Off-shell transport dynamics for strongly interacting systems

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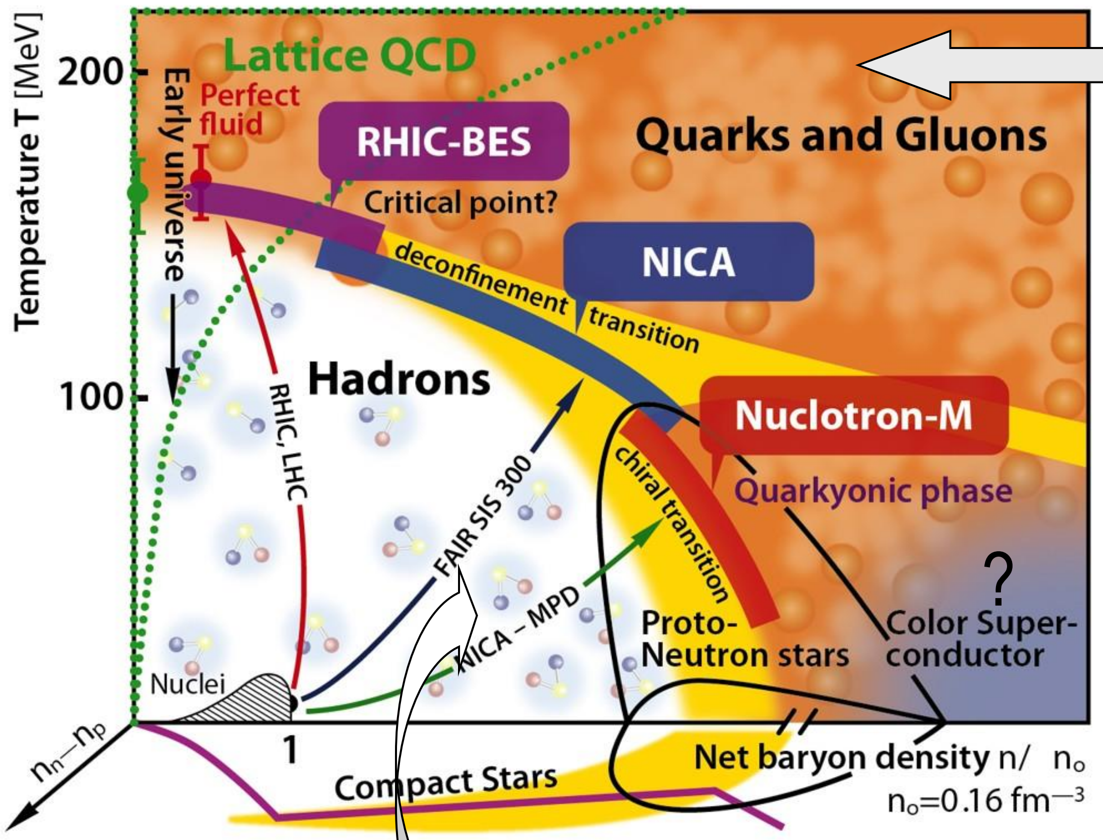
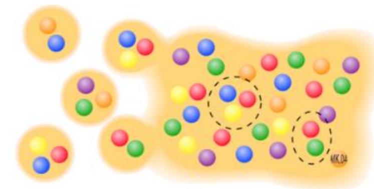
The ‚holy grail‘ of heavy-ion physics:

The phase diagram of QCD



- Search for the critical point



- Study of the phase transition from hadronic to partonic matter – Quark-Gluon-Plasma

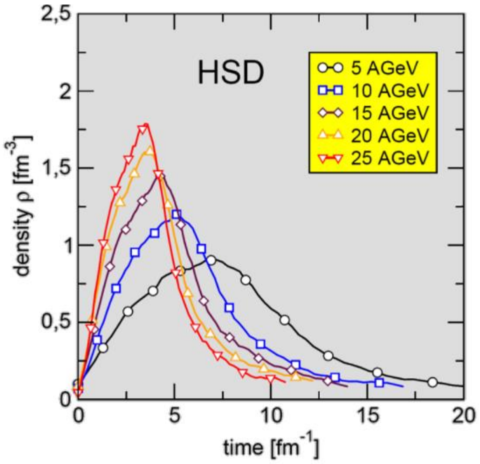
- Search for signatures of chiral symmetry restoration

- Search for the critical point

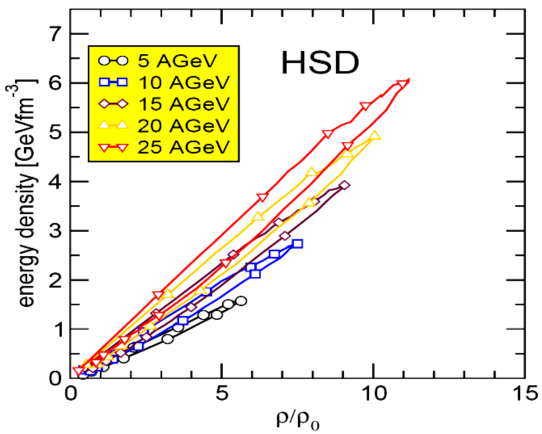
- Study of the in-medium properties of hadrons at high baryon density and temperature

Dense and hot matter created in HICs

Time evolution of baryon density ρ



Energy density vs. ρ/ρ_0

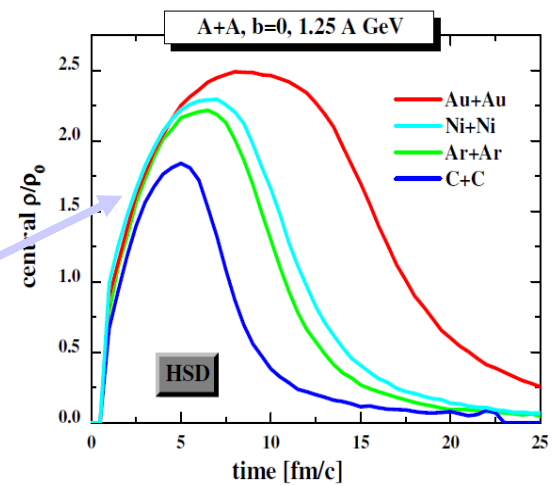


Large energy and baryon densities (even above critical $\varepsilon > \varepsilon_{crit} \sim 0.5$ GeV/fm^3) are reached in the central reaction volume at CBM and BM@N/NICA energies (> 5 A GeV)
➔ a phase transition to the **QGP**

❑ **At SIS energies: baryon density in central A+A collisions at 1.25 A GeV:**

- increases with nuclear size up to 2.5 ρ_0
- the reaction time is larger for heavy systems

➔ **Highly dense matter is created already at SIS energies!**



From weakly to strongly interacting systems

In-medium effects (on hadronic or partonic levels!) = changes of particle properties in the hot and dense medium

Examples: **hadronic medium** - vector mesons, strange mesons
QGP – dressing of partons

Many-body theory:
Strong interaction ➔ **large width** ➔ **broad spectral function** ➔ **quantum object**

Semi-classical on-shell BUU: applies for small width, i.e. for a weakly interacting systems of particles

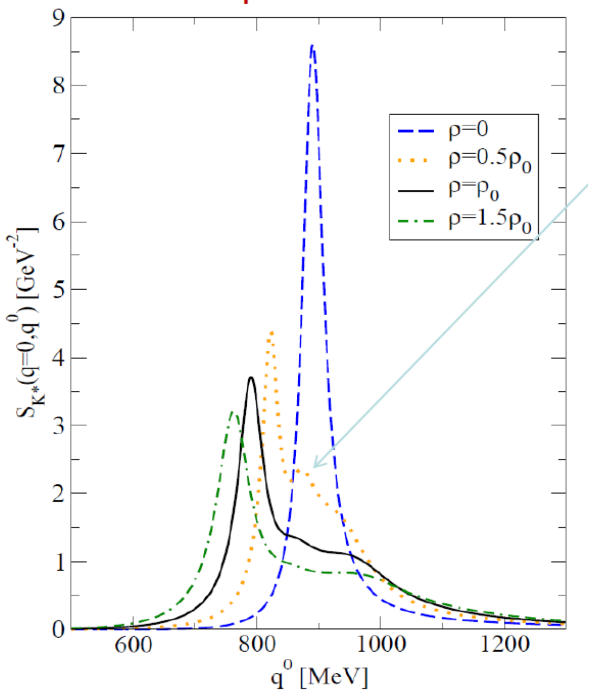
- **How to describe the dynamics of broad strongly interacting quantum states in transport theory?**

- ❑ **semi-classical BUU**

  first order gradient expansion of quantum Kadanoff-Baym equations

- ❑ **generalized transport equations based on Kadanoff-Baym dynamics**

Kbar* spectral function



Λ(1783)N^{-1} and Σ(1830)N^{-1} exitations

Barcelona / Valencia group

Dynamical description of strongly interacting systems

❑ **Quantum field theory ➔**
Kadanoff-Baym dynamics **for resummed single-particle Green functions $S^<$**

(1962)

$$\hat{S}_{0x}^{-1} \, S_{xy}^{<} = \Sigma_{xz}^{ret} \odot S_{zy}^{<} + \Sigma_{xz}^{<} \odot S_{zy}^{adv}$$

Integration over the intermediate spacetime

Green functions $S^<$ / self-energies Σ :

$$iS_{xy}^{<} = \eta \langle \{ \Phi^+(y)\Phi(x) \} \rangle$$

$$iS_{xy}^{>} = \langle \{ \Phi(y)\Phi^+(x) \} \rangle$$

$$iS_{xy}^{c} = \langle T^c \{ \Phi(x)\Phi^+(y) \} \rangle \quad - causal$$

$$iS_{xy}^{a} = \langle T^a \{ \Phi(x)\Phi^+(y) \} \rangle \quad - anticausal$$

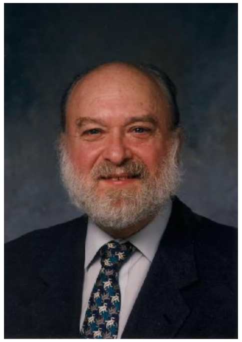
$$S_{xy}^{ret} = S_{xy}^{c} - S_{xy}^{<} = S_{xy}^{>} - S_{xy}^{a} \quad - retarded$$

$$S_{xy}^{adv} = S_{xy}^{c} - S_{xy}^{>} = S_{xy}^{<} - S_{xy}^{a} \quad - advanced$$

