Universität Zürich[UZH]

STRONG2020

MC developments for μe-scattering at 10ppm

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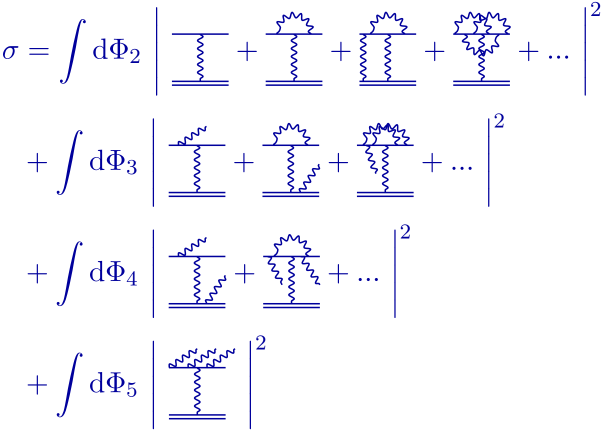
theory requirements



$+\,\mathrm{LL} + \mathrm{NLL} + \ldots$

fully-differential MC including

- ✓ dominant NNLO corr.
  → [Pavia 20, McMule 20]
- ✓ fermionic NNLO corr.
  → [Fael 18, Fael, Passera 19, Pavia 21]
- o full NNLO corr. w/o m^2/Q^2
  → first step [Bonciani et al. 21]
- o LL resummation
- o NLL resummation

} not this talk

- o dominant $\mathrm{N^3LO}$ w/o m^2/Q^2

- framework for fully-differential higher-order QED
- NNLO QED corrections for

$$\begin{aligned}
\mu \to e\nu\nu \quad &\longleftrightarrow \quad \text{MEG \& Mu3e} \\
\ell p \to \ell p \quad &\longleftrightarrow \quad \text{P2 \& MUSE} \\
e^- e^- \to e^- e^- \quad &\longleftrightarrow \quad \text{PRad} \\
e^+ e^- \to e^+ e^- \quad &\longleftrightarrow \quad \text{luminosity@}\ell\text{–colliders} \\
e^+ e^- \to \gamma\gamma \quad &\longleftrightarrow \quad \text{PADME \& luminosity@}\ell\text{–colliders} \\
e\mu \to e\mu \quad &\longleftrightarrow \quad \text{MUonE}
\end{aligned}$$

- also in McMule: rare and radiative lepton decay (NLO)
- planned: electroweak corrections, polarised leptons, ...

QED vs. QCD

QED and QCD calculations have many common issues, but . . .

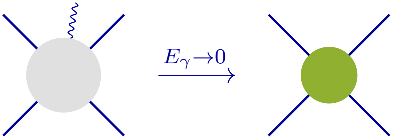
- The infrared structure of QED is much(!!) simpler [advantage]
  $\to$ ① FKS$^\ell$ subtraction scheme
- In QED we typically want to keep $m \neq 0$ since $\log(m)$ physical $\to$ [problem]
  $\to$ ② massification
- In QED we typically have to be exclusive w.r.t. hard collinear emission $\to$ [problem]
  $\to$ ③ next-to-soft stabilisation

① advantage: simple IR structure

$\rightarrow$ FKS$^\ell$

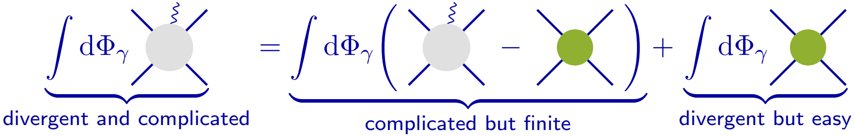
① advantage: FKS$^\ell$

only soft singularities



$$\mathcal{M}^{(\ell)}_{n+1} = \underbrace{\mathcal{E}\,\mathcal{M}^{(\ell)}_{n}}_{\mathcal{O}(E_\gamma^{-2})} + \mathcal{O}(E_\gamma^{-1})$$

$\Rightarrow$ subtraction scheme (FKS$^\ell$)



