Studies of LO and NLO FSR in the BABAR ISR $\mu^+\mu^-/\pi^+\pi^-/K^+K^-$ analyses

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The BaBar ISR method for $\mu\mu\gamma(\gamma)$, $\pi\pi\gamma(\gamma)$, $KK\gamma(\gamma)$

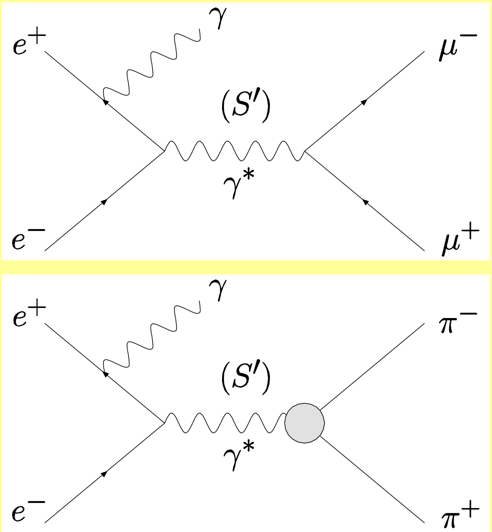
- $e^+ e^- \to \mu^+ \mu^- \gamma\ (\gamma)$ and $\pi^+ \pi^- \gamma\ (\gamma)$, $K^+K^- \gamma(\gamma)$ measured simultaneously and separated by PID
- ISR photon detected at large angle
- kinematic fits with additional ISR or FSR photon

$$x = 2E^*_\gamma / \sqrt{s}$$

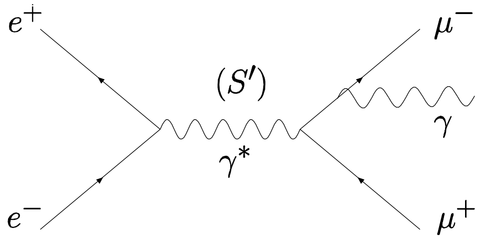
$$s' = s(1-x)$$

- measure ratios $\pi\pi/\mu\mu$ $KK/\mu\mu$
- ISR lumi drops out
- AfkQed +Phokhara generators
- Data/MC corrections for all efficiencies
- Loose cuts on kinematic fits to minimize inefficiency and related systematic effects

LO ISR

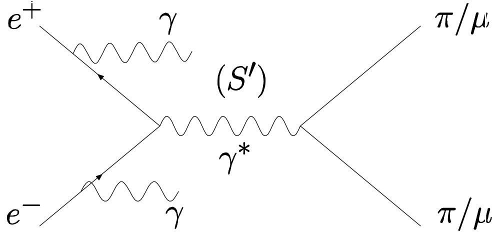


LO FSR

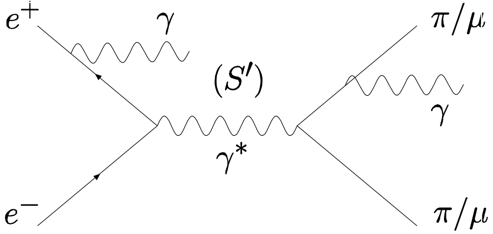


LO FSR expected to be very small for $\pi\pi$ at s~(10.6 GeV)^2
But how small?

LO + NLO ISR



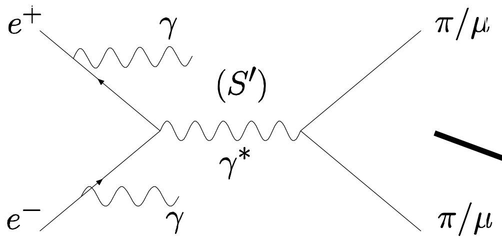
LO ISR + NLO FSR

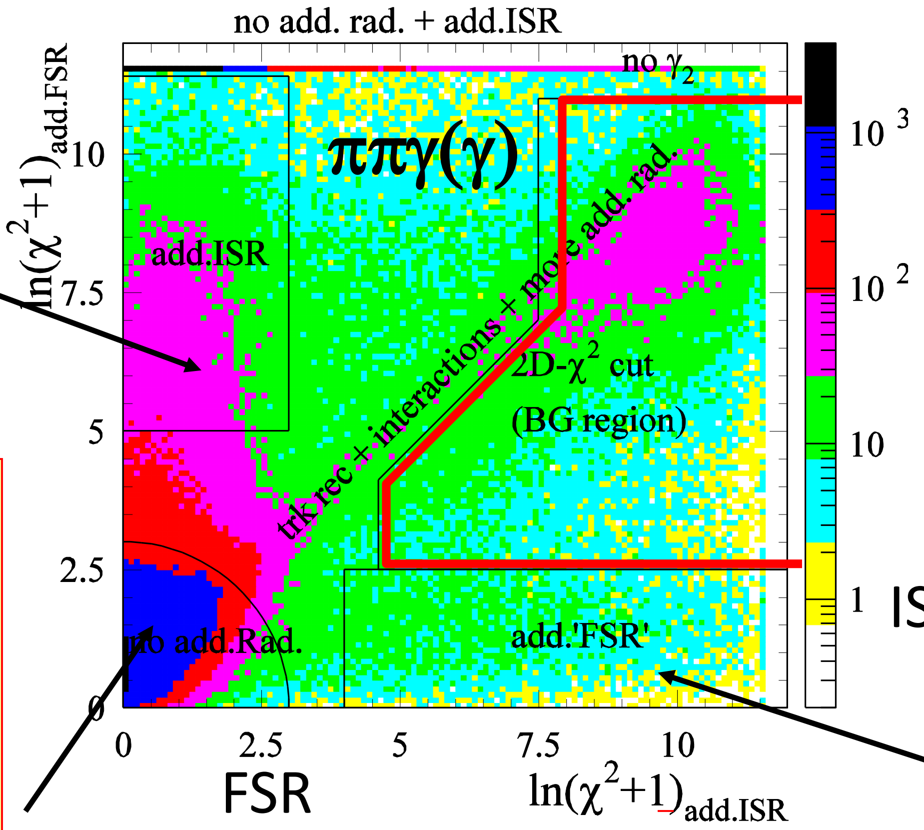


The BABAR ISR method for 2π (2001-2009)

