Dispersive approach to isospin-breaking corrections to $e^+e^- \to \pi^+\pi^-$ and $\pi^+\pi^- \to \pi^+\pi^-$

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Outline

Introduction

Dispersive approach to radiative corrections to $\pi\pi$ scattering

Dispersive approach to FSR in $e^+e^- \to \pi^+\pi^-$

Conclusions and outlook

Work done in collaboration with
Joachim Monnard and Jacobo Ruiz de Elvira

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HVP contribution to $(g-2)_\mu$

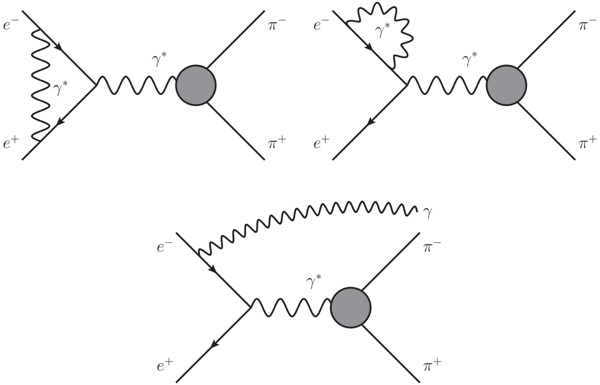
Contribution	Value $\times 10^{11}$
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 061(41)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)

HVP dominant source of theory uncertainty
rel. size $\sim 0.6\% \Rightarrow$ RC in $e^+e^- \to \pi^+\pi^-$ must be under control

RC evaluation based on models so far
A dispersive approach could lead to model-independent results

Radiative corrections to $e^+e^- \to \pi^+\pi^-$

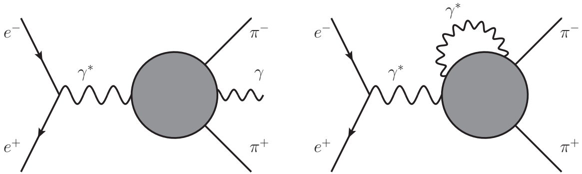
Initial State Radiation:



can be calculated in QED in terms of $F_\pi^V(s)$

Radiative corrections to $e^+e^- \to \pi^+\pi^-$

Final State Radiation:

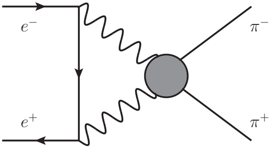


requires hadronic matrix elements beyond $F_\pi^V(s)$
known in ChPT to one loop

Kubis, Meißner (01)

Radiative corrections to $e^+e^- \to \pi^+\pi^-$

Interference terms:



also require hadronic matrix elements beyond $F_\pi^V(s)$

Radiative corrections to $e^+e^- \to \pi^+\pi^-$

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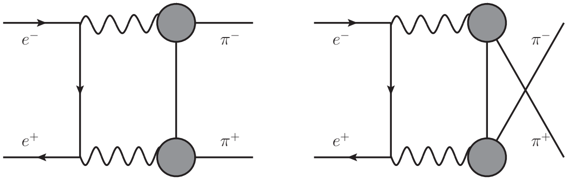
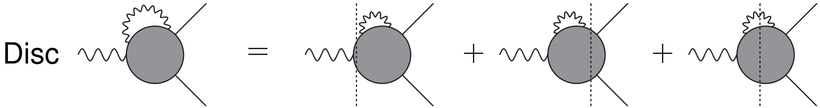


also require hadronic matrix elements beyond $F_\pi^V(s)$
other than in the 1π-exchange approximation;

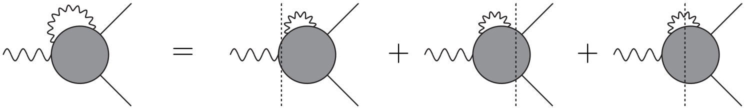
do not contribute to the total cross section and will be ignored
but have been evaluated and found to be small by J. Monnard

