Sub-percent determination of the HVP contribution to a_μ from BMWc

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on behalf of the Budapest-Marseille-Wuppertal collaboration

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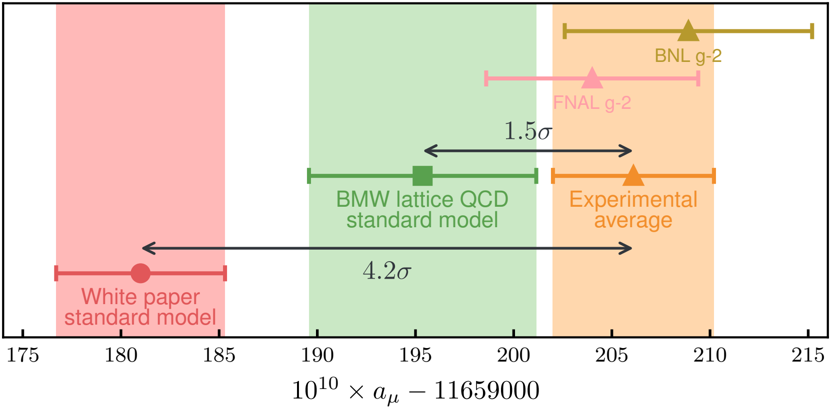
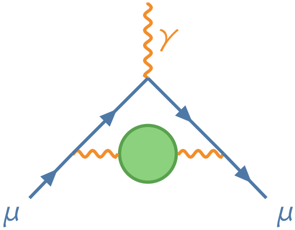
Overview

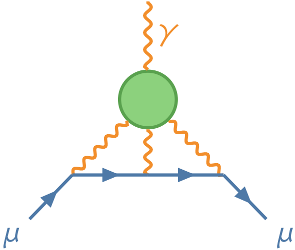


- Data-driven SM prediction differs from experiment by 4.2σ
- Our recent lattice result is 2.1σ higher than the data-driven value
- Consistent with experiment within 1.5σ

Standard Model contributions

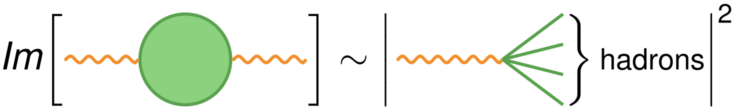
$a_\mu^{QED} \times 10^{10}$	11 658 471.893	$\pm$ 0.0104
$a_\mu^{EW} \times 10^{10}$	15.36	$\pm$ 0.10
$a_\mu^{HVP} \times 10^{10}$	684.5	$\pm$ 4.0
$a_\mu^{HLbL} \times 10^{10}$	9.2	$\pm$ 1.8
$a_\mu \times 10^{10}$	11 659 208.9	$\pm$ 6.3





- Errors dominated by QCD corrections
    - Hadronic vacuum polarisation
    - Hadronic light-by-light scattering

$a_\mu^{\text{LO-HVP}}$ from phenomenology



- Use optical theorem and dispersion relation to relate to coupling of virtual photon to hadrons
- Evaluate using e^+e^- scattering data
  - Predominantly the $\pi^+\pi^-$ channel

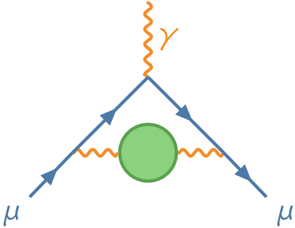
Dispersive approach

$$a_\mu^{\text{LO-HVP}} = \frac{m_\mu^2}{12\pi^3} \int_{M_\pi^2}^{\infty} ds \, \frac{\hat{K}(s)}{s} \sigma(e^+e^- \to \gamma^* \to \text{hadrons})$$

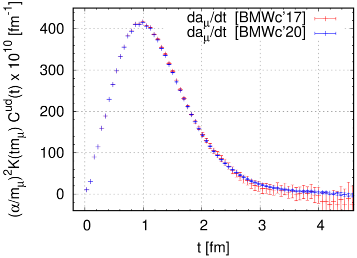
$a_\mu^{\text{LO-HVP}}$ from lattice QCD

