



Experimental asymmetry in CMD3 2π data vs prediction

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CMD3 at VEPP-2000 e+e- collider



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$e+e- \rightarrow \pi+\pi-by CMD3$

Very simple topology (just 2 track back to back), but the most challenging channel due to high precision requirement. Original plans was to reach systematic ~0.35-0.5%

Crucial pieces of analysis:

- × $e/\mu/\pi$ separation
- x radiative corrections
- × precise fiducial volume

events separation either by momentum or by energy deposition

Momentums works better at low energy 2E< 0.8 GeV



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Fiducial volume cross check

All events at ρ -peak : $E_{\text{beam}} = 350 - 410 \text{ MeV}$ $1 < \Theta < \pi$ -1 rad - good detector acceptance $\times 10^3$ 800 $N_{total} = 6.46 \times 10^7$ 700 600F sim mixed 500 data 400 : 47.4% N_{total} e+e-300 2π: 48.3% 200 100 :4% cosmic 0.0 0.8 2.2 $\theta_{average}, rad$

Sim mixed:

Generators spectra + all efficiencies/smearing extracted from data and full simulation $N_{\pi\pi/ee/\mu\mu,etc}$ - from event separation





Asymmetry



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Asymmetry $2\pi/e+e-/2\mu$

Asymmetry relative to generator prediction



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Scalar production

Could it be: $e+e- \rightarrow \rho \rightarrow \sigma\gamma$ or $a_1^{\pm}\pi^{\pm}$?



With help of FASTERD generator

O. Shekhovtsova, G. Venanzoni, G. Panccheri, Comp.Phys.C. 180 (2009) 1206-1218

Mixed in $\rho \rightarrow \sigma \gamma$ instead of $\phi \rightarrow (f_0 + \sigma) \gamma$ in non structure model with some rough σ production parameters

 $|\delta A| \sim 2 \times 10^{-5}$ effect only in far tails

Br $(\rho \rightarrow \sigma\gamma) \sim 1 \times 10^{-4}$ [x2 Br $(\rho \rightarrow \pi 0 \pi 0 \gamma)$] Interference with sQED e+e- $\rightarrow \pi^{+}\pi^{-}\gamma$: => ~ 1x10⁻³ x Collinearity selection cuts 1x10⁻² Total rate ~ 10⁻⁵ too small to affect something

 $\rho \rightarrow a_1 \pm \pi \pm$ effect should be same or less: Phys.Rev.D 76 (2007) 033001

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Asymmetry with $M\pi 2$

Asymmetry vs $M_{\pi\pi}^2$

Sample of 2π can be selected by energy deposition as MIP with E_{LXe}^{+-} (100 MeV (with some admixture of 2μ)

Comparison with full mixed simulation

Main difference comes from $M_{\pi\pi}^2/s \sim 1$: correspond to virtual/soft radiative corrections



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sQED assumptions

The radiative correction calculations is commonly done in the sQED approach, It's mean that the calculations are performed without form factor, then final Amplitude is scaled by $F(q^2)$



Proper way will be to put $F(q^2)$ to each vertex Thanks to Roman Lee, this calculations was done with above sQED STRONG2020 Virtual Workshop

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Virtual + soft corrections

Point Like formula consistent with A.B. Arbuzov et al., Mod.Ph.Lett.A 35 (2020) 25, 2050210, inconsistent with A.Hoefer et al. Eur.Ph.J.C 24 (2002) 51-69 $\delta_{odd}^{Virt}(\lambda) + \delta_{odd}^{Soft}(\lambda,\Delta) = \frac{\alpha}{\pi} \Big\{ 4\ln\left(\frac{1+\beta c}{1-\beta c}\right) \ln\frac{\sqrt{s}}{2\omega_0} + \ln\left(\frac{1+\beta}{1-\beta}\right) \ln\left(\frac{1+\beta c}{1-\beta c}\right) + \frac{c}{\beta(1-c^2)}\ln^2(1-\beta^2) - \frac{2c(1+\beta^2)}{(1-c^2)\beta^2} \Big[\frac{\pi^2}{12} + \frac{1}{4}\ln^2\left(\frac{1+\beta}{1-\beta}\right) + Li_2\left(\frac{1-\beta}{1+\beta}\right) \Big] + Li_2\left(\frac{1-\beta}{1-\beta}\right) \Big\}$ $+\left\{\frac{(1-\beta c)^2}{(1-c^2)\beta^2}\ln(1-\beta c)\ln\left(\frac{1-\beta c}{1-\beta^2}\right)+2Li_2\left(\frac{(1+c)\beta}{1+\beta}\right)+2Li_2\left(\frac{(1+c)\beta}{1+\beta c}\right)+\frac{1-2\beta c+\beta^2}{(1-c^2)\beta^2}\left[\frac{\pi^2}{12}+Li_2\left(\frac{1-2\beta c+\beta^2}{1-\beta^2}\right)\right]\right\}-\left\{c\rightarrow -c\right\}\right\}$

Double FF in box diagram addition:

 $|c \pi a (2 s^2 (-1 + b) (1 + 3 b - (-4 + c^2) b^2 + c^2 b^3)|$ 8 $\{-1 + c^2\} \beta^3 \Delta 2 \Delta 2 - s (1 + \beta)^2 \{-1 + c^2 \beta^2\} \{\Delta 2 + \Delta 2\}\}$ $(12(-1+c^2)\beta^2(-1+c\beta)(1+c\beta)(s-\Delta 2)(s-\Delta 2)) - \frac{1}{\pi\beta-c^2\pi\beta}2cs\alpha$ $Int\left[-\left[\left(2\left(ArcTan\left[\left\{s\left(-1+x\right)+\Delta 2-\Lambda 2\right\}\right)\left/\left(\sqrt{\left(-s^{2}\left(-1+x\right)^{2}-\left(\Delta 2-\Lambda 2\right)^{2}-2s\left(-1+x\right)\left(\Delta 2+\Lambda 2\right)\right)}\right)\right]-1\right)\right]\right]\right]$ $\frac{4 \cos \alpha (1+\beta)^2 (2 - \beta 2 - \beta 2) \log [1-\beta^2]}{2 (-1+\beta^2) \beta^2 (5-\beta^2) (5-\beta^2)}$ ArcTan[(s (-1+2 (1-x)+x)+A2-A2) / $\left(\sqrt{\left(-s^{2}(-1+x)^{2}-(\Delta 2-\Delta 2)^{2}-2s(-1+x)(\Delta 2+\Delta 2)\right)}\right)\right)$ $\left[\sqrt{-\mathbf{s}^{2}\left(-\mathbf{1}+\mathbf{x}\right)^{2}-\left(\Delta 2-\Delta 2\right)^{2}-2\mathbf{s}\left(-\mathbf{1}+\mathbf{x}\right)\left(\Delta 2+\Delta 2\right)}\right]\right), \ \left\{\mathbf{x}, \mathbf{0}, \mathbf{1}\right\}\right] + \frac{1}{\left[-\mathbf{1}+\mathbf{c}^{2}\right]\pi\beta}$ $(1 - 2 c \beta - \beta^2 + 2 c^2 \beta^2)$ $c s \alpha (1 + \beta^2) Int [(2 ArcTan] ((1 - x) \sqrt{(-(-s x + \Lambda^2)^2 - s (-1 + \beta^2)} (x (s (-1 + x) + \Lambda^2 - \Lambda^2) + \Lambda^2)))]/$ (s(-1+x)x+2xA2+A2-xA2)1)/(2 5 - 12 - 12) Log[1-cB] $\left(\int \left[-(-sx + h2)^2 - s(-1 + \beta^2)(s(-1 + x)x + xh2 + h2 - xh2)\right], (x, 0, 1)\right]$ Log [1-82]) / (+ (-1+ CB) (+ (1-2CB-B2+2C2B2) + (-1+2CB-B2) (A2+A2)) $(2(-1+c^2)\pi\beta^2(s-\Delta 2)(s-\Delta 2)) +$ Int[$(2 | ArcTan[(x (s + \triangle 2 - \triangle 2)) / (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + \beta^2) - (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + \beta^2) - (-1 + \beta^2) - (-1 + \beta^2)}) - (\sqrt{(-s^2 (-1 + \beta^2) - (-1 + \beta^2)}))$ $(\mathbf{s} \alpha (\mathbf{1} + \mathbf{2} \mathbf{c} \beta - \beta^2 + \mathbf{2} \mathbf{c}^2 \beta^2)$ $x (s^2 x + x (\Lambda 2 - \Lambda 2)^2 + s (-2 x \Lambda 2 - 2 x \Lambda 2 - 4 \sqrt{\Lambda 2 \Lambda 2} + 4 x \sqrt{\Lambda 2 \Lambda 2}))))$ (25-12-12) Log[1+c B] ArcTan[$(x (s - \Delta 2 + \Delta 2)) / (\sqrt{(-s^2 (-1 + x)^2 (-1 + \beta^2) - x (s^2 x + x (\Delta 2 - \Delta 2)^2 + (\Delta 2 - \Delta 2)^2 +$ Log [1 - 82]) / $s(-2xA2-2xA2-4\sqrt{A2A2}+4x\sqrt{A2A2}))))))/$ $(2(-1+c^2)\pi\beta^2(s-\Delta 2)(s-\Delta 2)) +$ $\left(\left(\frac{1}{4} \operatorname{s} \left(-1+x\right) \left(-1+2 \operatorname{c} \beta-\beta^{2}\right)-\frac{1}{4} \operatorname{s} \left(-1+\beta^{2}\right)+\frac{1}{4} \operatorname{s} x \left(-1+\beta^{2}\right)+x \sqrt{\delta 2 \wedge 2}\right)\right)$ $\frac{\mathbf{c}\,\mathbf{s}\,\alpha\,\left(\mathbf{1}+\beta\right)^2\,\left(\mathbf{2}\,\mathbf{s}-\Delta\mathbf{2}-\Delta\mathbf{2}\right)\,\mathbf{Log}\left[\mathbf{1}-\beta^2\right]^2}{\mathbf{4}\,\left(-\mathbf{1}+\mathbf{c}^2\right)\,\pi\beta^2\,\left(\mathbf{s}-\Delta\mathbf{2}\right)\,\left(\mathbf{s}-\Delta\mathbf{2}\right)}+$ 11-s2 (-1+x)2 (-1+82) $x (s^2 x + x (\Delta 2 - \Lambda 2)^2 + s (-2 x \Delta 2 - 2 x \Lambda 2 - 4 \sqrt{\Delta 2 \Lambda 2} + 4 x \sqrt{\Delta 2 \Lambda 2}))), (x, 0, 1))/$ $(1 - 2c\beta - \beta^2 + 2c^2\beta^2)$ $(4(-1+c^2)\pi\beta^2) - (s\alpha(1+c\beta)(s(1+2c\beta-\beta^2+2c^2\beta^2) - (1+2c\beta+\beta^2)(\delta^2+\delta^2)))$ (2 s - A2 - A2) Log [1 - B2] Int $\left[\left(2 \left[ArcTan \right] \left(x \left(s + \delta 2 - \delta 2 \right) \right) \right) / \left[\int \left[-s^2 \left(-1 + x \right)^2 \left(-1 + \beta^2 \right) - 1 + \beta^2 \right] \right] \right]$ $\frac{(1 - 1 - 1 - 1)(1 - 1 - 1)}{\log[1 - 2 c \beta + \beta^2]} / (2 (-1 + c^2) \pi \beta^2 (s - \Delta 2) (s - \Delta 2))$ $x (s^{2}x + x (h^{2} - h^{2})^{2} + s (-2xh^{2} - 2xh^{2} - 4\sqrt{h^{2}h^{2}} + 4x\sqrt{h^{2}h^{2}}))))] +$ ArcTan $\left(x(s-\Delta 2 + \Delta 2)\right) / \left(\sqrt{(-s^2(-1+x)^2(-1+\beta^2) - x(s^2x+x(\Delta 