic NNLO corrections: exact contributions

- Virtual NNLO photonic contributions are included exactly for electron or muon leg emission. 2-loop QED vertex from factors taken from Mastrolia and Remiddi.







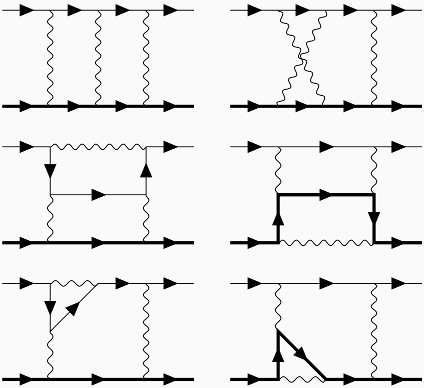
- 1-loop corrections to real photon emission exactly included: *e.g.* pentagon diagrams.
- Double real emission included exactly.

C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028.

P. Mastrolia and E. Remiddi, Nucl.Phys.B 664 (2003), 341-356.

P. Banerjee, T. Engel, A. Signer, Y. Ulrich. SciPost Phys. 9 (2020), 027; P. Banerjee *et al.*, Eur.Phys.J.C 80 (2020) 6, 591.

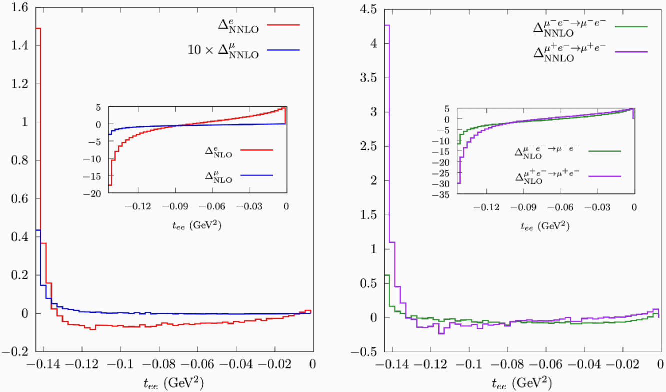
Photonic NNLO corrections: approximated contributions



- Of the 2-loop virtual diagrams with a virtual photon insertion on top of NLO boxes, only the IR part is included exactly (YFS).
- The non-IR remnants are approximate.

The gauge-invariant subset of NNLO corrections along the electron line cross-checked against McMule.

C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028.



$$\Delta_i^{\rm NNLO} = \frac{d\sigma_i^{\rm NNLO}}{d\sigma_i^{\rm LO}} \times 100$$

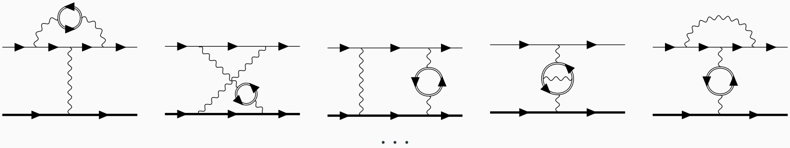
C. M. Carloni Calame, M. Chiesa, S. M. Hasan, G. Montagna, O. Nicrosini, and F. Piccinini, JHEP 11 (2020) 028.

Towards muon-electron scattering at NNLO in QED

NNLO Leptonic Pair Corrections

NNLO Lepton Pair Contributions: Virtual

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$



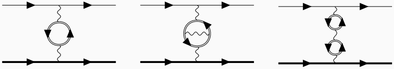
- Using the very same implementation of NLO corrections in MESMER by replacing the photon propagator as:

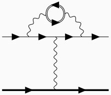
$$\frac{-ig_{\mu\nu}}{q^2 + i\varepsilon} \to -ig_{\mu\nu}\left(\frac{\alpha}{3\pi}\right)\int_{4m_\ell^2}^{\infty}\frac{dz}{z}\,\frac{1}{q^2 - z + i\epsilon}\left(1 + \frac{4m_\ell^2}{2z}\right)\sqrt{1 - \frac{4m_\ell^2}{z}}\,.$$

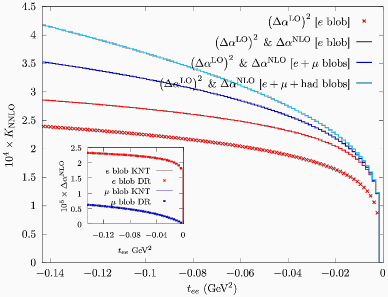
- Integration over z is performed numerically with MC techniques.
- Master Integral techniques for a subset of such diagrams to cross-check results.