Only experiment so far directly measuring cross section at NLO order $\pi\pi\gamma(\gamma)$ $\mu\mu\gamma(\gamma)$

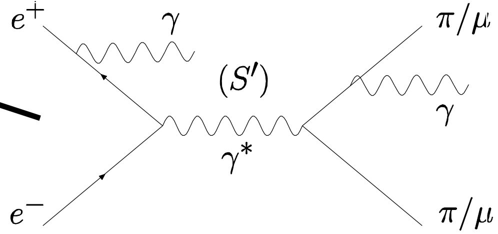
ISR + add. ISR



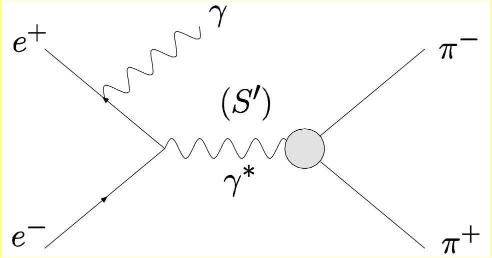


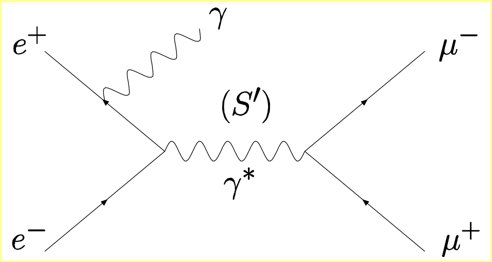
background

ISR + add. FSR (also large-angle ISR)

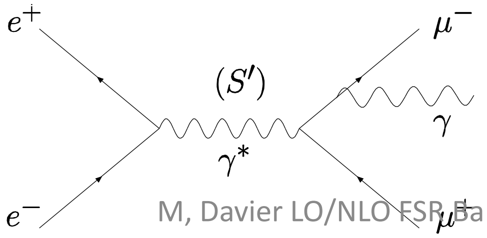


ISR



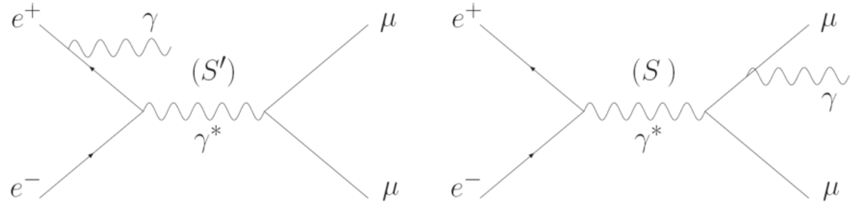


FSR



Subject 1: LO FSR contribution in ''ISR'' experiments

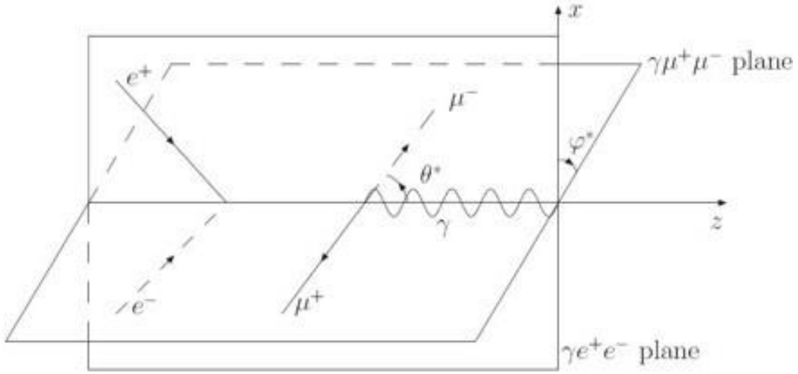
- ISR method used to measure hadronic cross sections: ee→Xγ X:QED ($\mu^+\mu^-$) or hadronic final states ($\pi^+\pi^-$)
- but radiation can be from initial state (LO ISR) or final state (LO FSR)



- LO FSR contribution (by theoretical prediction/estimation):
  - QED for $\mu\mu\gamma_{FSR}$ (use QED generators, AfkQed/Phokhara)
  - model dependent estimation for $\pi\pi\gamma_{FSR}$: very small if initial e^+e^- energy large (BABAR 10.58 GeV)
- how small is FSR for $\pi\pi\gamma$? BABAR analysis, *Phys.Rev.D* 92 (2015) 7, 072015; arxiv:1508.04008
- hard to do direct measurement, but the interference between the FSR and ISR amplitudes can be accessed through a charge asymmetry (C=± 1)