2 - \Delta 2)^2 + (-1+\beta^2) - x(s^2x+x(\Delta 2 - \Delta 2))^2 + (-1+\beta^2) + (-1+\beta^2) + x(s^2x+x(\Delta 2 - \Delta 2))^2 + (-1+\beta^2) + x(s^2x+x(\Delta 2 - \Delta 2))^2 + (-1+\beta^2) + x(s^2x+x(\alpha 2 - \Delta 2))^2 + (-1+\beta^2) + x(s^2x+x(\alpha 2 - \Delta 2))^2 + x(s^2x+x(\alpha 2 - \Delta$ sα (1-2cβ-β2+2c2β2) (2s-b2-b2) s (-2x A2-2x A2-4 VA2 A2 +4x VA2 A2))))))/ Log[1-2c8+82]2)/ $(4(-1+c^2)\pi\beta^2(s-\Delta 2)(s-\Delta 2))$ $\left(\left(-\frac{1}{4}s\left(-1+\beta^{2}\right)+\frac{1}{4}sx\left(-1+\beta^{2}\right)-\frac{1}{4}s\left(-1+x\right)\left(1+2c\beta+\beta^{2}\right)+x\sqrt{\delta^{2}\Lambda^{2}}\right)\right)$ $(s \alpha (1 + 2 c \beta - \beta^2 + 2 c^2 \beta^2) (2 s - \Delta 2 - \Delta 2)$ [-s2 (-1+x)2 (-1+62) - $\log[1-\beta^2] \log[1+2c\beta+\beta^2] /$ $x (s^2 x + x (\Delta 2 - \Lambda 2)^2 + s (-2 x \Delta 2 - 2 x \Lambda 2 - 4 \sqrt{\Delta 2 \Lambda 2} + 4 x \sqrt{\Delta 2 \Lambda 2}))), (x, 0, 1)])/$ $(2(-1+c^2)\pi\beta^2(s-\Delta 2)(s-\Delta 2))$ $(4(-1+c^2)\pi\beta^2) + (2\alpha(s(-1+2c\beta+\beta^2-2c^2\beta^2)+(1-2c\beta+\beta^2)\beta^2))$ (sa (1+2cB-B2+2c2B2) (2s-b2-b2) Int [- ((s (s (-1+x) + $\frac{1}{4}$ s (-2+x) (-1+2 c $\beta - \beta^2$) - $\frac{1}{4}$ s (2-3 x) (-1+ β^2)) Log[1+2c \$+ \$2]2)/ $\left(x\left(\frac{1}{2}s\left(-1+2c\beta-\beta^{2}\right)-\frac{1}{2}s\left(-1+x\right)\left(-1+\beta^{2}\right)\right)Log\left[-(-1+x)\left(s-\delta^{2}\right)\right]+$ (4 (-1+c²) πβ² (s - Δ2) (s - Δ2)) $= \frac{1}{4} \operatorname{sx} \left(-1 + \beta^2 \right) - \operatorname{x} \left(\operatorname{s} + \frac{1}{4} \operatorname{s} \left(-1 + 2 \operatorname{c} \beta - \beta^2 \right) \right) \operatorname{Log} \left[\frac{1}{4} \operatorname{sx}^2 \left(-1 + \beta^2 \right) + (-1 + \operatorname{x}) \Delta 2 \right] +$ (-1+c2) #2 (s-42) (s $\left(s(-1+x)+\frac{1}{4}sx^{2}(-1+\beta^{2})\right)\log\left[-\delta^{2}+x\left(\frac{1}{4}s(-1+2c\beta-\beta^{2})+\frac{1}{4}s(-1+\beta^{2})+\delta^{2}\right)\right]\right)$ $\left(4x\left(\frac{1}{4}s\left(-1+2c\beta-\beta^{2}\right)-\frac{1}{4}s\left(-1+x\right)\left(-1+\beta^{2}\right)\right)\left(s-sx-\frac{1}{4}sx^{2}\left(-1+\beta^{2}\right)\right)$ $\left(s - \frac{1}{4}sx(-1+\beta^2) - x(s + \frac{1}{4}s(-1+2c\beta-\beta^2))\right)$, (x, 0, 1) $((-1+c^2) \pi \beta^2 (s-\delta 2)) + (2 \alpha (s (1+2c\beta-\beta^2+2c^2\beta^2) - (1+2c\beta+\beta^2) \delta 2))$ Int[-((s (s (-1+x) - $\frac{1}{4}$ s (2-3x) (-1+ β^2) - $\frac{1}{4}$ s (-2+x) (1+2c β + β^2)) $\left(x\left(-\frac{1}{4}s(-1+x)(-1+\beta^{2})-\frac{1}{4}s(1+2c\beta+\beta^{2})\right)Log\left[-(-1+x)(s-\beta^{2})\right]+$ $\left(\mathbf{s} - \frac{1}{4}\mathbf{s} \times \left(-1 + \beta^2\right) - \times \left(\mathbf{s} - \frac{1}{4}\mathbf{s} \left(1 + 2\mathbf{c}\beta + \beta^2\right)\right)\right) \log\left[\frac{1}{4}\mathbf{s} \times^2 \left(-1 + \beta^2\right) + \left(-1 + \times\right)\beta^2\right] + \left(-1 + \infty\right)\beta^2\right] + \left(-1 + \infty\right)\beta^2$ $c \alpha (1 + \beta^2) \log \left[\frac{1}{4} (1 - \beta^2)\right] \log \left[-1 + \frac{2}{1 + \sqrt{\beta^2}}\right]$ $\left(\mathbf{s}\left(-1+\mathbf{x}\right)+\frac{1}{4}\mathbf{s}\,\mathbf{x}^{2}\left(-1+\beta^{2}\right)\right)\,\log\left[-\delta 2+\mathbf{x}\left(\frac{1}{4}\,\mathbf{s}\left(-1+\beta^{2}\right)-\frac{1}{4}\,\mathbf{s}\left(1+2\,\mathbf{c}\,\beta+\beta^{2}\right)+\delta 2\right)\right]\right)\right)/\delta \mathbf{x}^{2}$ (-1+c2) = 02 $\left(4x\left(s-sx-\frac{1}{4}sx^{2}\left(-1+\beta^{2}\right)\right)\left(-\frac{1}{4}s\left(-1+x\right)\left(-1+\beta^{2}\right)-\frac{1}{4}s\left(1+2c\beta+\beta^{2}\right)\right)\right)$ $(s - \frac{1}{2} s \times (-1 + \beta^2) - x (s - \frac{1}{2} s (1 + 2 c \beta + \beta^2)))), (x, 0, 1)])/$ a (1+ C A) $\left(\left(-1+\mathbf{c}^{2}\right)\pi\beta^{2}\left(\mathbf{s}-\Delta2\right)\right)+\left(2\alpha\left(\mathbf{s}\left(-1+2\mathbf{c}\beta+\beta^{2}-2\mathbf{c}^{2}\beta^{2}\right)+\left(1-2\mathbf{c}\beta+\beta^{2}\right)\Delta2\right)$ $Int\left[-\left(\left(s\left(s\left(-1+x\right)+\frac{1}{4}s\left(-2+x\right)\left(-1+2c\beta-\beta^{2}\right)-\frac{1}{4}s\left(2-3x\right)\left(-1+\beta^{2}\right)\right)\right)\right]$ $\left(x\left(\frac{1}{4}s\left(-1+2c\beta-\beta^{2}\right)-\frac{1}{4}s\left(-1+x\right)\left(-1+\beta^{2}\right)\right)Log\left[-\left(-1+x\right)\left(s-\Lambda^{2}\right)\right]+$ $\left(s - \frac{1}{4}sx(-1 + \beta^{2}) - x(s + \frac{1}{4}s(-1 + 2c\beta - \beta^{2}))\right) \log\left[\frac{1}{4}sx^{2}(-1 + \beta^{2}) + (-1 + x)A^{2}\right] +$ $\left(s(-1+x) + \frac{1}{4} s x^2 (-1+\beta^2) \right) \log \left[-\Lambda 2 + x \left(\frac{1}{4} s (-1+2c\beta-\beta^2) + \frac{1}{4} s (-1+\beta^2) + \Lambda 2 \right) \right] \right)$ $\left(4x\left(\frac{1}{4}s\left(-1+2c\beta-\beta^{2}\right)-\frac{1}{4}s\left(-1+x\right)\left(-1+\beta^{2}\right)\right)\left(s-sx-\frac{1}{4}sx^{2}\left(-1+\beta^{2}\right)\right)$ $\left(s - \frac{1}{4}sx(-1+\beta^2) - x(s + \frac{1}{4}s(-1+2c\beta-\beta^2)))\right), (x, 0, 1)\right)/$ $(\{-1 + c^2\} \pi \beta^2 (1 + c \beta) (s - \Delta 2) (s - \Delta 2)) +$ $((-1 + c^2) \pi \beta^2 (s - \Lambda 2)) + (2 \alpha (s (1 + 2 c \beta - \beta^2 + 2 c^2 \beta^2) - (1 + 2 c \beta + \beta^2) \Lambda 2)$ Int $\left[-\left(\left|s\left(s\left(-1+x\right)-\frac{1}{4}s\left(2-3x\right)\left(-1+\beta^{2}\right)-\frac{1}{4}s\left(-2+x\right)\left(1+2c\beta+\beta^{2}\right)\right)\right]\right]$ $\left\{x\left(-\frac{1}{4}s\left(-1+x\right)\left(-1+\beta^{2}\right)-\frac{1}{4}s\left(1+2c\beta+\beta^{2}\right)\right)Log\left[-\left(-1+x\right)\left(s-\Lambda^{2}\right)\right]+\right\}$ $\left(\mathbf{s} - \frac{1}{4} \mathbf{s} \mathbf{x} \left(-1 + \beta^2\right) - \mathbf{x} \left(\mathbf{s} - \frac{1}{4} \mathbf{s} \left(1 + 2 \mathbf{c} \beta + \beta^2\right)\right)\right) \log\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right] + \left(-1 + \mathbf{x}\right) \wedge 2\left[\frac{1}{4} \mathbf{s} \mathbf{x}^2 \left(-1 + \beta^2\right) + (-1 + \mathbf{x}) \wedge 2\right]$ $\log[1 - \sqrt{1 + \frac{8(-1+\beta^2)}{\lambda^2}}]$ $\left[s(-1+x)+\frac{1}{4}sx^{2}(-1+\beta^{2})\right] \log\left[-\Lambda^{2}+x\left(\frac{1}{4}s(-1+\beta^{2})-\frac{1}{4}s(1+2c\beta+\beta^{2})+\Lambda^{2}\right)\right]\right)/$ $\left(4 \times \left(\mathbf{s} - \mathbf{s} \times -\frac{1}{4} \mathbf{s} \times^2 \left(-1 + \beta^2\right)\right) \left(-\frac{1}{4} \mathbf{s} \left(-1 + \mathbf{x}\right) \left(-1 + \beta^2\right) - \frac{1}{4} \mathbf{s} \left(1 + 2 \mathbf{c} \beta + \beta^2\right)\right)\right)$ $1 = \frac{-\frac{1}{2} \pm \left(-\frac{1}{2} \pm 2 \pm (1 + 2 \pm (1 + 2 + 1)^2) + \frac{1}{2} \pm \left(-\frac{1}{2} \pm (1 + 2 \pm (1 + 2 \pm 1)^2) + \frac{1}{2} \pm \left(-\frac{1}{2} \pm (1 + 2 \pm 1)^2\right) + \frac{1}{2} \pm \left(-\frac{1}{2} \pm (1 + 2 \pm 1)^2\right)}$ $\left(s - \frac{1}{4}sx(-1 + \beta^2) - x(s - \frac{1}{4}s(1 + 2c\beta + \beta^2)))\right), (x, 0, 1)\right)/$ $\left(\left(-1+c^{2}\right)\pi\beta^{2}\left(s-\Lambda^{2}\right)\right)-\frac{ics\alpha\left(1+\beta^{2}\right)\left(2s-\Lambda^{2}-\Lambda^{2}\right)\log[2]}{\left(-1+c^{2}\right)\beta^{2}\left(s-\Lambda^{2}\right)\left(s-\Lambda^{2}\right)}+$ $\frac{\cos \alpha (1+\beta^2) (2s-\beta 2-\beta 2) \log (2)^2}{(-1+c^2) \cos^2 (n-\beta 2) (n-\beta 2)} +$ $(2(-1+c^2)\pi\beta^2(-1+c\beta)(s-\delta^2))$ $\frac{4 \, \mathrm{s} \, \alpha \, (1 - 2 \, \mathrm{c} \, \beta - \beta^2 + 2 \, \mathrm{c}^2 \, \beta^2) \, (2 \, \mathrm{s} - \beta 2 - \beta 2) \, \log(1 - \mathrm{c} \, \beta)}{2 \, (-1 + \mathrm{c}^2) \, \beta^2 \, (\mathrm{s} - \beta 2) \, (\mathrm{s} - \beta 2)} +$ $\frac{s\alpha \left(1 - 2c\beta - \beta^2 + 2c^2\beta^2\right) \left(2s - \beta^2 - \beta^2\right) \log \left(1 - c\beta\right)^2}{2\left(-1 + c^2\right) = \beta^2 \left(s - \beta^2\right) \left(s - \beta^2\right) \left(s - \beta^2\right)}$