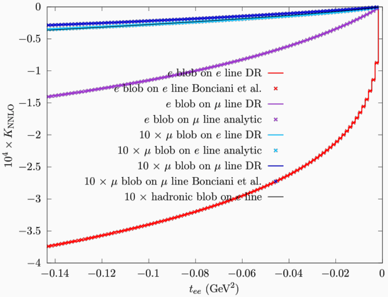
E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098.

Virtual NNLO Lepton Pair Contributions: Results









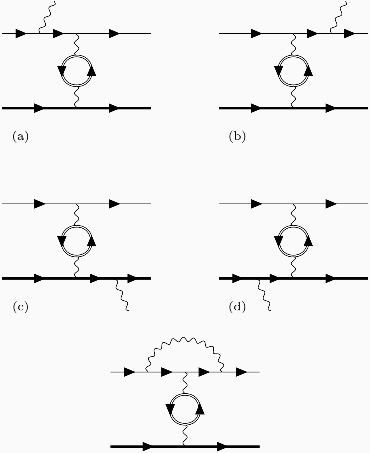
$$K_{\rm NNLO} = \frac{d\sigma_{N_f}^{\alpha^2}}{d\sigma_{\rm LO}}$$

E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098.

NNLO Lepton Pair Contributions: Mixed

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}}$$

- Interplay between real photon radiation and leptonic loop insertions.
- IR divergences are cancelled by a sub-set of the virtual contributions.



E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098.

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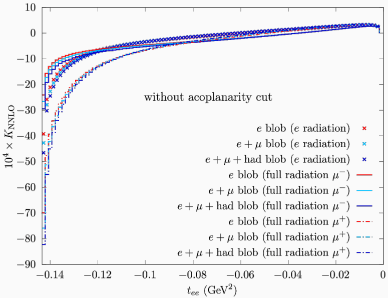

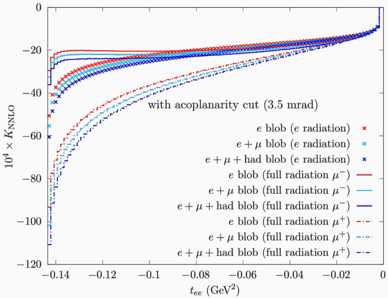

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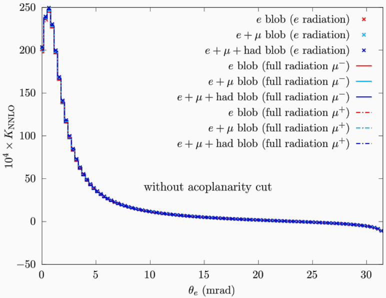

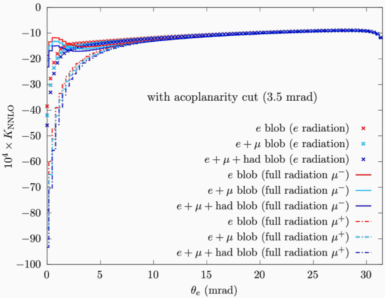

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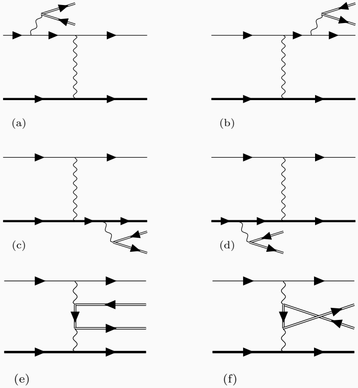
NNLO Lepton Pair Contributions: Real

$$d\sigma_{N_f}^{\alpha^2} = d\sigma_{\text{virt}}^{\alpha^2} + d\sigma_{\gamma}^{\alpha^2} + d\sigma_{\text{real}}^{\alpha^2}$$

- 2$\longrightarrow$ 4 LIPS.
- The QED matrix elements have been calculated with FORM and cross-checked with RECOLA.
- Cuts: a set of elasticity cuts must be imposed to reduce a potentially large background



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B. Ruijl, T. Ueda and J. Vermaseren, FORM version 4.2.

