Some Random Thoughts on

- Why is the $\mathbf{E_{sym}(\rho)}$ still so uncertain especially at high densities?
- What is the most fundamental but least known physics underlying the $\mathbf{E_{sym}(\rho)}$?
- What to do next to further constrain the $\mathbf{E_{sym}(\rho)}$?
- ……………………..
- The role of short-range correlations induced by the repulsive core and/or tensor force

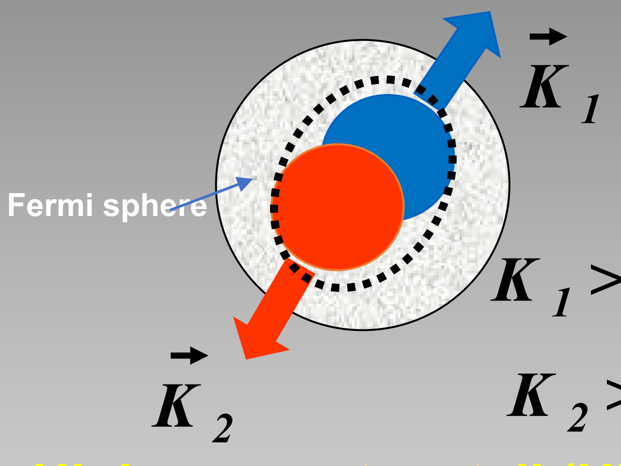
Bao-An Li

Physics & Astronomy
Texas A&M University-Commerce

What are the Short Range Correlations (SRC) in nuclei ?

(Modified from a slide by Eli Piasetzky)

In momentum space:



$K_1 > K_F$,

$K_2 > K_F$

High momentum tail (HMT): $(1.3 - 2.5)K_F$

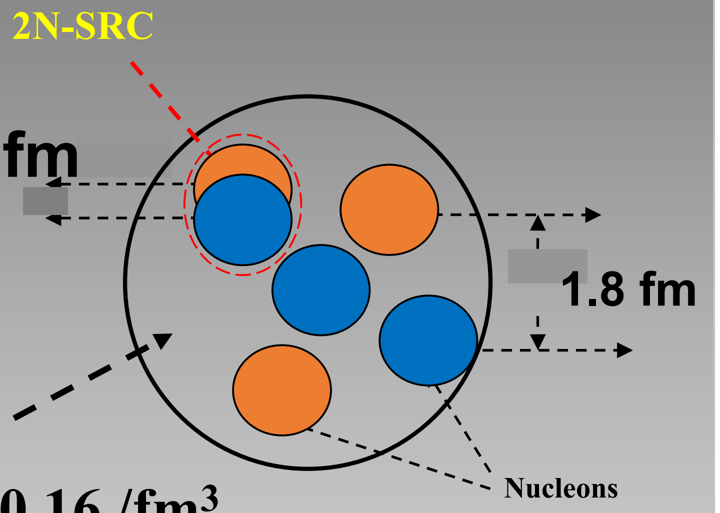
Nucleon pairs with large relative momenta and small CM momenta

In isospin space:

Dominated by the isosinglet (T=0) neutron-proton pairs

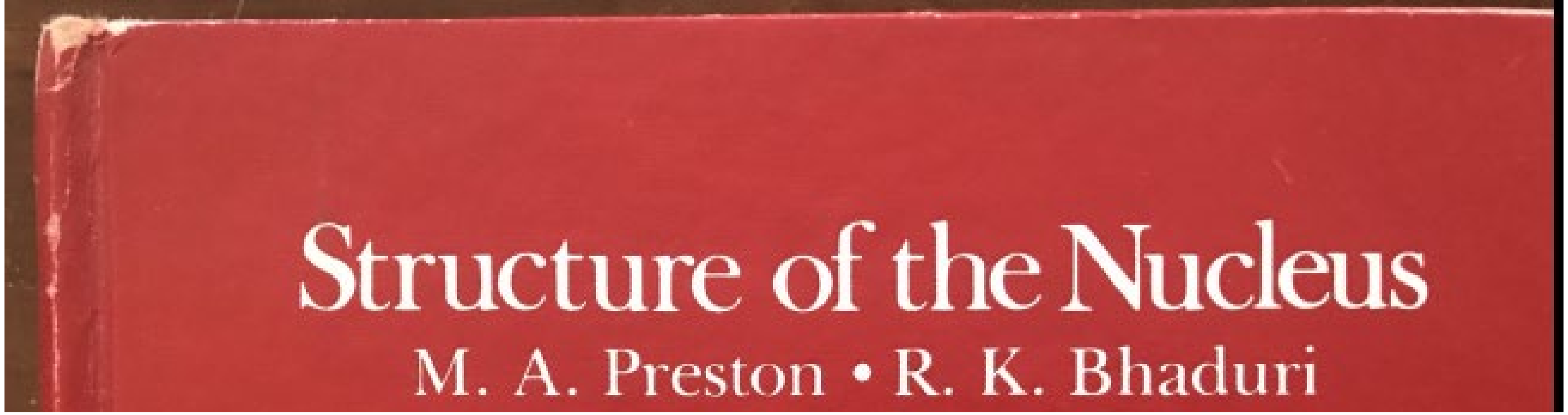
In coordinate space:

SRC $\sim R_N \sim 1$ fm



$\rho_o = 0.16\ /\text{fm}^3$

Short Range Correlated pairs: temporal fluctuations of strongly interacting nucleon pairs in close proximity



McMaster University
1975

where, as before, $\tilde{V}_1$ is half the volume integral of the potential V_1. An estimate in regard to nuclear matter ($I = 0$) indicates that $\langle V \rangle / A \approx -40$ MeV, with the $T = 1$ and $T = 0$ parts contributing about equally, that is, $3V_1\rho \approx V_0\rho \approx -80$ MeV. This shows that the coefficient of the I^2/A term in Eq. 6–19 is about 13 MeV and is repulsive. A more accurate estimate of this term yields about 10 MeV. Thus the symmetry term in the mass formula is repulsive and is given by $a_{\text{sym}}(I^2/A)$, where

$$a_{\text{sym}} = \tfrac{1}{3}t(k_F{}^0) + \tfrac{1}{6}k \left.\frac{\partial U}{\partial k}\right|_{k=k_F} + \tfrac{1}{4}\rho(\tilde{V}_1 - \tilde{V}_0). \qquad (6\text{–}20)$$

Fundamental physics underlying nuclear symmetry energy

Single-nucleon (Lane) potential in isospin-asymmetric matter: A. M. Lane, Nucl. Phys. 35, 676 (1962).

$$U_{n/p}(k,\rho,\delta) = U_0(k,\rho) \pm U_{sym1}(k,\rho)\cdot\delta + U_{sym2}(k,\rho)\cdot\delta^2 + o(\delta^3)$$

Hugenholtz-Van Hove (HVH) theorem:

N.M. Hugenholtz, L. Van Hove, Physica 24 (1958) 363.

$$E_F = \frac{d\xi}{d\rho} = \frac{d(\rho E)}{d\rho} = E + \rho\frac{dE}{d\rho} = E + P/\rho$$

$$E_{sym}(\rho) = \frac{1}{3}\frac{k_F^2}{2M} + \frac{1}{2}U_{sym,1}(\rho,k_F) + \frac{k_F}{6}\left(\frac{\partial U_0}{\partial k}\right)_{k_F} - \frac{1}{6}\frac{k_F^4}{2M^3}$$

S. Fritsch, N. Kaiser, W. Weise, Nuclear Phys. A 750 (2005) 259.

Using the K-matrix theory:

K.A. Brueckner, J. Dabrowski, Phys. Rev. B 134 (1964) 722.
J. Dabrowski, P. Haensel, Phys. Lett. B 42 (1972) 163;

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho,k_F),$$

$$m_0^*(\rho,k) = \frac{m}{1+\frac{m}{\hbar^2 k}\frac{\partial U_0(\rho,k)}{\partial k}},$$

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho,k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F}\cdot k_F + 3U_{sym,2}(\rho,k_F),$$

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011)

Bao-An Li, Bao-Jun Cai, Lie-Wen Chen and Jun Xu,
Progress in Particle and Nuclear Physics 99 (2018) 29–119

$$m_{n-p}^* \approx 2\delta\frac{m}{\hbar^2 k_F}\left[-\frac{dU_{sym,1}}{dk} - \frac{k_F}{3}\frac{d^2U_0}{dk^2} + \frac{1}{3}\frac{dU_0}{dk}\right]_{k_F}\left(\frac{m_0^*}{m}\right)^2$$

The most fundamental but lest known physics underlying the high-density symmetry energy

Spin-isospin dependence of nucleon interactions at short distance: $V_{np}(T_0) \neq V_{np}(T_1)$

- **Isospin dependence of strong interactions and correlations**

$$V_{T0} = V'_{np} \quad \text{(n-p pair in the T=0 state)}$$

$$V_{T1} = V_{nn} = V_{pp} = V_{np} \quad \text{(charge independence in the T=1 state)}$$

Tensor force due to pion and $\boldsymbol{\rho}$ meson exchange MAINLY in the T=0 channel

In the interacting Fermi gas model, the direct term of the symmetry potential:

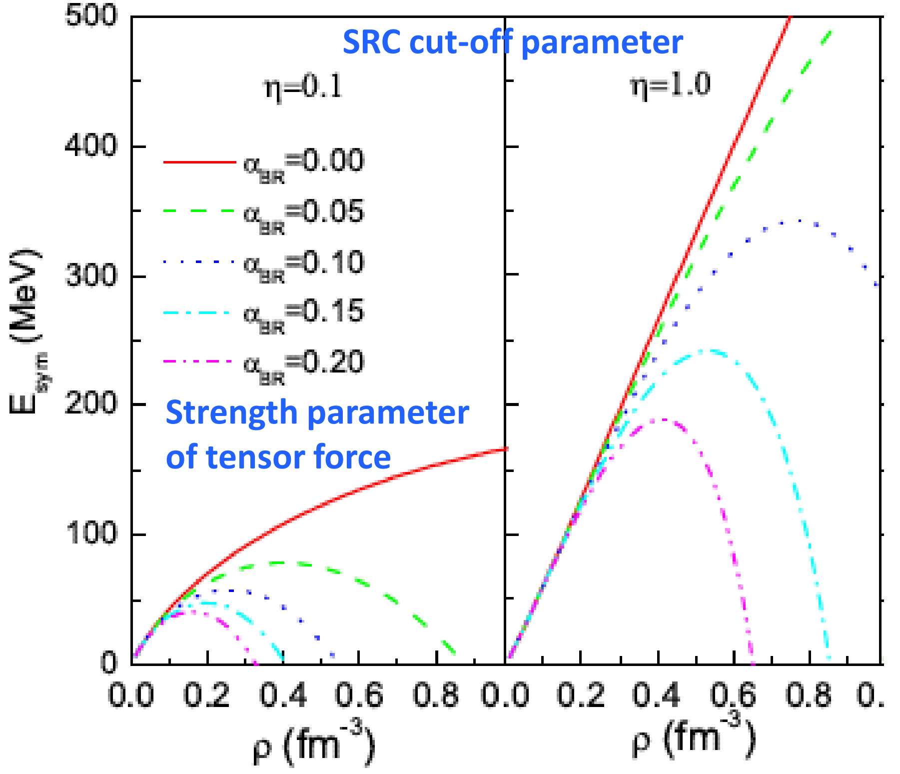
Isospin-dependent correlation function

$$U_{sym}(k_F, \rho) = \frac{1}{4}\rho \int [V_{T1}(r_{ij}) f^{T1}(r_{ij}) - V_{T0}(r_{ij}) f^{T0}(r_{ij})] d^3 r_{ij}$$

Isospin-dependent effective 2-body interaction

M.A. Preston and R.K. Bhaduri, Structure of the Nucleus, 1975

Effects of tensor force on E_{sym} ($\boldsymbol{\rho}$) in a deuteron cluster model (i.e. n-p dominance) of nuclear matter

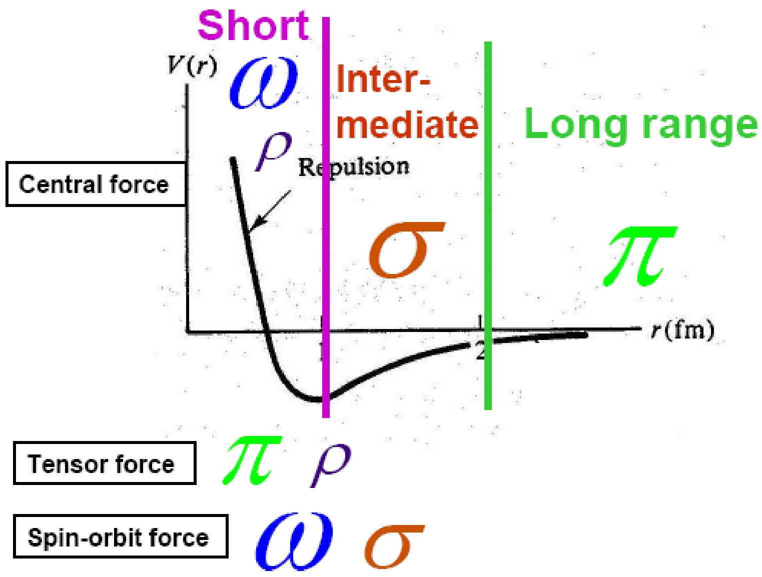


$$f(r) = 0, \text{ for } r < r_c \qquad r_c = \eta(3/4\pi\rho)^{1/3}$$

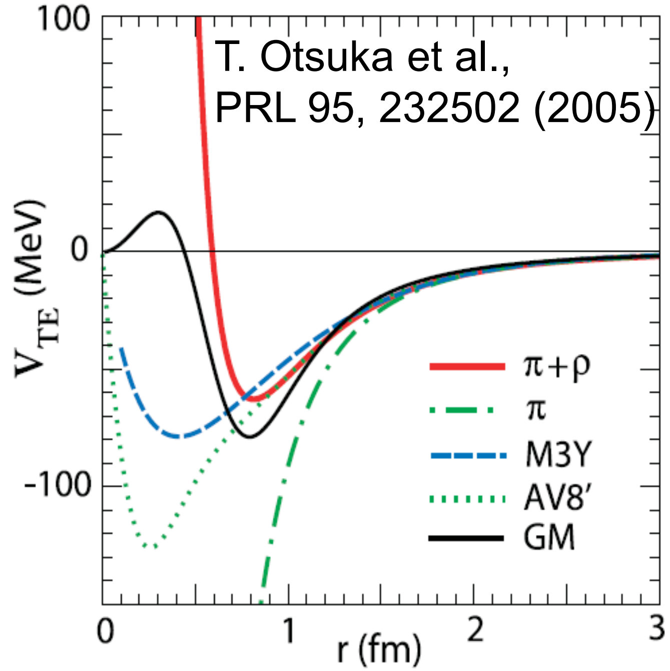
Chang Xu and Bao-An Li, PRC81, 064612 (2010)

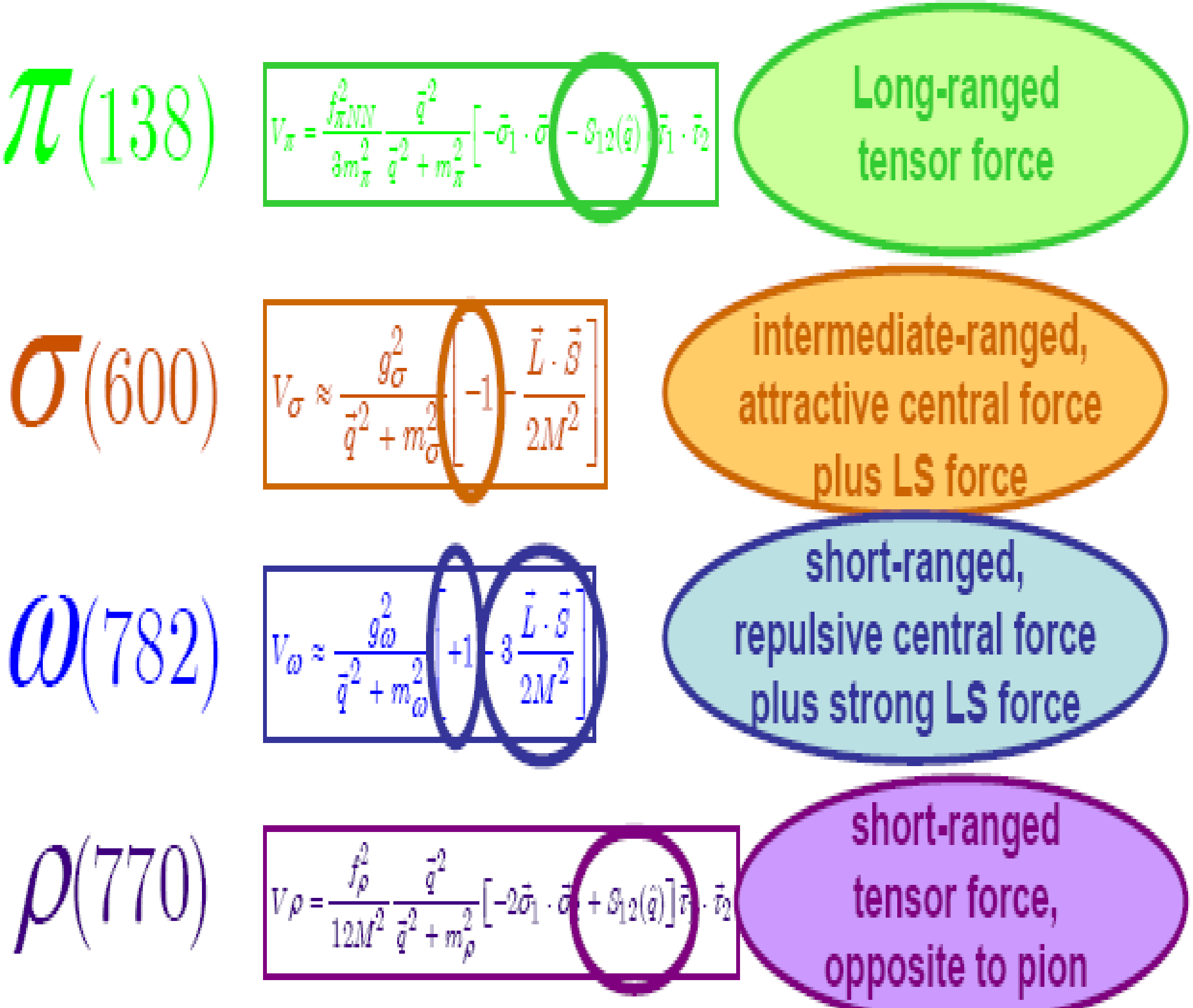
The short and long range tensor force

Lecture notes of R. Machleidt
CNS summer school, Univ. of Tokyo
Aug. 18-23, 2005



Strength of the tensor force





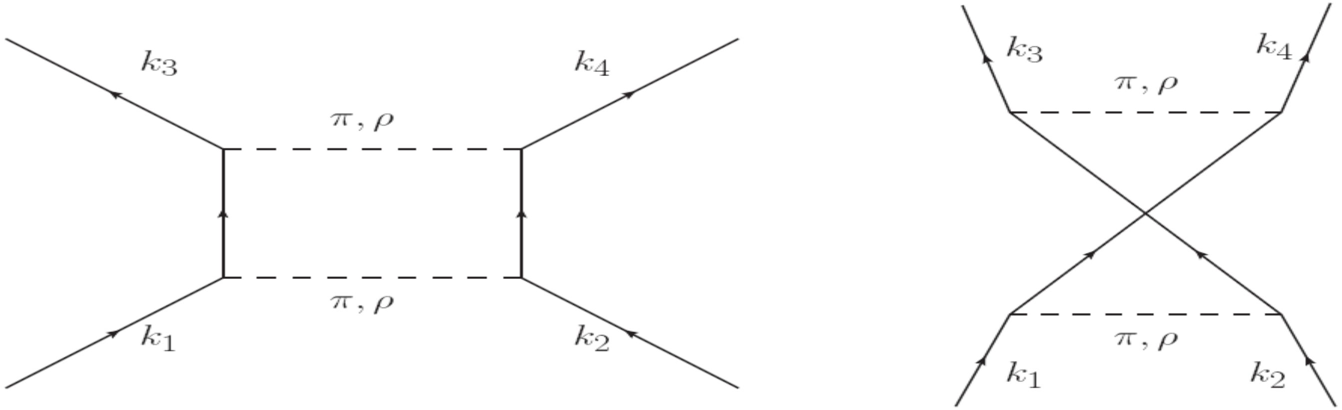
2nd order tensor force contribution to the potential part of the symmetry energy

G.E. Brown and R. Machleidt, Phys. Rev. C50, 1731 (1994).

