NN Models

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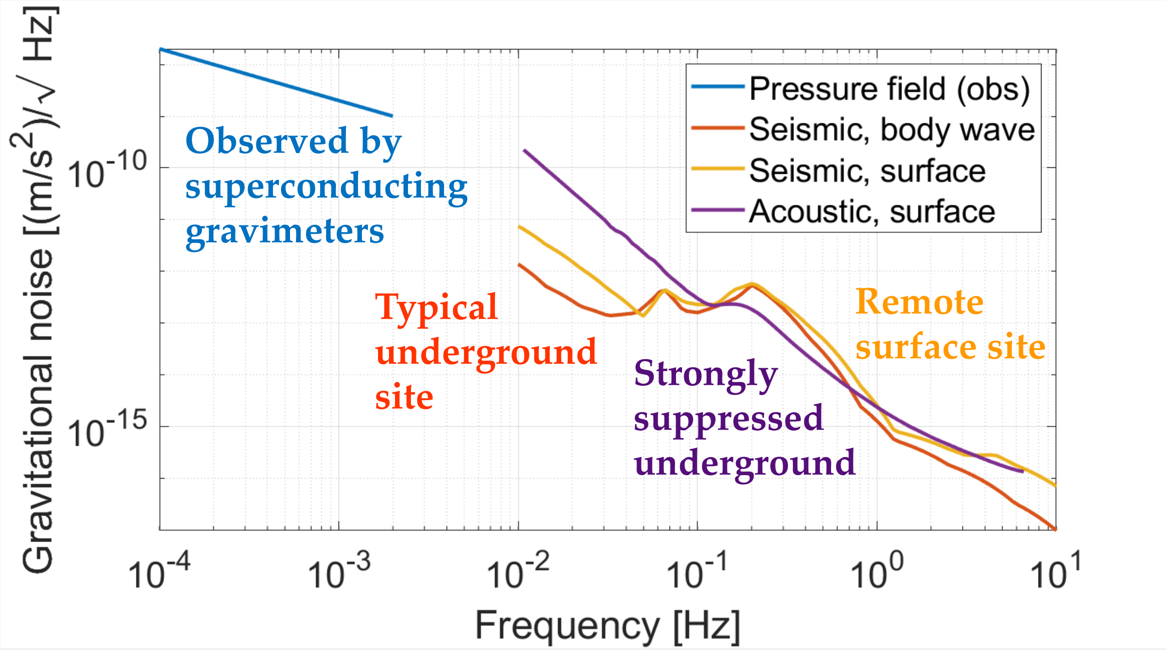
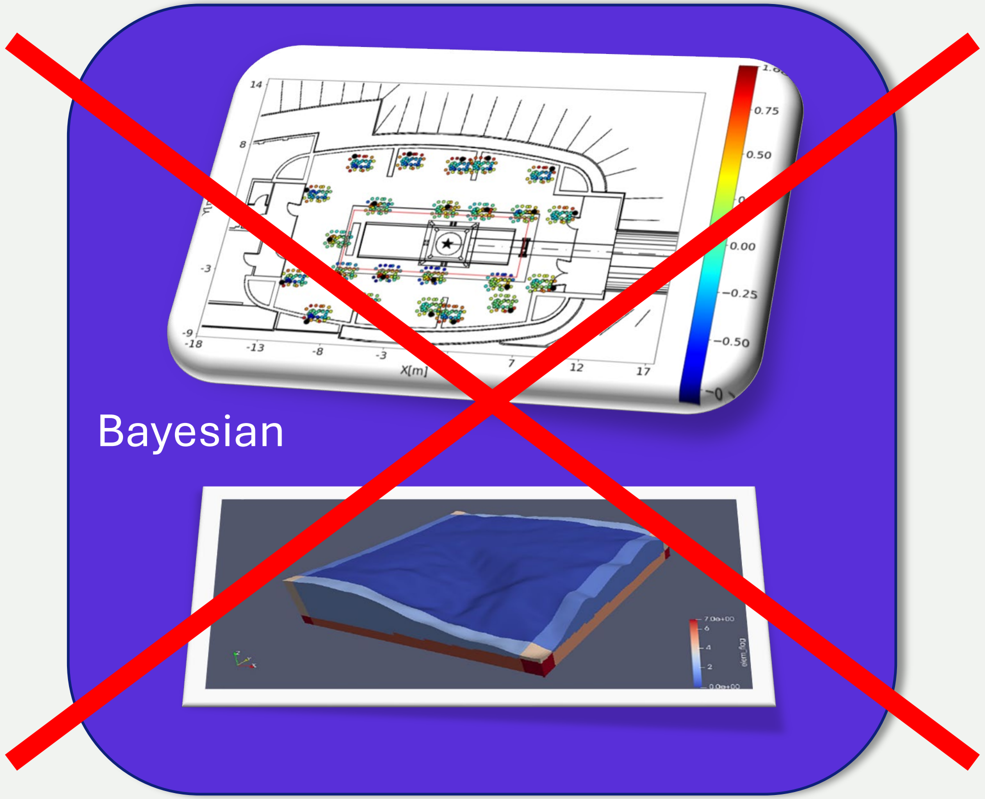
Noise estimate vs NNC design