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 $\frac{1 \operatorname{s} \alpha \left(1 + 2 \operatorname{c} \beta - \beta^{2} + 2 \operatorname{c}^{2} \beta^{2}\right) \left(2 \operatorname{s} - \beta 2 - \beta 2 - \beta 2}{2 \left(-1 + \operatorname{c}^{2}\right) \beta^{2} \left(\operatorname{s} - \beta 2\right) \left(\operatorname{s} - \beta 2\right)} \left(\operatorname{s} - \beta 2\right)}$ $\frac{s \alpha \left(1 + 2 c \beta - \beta^2 + 2 c^2 \beta^2\right) \left(2 s - \beta^2 - \beta^2\right) \log (1 + c \beta)^2}{2 \left(-1 + c^2\right) \beta^2 \left(s - \beta^2\right) \left(s - \beta^2\right) \left(s - \beta^2\right)} +$ $\frac{c \, s \, \alpha \, (1 + \beta^2) \, (2 \, s - \Delta^2 - \Delta^2) \, \log[2] \, \log[1 - \beta^2]}{(-1 + c^2) = \beta^2 \, (s - \Delta^2) \, (s - \Delta^2)} =$ $i c s \alpha (1 + \beta^2) (2 s - \beta 2 - \beta 2) Log \left[\frac{1}{2} (1 - \sqrt{\beta^2})\right]$ $\frac{2 \operatorname{cs} \alpha \left(1 + \beta^{2}\right) \left(2 \operatorname{s} - \beta 2 - \beta 2\right) \operatorname{Log}[2] \operatorname{Log}\left[\frac{1}{2} \left(1 - \sqrt{\beta^{2}}\right)\right]}{\left(-1 + \operatorname{c}^{2}\right) \pi \beta^{2} \left(\operatorname{s} - \beta 2\right) \left(\operatorname{s} - \beta 2\right)}$ $\frac{c s \alpha (1 + \beta^2) (2 s - \Delta 2 - \Delta 2) \log[1 - \beta^2] \log[\frac{1}{2} (1 - \sqrt{\beta^2})]}{(-1 + c^2) \pi \beta^2 (s - \Delta 2) (s - \Delta 2)}$ $\frac{\mathbf{c} \mathbf{s} \alpha \left(\mathbf{1} + \beta^2\right) \left(\mathbf{2} \mathbf{s} - \Delta \mathbf{2} - \Delta \mathbf{2}\right) \mathbf{Log} \left[\frac{1}{2} \left(\mathbf{1} - \sqrt{\beta^2}\right)\right]^2}{\left(-\mathbf{1} + \mathbf{c}^2\right) \pi \beta^2 \left(\mathbf{s} - \Delta \mathbf{2}\right) \left(\mathbf{s} - \Delta \mathbf{2}\right)},$ $(s^2 (1 - 2c\beta - \beta^2 + 2c^2\beta^2) + (1 - 2c\beta + \beta^2) \Delta 2 A 2 - s (-1 + c\beta)^2 (\Delta 2 + A 2))$ $\log \left[1 - \frac{-\frac{1}{4}s\left(-1 + 2c\beta - \beta^2\right) + \frac{1}{4}s\left(-1 + \beta^2\right)}{-\frac{1}{4}s\left(-1 + 2c\beta - \beta^2\right) - \frac{1}{4}s\left(-1 + \beta^2\right)}\right]^2\right] \Big/$ $((-1+c^2) \pi \beta^2 (-1+c \beta) (s-\Delta 2) (s-\Delta 2)) +$ $\alpha (-1 + c\beta) \left(s^{2} \left(1 + 2 c\beta - \beta^{2} + 2 c^{2} \beta^{2} \right) + \left(1 + 2 c\beta + \beta^{2} \right) \Delta 2 \Delta 2 - s \left(1 + c\beta \right)^{2} \left(\Delta 2 + \Delta 2 \right) \right)$ $log \left[1 - \frac{\frac{1}{4} s (-1 + \beta^2) + \frac{1}{4} s (1 + 2 c \beta + \beta^2)}{-\frac{1}{4} s (-1 + \beta^2) + \frac{1}{4} s (1 + 2 c \beta + \beta^2)}\right]^2 \right] / (1 + 2 c \beta + \beta^2)$ α (1 + c β) (s {1 - 2 c $\beta - \beta^2 + 2 c^2 \beta^2$ } + (-1 + 2 c $\beta - \beta^2$) $\Delta 2$ } $-\frac{\frac{1}{2} \frac{1}{8} \left(-\frac{1}{2} \frac{2}{2} \frac{1}{6} \frac{(-\frac{1}{2})^2}{(-\frac{1}{2})^2} + \frac{1}{8} \frac{1}{8} \left(-\frac{1}{2} \frac{(-\frac{1}{2})^2}{(-\frac{1}{2})^2} + \sqrt{\frac{1}{2} + \frac{1}{8} \frac{(-\frac{1}{2})^2}{(-\frac{1}{2})^2} + \frac{1}{8} \frac{(-\frac{1$ $x (1 + c\beta) (s (1 - 2c\beta - \beta^2 + 2c^2\beta^2) + (-1 + 2c\beta - \beta^2)\Delta 2)$