S.-O. Bacnman, G.E. Brown and J.A. Niskanen, Phys. Rep. 124, 1 (1985).

T.T.S. Kuo and G.E. Brown, Phys. Lett. 18, 54 (1965)

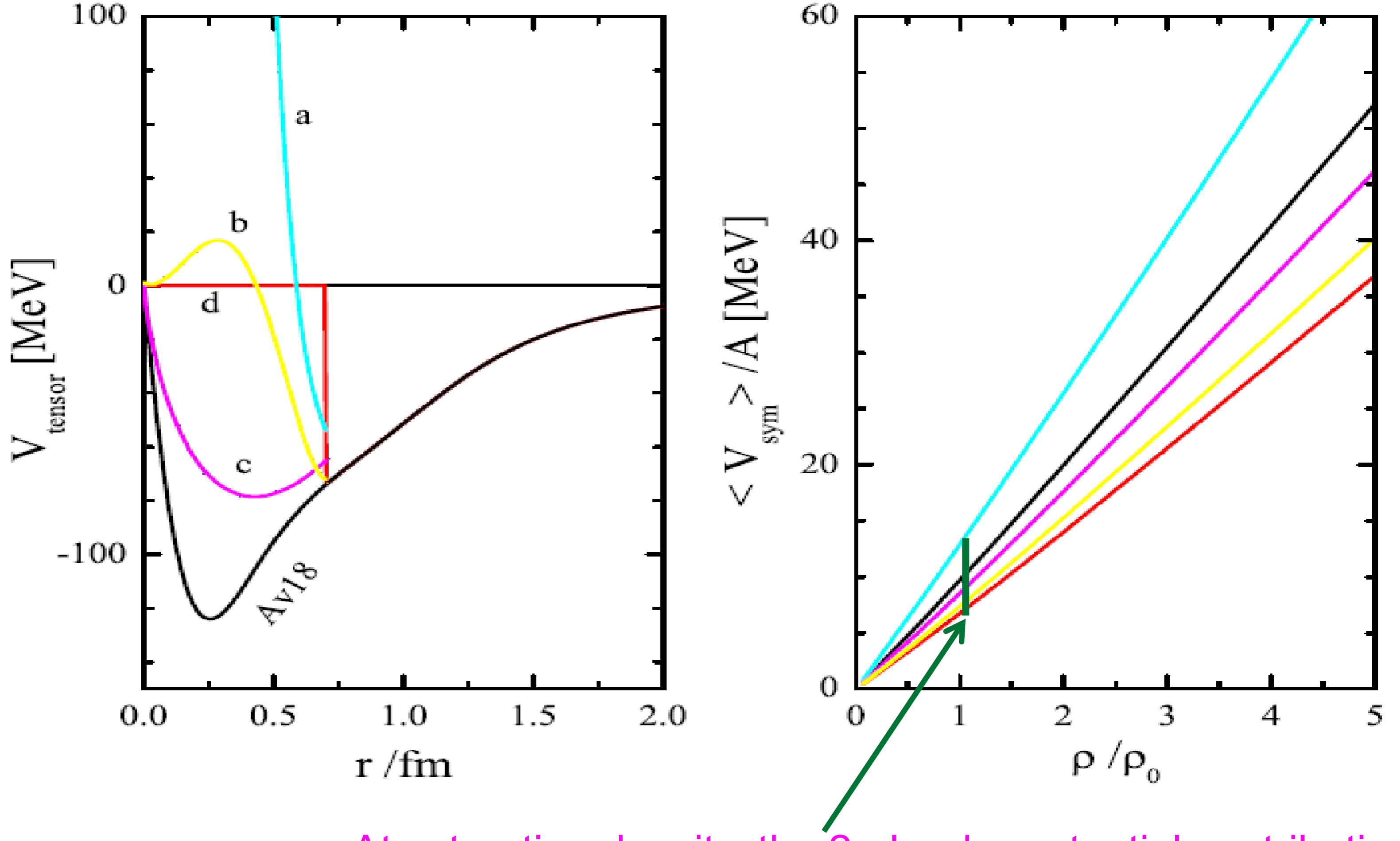
$$< V_{\rm sym} > = \frac{12}{e_{\rm eff}} < [V_t(\mathbf{r})]^2 >$$



$$\frac{< V_{\rm sym} >}{A} = \frac{12}{e_{eff}} \cdot \frac{k_F^3}{12\pi^2}\{\frac{1}{4}\int V_t^2(r)d^3r - \frac{1}{16}\int [\frac{3j_1(k_F r)}{k_F r}]^2 V_t^2(r)d^3r\}$$

Short-range tensor forces affects the high-density symmetry energy

C. Xu, A. Li and B.A. Li, Journal of Physics: Conference Series 420, 012190 (2013)



At saturation density, the 2nd order potential contribution
due to the tensor force is about 7-14 MeV, it is 9 MeV with Av18

Effects of the tensor force in T=0 neutron-proton interaction channel

(1) High-momentum tail (HMT) in nucleon momentum distribution due to the S-D coupling induced by the tensor force

H.A. Bethe, Ann. Rev. Nucl. Part. Sci., 21, 93-244 (1971)

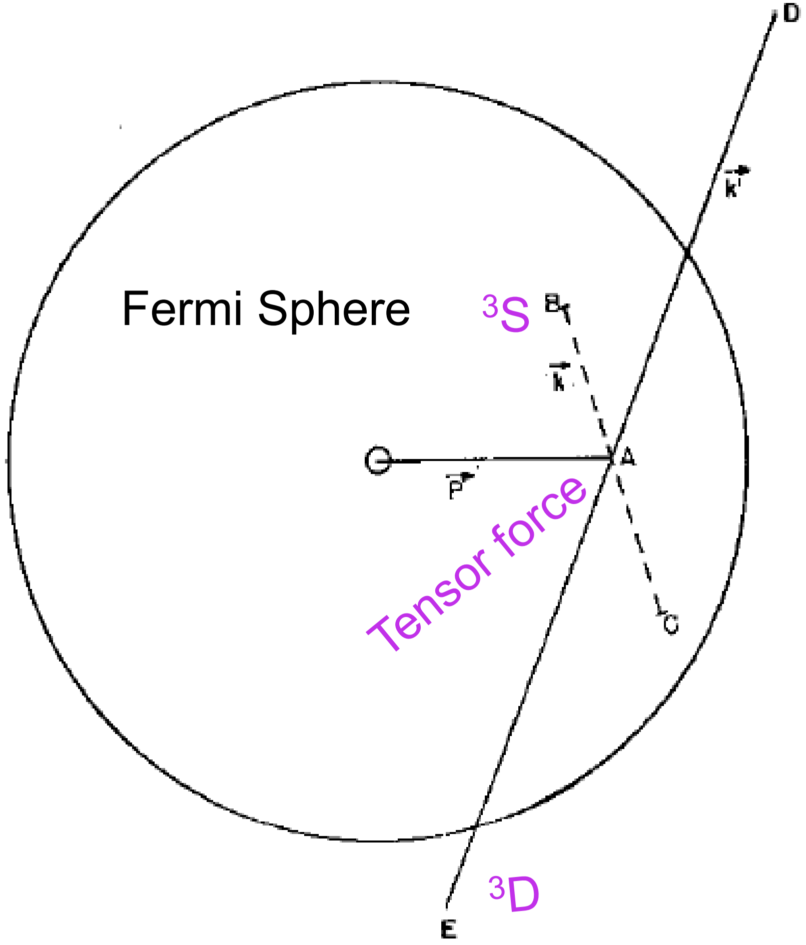
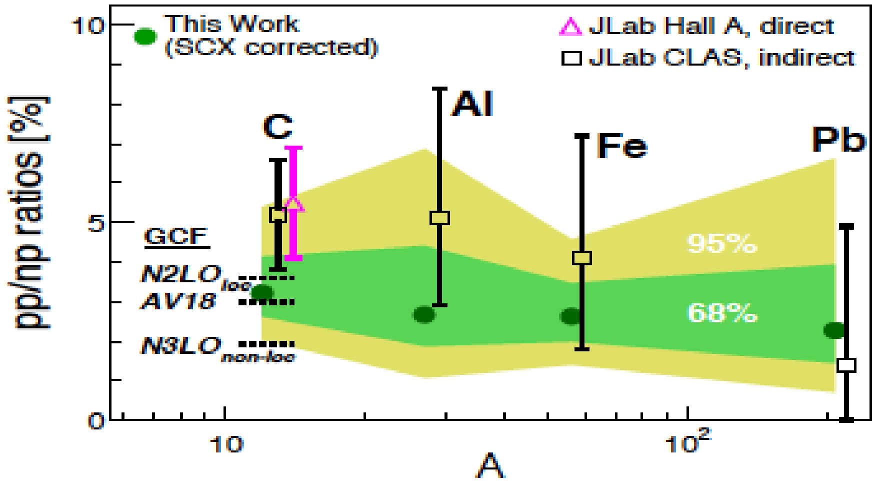


FIGURE 10. Two nucleons are initially in states *B* and *C*, having average momentum ***P*** and relative momentum ***k***. When they interact they are shifted to states *D* and *E* outside the Fermi sphere, with relative momentum ***k'***. If they are initially in a ^3S state and interact by tensor force, then they are in a $^3D_1$ state in *DE*.

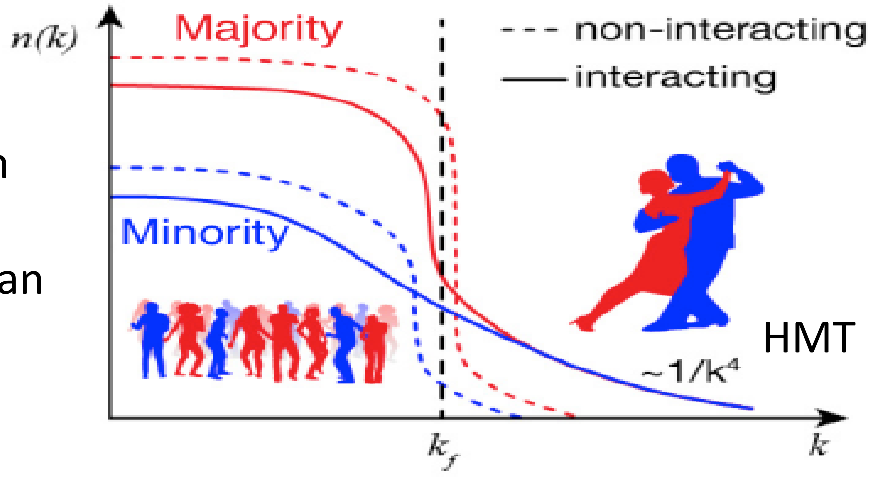
(2) isospin dependence of short-range correlation (SRC) in neutron-rich matter

M. Duer et al., PRL 122, 172502 (2019).

Evidence of np SRC dominance over pp (nn)



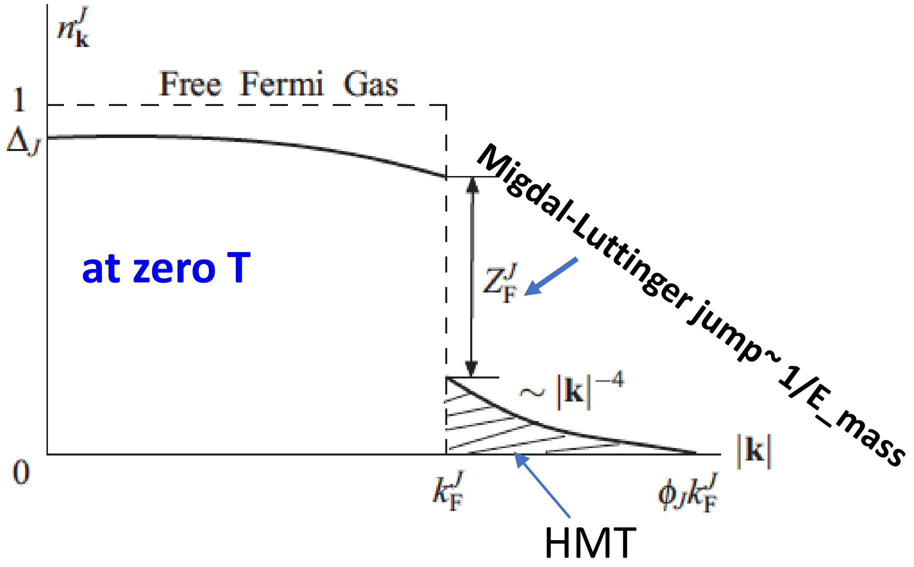
Implication: larger fraction of protons in the HMT → protons move faster than neutrons in n-rich matter



O. Hen et al., Science 346, 614 (2014)

The high momentum tail in dilute Fermi gas at zero temperature due to the repulsive core

A.B. Migdal, *The momentum distribution of interacting Fermi particles*, Sov. Phys. JETP, 333 (1957)



(1) From 2nd-order perturbation theory with a repulsive interaction

(2) The shape of HMT is universal-→ $(K_F/k)^4$

$$\rho_>(k)=\frac{\nu-1}{6\pi^2 x}(k_F c)^2\left\{(7x^3-3x-6)\ln\frac{x-1}{x+1}+(7x^3-3x+2)\ln 2\right.$$
$$-8x^3+22x^2+6x-24-4(x^2-2)^{3/2}$$
$$\left.\times\left[\tan^{-1}\frac{x+2}{(x^2-2)^{1/2}}+\tan^{-1}(x^2-2)^{-1/2}-2\tan^{-1}x(x^2-2)^{-1/2}\right]\right\}.$$

$x=k/k_F$

All HMT nucleons are on shell $\epsilon(k)=k^2/2m+V(k;\epsilon(k))$

(3) The fraction of HMT $N_>/N=\frac{8}{5}\frac{\nu-1}{\pi^2}(k_F c)^2$

depends on K_F, scattering length C and the spin-isospin degeneracy v

→ isospin dependence in neutron-rich matter

V. A. Belyakov, Sov. Phys. JETP 13, 850 (1961).

R. Sartor and C. Mahaux, Phys. Rev. C 21, 1546 (1981)

R. Amado, Phys. Rev. C 14, 1264 (1976).

S.N. Tan, Ann. Phys. 323, 2952 (2008); 323, 2971 (2008); 323, 2987 (2008).

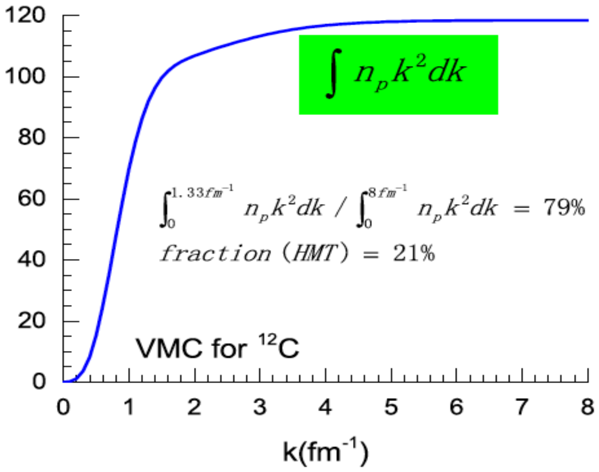
S. K. Bogner and D. Roscher, Phys. Rev. C 86, 064304 (2012)

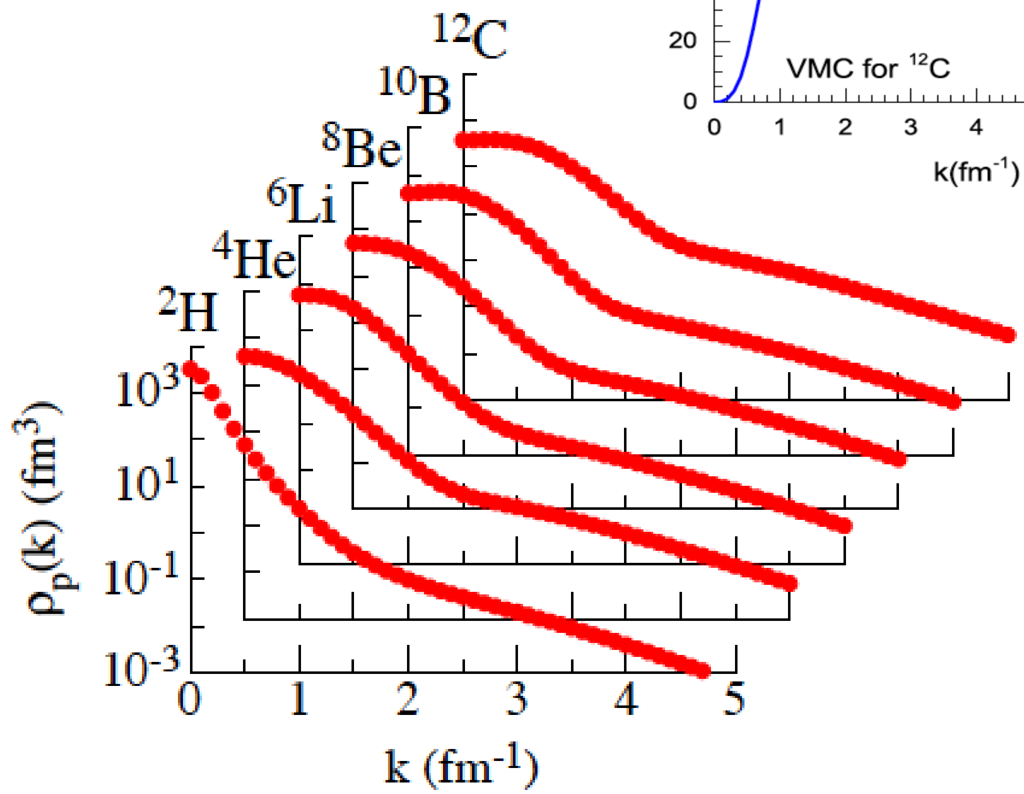
A. Rios, A. Polls, and W.H. Dickhoff, Phys. Rev. C 89, 044303 (2014).

The structure of ^{12}C in momentum space

R. B. Wiringa, R. Schiavilla, Steven C. Pieper, J. Carlson, PRC 89, 024305 (2014)

Variational Monte Carlo (VMC)





Strength and isospin dependence of SRC

R. Subedi et al. Science 320, 1475 (2008)

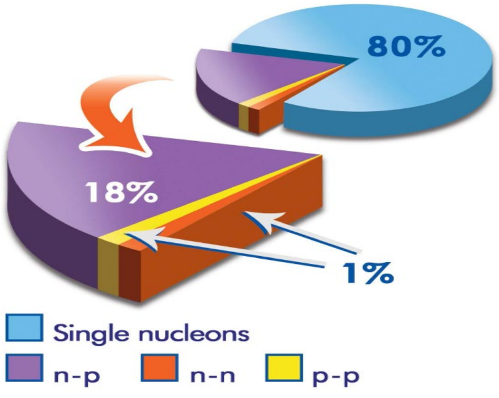
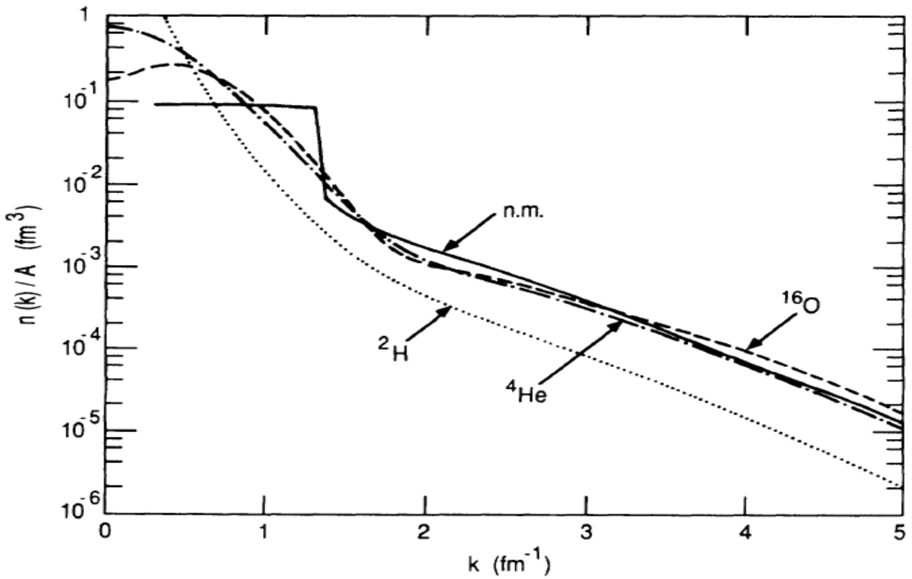


Figure 3: The average fraction of nucleons in the various initial state configurations of ^{12}C.

