



STRONG 2020 Virtual Workshop on "Spacelike and Timelike determination of the Hadronic Leading Order contribution to the Muon $g-2$ "

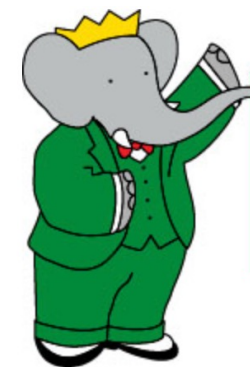
*Recent ISR measurements of hadronic spectra
with the BaBar*



Peter A. Lukin

**Budker Institute of Nuclear Physics and
Novosibirsk State University, Novosibirsk, Russia**

On behalf of BaBar Collaboration

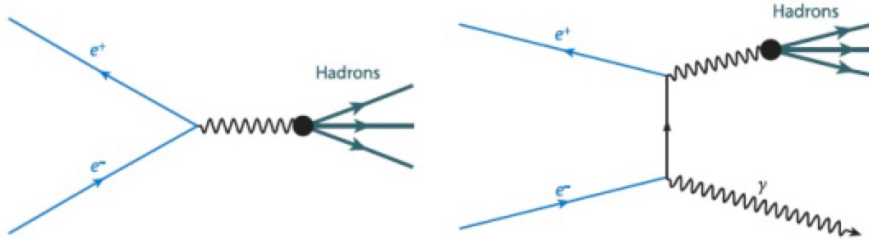


Outline

- New measurement of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section
2110.00520 [hep-ex] Submitted to PRD
- First measurements of $e^+e^- \rightarrow \pi^+\pi^-4\pi^0$ and $e^+e^- \rightarrow \pi^+\pi^-3\pi^0\eta$ cross sections
2110.00823 [hep-ex] Submitted to PRD
- First measurements of $e^+e^- \rightarrow 2(\pi^+\pi^-)3\pi^0$ and $e^+e^- \rightarrow 2(\pi^+\pi^-)2\pi^0\eta$ cross sections
Phys. Rev. D 103 092001 (2021)

BABAR hadronic cross section measurements using ISR

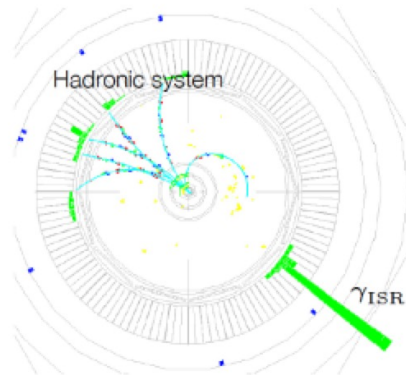
Initial State Radiation from e^+e^- allows to measure cross sections at all center-of-mass energies $\sqrt{s'}$ below the nominal \sqrt{s} of the beams:



$$\frac{d\sigma(s; s'; \theta_\gamma)}{ds' d\theta_\gamma} = W(s; s'; \theta_\gamma) \cdot \sigma_X(s')$$

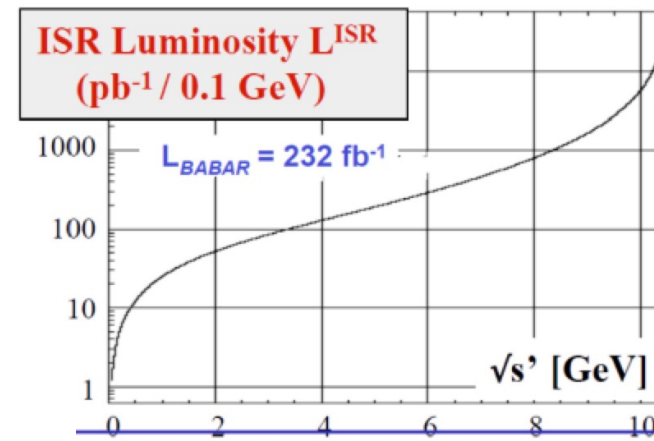
boost \Rightarrow harder momentum spectrum for daughter particles

tag photon to identify ISR events



- hadrons in fiducial detector region
- fully reconstruct the final state
- kinematic fit: energy resolution

- cross sections down to threshold
- measure σ at all \sqrt{s} simultaneously
- large "effective" luminosity



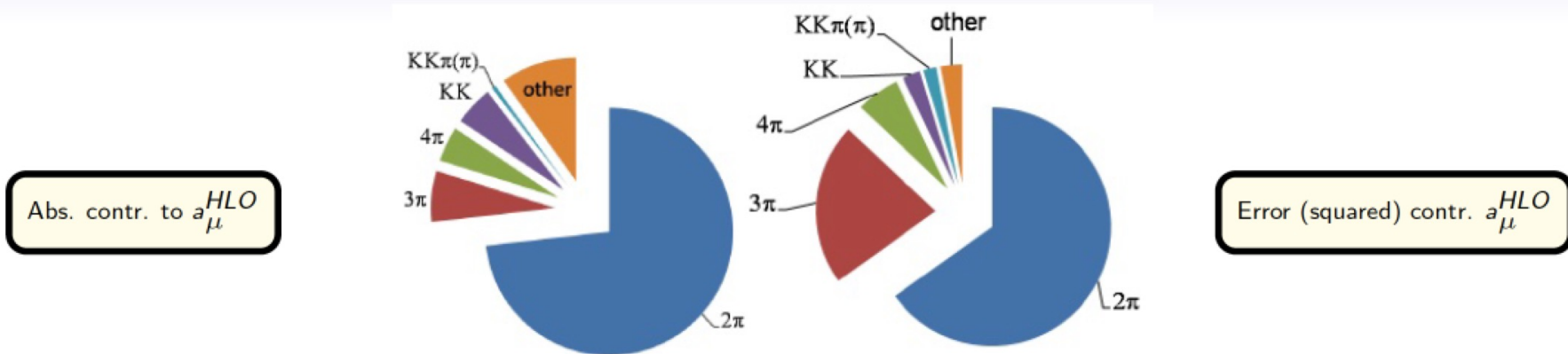
Light hadrons cross sections measured in ISR by *BABAR*

Many **first measurements**: (superseded results omitted)

$2(\pi^+\pi^-)\pi^0\pi^0\pi^0$ and $2(\pi^+\pi^-)\pi^0\pi^0\eta$	469 fb ⁻¹	Phys. Rev. D 103, 092001 (2021)
$\pi^+\pi^-\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0\pi^0\eta$	469 fb ⁻¹	Phys. Rev. D 98, 112015 (2018)
$\pi^+\pi^-\eta$	469 fb ⁻¹	Phys. Rev. D 97, 052007 (2018)
$\pi^+\pi^-\pi^0\pi^0$	454 fb ⁻¹	Phys. Rev. D 96, 092009 (2017)
$K_S^0 K^\pm \pi^\mp \pi^0$ and $K_S^0 K^\pm \pi^\mp \eta$	454 fb ⁻¹	Phys. Rev. D 95, 092005 (2017)
$K_S^0 K_L^0 \pi^0$, $K_S^0 K_L^0 \eta$, and $K_S^0 K_L^0 \pi^0 \pi^0$	469 fb ⁻¹	Phys. Rev. D 95, 052001 (2017)
$K^+ K^-$ (γ undetected)	469 fb ⁻¹	Phys. Rev. D 92, 072008 (2015)
$K_S^0 K_L^0$, $K_S^0 K_L^0 \pi^+ \pi^-$, $K_S^0 K_S^0 \pi^+ \pi^-$, and $K_S^0 K_S^0 K^+ K^-$	469 fb ⁻¹	Phys. Rev. D 89, 092002 (2014)
$K^+ K^-$	232 fb ⁻¹	Phys. Rev. D 88, 032013 (2013)
$p\bar{p}$	469 fb ⁻¹	Phys. Rev. D 87, 092005 (2013)
$p\bar{p}$ (E_{cm} : 3.0 ÷ 6.5 GeV)	469 fb ⁻¹	Phys. Rev. D 88, 072009 (2013)
$\pi^+\pi^-\pi^+\pi^-$	454 fb ⁻¹	Phys. Rev. D 85, 112009 (2012)
$K^+ K^- \pi^+ \pi^-$, $K^+ K^- \pi^0 \pi^0$, and $K^+ K^- K^+ K^-$	454 fb ⁻¹	Phys. Rev. D 86, 012008 (2012)
$\pi^+ \pi^-$	232 fb ⁻¹	Phys.Rev.Lett. 103, 231801 (2009)
$K^+ K^- \eta$, $K^+ K^- \pi^0$ and $K_S^0 K^\pm \pi^\mp$	232 fb ⁻¹	Phys. Rev. D 77, 092002 (2008)
$\Lambda\bar{\Lambda}$, $\Lambda\bar{\Sigma}^0$, and $\Sigma^0\bar{\Sigma}^0$	230, fb ⁻¹	Phys. Rev. D 76, 092006 (2007)
$2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+ K^- \pi^+ \pi^- \pi^0$ and $K^+ K^- \pi^+ \pi^- \eta$	232 fb ⁻¹	Phys. Rev. D 76, 092005 (2007)
$3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$ and $K^+ K^- 2(\pi^+\pi^-)$	232 fb ⁻¹	Phys. Rev. D 73, 052003 (2006)
$\pi^+\pi^-\pi^0$	89 fb ⁻¹	Phys. Rev. D 70, 072004 (2004)

$$e^+e^- \rightarrow \pi^+\pi^-\pi^0$$

It gives the second largest contribution to a_μ^{HLO} and its error



Previous *BABAR* measurement based on 1/5 of dataset
Cross section had been measured in $1.05 \div 3$ GeV

PRD 70, 072004 (2004)

New preliminary measurement using the whole dataset extends cross section below 1.05 GeV, in the region of ρ , ω and ϕ resonances