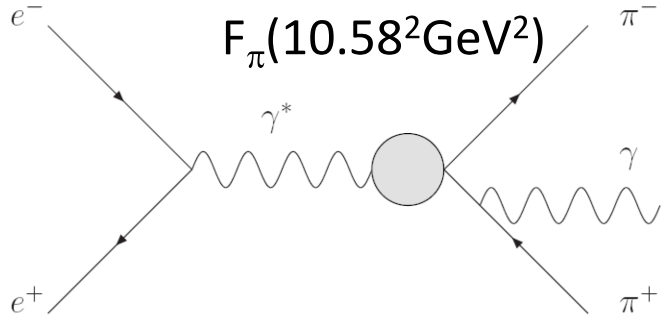
$$\sigma \propto |\mathcal{M}|^2 = |\mathcal{M}_{\rm ISR}|^2 + |\mathcal{M}_{\rm FSR}|^2 + 2\mathcal{R}e(\mathcal{M}_{\rm ISR}\mathcal{M}^*_{\rm FSR})$$

$$A = \frac{|\mathcal{M}|^2 - |\mathcal{M}_{x^+\leftrightarrow x^-}|^2}{|\mathcal{M}|^2 + |\mathcal{M}_{x^+\leftrightarrow x^-}|^2} = \frac{2\mathcal{R}e(\mathcal{M}_{\rm ISR}\mathcal{M}^*_{\rm FSR})}{|\mathcal{M}_{\rm ISR}|^2 + |\mathcal{M}_{\rm FSR}|^2} = A_0 \cos\phi^*$$

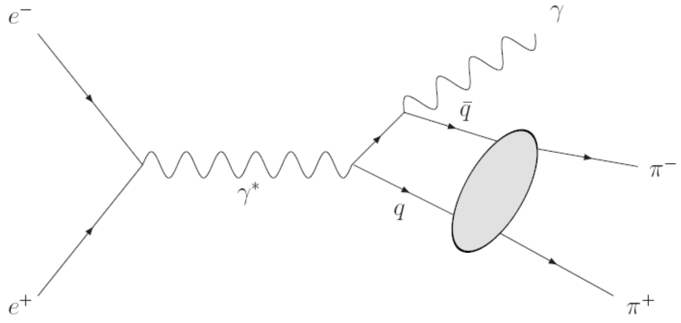


(b) In the x^+x^- c.m.

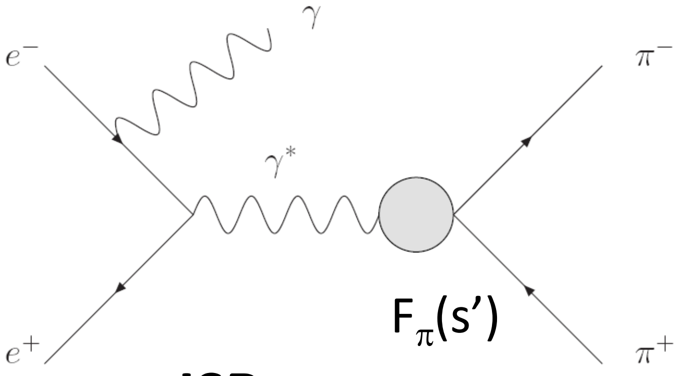
LO FSR models for $e^+e^- \rightarrow \pi^+\pi^-\gamma$



FSR model 1
(FSR from point-like pion)
used in Phokhara



FSR model 2
(FSR from quarks)



ISR
(from experiment)

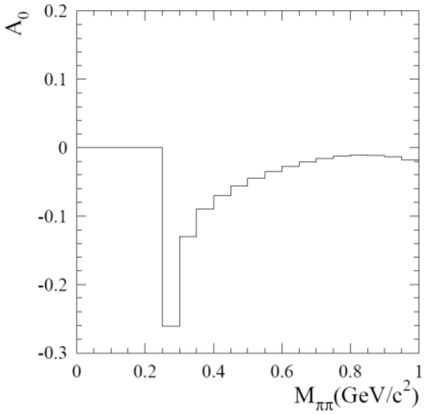
Z. Lu and I. Schmidt, Phys. Rev. D **73**, 094021 (2006); Erratum, Phy. Rev. D **75**, 099902(E) (2007)

M. Diehl, T. Gousset and B. Pire, Phys. Rev. D 62, 073014 (2000)

- pattern: A_0 crosses 0 at ρ peak
- $F_\pi(10.58^2 GeV^2)$ estimated from extrapolating CLEO and BaBar data
- $A_0 \sim 2\times10^{-3} (m_{\pi\pi}^2 - m_\rho^2)$ (too small to be measured)