- Use time-momentum representation [Bernecker & Meyer '11]
- Compute current-current correlator

$$C(t) = \frac{1}{3} \sum_{\mu=1,2,3} \langle J_\mu(t) J_\mu(0) \rangle$$



- Integrate with kernel function $K(t)$
- Above $Q^2_{\text{max}} = 3\,\text{GeV}^2$, use perturbation theory

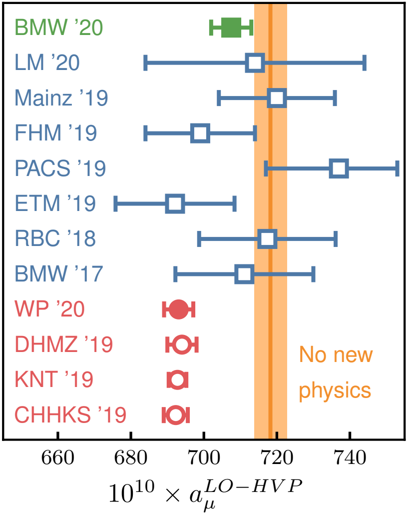


Lattice approach

$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty K(t) C(t) dt$$

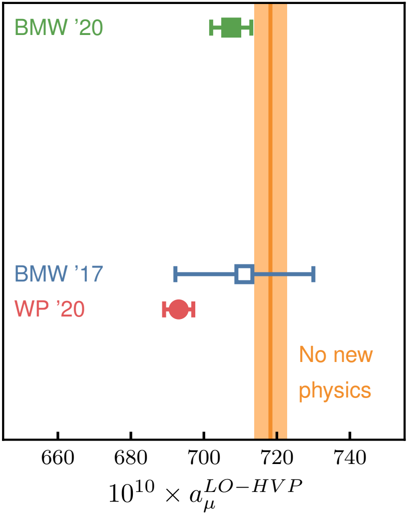
Hadronic vacuum polarisation

- Until now, lattice uncertainty larger than dispersive
- Results mostly consistent with both dispersive approach and experiment
- New, sub-percent lattice determination [BMWc '20]
- First lattice calculation with errors comparable to data-driven determinations

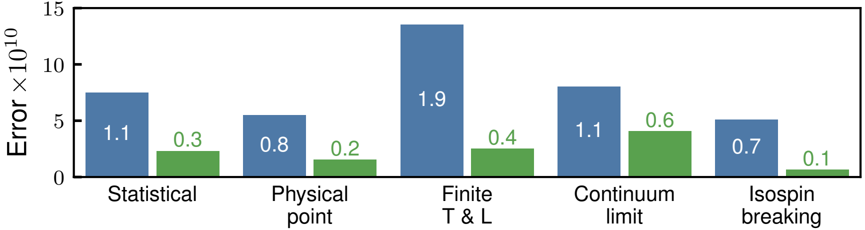


Three years of progress

- 3.4× increase in precision over our earlier work [BMWc '17]
- Many improvements needed to attain this precision
  - Statistical noise
  - Scale setting
  - Finite size effects
  - Continuum limit
  - Isospin breaking
- Made possible thanks to the work of many groups around the world

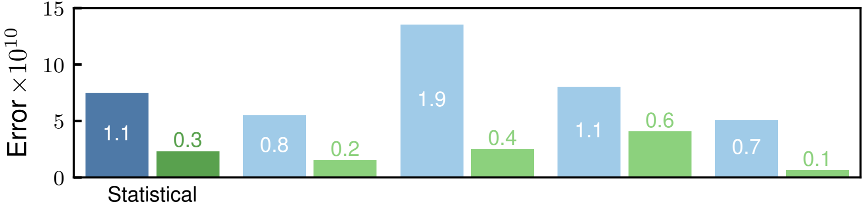


Key improvements



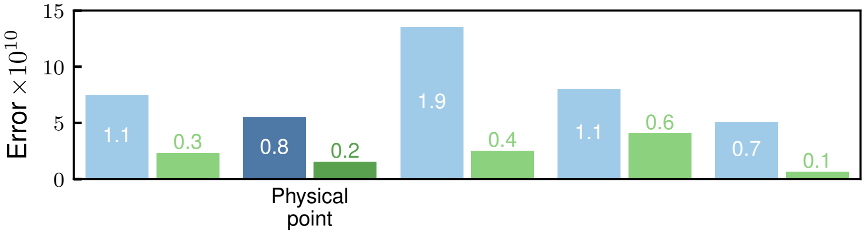
- Five major sources of error in our previous work
- Dominant error from finite size effects
- Once this was reduced, four other sources of error became important
- Significant reduction in all five for our recent publication