Dispersive approach to FSR



Dispersive approach to FSR

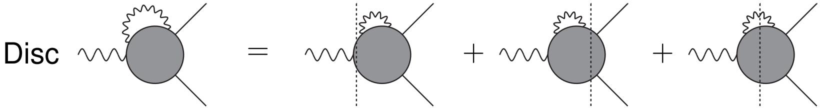
Disc



Neglecting intermediate states beyond 2π, unitarity reads

$$\frac{\mathrm{Disc}F_\pi^{V,\alpha}(s)}{2i} = \frac{(2\pi)^4}{2}\int d\Phi_2 F_\pi^V(s)\times T_{\pi\pi}^{\alpha *}(s,t) + \frac{(2\pi)^4}{2}\int d\Phi_2 F_\pi^{V,\alpha}(s)\times T_{\pi\pi}^{*}(s,t) + \frac{(2\pi)^4}{2}\int d\Phi_3 F_\pi^{V,\gamma}(s,t) T_{\pi\pi}^{\gamma *}(s,\{t_i\})$$

Dispersive approach to FSR



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$\Rightarrow$ need $T_{\pi\pi}^{\alpha}$ as well as $T_{\pi\pi}^{\gamma}$ and $F_\pi^{V,\gamma}$ as input

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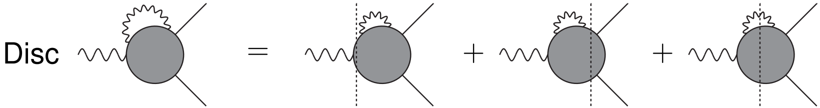

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$\Rightarrow$ need $T_{\pi\pi}^{\alpha}$ as well as $T_{\pi\pi}^{\gamma}$ and $F_\pi^{V,\gamma}$ as input

The DR for $F_\pi^{V,\alpha}(s)$ takes the form of an integral equation

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$\pi\pi$ scattering amplitude in the isospin limit

Phenomenological representation of $A(s,t,u)$ below ~ 1 GeV

$$A(s,t,u) = A(s,t,u)_{SP} + A(s,t,u)_d$$

where A_{SP} is the unitarity contribution of S and P waves

$$A(s,t,u)_{SP} = \frac{32\pi}{3}\left\{W^0(s) - W^2(s) + \frac{9}{2}(s-u)W^1(t) + \frac{3}{2}W^2(t) + (t \leftrightarrow u)\right\}$$

and (with $\sqrt{s_2} \sim 2$ GeV)

$$W^0(s) = \frac{a_0^0\, s}{4M_\pi^2} + \frac{s(s-4M_\pi^2)}{\pi}\int_{4M_\pi^2}^{s_2} ds' \, \frac{\mathrm{Im}\, t_0^0(s')}{s'(s'-4M_\pi^2)(s'-s)}$$

$$W^1(s) = \frac{s}{\pi}\int_{4M_\pi^2}^{s_2} ds' \, \frac{\mathrm{Im}\, t_1^1(s')}{s'(s'-4M_\pi^2)(s'-s)}$$

$$W^2(s) = \frac{a_0^2\, s}{4M_\pi^2} + \frac{s(s-4M_\pi^2)}{\pi}\int_{4M_\pi^2}^{s_2} ds' \, \frac{\mathrm{Im}\, t_0^2(s')}{s'(s'-4M_\pi^2)(s'-s)}$$

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where $t^I_\ell(s)$ are partial wave projections of isospin amplitudes

$$\begin{aligned} T^0(s,t,u) &= 3A(s,t,u) + A(t,u,s) + A(u,s,t) \\ T^1(s,t,u) &= A(t,u,s) - A(u,s,t) \\ T^2(s,t,u) &= A(t,u,s) + A(u,s,t) \end{aligned}$$

$\pi\pi$ scattering amplitude in the isospin limit

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and A_d is the “background amplitude”, due to higher waves and higher energies. Below $\sim$ 1 GeV it is a small and smooth contribution $\Rightarrow$ polynomial

$\pi\pi$ scattering amplitude away from the isospin limit

We need to consider three different effects: GC, Gasser, Rusetsky (09)