Universal high momentum tails

O. Benhar, V.R. Pandharipande, Steven C. Pieper, Rev. Modern Phys. 93 (1993) 817.



High momentum nucleons are on shell

Depletion of the nuclear Fermi sea,
Arnau Rios, Artur Polls, W.H. Dickhoff, PRC 79, 064308 (2009)

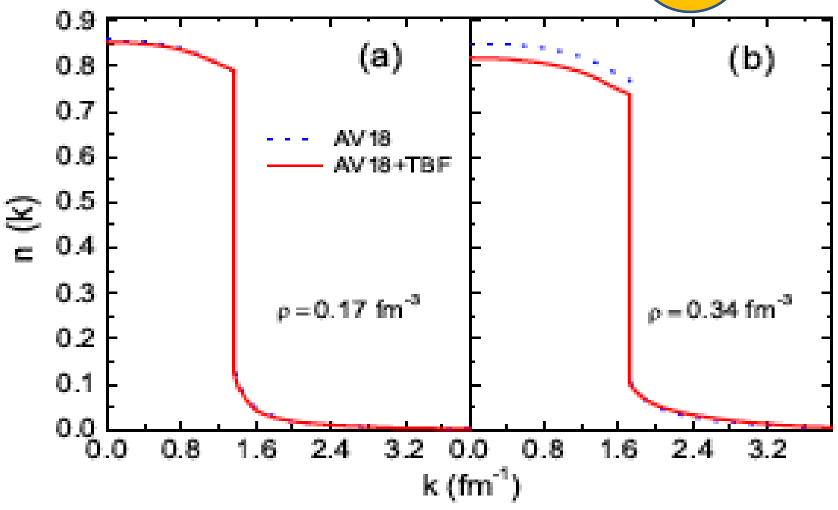
Since the peak of the dressed propagator will normally occur at the quasi-particle energy

$$\varepsilon(k) = \frac{k^2}{2m} + \mathrm{Re}\Sigma(k, \varepsilon(k)), \tag{39}$$

it is convenient to include an auxiliary potential

$$U(k) = \mathrm{Re}\Sigma(k, \varepsilon(k)), \tag{40}$$

$$n^1_\downarrow(k) = -\partial_\omega \mathrm{Re}\Sigma_\uparrow(k,\omega)|_{\omega=\varepsilon(k)}\,, \quad k > k_F\,.$$



Peng Yin, Jian-Yang Li, Pei Wang, Wei Zuo

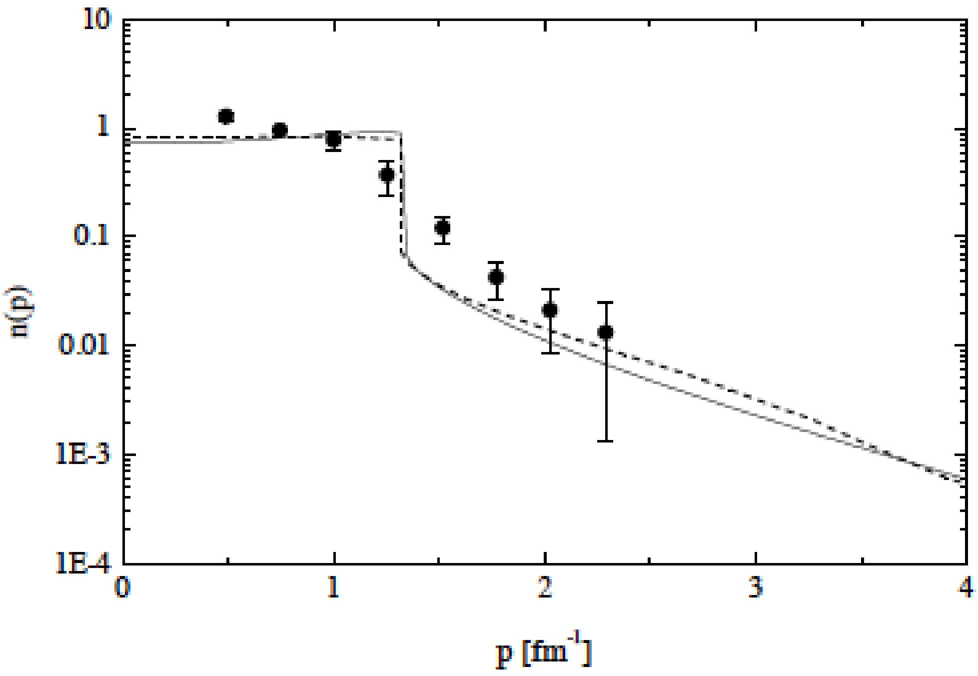
10.1103/PhysRevC.87.014314

Transport Theoretical Approach to the Nucleon Spectral Function in Nuclear Matter

G.F. Bertsch and P. Danielewicz, PLB367 (1996) 55.

J. Lehr, M. Effenberger, H. Lenske, S. Leupold, U. Mosel, PLB483 (2000) 324, *NPA* 703 (2002) 393

"Momentum distributions are dominated by phase space effects rather than by the off-shell momentum structure of interactions"



Effects of isospin-dependent SRC on the kinetic symmetry energy of quasi-nucleons

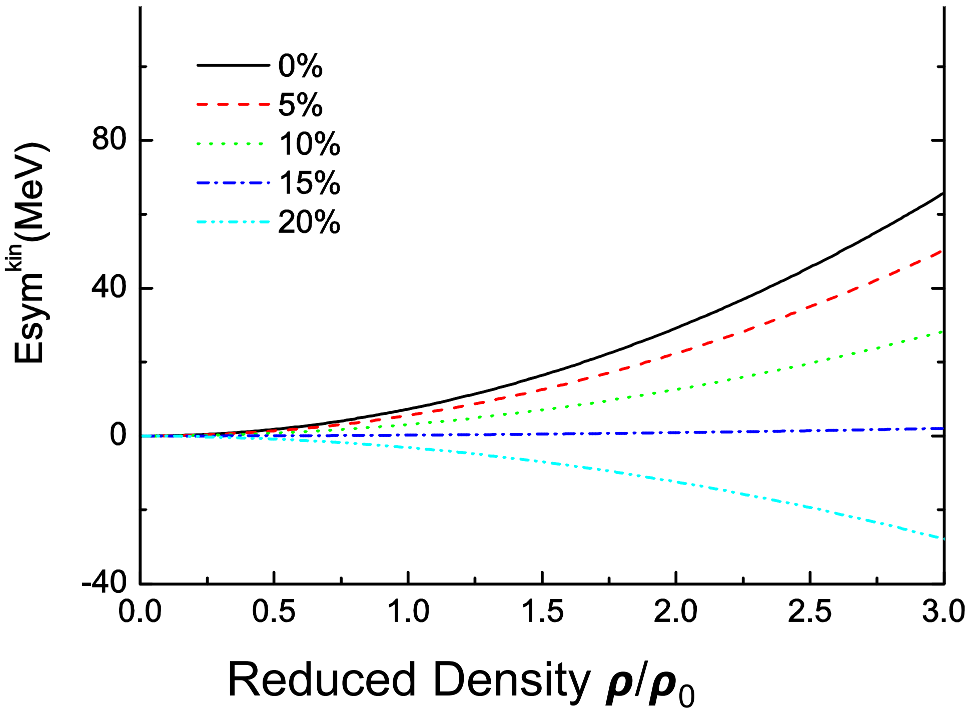
Chang Xu, Ang Li and Bao-An Li,
JPCS 420, 012190 (2013).

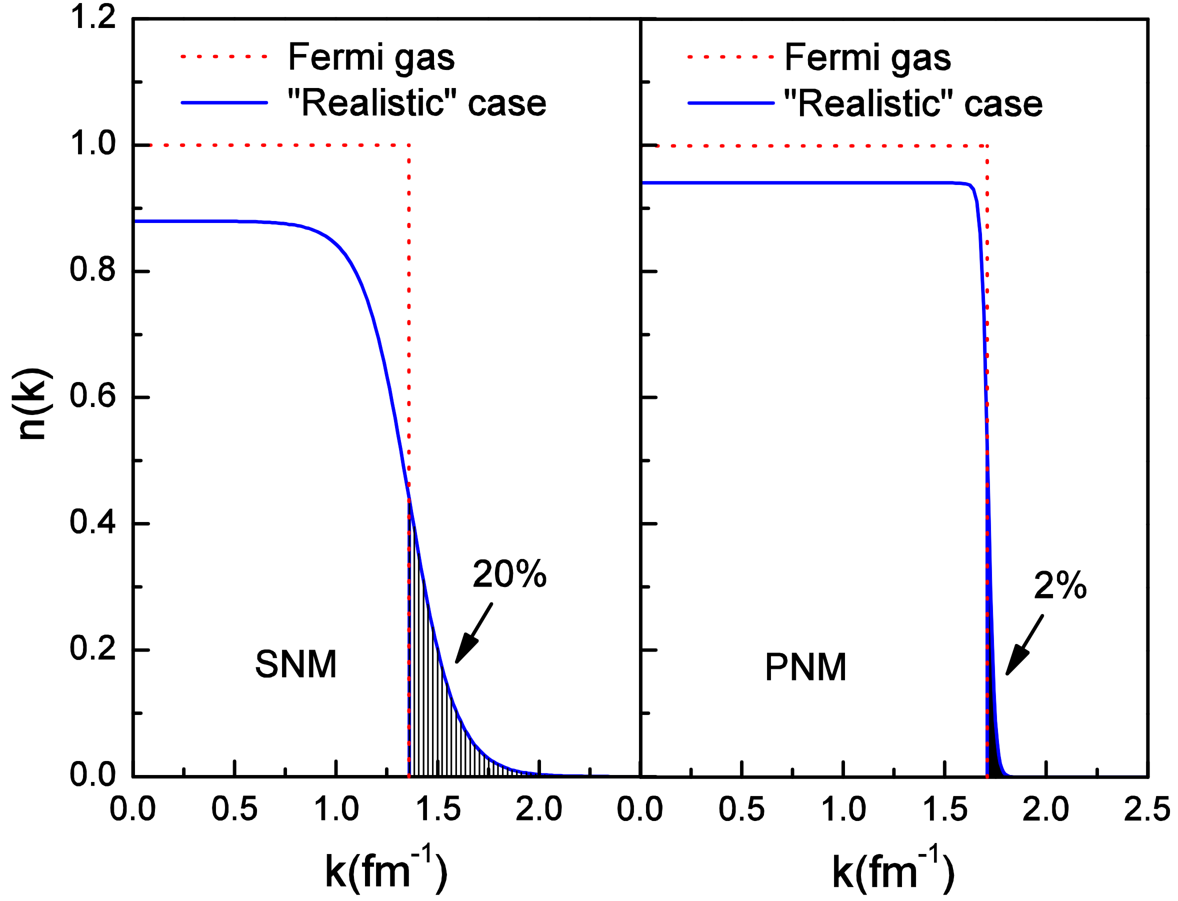
$$E_{kin} = \alpha \int_0^\infty \frac{\hbar^2 k^2}{2m} n(k) k^2 dk,$$

Free-Fermi Gas (FFG):
kinetic E_{sym}=12.3 MeV at ρ_0

$$E^{kin}_{sym} = E^{kin}_{PNM} - E^{kin}_{SNM} < 0$$

if more than 15% nucleons are in the high-momentum tail of SNM due to the tensor force for n-p T=0 channel, the kinetic symmetry energy becomes negative





Confirmation by Microscopic Many-Body Theories

1. Isaac Vidana, Artur Polls, Constanca Providencia
PRC84, 062801(R) (2011)

Brueckner--Hartree--Fock approach using the Argonne V18 potential
plus the Urbana IX three-body force

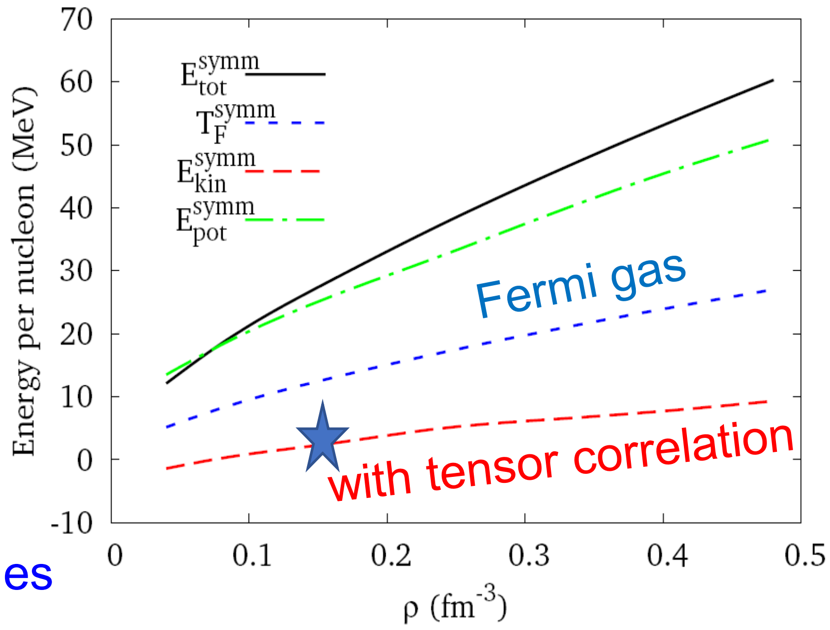
2. Arianna Carbone, Artur Polls, Arnau Rios, EPL 97, 22001 (2012)
A. Carbone, A. Polls, C. Providência, A. Rios, I. Vidaña, EPJA 50, 13 (2014)
Self-Consistent Green's Function Approach with Argonne Av18, CDBonn, Nij1, N3LO interactions

3. Alessandro Lovato, Omar Benhar et al.,
extracted from results already published in
Phys. Rev. C83:054003,2011
Using Argonne V'_6 interaction
Fermi-Hyper-Netted-Chain (FHNC)
Single Operator Chains (SOC)

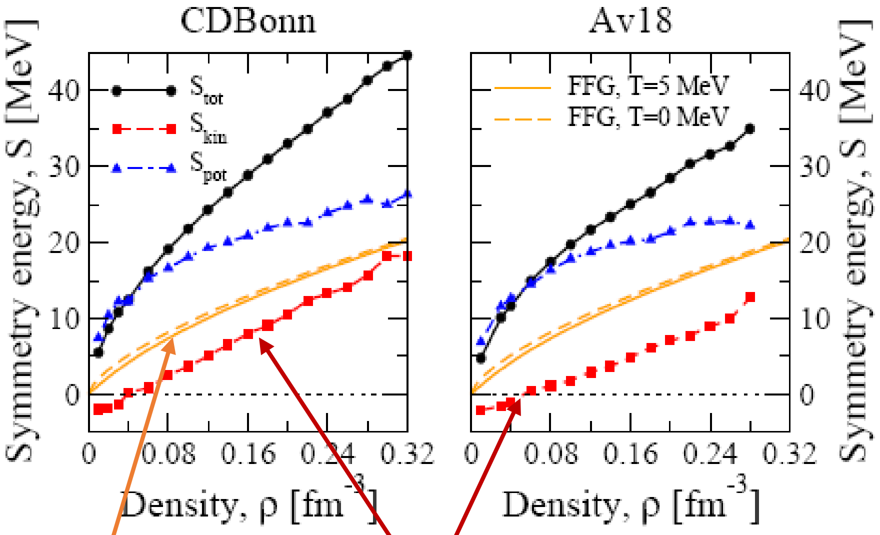
4. A. Rios, A. Polls, W. H. Dickhoff
PRC 89, 044303 (2014).
Ladder Self-Consistent Green Function

They all included the tensor force and
many-body correlations using different techniques

Brueckner--Hartree—Fock prediction



Self-Consistent Green's Function Approach (A. Rios et al.)



Free Fermi Gas

Actual kinetic symmetry E

	S_{tot} [MeV]	S_{kin} [MeV]	S_{pot} [MeV]	L [MeV]
Av18	25.1	4.9	20.2	37.7
Nij1	27.4	4.6	22.8	48.5
CDBonn	28.8	7.9	20.9	52.6
N3LO	29.7	7.2	22.4	55.2

Brueckner–Hartree–Fock approach (I. Vidana et al.)

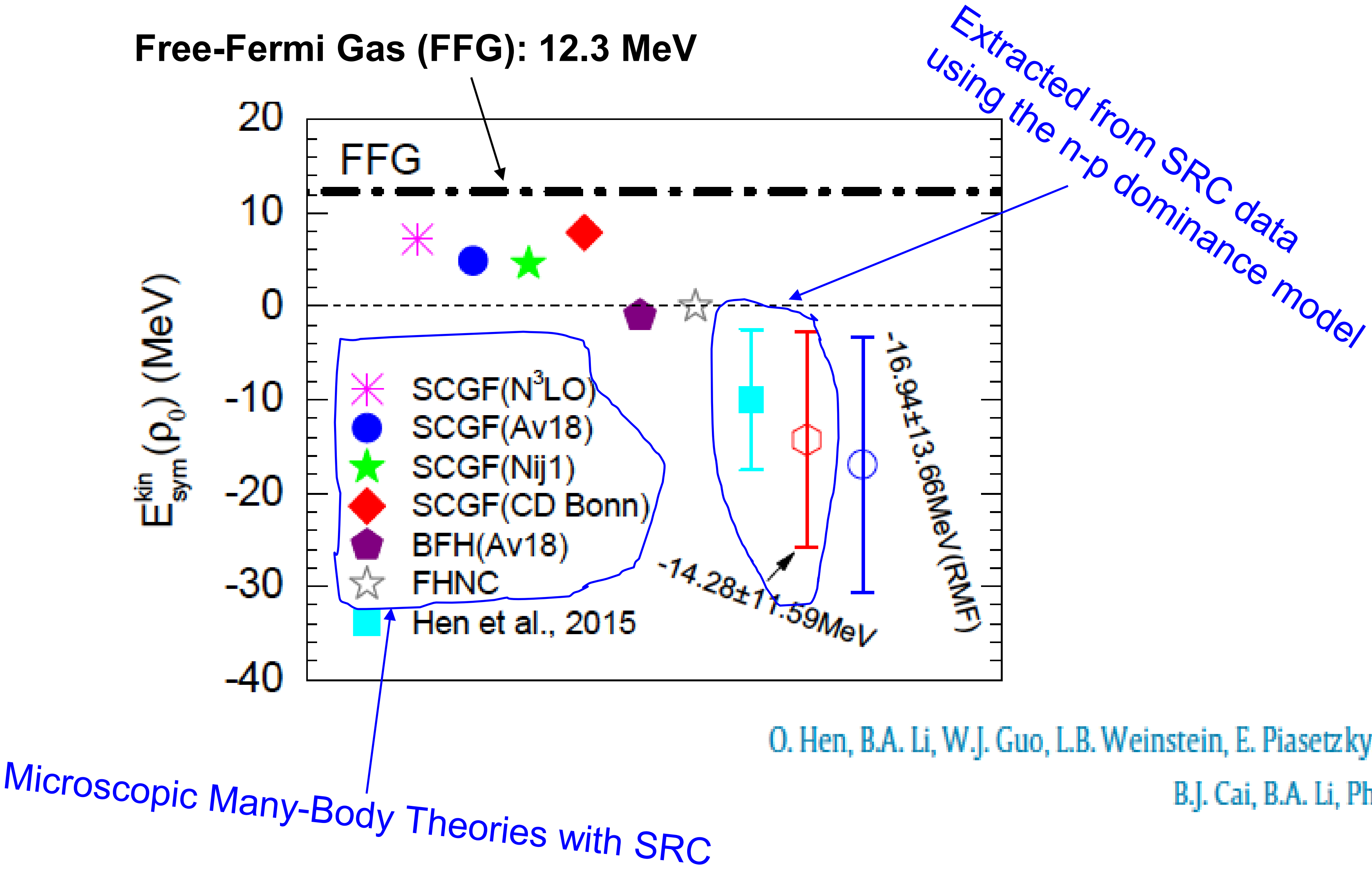
Using the Hellmann–Feynman theorem

V18 potential plus the Urbana IX three-body force.