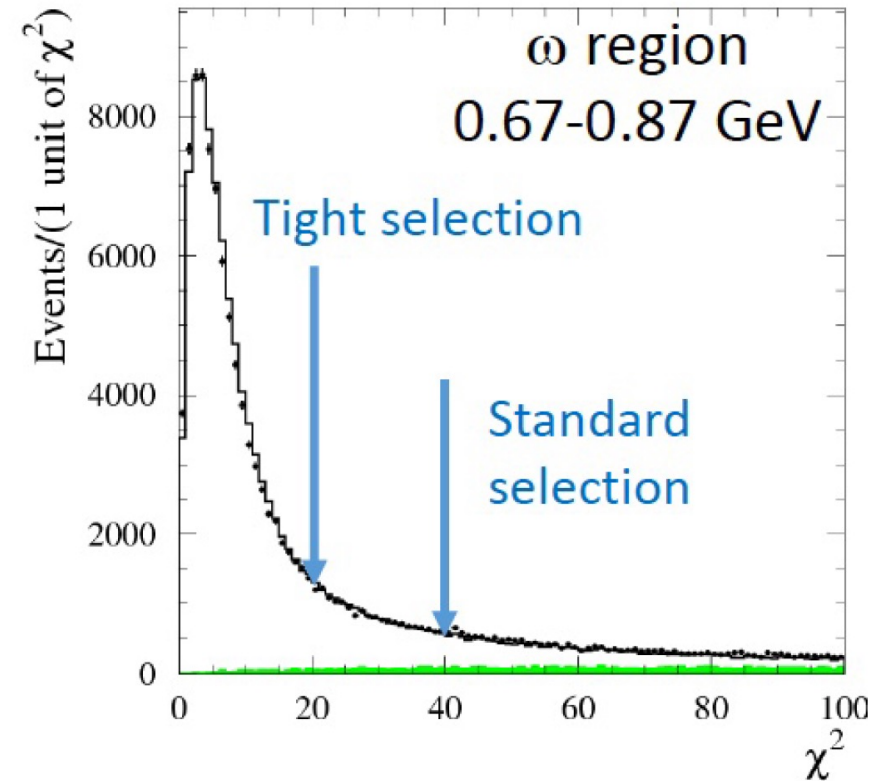
accuracy on a_μ^{HLO} contribution due to $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ currently $\approx 3\%$
new measurement will improve accuracy to $\approx 1.5\%$

$$e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma_{ISR}$$

Detect all final state particles
Select events using kinematic fit (cut on χ^2)
Several additional cuts reduce background by factor 2

Remaining ISR and $q\bar{q}$ background subtracted using simulation normalized to data.

Above 1.1 GeV sizeable FSR background from $e^+e^- \rightarrow a_1\gamma, a_2\gamma$ processes.
Estimated by pQCD with 100% uncertainty.
up to 8% contribution near 1.3 GeV



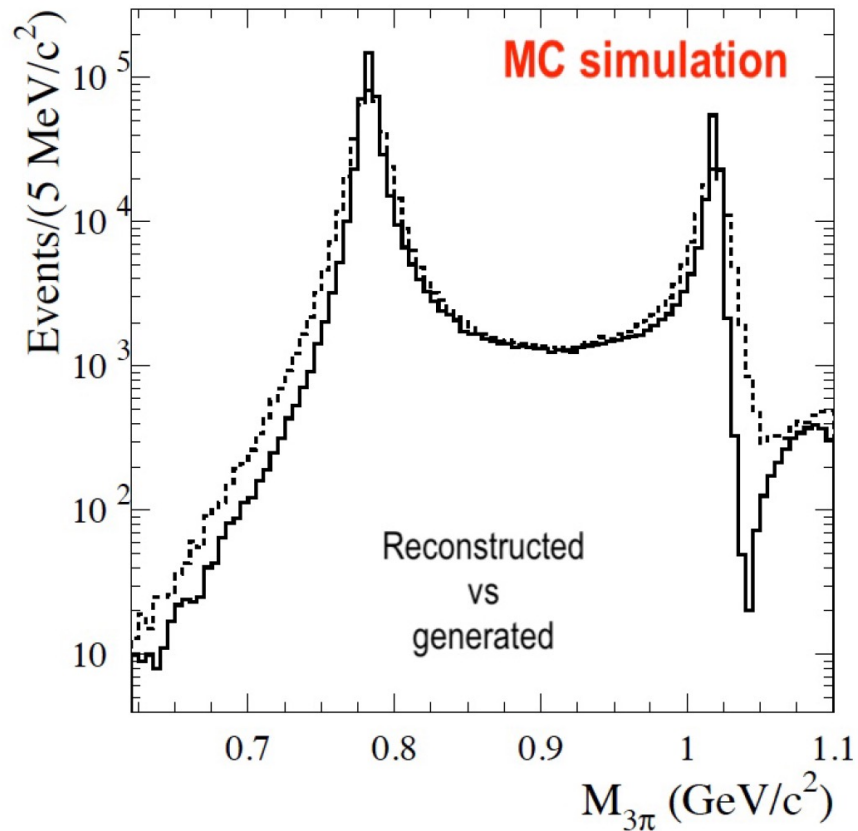
$\pi^+\pi^-\pi^0$ mass spectrum below 1.1 GeV

2110.00520 [hep-ex] Submitted to PRD

Below 1.1 GeV the mass spectrum has a sharp structure

unfolding required to determine true spectrum

cross section result depends on the assumed mass resolution



The ω and ϕ widths are well known

\Rightarrow use data to correct the simulated resolution function

Tails of the resolution depend on the χ^2 cut applied in selecting events:

\Rightarrow try more than one cut value

Fit to the $\pi^+\pi^-\pi^0$ mass spectrum

2110.00520 [hep-ex] Submitted to PRD

The mass spectrum fitted with VDM model including

$$\omega(782) + \omega(1420) + \omega(1680) + \phi(1020) \text{ resonances}$$

$\omega(782)$ and ϕ widths fixed to PDG average

+ the rare $\rho(770) \rightarrow 3\pi$ decay

For $\chi^2 < 20$ (nominal fit) the mass spectrum is well described by introducing a Gaussian smearing of parameters

$$\sigma_s = 1.5 \pm 0.2 \text{ MeV}$$

$$m_\omega - m_{PDG} = 0.042 \pm 0.055 \text{ MeV}$$

$$m_\phi - m_{PDG} = 0.095 \pm 0.084 \text{ MeV}$$

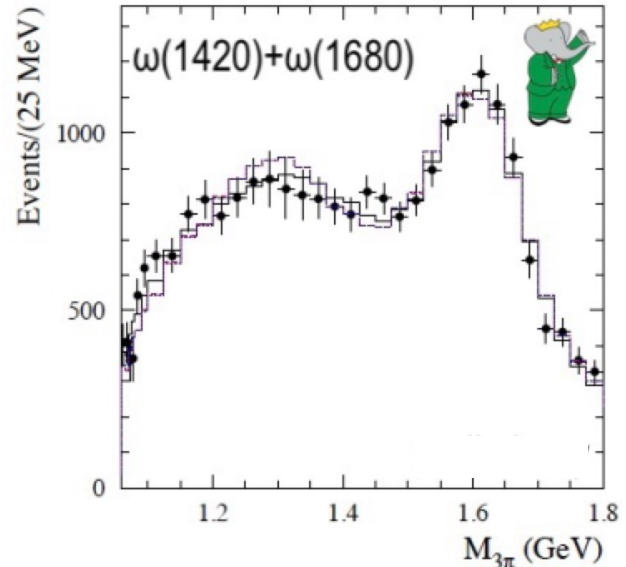
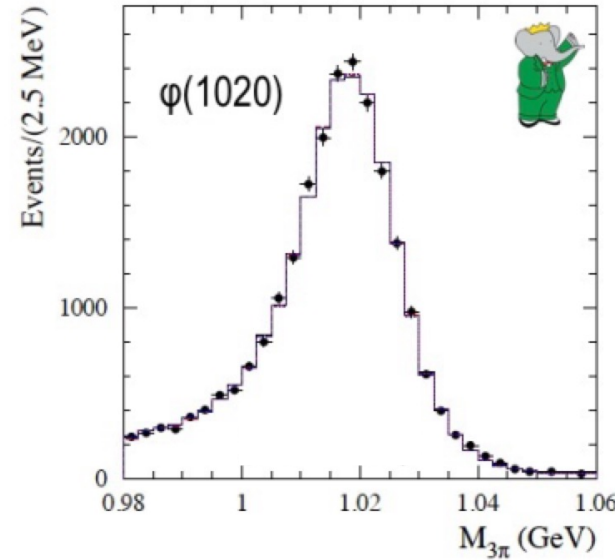
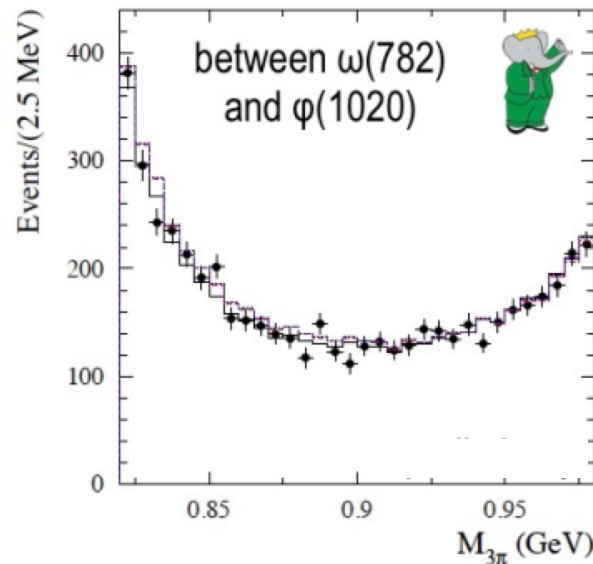
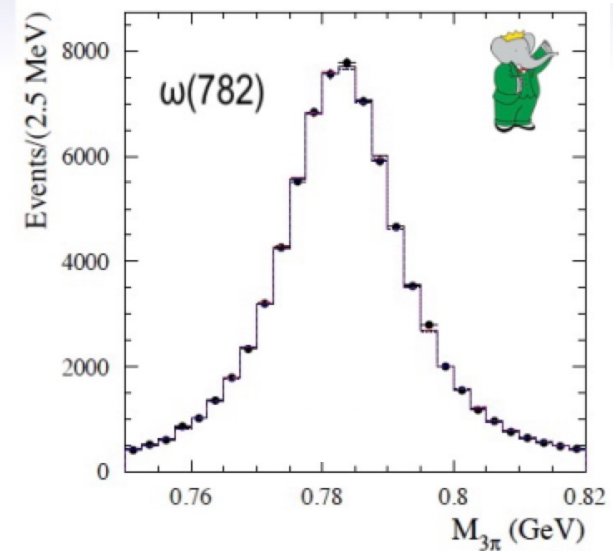
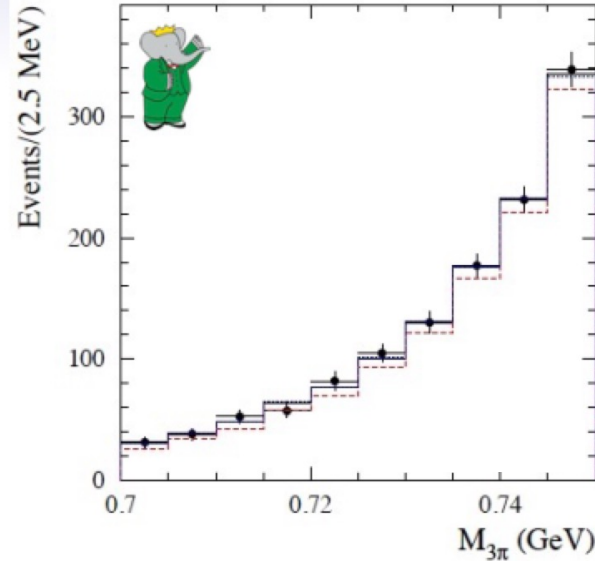
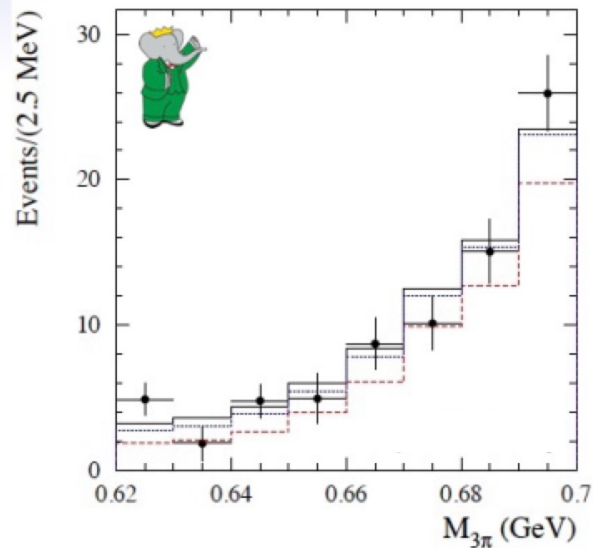
For $\chi^2 < 40$ (cross check): additional Lorentzian smearing required to describe tails

