Perspectives from theory on $e^+e^- \to 2\pi$ and $e^+e^- \to 3\pi$

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What can we say about "simple" hadronic channels?

- The HVP contribution to $g-2$ at few times 10^{-10} amounts to **precision measurements of ρ, ω, ϕ**

  $\hookrightarrow$ vector meson dominance clearly not good enough

- What can we still say from theory?
  - ω, ϕ parameters close to (reaction-independent) **pole parameters and residues**

$$\Delta M_\omega\big|_{\text{PDG}} = 0.13\,\text{MeV} \quad \text{vs.} \quad \frac{\Gamma_\omega^2}{M_\omega} \simeq 0.1\,\text{MeV} \qquad \left[\text{compare: } \frac{\Gamma_\rho^2}{M_\rho} \simeq 30\,\text{MeV!}\right]$$

$$\Delta M_\phi\big|_{\text{PDG}} = 0.016\,\text{MeV} \quad \text{vs.} \quad \frac{\Gamma_\phi^2}{M_\phi} \simeq 0.02\,\text{MeV}$$

$$\Delta\epsilon_{\rho\omega}\big|_{\text{fit from } \pi\pi \text{ data}} \gtrsim 3\times 10^{-5} \quad \text{vs.} \quad \epsilon_{\rho\omega}\frac{\Gamma_\omega}{M_\omega} \simeq 2\times 10^{-5}$$

  - Resonance shape from unitarity/analyticity via main decay channels

    Colangelo, MH, Stoffer 2018, MH, Hoid, Kubis 2019, 2020

Status of ω, ϕ parameters

- Need to specify **conventions regarding VP** (bar: include VP)

$$\Delta M_\omega = \bar{M}_\omega - M_\omega = 0.13\,\text{MeV} \qquad \Delta\Gamma_\omega = \bar{\Gamma}_\omega - \Gamma_\omega = -0.06\,\text{MeV}$$

$$\Delta M_\phi = \bar{M}_\phi - M_\phi = 0.26\,\text{MeV} \qquad \Delta\Gamma_\phi = \bar{\Gamma}_\phi - \Gamma_\phi \simeq 0$$

  $\hookrightarrow$ crucial at level of precision above

- PDG averages:
  - Most entries include VP, but not all!
  - Most entries based on VMD fits (with different assumptions for energy dependence)
- In the following: point out issues with PDG averages, and how this reflects tensions in $e^+e^- \to 2\pi, 3\pi, \bar{K}K$ data
- Will employ relations/corrections above, i.e., use both M_ω, $\bar{M}_\omega$ etc.

Start with the ω

- PDG: $\bar{M}_\omega|_{\text{average}} = 782.66(13)\,\text{MeV} \quad [S = 2.0]$ $\qquad \bar{\Gamma}_\omega|_{\text{average}} = 8.68(13)\,\text{MeV}$

$$\bar{M}_\omega|_{3\pi,\text{CMD2}} = 782.68(9)(4)\,\text{MeV} \qquad \bar{\Gamma}_\omega|_{3\pi,\text{CMD2}} = 8.68(23)(10)\,\text{MeV}$$

$$\bar{M}_\omega|_{3\pi,\text{SND}} = 782.79(8)(9)\,\text{MeV} \qquad \bar{\Gamma}_\omega|_{3\pi,\text{SND}} = 8.68(4)(15)\,\text{MeV}$$

$$\bar{M}_\omega|_{\pi^0\gamma,\text{CMD2}} = 783.20(13)(16)\,\text{MeV} \qquad \big[\bar{\Gamma}_\omega|_{3\pi,\text{ND}} = 8.4(1)\,\text{MeV}\big]$$

$$M_\omega|_{\bar{p}p\to\omega\pi^0\pi^0} = 781.96(13)(17)\,\text{MeV}$$

$\hookrightarrow$ **accidental cancellation** for the mass makes the result reasonable

- Our fits:

$$\bar{M}_\omega|_{3\pi} = 782.759(28)\,\text{MeV} \qquad \bar{\Gamma}_\omega|_{3\pi} = 8.65(6)\,\text{MeV}$$

$$\bar{M}_\omega|_{\pi^0\gamma} = 782.712(28)\,\text{MeV} \qquad \bar{\Gamma}_\omega|_{\pi^0\gamma} = 8.59(6)\,\text{MeV}$$

$$\bar{M}_\omega|_{\text{combination}} = 782.736(24)\,\text{MeV} \qquad \bar{\Gamma}_\omega|_{\text{combination}} = 8.63(5)\,\text{MeV}$$