A. Denner, *et al.* Recola2: REcursive Computation of One-Loop Amplitudes 2, Comput. Phys. Commun. 224 (2018) 346.

- Basic acceptance cuts
- When we have 4 particles in the final state we require that only 2 are detected ($E_i > 200$ MeV and $\vartheta_i < 100$ mrad).

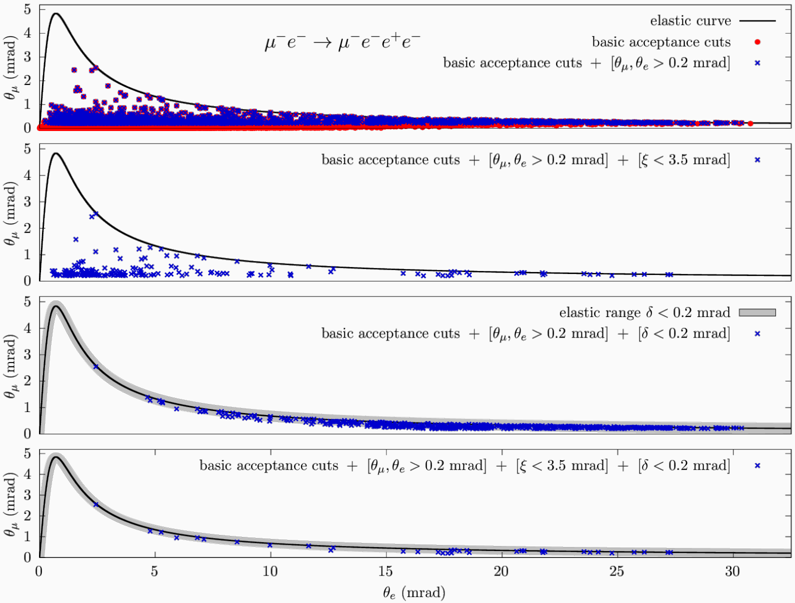
On top of it, we added 3 selection cuts to select elastic events:

- **cut 1**: $\vartheta_e > 0.2$ mrad and $\vartheta_\mu > 0.2$ mrad
- **cut 2**: $\xi = |\pi - |\phi_e - \phi_\mu|| < \xi_c = 3.5$ mrad
- **cut 3**: Elasticity distance $\delta < \delta_c = 0.2$ mrad. δ is defined as the distance from the elastic curve:

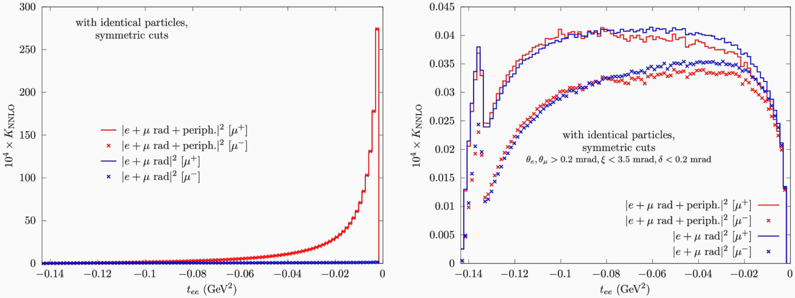
$$\delta = \min_{\theta_e} \sqrt{(\theta_e - \theta_e^0)^2 + (\theta_\mu(\theta_e) - \theta_\mu^0)^2}.$$

E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098.