1. strong isospin breaking: effects proportional to $(m_u - m_d)$
2. effects proportional to $M_{\pi^+} - M_{\pi^0}$
3. effects due to photon exchanges

Each of them can be considered separately from the other two

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At low energy effects 1. are small $\sim O((m_u - m_d)^2)$

At higher energies they generate π^0-η as well as ρ-ω mixing

These can be (and are) described phenomenologically
(and π^0-η mixing is not relevant for $F_\pi^V(s)$)

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The rest of the talk concerns the other two

Roy equations away from the isospin limit

First we need to switch from the isospin to the charge basis

$$
\begin{aligned}
T^c(s,t,u) &= \frac{1}{3}T^0(s,t,u) + \frac{1}{2}T^1(s,t,u) + \frac{1}{6}T^2(s,t,u) \\
T^x(s,t,u) &= \frac{1}{3}T^0(s,t,u) - \frac{1}{3}T^2(s,t,u) \\
T^n(s,t,u) &= \frac{1}{3}T^0(s,t,u) + \frac{2}{3}T^2(s,t,u)
\end{aligned}
$$

where

$$T^c := T(\pi^+\pi^- \to \pi^+\pi^-),\ T^x := T(\pi^+\pi^- \to \pi^0\pi^0),\ T^n := T(\pi^0\pi^0 \to \pi^0\pi^0)$$

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$$T^c := T(\pi^+\pi^- \to \pi^+\pi^-),\ T^x := T(\pi^+\pi^- \to \pi^0\pi^0),\ T^n := T(\pi^0\pi^0 \to \pi^0\pi^0)$$

and with crossed channels

$$\begin{aligned}
T^{++}(s,t,u) := T(\pi^+\pi^+ \to \pi^+\pi^+) &= T^c(t,u,s) \\
T^{+}(s,t,u) := T(\pi^+\pi^0 \to \pi^+\pi^0) &= T^x(t,u,s) \,.
\end{aligned}$$

Roy equations away from the isospin limit

First we need to switch from the isospin to the charge basis

$$\begin{aligned} T^c(s,t,u) &= \frac{1}{3}T^0(s,t,u) + \frac{1}{2}T^1(s,t,u) + \frac{1}{6}T^2(s,t,u) \\ T^x(s,t,u) &= \frac{1}{3}T^0(s,t,u) - \frac{1}{3}T^2(s,t,u) \\ T^n(s,t,u) &= \frac{1}{3}T^0(s,t,u) + \frac{2}{3}T^2(s,t,u) \end{aligned}$$

Then adapt unitarity relations

$$\begin{aligned} \mathrm{Im}\, T_S(s) &= T_S(s)\rho(s)T_S^*(s)\,, \quad \text{with} \quad T_S = \begin{pmatrix} t_{n,S}(s) & -t_{x,S}(s) \\ -t_{x,S}(s) & t_{c,S}(s) \end{pmatrix}\,, \\ \rho(s) &= \begin{pmatrix} \sigma_0(s)\theta(s-4M_{\pi^0}^2) & 0 \\ 0 & 2\sigma(s)\theta(s-4M_\pi^2) \end{pmatrix} \end{aligned}$$

where

$$\sigma_0(s) = \sqrt{1-4M_{\pi^0}^2/s}, \quad \sigma(s) = \sqrt{1-4M_\pi^2/s}$$

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T^c(s,t,u) &= \frac{1}{3}T^0(s,t,u) + \frac{1}{2}T^1(s,t,u) + \frac{1}{6}T^2(s,t,u) \\
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\end{aligned}
$$

Then adapt unitarity relations

$$
\begin{aligned}
\mathrm{Im}\, t_{n,S}(s) &= \sigma_0(s)|t_{n,S}(s)|^2 + 2\sigma(s)|t_{x,S}(s)|^2 \\
\mathrm{Im}\, t_{x,S}(s) &= \sigma_0(s)t_{n,S}(s)t^*_{x,S}(s) + 2\sigma(s)t_{x,S}(s)t^*_{c,S}(s) \\
\mathrm{Im}\, t_{c,S}(s) &= \sigma_0(s)|t_{x,S}(s)|^2 + 2\sigma(s)|t_{c,S}(s)|^2 \ .
\end{aligned}
$$

where

$$
\sigma_0(s) = \sqrt{1 - 4M^2_{\pi^0}/s}, \quad \sigma(s) = \sqrt{1 - 4M^2_{\pi}/s}
$$