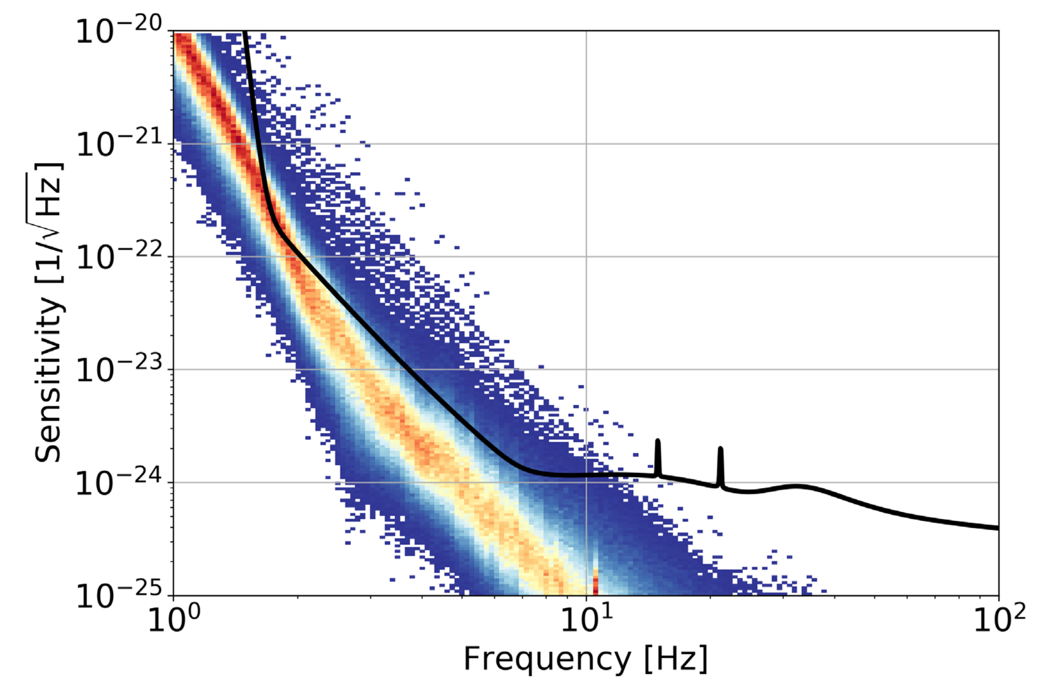


Simplest analytical models

Assumptions

- 2D isotropy of Rayleigh-wave fields
- Homogeneity of seismic fields
- Small caverns (<< $\lambda/(2\pi)$)
- Geology
  - Homogeneous (for body waves)
  - Stratified (for Rayleigh waves)
- Flat surface