Elasticity cut



Real NNLO Lepton Pair Contributions: Results



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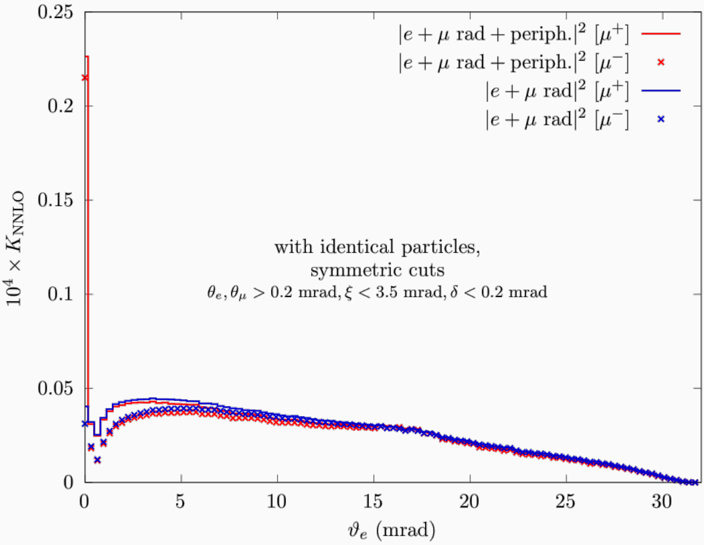

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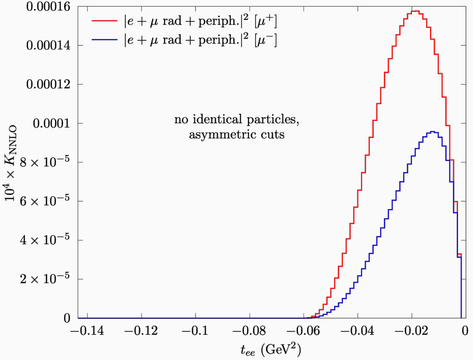
Muon pair production



- $\mu e \to \mu e + \mu^+ \mu^-$ contributions are well below 10 ppm without cuts.
- By imposing standard (symmetrical) cuts, the process is kinematically forbidden.

E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098.

Take-Home Message and Outlook

- Important efforts to develop NNLO fixed-order Monte Carlo event generators for μe scattering.

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- NNLO Virtual Lepton Pair contributions weigh 10^{-4} to 10^{-3}. e^+e^- emission on e^- leg is dominant.
  - virtual $\mu^+\mu^-$ bubble has contribution of the same order of hadronic VP.
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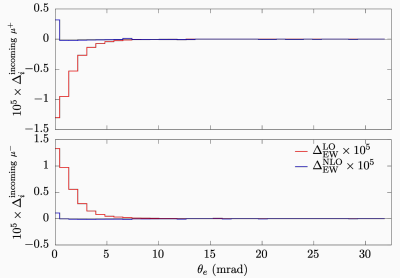
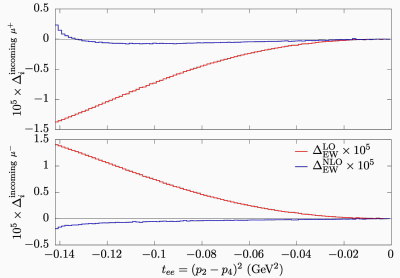
- MESMER code: *https://github.com/cm-cc/mesmer/*.