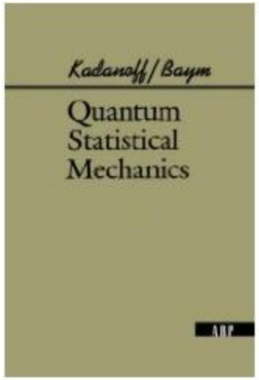
$$\hat{S}_{0x}^{-1} \equiv -(\partial_x^\mu \partial_\mu^x + M_0^2)$$

$$\eta = \pm 1 (bosons \,/\, fermions)$$

$$T^a (T^c) - (anti-) time - ordering \ operator$$



Leo Kadanoff



Gordon Baym

1st application for spacially homodeneous system with deformed Fermi sphere:
P. Danielewicz, Ann. Phys. 152, 305 (1984); … H.S. Köhler, Phys. Rev. 51, 3232 (1995); …

From Kadanoff-Baym equations to generalized transport equations

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

Generalized transport equations (GTE):

drift term — Vlasov term — backflow term — collision term = ‚gain' - ‚loss' term

$$\diamond\{P^2 - M_0^2 - Re\Sigma^{ret}_{XP}\}\{S^<_{XP}\} - \diamond\{\Sigma^<_{XP}\}\{ReS^{ret}_{XP}\} = \frac{i}{2}\left[\Sigma^>_{XP}S^<_{XP} - \Sigma^<_{XP}S^>_{XP}\right]$$

Backflow term incorporates the off-shell behavior in the particle propagation
! vanishes in the quasiparticle limit $A_{XP} \rightarrow \delta(p^2-M^2)$ → BUU equations

- GTE: Propagation of the Green's function $iS^<_{XP}=A_{XP}N_{XP}$, which carries information not only on the number of particles (N_{XP}), but also on their properties, interactions and correlations (via A_{XP})

Botermans-Malfliet (1990)

- Spectral function:

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

$\Gamma_{XP} = -Im\,\Sigma^{ret}_{XP} = 2p_0\Gamma$ – ‚width' of spectral function
= reaction rate of particle (at space-time position X)

4-dimentional generalizaton of the Poisson-bracket:

$$\diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

- Life time $\tau = \frac{\hbar c}{\Gamma}$

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

General testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

- Employ testparticle Ansatz for the real valued quantity $i\,S^{<}_{XP}$

$$F_{XP} = A_{XP} N_{XP} = i\,S^{<}_{XP} \sim \sum_{i=1}^{N} \delta^{(3)}(\vec{X} - \vec{X}_i(t))\ \delta^{(3)}(\vec{P} - \vec{P}_i(t))\ \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine equations of motion !

→ Generalized testparticle Cassing-Juchem off-shell equations of motion for the time-like particles:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_i + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

Note: the common factor $1/(1-C_{(i)})$ can be absorbed in an ‚eigentime' of particle (i) !

Collision term in off-shell transport models

Collision term for reaction 1+2->3+4:

$$I_{coll}(X,\vec{P},M^2) = Tr_2 Tr_3 Tr_4 A(X,\vec{P},M^2) A(X,\vec{P}_2,M_2^2) A(X,\vec{P}_3,M_3^2) A(X,\vec{P}_4,M_4^2)$$

$$|G((\vec{P},M^2)+(\vec{P}_2,M_2^2)\rightarrow(\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}}\ \delta^{(4)}(P+P_2-P_3-P_4)$$

$$[\,N_{X\vec{P}_3M_3^2}\,N_{X\vec{P}_4M_4^2}\,\bar{f}_{X\vec{P}M^2}\,\bar{f}_{X\vec{P}_2M_2^2}\ -\ N_{X\vec{P}M^2}\,N_{X\vec{P}_2M_2^2}\,\bar{f}_{X\vec{P}_3M_3^2}\,\bar{f}_{X\vec{P}_4M_4^2}\,]$$

‚gain‘ term ‚loss‘ term

with $\bar{f}_{X\vec{P}M^2} = 1+\eta\, N_{X\vec{P}M^2}$ and $\eta = \pm 1$ for bosons/fermions, respectively.

The trace over particles 2,3,4 reads explicitly

for fermions

$$Tr_2 = \sum_{\sigma_2,\tau_2}\frac{1}{(2\pi)^4}\int d^3P_2\frac{dM_2^2}{2\sqrt{\vec{P}_2^2+M_2^2}}$$

for bosons

$$Tr_2 = \sum_{\sigma_2,\tau_2}\frac{1}{(2\pi)^4}\int d^3P_2\frac{dP_{0,2}^2}{2}$$

additional integration

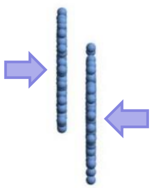
The transport approach and the particle spectral functions are fully determined once the in-medium transition amplitudes G are known in their off-shell dependence!

Parton-Hadron-String-Dynamics (PHSD)

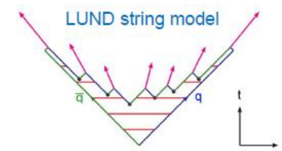
PHSD is a **non-equilibrium microscopic transport approach** for the description of **strongly-interacting hadronic and partonic matter** created in heavy-ion collisions

Dynamics: based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

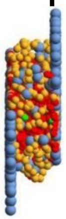
Initial A+A collision



- **Initial A+A collisions** :
  N+N → string formation → decay to pre-hadrons + leading hadrons

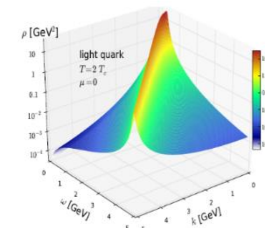


Partonic phase

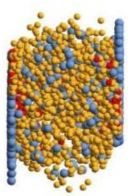


- **Formation of QGP stage** if local $\varepsilon > \varepsilon_{critical}$:
  dissolution of pre-hadrons → partons

- **Partonic phase - QGP:**
  QGP is described by the Dynamical QuasiParticle Model (DQPM) matched to reproduce lattice QCD EoS for finite T and μ_B (crossover)

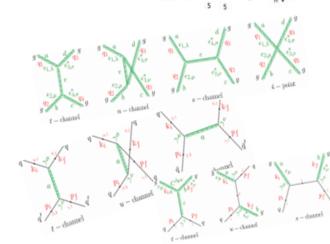


Hadronization

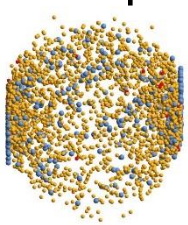


- **Degrees-of-freedom:** strongly interacting quasiparticles: massive quarks and gluons (g,q,q_{bar}) with sizeable collisional widths in a self-generated mean-field potential

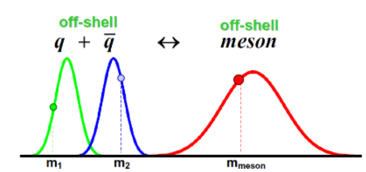
- **Interactions:** (quasi-)elastic and inelastic collisions of partons



Hadronic phase



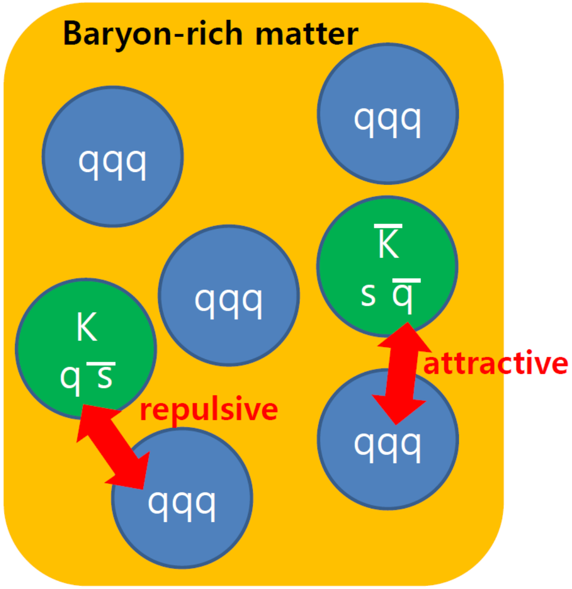
- **Hadronization** to colorless off-shell mesons and baryons:
  Strict 4-momentum and quantum number conservation



- **Hadronic phase: hadron-hadron interactions – off-shell HSD**

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3

In-medium effects at SIS energies:
I. Kaons – repulsive potential
II. Antikaons – G-matrix



cf. talks by Dan Cozma

In-medium effects

The hadrons - in particular strange mesons (K, Kbar and K*) - modify their properties in the dense and hot nuclear medium due to the strong interaction with the environment

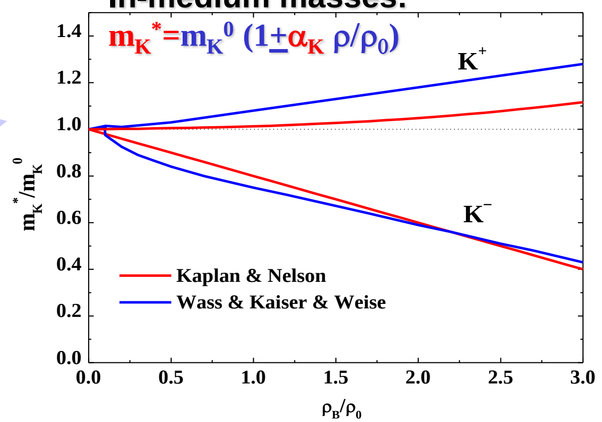
… long history …

Models:

- ❑ chiral SU(3) model, chiral perturbation theory, relativistic mean-field models: KN-potential ➔ ‚dropping' of K^- mass and ‚enhancement' of K^+ mass

Kaplan and Nelson, PLB 175 (1986) 57;
Weise, Brown, Schaffner, Krippa, Oset, Lutz, Mishra, ... et al.