① advantage: FKS$^\ell$

subtraction scheme

we do not write $\sigma_n^{(1)} = \sigma_n^{(v)}(\lambda) + \sigma_n^{(s)}(\lambda,\omega) + \sigma_{n+1}^{(h)}(\omega)$ photon mass λ, resolution ω

we do write $\sigma_n^{(1)} = \sigma_n^{(1)}(\xi_c) + \sigma_{n+1}^{(1)}(\xi_c)$ auxiliary unphysical parameter ξ_c

FKS$^{\ell=2}$

$$\sigma_n^{(2)}(\xi_c) = \int \mathrm{d}\Phi_n^{d=4}\left(\mathcal{M}_n^{(2)} + \hat{\mathcal{E}}(\xi_c)\,\mathcal{M}_n^{(1)} + \frac{1}{2!}\mathcal{M}_n^{(0)}\hat{\mathcal{E}}(\xi_c)^2\right) = \int \mathrm{d}\Phi_n^{d=4}\,\mathcal{M}_n^{(2)f}(\xi_c)$$

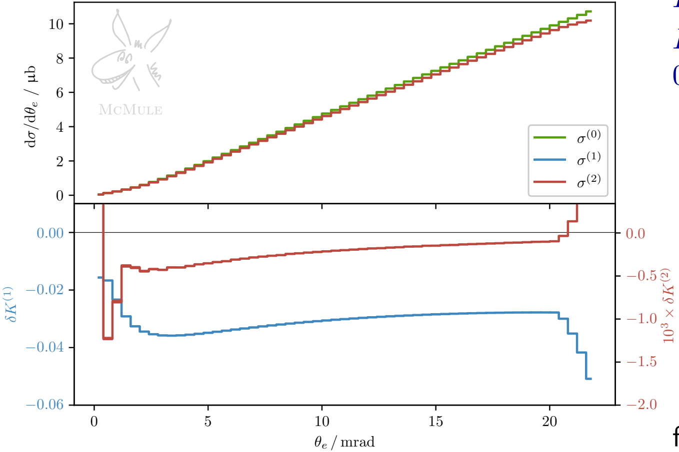
$$\sigma_{n+1}^{(2)}(\xi_c) = \int \mathrm{d}\Phi_{n+1}^{d=4}\left(\frac{1}{\xi_1}\right)_c\left(\xi_1\,\mathcal{M}_{n+1}^{(1)f}(\xi_c)\right),$$

$$\sigma_{n+2}^{(2)}(\xi_c) = \int \mathrm{d}\Phi_{n+2}^{d=4}\left(\frac{1}{\xi_1}\right)_c\left(\frac{1}{\xi_2}\right)_c\left(\xi_1\xi_2\,\mathcal{M}_{n+2}^{(0)f}\right)$$

$$\int_0^1 \mathrm{d}\xi_1\left(\frac{1}{\xi_1}\right)_c f(\xi_1) \equiv \int_0^1 \mathrm{d}\xi_1\,\frac{f(\xi_1) - f(0)\theta(\xi_c - \xi_1)}{\xi_1}$$

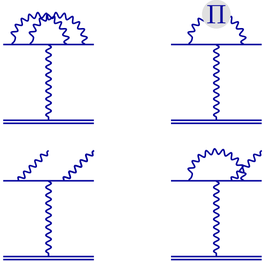
[Frixione, Kunszt, Signer 96, TE, Signer, Ulrich 19]

Dominant NNLO corrections for MUonE



[McMule 20] " = " [Pavia 20]

$E_\mu^{\text{beam}} = 150$ GeV with cuts:
$E_e > 1$ GeV, $\theta_\mu > 0.3$ mrad,
$0.9 < \theta_\mu/\theta_\mu^{\text{el}}(\theta_e) < 1.1$



form factors → [Bonciani, Mastrolia, Remiddi 03]
hyperspherical → [Fael 18]

② problem: loops with masses

$\to$ massification

full muone 2-loop amplitude with $M \neq 0$, $m = 0 \to$ [Bonciani et al. 21]
full muone 2-loop amplitude with $M \neq 0$, $m \neq 0 \to$ [??]

