The BabaYaga event generator: overview and future prospects

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Spacelike and Timelike determination
of the Hadronic Leading Order contribution to the Muon $g - 2$

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in collaboration with G. Montagna, O. Nicrosini and F. Piccinini

Outline

- References & motivations
- Theoretical formulation: QED PS and NLO matching
- Results
- Future developments (wish list)
- Conclusions

Relevant papers

⋆ **BabaYaga** core references:

- C.M.C.C et al. Phys. Lett. B **798** (2019), 134976 — $e^+e^- \to \gamma\gamma$ with EWK RCs for FCC-ee
- Barzè et al., Eur. Phys. J. C **71** (2011) 1680 — BabaYaga with dark photon
- Balossini et al., Phys. Lett. **663** (2008) 209 — BabaYaga@NLO for $e^+e^- \to \gamma\gamma$
- Balossini et al., Nucl. Phys. **B758** (2006) 227 — BabaYaga@NLO for Bhabha
- C.M.C.C. et al., Nucl. Phys. Proc. Suppl. **131** (2004) 48 — BabaYaga@NLO
- C.M.C.C., Phys. Lett. B **520** (2001) 16 — improved PS BabaYaga
- C.M.C.C. et al., Nucl. Phys. B **584** (2000) 459 — BabaYaga

⋆ Related work:

- S. Actis et al.
  "Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data", Eur. Phys. J. C **66** (2010) 585
  Report of the Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies
- C.M.C.C. et al., JHEP **1107** (2011) 126
  NNLO massive pair corrections

Motivations: reference processes for luminometry

- Reference processes for luminosity at flavour factories are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

⋆ **Large-angle QED processes** $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$ are golden processes at flavour factories to get typical precision at the 0.1% level

↪ QED radiative corrections are mandatory

↪ Fully-fledged exclusive Monte Carlo event generators are needed to account for complex experimental selection criteria

- `BabaYaga` was developed for high-precision simulation of QED processes at flavour factories, primarily for luminosity determination

- The focus is on integrated cross sections, within experimental cuts

⇝ Based on an *in-house* implementation of a **QED Parton Shower**, ***consistently matched with exact QED NLO matrix elements***

↪ An arbitrary number of (extra) photons can be generated

QED Parton Shower

⋆ Common methods used to account for multiple photon corrections are analytical collinear QED Structure Functions (SF), YFS exponentiation, QED Parton Shower (PS)

- The QED PS is an exact MC solution of the QED DGLAP equation for the non-singlet electron SF $D(x, Q^2)$

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dt}{t} P_+(t) D(\frac{x}{t}, Q^2)$$

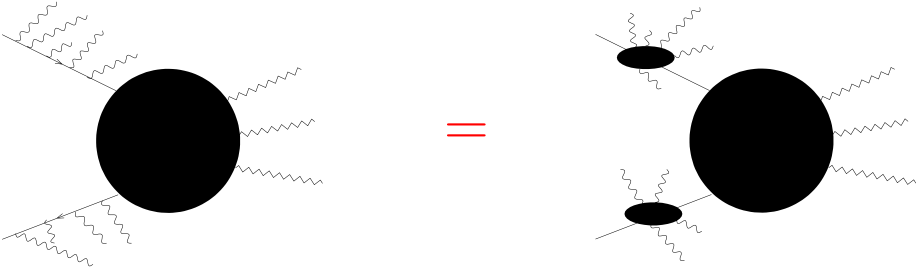
- The PS solution can be cast into the form

$$D(x, Q^2) \; = \; \Pi(Q^2, \epsilon) \sum_{n=0}^{\infty} \; \frac{1}{n!} \int \delta(x - x_1 \cdots x_n) \; \prod_{i=0}^{n} \left[\frac{\alpha}{2\pi} P(x_i) \; L \; dx_i \right]$$

$\rightarrow$ $\Pi(Q^2, \epsilon) \equiv e^{-\frac{\alpha}{2\pi} L I_+}$ Sudakov form factor, $I_+ \equiv \int_0^{1-\epsilon} P(x) dx$,
$L \equiv \ln Q^2/m^2$ collinear log, ϵ soft/hard separator and Q^2 virtuality scale

$\rightarrow$ the kinematics of the photon emissions can be recovered $\rightarrow$ exclusive photons generation

QED Parton Shower



- The PS catches the collinear and soft factorisable dynamics to all orders in α (exponentiation, resummation)
- By its nature, at any order corrections are approximate at Leading-Log (LL) level
- RCs are separated into soft+virtual emissions (virtual γ's or real γ's with $E_\gamma < \epsilon$) and hard emissions ($E_\gamma > \epsilon$).
  No IR-safe physical observable depends on ϵ.