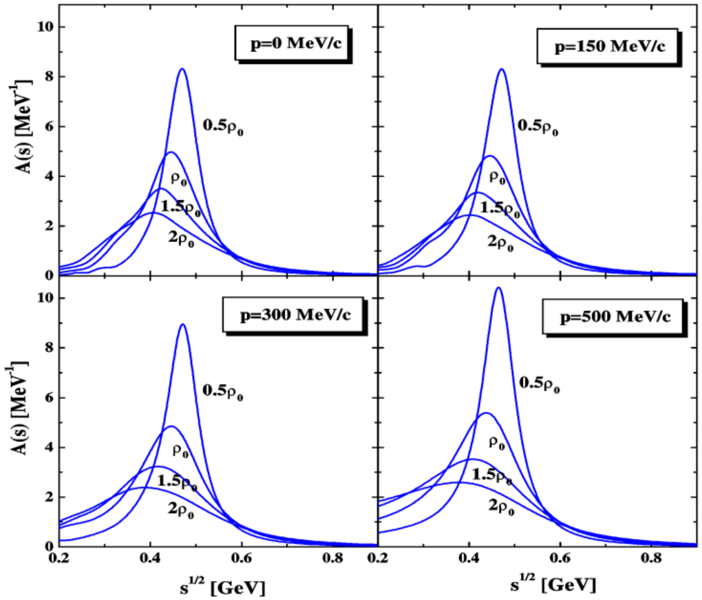
In-medium masses:



- ❑ self-consistent coupled-channel approach - G-matrix:

➔momentum, density and temperature dependent spectral function of antikaons $A(p_K,\rho,T)$: in-medium modification of the real and imaginary part of the self-energy (mass and width)

L. Tolos et al., NPA 690 (2001) 547



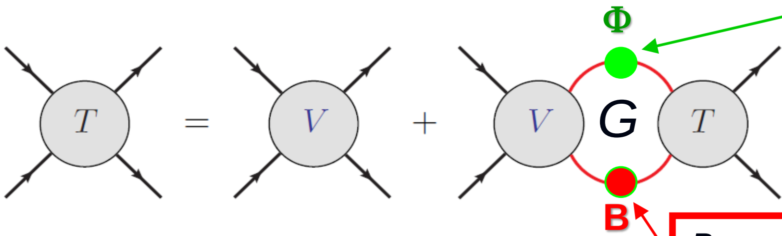
➔ off-shell HSD: W. Cassing et al., Nucl.Phys.A 727 (2003) 59

Cf. review: C. Hartnack et al., Phys.Rept. 510 (2012) 119

The coupled-channel G-matrix approach

Solution of the Bethe-Salpeter equation in coupled channels:

Medium:



Meson selfenergy and spectral function

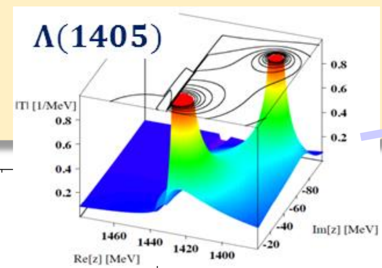
Baryons: Pauli blocking and potential dressing

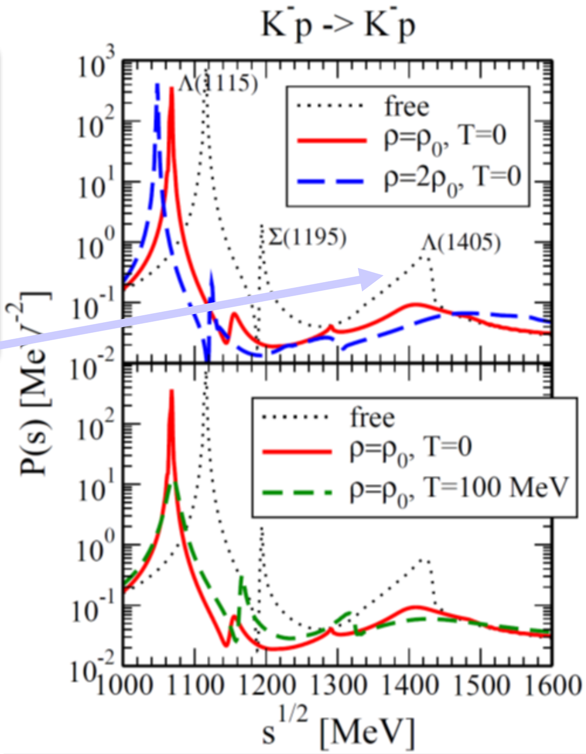
$$P \propto |T|^2$$

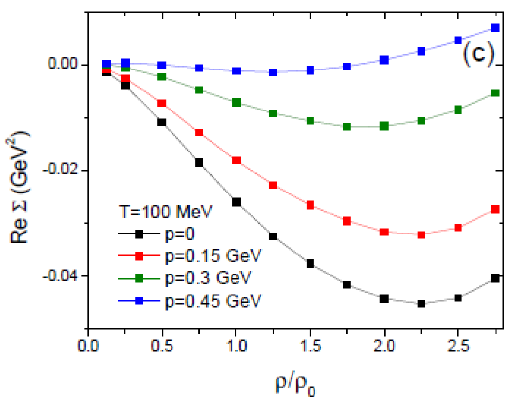
$$\mathcal{T}_{ij}(\rho,T) = V_{ij} + V_{il}\ G_l(\rho,T)\ \mathcal{T}_{lj}(\rho,T)$$

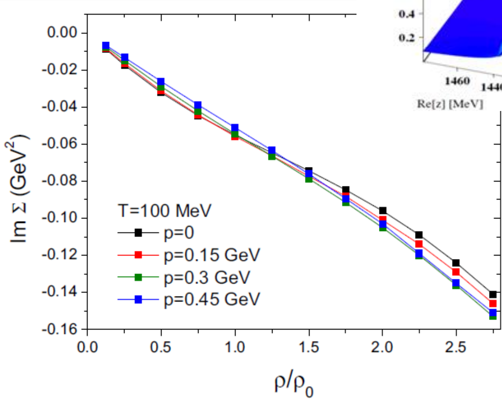
Coupled-channels [full $SU(3)$ basis, isospin $I = 0,1$]

- $S=-1$: $K^-p,\ \bar{K}^0n,\ \pi^0\Lambda,\ \pi^0\Sigma^0,\ \eta\Lambda,\ \eta\Sigma^0,\ \pi^+\Sigma^-,\ \pi^-\Sigma^+,\ K^+\Xi^-,\ K^0\Xi^0$
  $K^-n,\ \pi^0\Sigma^-,\ \pi^-\Sigma^0,\ \pi^-\Lambda,\ \eta\Sigma^-,\ K^0\Xi^-$
- $S=+1$: $K^+p;\ K^+n,\ K^0p$







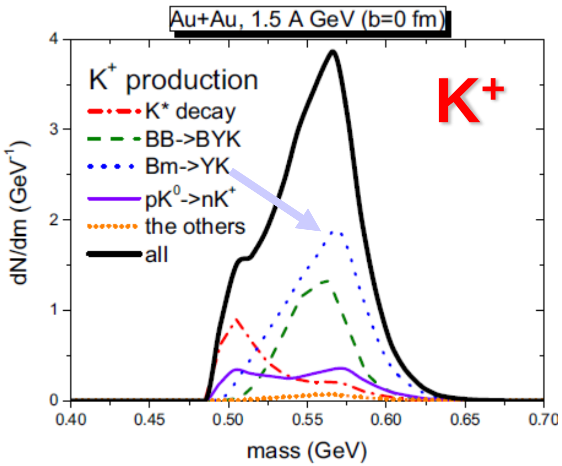


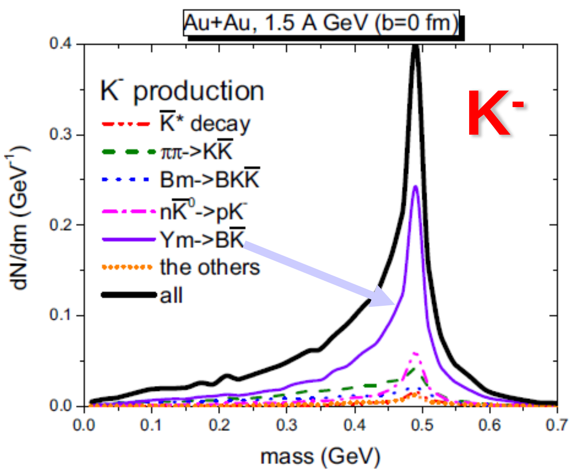
1) 1st G-matrix (based on the Jülich meson-exchange model): L. Tolos et al., NPA 690 (2001) 547

2) * Improved (based on SU(3) mB chiral Lagrangian): D. Cabrera, L. Tolos, J. Aichelin, E.B., PRC90 (2014) 055207

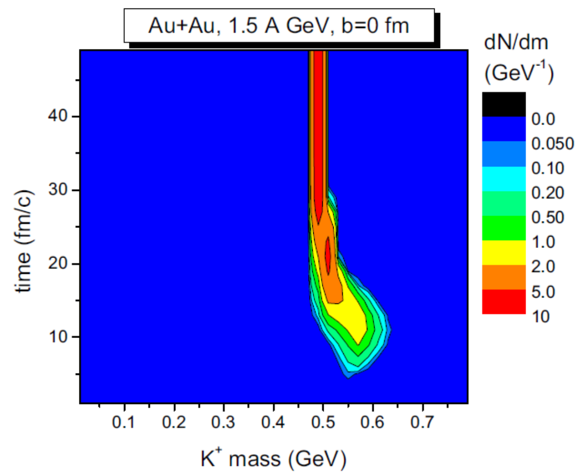
Time evolution of produced (anti)kaons

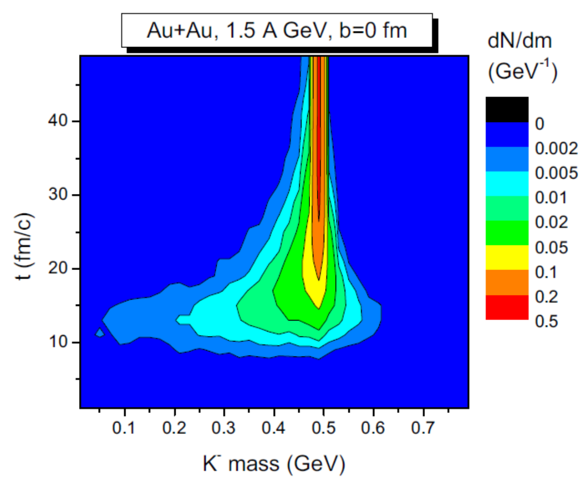
Mass distribution of K+, K- at the production points