1+ 1+ 1+ 1/2 -bs (-2+2 c 0+0)+bs (-2+0) + 12 + s (-2+0) $(4(-1+c^2)\pi\beta^2(-1+c\beta)(s-\beta^2))$ $x (1 + c\beta) (s (1 - 2c\beta - \beta^2 + 2c^2\beta^2) + (-1 + 2c\beta - \beta^2) \delta 2)$ $1 = \frac{1 + 5 \left(-2 + 2 \cos((d^2) + 5 + 5) + 1 + d^2 \right)}{1 + 5 + 5 \left(-2 + 2 \cos((d^2) + 5 + 5) + 5 + 5 + 1 + d^2 \right)}$ $Log[1 + \sqrt{1 + \frac{5(-1+\beta^2)}{\lambda^2}}] Log[- \frac{-\frac{1}{2}s\left(-\frac{1}{2}+2e^{\frac{1}{2}+e^{\frac{1}{2}}}\right)+\frac{1}{2}s\left(-\frac{1}{2}+e^{\frac{1}{2}}\right)}{-\frac{1}{2}s\left(-\frac{1}{2}+e^{\frac{1}{2}}\right)+\frac{1}{2}s\left(-\frac{1}{2}+e^{\frac{1}{2}}\right)+\frac{1}{2}s\left(-\frac{1}{2}+e^{\frac{1}{2}}\right)}{\sqrt{2}}+\sqrt{2}+\frac{s\left(-\frac{1}{2}+e^{\frac{1}{2}}\right)}{\sqrt{2}}$ $(2(-1+c^2)\pi\beta^2(-1+c\beta)(s-\Delta 2)) +$ $\left[\alpha (\mathbf{1} + \mathbf{c}\beta) \left(\mathbf{s} \left(\mathbf{1} - \mathbf{2}\mathbf{c}\beta - \beta^2 + \mathbf{2}\mathbf{c}^2\beta^2\right) + \left(-\mathbf{1} + \mathbf{2}\mathbf{c}\beta - \beta^2\right) \Delta \mathbf{2}\right)\right]$ $\log\left[-\frac{-\frac{1}{4}s\left(-1+2c\beta-\beta^2\right)+\frac{1}{4}s\left(-1+\beta^2\right)}{-\frac{1}{4}s\left(-1+2c\beta-\beta^2\right)-\frac{1}{4}s\left(-1+\beta^2\right)}+\sqrt{1+\frac{s\left(-1+\beta^2\right)}{\Delta 2}}\right]$ $\log \left[\frac{-\frac{1}{2} s \left(-1 + 2 c \beta - \beta^{2}\right) + \frac{1}{2} s \left(-1 + \beta^{2}\right)}{-\frac{1}{2} s \left(-1 + 2 c \beta - \beta^{2}\right) - \frac{1}{2} s \left(-1 + \beta^{2}\right)} + \sqrt{1 + \frac{8 \left(-1 + \beta^{2}\right)}{\beta 2}} \right] \right] / \frac{1}{\beta 2}$ $(2(-1+c^2)\pi\beta^2(-1+c\beta)(s-\alpha^2))$ $(-1+c\beta)$ (s $(1+2c\beta-\beta^2+2c^2\beta^2) - (1+2c\beta+\beta^2)\beta^2$) $1 = \frac{\frac{3}{2} x \left(-\frac{1}{2} x \right)^2 + \frac{3}{2} x \left(\frac{1}{2} x \right)^2$ $\log[1 - \sqrt{1 + \frac{5(-1+\beta^2)}{\beta^2}}] \log[\frac{\frac{1}{2} \frac{1}{2} \frac{$ $(2(-1+c^2)\pi\beta^2(1+c\beta)(s-\Delta 2))$ $(-1 + c\beta)$ (s $(1 + 2c\beta - \beta^2 + 2c^2\beta^2) - (1 + 2c\beta + \beta^2)\delta 2$) 1+ 1+ = (+1+1) $\frac{\frac{1}{2} \frac{1}{2} \left(\frac{1}{2} \frac{1}{2}$ $(4(-1+c^2)\pi\beta^2(1+c\beta)(s-\Delta^2))$ $(-1+c\beta)$ (s $(1+2c\beta-\beta^2+2c^2\beta^2) - (1+2c\beta+\beta^2) \Delta 2$) $1 = \frac{\frac{1}{2} s \left((1 + 1 + 1)^2 \right) + \frac{1}{2} s \left((1 + 2 + 1) + 1 \right)^2}{-\frac{1}{2} s \left((-2 + 1)^2 \right) + \frac{1}{2} s \left((2 + 2 + 1)^2 \right)}$ $\frac{\frac{3 \cdot s \left(-3 + t^2\right) + \frac{3}{2} \cdot s \left(3 + 2 \cdot c \left(3 + t^2\right)}{-\frac{3}{2} \cdot s \left(-1 + t^2\right) + \frac{3}{2} \cdot s \left(3 + 2 \cdot c \left(3 + t^2\right)}{+ \frac{3}{2} \cdot s \left(3 + 2 \cdot c \left(3 + t^2\right)\right)} + \sqrt{1 + \frac{s \left(-3 + t^2\right)}{+ 2}}$ $(2(-1+c^2)\pi\beta^2(1+c\beta)(s-\delta^2))$ $\alpha (-1 + c \beta) (s (1 + 2 c \beta - \beta^2 + 2 c^2 \beta^2) - (1 + 2 c \beta + \beta^2) \delta 2)$ $Log\left[-\frac{\frac{1}{2} s (-1 + \beta^{2}) + \frac{1}{2} s (1 + 2 c \beta + \beta^{2})}{-\frac{1}{2} s (-1 + \beta^{2}) + \frac{1}{2} s (1 + 2 c \beta + \beta^{2})} + \sqrt{1 + \frac{s (-1 + \beta^{2})}{\beta 2}}\right]$ $\log \Big[\frac{\frac{1}{4} \pm \left(-1 + \beta^2\right) + \frac{1}{4} \pm \left(1 + 2 \pm \beta + \beta^2\right)}{-\frac{1}{4} \pm \left(-1 + \beta^2\right) + \frac{1}{4} \pm \left(1 + 2 \pm \beta + \beta^2\right)} + \sqrt{1 + \frac{\pm \left(-1 + \beta^2\right)}{\Delta 2}} \Big] \Big] \Big/$ $(2(-1+c^2)\pi\beta^2(1+c\beta)(s-\Delta 2)) +$ $\frac{c \alpha (1+\beta^2) \log \left[-1+\frac{2}{1+\sqrt{\beta^2}}\right] \log \left[-\frac{c}{s-1}\right]}{(-1+c^2) \pi \beta^2}.$ α (1 + c β) (s (1 - 2 c $\beta - \beta^2 + 2 c^2 \beta^2$) + (-1 + 2 c $\beta - \beta^2$) \wedge 2) $1 - \frac{-\frac{1}{2}s\left(-\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}s\left(-\frac{1}{2}-\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)+\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)}{-\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)+\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)}{-\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)+\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)}{-\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)+\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)}{-\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)+\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)}{-\frac{1}{2}s\left(-\frac{1}{2}+\frac{1}{2}e^{\frac{1}{2}-\frac{1}{2}}\right)}$ $\int 1 + \frac{s(-1+\beta^2)}{1+\beta^2} |\log[-\beta^2]$ $\frac{-\frac{1}{2} \times \left(-\frac{1}{2} \times \frac{2}{2} \times \frac{1}{2} \times \frac{1}{2}$ $(2(-1+c^2)\pi\beta^2(-1+c\beta)(s-\Lambda 2)) +$ (-1+c²) πβ² (s-∆2) (s-∆2)