$$\text{fraction} = 0.7 \pm 0.2 \%; \quad \gamma = 63 \pm 35 \text{ GeV}$$

consistent results for all other parameters

The data spectrum cannot be adequately described with $\mathcal{B}(\rho \rightarrow 3\pi) \equiv 0$

Fits to the $\pi^+\pi^-\pi^0$ mass spectrum ($\chi^2 < 40$)



(solid): Lorentzian smearing and $\rho \rightarrow 3\pi$

(dash): No Lorentzian smearing and no $\rho \rightarrow 3\pi$

(dot): Lorentzian smearing and no $\rho \rightarrow 3\pi$

$\chi^2/\nu = 136/127$

$\chi^2/\nu = 201/131$

$\chi^2/\nu = 180/129$

For $\omega(782)$ and $\phi(1020)$ the products $\Gamma_{ee} \times \mathcal{B}_{3\pi}$ are in reasonable agreement with world average values:

$$\Gamma(\omega \rightarrow e^+ e^-) \cdot \mathcal{B}(\omega \rightarrow \pi^+ \pi^- \pi^0) = (0.5698 \pm 0.0031 \pm 0.0082) \text{ keV}$$

world average: $(0.557 \pm 0.011) \text{ keV}$

$$\Gamma(\phi \rightarrow e^+ e^-) \cdot \mathcal{B}(\phi \rightarrow \pi^+ \pi^- \pi^0) = (0.1841 \pm 0.0021 \pm 0.0080) \text{ keV}$$

world average: $(0.1925 \pm 0.0043) \text{ keV}$

The rare decay $\rho \rightarrow \pi^+ \pi^- \pi^0$ is observed with significance greater than 6σ the value and the relative phase wrt to the $\omega(782)$ amplitude are in agreement with the only previous measurement by SND

SND: Phys.Rev.D 63,07002 (2001)

$$\mathcal{B}(\rho \rightarrow \pi^+ \pi^- \pi^0) = (0.88 \pm 0.23 \pm 0.30) \times 10^{-4}$$

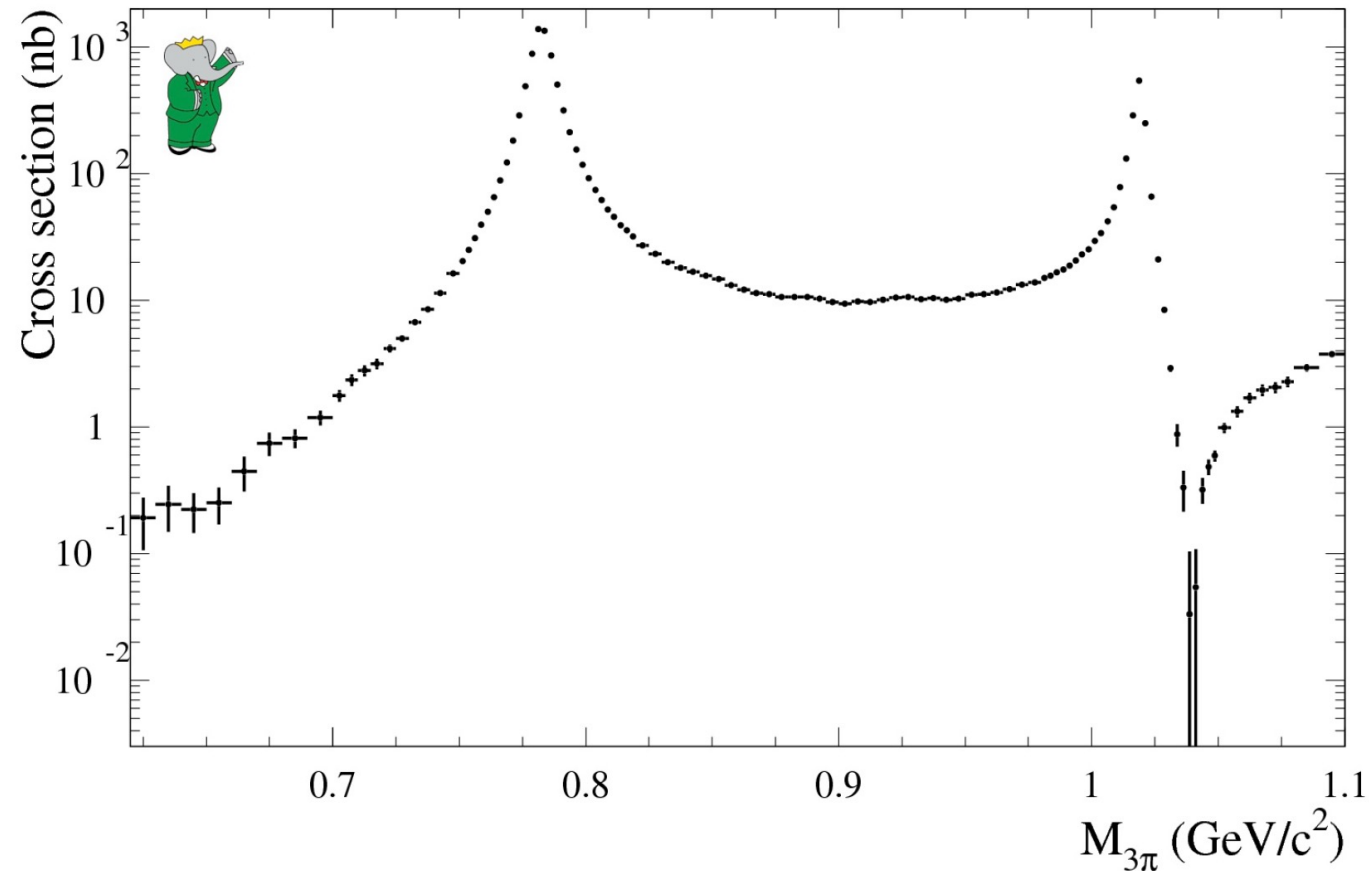
SND: $(1.01^{+0.54}_{-0.34} \pm 0.34) \times 10^{-4}$

$$\phi_\rho - \phi_\omega = -(99 \pm 9 \pm 15)^\circ$$

SND: $-(135^{+17}_{-13} \pm 9)^\circ$

$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section below 1.1 GeV

The parameters of the smearing function determined in the VDM fit are used to correct the simulated resolution function



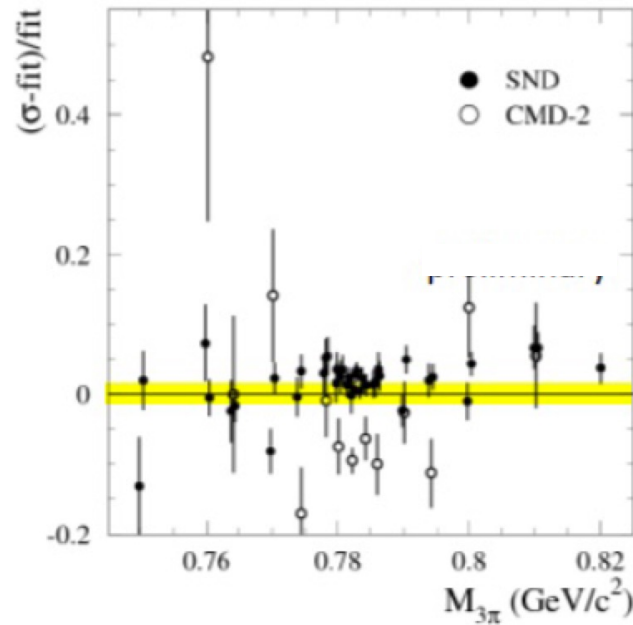
The unfolding is performed using the IDS (iterative dynamically stabilized) method (B. Malaescu, arXiv:0907.3791)