Example: P2 borehole



Simplest analytical models

Body-wave NN (simple: weak dependence on geology and topography)

$$C_{\mathrm{NN}} = \left(\frac{4}{3}\pi G\rho_0\right)^2 \left[4\langle(\vec{e}_{\mathrm{tm}}\cdot\vec{\xi}^{\mathrm{P}}(\vec{r}_0,\omega))^2\rangle + \langle(\vec{e}_{\mathrm{tm}}\cdot\vec{\xi}^{\mathrm{S}}(\vec{r}_0,\omega))^2\rangle\right]$$
$$= \left(\frac{4}{3}\pi G\rho_0\right)^2 S(\xi;\omega)(3p+1)$$

Surface and Rayleigh-wave NN (complicated: topography, geology)

$$S_{\mathrm{NN}}(\delta a;\omega) = \frac{1}{2}\left(2\pi\gamma G\rho_0 \exp(-kh)\right)^2 S(\xi_z;\omega)$$

Method

- Calculate the gravitational perturbation caused by a single plane wave
- Rayleigh-wave NN: average over wave-propagation directions

Bulk term: compressional and Rayleigh waves

$$\delta\phi_{\mathrm{bulk}}(\mathbf{r}_0,t) = G\rho_0 \int_{\mathcal{V}} \mathrm{d}V \frac{\nabla\cdot\xi(\mathbf{r},t)}{|\mathbf{r}-\mathbf{r}_0|}$$

Surface term: all wave types

$$\delta\phi_{\mathrm{surf}}(\mathbf{r}_0,t) = -G\rho_0 \int \mathrm{d}S \frac{\mathbf{n}(\mathbf{r})\cdot\xi(\mathbf{r},t)}{|\mathbf{r}-\mathbf{r}_0|}$$

Body-wave NN

Body-wave NN model (for individual test mass) does not make any assumption about isotropy!

$$C_{\rm NN} = \left(\frac{4}{3}\pi G\rho_0\right)^2 \left[4\langle(\vec{e}_{\rm tm}\cdot\vec{\xi}^{\rm P}(\vec{r}_0,\omega))^2\rangle + \langle(\vec{e}_{\rm tm}\cdot\vec{\xi}^{\rm S}(\vec{r}_0,\omega))^2\rangle\right]$$
$$= \left(\frac{4}{3}\pi G\rho_0\right)^2 S(\xi;\omega)(3p+1)$$

Cavern-wall contribution (3p → 0) defines a **lower limit to NN**, which only depends on seismic spectra (weak dependence on geology and cavern size/geometry).

Weakly depending on geology:
For a P-wave content of p=1/3 (assuming equal partition between the three polarizations), this means that the cavern-wall contribution is a factor 0.7 smaller than the full body-wave NN (in amplitude).

Potentially significant dependence on propagation direction:

1) Common-mode between arms
2) For shallow detectors (depth $\leq \lambda_{seis}$), Earth surface can significantly impact body-wave NN

Simplest numerical models

Dipole equation (oscillating mass elements)

$$\delta\mathbf{a}(\mathbf{r}_0,t) = -G\int \mathrm{d}V\,\rho(\mathbf{r})(\xi(\mathbf{r},t)\cdot\nabla_0)\cdot\frac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|^3}$$
$$= G\int \mathrm{d}V\,\rho(\mathbf{r})\frac{1}{|\mathbf{r}-\mathbf{r}_0|^3}\left(\xi(\mathbf{r},t) - 3(\mathbf{e}_{rr_0}\cdot\xi(\mathbf{r},t))\mathbf{e}_{rr_0}\right)$$