Statistical noise



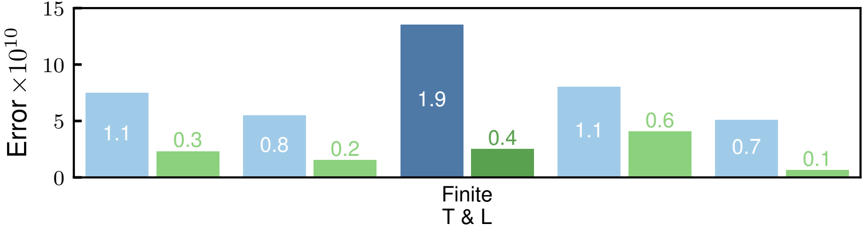
- Statistical noise in u/d contributions grows exponentially at large t
- Algorithmic improvements (EigCG, solver truncation [Bali et al 09], all mode averaging [Blum et al 13]) to generate more statistics
- Exact treatment of IR modes to reduce long-distance noise (low mode averaging [Neff et al 01, Giusti et al 04, ...])
- Rigorous upper/lower bounds on long-distance contribution [Lehner 16, BMWc 17]

Scale setting



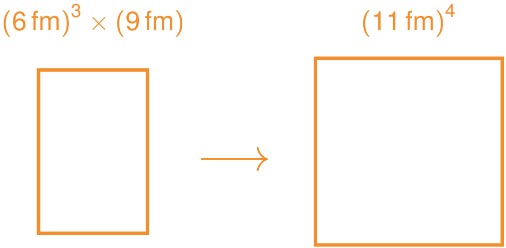
- Naïvely, relative errors in lattice spacing are doubled
- Requires permille determination of scale
- Use Ω^- baryon mass computed with 0.2% error
  - Partially subsumed into statistical error
- Wilson-flow scale [Lüscher 10, BMWc 12] for isospin decomposition

Finite-size effects



- Even in our large volumes ($L \gtrsim 6.1\,fm$, $T \geq 8.7\,fm$), exponentially suppressed FV effects are significant
- One-loop SU(2) χPT [Aubin et al 16] suggests $\sim 2\%$ effect
- Perform dedicated FV study with even larger volumes: $(\sim 11\,fm)^4$
- χPT & other models validated by comparing to lattice data
- Use two-loop χPT [Aubin et al 20] for tiny, residual correction

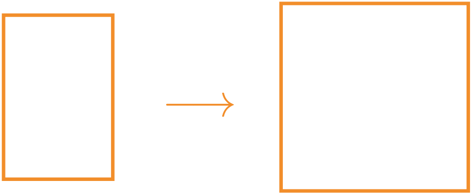
Dedicated finite-volume study



- Perform continuum extrapolation at reference volume
- Apply finite-size corrections in continuum
- Taste breaking distorts finite-size effects
- Large volumes only practical with coarse lattices ($a = 0.112\,\mathrm{fm}$)
- Perform dedicated simulations with reduced taste breaking
  - DBW2 action [Takaishi et al '96] and 4HEX smearing [Capitani et al '06] to suppress UV fluctuations
  - Tune pion masses with HMS mass instead of Goldstone pion

Dedicated finite-volume study

$(6\,\mathrm{fm})^3 \times (9\,\mathrm{fm})$ $\longrightarrow$ $(11\,\mathrm{fm})^4$



4HEX	18.1 ± 2.4

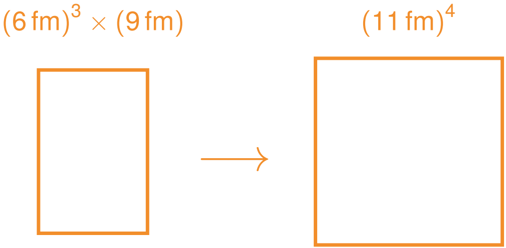
$\times 10^{-10}$

Model comparison

- Two models for finite L & T
  - Chiral perturbation theory to two loops
  - Meyer-Lellouch-Lüscher-Gounaris-Sakurai model [Bernecker & Meyer '11] (MLLGS), a phenomenological model of interacting two-pion states
- Two more models for finite L (but not T)
  - Generic field-theory approach [Hansen & Patella '19, '20] (HP) relates the finite-size effect to $F_\pi(k)$
  - Rho-pion-gamma model [Chakraborty et al '17] (RHO) incorporates the $\rho(770)$ resonance directly into a χPT-like framework
- Compare finite L corrections for reference volume in infinite-T limit
- All four models agree within $\sim 2.5 \times 10^{-10}$