Systematic uncertainty at $\omega(782)$ and $\phi(1020)$ peak $\approx 1.3\%$

$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section below 1.1 GeV: comparison with previous measurements

SND: Phys.Rev.D 68,052006 (2003)

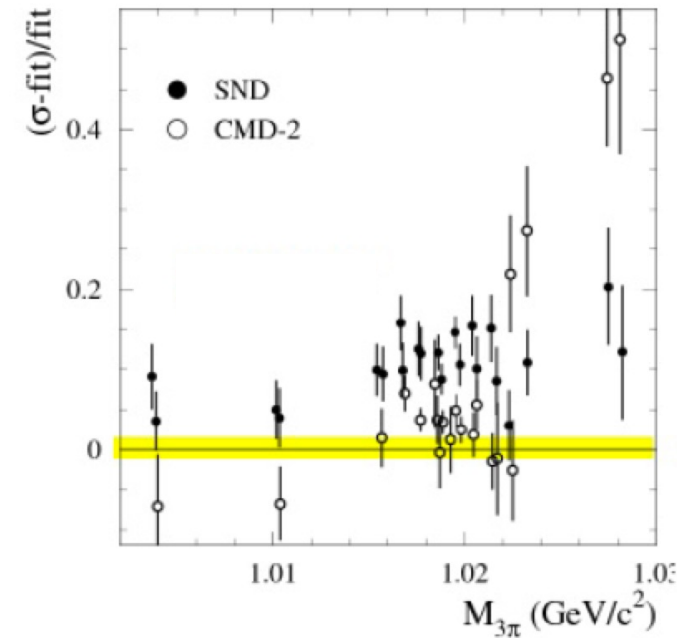
CMD-2: Phys.Lett.B 578,285 (2004)



SND-*BABAR* difference $\simeq 2\%$
 below syst. (3.4% SND, 1.4% *BABAR*)
 CMD-2 (1.8% stat and 1.3% syst) is
 $\simeq 7\%$ smaller than *BABAR*
 $\approx 2.7\sigma$ difference

SND: Phys.Rev.D 63,072002 (2001)

CMD-2: Phys.Lett.B 642,203 (2006)

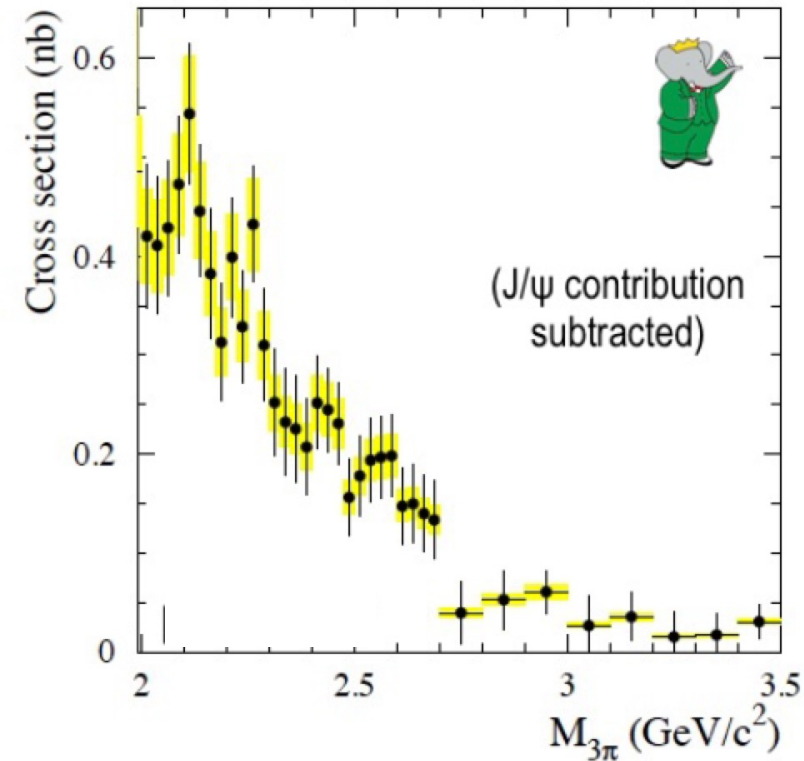
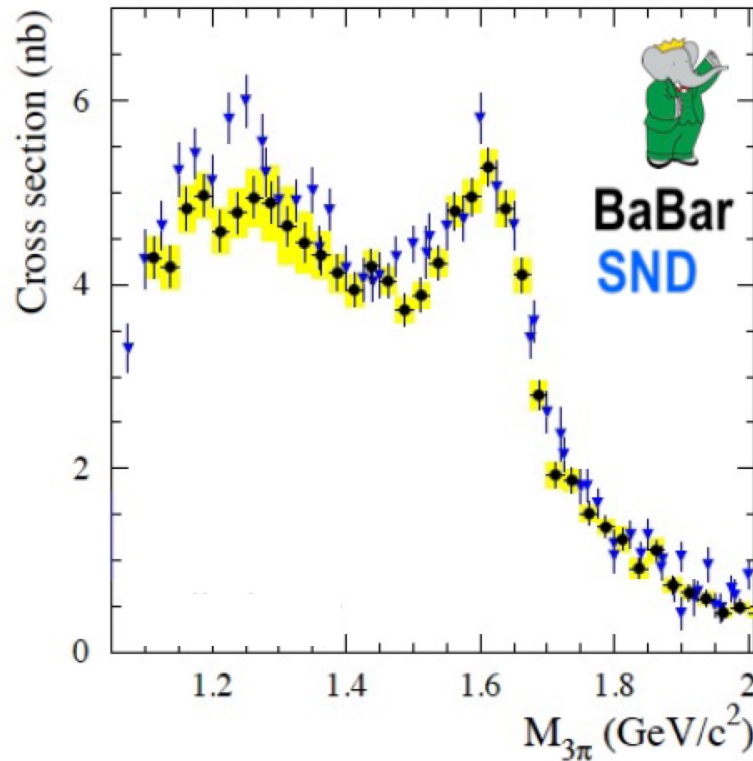


SND-*BABAR* difference $\simeq 11\%$
 syst: 5% (SND); 1.4% (*BABAR*)
 CMD-2-*BABAR* difference $\simeq 3\%$
 syst: 2.5% (CMD-2); 1.4% (*BABAR*)


$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section above 1.1 GeV

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No smearing is needed above 1.1 GeV. Syst. uncertainty: $4 \div 15\%$
dominated by background subtraction



Significant localized differences around 1.25 GeV and 1.5 GeV between *BABAR* and SND (Eur.Phys.J. C 80, 993 (2020))

$M_{3\pi} \text{ GeV}/c^2$		$a_\mu^{3\pi} \times 10^{10}$
0.62–1.10		$42.91 \pm 0.14 \pm 0.55 \pm 0.09$
1.10–2.00		$2.95 \pm 0.03 \pm 0.16$
< 2.00		$45.86 \pm 0.14 \pm 0.58$
< 1.80 [A]		$46.21 \pm 0.40 \pm 1.40$
< 1.97 [B]		46.74 ± 0.94
< 2 [C]		44.32 ± 1.48

[A] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur.Phys.J. C 80, 241 (2020)

[B] A. Keshavarzi, D. Nomura and T. Teubner, Phys.Rev.D 101, 014029 (2020)

[C] F. Jegerlehner, Springer Tracts Mod. Phys. 274, 1 (2017)

The value of $a_\mu^{3\pi}$ calculated using the preliminary $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross-section is in reasonable agreement with earlier calculations

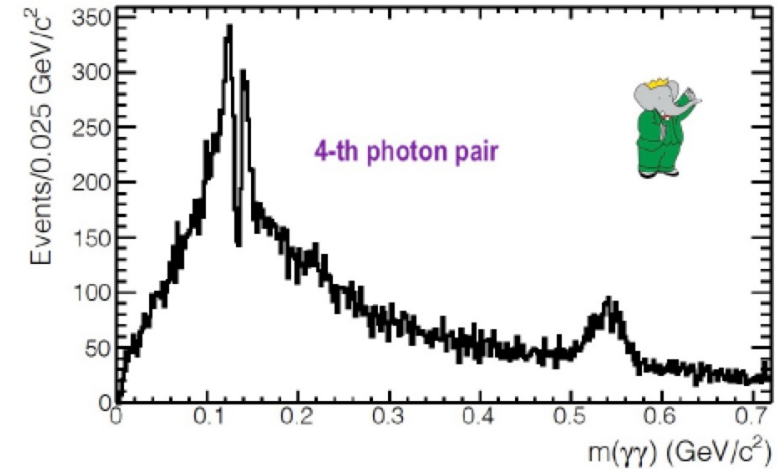
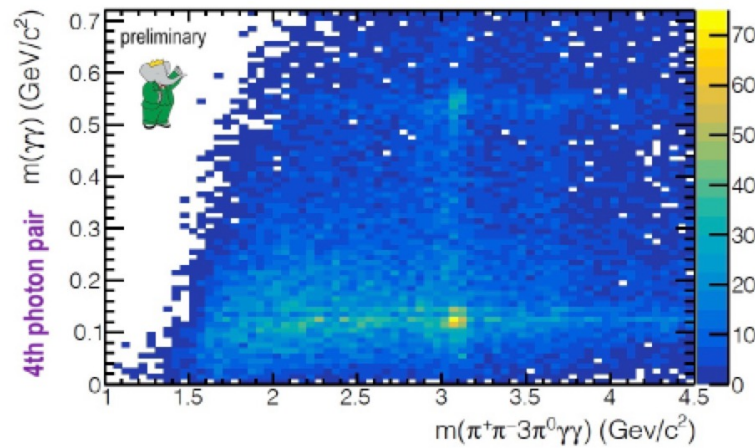
the error on this contribution is reduced by a factor ≈ 2

$e^+e^- \rightarrow \pi^+\pi^-4\pi^0$ and $\pi^+\pi^-3\pi^0\eta$ (first measurement)