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\alpha (1 + c\beta) (s (1 - 2c\beta - \beta^2 + 2c^2\beta^2) + (-1 + 2c\beta - \beta^2) \wedge 2)
                                                                                                                                                                             1+ 1+ 5(110)
                                                                                           \frac{-\frac{1}{2} \frac{1}{2} \left( -\frac{1}{2} \frac{1}{2} \frac{1}{
                   (4(-1+c^2)\pi\beta^2(-1+c\beta)(s-\beta^2))
                   \alpha (1 + c\beta) (s (1 - 2c\beta - \beta^2 + 2c^2\beta^2) + (-1 + 2c\beta - \beta^2) \wedge 2)
                                                                                                                                                                                                                                                                                                                                                               1 = \frac{-\frac{1}{2} \pi \left( -1 + 2 \, c \, (1 + i)^2 \right) + \frac{1}{2} \pi \left( -1 + i \right)^2}{-\frac{1}{2} \pi \left( -1 + 2 \, c \, (1 + i)^2 \right) + \frac{1}{2} \pi \left( -1 + i \right)^2}
                                      Log 1+ 1+ 5 (-1+82) Log -
                   (2(-1+c^2)\pi\beta^2(-1+c\beta)(s-\beta^2)) +
                   \alpha (1+c\beta) (s (1-2c\beta-\beta<sup>2</sup>+2c<sup>2</sup>\beta<sup>2</sup>) + (-1+2c\beta-\beta<sup>2</sup>) \wedge2)
                                      Log \left[ -\frac{\frac{1}{2} s \left(-1 + 2 c \beta - \beta^{2}\right) + \frac{1}{2} s \left(-1 + \beta^{2}\right)}{-\frac{1}{2} s \left(-1 + 2 c \beta - \beta^{2}\right) - \frac{1}{2} s \left(-1 + \beta^{2}\right)} + \sqrt{1 + \frac{s \left(-1 + \beta^{2}\right)}{\beta 2}} \right]
                                 \log \left[ \frac{-\frac{1}{2} s \left(-1+2 c \beta - \beta^{2}\right) + \frac{1}{2} s \left(-1+\beta^{2}\right)}{-\frac{1}{2} s \left(-1+2 c \beta - \beta^{2}\right) - \frac{1}{2} s \left(-1+\beta^{2}\right)} + \sqrt{1 + \frac{s \left(-1+\beta^{2}\right)}{\Lambda^{2}}} \right] \right] / \frac{1}{2}
                   (2(-1+c^2)\pi\beta^2(-1+c\beta)(s-\Delta 2)) -
                        (-1 + c\beta) (s (1 + 2c\beta - \beta^2 + 2c^2\beta^2) - (1 + 2c\beta + \beta^2) \wedge 2)
                                                                                                                                                                                                                                                                                                                                                                         1 = \frac{\frac{1}{2} s \left(-1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + 2 c \left(1 + i \beta^2\right) + \frac{1}{2} s \left(1 + i \beta^2\right) + \frac{1}{2} s
                                      \log[1 - \sqrt{1 + \frac{8(-1 + \beta^2)}{12}}] \log[-
                   (2(-1+c^2)\pi\beta^2(1+c\beta)(s-\Lambda 2))
                        (-1 + c\beta) (s (1 + 2c\beta - \beta^2 + 2c^2\beta^2) - (1 + 2c\beta + \beta^2) \land 2)
                                                                                                                                                                   1 + \sqrt{1 + \frac{3 \cdot (-1 + \beta^2)}{\beta^2}}
                                                                                                \frac{b_{R}(-1,c^{2})+b_{R}(1+2\pi)(c^{2})}{-b_{R}(-1,c^{2})+b_{R}(1+2\pi)(c^{2})} + \sqrt{1 + \frac{n(-1,c^{2})}{\sqrt{2}}}
                        (4(-1+c^2)\pi\beta^2(1+c\beta)(s-\Lambda 2))
                        (-1 + c\beta) (s (1 + 2c\beta - \beta^2 + 2c^2\beta^2) - (1 + 2c\beta + \beta^2) \wedge 2)
                                                                                                                                                                                                                                                                                                                                                           \mathbf{1} = \frac{\frac{1}{2} \frac{1}{2} \frac{1
                                      \log[1 + \sqrt{1 + \frac{s(-1+\beta^2)}{2}}] \log[-
                   (2(-1+c^2)\pi\beta^2(1+c\beta)(s-\Lambda 2)) -
              \alpha (-1+c \beta) (s (1+2 c \beta - \beta<sup>2</sup> + 2 c<sup>2</sup> \beta<sup>2</sup>) - (1+2 c \beta + \beta<sup>2</sup>) \wedge2)
                                 Log\left[-\frac{\frac{1}{2} s \left(-1 + \beta^{2}\right) + \frac{1}{2} s \left(1 + 2 c \beta + \beta^{2}\right)}{-\frac{1}{2} s \left(-1 + \beta^{2}\right) + \frac{1}{2} s \left(1 + 2 c \beta + \beta^{2}\right)} + \sqrt{1 + \frac{s \left(-1 + \beta^{2}\right)}{A2}}\right]
                            \log\left[\frac{\frac{1}{4}s\left(-1+\beta^{2}\right)+\frac{1}{4}s\left(1+2c\beta+\beta^{2}\right)}{-\frac{1}{4}s\left(-1+\beta^{2}\right)+\frac{1}{4}s\left(1+2c\beta+\beta^{2}\right)}+\sqrt{1+\frac{s\left(-1+\beta^{2}\right)}{\Lambda^{2}}}\right]\right]
              (2(-1+c^2)\pi\beta^2(1+c\beta)(s-\Lambda 2)) +
\frac{\mathbf{c} \alpha \left(\mathbf{1} + \beta^2\right) \log \left[-\mathbf{1} + \frac{2}{\mathbf{1} + \left[\beta^2\right]}\right] \log \left[-\frac{5}{6 - 12}\right]}{\left(-\mathbf{1} + \mathbf{c}^2\right) \pi \beta^2}
    \left(\mathbf{s} \alpha \left(\mathbf{1} - \mathbf{2} \mathbf{c} \beta - \beta^2 + \mathbf{2} \mathbf{c}^2 \beta^2\right) \left(\mathbf{2} \mathbf{s} - \beta \mathbf{2} - \beta^2\right) \operatorname{PolyLog}\left[\mathbf{2}, -\frac{-\mathbf{1} + \beta^2}{-\mathbf{1} + \mathbf{2} \mathbf{c} \beta} - \beta^2\right]\right)
              (2(-1+c^2)\pi\beta^2(s-\Delta 2)(s-\Delta 2)) +
    \left(\mathbf{s} \alpha \left(\mathbf{1} + \mathbf{2} \mathbf{c} \beta - \beta^2 + \mathbf{2} \mathbf{c}^2 \beta^2\right) \left(\mathbf{2} \mathbf{s} - \beta \mathbf{2} - \beta \mathbf{2}\right) \operatorname{PolyLog}\left[\mathbf{2}, \frac{-\mathbf{1} + \beta^2}{\mathbf{1} + \mathbf{2} \mathbf{c} \beta + \beta^2}\right]\right) \right)
    (2(-1+c^2)\pi\beta^2(s-\Delta 2)(s-\Delta 2)) +
    c s \alpha (1 + \beta^2) (2 s - \Delta 2 - \Delta 2)  PolyLog\left[2, \frac{-1-\beta^2}{\left(-1 - \sqrt{\beta^2}\right)^2}\right]
```