NNLO χPT	16.7	$\times 10^{-10}$
MLLGS	18.8	$\times 10^{-10}$
HP	17.7	$\times 10^{-10}$
RHO	16.2	$\times 10^{-10}$

Model cross-check



4HEX	18.1 ± 2.4	$\times 10^{-10}$
NNLO χPT	15.7	$\times 10^{-10}$
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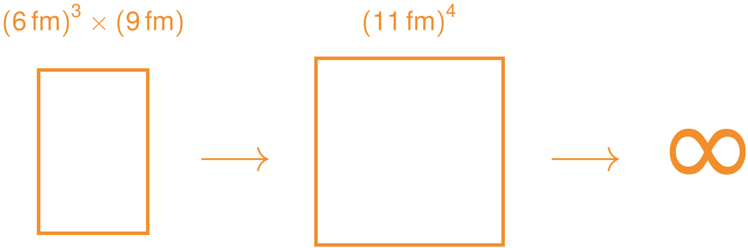
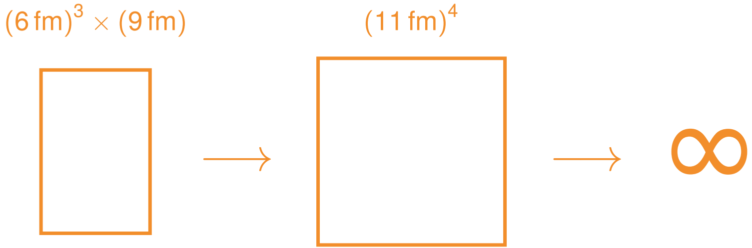
Residual correction



4HEX	18.1 ± 2.4		$\times 10^{-10}$
NNLO χPT	15.7	0.6 ± 0.3	$\times 10^{-10}$
MLLGS	17.8		$\times 10^{-10}$

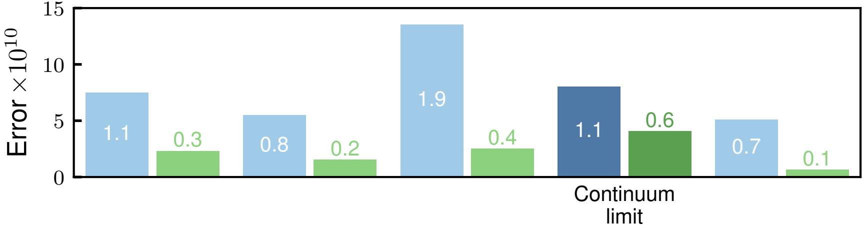
Residual correction



4HEX	18.1 ± 2.4		$\times 10^{-10}$
NNLO χPT	15.7	0.6 ± 0.3	$\times 10^{-10}$
MLLGS	17.8		$\times 10^{-10}$

- Estimated $I = 0$ effect: 0.0(0.6)
- Estimated isospin-breaking correction: 0.0(0.1)

Continuum limit



- Need controlled continuum ($a \to 0$) limit
- Perform all calculations at 6 lattice spacings: $0.134 fm$–$0.064 fm$
- Statistical error at finest a reduced from 1.9% to 0.3%!
- Improve continuum limit w/ EFTs and phenomenological models (SRHO) [Sakurai 60, Jegerlehner et al 11, Chakraborty et al 17, BMWc 20]
  - 2-loop SU(2) SχPT for systematic error [Bijnens et al 99, BMWc 20]
  - Models validated with data