	E_{NM}	E_{SM}	E_{sym}	L
$\langle T \rangle$	53.321	54.294	−0.973	14.896
$\langle V \rangle$	−34.251	−69.524	35.273	51.604
Total	19.070	−15.230	34.300	66.500

At saturation density,
the Free Fermi Gas (FFG) model
prediction is about 12.5 MeV

Reduced Kinetic symmetry energy of quasi-nucleons due to the isospin dependence of SRC



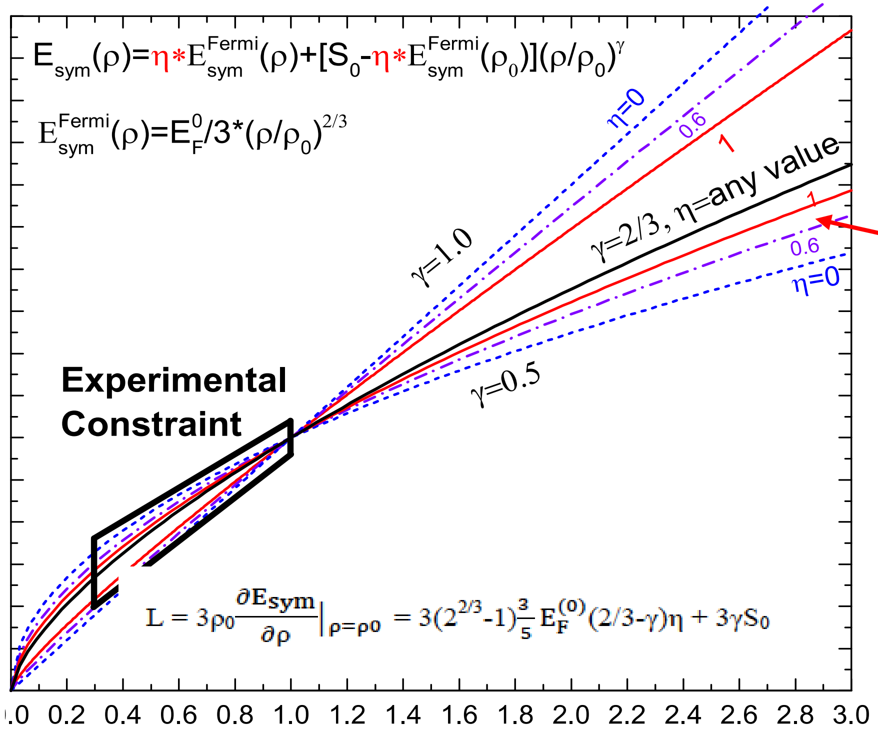
O. Hen, B.A. Li, W.J. Guo, L.B. Weinstein, E. Piasetzky, Phys. Rev. C 91 (2015) 025803.

B.J. Cai, B.A. Li, Phys. Rev. C 92 (2015) 011601(R).

Incorporating the SRC-modified E_{sym} in heavy-ion collisions and neutron star mergers

The kinetic and potential E_{sym} have different effects on observables in heavy-ion reactions and neutron stars

Existing constraints Do NOT exclude negative kinetic symmetry energy



Bao-An Li, Wen-Jun Guo, Zhaozhong Shi
Phys. Rev. C 91, 044601 (2015)

Widely used:

$$E_{sym}(\rho)=E_{sym}^{kin}(FFG)(\rho)+\left[S_0-E_{sym}^{kin}(FFG)(\rho_0)\right]\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Reducing the kinetic E_{sym} with η

$$E_{sym}(\rho)=\eta E_{sym}^{FFG}(\rho)+[S_0-\eta E_{sym}^{FFG}(\rho_0)](\rho/\rho_0)^{\gamma}$$

Adopted recently in:

Finite-temperature extension for cold neutron star equations of state
Carolyn A. Raithel, Feryal Ozel, Dimitrios Psaltis
Astrophys. J. 875 (2019) 1

Realistic finite-temperature effects in neutron star merger simulations
Carolyn A. Raithel, Dimitrios Psaltis, Feryal Ozel
Phys. Rev. D 104 (2021) 6

Incorporating the SRC-induced high-momentum tail in RMF

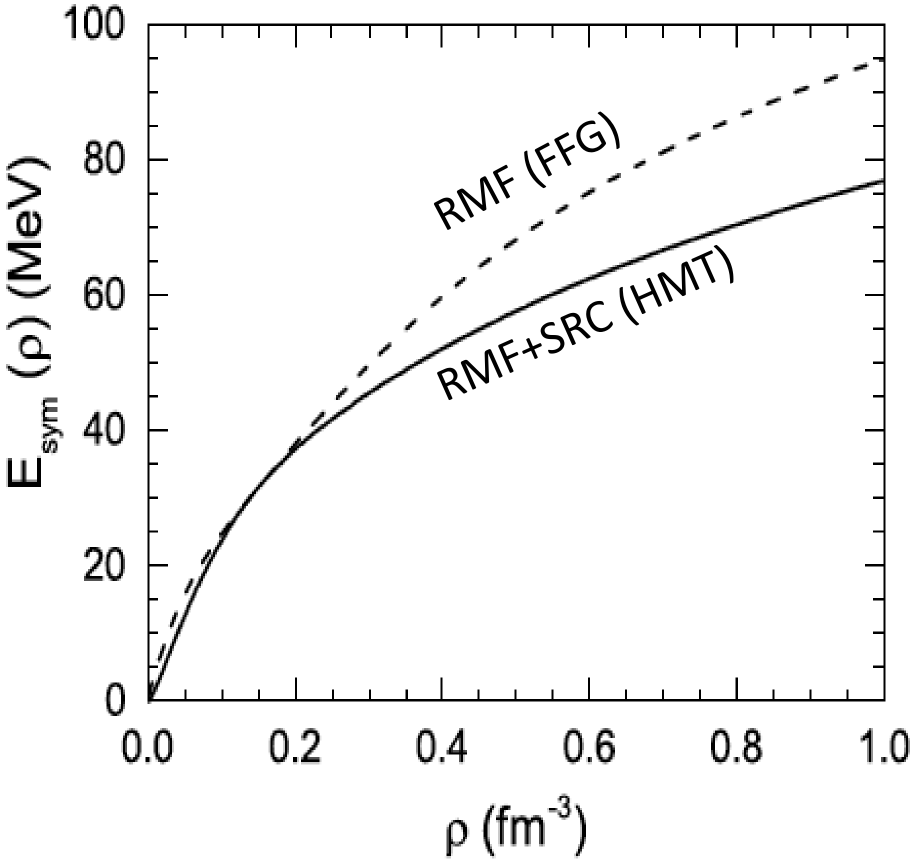
(via kinetic energy and scalar density)

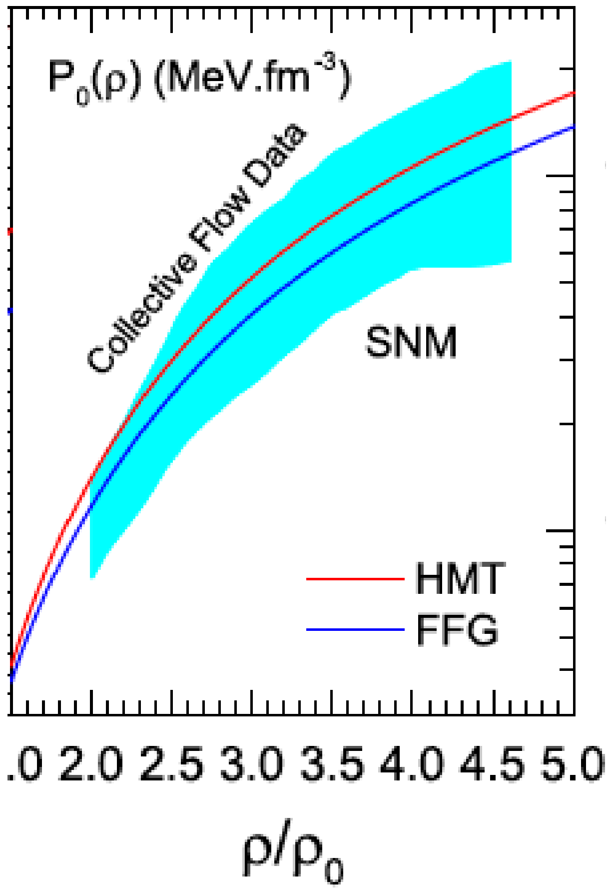
PHYSICAL REVIEW C 93, 014619 (2016)

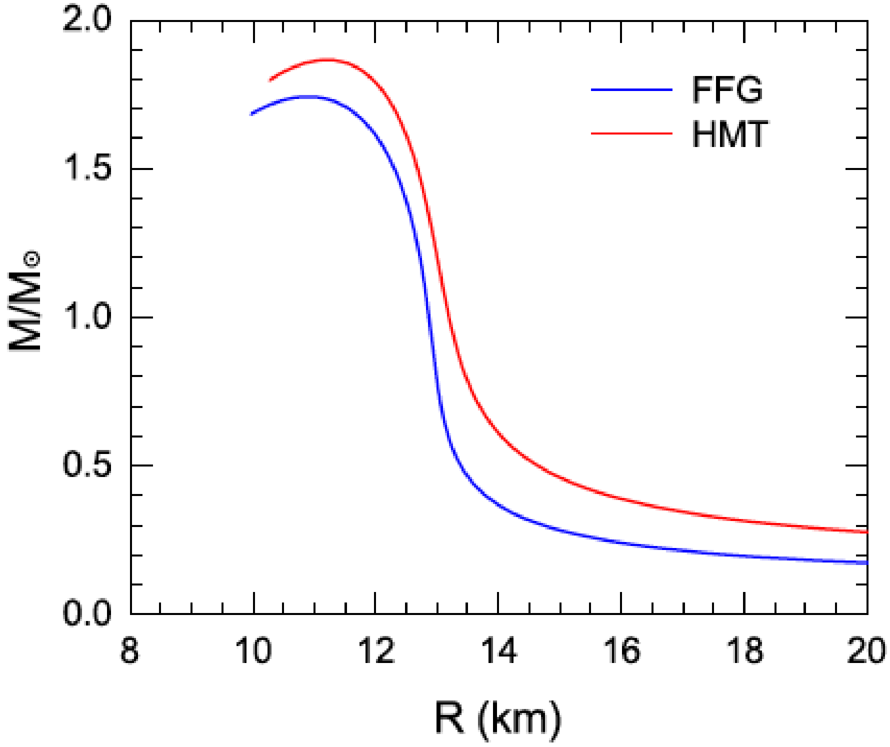
Bao-Jun Cai and Bao-An Li

$$\int_0^{k_F^J} (\text{FFG step function}) f\, d\mathbf{k} \longrightarrow \int_0^{\phi_J k_F^J} n_{\mathbf{k}}^J\, (\text{HMT}) f\, d\mathbf{k}.$$

e.g., scalar density:
$$\rho_{S,J} = \frac{2}{(2\pi)^3} \int_0^{\phi_J k_F^J} n_{\mathbf{k}}^J d\mathbf{k} \frac{M_J^*}{\sqrt{|\mathbf{k}|^2 + M_J^{*2}}}$$



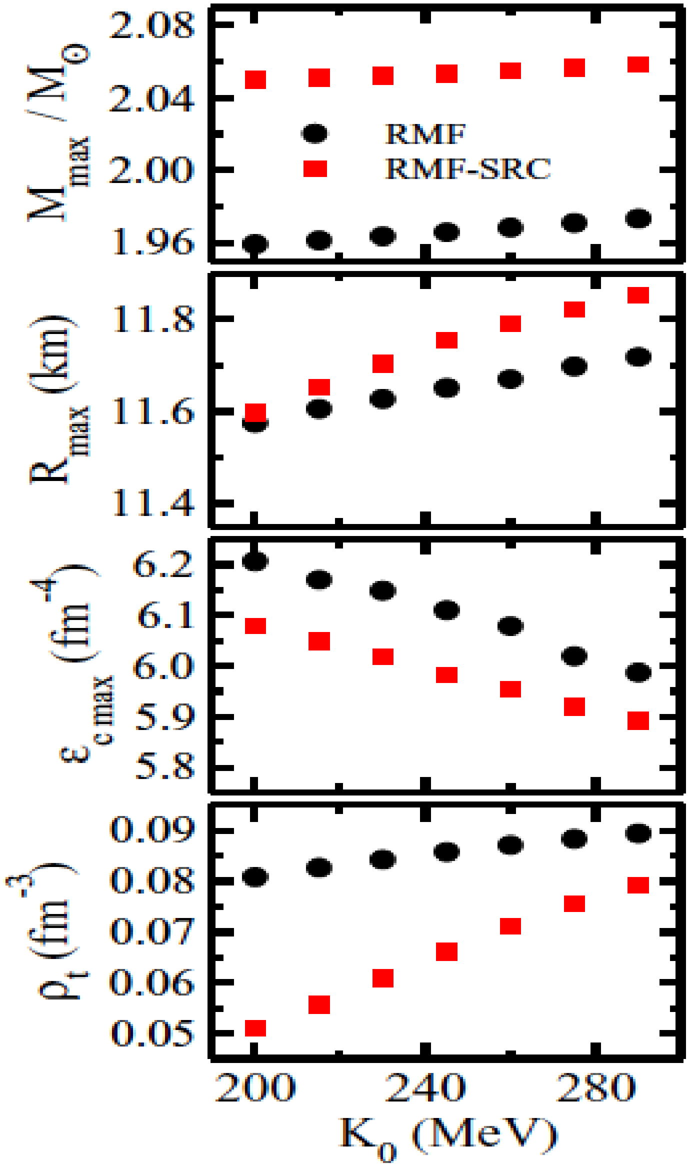




Effects of short-range nuclear correlations on the deformability of neutron stars

PHYSICAL REVIEW C **101**, 065202 (2020)

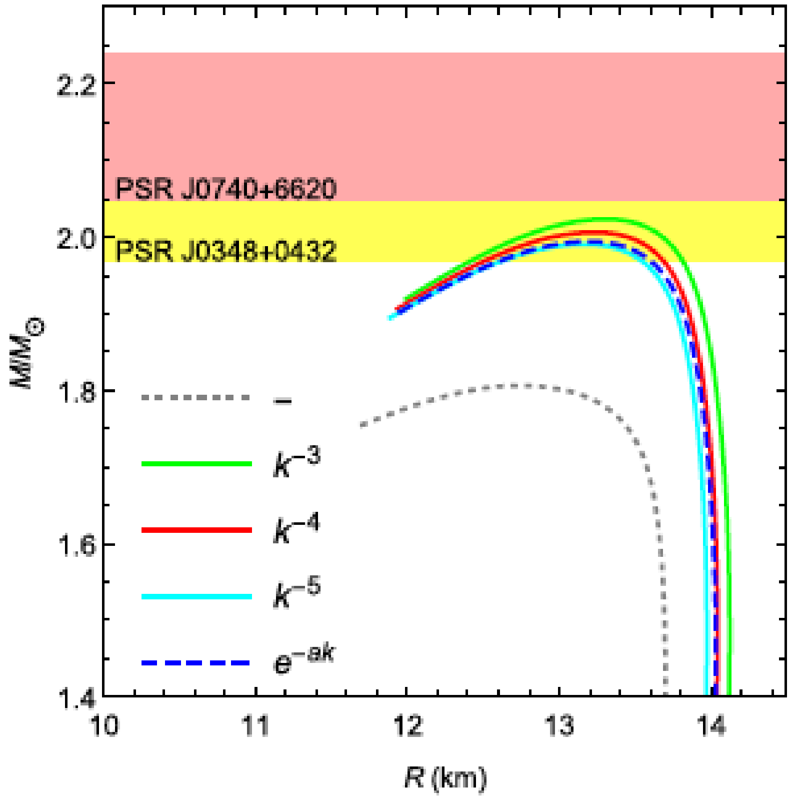
Lucas A. Souza, Mariana Dutra, César H. Lenzi, and Odilon Lourenço
Departamento de Física, Instituto Tecnológico de Aeronáutica, DCTA, 12228-900, São José dos Campos, SP, Brazil



Impacts of nucleon-nucleon short-range correlations on neutron stars

Hao Lu [a], Zhongzhou Ren [b,c,*], Dong Bai [b]

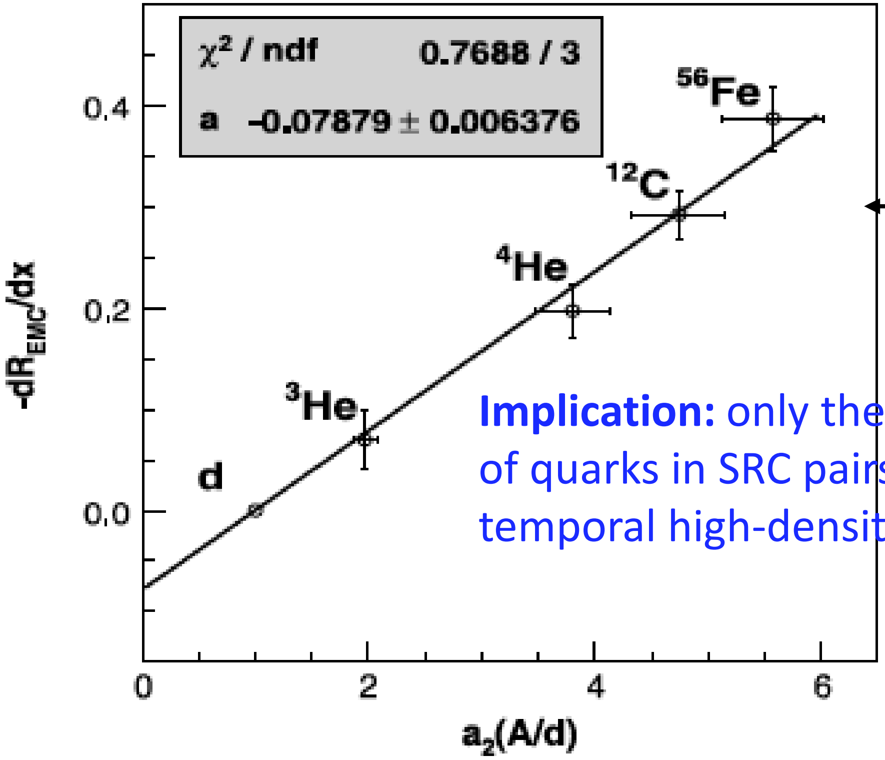
Nuclear Physics A 1011 (2021) 122200



Relationship between the SRC and EMC effect

L.B. Weinstein et al., PRL 106, 052301 (2011)

EMC: Medium modifications of the momentum distribution (structure functions) of quarks in nuclei w.r.t free nucleons



Strength of SRC

unscaled

Strength of EMC

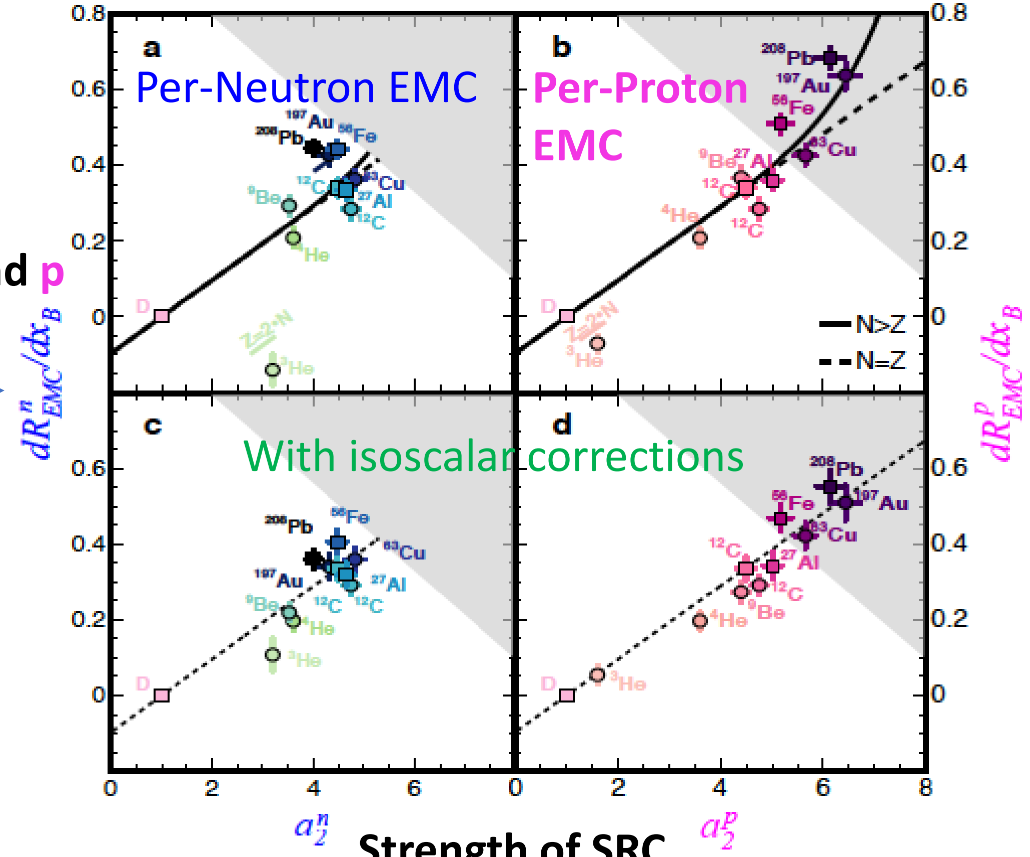
Implication: only the momentum distribution of quarks in SRC pairs are modified when a local temporal high-density is formed in nuclei