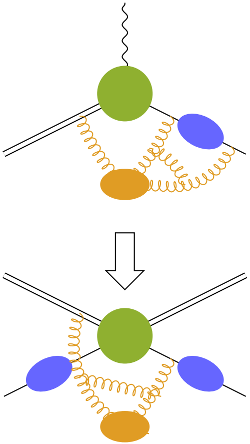
$\to$ exploit scale hierarchy $m^2 \ll M^2, Q^2$

② problem: massification

simple process ($\mu \to e\nu\nu$ or $t \to b\ell\nu$)

- $\mathcal{A}_\mu(m) = \mathcal{S} \times Z \times \mathcal{A}_\mu(0) + \mathcal{O}(m)$
- $Z \supset \log(m)$: process indep. jet fct.
- $\mathcal{S} \supset \log(m)$: process dep. soft fct. (easy)

[Penin 06, Becher, Melnikov 07; TE, Gnendiger, Signer, Ulrich 18]



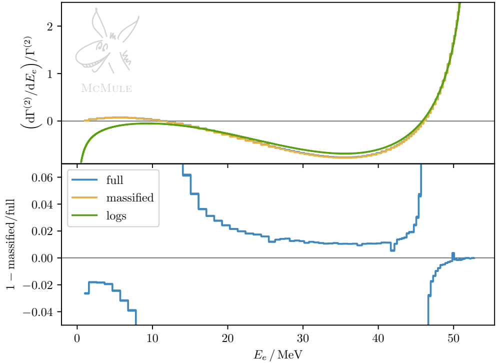
different process ($\mu e \to \mu e$)

- $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$

based on SCET and
method of regions as calculational tool

$\to$ massify [Bonciani et al. 21] $\to$ enhanced + constant terms

NNLO corrections to electron energy spectrum



- full m_e effects
  $\to$ [Chen 18, McMule 18] (analytic)
  $\to$ [Anastasiou et al. 07] (numerical)
- log. approx. $\to$ [Arbuzov et al. 02]
- massification $\to$ [McMule 18]

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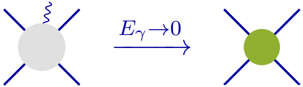
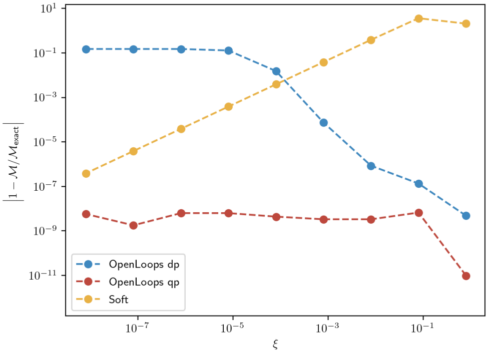
③ problem: numerics

$\rightarrow$ next-to-soft stabilisation real-virtual corrections 'trivial' in principle, extremely delicate numerically

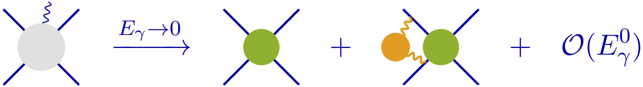


$+ \quad \mathcal{O}(E_\gamma^{-1})$

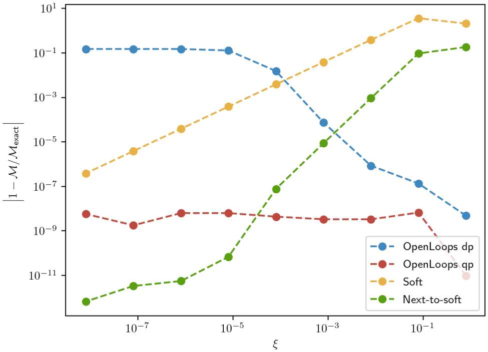
- soft limit (of collinear emission) $E_\gamma = \xi \sqrt{s}/2$
- Bhabha scattering (as example) [McMule, 2106.07469]
- M_{exact} Mathematica expression
- full M vs soft limit
- stability problem