- Reasonable agreement with PDG averages, but precision increases if
  - Hadronic $\bar{p}p$ reaction is tossed
  - $\pi^0\gamma$ included avoiding unphysical phase (then good consistency with 3π)
  - **Energy resolution/calibration not added as additional uncertainty**

Next: the ϕ

- PDG: $\bar{M}_\phi|_{\text{average}} = 1019.461(16)\,\text{MeV}$ $\bar{\Gamma}_\phi|_{\text{average}} = 4.249(13)\,\text{MeV}$

$$\bar{M}_\phi|_{K^+K^-,K_S^0K_L^0,\text{CMD3}} = 1019.463(61)\,\text{MeV} \qquad \bar{\Gamma}_\phi|_{K^+K^-,K_S^0K_L^0,\text{CMD3}} = 4.245(13)\,\text{MeV}$$

$$\bar{M}_\phi|_{K_S^0K_L^0,\text{BaBar}} = 1019.462(42)(56)\,\text{MeV} \qquad \bar{\Gamma}_\phi|_{K_S^0K_L^0,\text{BaBar}} = 4.205(103)(67)\,\text{MeV}$$

$$\bar{M}_\phi|_{K^+K^-,\text{BaBar}} = 1019.51(2)(5)\,\text{MeV} \qquad \bar{\Gamma}_\phi|_{K^+K^-,\text{BaBar}} = 4.29(4)(7)\,\text{MeV}$$

$$\bar{M}_\phi|_{K_S^0K_L^0,\text{CMD2}} = 1019.483(11)(25)\,\text{MeV} \qquad \bar{\Gamma}_\phi|_{K_S^0K_L^0,\text{CMD2}} = 4.280(33)(25)\,\text{MeV}$$

$\hookrightarrow$ **totally dominated by $\bar{K}K$** (other channels included, but less precise)

- Our fits: $\bar{M}_\phi|_{3\pi} = 1019.456(21)\,\text{MeV}$ $\bar{\Gamma}_\phi|_{3\pi} = 4.23(4)\,\text{MeV}$

$$\bar{M}_\phi|_{\pi^0\gamma} = 1019.465(55)\,\text{MeV} \qquad \bar{\Gamma}_\phi|_{\pi^0\gamma} = 4.07(13)\,\text{MeV}$$

$$\bar{M}_\phi|_{\text{combination}} = 1019.457(20)\,\text{MeV} \qquad \bar{\Gamma}_\phi|_{\text{combination}} = 4.22(5)\,\text{MeV}$$

$\hookrightarrow$ **dominated by 3π**

- In general, good agreement between 3π and $\bar{K}K$ at similar level of precision

Why does this matter?

- So far: 3π around ω dominated by SND data
- Out fits (preliminary!) to **recent new 3π data sets**
  - BESIII 1912.11208

$$\bar{M}_\omega|_{\text{BESIII}} = 783.30(6)\,\text{MeV} \qquad \bar{M}_\phi|_{\text{BESIII}} = 1019.73(7)\,\text{MeV}$$

    $\hookrightarrow$ both ω ($+0.56\,\text{MeV}$) and ϕ ($+0.27\,\text{MeV}$) masses much larger than before

  - BaBar 2110.00520

$$\bar{M}_\omega|_{\text{BaBar}} = 782.66(1)\,\text{MeV} \qquad \bar{M}_\phi|_{\text{BaBar}} = 1019.54(1)\,\text{MeV}$$

    $\hookrightarrow$ $\bar{M}_\omega$ slightly smaller ($-0.08\,\text{MeV}$) and $\bar{M}_\phi$ slightly larger ($+0.08\,\text{MeV}$) than before

  - For the ϕ, the **nice agreement between 3π and $\bar{K}K$ seems jeopardized** (but note K^+K^- channel from BaBar)
- Concerns when resonance positions do not match (exactly):
  - To what extent can we combine data sets?
  - To what extent is VP removal compromised?

Why does this matter?