Modified Structure of Protons and Neutrons in Correlated Pairs

Nature 566, 354 (2019)

B. Schmookler, M. Duer, A. Schmidt, O. Hen, S. Gilad, E. Piasetzky, M. Strikman, L.B. Weinstein et al. (The CLAS Collaboration)

Scaled by n and p numbers



Strength of SRC

Implications:

Due to n-p SRC, protons in n-rich nuclei move faster than neutrons

→ the structure function of protons are modified more→ u quarks will be modified more→ DIS neutrino/antineutrino-nuclei interactions will be affected differently as they scatter primarily from d/u quark separately

Planned SRC experiments at the Electron-Ion Collider

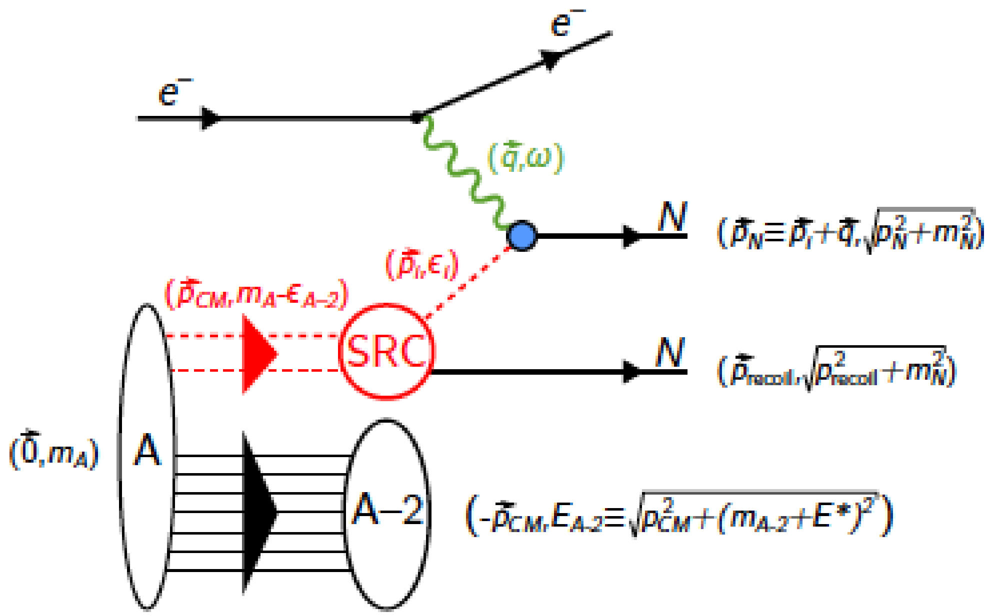
(1) The EIC Science Paper, arXiv:1108.1713v2

(2) Probing short-range correlations in the deuteron via incoherent diffractive J/ψ production with spectator tagging at the EIC

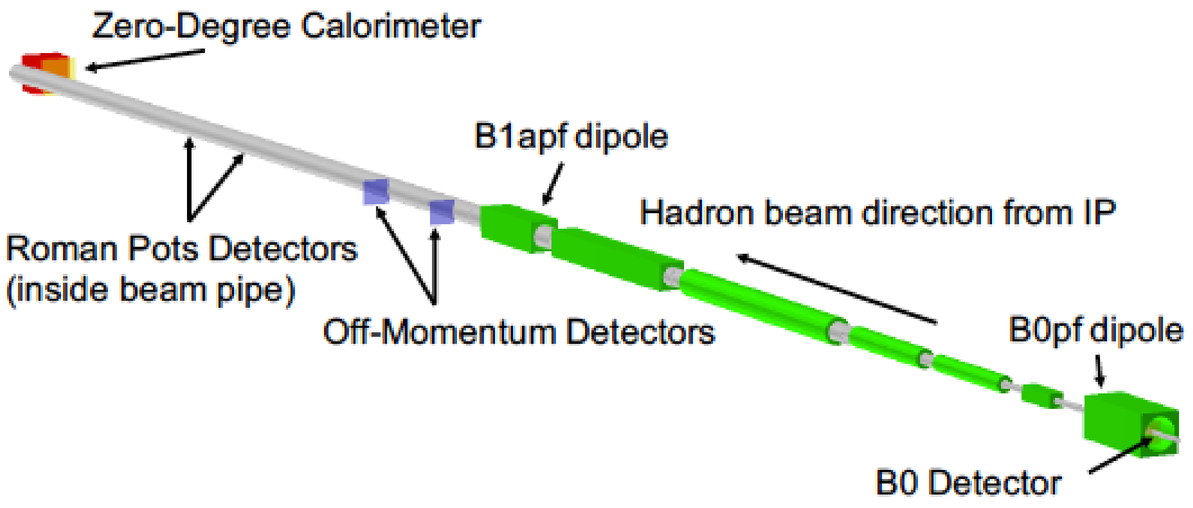
Z. Tu et al., Physics Letters B 811 (2020) 135877

(3) Measuring Recoiling Nucleons from the Nucleus with the Electron Ion Collider

F. Hauenstein et al., arXiv:2109.09509v1



Far-Forward detector system for recoiling nucleons



Summary

The most fundamental but lest known physics underlying the high-density symmetry energy

Spin-isospin dependence of nucleon interactions at short distance

$V_{np}(T_0) \neq V_{np}(T_1)$

Tensor force and short-range repulsive core

Isospin dependent short range nucleon-nucleon correlation

(1) Modification of nucleon momentum distribution at zero temperature w.r.t. the step function for Free Fermi Gas
(2) Protons move faster than neutrons in n-rich nuclei/matter

Reduced kinetic symmetry energy w.r.t. the Free Fermi Gas

Potential symmetry energy

Structures and collisions of nuclei and neutron stars

Dynamical origin of the EMC effect

(1) Only the momentum distributions of quarks in SRC nucleon pairs in nuclei are modified w.r.t. to free nucleons
(2) Quarks in protons of n-rich nuclei are modified more
→ u quarks move faster than d quarks in n-rich nuclei

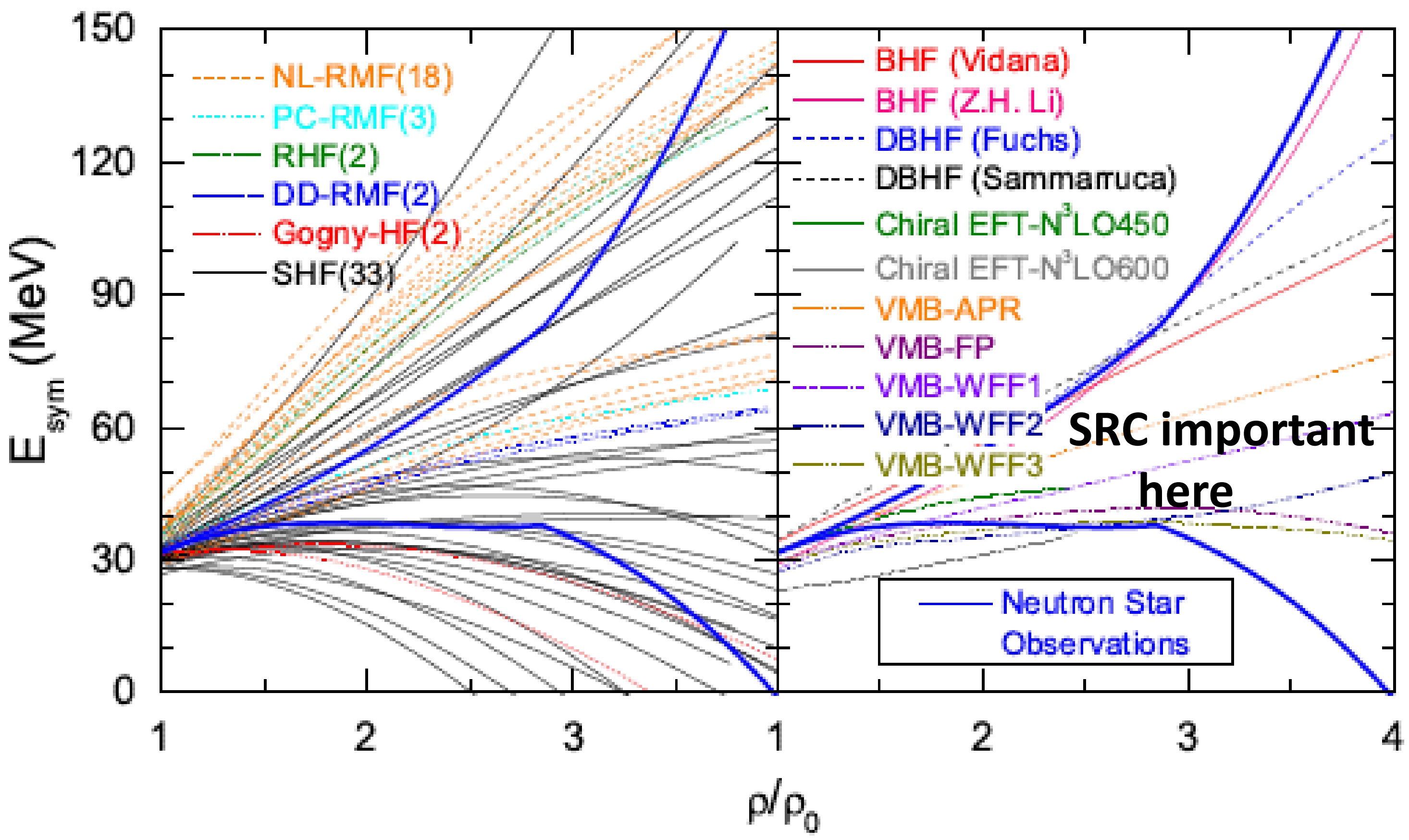
nucleons ? quarks

Frontiers of QCD

The flavor and spin dependent momentum distributions of quarks and gluons in nuclei and nuclear matter

Phenomenological Models
60 examples

Microscopic & *ab initio* Theories
11 examples



Needs data from post-mergers with high-frequency gravitational waves, radii of massive NSs or messengers directly from the core, e.g., neutrinos

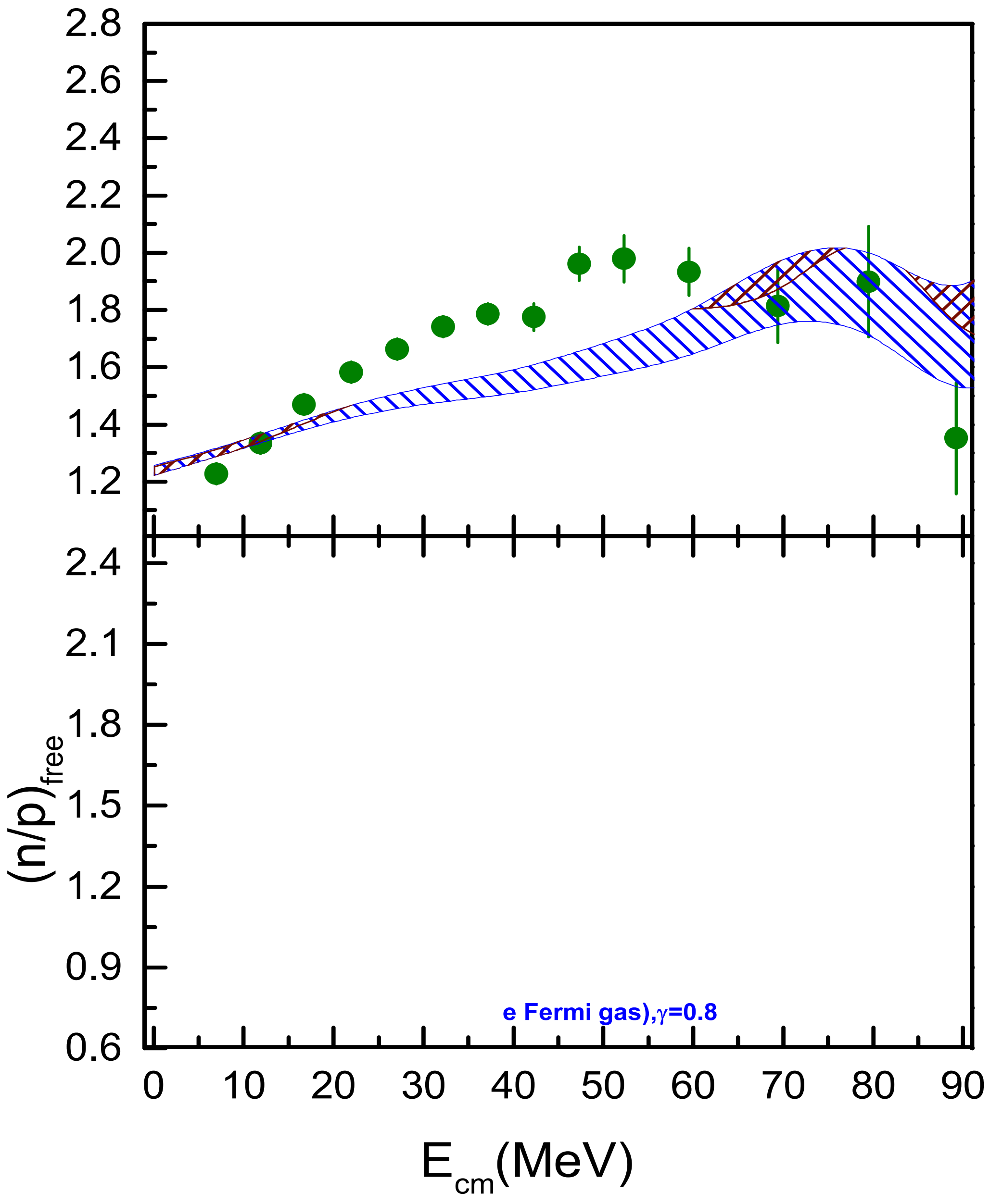
- E_{sym} around (1-2)ρ_0 is most relevant for determining the radii of canonical neutron stars, existing 1.4M_{sun}NS observations do NOT constrain much E_{sym} above 2ρ_0 where SRC effects are important.

Effects of reduced kinetic symmetry energy on heavy-ion collisions

Or Hen, Bao-An Li, Wen-Jun Guo,
L.B. Weinstein, Eli Piasetzky
Phys. Rev. C 91, 025803 (2015)

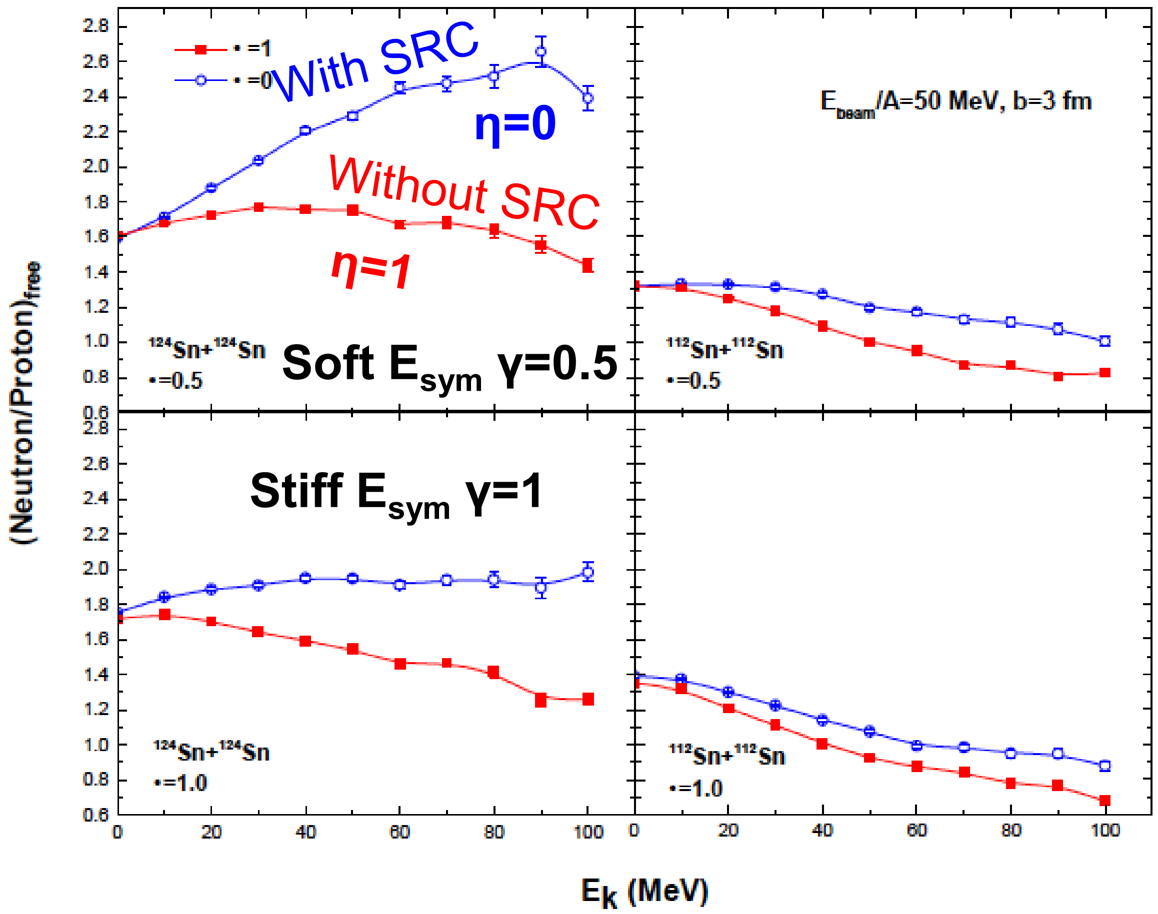
$$E_{sym}^{pot}(\rho) = [E_{sym}(\rho_0) - \eta \cdot E_{sym}^{kin}(\rho_0)|_{FG}] \cdot (\rho/\rho_0)^{\gamma}.$$

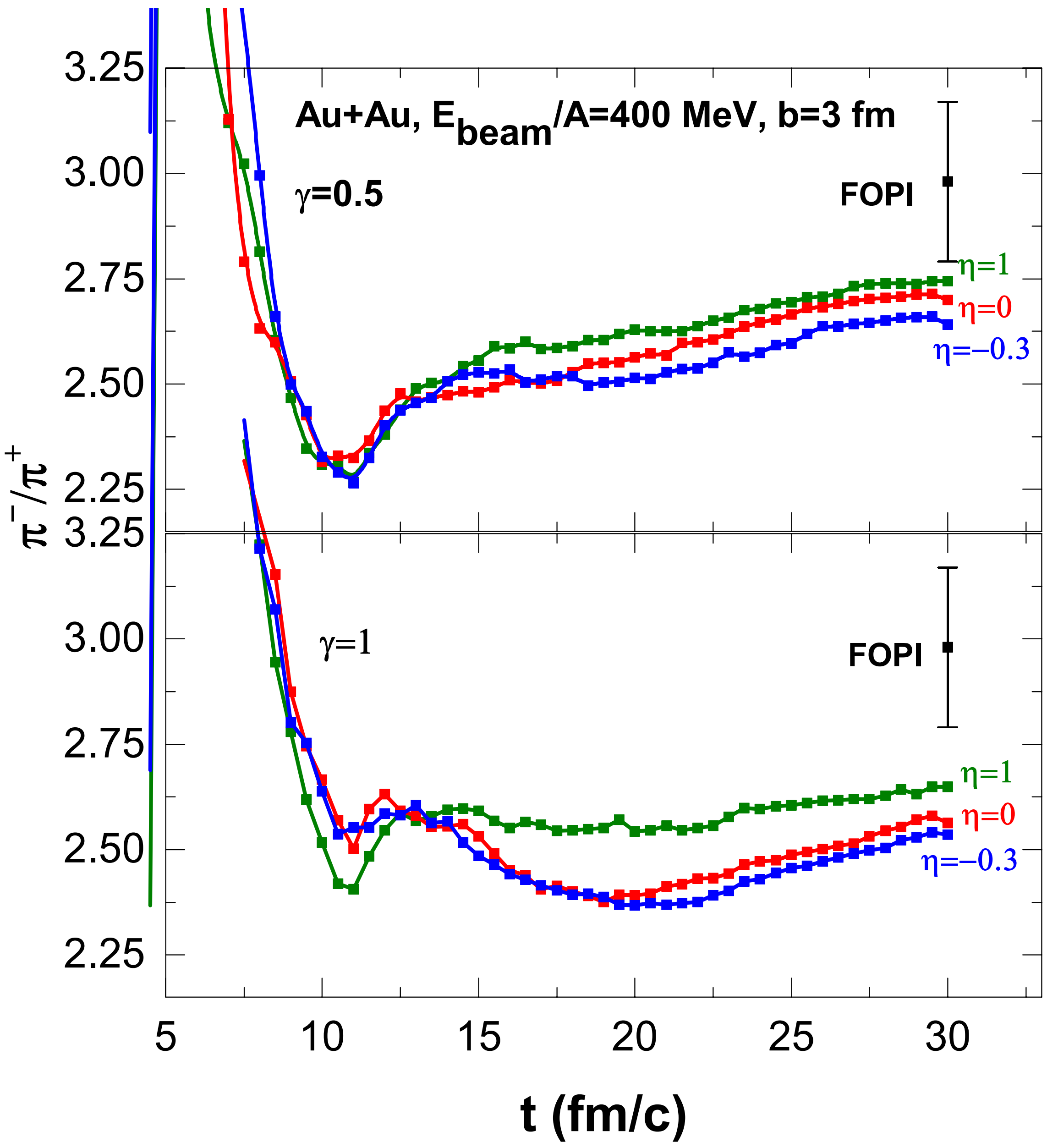
$$V_{sym}^{n/p}(\rho,\delta) = [E_{sym}(\rho_0) - \eta \cdot E_{sym}^{kin}(\rho_0)|_{FG}](\rho/\rho_0)^{\gamma} \times [\pm 2\delta + (\gamma-1)\delta^2]. \quad (10)$$