Roy equations away from the isospin limit

This leads to the following Roy eqs.

$$T^n_{SP}(s,t,u) = 32\pi \left(W^{00}_{n,S}(s) + W^{+-}_{n,S}(s) + (s \leftrightarrow t) + (s \leftrightarrow u) \right)$$

$$\begin{aligned} W^{00}_{n,S}(s) &= \frac{a^{00}_n\, s}{4M^2_{\pi^0}} + \frac{s(s-4M^2_{\pi^0})}{\pi} \int_{4M^2_{\pi^0}}^{s_2} ds' \frac{\mathrm{Im}\, t^{00}_{n,S}(s')}{s'(s'-4M^2_{\pi^0})(s'-s)} \\ W^{+-}_{n,S}(s) &= \frac{s(s-4M^2_{\pi^0})}{\pi} \int_{4M^2_{\pi}}^{s_2} ds' \frac{\mathrm{Im}\, t^{+-}_{n,S}(s')}{s'(s'-4M^2_{\pi^0})(s'-s)} \ , \end{aligned}$$

Roy equations away from the isospin limit

This leads to the following Roy eqs.

$$
\begin{aligned}
T_{SP}^{++}(s,t,u) &= 32\pi\Big[W_S^{++}(s) + W_{c,S}^{00}(t) + W_{c,S}^{+-}(t) + W_{c,S}^{00}(u) + W_{c,S}^{+-}(u) \\
&\quad +(s-u)W_{c,P}^{+-}(t) + (s-t)W_{c,P}^{+-}(u)\Big]
\end{aligned}
$$

$$
\begin{aligned}
W_S^{++}(s) &= \frac{a^{++}\,s}{4M_\pi^2} + \frac{s(s-4M_\pi^2)}{\pi}\int_{4M_\pi^2}^{s_2} ds' \frac{\mathrm{Im}\,t_S^{++}(s')}{s'(s'-4M_\pi^2)(s'-s)} \\
W_{c,S}^{+-}(s) &= \frac{a_c^{+-}\,s}{4M_\pi^2} + \frac{s(s-4M_\pi^2)}{\pi}\int_{4M_\pi^2}^{s_2} ds' \frac{\mathrm{Im}\,t_{c,S}^{+-}(s')}{s'(s'-4M_\pi^2)(s'-s)} \\
W_{c,S}^{00}(s) &= \frac{s(s-4M_\pi^2)}{\pi}\int_{4M_{\pi^0}^2}^{s_2} ds' \frac{\mathrm{Im}\,t_{c,S}^{00}(s')}{s'(s'-4M_\pi^2)(s'-s)} \\
W_{c,P}^{+-}(s) &= \frac{s}{\pi}\int_{4M_\pi^2}^{s_2} ds' \frac{3\,\mathrm{Im}\,t_{c,P}^{+-}(s')}{s'(s'-4M_\pi^2)(s'-s)}\,.
\end{aligned}
$$

Via crossing this provides also a representation for T^c

Roy equations away from the isospin limit

This leads to the following Roy eqs.