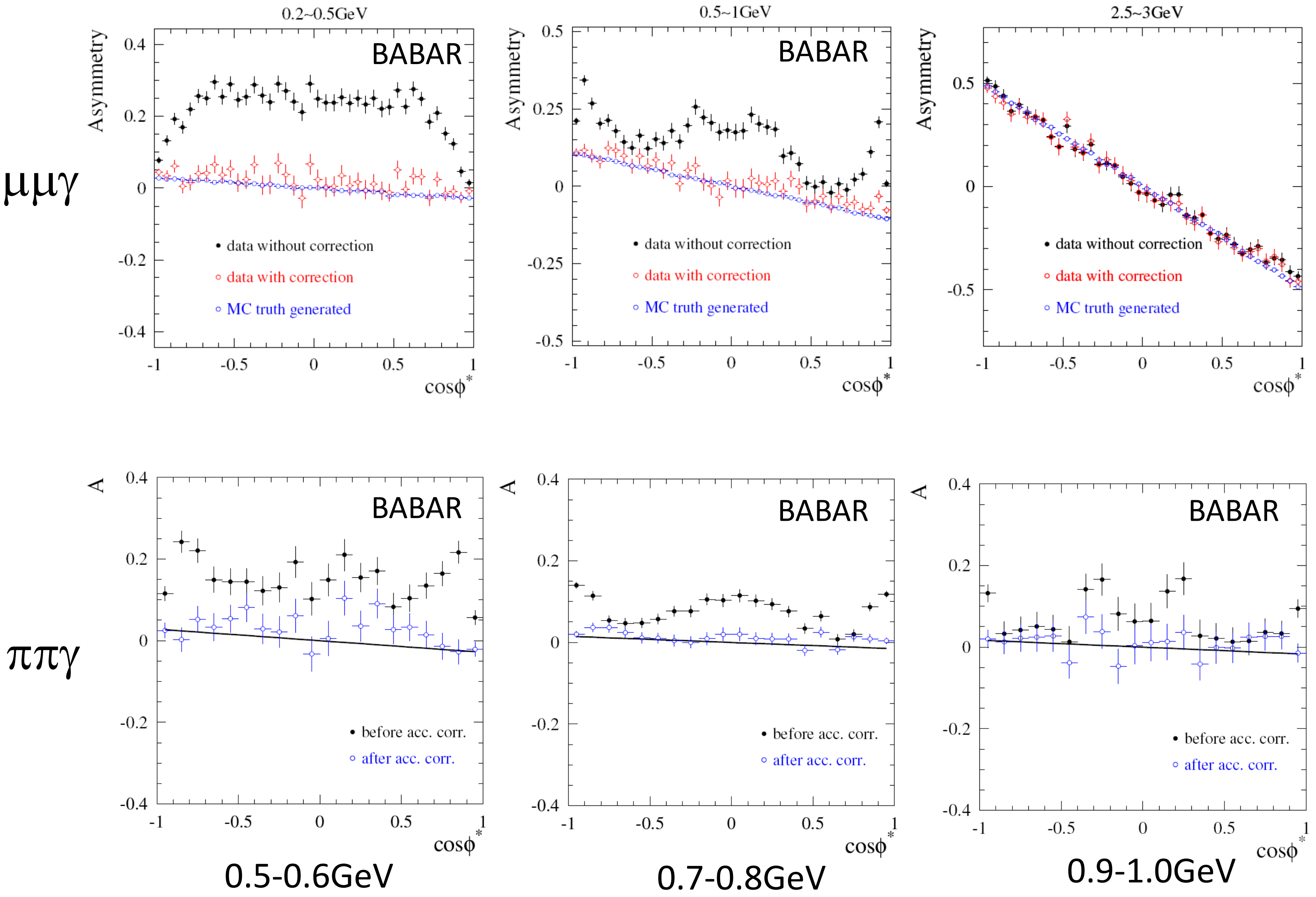
quark-antiquark $\rightarrow\pi\pi$ amplitude dominated by C=+1 states: $f_0(500)$, $f_2(1270)$

- $A_0 \sim$ -1% around ρ (measureable)
- This analysis => independent information about C even S-wave and D-wave states