Effects of the η-parameter on the free neutron/proton ratio in heavy-ion collisions

IBUU transport model calculations with momentum-independent mean-fields

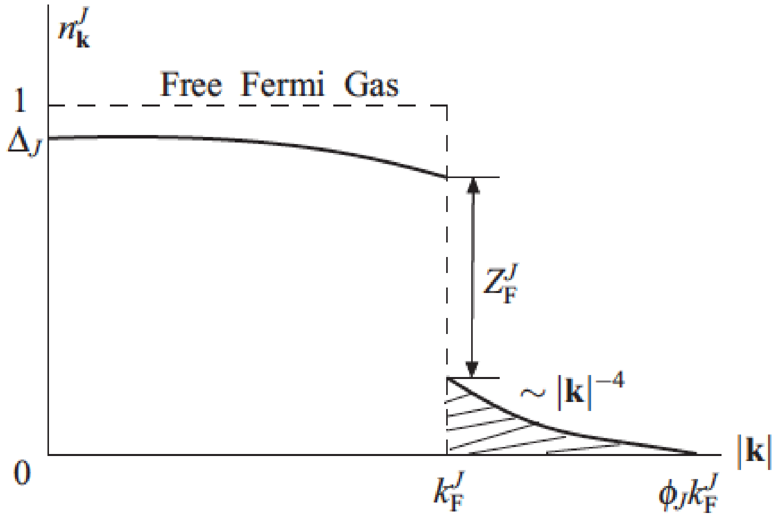




Au+Au, E_beam/A=400 MeV, b=3 fm
γ=0.5
FOPI
η=1
η=0
η=−0.3
γ=1
FOPI
η=1
η=0
η=−0.3
π⁻/π⁺
t (fm/c)
3.25
3.00
2.75
2.50
2.25
5
10
15
20
25
30

Phenomenological nucleon momentum distribution n(k) including SRC effects guided by microscopic theories and experimental findings

B.J. Cai and B.A. Li, PRC92, 011601(R) (2015).



$$n_{\mathbf{k}}^J(\rho,\delta) = \begin{cases} \Delta_J + \beta_J I\left(|\mathbf{k}|/k_F^J\right), & 0 < |\mathbf{k}| < k_F^J, \\ C_J\left(k_F^J/|\mathbf{k}|\right)^4, & k_F^J < |\mathbf{k}| < \phi_J k_F^J. \end{cases}$$

All parameters are assumed to have a linear dependence on isospin asymmetry as indicated by SCGF and BHF calculations

$$Y_J = Y_0(1 + Y_1^J \delta).$$

Fraction of high-k nucleons

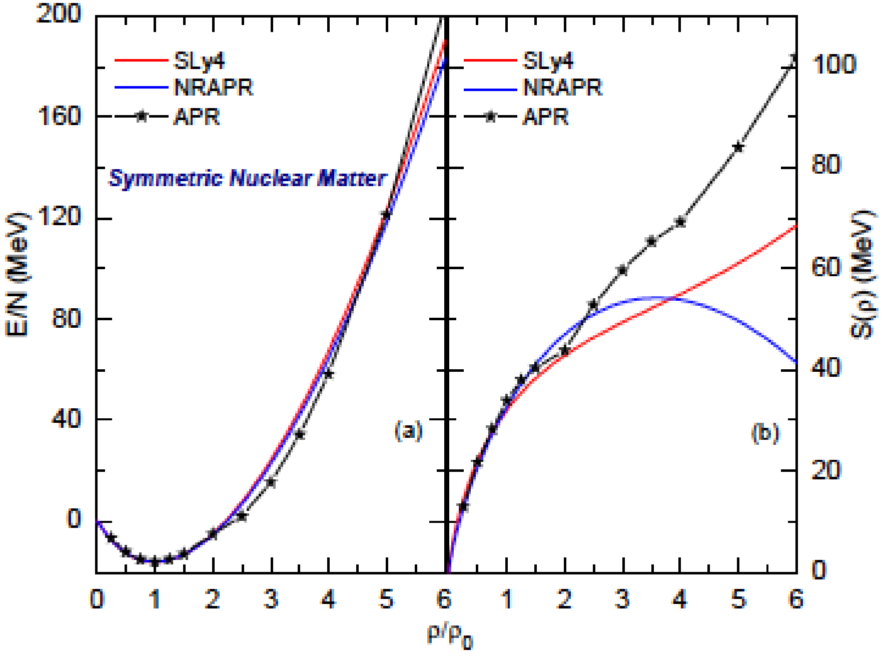
$$x_J^{\mathrm{HMT}} = 3C_J\left(1 - \frac{1}{\phi_J}\right)$$

All parameters are fixed by

(1) Jlab data: HMT in SNM=25%, 1.5% in PNM,
(2) Contact C for SNM from deuteron wavefunction
(3) Contact C in PNM from microscopic theories

Microscopic diagnosis of n-skins in two Skyrme-Hartree-Fock models with similar EOSs for SNM and E_{sym} as the APR up to $1.5\rho_0$

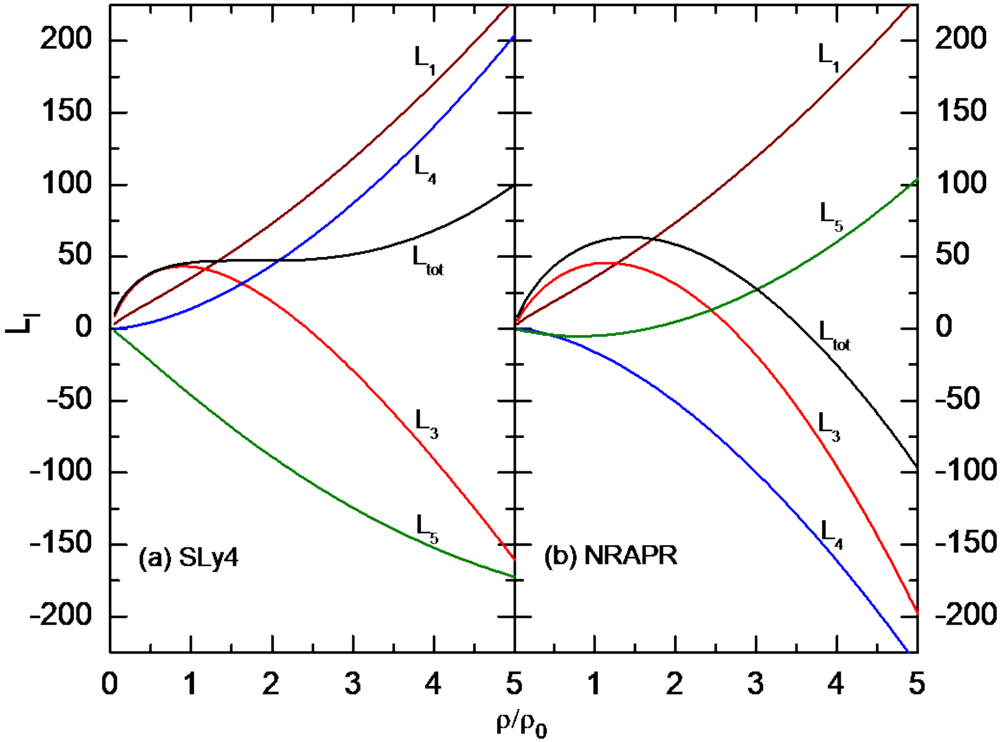
$$S_1(\rho) = \frac{\hbar^2 k_F^2}{6m_0^*(\rho, k_F)}$$
$$S_2(\rho) = \frac{1}{2}U_{sym,1}(\rho, k_F) \ ,$$
$$L_1(\rho) = \frac{2\hbar^2 k_F^2}{6m_0^*(\rho, k_F)} \equiv 2S_1(\rho)$$
$$L_2(\rho) = -\frac{\hbar^2 k_F^3}{6m_0^{*2}(\rho, k_F)} \frac{\partial m_0^*(\rho, k)}{\partial k}\bigg|_{k=k_F}$$
$$L_3(\rho) = \frac{3}{2}U_{sym,1}(\rho, k_F) \equiv 3S_2(\rho)$$
$$L_4(\rho) = \frac{\partial U_{sym,1}(\rho, k)}{\partial k}\bigg|_{k=k_F} \cdot k_F$$
$$L_5(\rho) = 3U_{sym,2}(\rho, k_F) \ .$$



L_Sly4 = 45.9 MeV
L_NRAPR = 59.6 MeV

For [208]Pb
Rskin_Sly4 = 0.157 fm
Rskin_NRAPR = 0.184 fm

F. Fattoyev, W.G. Newton and Bao-An Li
PRC 90, 022801(R) (2014)



$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho, k_F),$$

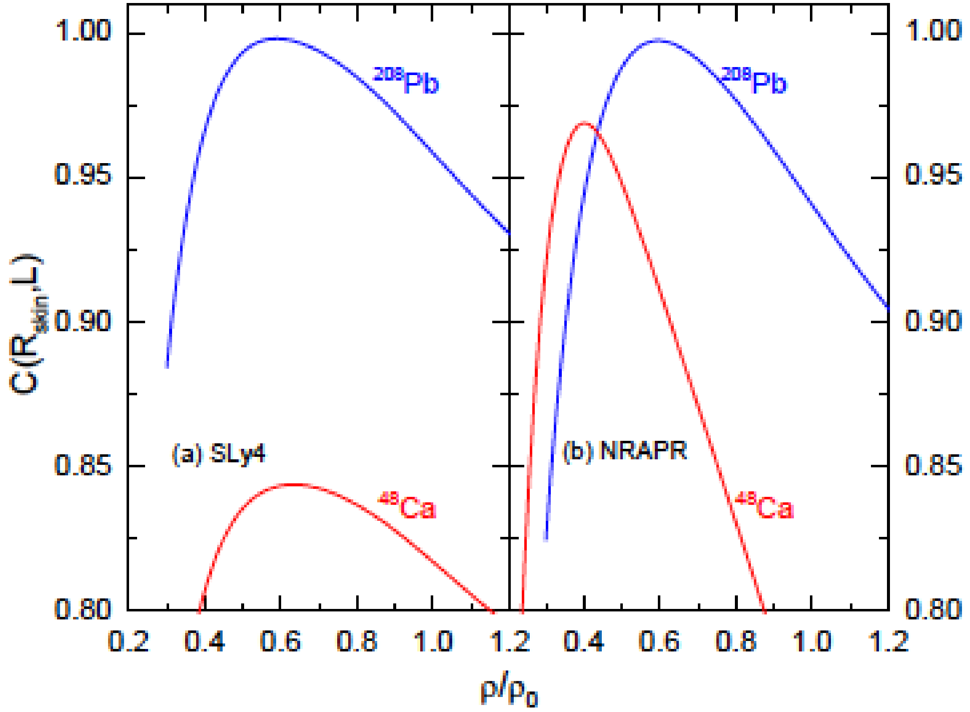
$$U_{n/p}(k,\rho,\delta) = U_0(k,\rho) \pm U_{sym1}(k,\rho)\cdot\delta + U_{sym2}(k,\rho)\cdot\delta^2 + o(\delta^3)$$

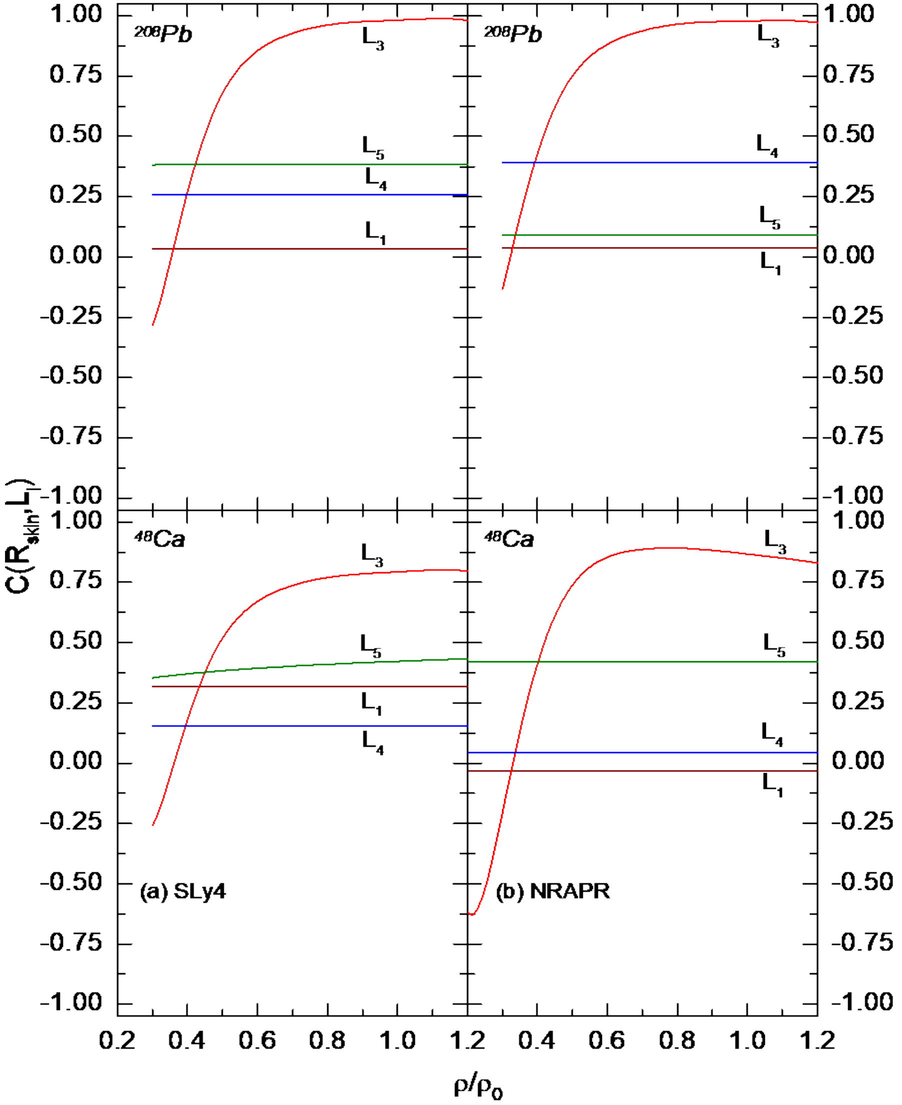
$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho, k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F}\cdot k_F + 3U_{sym,2}(\rho, k_F),$$

L_1 L_2 L_3 L_4 L_5

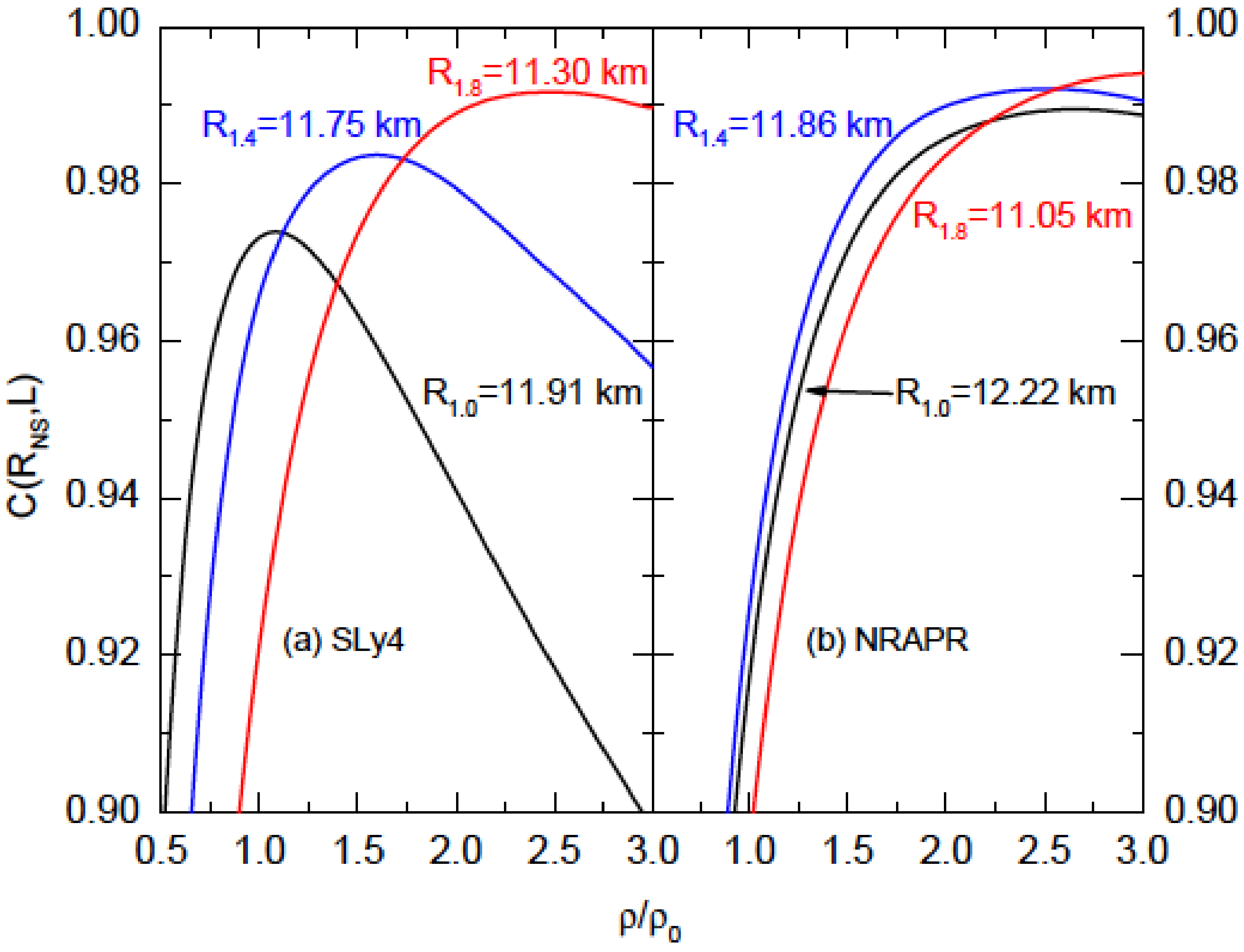
Covariance analysis of the correlation between n-skin and $L_i(\rho)$

$$C(X,Y) = \frac{\langle (X - \langle X \rangle)(Y - \langle Y \rangle) \rangle}{\sqrt{\langle (X - \langle X \rangle)^2 \rangle \langle (Y - \langle Y \rangle)^2 \rangle}}$$