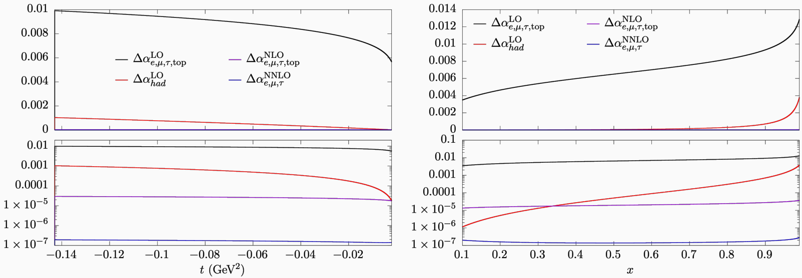
Backup slides

Backup: NLO EW corrections

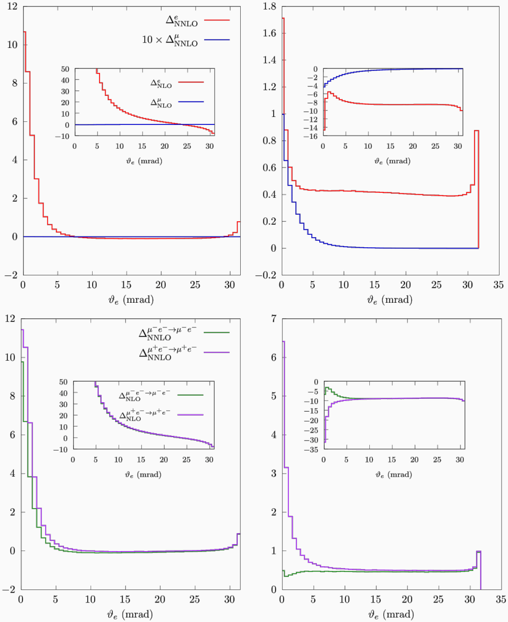




Backup: $\Delta\alpha$



Backup: Photonic NNLO corrections: Results



Backup: YFS approximation

At NLO for virtual box diagrams, YFS misses terms of order:

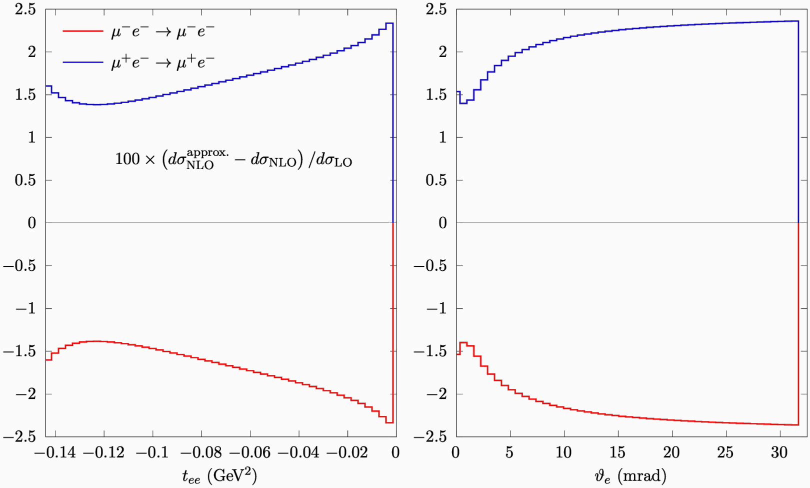
$$\frac{\alpha}{\pi} \ln \frac{m_\mu^2}{m_e^2} \simeq 0.025.$$

Therefore, for NNLO boxes YFS is expected to be accurate up to terms of order:

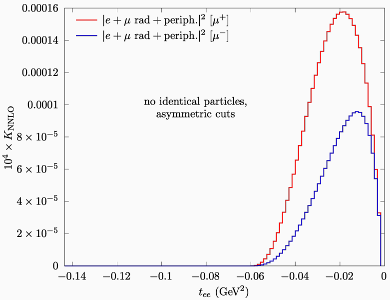
$$\left(\frac{\alpha}{\pi}\right)^2 \ln^2 \frac{m_\mu^2}{m_e^2} \simeq 6 \times 10^{-4}.$$

Improving the accuracy requires the inclusion of exact NNLO boxes, at least their leading terms in m_e.

Backup: YFS @ NLO

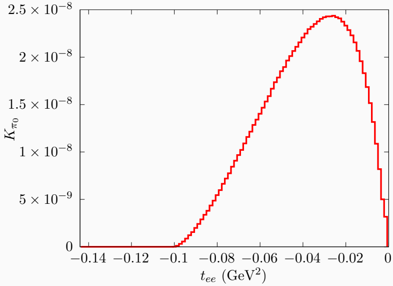


Backup: $\mu e \to \mu e \pi_0$



- $\mu^+\mu^-$ production is negligible without cuts & goes to zero with acceptance cuts.
- Hadronic production: $\pi^+\pi^-$ production and π^0 production.
- $\pi^+\pi^-$ is more suppressed than $\mu^+\mu^-$ production since $m_\pi > m_\mu$.

$$\mathcal{L}_{\text{int}} = \frac{g}{2!} F^{\mu\nu} \tilde{F}_{\mu\nu}.$$



π_0 production is well under the experimental resolution of ~ 10 ppm.