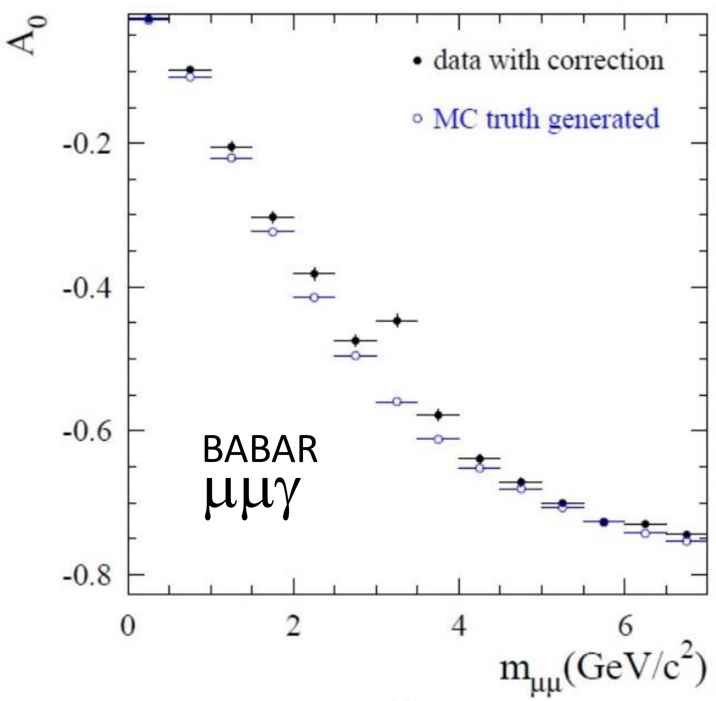


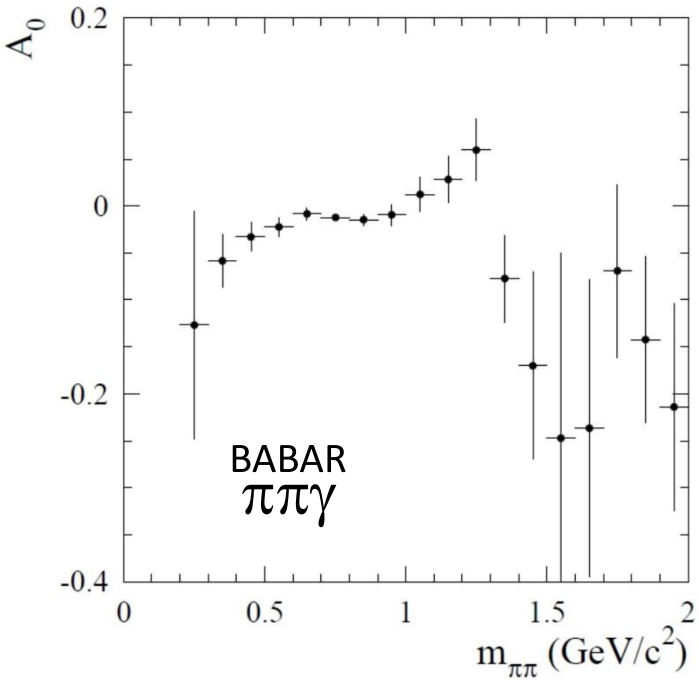
Measured Asymmetry

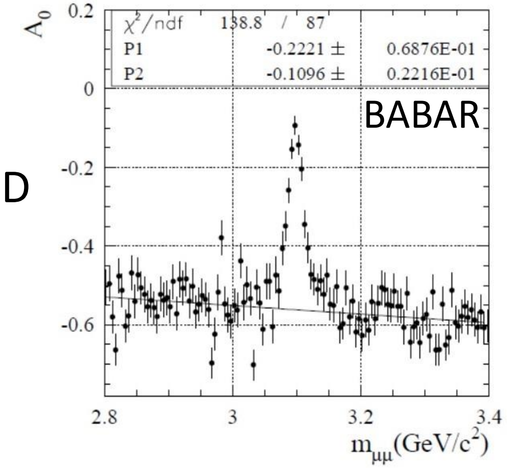
- Many detailed studies to track biases and obtain efficiency corrections (see paper)
- Corrections large at low masses (track overlap in drift chamber), but small at large mass
- cosϕ* slope A_0 (asymmetry of the asymmetry) insensitive to corrections

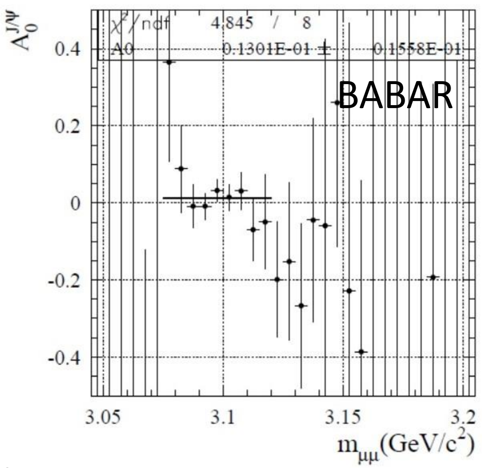


Slope of the charge asymmetry A_0





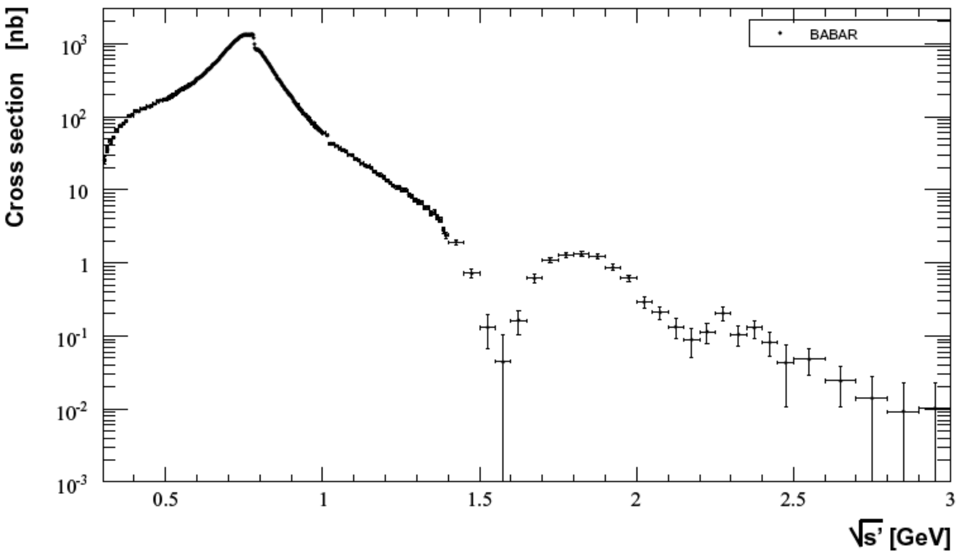


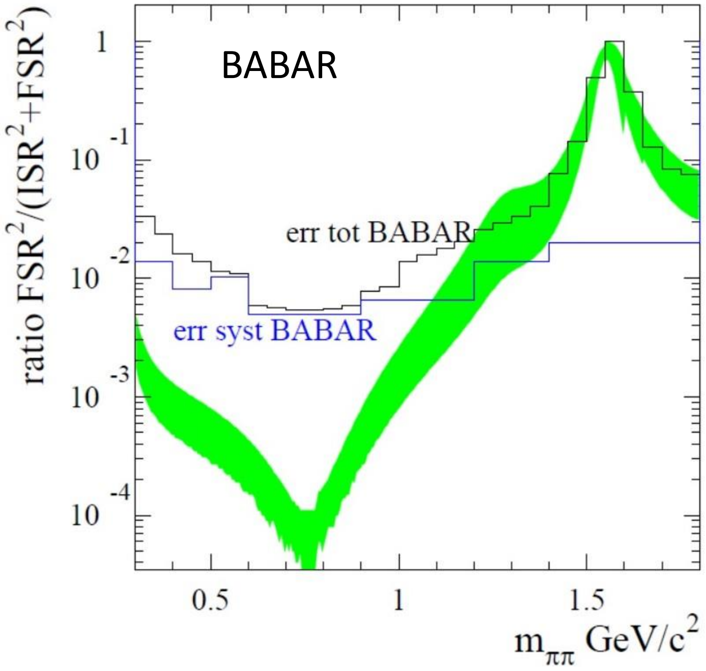


- Fair agreement with QED
- Origin of deviation
- Test of null asymmetry on $J/\psi \to \mu\mu$
  $A_0(J/\psi) = (1.3 \pm 1.6)\%$