OpenLoops $\to$ [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19] real-virtual corrections ‘trivial’ in principle, extremely delicate numerically

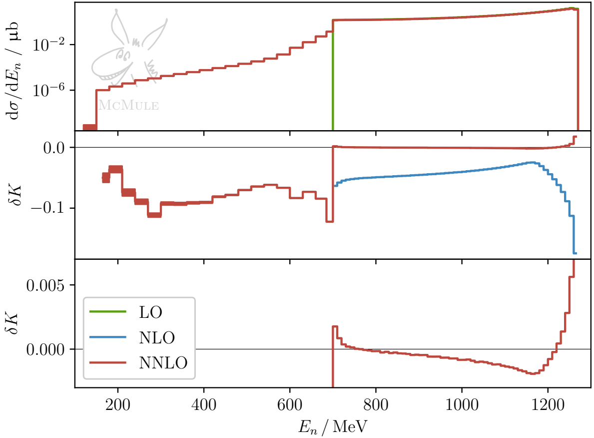


- soft limit (of collinear emission)
- Bhabha scattering (as example) [McMule, 2106.07469]
- M_{exact} Mathematica expression
- full M vs next-to-soft limit
- stability problem solved



OpenLoops → [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19]

Møller for PRad



beam energy $E_b = 1.4$ GeV and kinematical cuts on angles of narrow/wide electron, inelasticity $\eta = E_b + m - E_n - E_w$ and coplanarity $\zeta = \big|180^\circ - |\phi_n - \phi_w|\big|$

$$0.5^\circ < \theta_n, \theta_w < 6.5^\circ$$

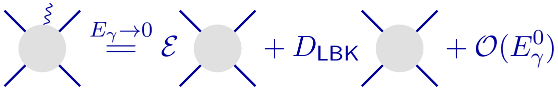
$$\eta < 3.5\sigma_E$$

$$\zeta < 3.5\sigma_\phi$$

with $\sigma_E = 37.7$ MeV, $\sigma_\phi = 2.1^\circ$

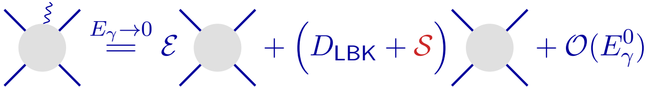
very recent: NTS-stabilisation with LBK

LBK theorem @ tree-level [Low 58, Burnett, Kroll 67]



LBK theorem @ one-loop [TE, Signer, Ulrich, *in preparation*]

→ D_{LBK} yields hard contribution in language of MoR
→ generic soft contribution $\mathcal{S}$



outlook: muon-electron scattering

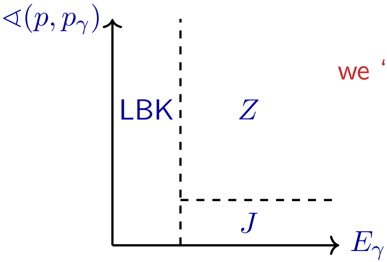
short term

full set of NNLO corr. with FKS^2, massification, and next-to-soft stabilisation

$\rightarrow$ rely on external input: 2-loop amplitude with $m = 0$

long term

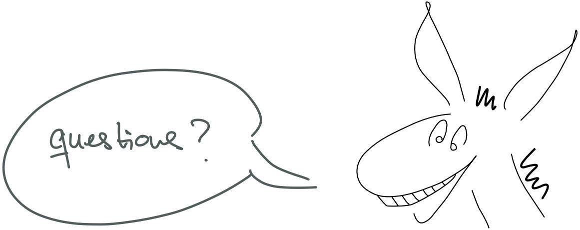
dominant (electronic) N^{3}LO QED corrections



we 'only' need

- ✓ subtraction scheme at N^{3}LO $\rightarrow$ FKS^3
- o massification @ 3-loops
- o massification of radiative amplitudes @ 2-loops

The End



McMule

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