Preliminary

Backup: elasticity cut

Elasticity curve can be parametrised as follows:

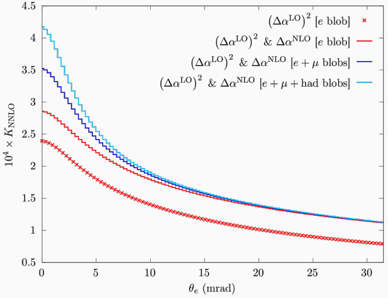
$$\theta_\mu(\theta_e) = \arctan\left[\frac{2m_e r \cos\theta_e \sin\theta_e}{E^i_\mu - r\left(rE^i_\mu + 2m_e\right)\cos^2\theta_e}\right],$$

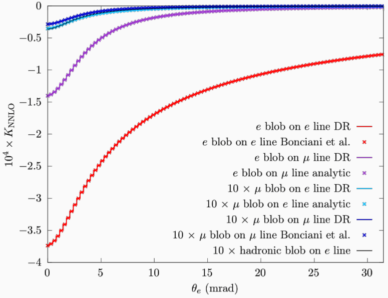
where r is defined as:

$$r = \frac{\sqrt{(E^i_\mu)^2 - m^2_\mu}}{E^i_\mu + m_e}$$

and E^i_μ is the incident muon energy in the laboratory reference frame.

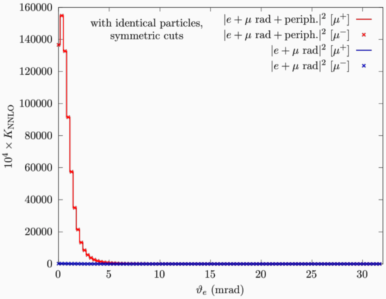
Backup: Virtual NNLO Lepton Pair Contributions: Results





E. Budassi, C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini, F. Piccinini. JHEP 11 (2021), 098. arXiv:2109.14606, doi:10.1007/JHEP11(2021)098.

Backup: Real NNLO Lepton Pair Contributions: Results



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Backup: Sketch of the NLO calculation

NLO contributions:

$$\sigma_{\text{NLO}} = \sigma_{2\to 2} + \sigma_{2\to 3}$$

- Leading Order and NLO virtual contributions::

$$\sigma_{2\to 2} = \sigma_{\text{LO}} + \sigma_{\text{NLO}}^{\text{v}} = \frac{1}{F}\int d\Phi_2 \left\{ \left|\mathcal{M}_{\text{LO}}\right|^2 + 2\,\text{Re}\left[\mathcal{M}_{\text{LO}}^{\dagger}\mathcal{M}_{\text{NLO}}^{\text{v}}(\lambda)\right]\right\}$$

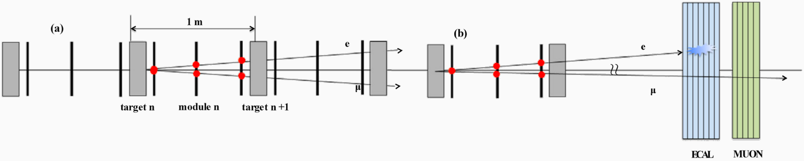
- NLO Real contributions:

$$\sigma_{2\to 3} = \frac{1}{F}\left(\int_{\lambda < E_\gamma < \Delta E} d\Phi_3 \left|\mathcal{M}_{\text{NLO}}^{\gamma}\right|^2 + \int_{E_\gamma > \Delta E} d\Phi_3 \left|\mathcal{M}_{\text{NLO}}^{\gamma}\right|^2\right)$$

- Same strategy used at NNLO

Backup: MUonE Apparatus

- Measure μ and e angles with very high precision.
- Modular: 40 tracking stations with silicon detectors built for HL-LHC upgrade.
- ECAL and μ filter downstream.
- Precision required on the differential cross sections: 10 ppm (!)



MUonE Collaboration, G. Abbiendi et al.,Letter of Intent: the MUonE project, Tech. Rep.CERN-SPSC-2019-026, SPSC-I-252, CERN, Geneva, Jun, 2019.

Backup: test run picture