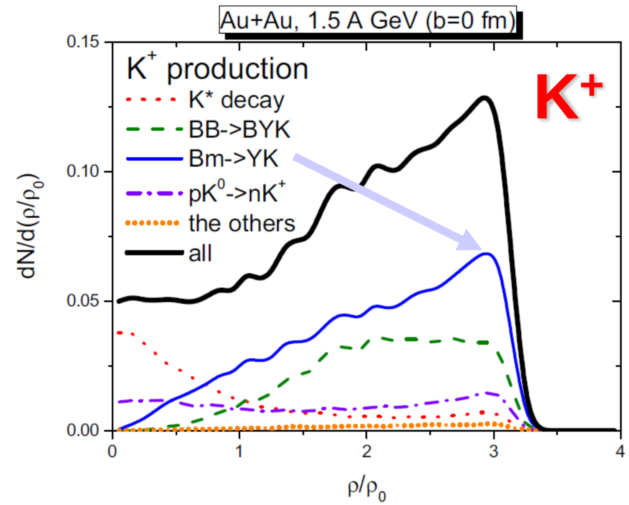


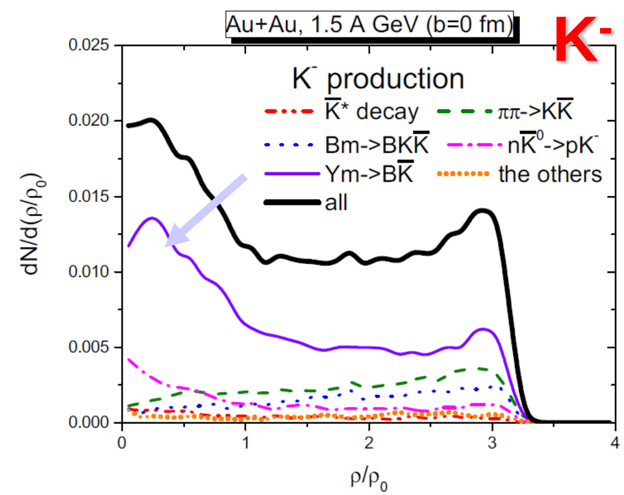
Time evolution of the K+, K- masses





Density distribution of K+, K- at the production point

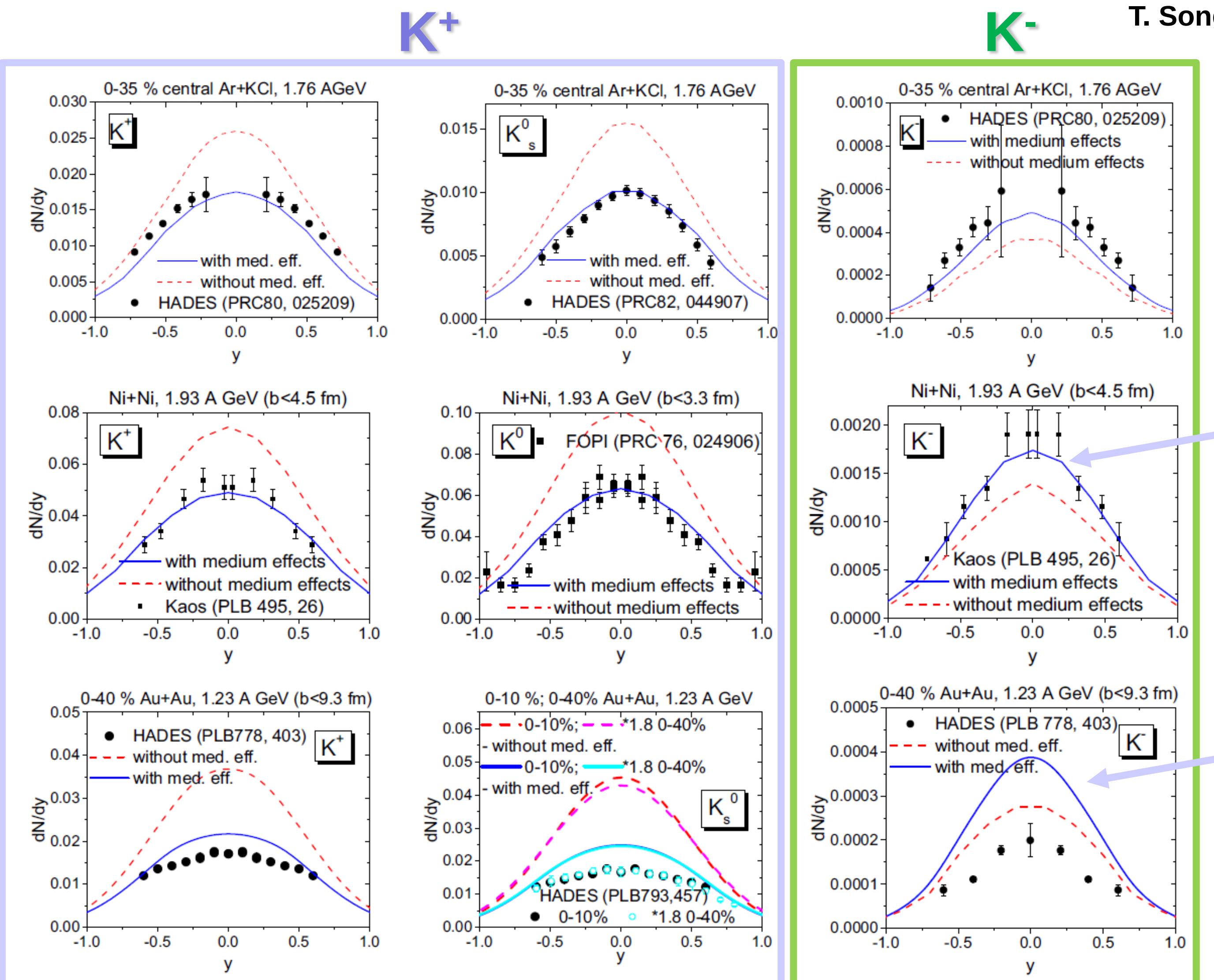




T. Song et al., PRC 103, 044901 (2021)

Rapidity distributions of (anti)kaons

T. Song et al., PRC 103, 044901 (2021)



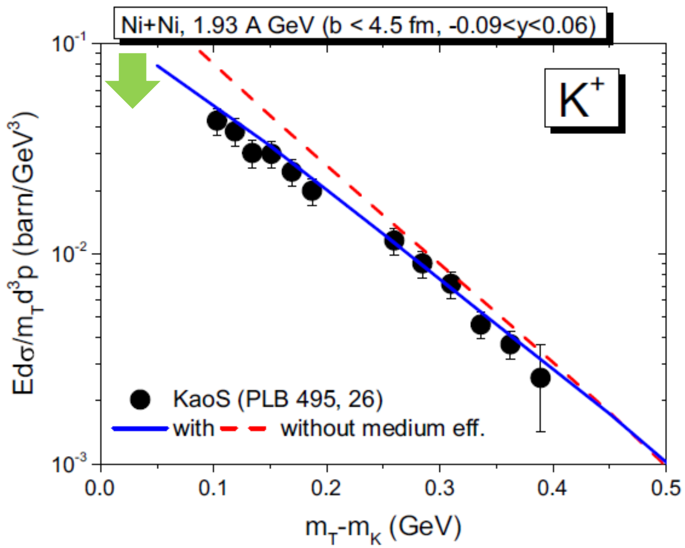
Good agreement with FOPI and KaoS data for K+ and K- for light and heavy A+A systems, as well as with HADES data for semi-heavy systems

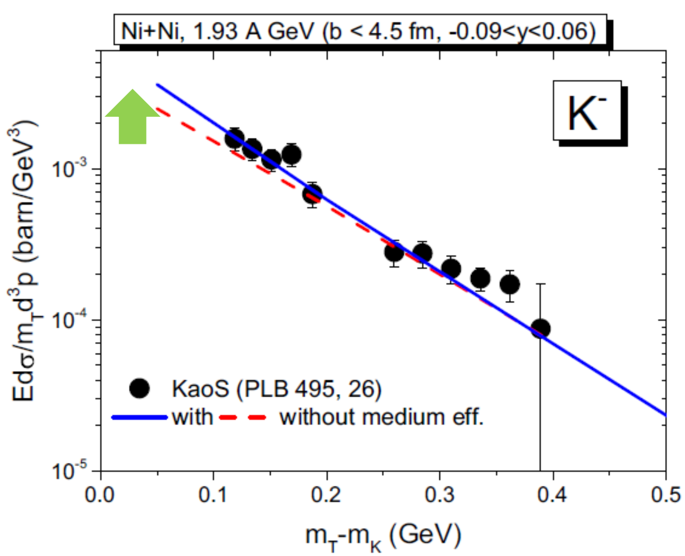
Tension with HADES data for K- for Au+Au at 1.23 A GeV

- Nuclear matter effects suppress kaon production
- Nuclear matter effects enhance antikaon production

m_T spectra of (anti)kaons in central Ni+Ni collisions at 1.93 A GeV

T. Song et al., PRC 103, 044901 (2021)





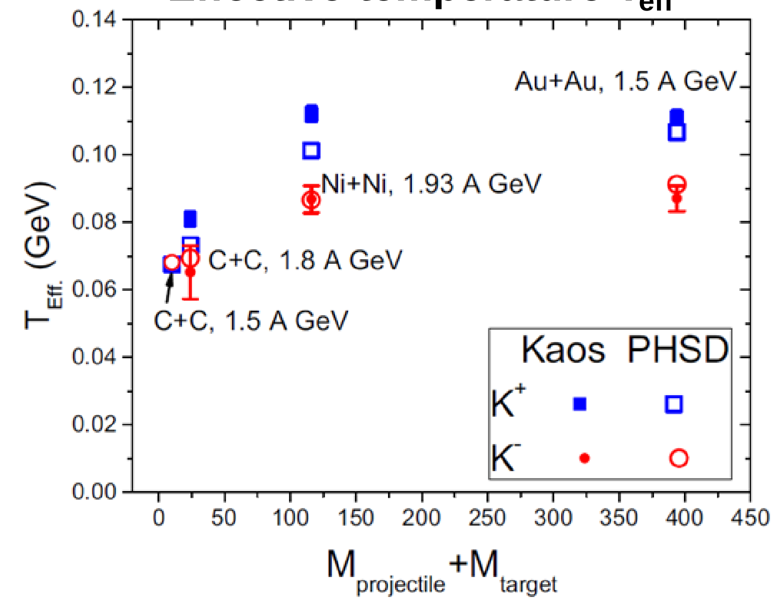
In-medium effects:

- **suppresses kaon production**
- **hardens kaon spectrum**
- **enhances antikaon production**
- **softens antikaon spectrum**

since for $M<M_0$, $\mathrm{Re}\Sigma \rightarrow 0$ and

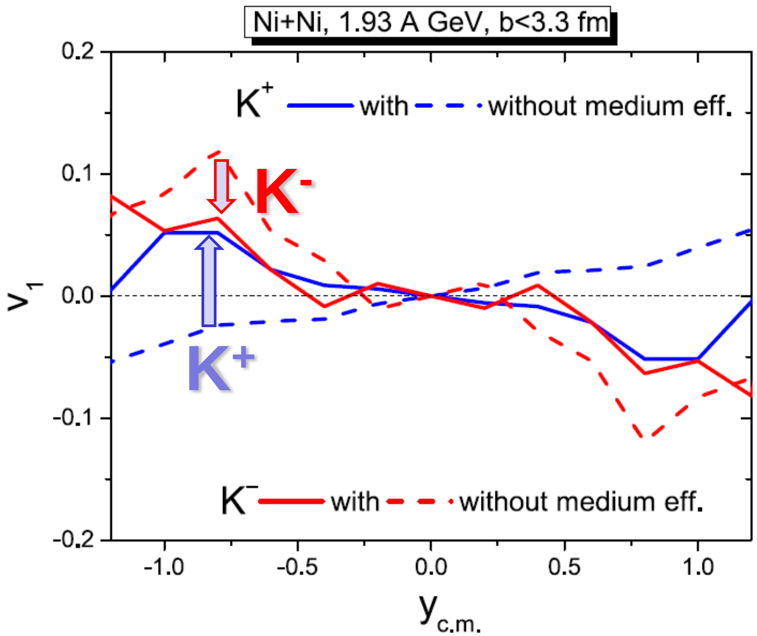
$$\frac{dp_i}{dt} \approx -\frac{1}{2E}\frac{M^2-M_0^2}{\mathrm{Im}\Sigma}\nabla_r \mathrm{Im}\Sigma,$$

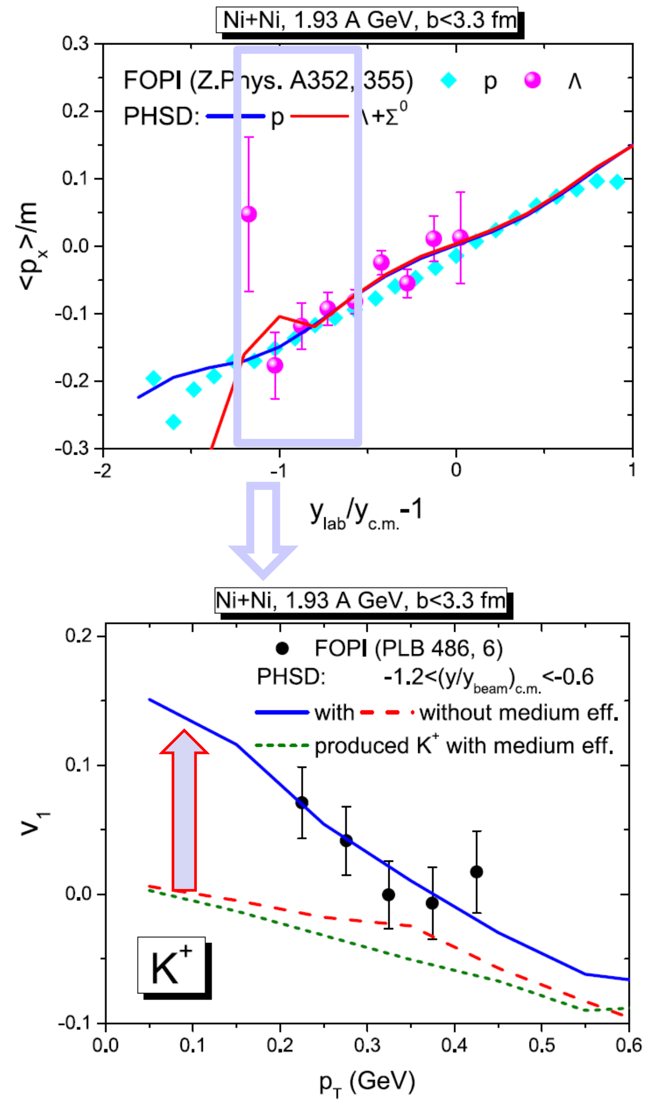
Effective temperature T_{eff}



Directed flow (v_1)

$$\frac{dN(p_T, y)}{d\phi} = C[1 + 2v_1(p_T, y)\cos\phi + 2v_2(p_T, y)\cos(2\phi) + \cdots]$$

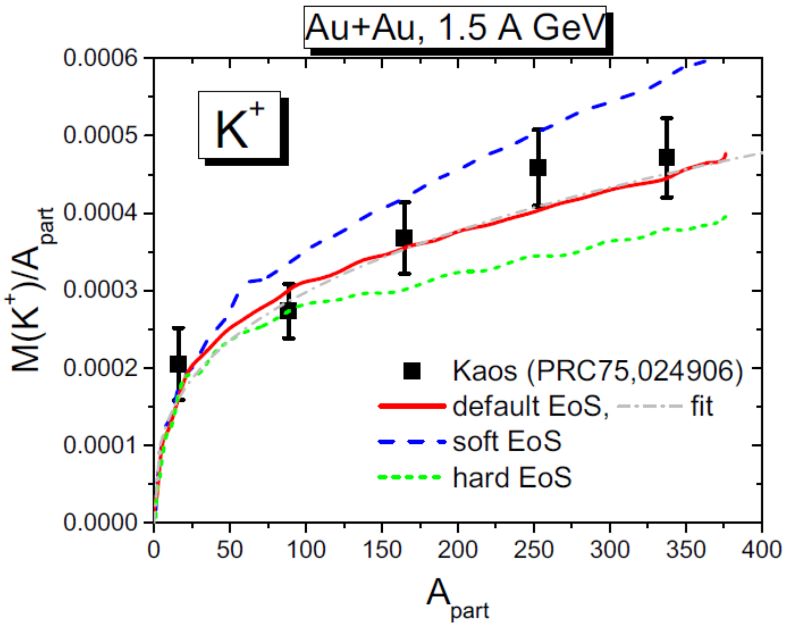


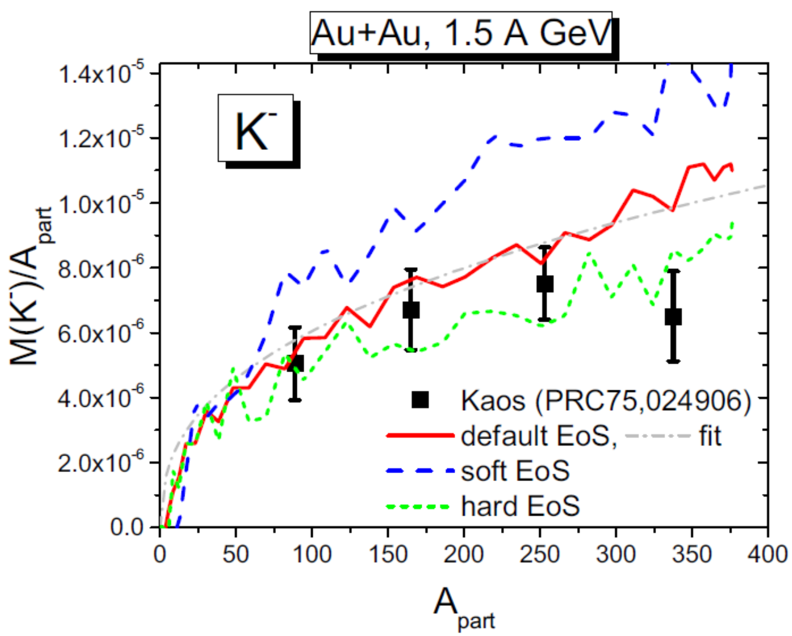


- v_1 of initial kaon follows that of nucleons while kaon is mostly produced by NN scattering
- repulsive force pushes v_1 of kaons upward
- attractive force pulls down v_1 of antikaon

T. Song et al., PRC 103, 044901 (2021)

Equation of State (EoS) of nuclear matter





Skyrme potential

$$U(\rho) = a\left(\frac{\rho}{\rho_0}\right) + b\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

where $a = -153$ MeV, $b = 98.8$ MeV, $\gamma = 1.63$.

Compression modulus K :

$$K = -V\frac{dP}{dV} = 9\rho^2 \frac{\partial^2(E/A)}{\partial \rho^2}\bigg|_{\rho_0}$$

Hard EoS: K=380 MeV
→ hard to be compressed,
less NN collisions to produce (anti)kaons

Default EoS: K=300 MeV

Soft EoS: K= 210 MeV
→ easy to be compressed,
more NN collisions to produce (anti)kaons

T. Song et al., PRC 103, 044901 (2021)

Summary – I

Dynamical description of strongly interaction hadronic (and partonic) matter:
→ **off-shell dynamics based on Kadanoff-Baym equations**
→ **Parton-Hadron-String Dynamics (PHSD)**

Application: study of the in-medium effects within a G-matrix approach for antikaons and by a linear repulsive nuclear potential for kaons:

T. Song et al., PRC 103, 044901 (2021)

- **The repulsive kaon nuclear potential increases the threshold energy for kaon production → suppression of kaon production, hardening of m_T spectra**
- **The broadening of Kbar spectral function in a medium decreases the threshold energy for kaon production → enhancement of Kbar production, softening of m_T spectra**
- **Modification of v_1, v_2 of (anti-)kaons due to the in-medium effects**
- **Selectivity to EoS: soft EoS enhances and hard EoS suppresses the production of (anti)kaons; moderate EoS (K=300 MeV) reproduces experimental data better within the PHSD**
- **… still tension in the description of HADES data for Au+Au at 1.23 A GeV Further robust experimental data are needed (HADES, CBM, BMN,…)!**

Outlook – I :

Consistent transport description of strongly interaction systems based on Kadanoff-Baym theory

requires

a knowledge on the in-medium properties of all degrees-of-freedom (complex self-energies) and
their interactions (in-medium cross sections)

➔ 'Ab initio' many-body calculations are needed!
Cf. Bruckner theory for G-matrix