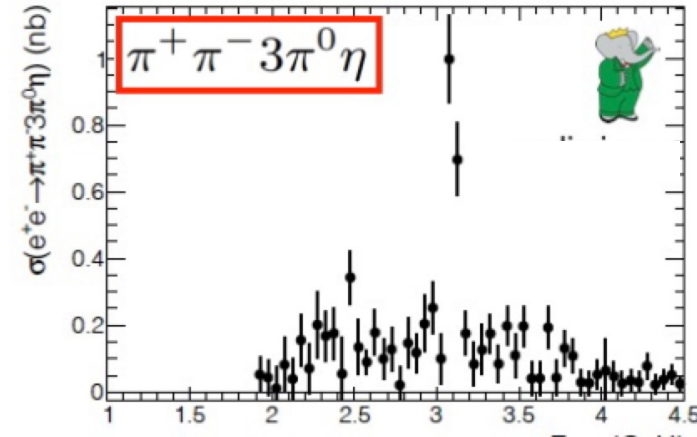
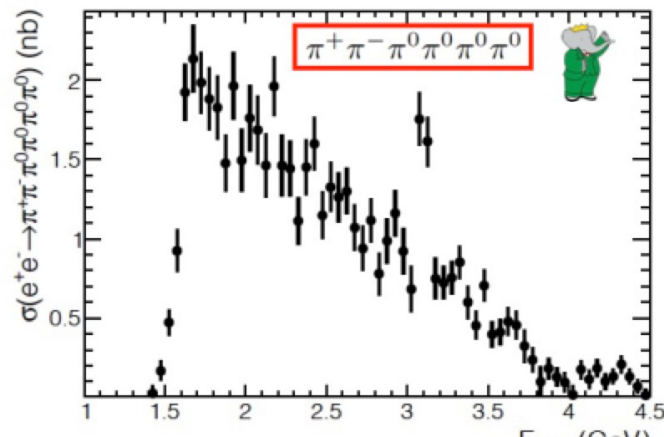
Events with 2 oppositely charged tracks, one γ_{ISR} photon candidate, 3 photon pairs with $m_{\gamma\gamma}$ compatible with π^0 and a 4-th photon pair

Signal events selected based on $\chi^2 < 70$; background from χ^2 sidebands

some additional cuts to reduce background



Fit the 4-th photon pair invariant mass distribution in bins of $m(\pi^+\pi^-3\pi^0\gamma\gamma)$ to determine the $\pi^+\pi^-4\pi^0$ and $\pi^+\pi^-3\pi^0\eta$ yields to determine cross sections



Substructures in $e^+e^- \rightarrow \pi^+\pi^-4\pi^0$

Intermediate states:

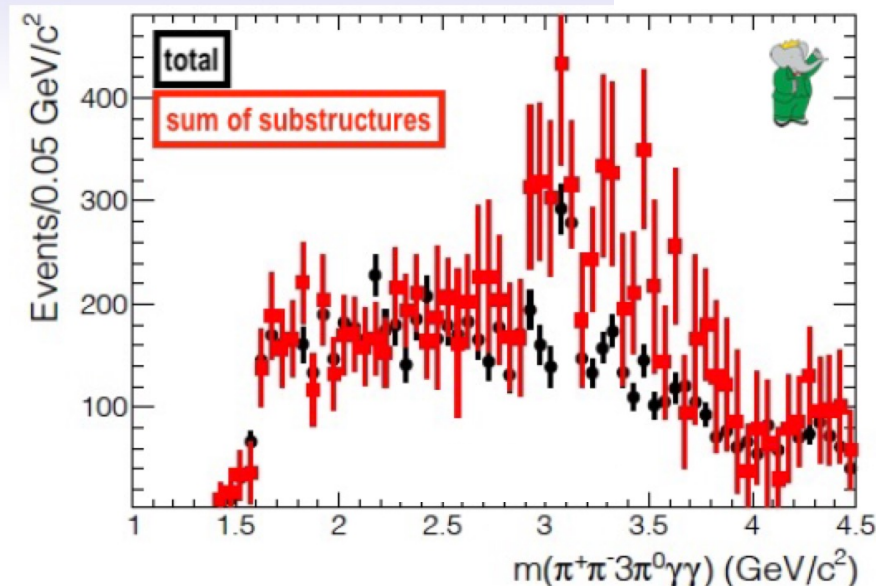
$$\pi^+\pi^-\pi^0\eta$$

$$\omega 3\pi^0$$

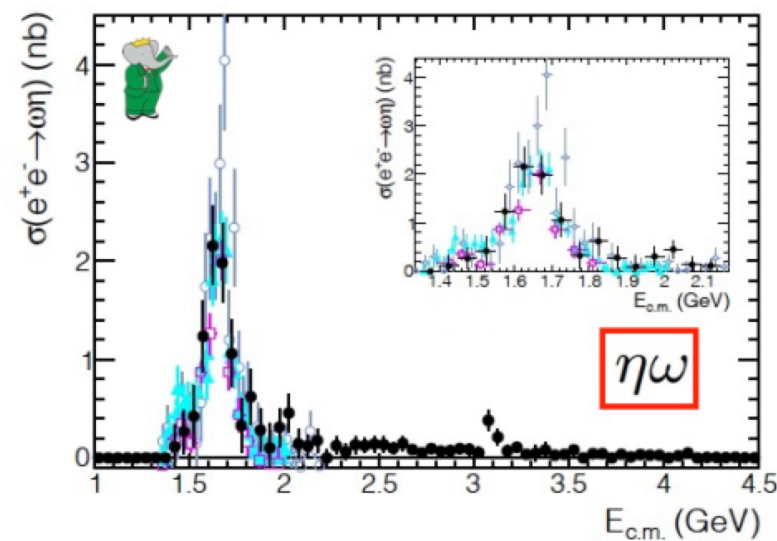
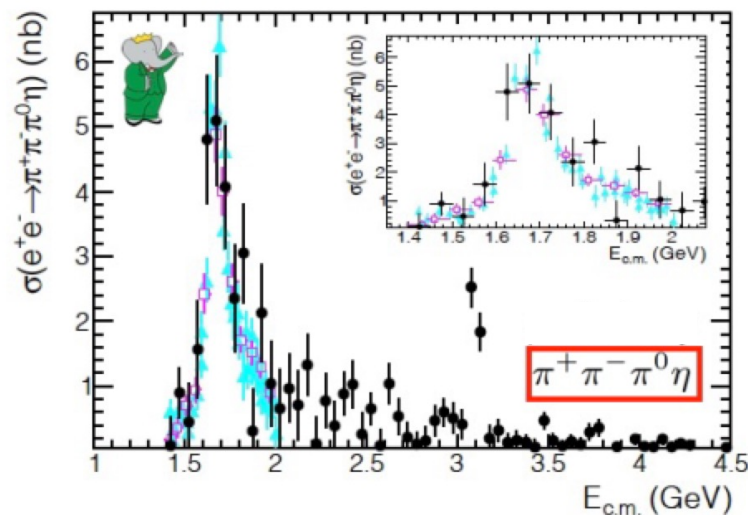
$$(\rho\pi)3\pi^0$$

(possibly $\rho^+\rho^-2\pi^0$ above 2.9 GeV)

Sum of intermediate states seem to saturate the observed cross section



Below 2 GeV agreement with SND and CMD-2 measurements of $\pi^+\pi^-\pi^0\eta$ and $\omega\eta$:



$e^+e^- \rightarrow \pi^+\pi^-4\pi^0$ and $\pi^+\pi^-3\pi^0\eta$: charmonium

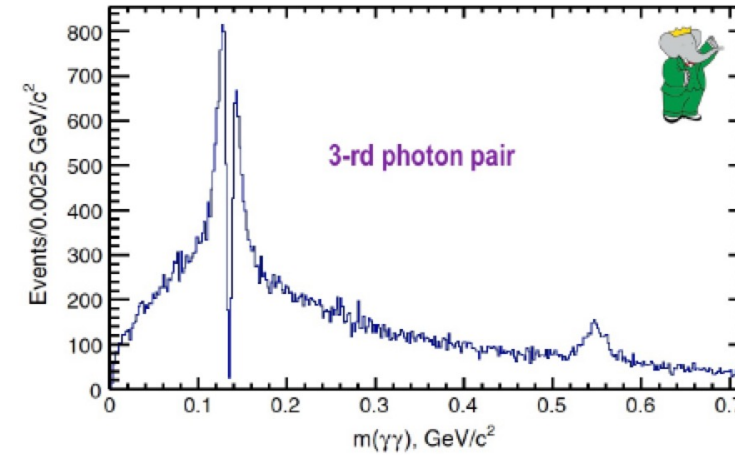
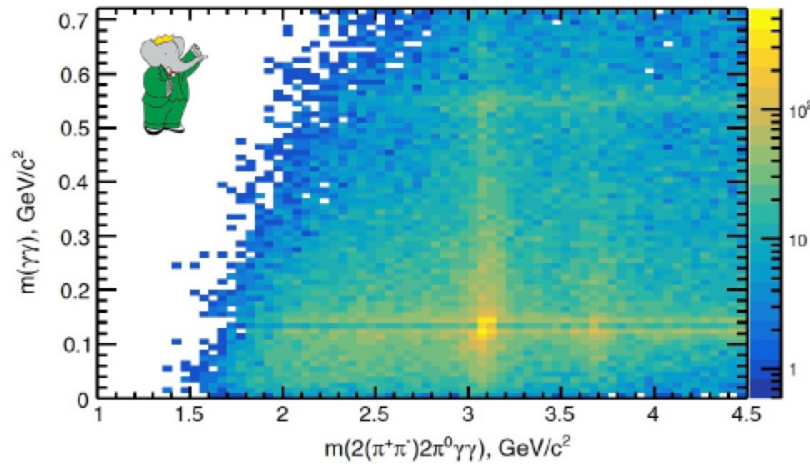
2110.00823 [hep-ex] Submitted to PRD