- Small FSR in the ρ region
- Agreement with model 2
- FSR from quarks
- Consistency with $f_0 + f_2$ contributions

Impact on ISR $\sigma(ee\rightarrow\pi\pi)$measurement and a_μ





- FSR contribution negligible in ρ mass region
- comparable with the total error of the cross section measurement above 1.2GeV
- Measured F_π between 1.5 and 1.6 GeV very small ~0

	$\pi\pi$ ($2m_\pi$~1.8GeV)	$\pi\pi$ FSR($2m_\pi$~1.8GeV)
$a_\mu^{had}(10^{-10})$	$514.09\pm2.22\pm3.11$	0.26 ± 0.12

negligible effect

Subject 2: additional FSR radiation (NLO) for $\mu\mu\gamma$ and $\pi\pi\gamma$

Phys.Rev.Lett. 103 (2009) 231801; arxiv: 0908.3589 *Phys.Rev.D* 86 (2012) 032013; arxiv: 1205.2228

- Add ISR: assumed to be along one of the beams and fitted
  should be well predicted for both channels NLO QED (initial state)
  common to $\mu\mu\gamma\gamma$ and $\pi\pi\gamma\gamma$ (part of ISR lumi which drops out in the $\pi\pi\gamma/\mu\mu\gamma$ ratio)
  will not be covered in this talk

- Add FSR: here the add γ is detected, can be either true FSR emitted by μ/π or a large-angle ISR
  the 2 contributions can be separated by looking at the angular distribution of add γ with respect to the closest μ/π

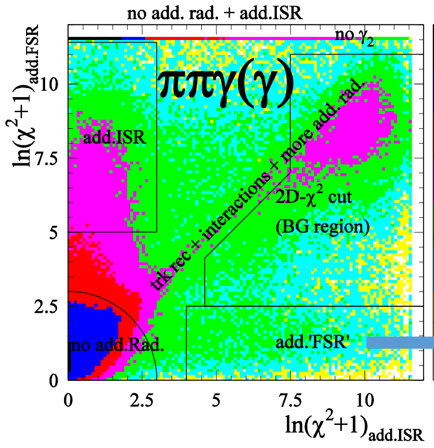
- The study of additional radiation is an important test for the consistency of our measurement in the case of ISR (small and large angle) since it can be compared to NLO QED.

- For add FSR the interest is threefold:
  (1) for muons it is again a test of NLO QED
  (2) for pions, it is a test of the model used in AfkQed (PHOTOS) and Phokhara (scalar QED assuming point-like pions)
  (3) as such, any discrepancy data/MC will affect the analysis and must be corrected for.

Additional large-angle radiation (FSR and large-angle ISR)

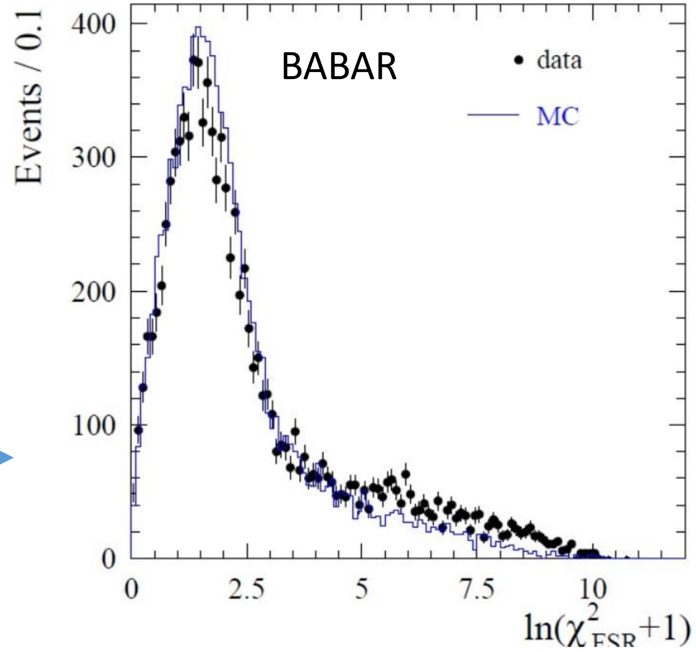
- Kinematic fit using 2 charged tracks + large-angle energetic ISR + additional large-angle measured photon (4 constraints)