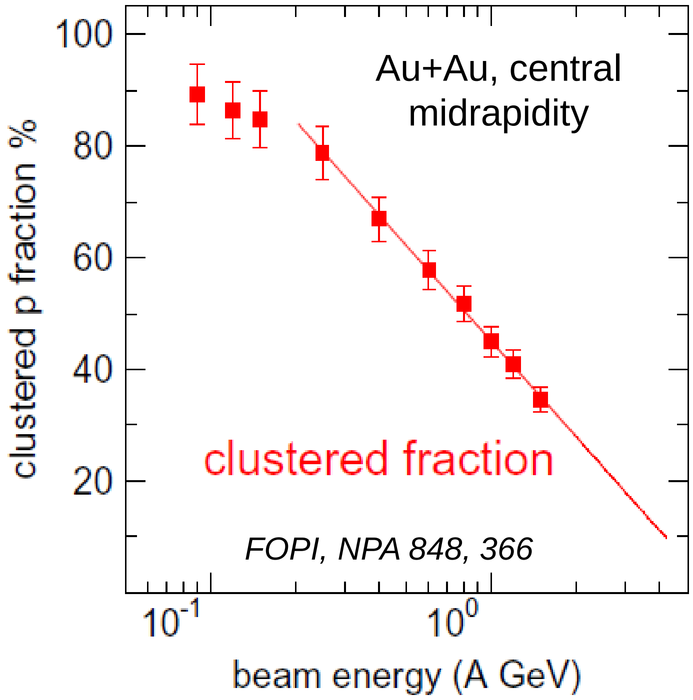
Clusters and hypernuclei in PHQMD

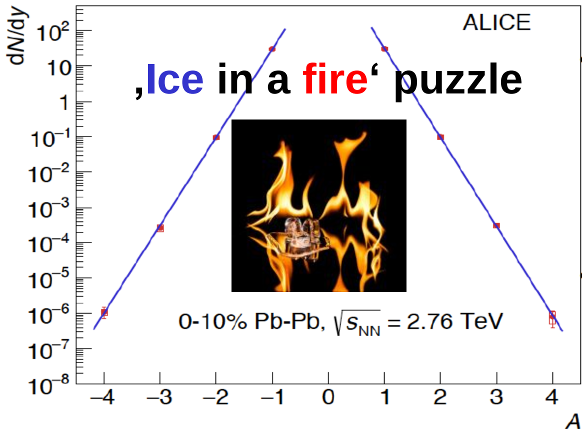
MF vs QMD

PHQMD

Jörg Aichelin, E.B., Arnaud Le Fèvre, Yvonne Leifels, Viktar Kireyeu, Vadim Kolesnikov, Vadim Voronyuk, Gabriele Coci, Michael Winn, Susanne Gläßel, Christoph Blume
(SUBATECH, Nantes & GSI, Darmstadt & JINR, Dubna & Uni. Frankfurt)

Clusters and hypernuclei in HICs

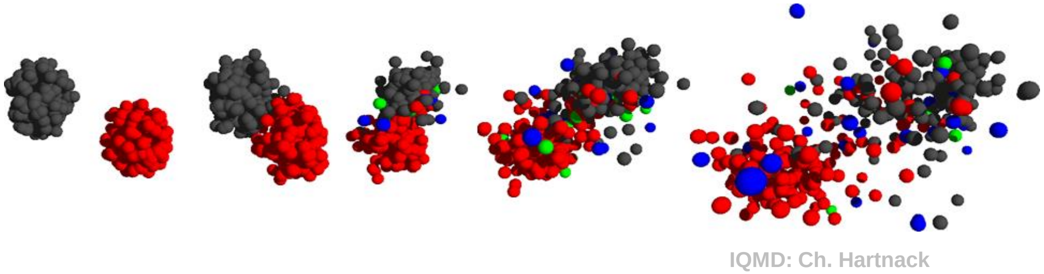




- Clusters are very **abundant at low energy:**
- at **3 AGeV** in central Au+Au collisions
  ~**20% of the baryons are in clusters!**

➔ **Understanding of cluster formation is needed:**

- for proper description of nucleon observables (v_1 ,v_2, dn/dp_T)
- to **probe EoS**
- to explore new physics opportunities like
  - hypernucleus formation
  - possible signals of the 1st order phase transition
  - cluster formation at midrapidity (RHIC, LHC)



PHQMD

PHQMD: a unified n-body microscopic transport approach for the description of heavy-ion collisions and dynamical cluster formation from low to ultra-relativistic energies

Realization: combined model PHQMD = (PHSD & QMD) & (MST/SACA)

Parton-Hadron-Quantum-Molecular Dynamics

↓

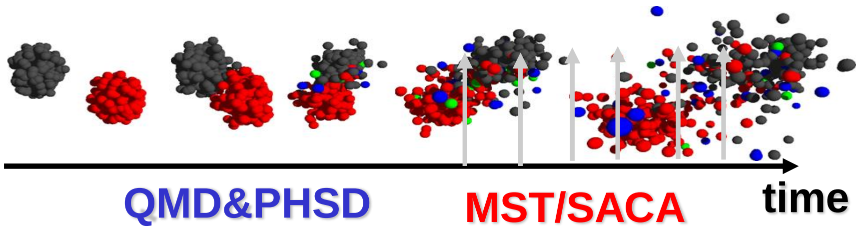
Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

↓

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

↓

Cluster recognition:
SACA (Simulated Annealing Clusterization Algorithm)
or MST (Minimum Spanning Tree)



J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläßel et al., arXiv:2106.14839

QMD propagation

- **Generalized Ritz variational principle**: $$\delta \int_{t_1}^{t_2} dt < \psi(t)| i\frac{d}{dt} - H|\psi(t) >= 0.$$

Assume that $$\psi_N = \prod_{i=1}^{N} \psi_i(q_i, q_{0i}, p_{0i})$$ **for N particles (neglecting antisymmetrization !)**

Ansatz: trial wave function for one particle "*i*" :
Gaussian with width *L* centered at r_{i0}, p_{i0}

$$\psi_i(q_i, q_{0i}, p_{0i}) = C exp[-(q_i - q_{0i} - \frac{p_{0i}}{m}t)^2/4L] \cdot exp[ip_{0i}(q_i - q_{0i}) - i\frac{p_{oi}^2}{2m}t]$$

L=4.33 fm^2

- **Equations-of-motion (EoM) for Gaussian centers in coordinate and momentum space:**

$$\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}$$

Hamiltonian: $$H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j \neq i} V_{i,j})$$

$$V_{i,j} = V(\mathbf{r_i}, \mathbf{r_j}, \mathbf{r_{i0}}, \mathbf{r_{j0}}, t) = V_{\text{Skyrme}} + V_{\text{Coul}}$$

QMD interaction potential and EoS

The expectation value of the Hamiltonian:

$$\langle H \rangle = \langle T \rangle + \langle V \rangle = \sum_i \left(\sqrt{p_{i0}^2 + m^2} - m\right) + \sum_i \langle V_{Skyrme}(\mathbf{r}_{i0}, t) \rangle$$

- **Skyrme potential - scalar ('static') * :**

$$\langle V_{Skyrme}(\mathbf{r}_{i0}, t) \rangle = \alpha \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right) + \beta \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right)^{\gamma}$$

	α (MeV)	β (MeV)	γ	K [MeV]
S	-390	320	1.14	200
H	-130	59	2.09	380

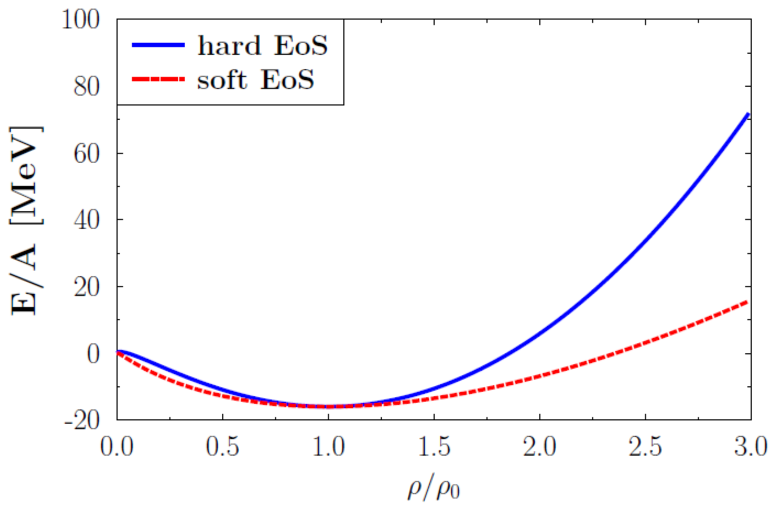
- **modifed interaction density (with relativistic extension):**

$$\rho_{int}(\mathbf{r}_{i0}, t) \rightarrow C \sum_j \left(\frac{4}{\pi L}\right)^{3/2} e^{-\frac{4}{L}(\mathbf{r}^{T}_{i0}(t) - \mathbf{r}^{T}_{j0}(t))^2} \times e^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}^{L}_{i0}(t) - \mathbf{r}^{L}_{j0}(t))^2},$$

- **HIC ←→ EoS for infinite matter at rest**
  - **compression modulus K of nuclear matter:**

$$K = -V \frac{dP}{dV} = 9\rho^2 \frac{\partial^2 (E/A(\rho))}{(\partial\rho)^2}\Big|_{\rho=\rho_0}.$$

EoS for infinite matter at rest



*Work in progress: implementation of momentum dependent potential + symmetry energy (M. Winn)

Cluster recognition: Minimum Spanning Tree (MST)

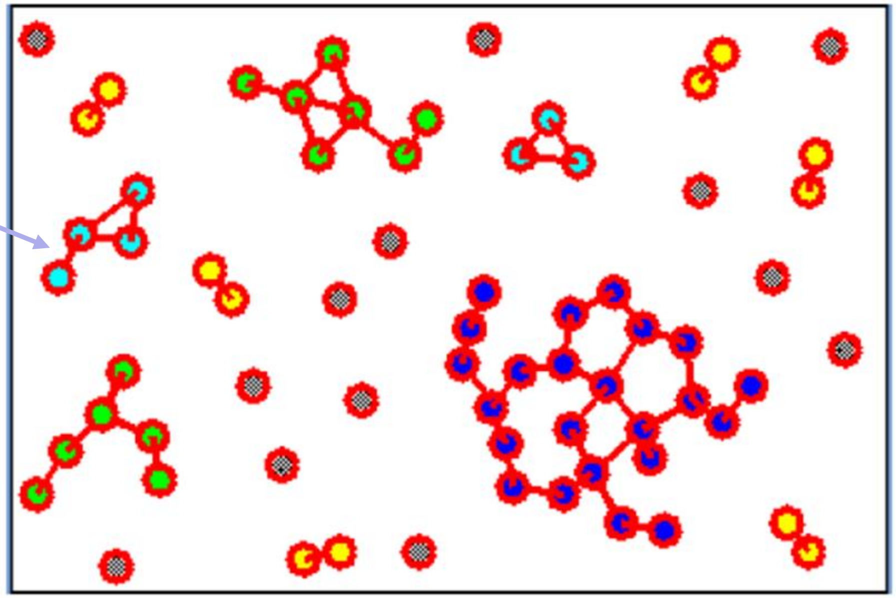
The Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final states where coordinate space correlations may only survive for bound states.