Measured Quantity	Measured Value (eV)	J/ψ or $\psi(2S)$ Branching Fraction (10^{-3})	
		Calculated, this work	PDG [28]
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0}$	$35.8 \pm 4.4 \pm 5.4$	$6.5 \pm 0.8 \pm 1.0$	no entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \eta\pi^+\pi^-\pi^0} \cdot \mathcal{B}_{\eta \rightarrow \pi^0\pi^0\pi^0}$	$21.1 \pm 1.7 \pm 3.2$	$11.9 \pm 0.9 \pm 2.3$	no entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \omega\eta} \cdot \mathcal{B}_{\omega \rightarrow \pi^+\pi^-\pi^0} \cdot \mathcal{B}_{\eta \rightarrow \pi^0\pi^0\pi^0}$	$4.9 \pm 2.1 \pm 0.7$	$3.0 \pm 1.3 \pm 0.5$	1.74 ± 0.20
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \omega\pi^0\pi^0\pi^0} \cdot \mathcal{B}_{\omega \rightarrow \pi^+\pi^-\pi^0}$	$9.4 \pm 2.3 \pm 1.5$	$1.9 \pm 0.5 \pm 0.3$	no entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0\eta} \cdot \mathcal{B}_{\eta \rightarrow \gamma\gamma}$	$10.6 \pm 1.6 \pm 1.6$	$4.9 \pm 0.8 \pm 0.8$	no entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0}$	$3.3 \pm 2.3 \pm 0.5$	$1.4 \pm 1.0 \pm 0.2$	no entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \eta\pi^+\pi^-\pi^0} \cdot \mathcal{B}_{\eta \rightarrow \pi^0\pi^0\pi^0}$	< 3.0 at 90% C.L.	< 3.5 at 90% C.L.	no entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \omega\eta} \cdot \mathcal{B}_{\omega \rightarrow \pi^+\pi^-\pi^0} \cdot \mathcal{B}_{\eta \rightarrow \pi^0\pi^0\pi^0}$	< 1.1 at 90% C.L.	< 1.4 at 90% C.L.	< 0.11 at 90% C.L.
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \omega\pi^0\pi^0\pi^0} \cdot \mathcal{B}_{\omega \rightarrow \pi^+\pi^-\pi^0}$	< 1.6 at 90% C.L.	< 0.8 at 90% C.L.	no entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0\eta} \cdot \mathcal{B}_{\eta \rightarrow \gamma\gamma}$	< 1.9 at 90% C.L.	< 2.0 at 90% C.L.	no entry

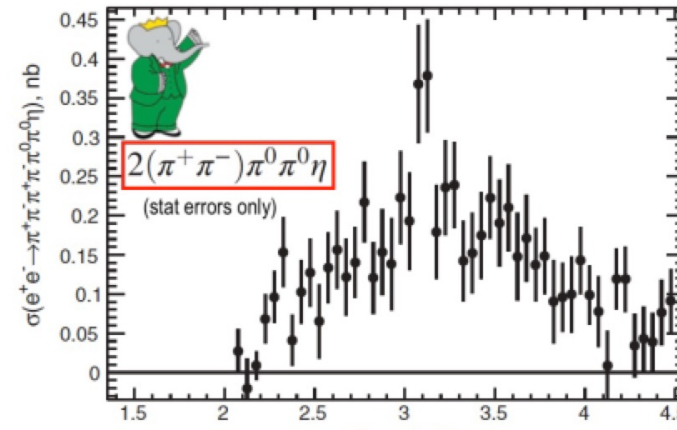
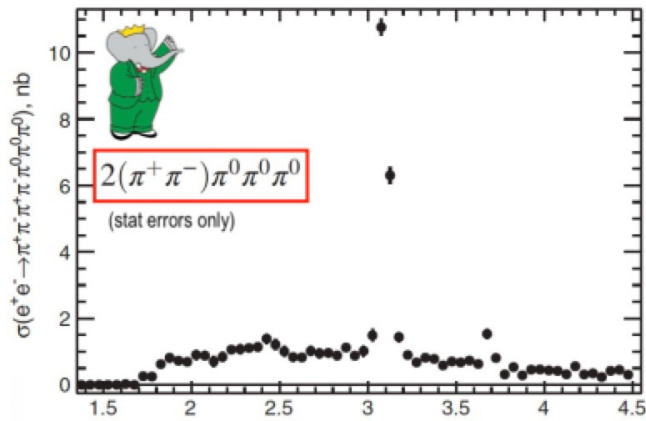
$$e^+e^- \rightarrow 2(\pi^+\pi^-)3\pi^0 \text{ and } 2(\pi^+\pi^-)2\pi^0\eta \text{ (first measurement)}$$

Events with 4 oppositely charged tracks, one γ_{ISR} photon candidate, 2 photon pairs with $m_{\gamma\gamma}$ compatible with π^0 and a 3-rd photon pair

Signal events selected based on $\chi^2 < 50$; background from χ^2 sidebands
some additional cuts to reduce background

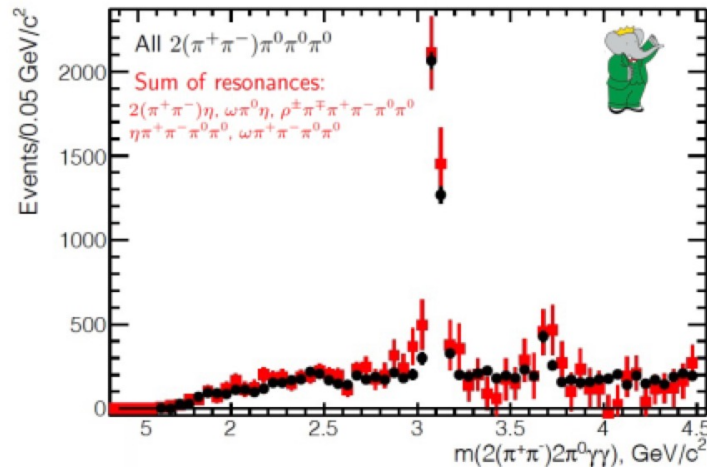
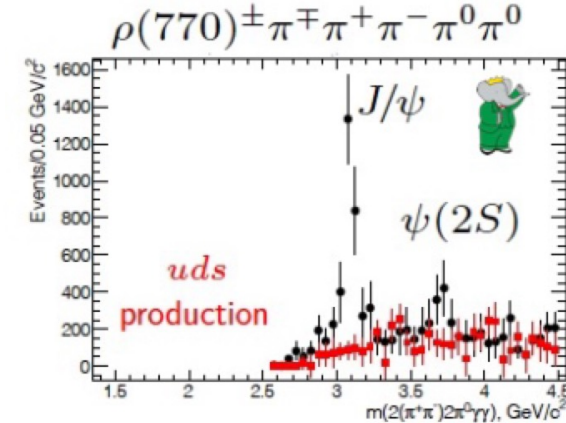
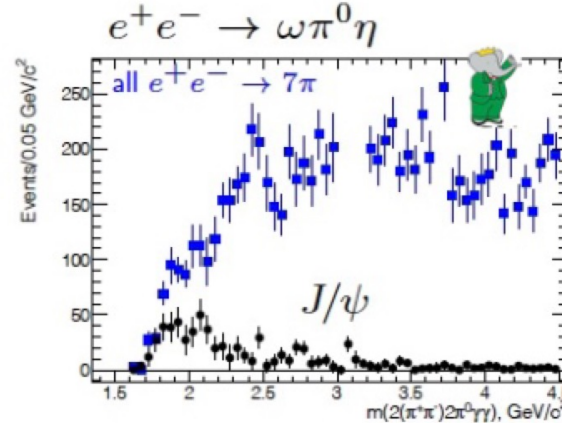
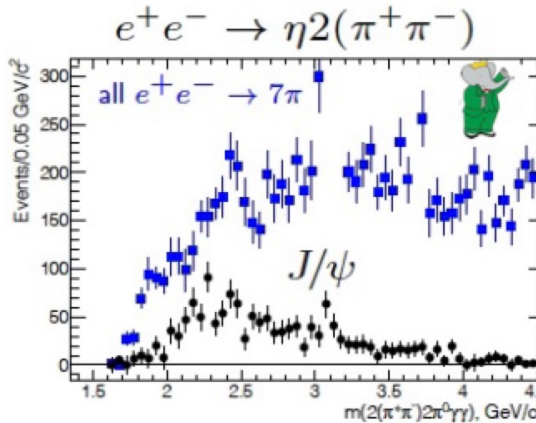
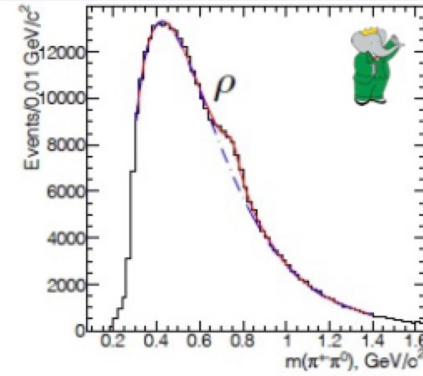
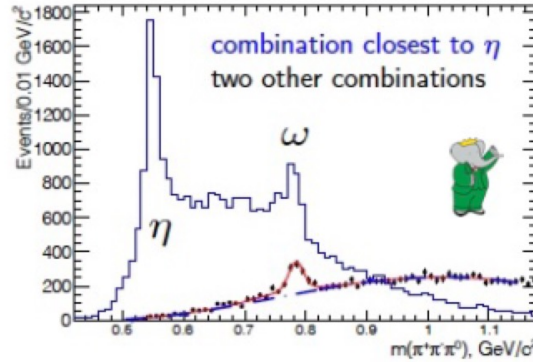
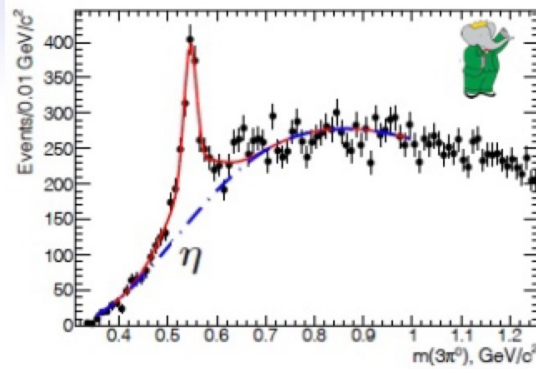


Fit the 3-rd photon pair mass distribution in bins of $m(2(\pi^+\pi^-)2\pi^0\gamma\gamma)$ to determine the $2(\pi^+\pi^-)3\pi^0$ and $2(\pi^+\pi^-)2\pi^0\eta$ yields to determine cross sections



Substructures in $e^+e^- \rightarrow 2(\pi^+\pi^-)3\pi^0$

Phys. Rev. D 103 092001 (2021)