- fitted $E_{\gamma\ add} > 0.2$ GeV

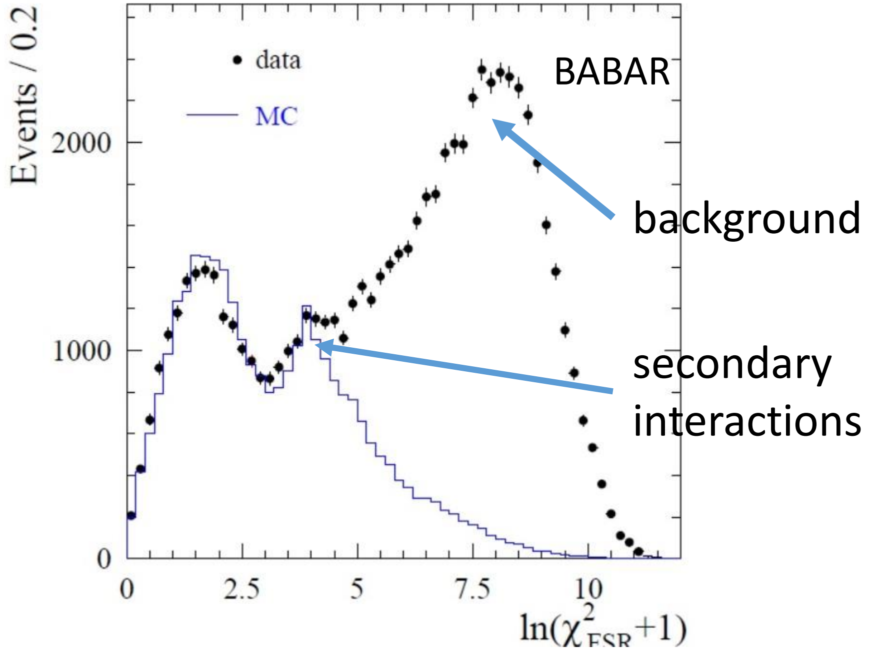


before 2D χ^2 cut

μμγγ



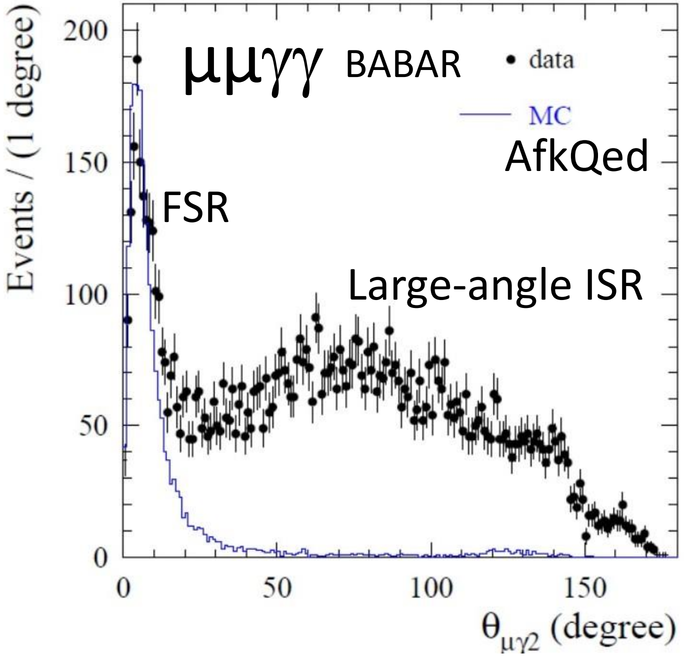
ππγγ



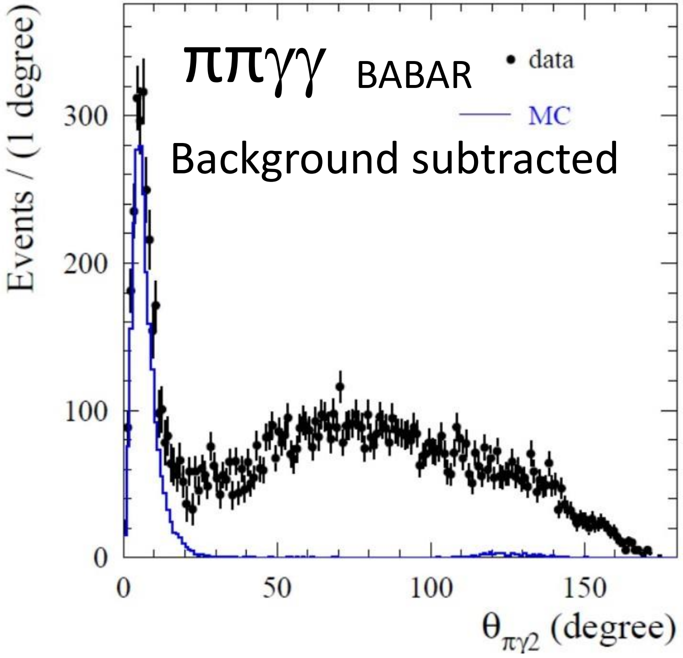
Additional large-angle radiation: separating FSR/large angle ISR