Method

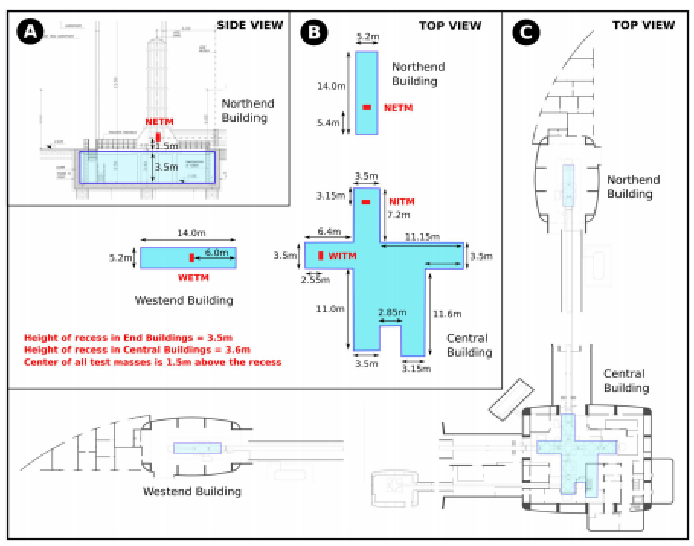
- **Inject waves of known analytical form** into a finite-element model
- Numerically integrate the associated gravity perturbation using the dipole equation

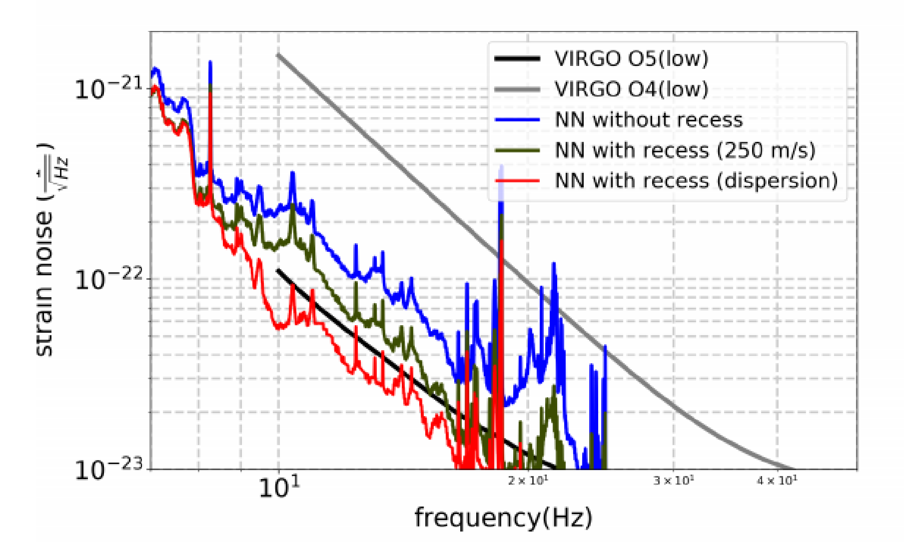
Simplest numerical models

Improvement

- Arbitrary distributions of seismic sources
- Arbitrary ground density
  - ➢ However, analytical form of seismic waves likely inconsistent with density model
- Arbitrary topography
  - ➢ However, analytical form of seismic waves likely inconsistent with topography
- Waves not necessarily plane

Example: Virgo NN estimate





Singha et al

Numerical models based on seismic correlations

Example: surface NN, homogeneous model

$$S(\delta a_x;\omega) = (2\pi G\rho_0\gamma(\nu))^2\frac{1}{2\pi}\int \mathrm{d}^2\varrho\left[\frac{x^2}{\varrho^2}\frac{2h}{((2h)^2+\varrho^2)^{3/2}} + \frac{y^2-x^2}{\varrho^4}\left(1-\frac{2h}{((2h)^2+\varrho^2)^{1/2}}\right)\right]C(\xi_z;\varrho,\omega)$$