Lattice spacing dependence

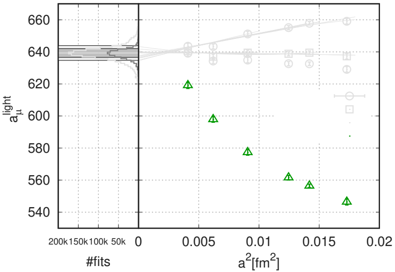
- Naïvely, staggered scaling is a^2
- Can be modified by logarithmic corrections

$$a^2\,\alpha_s^\Gamma\left(\tfrac{1}{a}\right) \sim \frac{a^2}{\log(a)^\Gamma}$$

- Systematic from comparing $\Gamma = 0\ \&\ 3$
- a_μ still non-linear
- Only apparent with high precision data
- Taste-splitting in loops
- Effectively a combined chiral-continuum limit

Two types of power series

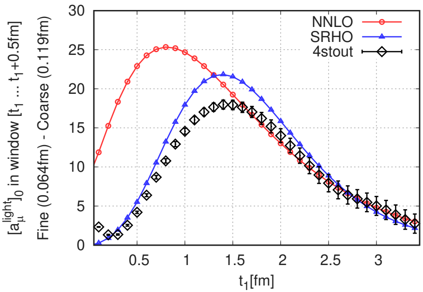
1. $A_0 + A_1\left[a^2\right] + A_2\left[a^2\right]^2$
2. $A_0 + A_1\left[a^2\alpha_s^3\left(\tfrac{1}{a}\right)\right] + A_2\left[a^2\alpha_s^3\left(\tfrac{1}{a}\right)\right]^2$



Taste improvement

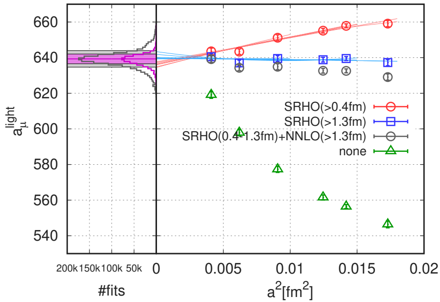
Incorporate taste splitting into models to describe taste violation:

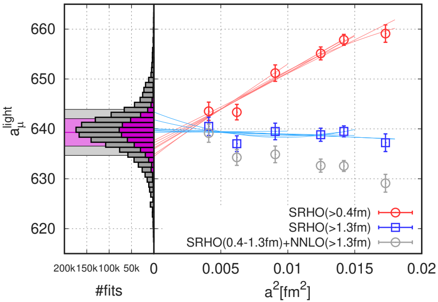
1. Rho-pion-gamma model [Jegerlehner, Szafron '11; HPQCD '16] (SRHO)
   $\rightarrow$ depends on rho parameters
2. Chiral perturbation theory [Lee, Sharpe, Van de Water, Bailey] (SχPT)
   $\rightarrow$ depends on one LEC (l_6)



- At large euclidean times, both models reproduce lattice artefacts well
- These are not fits
- SχPT breaks down below 1.3 fm
  - Resonance contribution is missing
- SRHO seems to work well over the whole range

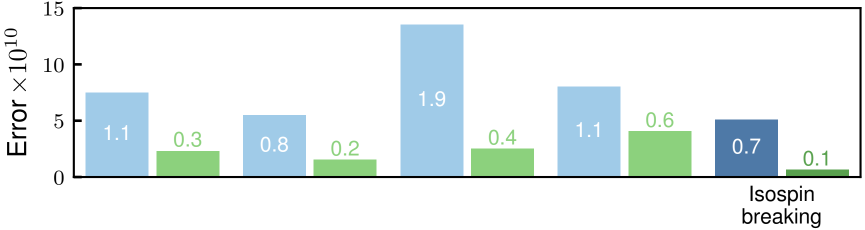
Continuum extrapolation





- Reduced lattice artefact and linear a^2 dependence
- Central value from SRHO improvement. For systematic errors:
  1. Change starting point of improvement $t = 0.4 \to 1.3\,\text{fm}$
  2. Skip coarse lattices & change fit form (linear, quadratic, cubic)
  3. Interchange $\Gamma = 0$ and $\Gamma = 3$
  4. Replace SRHO by NNLO SχPT above 1.3 fm