Correlations between radii of neutron stars of different masses and L(ρ)



$$S_1(\rho) = \frac{\hbar^2 k_F^2}{6 m_0^*(\rho, k_F)}$$

$$S_2(\rho) = \frac{1}{2} U_{\mathrm{sym},1}(\rho, k_F) \ ,$$

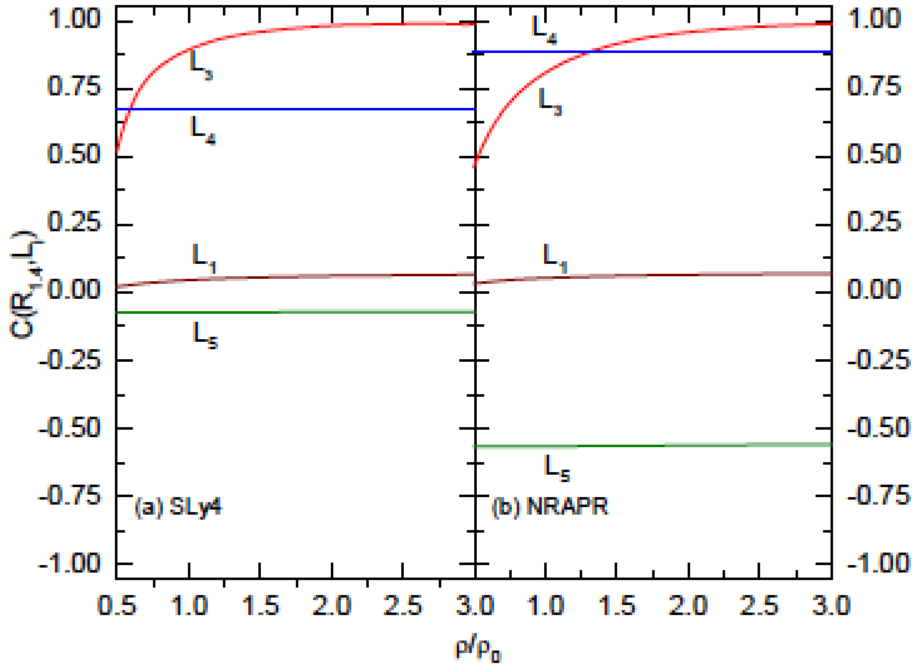
$$L_1(\rho) = \frac{2\hbar^2 k_F^2}{6 m_0^*(\rho, k_F)} \equiv 2 S_1(\rho)$$

$$L_2(\rho) = -\frac{\hbar^2 k_F^3}{6 m_0^{*2}(\rho, k_F)} \frac{\partial m_0^*(\rho, k)}{\partial k}\bigg|_{k=k_F}$$

$$L_3(\rho) = \frac{3}{2} U_{\mathrm{sym},1}(\rho, k_F) \equiv 3 S_2(\rho)$$

$$L_4(\rho) = \frac{\partial U_{\mathrm{sym},1}(\rho, k)}{\partial k}\bigg|_{k=k_F} \cdot k_F$$

$$L_5(\rho) = 3 U_{\mathrm{sym},2}(\rho, k_F) \ .$$



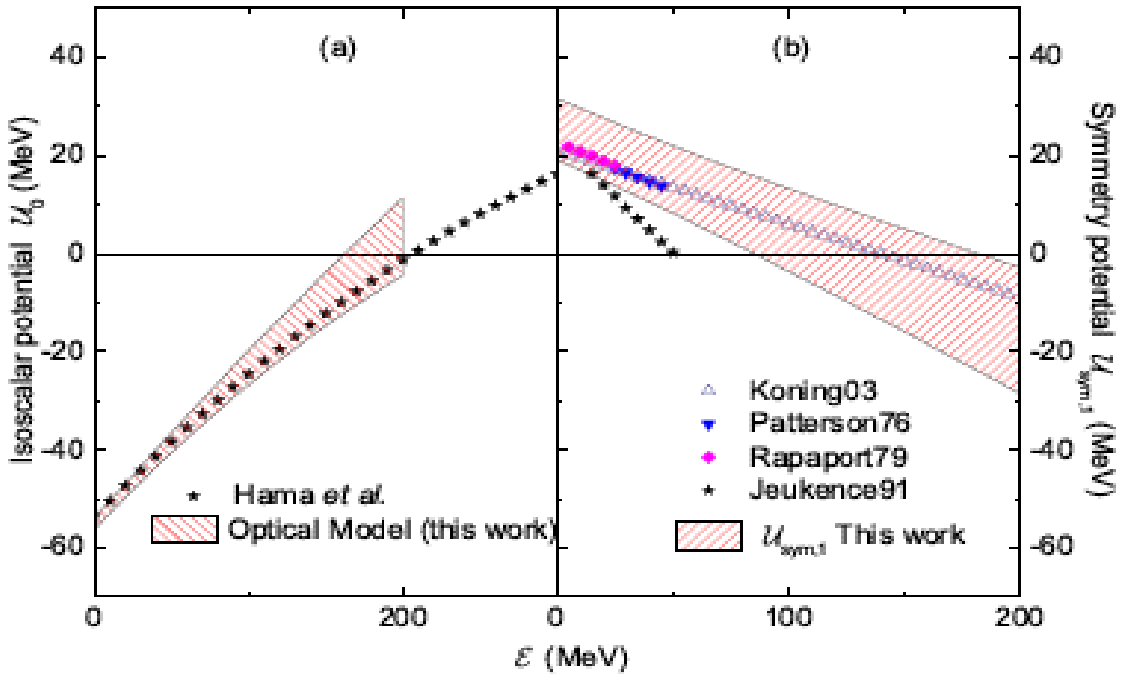
Constraints on $L_n(\rho_0)$ using optical potentials

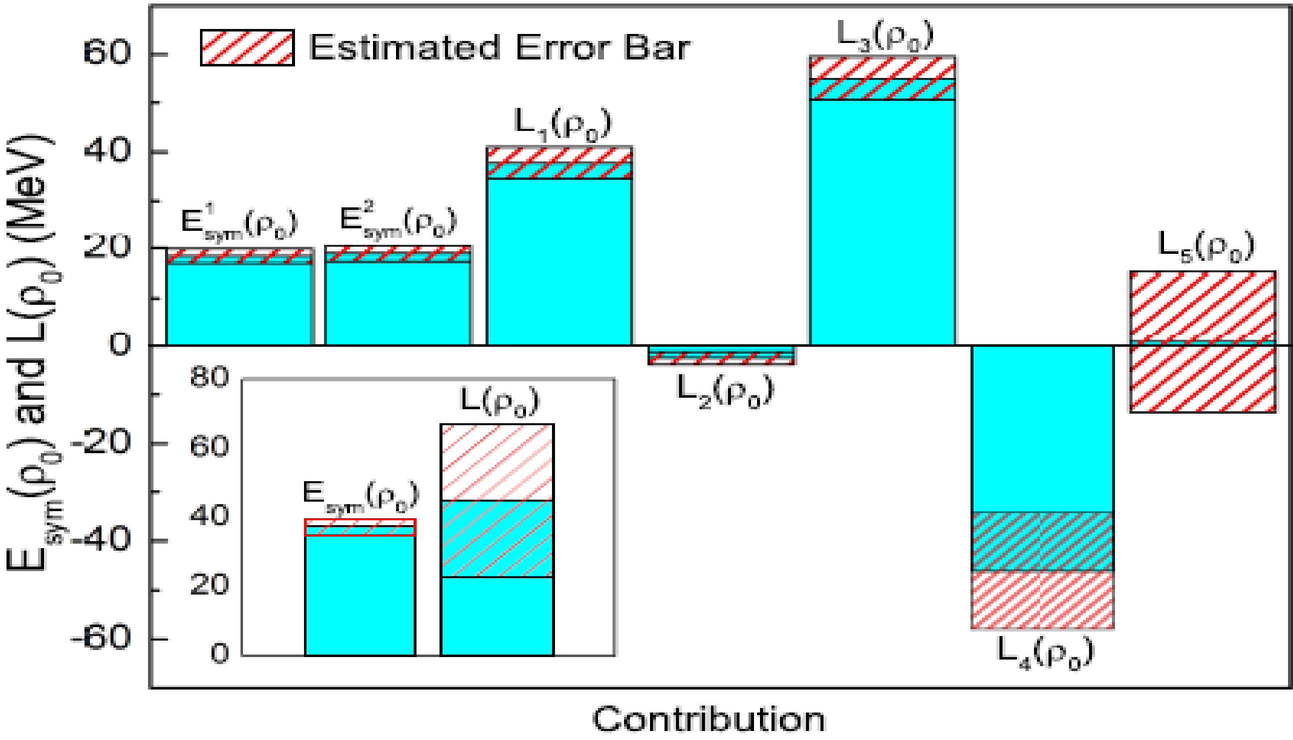
from nucleon-nucleus elastic scatterings and (p,n) charge exchange reactions

Chang Xu, Bao-An Li and Lie-Wen Chen, PRC 82, 054607 (2010)

XH Li, BJ Cai, LW Chen, R Chen, BA Li, C Xu, PLB721 (2013) 101;
XH Li, WJ Guo, BA Li, LW Chen, FJ Fattoyev, WG Newton, PLB743 (2015) 408

$$
\begin{aligned}
L_1(\rho) &= \frac{2}{3}\frac{\hbar^2 k_F^2}{2m_0^*(\rho,k_F)} \\
L_2(\rho) &= -\frac{1}{6}\frac{\hbar^2 k_F^3}{m_0^{*2}(\rho,k_F)}\frac{\partial m_0^*(\rho,k)}{\partial k}|_{k_F} \\
L_3(\rho) &= \frac{3}{2}U_{sym,1}(\rho,k_F) \\
L_4(\rho) &= \frac{\partial U_{sym,1}(\rho,k)}{\partial k}|_{k_F}\cdot k_F \\
L_5(\rho) &= 3U_{sym,2}(\rho,k_F).
\end{aligned}
$$



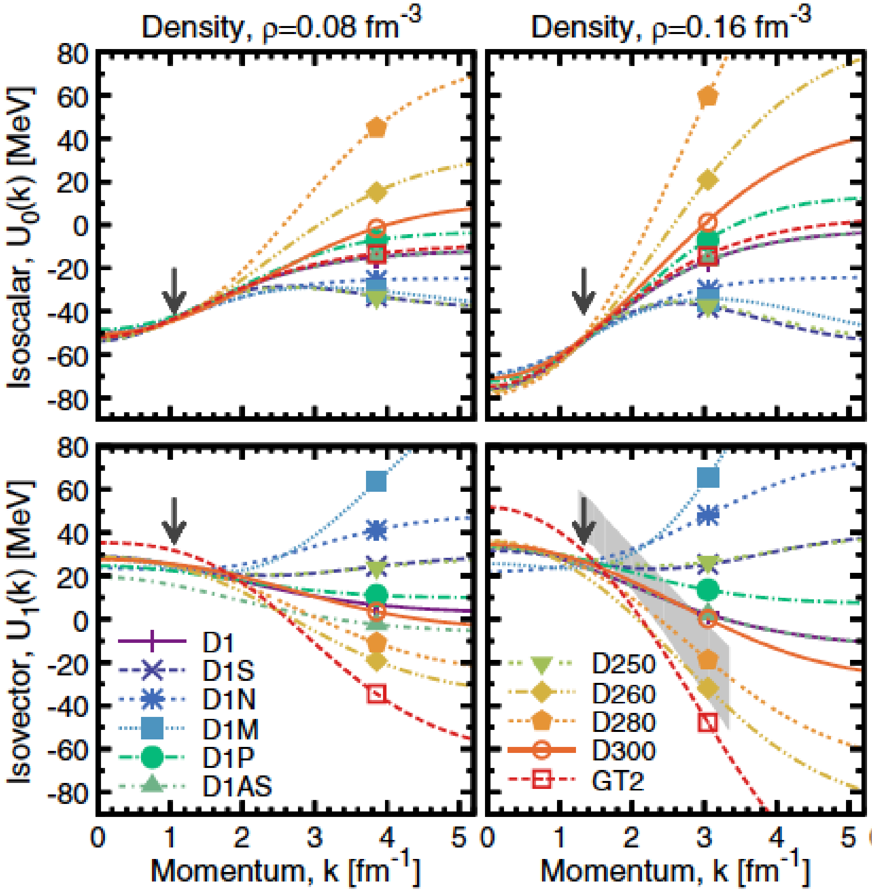


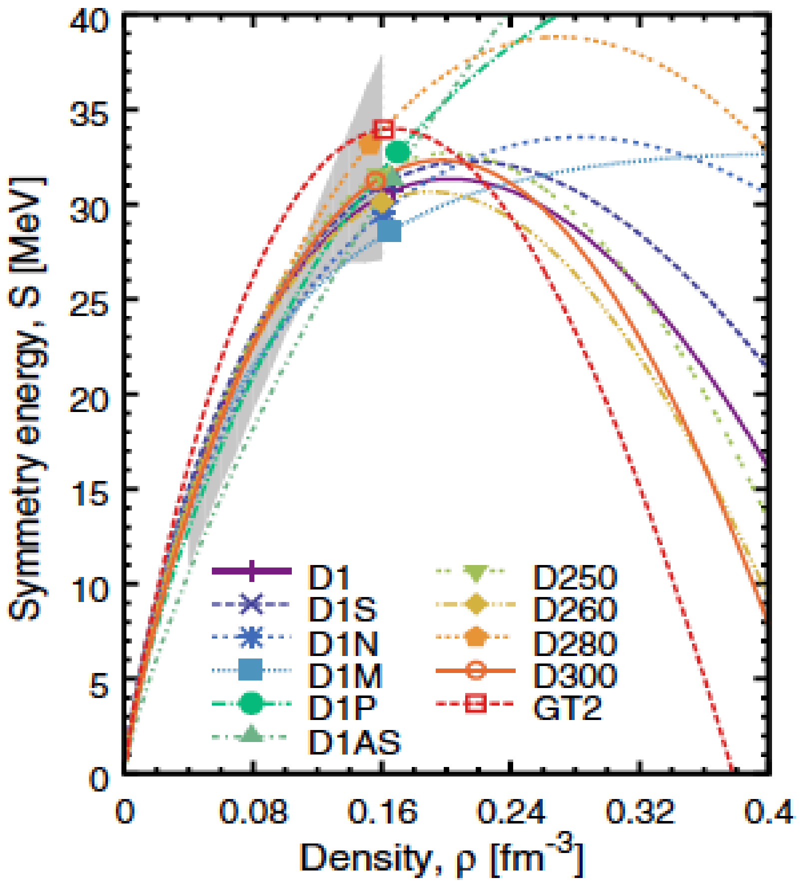
Density and momentum dependence of Isoscalar and Isovector potentials
Gogny Hartree-Fock predictions using 11 popular Gogny (finite-range) forces

PHYSICAL REVIEW C **90**, 054327 (2014)

Isovector properties of the Gogny interaction

Roshan Sellahewa and Arnau Rios





Nuclear Physics **A181** (1972) 33—48

VARIATIONAL CALCULATION OF NUCLEAR MATTER

V. R. PANDHARIPANDE
Tata Institute of Fundamental Research, Bombay-5, India

2. The tensor correlations

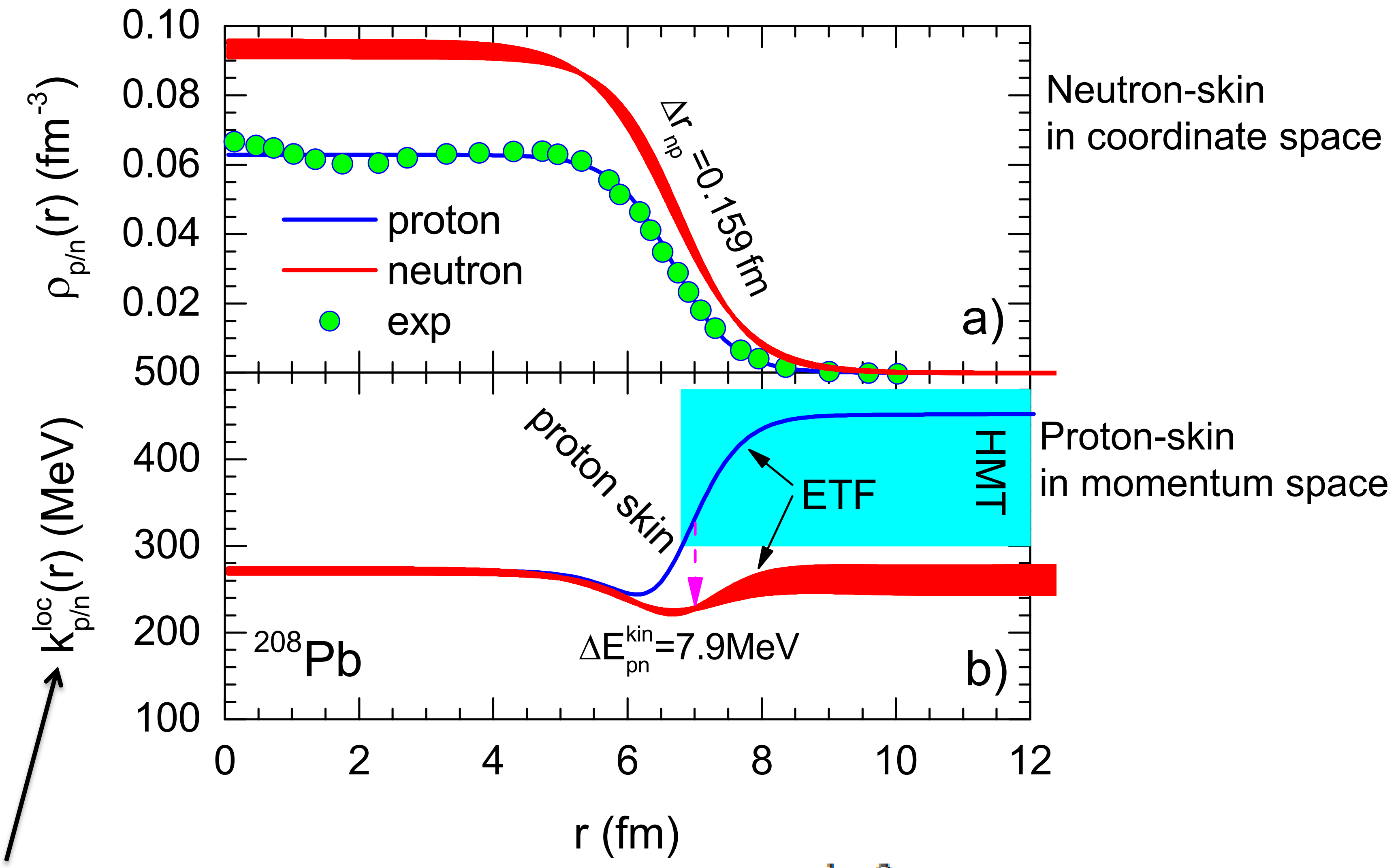
When correlations are restricted to the nearest neighbours their range is sufficiently small to neglect the presence of model states with $L \geqq 2$ within the correlation volume. Hence in this approximation the tensor correlations are important only for the $^3S_1$-$^3D_1$, and possibly $^3P_2$-$^3F_2$ coupled states. The $^3P_2$–$^3F_2$ tensor force of the Reid potential is weak and initially tensor correlations in this state are neglected. In sect. 3 its effect on the nuclear matter binding energy is shown to be very small. This also implies that only neutron-proton pairs can be tensor correlated. The $^3S_1$-$^3D_1$ potential is given by

$$v^{S=1,\,L=\mathrm{even},\,J=1} = v_C(r)+v_T(r)S_{12}+v_{LS}(r)(L\cdot S), \tag{6}$$

Protons move much faster than neutrons in neutron-skins

--consistent with the Liouville theorem

Bao-Jun Cai, Bao-An Li and Lie-Wen Chen, PRC 94, 061302 (R) (2016)



The average local momentum is defined via $k_J^{\mathrm{loc},2}(r)/2M = \varepsilon_J^{\mathrm{kin}}(r)/\rho_J(r)$

Efforts (not necessarily consistent or even correct!) to incorporate SRC in transport models

(1) Include spin explicitly, too little information about the spin-isospin-energy dependent elementary cross sections available

Jun Xu, Bao-An Li, PLB 724, 346 (2013)

(2) **Incorporating the high-momentum nucleons in the momentum-dependent mean-field**

Bao-Jun Cai, Bao-An Li and Lie-Wen Chen, AIP Conference Proceedings 2038, 020041 (2018)

(3) **Hard photons from two colliding nuclei at Fermi energy**
to probe the strength and shape of the SRC/HMT in the initial state

The interplay of short-range correlations and nuclear symmetry energy in hard photon productions from heavy-ion reactions at Fermi energies
Gao-Chan Yong, Bao-An Li, PRC 96, 064614 (2017)

Imprints of high-momentum nucleons in nuclei on hard photons from heavy-ion collisions around the Fermi Energy
Wen-Mei Guo, Bao-An Li, Gao-Chan Yong, PRC 104, 034603 (2021)

Modified Gogny Hartree-Fock energy density functional incorporating SRC-induced high momentum tail in the single nucleon momentum distribution

Kinetic — Zero-range Two-body force — Three-body force

$$E(\rho,\delta) = E^{\mathrm{kin}}(\rho,\delta) + \frac{A_\ell(\rho_{\mathrm{p}}^2+\rho_{\mathrm{n}}^2)}{2\rho\rho_0} + + \frac{A_{\mathrm{u}}\rho_{\mathrm{p}}\rho_{\mathrm{n}}}{\rho\rho_0} + \frac{B}{\sigma+1}\left(\frac{\rho}{\rho_0}\right)^{\sigma}(1-x\delta^2)$$
$$+\sum_{J,J'}\frac{C_{J,J'}}{\rho\rho_0}\int \mathrm{d}\mathbf{k}\mathrm{d}\mathbf{k}' f_J(\mathbf{r},\mathbf{k}) f_{J'}(\mathbf{r},\mathbf{k}')\Omega(\mathbf{k},\mathbf{k}'),$$