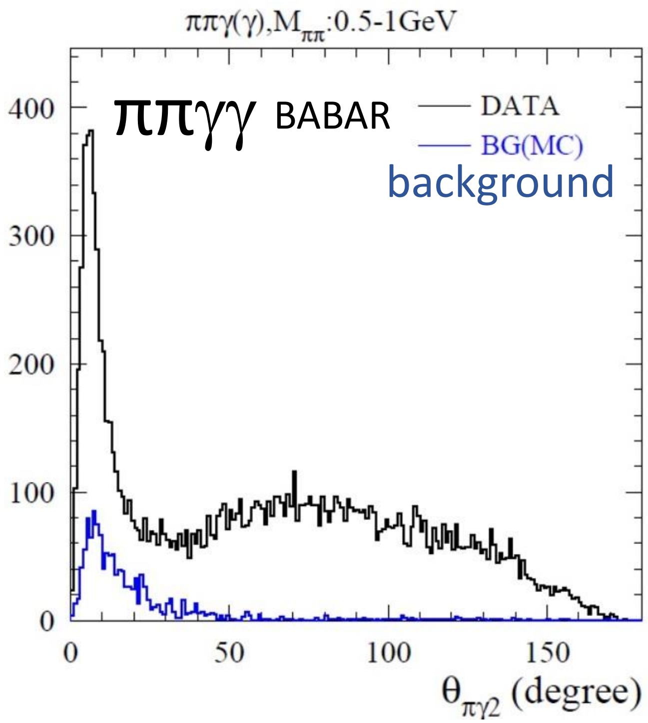
- Distributions of the minimum angle between γ_{add} and muons/pions, $E_{add\ \gamma\ FSR} > 0.2$ GeV

0.2<mass<1.0 GeV



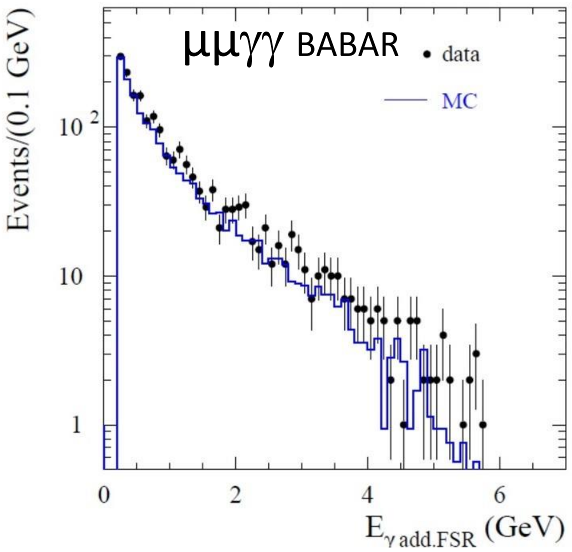
0.5<mass<1.0 GeV

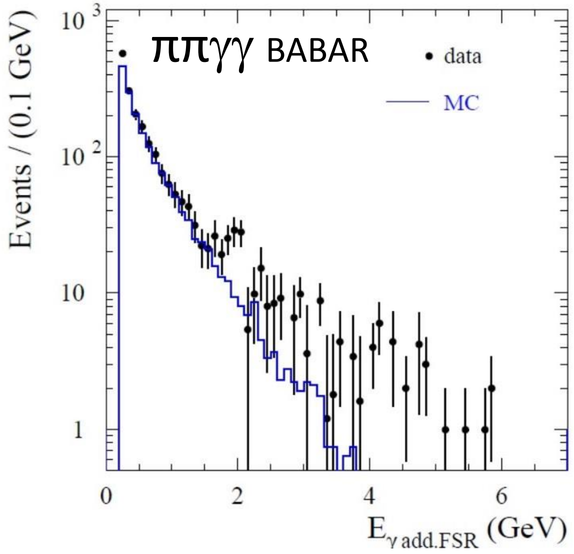




- Some background in the FSR peak (only for pions), mainly from $\pi^+\pi^-\pi^0\gamma$, subtracted
- Phokhara reproduces well the large-angle ISR distribution, absent in AfkQed

Results on additional FSR





- FSR energy distributions in fair agreement with prediction (AfkQed: QED for muons, PHOTOS for pions) above 0.2 GeV
- FSR rate obtained from event statistics for $\theta_{min} < 20^0$ after subtracting an estimated small large-angle ISR contribution
- FSR rate in agreement with QED for muons: data/QED = 0.96 ± 0.06
- Excess observed for pions above the prediction: data/PHOTOS = 1.21 ± 0.05 taking into account systematic uncertainty from background subtraction from well-studied $\pi^+\pi^-\pi^0\gamma$ process
- However, in our experiment, due to the loose cut on the χ^2 of the kinematical fits (still rejecting 35% of the FSR events for $E_{add\,\gamma\,FSR} > 0.2$ GeV), this excess modifies the selection efficiency by only (0.6 ± 0.2) per mil
- LO experiments without direct access to FSR would be affected more significantly.

Summary and Conclusions

- In this short talk I presented two published original studies on radiative effects performed with BABAR which are relevant to the measurement of $e^+e^- \to \pi^+\pi^-(\gamma)$ cross section and the muon g-2 HVP prediction by the dispersive method.

- Through a charge asymmetry measurement it has been possible to extract the LO FSR amplitude at the CM energy of 10.58 GeV and show that it is well explained by radiation from quarks which recombine into C=+1 resonances. We computed its effect on the measured $\pi^+\pi^-$ spectrum obtained by the ISR method. In this way we estimated that the previous neglect of the $|FSR|^2$ contribution corresponds to an increase of $a_\mu^{2\pi}$ by $(2.6 \pm 1.2)\text{x}10^{-11}$, i.e. one order of magnitude less that the uncertainty on this quantity.

- The BABAR ISR analysis of $\mu\mu\gamma(\gamma)$, $\pi\pi\gamma(\gamma)$, $KK\gamma(\gamma)$ is so far the only direct NLO measurement of these cross sections. Hence it has been possible to study the radiative processes with additional radiation beyond LO ISR. Here we have emphasized the NLO FSR production which relies on a model using scalar QED with pointlike pions in all existing MC generators. Whereas NLO FSR is well described by QED for muons, an excess of $(21 \pm 5)\%$ is found for pions, which has implications for the cross section measurement.