⋆ The accuracy is improved by matching leading-log PS with exact NLO matrix elements (NLOPS)

$\rightsquigarrow$ theoretical error starts then at $\mathcal{O}(\alpha^2)$ (NNLO)

Matching NLO and PS in BabaYaga@NLO

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual (SV) corrections and hard-bremsstrahlung (H) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma_{PS}^{\infty} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$
- $d\sigma_{PS}^{\alpha} = [1 + C_{\alpha,PS}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_{1,PS}|^2 d\Phi_3 \equiv d\sigma_{PS}^{SV}(\varepsilon) + d\sigma_{PS}^{H}(\varepsilon)$
- $d\sigma_{\rm NLO}^{\alpha} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_1|^2 d\Phi_3 \equiv d\sigma_{\rm NLO}^{SV}(\varepsilon) + d\sigma_{\rm NLO}^{H}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,PS})$ $\qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,PS}|^2}{|\mathcal{M}_{1,PS}|^2}$

Master formula

$$d\sigma_{\rm matched}^{\infty} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,PS}|^2 \prod_{i=0}^{n} F_{H,i}\ d\Phi_n$$

$d\Phi_n$ is the exact phase space for n final-state particles

Any approximation is confined into matrix elements

$\rightsquigarrow$ F_{SV} and F_H are built-up with NLO "ingredients"

- F_{SV} and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of leading-logs
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\mathrm{NLO}}$
- resummation of higher orders LL (PS) contributions is preserved
- the cross section is still fully differential in the momenta of the final state particles (F's correction factors are calculated and applied on an event-by-event basis)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV\ |\ H,i} \otimes$ [leading-logs]

G. Montagna et al., **PLB** 385 (1996)

- the theoretical error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared, singly collinear terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m_e^2} \simeq 5\times 10^{-4} \text{ at 1 GeV}$$

Summary of QED (photonic) radiative corrections

Loosely and schematically, the corrections to the LO cross section can be arranged as:

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\dots$

Blue: Leading-Log PS, SF

$L \simeq 14$ at 1 GeV

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h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Red: matched PS, SF + NLO

$L \simeq 14$ at 1 GeV

Summary of QED (photonic) radiative corrections

Loosely and schematically, the corrections to the LO cross section can be arranged as:

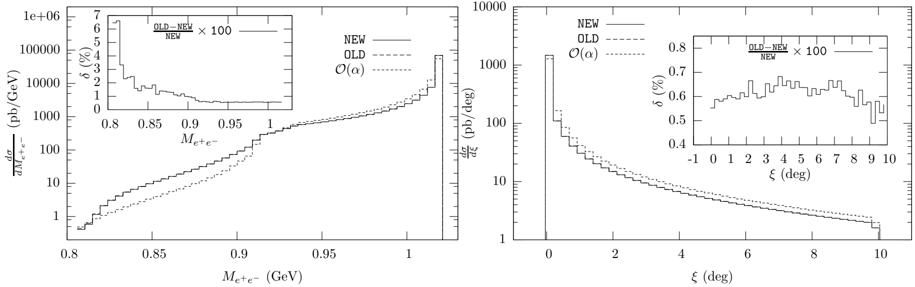
LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	$\cdots$	$\cdots$

Typically at flavour factories (on integrated Bhabha σ)

$L \simeq 14$ at 1 GeV

from Balossini et al., Nucl. Phys. **B758** (2006) 227

↦ $M_{e^+e^-}$ invariant mass and acollinearity distributions, at KLOE



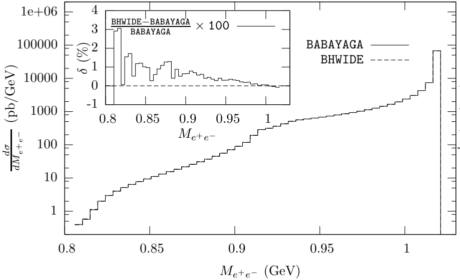
`OLD` $\rightarrow$ pure PS `NEW` $\rightarrow$ matched PS with NLO $\mathcal{O}(\alpha) \rightarrow$ exact NLO