The MST algorithm searches for accumulations of particles in coordinate space:

1. Two particles are 'bound' if their distance in coordinate space fulfills

$$\left|\vec{r}_i - \vec{r}_j\right| \leq 2.5\, fm$$

2. Particle is bound to a cluster if it binds with at least one particle of the cluster.



* Remark:
inclusion of an additional momentum cut (coalescence) lead to small changes: particles with large relative momentum are mostly not at the same position
(Cf. V. Kireyeu, Phys.Rev.C 103 (2021))5

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

Simulated Annealing Clusterization Algorithm (SACA)

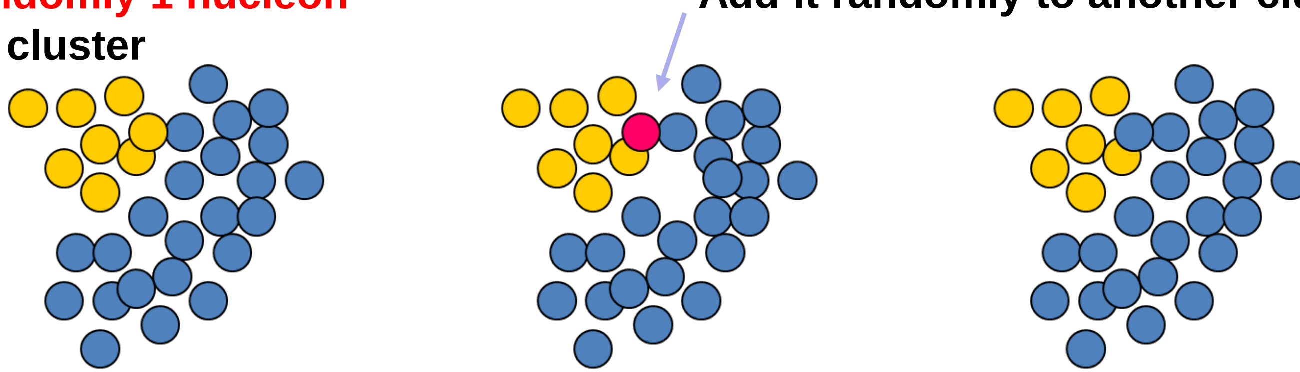
Basic ideas of clusters recognition by SACA:

Based on idea by Dorso and Randrup (Phys.Lett. B301 (1993) 328)

- **Take the positions and momenta of all nucleons at time t**
- **Combine them in all possible ways into all kinds of clusters or leave them as single nucleons**
- **Neglect the interaction among clusters**
- **Choose that configuration which has the highest binding energy:**

Take randomly 1 nucleon out of a cluster

Add it randomly to another cluster



$E=E^1_{kin}+E^2_{kin}+V^1+V^2$

$E'=E^{1'}_{kin}+E^{2'}_{kin}+V^{1'}+V^{2'}$

If E' < E take a new configuration
If E' > E take the old configuration with a probability depending on E'-E
Repeat this procedure many times

→ **Leads automatically to finding of the most bound configurations**

(realized via a Metropolis algorithm)

R. K. Puri, J. Aichelin, PLB301 (1993) 328, J.Comput.Phys. 162 (2000) 245-266;
P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390

Cluster formation: QMD vs MF

- Cluster formation is sensitive to nucleon dynamics

➔ One needs to keep the nucleon correlations (initial and final) by realistic nucleon-nucleon interactions in transport models:

- QMD (quantum-molecular dynamics) – allows to keep correlations
- MF (mean-field based models) – correlations are smeared out
- Cascade – no correlations by potential interactions

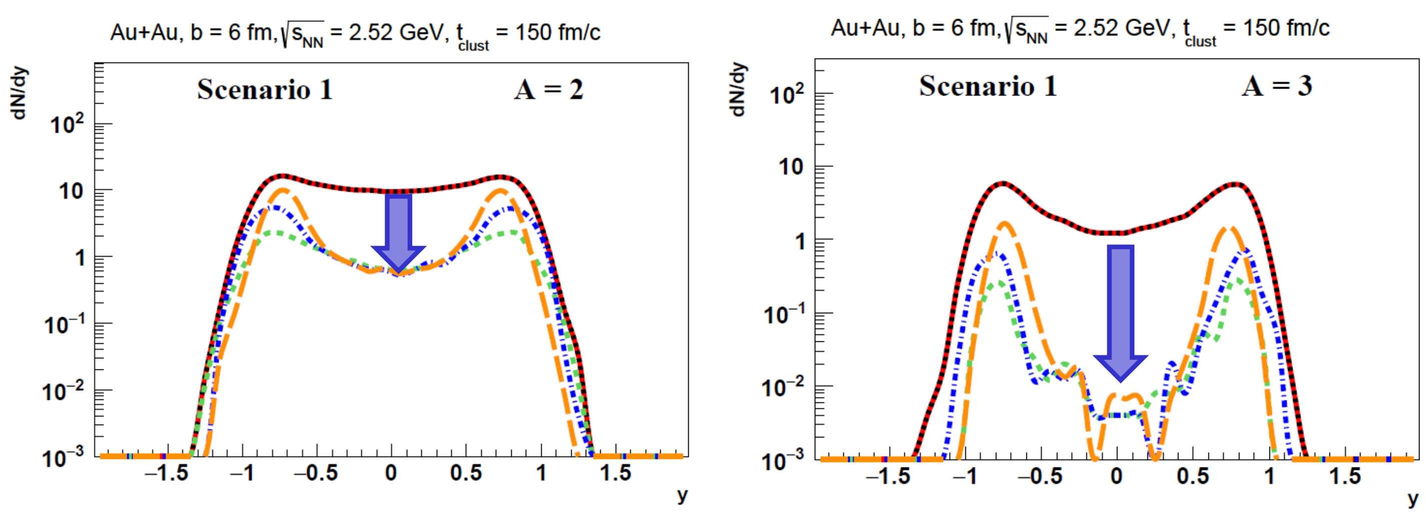
Example: Cluster stability over time:

V. Kireyeu, Phys.Rev.C 103 (2021) 5

QMD:
PHQMD + psMST

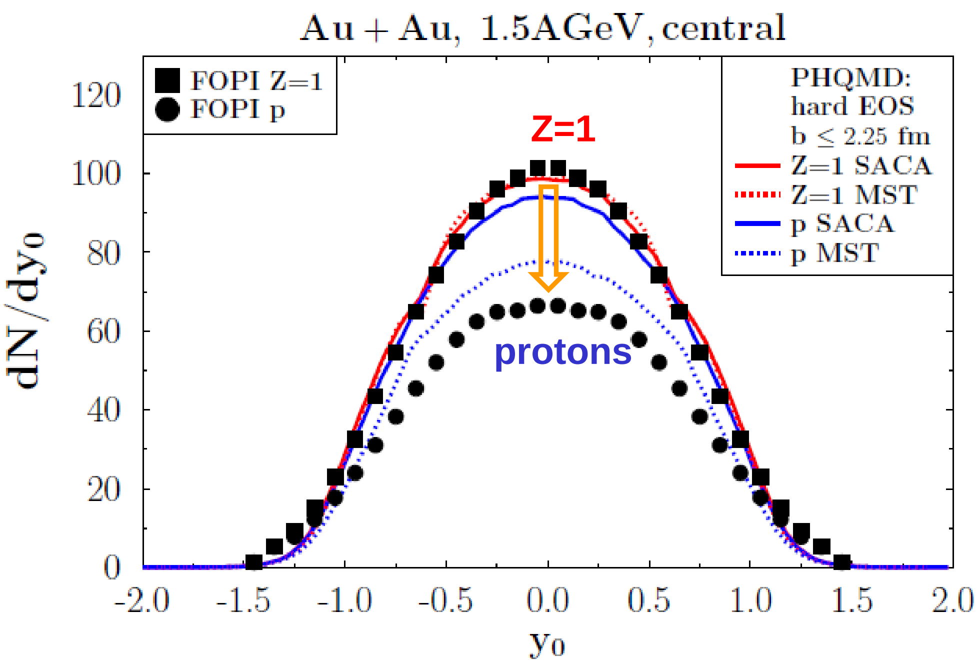
MF:
PHSD + psMST

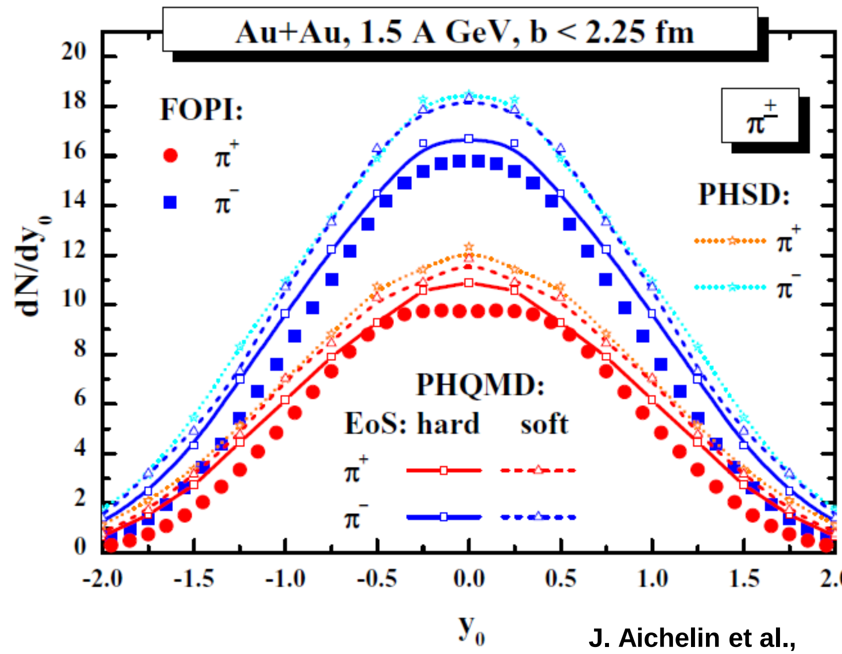
Cascade:
SMASH + psMST
UrQMD + psMST



PHQMD: light clusters and ‚bulk' dynamics at SIS

Scaled rapidity distribution $y_0 = y/y_{proj}$ in central Au+Au reactions at 1.5 AGeV