Seismic correlations

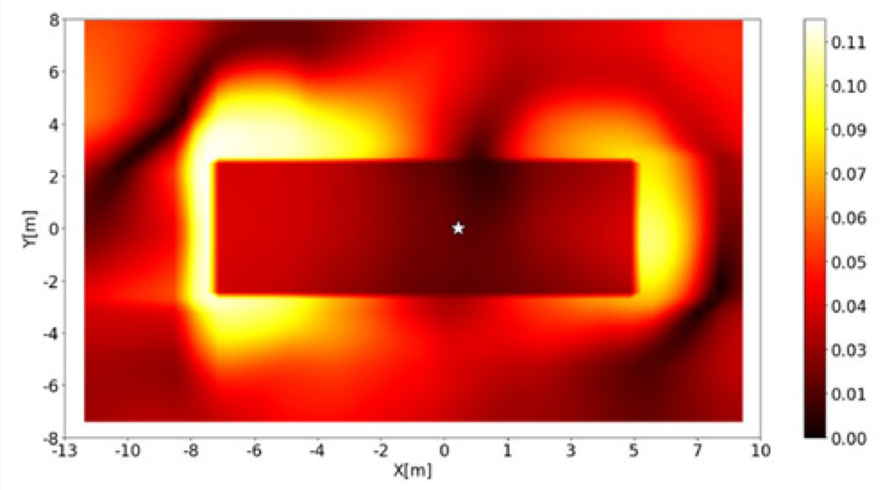
Method

- Use **observed seismic correlations**
- Integrate numerically over surface area using an appropriate kernel

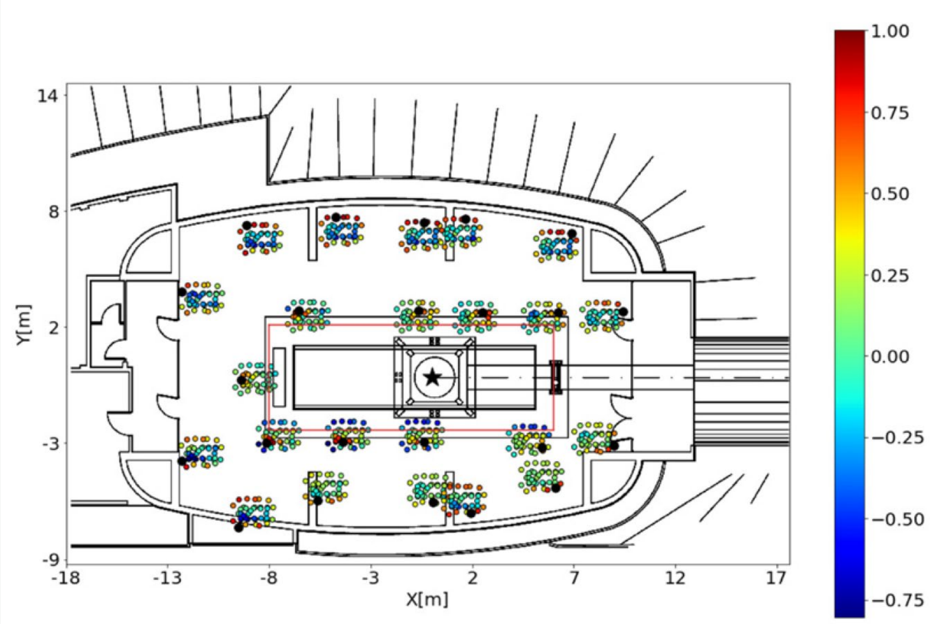
Numerical models based on seismic correlations

Improvement

- Works for inhomogeneous fields
- Does not require analytical models of seismic waves