(not shown: $\eta\pi^+\pi^-\pi^0$, $\omega\pi^+\pi^-\pi^0$)

No significant evidence for intermediate states other than $2(\pi^+\pi^-)\eta$, $\omega\pi^0\eta$, $\rho^\pm\pi^\mp\pi^+\pi^-\pi^0$, $\eta\pi^+\pi^-\pi^0$, $\omega\pi^+\pi^-\pi^0$

$$e^+e^- \rightarrow 2(\pi^+\pi^-)3\pi^0 \text{ and } 2(\pi^+\pi^-)2\pi^0\eta: \text{ charmonium}$$

Phys. Rev. D 103 092001 (2021)

Measured quantity	Measured value (eV)	J/ψ or $\psi(2S)$ branching fraction (10^{-3})	
		Calculated, this work	PDG [22]
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0}$	$345.0 \pm 10.0 \pm 50.0$	$62.0 \pm 2.0 \pm 9.0$	No entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \omega\pi^+\pi^-\pi^0\pi^0} \cdot \mathcal{B}_{\omega \rightarrow \pi^+\pi^-\pi^0}$	$165.0 \pm 9.0 \pm 25.0$	$33.0 \pm 2.0 \pm 5.0$	No entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \eta\pi^+\pi^-\pi^0\pi^0} \cdot \mathcal{B}_{\eta \rightarrow \pi^+\pi^-\pi^0}$	$6.0 \pm 4.0 \pm 1.0$	$4.8 \pm 3.2 \pm 0.8$	2.3 ± 0.5
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \pi^+\pi^-\pi^+\pi^-\eta} \cdot \mathcal{B}_{\eta \rightarrow \pi^0\pi^0\pi^0}$	$5.6 \pm 2.6 \pm 0.8$	$2.6 \pm 1.2 \pm 0.5$	2.26 ± 0.28
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \rho^+\pi^-\pi^+\pi^-\pi^0\pi^0}$	$155.0 \pm 26.0 \pm 36.0$	$28.0 \pm 4.7 \pm 6.6$	No entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \rho^+\rho^-\pi^+\pi^-\pi^0}$	$32.0 \pm 13.0 \pm 15.0$	$5.7 \pm 2.4 \pm 2.7$	No entry
$\Gamma_{ee}^{J/\psi} \cdot \mathcal{B}_{J/\psi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0\eta} \cdot \mathcal{B}_{\eta \rightarrow \gamma\gamma}$	$9.1 \pm 2.6 \pm 1.4$	$4.2 \pm 1.2 \pm 0.6$	No entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0}$	$33.0 \pm 5.0 \pm 5.0$	$14.0 \pm 2.0 \pm 2.0$	No entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow J/\psi\pi^0\pi^0} \cdot \mathcal{B}_{J/\psi \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0}$	$14.8 \pm 2.6 \pm 2.2$	$34.7 \pm 6.1 \pm 5.2$	33.7 ± 2.6
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow J/\psi\pi^+\pi^-} \cdot \mathcal{B}_{J/\psi \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0}$	$19.2 \pm 4.5 \pm 3.2$	$23.8 \pm 5.6 \pm 3.6$	27.1 ± 2.9
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \omega\pi^+\pi^-\pi^0\pi^0} \cdot \mathcal{B}_{\omega \rightarrow \pi^+\pi^-\pi^0}$	$18.0 \pm 4.0 \pm 3.0$	$8.7 \pm 1.9 \pm 1.5$	No entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0\eta} \cdot \mathcal{B}_{\eta \rightarrow \gamma\gamma}$	$<1.9 \text{ at } 90\% \text{ C.L.}$	$<2.0 \text{ at } 90\% \text{ C.L.}$	No entry
$\Gamma_{ee}^{\psi(2S)} \cdot \mathcal{B}_{\psi(2S) \rightarrow \pi^+\pi^-\pi^+\pi^-\eta} \cdot \mathcal{B}_{\eta \rightarrow \pi^0\pi^0\pi^0}$	$<2.3 \text{ at } 90\% \text{ C.L.}$	$<2.4 \text{ at } 90\% \text{ C.L.}$	1.2 ± 0.6

Conclusions

New measurement of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section

- based on the entire *BABAR* dataset
- measured in the range $0.62 \div 3.5$ GeV
- 1.3% systematic uncertainty near the maxima of $\omega(782)$ and $\phi(1020)$
- the error on the leading order contribution to muon magnetic anomaly from $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ ($E < 2$ GeV) reduced by a factor ≈ 2

2110.00520 [hep-ex] Submitted to PRD

First measurements of $e^+e^- \rightarrow \pi^+\pi^-4\pi^0$ and $e^+e^- \rightarrow \pi^+\pi^-3\pi^0\eta$ cross sections

2110.00823 [hep-ex] Submitted to PRD

- $e^+e^- \rightarrow \pi^+\pi^-4\pi^0$ cross section seem to be saturated by intermediate states:
 $\pi^+\pi^-\pi^0\eta$, $\omega 3\pi^0$, $(\rho\pi)3\pi^0$ and possibly $\rho^+\rho^-2\pi^0$ intermediate states
- new J/ψ and $\psi(2S)$ decay modes

First measurements of $e^+e^- \rightarrow 2(\pi^+\pi^-)3\pi^0$ and $e^+e^- \rightarrow 2(\pi^+\pi^-)2\pi^0\eta$ cross sections

Phys. Rev. D 103 092001 (2021)

- $e^+e^- \rightarrow 2(\pi^+\pi^-)3\pi^0$ cross section seem to be saturated by intermediate states:
 $2(\pi^+\pi^-)\eta$, $\omega\pi^0\eta$, $\rho^\pm\pi^\mp\pi^+\pi^-2\pi^0$, $\eta\pi^+\pi^-2\pi^0$, $\omega\pi^+\pi^-2\pi^0$
- new J/ψ and $\psi(2S)$ decay modes

BACKUP



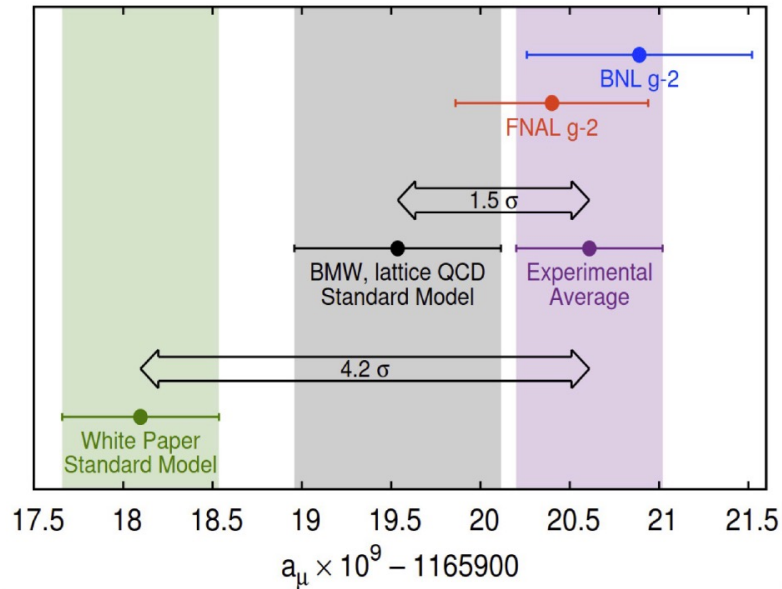
$a_\mu = \frac{1}{2}(g - 2)_\mu$ precision test of SM

SM prediction for muons: $a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{hadr}$

absolute value dominated by $a_\mu^{QED} + a_\mu^{EW}$

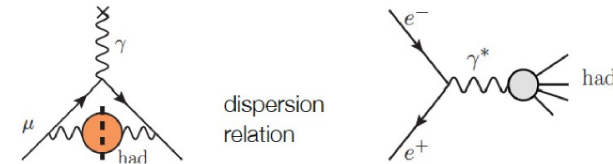
ERROR dominated by a_μ^{hadr} : not calculable perturbatively

a_μ^{hadr} : LQCD or data-driven dispersive approach



4.2 σ (WP/SM)

or 1.5 σ (LQCD/SM)



dispersion relation

$K(s)$: analytically known kernel function

$$a_\mu^{\text{had, LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \text{ experimental input}$$

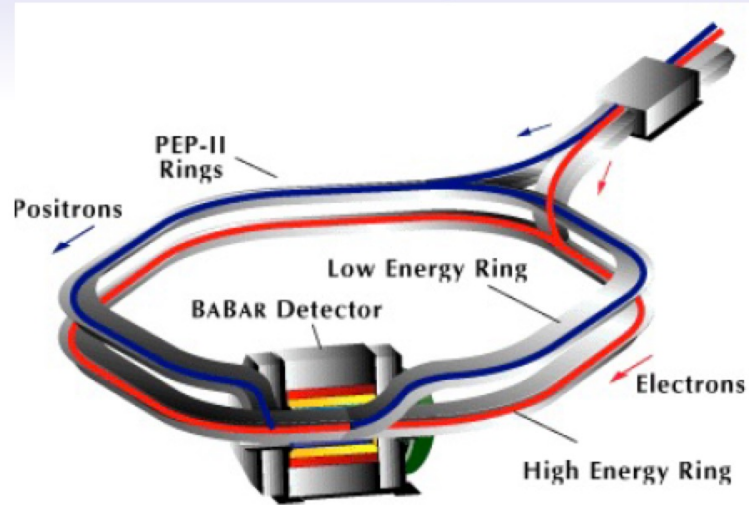
relies on hadronic cross section measurements

Largest contribution to the integral from hadronic cross section at low energies

For $\sqrt{s} \lesssim 2 \text{ GeV}$ finite number of final states contribute:

$\sigma(e^+e^- \rightarrow \text{hadrons})$ can be obtained as sum of all exclusive cross sections