 $\frac{2 \operatorname{c} \alpha \left(1 + \beta^{2}\right) \operatorname{PolyLog}\left[2, -\frac{\operatorname{s}\left(1 + \beta^{2}\right)}{2 \left(\operatorname{s} - \operatorname{s} \sqrt{\beta^{2}}\right)}\right]}{\left(-1 + \operatorname{c}^{2}\right) \pi \beta^{2}}$ $2 c \alpha (1 + \beta^2) \operatorname{PolyLog} \left[2, -\frac{s (1 + \beta^2)}{2 \left(s + s \sqrt{\beta^2}\right)}\right]$ (-1+c2) = B2 $\alpha (\mathbf{1} + \mathbf{c} \beta) (\mathbf{s}^2 (\mathbf{1} - \mathbf{2} \mathbf{c} \beta - \beta^2 + \mathbf{2} \mathbf{c}^2 \beta^2) + (\mathbf{1} - \mathbf{2} \mathbf{c} \beta + \beta^2) \Delta 2 \Delta 2 - \mathbf{s} (-\mathbf{1} + \mathbf{c} \beta)^2 (\Delta 2 + \Delta 2)$ $\operatorname{PolyLog}\left[2, \frac{1 + \frac{-5 \cdot x \left(-1 + 2 \cdot c \cdot d \cdot d^2\right) + 5 \cdot x \left(-1 + d^2\right)}{-1 + \frac{-5 \cdot x \left(-1 + 2 \cdot c \cdot d \cdot d^2\right) + 5 \cdot x \left(-1 + d^2\right)}{-1 + \frac{-5 \cdot x \left(-1 + 2 \cdot c \cdot d \cdot d^2\right) + 5 \cdot x \left(-1 + d^2\right)}{-5 \cdot x \left(-1 + 2 \cdot d^2\right) + 5 \cdot x \left(-1 + d^2\right)}}\right]\right] \right)$ $(\{-1+c^2\} \pi \beta^2 (-1+c\beta) (s-\Delta 2) (s-\Delta 2)) +$ $(-1 + c\beta)$ $(s^2 (1 + 2c\beta - \beta^2 + 2c^2\beta^2) + (1 + 2c\beta + \beta^2) \Delta 2 \Delta 2 - s (1 + c\beta)^2 (\Delta 2 + \Delta 2))$ $\frac{-\frac{1}{2} \left(\frac{1}{2} - \frac{2}{2} \left(\frac{1}{2} + \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2}$ $PolyLog[2, \frac{1 + \frac{\frac{1}{2} S (-1 + i^2) + \frac{1}{2} S (1 + 2 e i + i^2)}{-\frac{1}{2} S (-1 + i^2) + \frac{1}{2} S (1 + 2 e i + i^2)}}{-1 + \frac{\frac{1}{2} S (-1 + i^2) + \frac{1}{2} S (1 + 2 e i + i^2)}{-\frac{1}{2} S (-1 + i^2) + \frac{1}{2} S (2 + 2 e i + i^2)}}$ $((-1+c^2)\pi\beta^2(1+c\beta)(s-\Delta 2)(s-\Delta 2)) +$ $x (1 + c\beta) (s (1 - 2c\beta - \beta^2 + 2c^2\beta^2) + (-1 + 2c\beta - \beta^2) \Delta 2)$ PolyLog[2, $\frac{-\frac{1}{2}S\left(\frac{1}{2}S\left(\frac{1}{2}S\left(\frac{1}{2}S\left(\frac{1}{2}S\right)\right)^{2}\right)+\frac{1}{2}S\left(\frac{1}{2}S\left(\frac{1}{2}S\right)^{2}\right)}{\frac{1}{2}S\left(\frac{1}{2}S\left(\frac{1}{2}S\right)^{2}\right)+\frac{1}{2}S\left(\frac{1}{2}S\left(\frac{1}{2}S\right)^{2}\right)}+\sqrt{1+\frac{S\left(\frac{1}{2}S\left(\frac{1}{2}S\right)^{2}\right)}{22}}$ 1+ /1+ = (-1/2) $(2(-1+c^2) \pi \beta^2(-1+c\beta)(s-\Delta 2))$ α (1+c β) (s (1-2c β - β^2 +2c² β^2) + (-1+2c β - β^2) δ 2) $-\frac{\frac{1}{2} \frac{1}{2} \left(\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} + \frac{1}{2} \frac{1}{$ $(2(-1+c^2)\pi\beta^2(-1+c\beta)(s-\Delta 2))$ α (-1+c β) (s (1+2c β - β ²+2c² β ²) - (1+2c β + β ²) Δ 2) PolyLog[2, $\frac{-\frac{1-5(-1+0^2)+\frac{5}{2}+5(1+2+5)+0^2}{-\frac{1}{2}+5(-1+0^2)+\frac{1}{2}+5(1+2+5)+0^2} + \sqrt{1+\frac{5(-1+0^2)}{-2}}$ $-1/(2(-1+c^2)\pi \theta^2(1+c^3)(s-(2)))$ $\alpha (-1 + c\beta) (s (1 + 2c\beta - \beta^2 + 2c^2\beta^2) - (1 + 2c\beta + \beta^2) \Delta 2$ $\frac{b_{R}(-1+i^{2})+b_{R}(1+2\pi i(+i^{2}))}{-b_{R}(-1+i^{2})+b_{R}(1+2\pi i(+i^{2}))} + \sqrt{1 + \frac{n(-1+i^{2})}{i^{2}}}$ -1+ 1+ *(-1-1)/2 $\left| \left| \left(2 \left(-1 + c^2 \right) \pi \beta^2 \left(1 + c \beta \right) \left(s - \Delta 2 \right) \right) \right| \right| \right|$ $\frac{\frac{1}{2} s \left(-1 + d^{2}\right) + \frac{1}{2} s \left(1 + 2 c d + d^{2}\right)}{-\frac{1}{2} s \left(-1 + d^{2}\right) + \frac{1}{2} s \left(1 + 2 c d + d^{2}\right)} + \sqrt{1 + \frac{s \left(-1 + d^{2}\right)}{d2}}$ α (1 + c β) (s (1 - 2 c β - β^2 + 2 c² β^2) + (-1 + 2 c β - β^2) $\triangle 2$) PolyLog [2, $\frac{s(-1+\beta^2)}{\left(1 - \frac{(-\frac{1}{2}+a(-1)+2s(-b(-\beta^2)+\frac{1}{2}+a(-1+\beta^2))^2}{(-\frac{1}{2}+a(-1+\beta^2)+a(-1+\beta^2)+a(-1+\beta^2)}\right)} d2$] $(2 \{-1+c^2\} \pi \beta^2 (-1+c\beta) (s-\Delta 2)) =$ α (-1+c β) (s (1+2c β - β^2 +2c² β^2) - (1+2c β + β^2) δ 2) $\operatorname{PolyLog}[2, \frac{s(-2+\beta^2)}{\left(1 - \frac{(\frac{1}{2} + (-2+\beta^2) + \frac{1}{2} + (1+2+\beta+\beta^2))^2}{(-\frac{1}{2} + (-1+\beta^2) + \frac{1}{2} + (1+2+\beta+\beta^2))^2} + \frac{s(-2+\beta^2)}{(-2}\right) \delta 2}\right]$ $(2(-1+c^2) \pi \beta^2 (1+c\beta) (s-\Delta 2))$ $c \alpha (1 + \beta^2) \operatorname{PolyLog}[2, \frac{5 + 5 \sqrt{\beta^2} - (2 - \sqrt{1 + 5 \lfloor \frac{1}{\beta} \rfloor^2} + 2}{\sqrt{1 + 5 \lfloor \frac{1}{\beta} \rfloor^2} + 2}]$ (-1+c²) πβ² STRONG2020 Virtual Workshop



Calculations σ Roman 0 Ψ P

FormFactor parametrization

Analytical calculation was done with constant BW parametrization: (off mass shell effect in FF was out of scope)

$$F_{i}(q^{2}) = \frac{\Lambda_{i}^{2}}{\Lambda_{i}^{2} - q^{2}}, \Lambda^{2} = M^{2} - i M \Gamma$$

$$F_{j}(q^{2})$$

Full GS function was re-parametrized by sum of constant BW:

$$F(s) = \sum \alpha_i \frac{\Lambda_i^2}{\Lambda_i^2 - s}$$

3 BW gives ~ 5% precision



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Virtual + soft corrections

 $d\sigma/d\theta = d\sigma^{Born}/d\theta * (1 + \delta^{PL}_{odd}(s, \theta) + \delta^{vFF}(s, \theta))$ δ_{odd}(at θ=1 rad) pe 0.08 virt pointlike δvFF correction -0.005 δvFF single BW 0.02 soft+virt pointlike pion(without log() term) -0.01 x5 virt pointlike δvFF correction -0.02 -0.015δvFF single BW 500 1000 2000 2500 1500 300 0 √s. MeV with PDG M, G -0.02 45 **GS** form 40E х9 sum of BW 35E -0.025 single $\left|\frac{\Lambda^2}{\Lambda^2-S}\right|^2$ 400 600 800 1000 1200 1400 30 with PDG M.J s, MeV 25E 20 Red line - with sum of BW, 15 for comparison (black, grey) with single BW: result stable at p-peak 10

Enhancement of virtual correction by x5-10 factor!

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700

800

900 1000 1100

12 Vs, MeV

300

400

500

600

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Asymmetry

After plugging δvFF in MCGPJ generator



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Asymmetry

After plugging δvFF in MCGPJ generator



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Final angle spectra

Still some disagreement in $dN/d\theta$ between data and prediction at level ~ 0.1%: 1) Bhabha generator or Asym. in 2π 2) detector inefficiencies 3) $N_{\pi\pi}$ / N_{ee}

But already it allow to fit angle spectra with released $N_{\pi\pi}$ / N_{ee} , Asym parameters. For sum of 350-410 MeV points Event separation <u>by momentums</u>:

	$N_{\pi\pi} / N_{ee} =$		1.0187 +- 0.00028
by energies in LXe	$\Delta N_{\pi\pi} / N_{ee}$	=	+0.05 +- 0.033%
from theta with free δA:		=	-0.23 +- 0.12%
with fixed δA =0:		=	+0.20 +- 0.08%

We have 3 fully independent methods for $N_{\pi\pi}$ /N_{ee} determination, they are consistent at ~ 0.2% 24 November 2021



How it can affect pion form factor measurements?

Usually event selections in analyses are charge/angle symmetric

Main effect at lowest order comes from: Interference of box vs born diagrams

=> only charge-odd contribution effect is integrated out in full cross-section

Interference of ISR & box vs FSR (or v.v.) => charge-even

ISR measurements



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sQED assumption

Model assumptions

Henryk Czyz the Muon g-2 Theory Initiative Worksop 2019





H. Czyż

Radiative corrections in PHOKHARA and EKHARA MC generators,

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ISR measurements

Complete NLO: KLOE-large



KLOE-2010 with tag photon measurement can be affected

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Complete NLO: KLOE-small



Complete NLO: BaBar



Summary

It seen ~1% disagreement in the asymmetry between 2π CMD3 data vs prediction based on sQED assumption

Proper account of the Form Factor in the box diagrams gives x5-10 enhancement of them.

It can gives sizeable effect both in charge-odd and in some cases in the charge-even parts of radiative corrections.

Inclusion of double FF in box diagram describe well seen effect in the 2π Asymmetry with CMD3 data at the current precision.

backups

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Generators MCGPJ/Phokhara



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Generators MCGPJ/BabaYaga@NLO

Для µ+µ- интегральная асимметрия совпадает между MCGPJ/BabaYaga@NLO с абс. точностью ~0.05% (5% относительная точность)



ВаbaYaga@NLO моделирует фотоны рекурсивно У нас только один фотон на большой угол Поведение BabaYaga около q2~1 более физично Скорее всего это отличие дает эффект в систематику разделения по Р из-за разницы генераторов 24 November 2021



Asymmetry with q2