- Discussion not restricted to 3π and $\bar{K}K$

  $\hookrightarrow$ affects 2π channel via **ρ–ω mixing**

- 3π channel parameterized by **ρ–ω mixing parameter** $\epsilon_{\rho\omega}$

$$F_\pi^V(s) = \left(1 + \frac{s}{M_\omega^2 - s - iM_\omega\Gamma_\omega}\epsilon_{\rho\omega}\right)\Omega(s)G_{\text{in}}(s)$$

- From analytic continuation to ω pole: $\delta_\epsilon \simeq \arctan\frac{\Gamma_\omega}{M_\omega} = 0.6^\circ$

  $\hookrightarrow$ up to tiny corrections $\epsilon_{\rho\omega}$ has to be real

- However BaBar 2012:
  - M_ω in 2π fits tends to come out (significantly) below M_ω from 3π
  - M_ω strongly correlated with δ_ϵ

  $\hookrightarrow$ points to 2π channel not being understood at required level

On the isospin-violating phase of ρ–ω mixing

- Can produce a more sizable phase in $\epsilon_{\rho\omega}$ via **isospin-breaking corrections**

$$\omega \to X \to \rho \qquad X = \pi^0\gamma, \pi\pi\gamma, \eta\gamma$$

- Implemented via

$$\epsilon_{\rho\omega} \to \text{Re}\,\epsilon_{\rho\omega} + i\text{Im}\,\epsilon_{\rho\omega}\frac{\left(1-\frac{M_{\pi^0}^2}{s}\right)^3}{\left(1-\frac{M_{\pi^0}^2}{M_\omega^2}\right)^3}\theta\big(s-M_{\pi^0}^2\big)$$

  to ensure correct thresholds

- **Narrow-width estimate**

$$|\text{Im}\,\epsilon_{\rho\omega}^X| = \frac{\sqrt{\Gamma(\omega\to X)\Gamma(\rho\to X)}}{3M_V}$$

  leads to 2.7° ($\pi^0\gamma$), 0.7° ($\pi\pi\gamma$), and 0.2° ($\eta\gamma$)

  $\hookrightarrow$ $\delta_\epsilon \gtrsim 4^\circ$ hard to understand

On the isospin-violating phase of ρ–ω mixing

	χ^2/dof	M_ω [MeV]	$10^3 \times$ Re $\epsilon_{\rho\omega}$	δ_ϵ [°]	$10^{10} \times a_\mu^{\pi\pi}\|_{\leq 1\text{ GeV}}$
SND06 + CMD2	1.28	781.75(22)(1)	1.97(4)(2)		498.5(3.4)(2.6)
	1.05	782.39(23)(2)	1.92(4)(3)	9.9(1.8)(0.4)	497.3(3.1)(3.9)
BaBar	1.14	781.86(14)(1)	2.04(3)(2)		501.9(3.3)(2.0)
	1.14	781.93(18)(4)	2.03(4)(1)	1.3(1.9)(0.7)	501.9(3.3)(1.8)
KLOE''	1.20	781.81(16)(3)	1.98(4)(1)		491.8(2.1)(1.8)
	1.13	782.42(23)(5)	1.95(4)(2)	6.1(1.7)(0.6)	490.8(2.1)(1.7)
BESIII	1.12	782.17(33)(7)	2.01(19)(9)		490.8(4.8)(3.9)
	1.02	783.05(48)(2)	1.98(19)(7)	17.7(7.0)(1.2)	490.3(4.6)(3.0)
SND20	2.93	781.79(30)(6)	2.04(6)(3)		494.2(6.7)(9.0)
	1.87	782.37(28)(6)	2.01(5)(2)	10.2(2.4)(1.4)	494.9(5.3)(3.1)
Combination w/o SND20	1.25	781.72(8)(3)	2.02(2)(3)		494.5(1.5)(2.3)
	1.20	782.12(12)(4)	1.96(2)(2)	4.6(9)(8)	494.2(1.4)(2.1)

- Most fits improve moderately by allowing for a phase δ_ϵ

  $\hookrightarrow$ **huge effect for SND20**

- Preferred value of δ_ϵ **varies a lot** among the different data sets

- δ_ϵ correlated with M_ω BaBar 2012, $\Delta M_\omega \simeq \frac{\Gamma_\omega}{2}\delta_\epsilon$, but required value of $\delta_\epsilon \simeq 10^\circ$ for agreement with $e^+e^- \to 3\pi, \pi^0\gamma$ too large to be explained by $X = \pi^0\gamma, \pi\pi\gamma, \eta\gamma$

Conclusions

- To claim subpercent precision in the HVP contribution, we ought to have a **consistent understanding of the simplest hadronic channels**
- This does not seem to be the case:
  - Tensions in ω (and ϕ?) parameters in isospin-conserving case (3π, $\bar{K}K$)
  - Tensions in ω parameters in isospin-violating case (2π), both among data sets and with isospin-conserving case

  $\hookrightarrow$ these parameters should not be **reaction dependent**!
- This is relevant also for direct-integration techniques: **removal of VP depends critically on ω, ϕ parameters**

  $\hookrightarrow$ do we need to pay more attention to **energy resolution/calibration** when combining data sets?