J. Aichelin et al., PRC 101 (2020) 044905

- **30% of protons are bound in clusters at 1.5 A GeV**
- **Presently MST is better identifying light clusters than SACA**
  ➔ To improve in SACA: more realistic potentials for small clusters, quantum effects

- **Pion spectra are sensitive to EoS: better reproduced by PHQMD with a 'hard' EoS**
- **PHQMD with soft EoS is consistent with PHSD (default – middle soft EoS)**

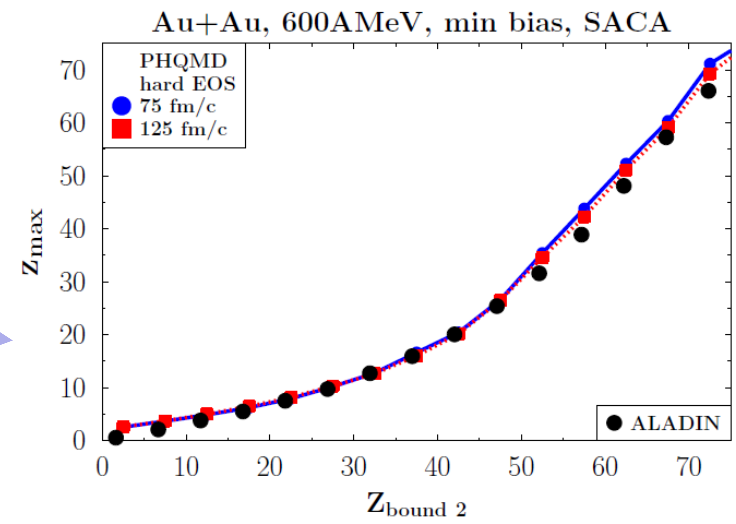
*** To improve in PHQMD: momentum dependent potentials + symmetry energy (M. Winn)**

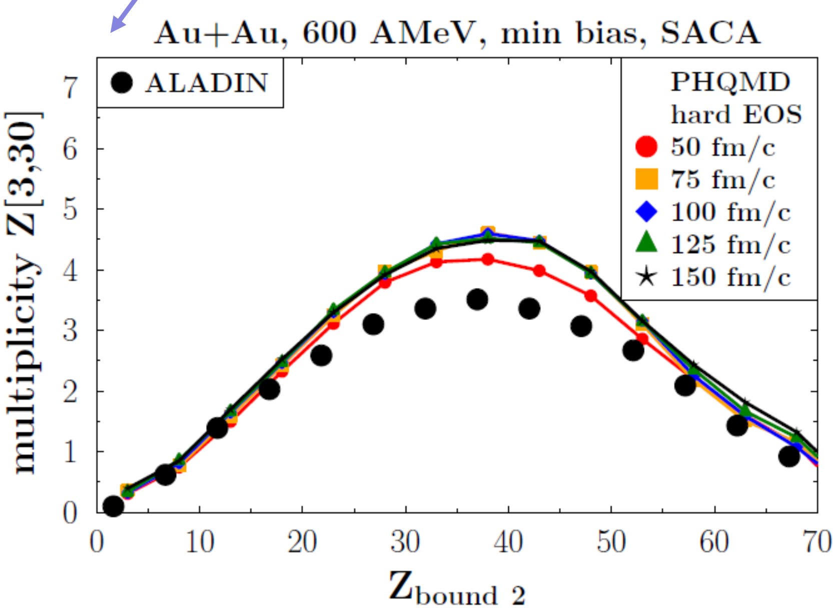
PHQMD: heavy clusters

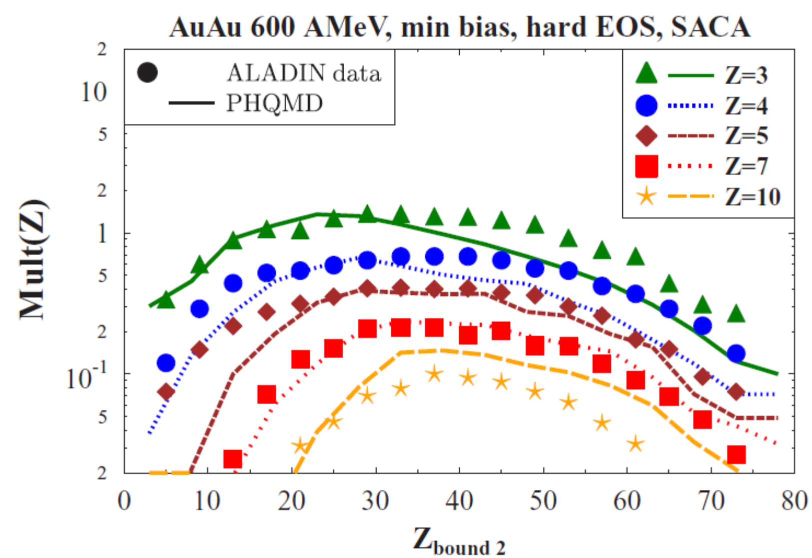
Heavy clusters (spectator fragments): experim. measured up to E_{beam} =1 AGeV (ALADIN Collab.)

PHQMD with SACA shows an agreement with ALADIN data for very complex cluster observables as

- Largest clusters (Z_{bound})
- Multiplicity (Zbound)
- Energy independent 'rise and fall'







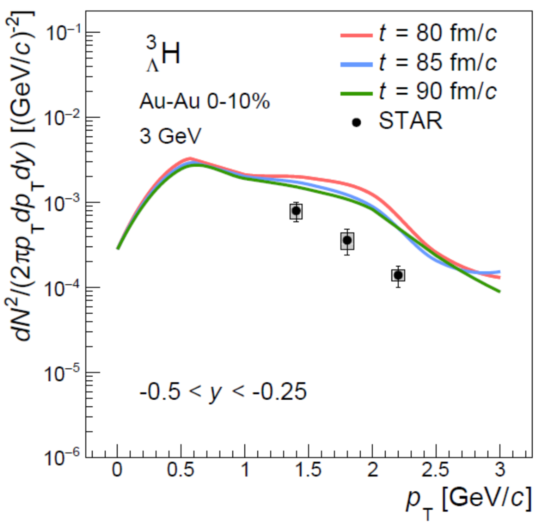
$$Z_{bound\ 2} = \sum_i Z_i\, \Theta(Z_i - (1+\epsilon)) \quad (\epsilon < 1)$$

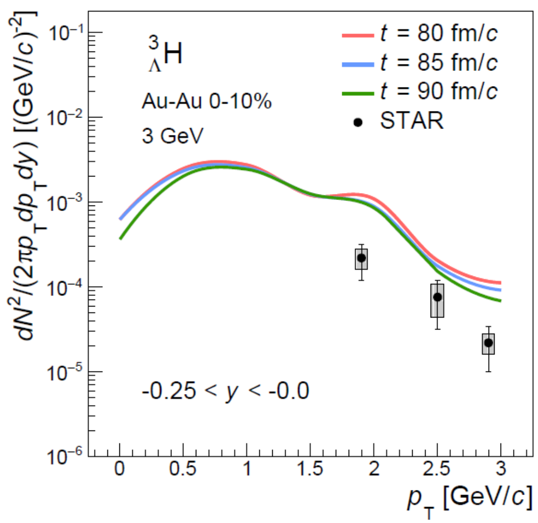
J. Aichelin et al., PRC 101 (2020) 044905

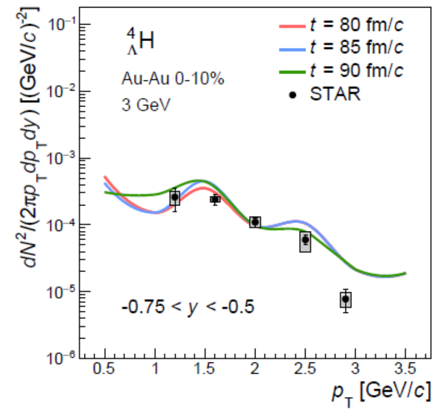
Hypernuclei production at $s^{1/2}$ = 3 GeV

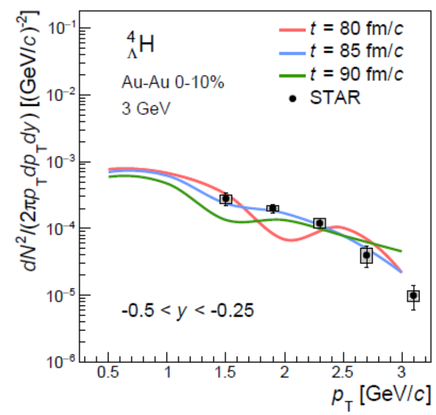
The PHQMD comparison with most recent (preliminary!) STAR fixed target p_T distribution of $^3H_\Lambda$, $^4H_\Lambda$ from Au+Au central collisions at $\sqrt{s}$ =3 GeV

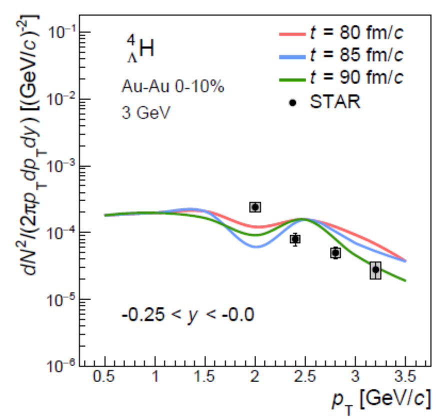
- **Assumption for nucleon-hyperon potential: $V_{N\Lambda} = 2/3\ V_{NN}$**











➔ **Reasonable description of hypernuclei production at $\sqrt{s}$ =3 GeV**

S. Gläßel et al., arXiv:2106.14839

Summary-II

The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster formation

combined model PHQMD = (PHSD & QMD) & (MST | SACA)

- provides a good description of hadronic 'bulk' observables from SIS to RHIC energies
- predicts the dynamical formation of clusters from low to ultra-relativistic energies due to the interactions
- allows to study the origin as well as the properties of cluster formation (rapidity and p_T spectra)
- allows to study the formation of hypernuclei originating from ΛN interactions
- QMD dynamics allows to keep clusters 'bound' better than MF

Outlook-II

*Work in progress: implementation of momentum dependent potential + symmetry energy (M. Winn)