The *BABAR* experiment



PEP-II **asymmetric e^+e^- collider** operating at center of mass energies near the $\Upsilon(4S)$ (for most of the time)

$$\sqrt{s} = 10.58 \text{ GeV}/c^2$$

General-purpose detector

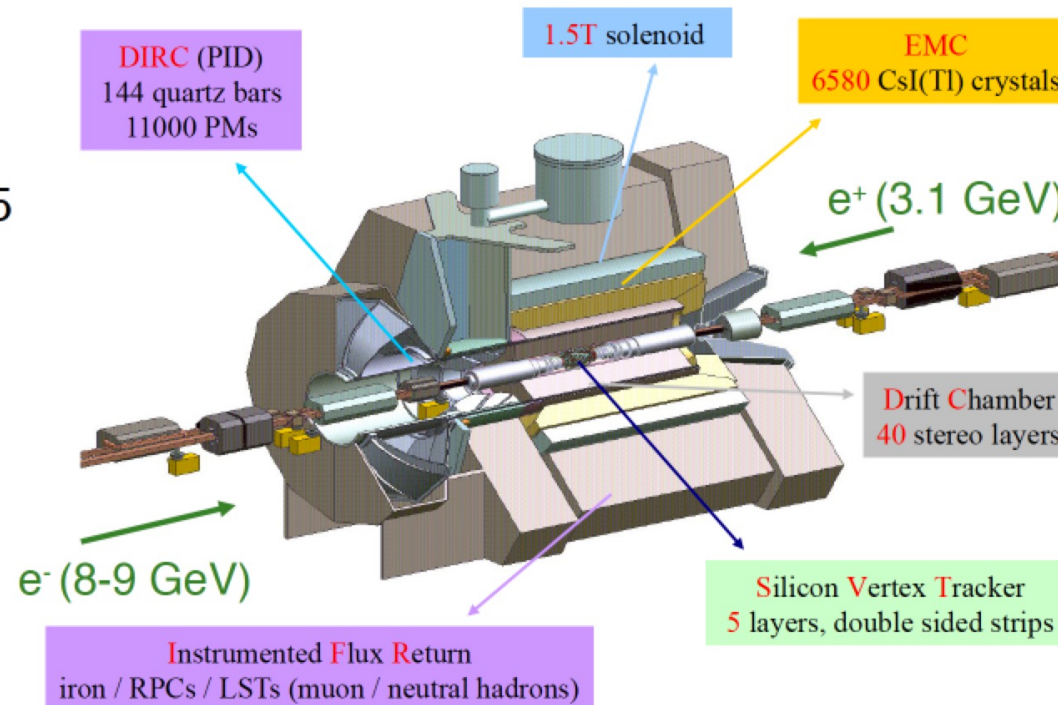
Asymmetric detector:

$$-0.9 < \cos \theta^* < 0.85$$

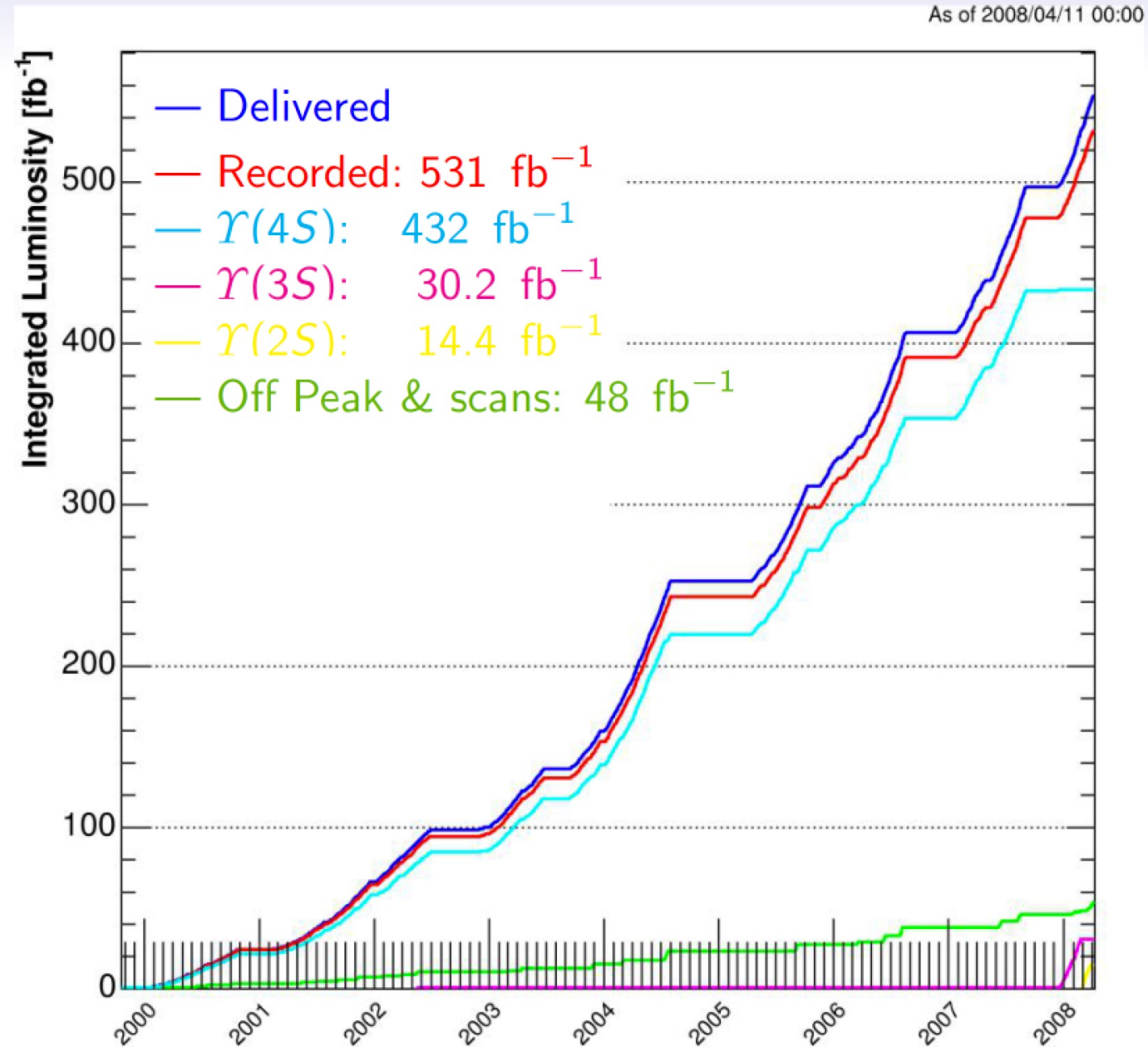
wrt electron beam

excellent performance:

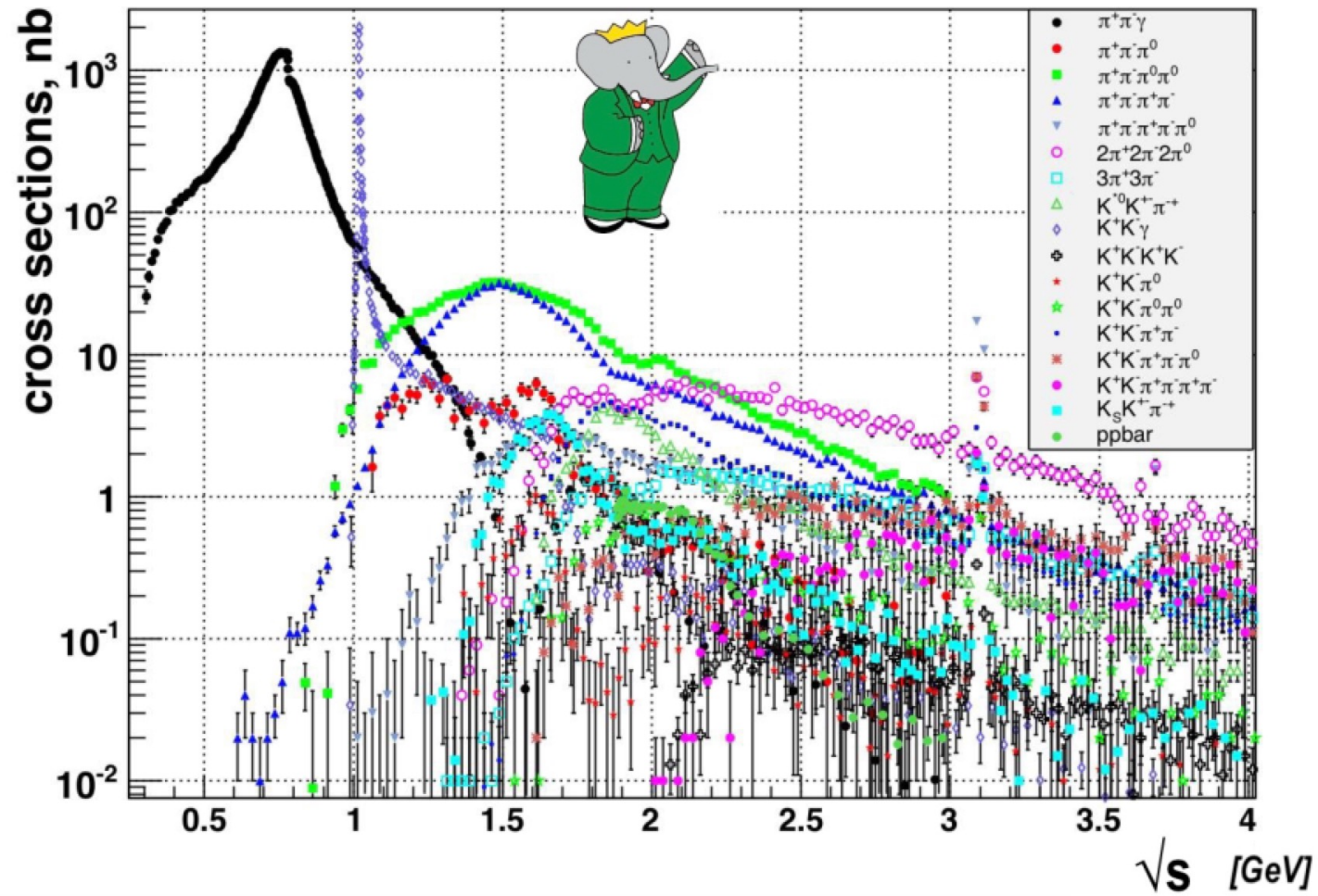
- vertexing
- tracking
- PID
- calorimeter



Data samples



Light hadrons cross sections measured by *BABAR*



Substructures in $e^+e^- \rightarrow \pi^+\pi^-\pi^0$