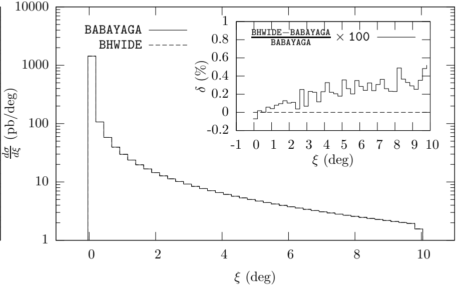
⇝ both exact QED NLO and higher orders resummation are essential for high-precision simulations

Estimating the theoretical accuracy

from S. Actis et al. Eur. Phys. J. C **66** (2010) 585

- It is extremely important to compare independent calculations, implementations, codes, in order to
  - ↦ asses the technical precision, spot bugs (with the same "theory ingredients")
  - ↦ estimate theoretical errors when including partial/incomplete higher-order corrections
- E.g. comparison `BabaYaga@NLO` vs. `BHWIDE` (Jadach, Płaczek, Ward) at KLOE





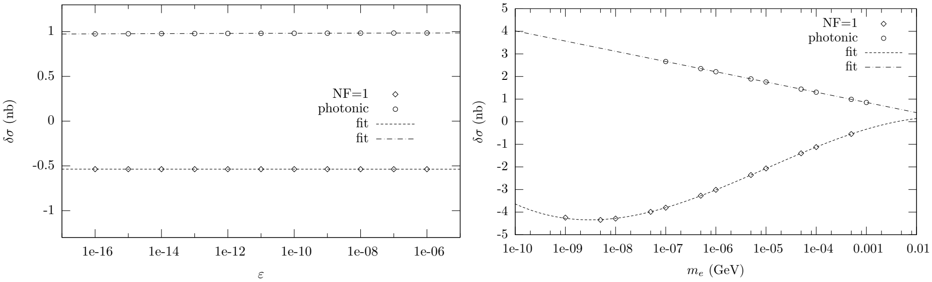
Estimating the theoretical accuracy by measuring missing NNLO

from Balossini et al., Nucl. Phys. **B758** (2006) 227

Using realistic cuts for luminosity at KLOE

The BabaYaga@NLO master formula can be expanded up to NNLO and consistently compared to exact results

- e.g., vs the class of exact photonic soft+virtual QED NNLO corrections, function of the soft photon cut-off ε and m_e



★ differences are infrared safe and $\delta\sigma(\text{photonic})/\sigma^{\text{LO}} \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\text{SV}}^{\text{NNLO}}(\text{photonic}) - \sigma_{\text{SV}}^{\text{NNLO}}(\texttt{BabaYaga@NLO}) < 0.02\% \times \sigma^{\text{LO}}$$

from P. Banerjee *et al.*, Bhabha scattering at NNLO with next-to-soft stabilisation

Phys. Lett. B **820** (2021), 136547

- Recently, NNLO RCs to Bhabha were included in **McMule**
- A (quick) tuned comparison to **BabaYaga@NLO** expanded up to $\mathcal{O}(\alpha^2)$ was performed (by including only photonic corrections)

	σ/μb McMule	σ/μb BABAYAGA	$\delta K^{(i)}/\%$ McMule
$\sigma^{(0)}$	6.8557	6.8557	
$\sigma^{(1)}$	-0.7957	-0.7957	-11.606
$\sigma^{(2)}$	0.0312	0.0267	0.515
σ_2	6.0912	6.0868	

Table 1: Comparison of our exact fixed-order calculation for the total cross section with the full LO and NLO as well as the approximate NNLO results from BABAYAGA [50]. All digits given are significant compared to the error of the numerical integration.

$$\rightsquigarrow \quad \sigma^{\text{NNLO}}_{\texttt{McMule}} - \sigma^{\text{NNLO}}_{\texttt{BabaYaga@NLO}} = 0.06\% \times \sigma^{\text{LO}}$$

including all NNLO (photonic) contributions

Error budget for Bhabha luminometry at flavour factories

main conclusion of the Luminosity section of S. Actis et al., Eur. Phys. J. C **66** (2010) 585

- Putting the sources of uncertainties (in large-angle Bhabha) all together:

Source of error (%)	$\Phi-$factories	$\sqrt{s}=3.5$ GeV	$B-$factories
$\vert\delta^{\rm err}_{\rm VP}\vert$ [Jegerlehner]	0.00	0.01	0.03
$\vert\delta^{\rm err}_{\rm VP}\vert$ [HMNT]	0.02	0.01	0.02
$\vert\delta^{\rm err}_{{\rm SV},\alpha^2}\vert$	0.02	0.02	0.02
$\vert\delta^{\rm err}_{{\rm HH},\alpha^2}\vert$	0.00	0.00	0.00
$\vert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\vert$	0.05	0.05	0.05
$\vert\delta^{\rm err}_{\rm pairs}\vert$	0.03	0.016	0.03
$\vert\delta^{\rm err}_{\rm total}\vert$ linearly	0.12	0.1	0.13
$\vert\delta^{\rm err}_{\rm total}\vert$ in quadrature	0.07	0.06	0.06

★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements at the 0.1% level.
It is comparable with that achieved for small–angle Bhabha luminosity monitoring at LEP

- For the experiments on top of and closely around the narrow resonances (J/ψ, Υ,...), the accuracy quickly deteriorates, because of the differences between the predictions of independent $\Delta\alpha^{(5)}_{\rm had}(q^2)$ parameterizations and/or their intrinsic error

Wish-list for the future

- Since a long time in the wish-list: **QED PS matching with exact NNLO**
  - $\rightsquigarrow$ NNLO calculations are now numerically more reliable, more independent cross-checks are available
  - $\rightsquigarrow$ Can be boosted by parallel efforts and developments on MUonE side (`Mesmer` code)
- Include full mass-dependent effects in RCs
  (needless for Bhabha and $e^+e^- \to \gamma\gamma$, but perhaps needed for $e^+e^- \to \mu^+\mu^-$ at low energies, see Fedor's talk)
- Make simulations more reliable in proximity of narrow resonances

Conclusions

- `BabaYaga@NLO` matches a QED PS with NLO matrix elements
- It was developed to provide high-precision cross section predictions for luminometry measurements at flavour factories
- The natural next-step is to include NNLO matrix elements to improve accuracy
- Efforts on the MUonE side are expected to produce a positive boost towards inclusion of exact NNLO corrections also in `BabaYaga@NLO`, i.e. `BabaYaga@NLO` $\rightarrow$ `BabaYaga@NNLO`
- For easier maintenance, we plan to make the code public on a **github**/**gitlab** repository

SPARES

Collinear log

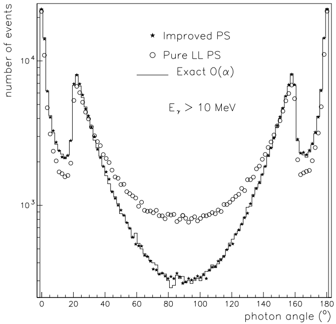
CMCC, Phys. Lett. B **520** (2001), 16

- The collinear log L comes from the integration over angular variables of the photons
- In the spirit of exclusive generation, it can be improved to include interference effects among charged legs (at least in the soft-limit)

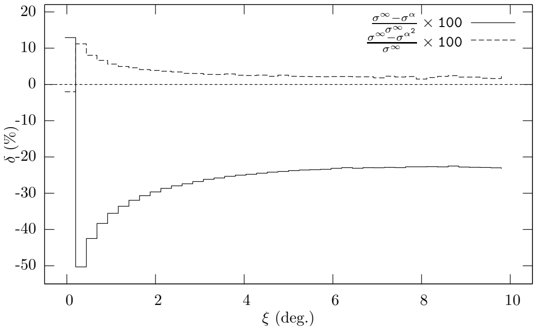
$$L = \int d\Omega_k I(k) \qquad\qquad I(k) = \sum_{i,j} \eta_i \eta_j \frac{p_i \cdot p_j}{(p_i \cdot k)\,(p_j \cdot k)} E_\gamma^2$$

- Generating photons' angular variables according to $I(k)$ improves exclusive description of the event, e.g. w.r.t. exact NLO

$$L^{\text{Bhabha}} \propto \ln \frac{st}{u\, m_e^2} - 1$$
$$\simeq 14 \text{ at } 1 \text{ GeV}$$



- with a complete 2-loop generator at hand, (leading-log) resummation beyond α^2 can be neglected?



Impact of α^2 (solid line) and resummation of higher order ($\geq \alpha^3$) (dashed line) corrections on the acollinearity distribution

- Resummation beyond α^2 still important!