$$T^x_{SP}(s,t,u) = 32\pi\Big[W^{+-}_{x,S}(s) + W^{00}_{x,S}(s) + W^{+0}_S(t) + W^{+0}_S(u) + \left(t(s-u)+\Delta^2_\pi\right) W^{+0}_P(t) + \left(u(s-t)+\Delta^2_\pi\right) W^{+0}_P(u)\Big]$$

$$W^{+-}_{x,S}(s) = \frac{a^{+-}_x\, s}{4M^2_\pi} + \frac{s(s-4M^2_\pi)}{\pi}\int_{4M^2_\pi}^{s_2} ds' \frac{\mathrm{Im}\, t^{+-}_{x,S}(s')}{s'(s'-4M^2_\pi)(s'-s)}$$

$$W^{00}_{x,S}(s) = \frac{s(s-4M^2_\pi)}{\pi}\int_{4M^2_{\pi^0}}^{s_2} ds' \frac{\mathrm{Im}\, t^{00}_{x,S}(s')}{s'(s'-4M^2_\pi)(s'-s)}$$

$$W^{+0}_S(s) = \frac{a^{+0}_c\, s}{4\bar{M}^2_\pi} + \frac{s(s-4\bar{M}^2_\pi)}{\pi}\int_{4\bar{M}^2_\pi}^{s_2} ds' \frac{\mathrm{Im}\, t^{+0}_S(s')}{s'(s'-4\bar{M}^2_\pi)(s'-s)}$$

$$W^{+0}_P(s) = \frac{1}{\pi}\int_{4\bar{M}^2_\pi}^{s_2} ds' \frac{3\mathrm{Im}\, t^{+0}_P(s')}{\lambda(s',M^2_\pi,M^2_{\pi^0})(s'-s)} \, ,$$

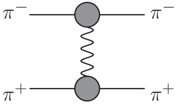
$$\Delta_\pi := M^2_\pi - M^2_{\pi^0} \quad \bar{M}_\pi := (M_\pi + M_{\pi^0})/2$$

Roy eqs. and $M_\pi^2 - M_{\pi^0}^2$ effects

- Roy eqs. rely on input above $\sqrt{s_1} \sim 1.15$ GeV and for the scattering lengths
- the numerical solution of the equations provides the partial waves for $4M_\pi^2 \leq s \leq s_1$
- we assume that the input above s_1 does not change for $\Delta_\pi \neq 0$
- taking as starting point the solutions in the isospin limit and simply reevaluating the dispersive integrals after having shifted the thresholds will provide the desired effects
- the procedure can be iterated
- the effect on $F_\pi^V(s)$ is small (the $\pi^0\pi^0$ only appears in the t-channel of the $\pi\pi$ amplitude in the unitarity relation)

Roy eqs. and photon-exchange effects

Photon-exchange diagrams are $O(\alpha)$ effects not included in the Roy eqs.

$T_B(t, s, u) :=$ 

$= 4\pi\alpha \frac{s-u}{t} F_\pi^V(t)^2$

$$T_B^c(s, t, u) = T_B(t, s, u) + T_B(s, t, u)$$

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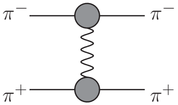
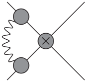
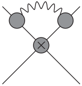
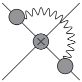


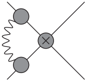
$$= 4\pi\alpha \frac{s-u}{t} F_\pi^V(t)^2$$

$$T_B^c(s,t,u) = T_B(t,s,u) + T_B(s,t,u)$$

- Adding such a contribution to the T^c amplitude upsets the unitarity relations for all amplitudes
- we are interested in corrections only up to $O(\alpha)$
  $\Rightarrow$ set up an iterative scheme

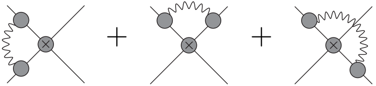
Roy eqs. and photon-exchange effects: 1. iteration

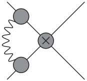
$T_D^c(s,t,u) :=$  $+$  $+$  $+$ flipped diags.

$T_D^x(s,t,u) :=$ 

“Triangle diagrams” $\Rightarrow$ topology of box diagrams and expressed through a double-spectral representation

Roy eqs. and photon-exchange effects: 1. iteration

$T_D^c(s,t,u) :=$  + flipped diags.