Momentum-dependent potential energy due to finite-range 2-body interaction

$$E^{\mathrm{kin}}(\rho,\delta) = \sum_{J=\mathrm{n,p}}\frac{1}{\rho_J}\int_0^\infty \frac{\mathbf{k}^2}{2M} n_{\mathbf{k}}^J(\rho,\delta)\mathrm{d}\mathbf{k}$$

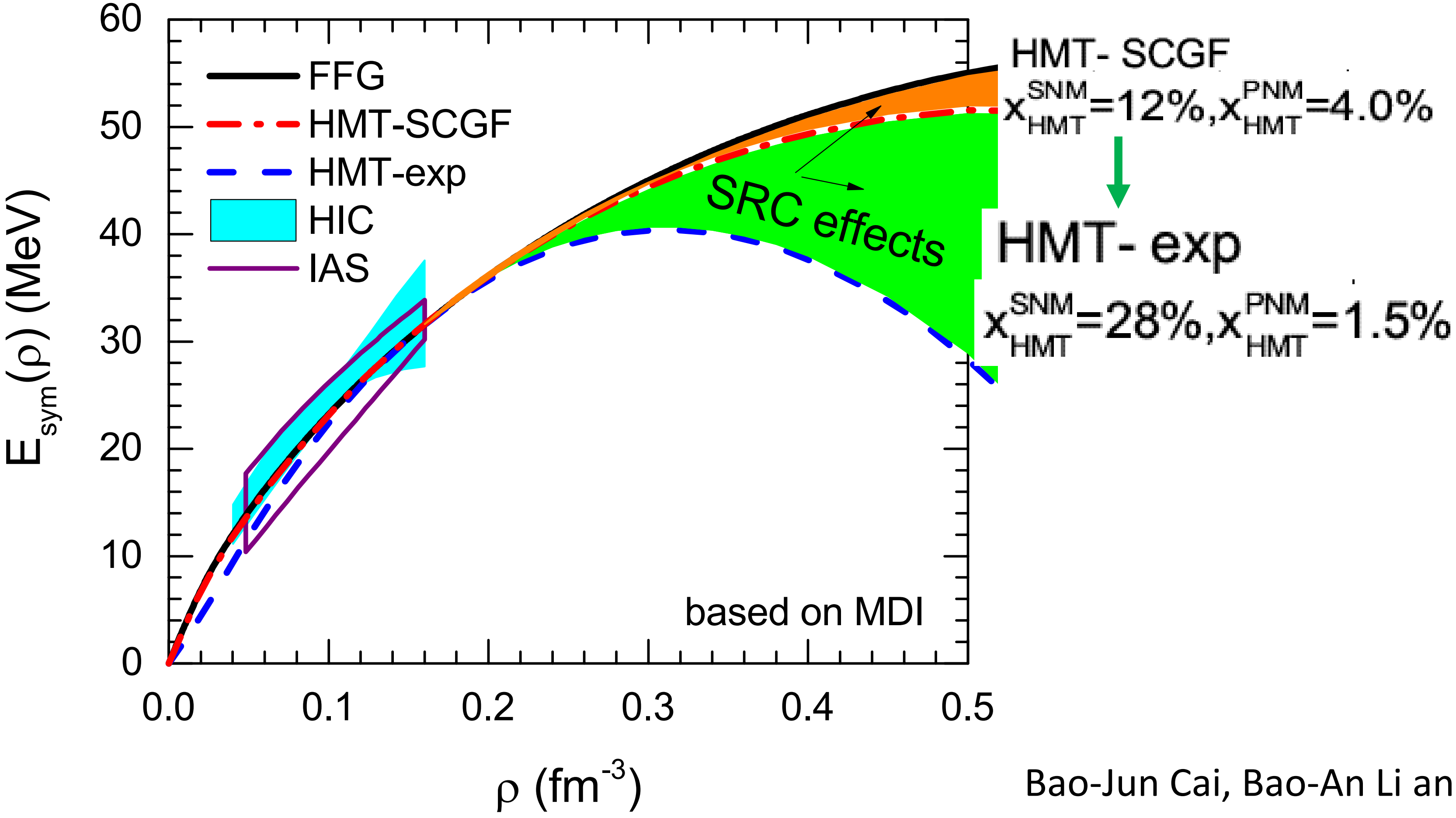
$$\Omega(\mathbf{k},\mathbf{k}') = \left[1+\frac{(\mathbf{k}-\mathbf{k}')^2}{\Lambda^2}\right]^{-1}$$

SRC-induced HMT in the single-nucleon momentum distribution affects both the kinetic energy and the momentum-dependent part of the potential energy

C. B. Das, S. Das Gupta, C. Gale, Bao-An Li Phys.Rev.C67:034611,2003

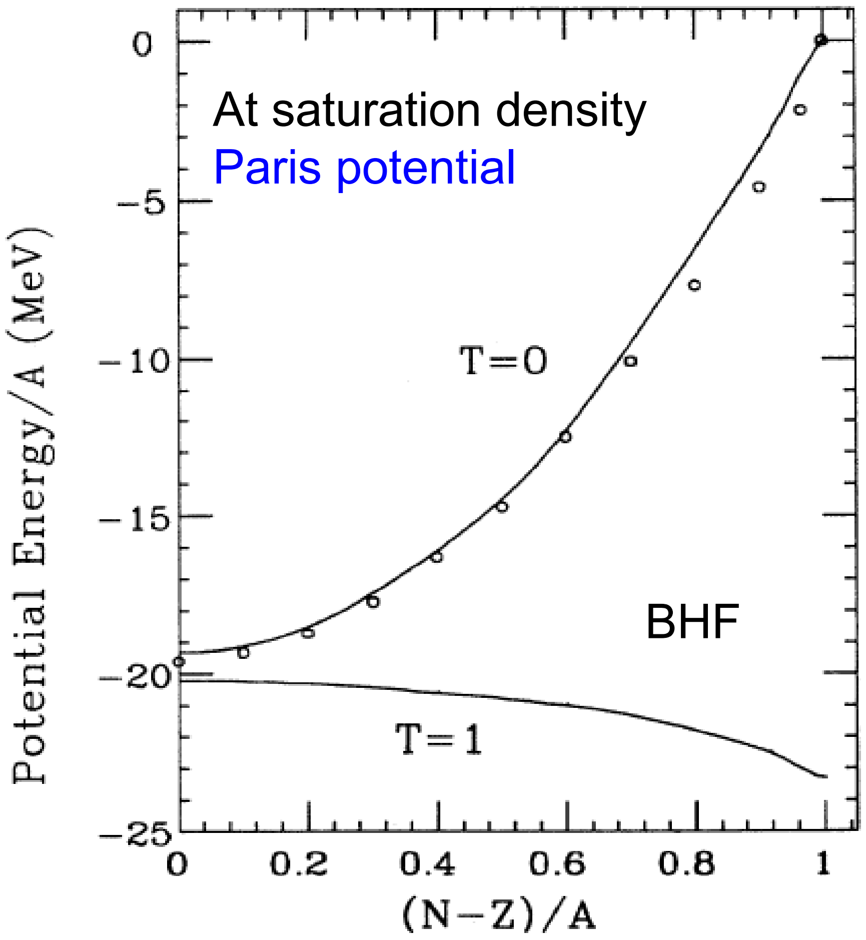
Readjusting model parameters to reproduce the same saturation properties of nuclear matter as well as $E_{sym}(\rho_0)$=31.6 MeV and $L(\rho_0)$=58.9 MeV

Consequence: Symmetry energy gets softened at both low and high densities

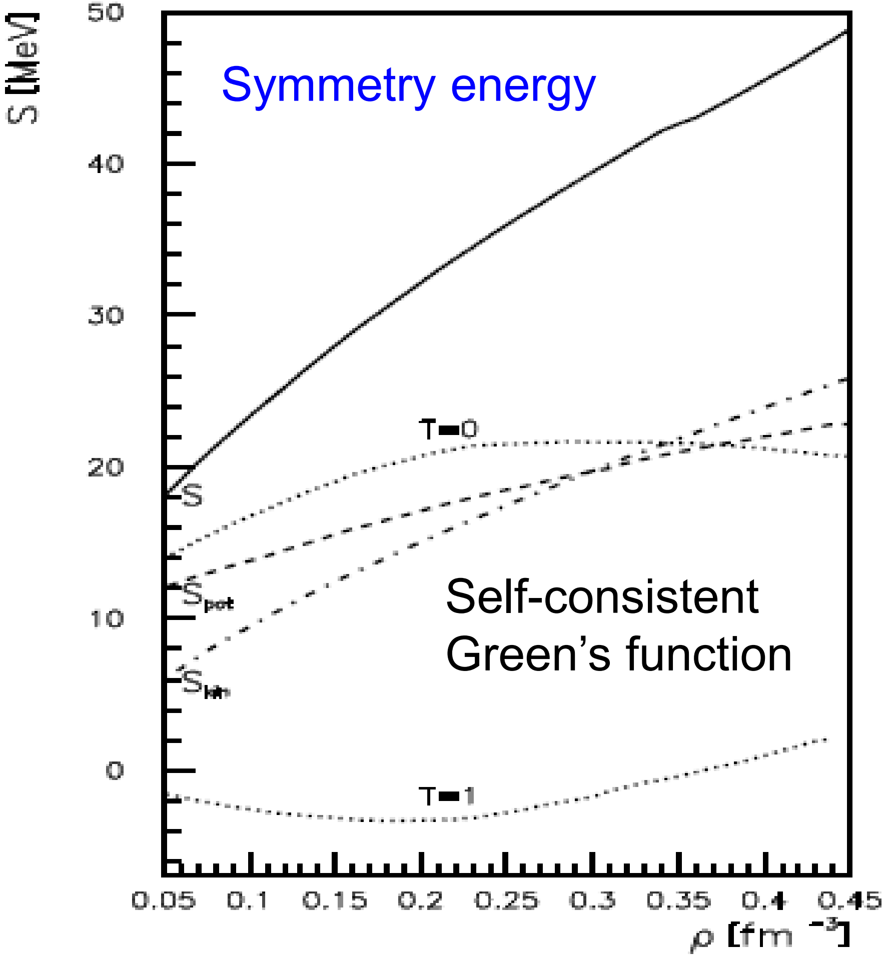


Bao-Jun Cai, Bao-An Li and Lie-Wen Chen,
AIP Conference Proceedings 2038, 020041 (2018)

Dominance of the isosinglet (T=0) interaction



I. Bombaci and U. Lombardo PRC 44, 1892 (1991)

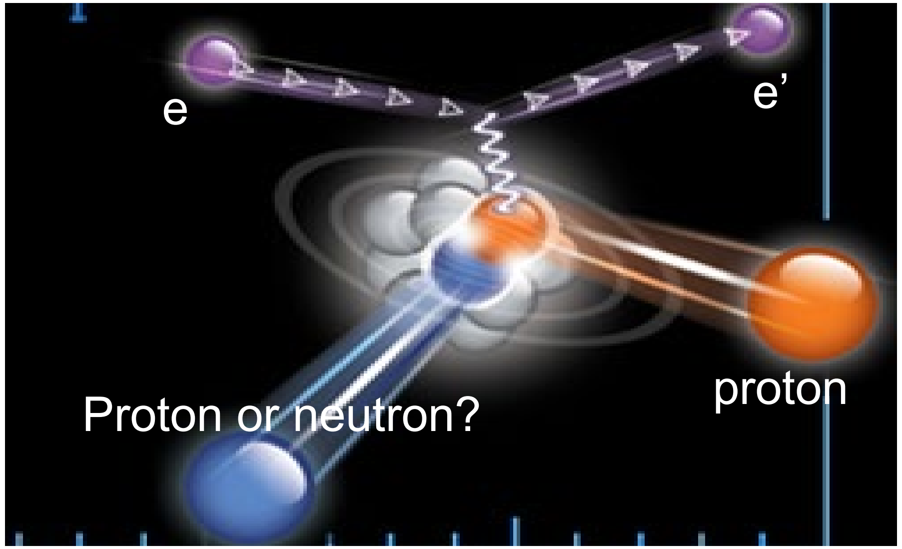


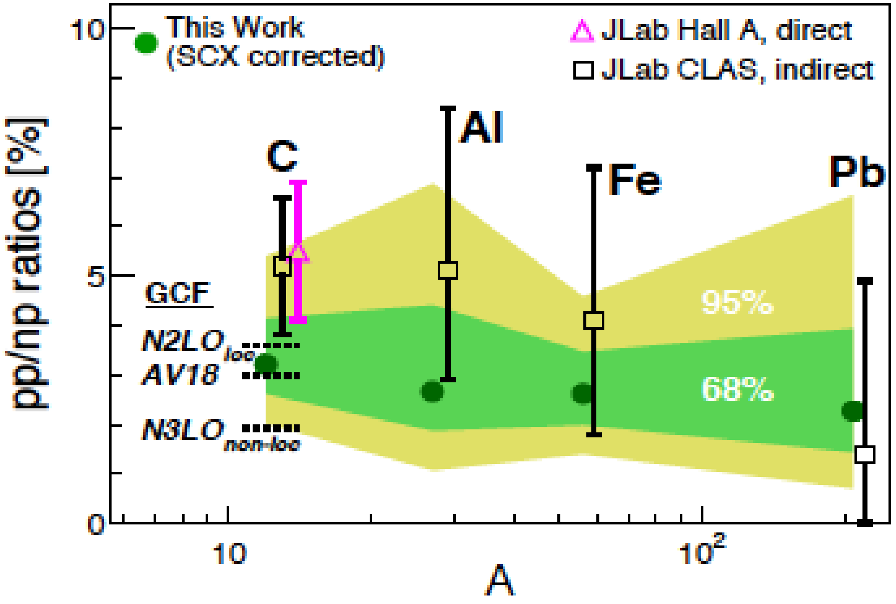
A.E.L. Dieperink,[1] Y. Dewulf,[2] D. Van Neck,[2] M. Waroquier,[2] and V. Rodin[3]

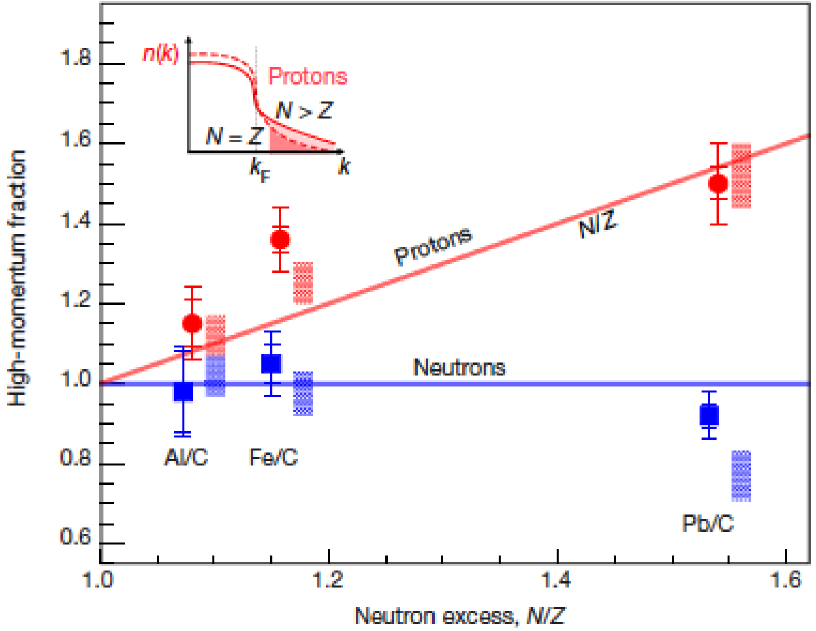
PRC68, 064307 (2003)

$$E_{sym}(\rho) = \frac{1}{2}\frac{\partial^2 E}{\partial \delta^2} \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$

Experimental evidence of isospin-dependent nucleon momentum distribution:
Deformed-Fermi distributions in neutron-rich matter







M. Duer et al., Nature 560, 617 (2018).

M. Duer et al., PRL 122, 172502 (2019).

SRC Effects on Nuclear Symmetry Energy

Bao-An Li

Physics & Astronomy
Texas A&M University-Commerce

Collaborators: Bao-Jun Cai, Lie-Wen Chen, Wenjun Guo, Ang Li, Wen-Jie Xie, Chang Xu, Nai-Bo Zhang, Or Hen, Eli Piasetzky and Larry Weinstein

NSF

Office of Science
U.S. DEPARTMENT OF ENERGY

Empirical parabolic law of the EOS of cold, neutron-rich nucleonic matter

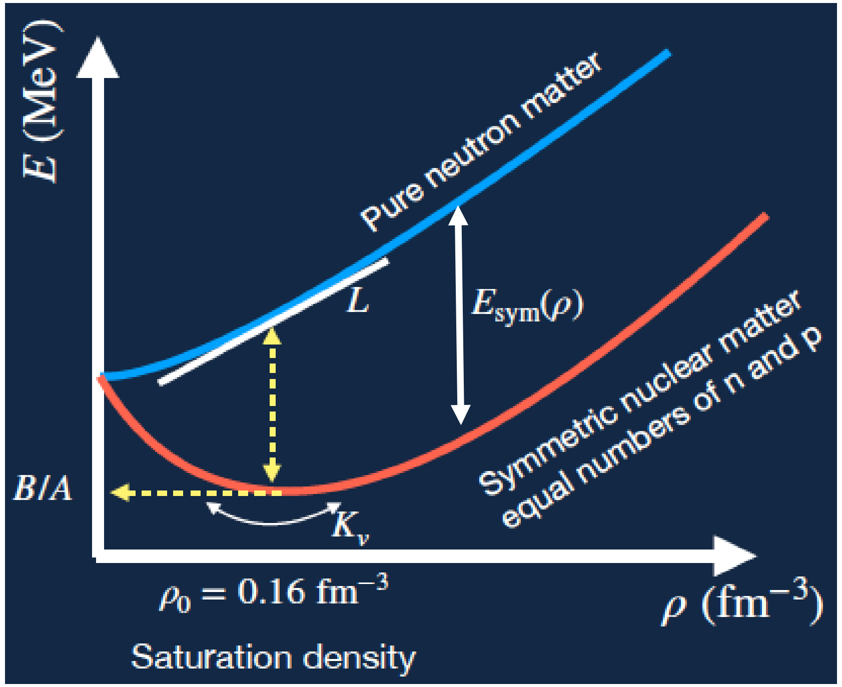
symmetry energy

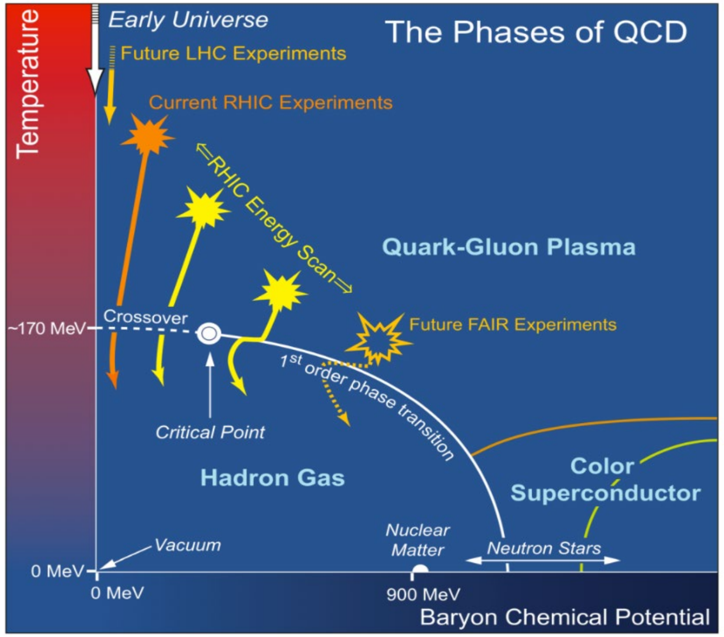
Isospin asymmetry δ

$$E(\rho_n,\rho_p)=E_0(\rho_n=\rho_p)+E_{sym}(\rho)\left(\frac{\rho_n-\rho_p}{\rho}\right)^2+o(\delta^4)$$

Energy per nucleon in symmetric matter

Energy in asymmetric nucleonic matter





density

Important SRC roles

New opportunities

Isospin asymmetry

$\delta = (\rho_n - \rho_p)/\rho$

Structures and collisions
of neutron stars & heavy nuclei