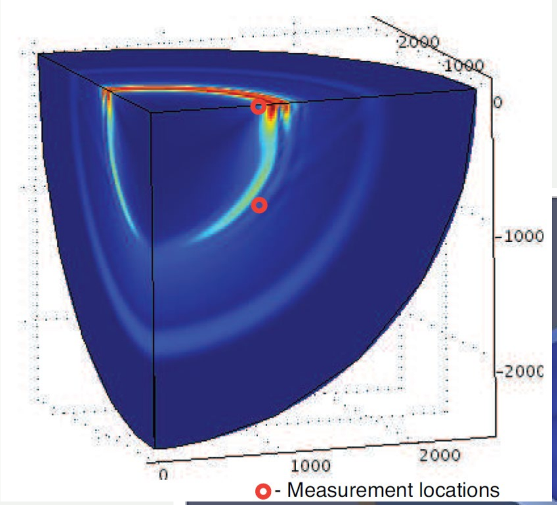


Example: Virgo NNC optimization

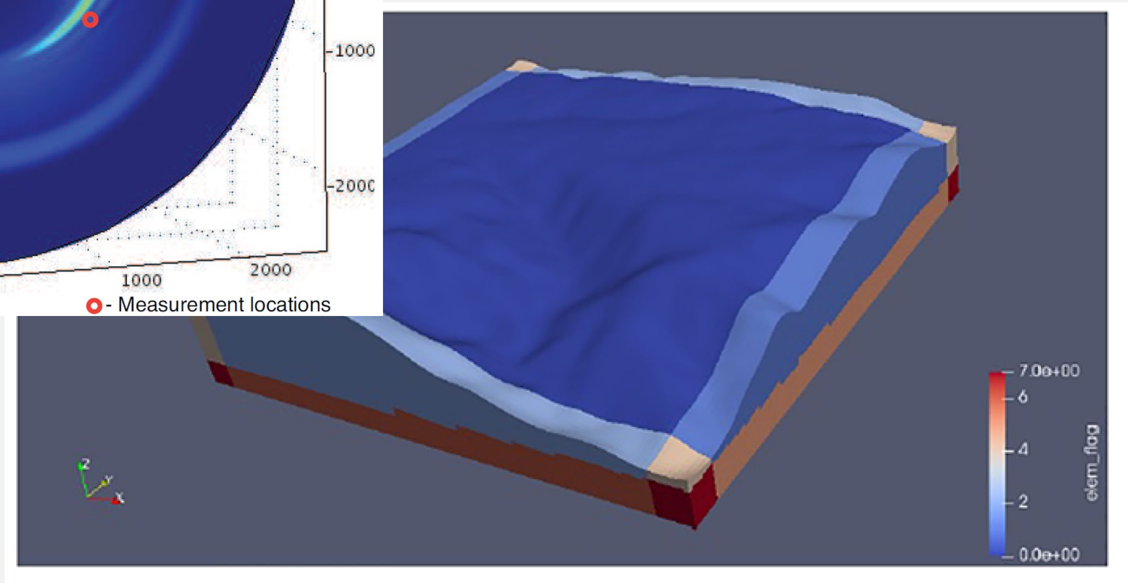


Badaracco et al

Dynamical finite-element simulations



Beker



Andric

Method

- Implement arbitrary inhomogeneous models
- Leave the calculation of the wavefield to Comsol, ANSYS, SPECFEM3D,…
- SPECFEM3D can directly solve for seismic correlations
- Numerically demanding and prone to systematic errors

Summary

	Required/recommended input	Purpose
Analytical models	Seismic spectra, Rayleigh-wave dispersion	Estimate NN spectra in simple geological/topographic settings, and for simple seismic fields
Simplest numerical models	Seismic spectra, anisotropy of seismic field, Rayleigh-wave dispersion, topography, geology	Estimate NN spectra in geological/topographic settings weakly different from flat/homogeneous
Correlation-based models	Two-point spatial correlations, topography, geology	Estimate NN spectra in complicated geological/topographic settings
Dynamical FEM simulation	Information about seismic sources, geology, topography	Study specific phenomena relevant to NN estimation and cancellation
Bayesian methods	Two-point spatial correlations, geology, topography, information about seismic sources	NNC design