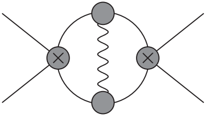
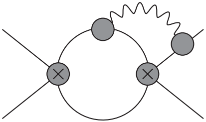
$T_D^x(s,t,u) :=$ 

"Triangle diagrams" $\Rightarrow$ topology of box diagrams and expressed through a double-spectral representation

Starting point for further iterations:

$$\begin{aligned} T_1^c(s,t,u) &= T_0^c(s,t,u) + T_B^c(s,t,u) + T_D^c(s,t,u) \\ T_1^x(s,t,u) &= T_0^x(s,t,u) + T_D^x(s,t,u) \\ T_1^n(s,t,u) &= T_0^n(s,t,u) \end{aligned}$$

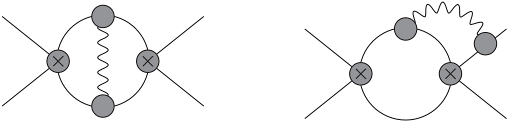
Roy eqs. and photon-exchange effects: 2. iteration

Diagrams have to be cut in all possible ways:

$\Rightarrow$ contributions from subamplitudes with real photons

Roy eqs. and photon-exchange effects: 2. iteration



Diagrams have to be cut in all possible ways:
⇒ contributions from subamplitudes with real photons

Expression after further iterations:

$$
\begin{aligned}
T_1^c(s,t,u) &= T_0^c(s,t,u) + T_B^c(s,t,u) + T_D^c(s,t,u) + \sum_{k=2} R_k^c(s,t,u) \\
T_1^x(s,t,u) &= T_0^x(s,t,u) + T_D^x(s,t,u) + \sum_{k=2} R_k^x(s,t,u) \\
T_1^n(s,t,u) &= T_0^n(s,t,u) + \sum_{k=2} R_k^n(s,t,u)
\end{aligned}
$$

Roy eqs. and photon-exchange effects: comments

- starting from the 2. iteration the evaluation of the R^i_{k+1} is done as follows:
  1. project the R^i_k amplitudes onto partial waves
  2. insert these into the unitarity relations combined with the projections of T^i_0
  3. add the contribution of subdiagrams with real photons
  4. solve the corresponding dispersion relation
- subtraction constants can be fixed by matching to ChPT
- iteration number k corresponds to chiral $O(p^{2k})$
- ChPT $\pi\pi$ amplitude with RC known to one loop Knecht, Nehme (02)
  $\Rightarrow$ subtraction constants for all R^i_k, $k \geq 2$ can be set to zero

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Dispersive approach to FSR in $e^+e^- \to \pi^+\pi^-$

Conclusions and outlook

Dispersive treatment of FSR in $e^+e^- \to \pi^+\pi^-$

$$\begin{aligned}\frac{\text{Disc}F_\pi^{V,\alpha}(s)}{2i} &= \frac{(2\pi)^4}{2}\int d\Phi_2 F_\pi^V(s)\times T_{\pi\pi}^{\alpha *}(s,t)\\ &+ \frac{(2\pi)^4}{2}\int d\Phi_2 F_\pi^{V,\alpha}(s)\times T_{\pi\pi}^{*}(s,t)\\ &+ \frac{(2\pi)^4}{2}\int d\Phi_3 F_\pi^{V,\gamma}(s,t) T_{\pi\pi}^{\gamma *}(s,\{t_i\})\end{aligned}$$

After this long digression we have obtained $T_{\pi\pi}^{\alpha}$

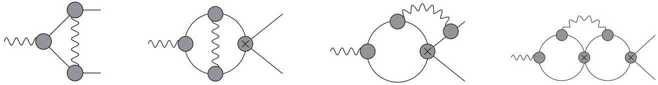
For $F_\pi^{V,\gamma}$ and $T_{\pi\pi}^{\gamma}$ the approximation no heavier intermediate states than two pions means:

All subamplitudes known $\Rightarrow F_\pi^{V,\gamma}$ and $T_{\pi\pi}^{\gamma}$ ✓