Isospin breaking

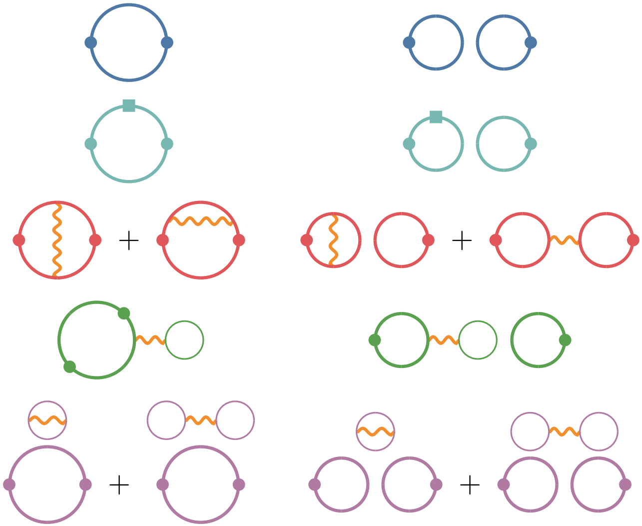


- $\mathcal{O}(\alpha)$ and $\mathcal{O}(\delta m = m_d - m_u)$ effects are $\mathcal{O}(1\%)$
- Include all relevant isospin-breaking effects
- Compute effects on *all* quantities needed
  1. Correlator $C(t)$
  2. Meson masses M_π, M_K
  3. Scale setting w_0

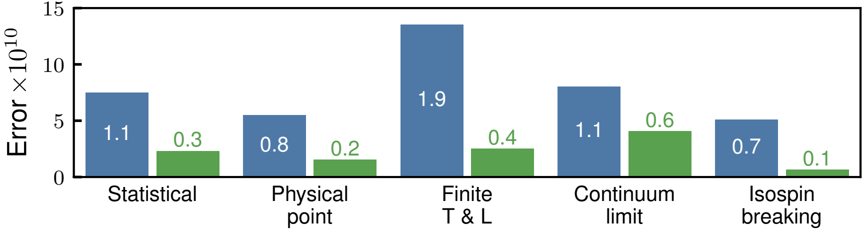
Isospin breaking contributions

Expand $\langle O\rangle$ to first order in $\alpha = \frac{e^2}{4\pi}$ and $\delta m = m_d - m_u$:

$$\langle O\rangle \approx \langle O\rangle_0 + \frac{\delta m}{m_l}\langle O\rangle'_m + e_v^2\langle O\rangle''_{20} + e_v e_s\langle O\rangle''_{11} + e_s^2\langle O\rangle''_{02}$$



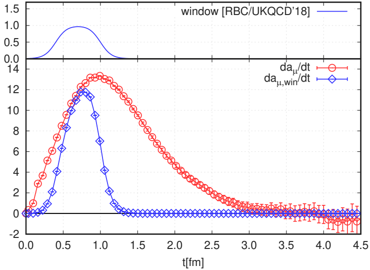
Error budget

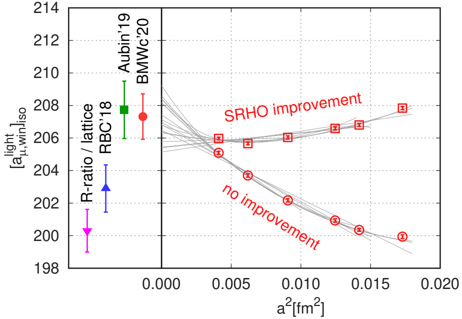


- Thorough & robust determination of statistical & systematic errors
- Statistical error: resampling methods
- Systematic error: extended frequentist approach [BMWc 08, 14]
  - Hundreds of thousands of different analyses of correlation functions
  - Weighted by AIC weight
  - Use median of distribution for central values & 68% confidence interval for total error

Window Observable

- Most systematics come from short or long distances
- Idea: Perform cross-check by restricting correlator to window 0.4–1.0 fm [RBC '18]
- Much easier to compute





Systematic errors:

1. SRHO vs no improvement
2. Skip coarse lattices, and linear, quadratic, or cubic
3. $\Gamma = 0$ or 3

Conclusion

- Significant improvement in
  - Statistical noise
  - Scale setting
  - Finite size effects
  - Continuum limit
  - Isospin breaking
- Reduction in total error from 2.7% to 0.8%
- Shows surprising agreement with no-new-physics scenario
- Important to have lattice cross-checks
  - Particularly of $a_{\mu,win}$
- Important to understand disagreement with R-ratio