Evaluation of $F_\pi^{V,\alpha}$

Having evaluated all the following diagrams

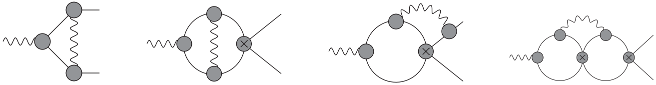
J. Monnard, PhD thesis 2021



Evaluation of $F_\pi^{V,\alpha}$

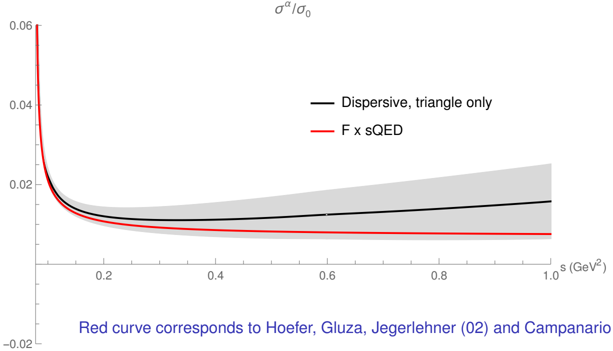
Having evaluated all the following diagrams

J. Monnard, PhD thesis 2021



the results for $\sigma(e^+e^- \to \pi^+\pi^-(\gamma))$ look as follows:

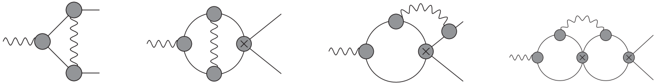
Preliminary!



Red curve corresponds to Hoefer, Gluza, Jegerlehner (02) and Campanario et al. (19) (?)

Evaluation of $F_\pi^{V,\alpha}$

Having evaluated all the following diagrams — J. Monnard, PhD thesis 2021



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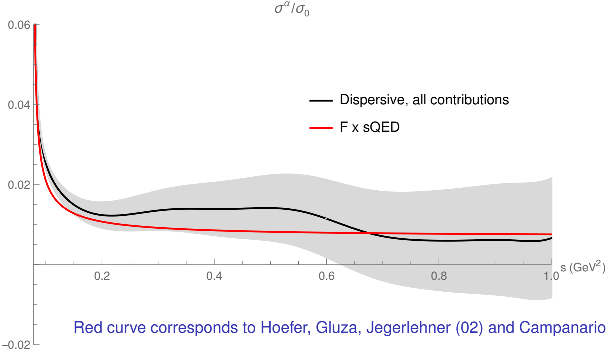

Red curve corresponds to Hoefer, Gluza, Jegerlehner (02) and Campanario et al. (19) (?)

Impact on a_μ^{HVP}

Ideally one would use the calculated RC directly in the data analysis (future?). To get an idea of the impact we did the following:

thanks to M. Hoferichter and P. Stoffer

1. remove RC from the measured $\sigma(e^+e^- \to \pi^+\pi^-(\gamma))$
2. fit with the dispersive representation for $F_\pi^V(s)$
3. insert back the RC

The impact on a_μ^{HVP} is evaluated by comparing to the result obtained by removing RC with $\eta(s)$ calculated in sQED

$$10^{11}\Delta a_\mu^{\mathrm{HVP}} = \begin{cases} 10.2 \pm 0.5 \pm 5 & \text{FsQED} \\ 10.5 \pm 0.5 & \text{triangle} \\ 13.2 \pm 0.5 & \text{full} \end{cases}$$

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Conclusions and outlook

- We have developed the formalism for evaluating dispersively RC to the $\pi\pi$ scattering amplitude and $F_\pi^V(s)$

  work in progress GC, J. Monnard, J. Ruiz de Elvira
- the possibility to obtain a finite system of equations and solve them relies on the approximation of including only up to 2π intermediate states
- our preliminary evaluation of the corrections to $F_\pi^V(s)$ shows no unexpectedly large effects J. Monnard, PhD thesis, 2021
- our preliminary estimate of the impact on $a_\mu^{\rm HVP}$ also shows moderate effects J. Monnard, PhD thesis, 2021
- the final goal is to provide a ready-to-use code which can be implemented in MC and used in data analysis
- we plan to apply the same approach to $\tau \to \pi\pi\nu_\tau$